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EXTENDED WARRANTY: AN APPROACH VIA STACKELBERG GAME

H. P. Z. SANTOS*, C. CRISTINO
Federal Rural University of Pernambuco, Recife, Brazil[a]

B. GUEDES
Federal University of Pernambuco, Recife, Brazil[b]

This paper aims at showing how the game theory can be applied to the quantitative study of extended warranty (EW). This kind of after-sales service represents an additional non-compulsory coverage, which the consumer can or not buy after the acquisition of some device, starting subsequently to the end of the base warranty (BW) period. Thus, a model, in which two players interact, is shown, i. e. the original equipment manufacturer (OEM), in charge of assigning prices to the equipment and to the extended warranty service; and the customer (owner of the device). The way, in which the parts interact, is modeled through the Stackelberg leadership game, which was originally applied in the analysis of the oligopolies competition market. In this particular model, the leader represents the OEM and the follower is the customer, where the OEM's goal is to find a certain price structure to maximize his profit and consequently influence the buyer's decision. Through this adaptation, the strategies and payoffs of players are defined. The robustness of the model is due to incorporating elements of the economic theory (consumer's surplus, producer's surplus, consumer's reservation price, choice under uncertainty, maximum profit and contingent consumption plan) and elements of the reliability theory (probability of failure and non-repairable systems). Additionally, a numerical example and a sensitivity analysis of parameters are presented to highlight the model. Finally, this research systematizes the steps of Stackelberg game under the modeling of extended warranty (EW) as well.

Keywords: extended warranty, game theory, Stackelberg game, consumer's reservation price.

JEL: C7, M3.

The process of globalization has increased the competition among companies, which have sought strategic advantages to continue in the market. On the other hand, the customers have become very rigorous in the purchase of their products, once their decision

* Corresponding author: santos.henrique624@gmail.com
[a] Postal Address: Rua Dom Manoel de Medeiros, s/n, Dois Irmãos, Recife, PE, 52171-900, Brazil.
[b] Postal Address: Av. Prof. Moraes Rego, 1235, Cidade Universitária, Recife, PE, 50670-901, Brazil.

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process includes price, payment method, quality, the degree of product substitution, reliability, practicality and after-sales services offered. Hence, companies need to define their goals with the buyers' necessity to keep competitive and increase their profits.

Among the points that influence the customer's decision to buy, the after-sales services have some special features. A good execution of services can increase the success rate in inserting new products in the market, a way to find new customers and retain older ones; these actions can increase the profitability of the companies [Clark, Armistead, 1991; Jonke, 2012]. From this broad set of after-sales services activities, the extended warranty (EW) has been a trend widely adopted by firms over the recent decades [Murthy, 2000].

EW is defined as a voluntary agreement between the provider (manufacturer, retailer or a third party) and the buyer of the equipment, which he can or not buy [Murthy, Djamaludin, 2002]. This kind of service is usually purchased at the moment of the sale of an item and can be seen as insurance, starting after the base warranty period (mandatory coverage) [Rahman, Chattopadhyay, 2015; Murthy, Jack, 2014]. If the device fails, the EW provider must take a specific action, such as repair activities, replacement of a broken item and/or financial compensation to the owner of the equipment [Pope et al., 2014].

From the moment when the EW provider delivers this service to his buyers, his costs will be stochastic, as failures are not deterministic events. They are called "warranty servicing" costs and represent the cost of rectifying a broken item along the EW period [Shafiee, Chukova, 2013]. The four main factors that affect this kind of cost are the coverage period to be determined, the benefits offered to consumers, the claim frequency (quantity of times that the EW will be required) and the claim severity degree [Rai, Singh, 2004].

The economic weight of the warranty can be seen, for instance, in the annual report of General Motors [Annual Report..., 2015]. In 2015, this company got a global revenue of 152,36 billion dollars, while its warranty costs were 9,279 billion dollars, that is, 6% of its billing. Thus, it is fundamental to know how to project warranty policies, balancing the benefits and costs associated to this kind of service due to the impact on the profitability of firms.

Thomas and Rao argue that the building of warranty models has been associated to: (1) the establishment of conditions and parameters that influence the cost of producing the products and (2) predicting the monetary amount that must be destined to warranty actions, for instance, financial compensation to the buyer and repair (maintenance) costs [Thomas, Rao, 1999].

It is necessary to emphasize that the insertion of the EW produces different perceptions among the agents. The customer when buying additional service wants to get an extra protection after the end of the base warranty [Murthy, Djamaludin, 2002]. On the other hand, the EW provider tries to maximize his profit by adding extra activities to his set of after-sales services [Maronick, 2007; Murthy, Jack, 2014].

A way of seeing the customer-EW provider relationship is through the game theory [Murthy, Karim, Ahmadi, 2015]. A game represents a strategic mutual interaction between more than one person as his/her action affects the payoff of another person [Gibbons, 1997]. Thus, there is a big involvement between decision-makers (or players), who need to develop strategies and action plans considering this reciprocal effect. Additionally, this tool is useful for suggesting or advising the players about best way to take their actions [Morris, 1994].

Among the types of games used in the literature that model the relationship between the decision-makers [Fujiwara-Greve, 2015], the Stackelberg game (SG), originally applied in oligopolistic competition, has been used in quantitative modeling of the after-sales services. This game has been adapted to many situations, especially to extended warranty, maintenance outsourcing, maintenance service contracts and lease contracts [Ashgarizadeh, Murthy, 2000; Esmaeili, Gamchi, Asgharizadeh, 2014; Hamidi, Liao, Szidarovszky, 2016; Kurata, Nam, 2013; Li, Mallik, Chhajed, 2012; Murthy, Asgharizadeh, 1998; Murthy, Asgharizadeh, 1999; Murthy, Yeung, 1995; Wei, Zhao, Li, 2015].

Another key point in after-sales modeling is about characteristics of the system to be considered, for instance whether the device is a repairable or a non-repairable system. If the equipment fails and cannot return to its operational state, that is, the item does not receive maintenance actions after breaking down, it can be defined as a non-repairable system (such as bulbs or batteries). On the other hand, if the equipment fails and it can receive maintenance actions, such as restoration/renovation of its components, it is defined as a repairable system (for instance, vehicle maintenance) [Crow, 1975]. This premise is fundamental because, based on it, maintenance actions, warranty policies and failure repair processes are defined [Yanez, Joglar, Modarres, 2002].

Therefore, this paper explores the mathematical model on extended warranty applied to non-repairable systems as seen by [Murthy, Jack, 2014]. The main contributions to this research are related to the microeconomic ambiance in the extended warranty study, to the inclusion of a numerical example and to a sensitivity analysis of the probability of equipment. Additionally, the present paper outlines Stackelberg game steps for the proposed study, serving as a framework to further studies involving the SG in the modeling of after-sales services.

Table 1

Payoff Matrix

		Player Column	
		A	***B***
Player Line	***A***	(3, 1)	(5, 0)
	B	(2, 0)	(4, 2)

The paper is structured in 4 sections. In Section 1 a theoretical background is presented emphasizing the application of the Stackelberg game in modeling after-sales services. Subsequently, in Section 2 the model is illustrated under microeconomic ambiance, inserting Stackelberg game as a way to solve the research problem exposed. In Section 3 a numerical example and its extension are evidenced. Finally, the last part of this study summarizes all information presented and proposes future contributions.

1. THEORETICAL BACKGROUND

The Stackelberg game was developed by Heinrich von Stackelberg, being applied originally in the oligopolistic competition (see, f. e. [von Stackelberg, 2011]), based on which the firms define the amount of a homogeneous product to be put on the market hierarchically over an established period of time [Bagchi, 1984; Gibbons, 1992]. This sequential structure has an important effect on the definition of equilibrium of the Stackelberg game. For instance, table 1 represents a payoff matrix of a bimatrix game, in which the players (Line and Column) have two dichotomous strategies (A, B).

If this game is simultaneous, then the Nash equilibrium will be the pair of strategies (A, A) and the payoff of the Line player will correspond to 3. However, if this game is sequential and the Line player is the first to take action, then the Nash equilibrium will be the pair of strategies (B, B)

and his payoff will correspond to 4. This situation shows a peculiar characteristic, the "first mover advantage", that is, the first player (the leader) presents a greater power of choice and decision over the second player (the follower) [Pindyck, Rubinfeld, 2001].

It is also necessary to reinforce the way of solving the Stackelberg game, i. e., the backward induction. This procedure involves initially solving the decision problem of the follower and posteriorly solving the decision problem of the leader [Gibbons, 1992; Pindyck, Rubinfeld, 2001; Fujiwara-Greve, 2015]. Thus, the Stackelberg game is viewed as a conditioned optimization problem in which the follower maximizes his payoff via the previous movement of the leader through his reaction function [Varian, 1992]. Finally, the action ("best response") of the second player corresponds to his better strategy given any action taken by the leader [Bagchi, 1984].

The two points presented above (the sequential two-step structure and the backward induction) have been adopted in the design of quantitative models of outsourcing of maintenance, of maintenance service contracts, of warranty, of extended warranty and of leasing contracts.

The starting point of the after-sales service quantitative modeling that applies the Stackelberg game is the study by [Murthy, Yeung, 1995]. These authors developed a strategic decision-making framework involving the service agent and the owner of the equipment under the service contract considering either planned or failure replacement.

Murthy and Asgharizadeh developed three papers that deal with the interaction between the owner of the equipment and the service agent [Murthy, Asgharizadeh, 1998; 1999; Ashgarizadeh, Murthy, 2000] applied in the after-sales services ambiance. In [Murthy, Asgharizadeh, 1998] they developed a game of two players (the service agent and the owner of the equipment). For this model, the service agent is in charge of assigning prices to a service contract (warranty) and to a non-additional coverage service (without warranty), while the owner of the equipment needs to choose whether to buy the device or not, as well as whether to buy the warranty service or not. The EW provider's goal is to maximize his profit under his buyer's best option. The authors expanded the problem by considering an ambiance of multiple customers [Ashgarizadeh, Murthy, 2000] and multiple service channels which would provide maintenance of the devices [Murthy, Asgharizadeh, 1999]. Thus, papers [Murthy, Asgharizadeh, 1999; Ashgarizadeh, Murthy, 2000] incorporated the queueing theory in after-sales service modeling.

In [Murthy, Yeung, 1995; Murthy, Asgharizadeh, 1998; Murthy, Asgharizadeh, 1999; Ashgarizadeh, Murthy, 2000] the leader is the service agent, that is, he takes the first action, showing the price structure to his customers, while the buyers are followers, that is, they respond optimally to the leader's actions (second action). From this interaction, the players' payoffs are generated and the Nash equilibrium is reached.

Authors [Esmaeili, Gamchi, Asgharizadeh, 2014] adopted a game with three players: the OEM, the service agent and the customer. In this model, the authors showed the interaction of the players by means of non-cooperative and semi-cooperative games. Under one of these scenarios, the manufacturer has more power than the service agent does and the service agent has more power than the customer does, i. e., there are two Stackelberg games, the first with the manufacturer and the service agent as players, whereas the second has the service agent and the buyer. Another scenario considers the OEM and the service agent cooperating (as leaders) while the customer is the follower (semi-cooperative game).

In [Kurata, Nam, 2013; Wei, Zhao, Li, 2015; Li, Mallik, Chhajed, 2012] incorporated the retailer into the after-sales service modeling. These authors developed the SG as a supply chain where both the retailer and the manufacturer can sell the after-sales services. In such models, the leader is OEM and the follower is the retailer. Moreover, [Hamidi, Liao, Szidarovszky, 2016] studied after-sales services to rental equipment, that is, a leasing contract where the lessor (the owner of the device) rents to lessee (the user of the equipment). In this context, the lessor is the leader who specifies the maintenance policy first, whilst the lessee, as the follower, decides on the lease period and usage rate accordingly.

Backward induction applied to the after-sales services models

It is also essential to explain how the backward induction is implemented in the after-sales services models through the SG. The key point to this process is when the EW provider can maximize his profit by the consumer's reservation price. This term corresponds to the maximum price a consumer is willing to pay to consume one unit of a product or a service, i. e., it is the consumer's indifference price between purchasing or not purchasing the product [Varian, 1992; Pindyck, Rubinfeld, 2001; Breidert, 2006]. The design for this kind of price in after-sales services models is linked to the characteristics of the consumer's utility function and his degree of risk aversion, since failure is a random variable [Murthy, Jack, 2014].

In [Murthy, Asgharizadeh, 1998; 1999], the reservation price has a fundamental role since when the EW provider knows it (situation without asymmetric information), he elaborates his strategy based on the consumer's reservation prices. When the EW provider performs this action plan, he influences the buyer's decision and his profit becomes maximum. This whole process implies in the absorption of all consumer surplus, similar to the monopolist when applying the perfect price discrimination (or first-degree price differentiation) [Murthy, Jack, 2014]. Finally, this process of transference of consumer surplus to the EW provider implies in the buyer's expected utility corresponding to zero.

From the moment the EW provider uses the consumer's reservation price to get maximum profit, the backward induction can be seen easily. There are two well-defined subgames, which are interconnected. The former is related to the decision by the owner of the equipment (follower), whereas the latter is related to the decision by the EW provider (leader) [Murthy, Asgharizadeh, 1998; 1999].

First Subgame: Owner of the equipment. In the first subgame, it is necessary to find the consumer's reservation prices of the after-sales services based on the objective function of the buyer. Considering those prices, the second subgame can be solved.

Second Subgame: EW provider. In the second subgame, it is necessary to define the pricing strategy that influences the consumer's decision, ensuring the maximum profit to the EW provider.[1] For instance, if the EW provider wants the buyer not to choose to buy the equipment, he will show a higher sale price than the consumer's reservation price for the acquisition of the device. As consequence, the consumer does not buy the item. The consumer's reservation prices are like the reaction function of the follower firm in the SG.

[1] In models in [Murthy, Asgharizadeh, 1999] there are other decision variables that the EW provider must consider, such as the number of customers and of service channels.

Therefore, from the explanation of the role of the Stackelberg game in the modeling of after-sales service, the extended warranty model is to be presented in the next section.

2. EXTENDED WARRANTY MODEL

The model to be explained in this paper can be seen in [Murthy, Jack, 2014]. It is characterized by the uncertainty that surrounds the players in their decisions. Each decision-maker will only take one action at a time without any negotiation (possibility of bargaining) between the parties involved, that is, the players will not share payoffs.

The key elements in this modeling are the following: the players involved (the OEM and the customer), the characteristics of the system (a non-repairable item), the probability of equipment failure, the equipment price, and the extended warranty plan shown to the buyer. The model is presented in a non-cooperative environment, i. e., each player acts individually, seeking to obtain the highest possible payoff.

Finally, the singularity of this model is due to the OEM making a maximum profit if the consumer's reservation prices (insert acronyms) are used to define the price portfolio. On the other hand, when the OEM implements this strategy, the customer's expected profit is zero.

Extended warranty model description

Notation:

- Π_X^c — Customer's profit, when $X=0$, 1 or 2;
- Π_X^{oem} — OEM's profit, when $X=0$, 1 or 2;
- C_m — unit cost to produce one piece of equipment;
- P_{ew} — the extended warranty price for the equipment;
- P_{ew}^* — the consumer's reservation price for the extended warrant;
- P_e — the selling price of the equipment to the customer;
- P_e^* — the consumer's reservation price for the acquisition of the equipment;
- k — financial loss suffered by the equipment owner if the device fails and it is in a non-operational state;
- m — revenue generated by the equipment owner when the device is in an operational state;
- p — the probability of the equipment not failing;
- s — the level of coverage of the extended warranty for the equipment. Its value can be between (0, 1];
- X — the decision variable of the equipment owner (or customer). Its value can be 0, 1 or 2;
- $\mathbf{Y}$ — the vector of the decision variables of the OEM.

The OEM shows to the customer the equipment. If the buyer purchases the item, he will be given revenue m over the extended warranty period; however, if the equipment fails, the consumer will have financial loss k, due to a non-operational state of the device. In order to mitigate the effect of k, the OEM also presents an extended warranty plan that attenuates the financial loss effect. This additional coverage works as insurance (or contingent consumption plan) when the customer pays the P_{ew} and receives financial compensation if the equipment fails during the EW period. This amount is a fraction of k, that is, sk. The greater the value of s, the greater the compensation to be paid to the customer.

The decision-making process that surrounds the customer is linked to making two interconnected decisions, which are to purchase or not the equipment and to acquire or not the extended warranty. The buyer must take these decisions at the same time.

The stochastic approach of this paper is related to the probability of failure $(1-p)$, which is inherent to the equipment and affects the decisions made by the players. The device has two states of nature mutually exclusive and collectively exhaustive, the operational and the non-operational states, which are linked to the probability of failure. For each situation, the players' payoff changes and each event is evaluated through their respective probabilities of occurrence to each state. For instance, if the probability of failure is low, the customer can choose not to have the contingent consumption plan.

The hierarchical sequence of steps of the model is as follows:

First Step. The OEM presents the equipment (P_e) and the extended warranty (P_{ew}) prices to the customer.

Second Step. The customer analyzes those prices and makes a double decision: whether or not to buy the equipment and the extended warranty.

Second Step. The players' payoffs are generated and the equilibrium is reached through the actions taken by players.

Assumptions

1. The equipment is a non-repairable system. If the device fails, it will be non-operational and will not receive maintenance.
2. The customer purchases only one unit of the equipment.
3. There is no asymmetry of information between players. The customer knows the probability of equipment failure and the OEM knows the consumer's reservation prices.
4. Both players are risk neutral, seeking just an expected maximum monetary return (profit).
5. The objective function of each player ponders the occurrence of events by means of their respective probabilities.[2] Thus, the profit also becomes a random variable.
6. The OEM is considered the leader (the dominant entity), while the customer is designated as follower (the dominated entity).

Consumer's perspective

The customer analyzes the prices presented by the OEM, the probability of equipment failure, the financial revenue derived from the use of the device and the financial compensation expected. Based on all these elements, the buyer takes a decision X which can present the following possibilities:

$X = 0$, the device is not bought;

$X = 1$, the device is bought; however, without extended warranty plan;

$X = 2$, the device is bought with extended warranty plan.

Fig. 1 summarizes the consumer's decision problem through his decision tree.

For each decision taken by the customer, the following possibilities of profit are presented:

$$X = 0,\ \Pi_0^c = 0; \tag{1}$$

$$X = 1,\ \Pi_1^c = m - P_e - (1 - p)k; \tag{2}$$

$$X = 2,\ \Pi_2^c = m - P_e - P_{ew} - (1 - p)(1 - s)k. \tag{3}$$

In equation (2), the revenue generated by the use of the equipment is a deterministic component, i. e., the customer always gains m if he buys the item. On the other hand, if the customer chooses to have the extended warranty plan (equation (3)), the revenue will be a random variable due to the inclusion of the financial compensation expected. In relation to financial loss, it will always be stochastic, as uncertainly refers to the probability of equipment failure.

[2] The objective function of the players involved has the same features of the expected utility function by von Neumann — Morgenstern (see [von Neumann, Morgenstern, 1947]) since it is separable in the consumption of the various states of nature and linear in the probabilities.

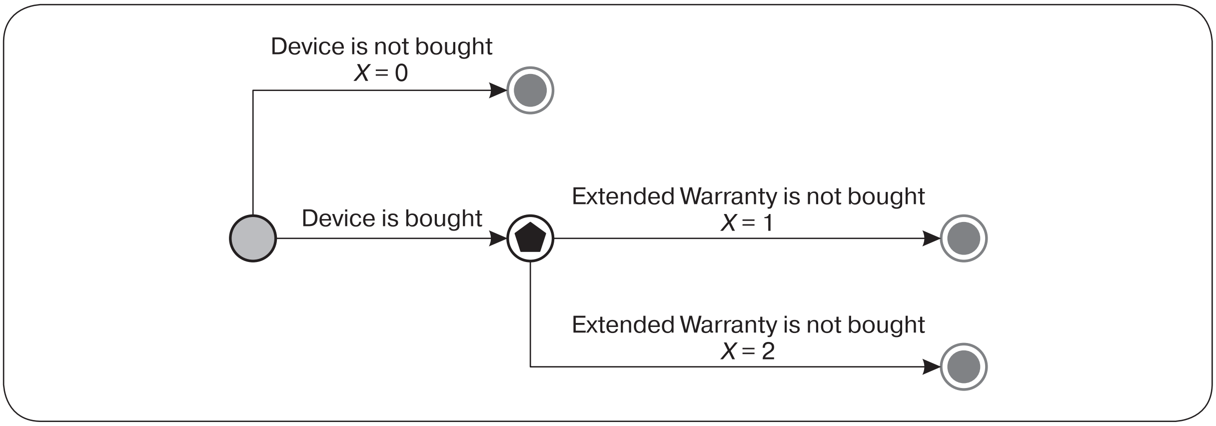

Fig. 1. Consumer's decision tree

OEM's perspective

While the customer has a decision variable, the OEM has a vector **Y** of decision variables composed of three elements (s, P_e, P_{ew}), by means of which the manufacturer defines the portfolio of prices. In addition, it should be emphasized that the OEM's payoff is conditioned to the previous action of the buyer. Thus, three possibilities of profit can also be generated given the customer's choice:

$$X = 0,\ \Pi_0^{oem} = 0; \quad (4)$$

$$X = 1,\ \Pi_1^{oem} = P_e - C_m; \quad (5)$$

$$X = 2,\ \Pi_2^{oem} = P_e + P_{ew} - C_m - (1 - p)sk. \quad (6)$$

In relation to the structuring of the OEM's profit, in equations (5) and (6), the revenue is a deterministic component whereas the financial loss is a random component due to the probability of equipment failure (same situation as for the customer's payoff). In equation (6), the OEM's profit is higher if he sells the extended warranty plan and the device does not fail, thus the manufacturer does not need to pay the buyer the financial compensation.

The way the players' actions interact results in the possibility of the problem of the decision of the extended warranty being modeled via a game theory framework.

Adaptation of the extended warranty problem as a Stackelberg game

Since the model is presented under a sequential structure that makes an interconnection between the players' actions and payoffs and also there is a relation of heterogeneous power, the Stackelberg game can be applied as a solution method. Each decision-maker wants to get the highest possible payoff; however, they know that their decisions interfere reciprocally (mutual dependence). In this way, the strategic plan to be developed by the players is conditioned to what the other can do (reaction function). For example, if the manufacturer presents a high P_e to the customer in order to try to obtain a higher profit, there is the possibility that the buyer does not purchase of the equipment. In this case, both players' payoff is zero.

Table 2 systematizes how the extended warranty model can be interpreted as a Stackelberg game, highlighting the common components of both environments.

Equilibrium of the extended warranty model

The key point of this model is to find the consumer's reservation prices (P_e^*, P_{ew}^*), due to the role they have in players' decision

Table 2

Comparison between the Stackelberg game and the extended warranty model

Element	Stackelberg game	Extended warranty model
Players	The leader and the follower	The OEM and the customer
Power structure	The leader has the power	The OEM has the power
Action taken by the players	The companies choose the optimal level of output for the market	The OEM determines the s, P_e and P_{ew}, while the customer determines his decision X
Game dynamics	Initially, the leader determines how much to produce. Then, the follower determines how much to produce in order to maximize his own profit (considering the leader's action)	Initially, the OEM determines the s, P_e and P_{ew}. Then, the customer assesses the price structure and takes his action
Solution	Backward induction. Initially, the follower's production amount must be determined and his profit maximized considering the leader's previous action. Next, the leader's profit is maximized by his own production amount	Backward induction. Initially, the consumer's reservation prices are found (P_e and P_{ew}). Next, the OEM analyzes those prices and defines the amount for vector **Y** in order to maximize his profit
Equilibrium	It is reached when the optimal quantity to be produced for the market is determined, considering the sum of the production amounts by both companies. The payoffs of each company derive from their actions	It is reached when the customer takes his action, resulting in the maximum profit for the OEM. For this situation, the customer's expected profit is zero

S o u r c e: The authors' research.

process. To explore such prices, it is necessary to understand firstly the strategic design of each player.

Consumer's strategy

The composition of the customer's decision-making process can be divided into two parts: the first one is related to the decision to purchase the equipment and the second one is related to the adhesion to the contingency plan.

The key point for the buyer to purchase the device is that his gain must be equal to or bigger than the costs related to the equipment. Thus, the consumer evaluates the decision to have the equipment according to the following constraint (7). If the gain (G), revenue obtained due to using the equipment, is smaller than the expected loss (L), monetary loss due to the failure of the equipment, then the customer will not buy the device since it is preferable to have a zero profit than to have an expected financial loss.

$$P_e \le m - (1 - p)k. \tag{7}$$

Note that if such a constraint is represented by an equality (equation (8)), the customer is indifferent regarding whether or not to buy the equipment, so $\Pi_0^c = \Pi_1^c$.

$$P_e = m - (1 - p)k. \tag{8}$$

The extended warranty is seen as a process of risk mitigation between different states of nature. Thus, the buyer must again consider the gains and losses related to the contingency plan presented to evaluate its economic viability. From this perspective, fig. 2 shows the budget line related to the purchase of the extended warranty, given by following equation:

$$G = m - P_e - \left(\frac{P_{ew}}{-P_{ew} + sk}\right) \times \times \left(m - P_e - k - P_{ew} + sk\right) - \left(\frac{P_{ew}}{-P_{ew} + sk}\right) L. \tag{9}$$

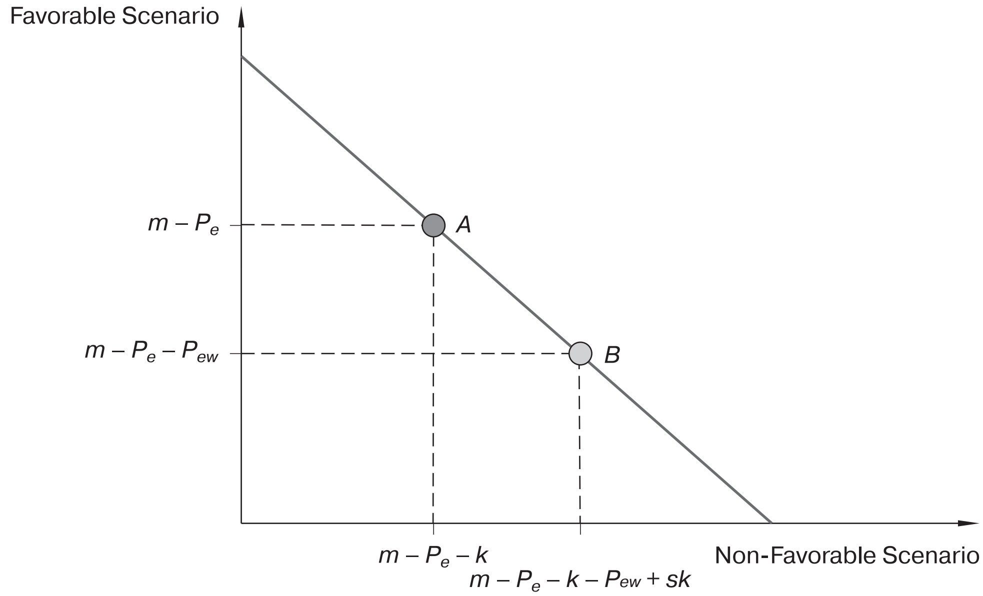

Fig. 2. Consumer's budget line to the extended warranty

The budget line presented in fig. 2 allows seeing the purchase of the insurance, the Y-axis represents the favorable scenario (when the equipment does not fail), while the X-axis represents the non-favorable scenario (when the equipment fails). Point A represents the situation in which the customer decides to buy the equipment without acquiring the extended warranty, i. e. in case a failure occurs his payoff will be discounted by k monetary units and he will not be refunded of any fraction s by the extended warranty provider, i. e. $G = m - P_e - k$. On the other hand, when the extended warranty is acquired, situation represented by point B, the monetary loss is mitigated, but at a cost of P_{ew} monetary units, i. e. $G = m - \mathrm{P}_e - P_{ew}$ at the favorable scenario, in order to alleviate the loss in case the non-favorable scenario occurs by sk monetary units. The slope of this budget line is seen in (10)

$$-\left(\frac{P_{ew}}{-P_{ew} + sk}\right). \tag{10}$$

The negative slope of the budget line implies that if the consumer wants to obtain higher insurance coverage, it is necessary to pay more for it; consequently, the buyer will change the consumption in the different states of nature (payoff). Therefore, it is impracticable to have high coverage by paying less for P_{ew}.

In case of purchasing the extended warranty, this is only justified if it is smaller than the expected financial loss due to the failure of the equipment. For instance, it would not be feasible to have a contingency plan if the expected loss corresponds to 10 monetary units, while the price of the extended warranty corresponds to 50 monetary units, that is, that insurance would not be economically viable (if based only on the consumer's profit function).

Equation (11) represents the constraint that satisfies the customer's decision to have the extended warranty.

$$P_{ew} \leq (1 - p)ks. \tag{11}$$

Note that if such a constraint is represented by an equality (equation (12)), the customer is indifferent regarding whether or not to buy the extended warranty.

$$P_{ew} = (1 - p)ks. \tag{12}$$

From equation (12), it is possible to observe that the higher the level of coverage s, the more willing to pay for the contingency plan the buyer will be.

The consumer's reservation prices can be seen in equations (8) and (12). Such prices are understood as the prices that will not bring expected financial loss to the consumer if the equipment is acquired (equation (8)) and the extended warranty is acquired (equation (12)). If the manufacturer presents lower prices, the buyer will have a higher profit.

OEM's strategy

The starting point for the manufacturer's strategy design is that the equipment will only be sold if its price is higher than its cost of production:

$$P_e > C_m. \tag{13}$$

Equation (13) shows that if the cost of production is too high, it may not be feasible to sell the equipment and consequently the payoff of both players is zero. The manufacturer will firstly need to define the vector of decision variables that the customer will face to make a decision. It is important to highlight that the OEM knows the customer's demand curves (shown in equations (8) and (12)), which will be used to model the price portfolio, maximizing the manufacturer's expected profit and capturing all consumer surplus. The value of s is implicit in the extended warranty price and does not need to be determined analytically. Therefore, for any level of s, a P_{ew} will be defined.

Note that if the manufacturer proposes prices higher than the consumer's reservation prices, the costumer will not buy either the equipment or the extended warranty plan, since it is preferable to have a zero payoff to a negative profit.

Table 3

Consumer's reservation prices

P_e^*	P_{ew}^*
$m - (1 - p)k$	$(1 - p)ks$

Table 3 presents the consumer's reservation prices, which will be used by the OEM to maximize the manufacturer's decision. The conclusion of this process takes place when the OEM induces the customer to buy the item, but the buyer is indifferent to whether to buy or not the extended warranty plan ($\Pi_1^c = \Pi_2^c$).

Finally, the OEM's optimal strategy is the definition of a prices vector, which is represented by the following equation (14):

$$\Pi_{oem}^* = m - (1 - p)k - C_m. \tag{14}$$

3. NUMERICAL EXAMPLE

The customer's and the manufacturer's optimal strategies were previously discussed for the extended warranty model. As a way of emphasizing the modeling, the following data to the numerical example of the model are taken into consideration: m = \$1000, k = \$600, C_m = \$450, p = 0,4 and s = 0,3.

In order to obtain the equilibrium of the extended warranty model and to see the players' strategy, four steps should be taken, namely:

Step 1. The OEM needs to define a selling price of the equipment above \$450; otherwise, the manufacturer will suffer a financial loss.

Step 2. Based on the data, it is possible to find the consumer's reservation prices. From the equation (8), the P_e^* = \$640, that is, it is the maximum price that the buyer will pay to have the equipment. From the equation (12), the P_{ew}^* = \$108,

Table 4

Consumer's reservation prices and OEM's payoff for a range of p

p	Π^{oem}, \$	P_e^*, \$	P_{ew}^*, \$
0,001	–49	401	180
0,010	–44	406	178
0,100	10	460	162
0,150	40	490	153
0,300	130	580	126
0,400	190	640	108
0,500	250	700	90
0,800	430	880	36
0,900	490	940	18
0,999	544	994	2

that is, it is the maximum price that the buyer will pay to have the extended warranty plan. It is necessary to emphasize that P_{ew}^* is lower than expected financial loss k.

Step 3. The manufacturer analyzes these prices (P_e^*, P_{ew}^*) and takes a decision whether or not to sell the equipment and the extended warranty plan to the customer. As the $P_e^* > C_m$, so the OEM can use this price in his portfolio prices, on the other hand, the P_{ew}^* is associated with level s of the coverage. The P_e^* and the P_{ew}^* are the OEM's maximum limits, which, if exceeded, will result in the buyer not acquiring the item or the warranty plan.

Step 4. When the P_e^* and the P_{ew}^* are shown to the customer, the OEM's expected profit is \$190. If the consumer buys the equipment without the extended warranty ($X = 1$), he will have 40 percent chance of gaining \$36 000 profit and will have 60 percent chance of obtaining \$24 000 financial loss. Thus, the buyer's expected profit will be 0. For the situation in which the consumer buys the equipment with the extended warranty ($X = 2$), he will have 40 percent chance of gaining \$25 200 profit and will have 60 percent chance of obtaining \$18 000 financial loss. Again, his expected profit will be 0. From the moment the consumer buys the device, but is indifferent to whether or not to buy the extended warranty (both cases ensure the same payoff), the manufacturer's profit will be maximum.

Extension

Once the mathematical analysis of the model is presented, one extension will be shown to expand the modeling dynamics. Thus, a variation related to the probability of the equipment operating without failure is exposed in order to analyze the change of the players' payoffs according to p. The analysis related to the probability of the equipment operating without failure is made for a range of ten possibilities of p. So, the P_e^*, the P_{ew}^* and the OEM's profit change according to the change of p. Table 4 summarizes all these results.

As can be seen, the P_{ew}^* decreases according to the increase in p, i. e., these variables have opposite trajectories of growth. The customer reduces his willingness to pay for the extended warranty since the knowledge of p is known to the players. Moreover, it does not make sense to pay a high amount of coverage if the equipment has a high reliability. On the other hand, the P_{ew}^* has the same trajectory of p, a high level of reliability implying that the buyer is willing to pay a high amount for the acquisition of the item, due to the possibility of the consumer's expected profit to increase.

The OEM's profit changes according to the P_{ew}^*. For high values of the P_{ew}^*, the manufacturer's profit increases, i. e., the higher the customer's willingness to pay for the equipment, the higher the payoff of the EW provider. It is necessary to highlight that for low levels of p, the OEM chooses not to sell the equipment to the

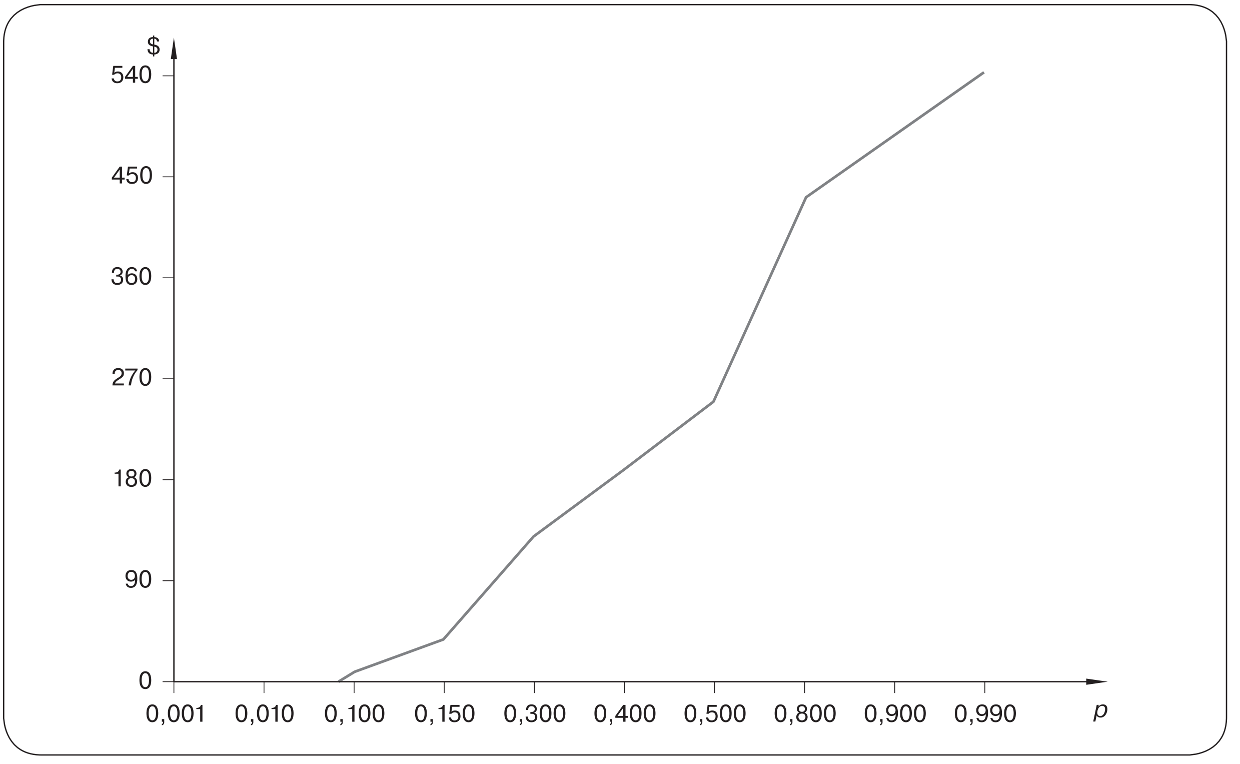

Fig. 3. OEM's profit associated to p

buyer since his profit will be negative (situations in which the cost is higher than the revenue, $P^*_{ew} < C_m$). Thus, it is better to get zero payoff than a negative payoff. Fig. 3 shows the OEM's profit trajectory related to the probability of the equipment operating without failure.

4. CONCLUSION

This paper emphasizes the role of the game theory as a useful tool to model problems related to the quantitative study of the warranty and the extended warranty, due to the existence of multi-players (EW provider — consumer) with different strategies and goals. Moreover, this study details how the Stackelberg game has been applied in the quantitative literature on after-sale services.

The model presented is based on the definition of the consumer's reservation prices, which are the key components in the strategy employed by the OEM to define his portfolio prices, thus maximizing his profit and extracting all the consumer surplus from the customer, leaving the latter with zero profit. The model is solved through an adaptation of the Stackelberg game, where the leader is the OEM and the follower is the consumer. Since there is no asymmetric information in the model, the OEM uses the consumer's reservation prices to make his decision and to influence the customer.

The model can be extended in several ways to capture the reality more meaningfully:

- By removing the hypothesis of information asymmetry. Thus, the OEM would not know the consumer's reservation prices;
- By changing roles between the players, i. e., the OEM can be the follower and the buyer can be the leader. Thus, new

strategies would be defined among decision-makers.

- By considering many different customers, that is, the manufacturer would face a heterogeneous population of buyers. Thus, the OEM would need to define a portfolio of strategies in order to maximize his profit.
- By incorporating an expected utility function to capture the risk aversion coefficient. As the failure is a random event, it is possible to consider an expected utility function to model the preference of players under uncertainty. Moreover, it is also possible to determine the Arrow–Pratt risk aversion function to classify the players' preference related to risk (averse, lover or neutral to the risk) [Eeckhoudt, Gollier, Schlesinger, 2005].

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Расширенная гарантия: анализ с использованием модели Штакельберга

Э. Сантос
Федеральный сельскохозяйственный университет штата Пернамбукано, Ресифе, Бразилия
E-mail: santos.henrique624@gmail.com

К. Кристино
Федеральный сельскохозяйственный университет штата Пернамбукано, Ресифе, Бразилия
E-mail: claudio.cristino@ufrpe.br

Б. Гуэджис
Федеральный университет штата Пернамбукано, Ресифе, Бразилия
E-mail: faleconosco@ufpe.br

Статья призвана продемонстрировать применение теории игр к количественному исследованию расширенной гарантии (extended warranty). Этот вид послепродажного обслуживания представляет собой дополнительное необязательное покрытие, которое потребитель может приобрести после покупки какого-либо устройства, начиная с момента окончания периода базовой гарантии. Продемонстрирована модель, в которой взаимодействуют два игрока: изготовитель оборудования (OEM-производитель), отвечающий за назначение цен на устройство и расширенное гарантийное обслуживание, и покупатель (владелец устройства). Способ их взаимодействия моделируется с помощью модели Штакельберга, которая изначально была разработана для анализа конкуренции на олигопольном рынке. В представленной модели «лидер» представлен OEM, а «последователем» является клиент, где цель OEM — найти некоторую ценовую структуру, чтобы максимизировать свою прибыль и влиять на решение покупателя. С помощью таких уточнений определяются стратегии и выигрыши игроков. Надежность модели обусловлена включением ряда понятий экономической теории (излишек потребителя, излишек производителя, резервная цена потребителя, выбор в условиях неопределенности, максимальная прибыль и план потребления в зависимости от условий) и элементов теории надежности (вероятность отказа и не поддающаяся ремонту система). Кроме того, для наглядности предложенной модели приведены числовой пример и анализ чувствительности. В статье систематизируются этапы применения модели Штакельберга при моделировании расширенной гарантии.

Ключевые слова: расширенная гарантия, теория игр, модель Штакельберга, резервная цена потребителя.

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