St. Petersburg University
Graduate School of Management

Master in Corporate Finance

CREDIT RISK MANAGEMENT, MODELING LOAN PORTDOLIO LOSS DISTRIBUTION, A CASE STUDY IN BANKING

Master's Thesis by the 2nd year student
Corporate Finance
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АННОТАЦИЯ

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Название магистерской диссертации	Моделирование распределения убытка кредитного портфеля, исследование в банковской отрасли
Факультет	Высшая школа менеджмента
Направление подготовки	Корпоративные Финансы
Год	2016
Научный руководитель	Александр Васильевич Бухвалов
Описание цели, задач и основных результатов	В этой работе проводится исследование в области управления кредитными рисками и предлагается модель для распределения убытков кредитного портфеля в банковской индустрии. Было определено, что модель в основе требований системы «Базель» работает хорошо в благоприятных экономических условиях, но не была доказана эффективность модели в периоды кризисов и экономических спадов. В основе системы «Базеля» лежит однофакторная модель Гаусса, которая определяет корреляцию между дефолтами кредитов в портфеле, а также модель Васичека для определения размера резервов. В работе была предложена модель распределения убытков портфеля в периоды экономического спада и кризисов, в основе которой лежит однофакторная копула распределения Стьюдента со стохастическим распределением убытков. Более того модель предоставляет кривую зависимости кредитного риска временной структуры и расширяет модель «Базеля» внедряя также корреляционную связь между вероятностью дефолта и убытком в случае дефолта. Исследование было проведено с помощью метода Монте-Карло и предлагает анализ влияния предложенной модели на стратегию кредитования в сравнении с моделью «Базель».
Ключевые слова	Базель, кредитный риск, корреляция дефолтов, копула, распределение убытков кредитного портфеля, метод Монте-Карло

Abstract

Master Student's Name	Amir Azamtarrahian
Master Thesis Title	Modeling loan portfolio loss distribution, a case study in banking
Faculty	Graduate School of Management
Main field of study	Master in Corporate Finance
Year	2016
Academic Advisor's Name	Alexander Bukhvalov
Description of the goal, tasks and main results	This thesis studies credit risk management and proposes a generic model for loan portfolio loss distribution in banking industry. Basel model works acceptably well in normal economic situations but not in downturns. It assumes one-factor Gaussian copula for default correlations and introduces capital reserve on the ground of Vasicek process. To model portfolio loss distribution in downturn economy, a one-factor t-student copula with stochastic default correlations is proposed, moreover, the model determines specific credit curve for each counterparty and extends Basel by correlated PD and LGD as well. The analysis is done through Monte Carlo simulation and studies the influence of the proposed model in lending strategy comparing to Basel.
Keywords	Basel, credit risk, default correlation, copula, portfolio loss distribution, Monte Carlo

Introduction

Credit risk can be considered as the most critical of all types of risk. It is estimated that financial institutions allocate about 60% of the regulatory capital to credit risk, about 15% to market risk and about 25% to operational risk[1]. There are two types of credit risk, migration risk and default risk whereas the latter is the subject of this thesis.

On the regulatory framework of Basel II & III, the required reserves are categorized in two Tiers. Tier 1 generally represents shareholders' equity and retained earnings while Tier 2 determines the subordinated long-term debt, general loan-losses and undisclosed reserves. Banks have to maintain a total capital ratio of 8% regarding risk-based assets[2] in the balance sheet, broken into 6% for Tier 1 and at least 2% for Tier 2. Apparently, putting aside a specific part of capital to hedge against probable losses limits the potential interest incomes. On the other hand, banks avoid unhedged risk and it prompts them to demand for quantifying accurate reserves to trade-off between risk and return and formulate a more efficient hedging strategy. Likewise, maintaining the net-interest income within a steady state range and decisions concerning the bank capital structure or the service fees are highly contingent on the amount of these reserves as well.

Among the major genres of risks that banks are exposed to, such as market, operational, credit, and liquidity risk, this thesis concentrates on credit default risk of the counterparty corporations in a loan portfolio and provides the bank with a quantitative figure of loss distribution and the required economic capital. It comes up with a generic model for credit risk and extends Basel to model loan portfolio loss distribution. Basel capital adequacy model generally works well in normal economic situation, however, it does not take into account some types of risks appearing in economy downturns and recessions, such as default contagion and tail dependence of default rates. Moreover, empirically it is evidenced that default rates and recovery rates depend nonlinearly in a manner that expected recoveries are tending to decrease more in recession comparing their likely increase in expansionary economy.

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[1] Correlation risk modelling and management by Gunter Meissner, 2013
[2] Weighted sum of assets based on their corresponding risks

Research goal

The thesis primarily involves “extending Basel credit risk capital adequacy model for economy downturns through modeling and incorporating empirical evidences of the risk factors and their interactions and analyzing how it influences the bank lending strategy”.

Motivation and research questions

Should banks operate smoothly, Basel regulates to keep some capital as reserves to absorb probable losses. In this regard, Basel II recommends Vasicek model as an industry standard. However, the model comes with unrealistic assumptions such as similar probability of default for all counterparties in portfolio with the same default correlation and constant through time. Moreover, contrary to empirical evidences, it assumes independence between recovery rates and default rates. This thesis extends the model to approach the problem regarding the corresponding realities based on empirical and the stylized facts. It comes up with the consequences of applying Basel model and answers to the following questions,

1- How reliable is it to apply Basel in economic downturns?
2- Does it matter to apply a more accurate model?
3- How much is the difference between Economic Capital (EC) in Basel and the model?
4- How does it influence the bank loan portfolio structure and lending strategy?

Research gap

Scholars in credit risk modeling devote their work on particular components of credit risk modeling such as probability of default, recovery rates or default correlations. Moreover, they chiefly concentrate on Analytically Tractable (AT) models via assuming independence or Gaussian processes to come up with mathematical closed form expressions; this limits the flexibility and also the applicability of the model in regard with real behavior of risk factors. The extensions to Vasicek model are mostly applied in pricing credit derivative products like Collateralized Debt Obligations (CDO), Credit Default Swaps (CDS) which deal with a portfolio of credit assets and defaultable counterparties; the study for loan portfolio starts with Vasicek 1987 who correlated default rates by one-factor Gaussian copula and introduced analytical model for Large Homogeneous Portfolio (LHP). In his model correlations and recoveries are assumed deterministic and constant. Giese 2005 extends stochastic recoveries and comes up with correlated default rates and loss give defaults, furthermore, Fray 2013 predicts loss given defaults as a function of default rates. Moreover, Gregory, Burtschell and Laurent 2005 carry on a comparative study of different copulas in pricing of synthetic CDO tranches. Hull-White 2004 applies double t-student copula to CDO and n^{th} to default CDS. They extend their work in Hull-White 2010 and propose stochastic correlations as well as recovery rates correlated to default rates through Gaussian copulas.

In loan portfolios there is a need to incorporate all realities together to model credit risk through considering not only appropriate models for each risk factor but also taking into account their empirical interactions and individual characteristics in economic downturns as well. Although Hull-White 2010 fulfilled this objective to some extent, but still they simplified some realized facts and ignored tail dependence, a frequently observed phenomena in economic downturns, to model default correlations. Furthermore they did not account for negative-negative tail dependence of recovery rates and business cycles. Moreover, some commercial credit risk models like “Credit Metrics”, “Credit Risk+” and “Moody’s KMV” propose models to forecast credit risk with better accuracy. Credit Swiss recommends “Credit Risk+” and correlates default rates by introducing default rate volatilities rather than some background common factor to model default correlations; Moody’s KMV tries to model default correlations through correlating the assets’ processes of counterparties in a portfolio and JP Morgan “Credit Metrics” model concentrates on transition matrixes of default correlations and tries to simulate portfolio behavior in terms of a Markov chain. Although each product has an advantage in some aspects but none of them thoroughly address the problem via incorporating all considerations.

Research design

In order to extend Basel for economic downturns and benefit from the previous proposed methodologies in loan portfolio, this thesis focuses on modeling interactions and gets Merton model to calculate probability of default, besides, assumes Vasicek process for Counterparty’s assets value. The default rates are correlated through t-copula taking into account tail dependence to model systematic risk recessions. Moreover, it correlates recoveries with default rates through a Clayton copula to capture the negative-negative tail dependency as the stylized fact in market. In addition, it releases the constant correlation assumption and comes up with stochastic correlations negatively correlated with market performance through Gaussian copula. Finally, it takes a sample portfolio of loans and compares the economic capital with Basel. The main steps in the modeling process is summarized in the following flow chart,

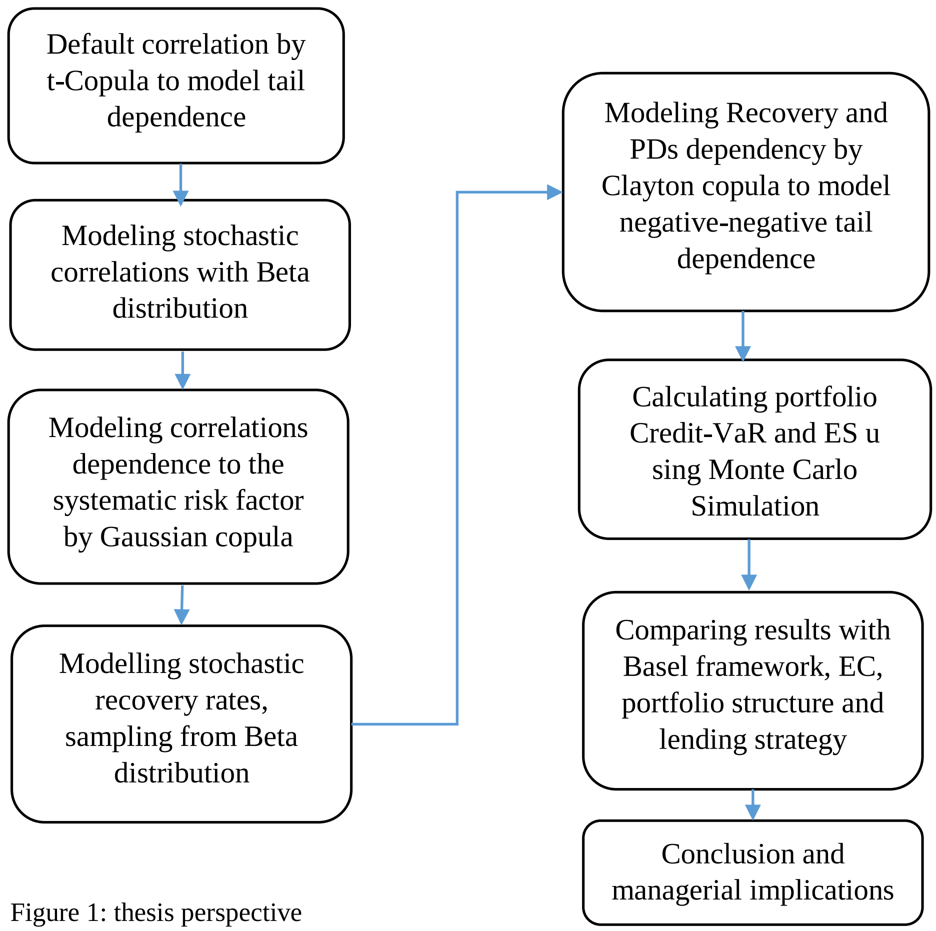

Figure 1: thesis perspective

Chapter one introduces research objective and poses thesis questions. Subsequent to a concise literature review, the methodology and problem formulation is presented in the next chapter where inputs and risk factors are described in detail with the associated characteristics of interactions for Monte Carlo simulation. Chapter 2 continues with modeling loss distribution in portfolio level and proposes credit-VaR. Finally, the third chapter represents implementation and managerial implication and provides conclusions through evaluating results regarding Basel capital adequacy accord and the strategies proposed by the model.

CHAPTER 1: LITERATURE REVIEW

This chapter illustrates thesis overview and the evolution of literature about the subject. It studies the previous works of credit risk models and the associated stylized facts for each component of the portfolio loss risk factor is presented.

1. Literature review

This section starts with a quick definition of credit risk and Basel regulatory requirements, it reviews the frequently cited default risk models focusing on firm-value models'[3] literature. Subsequently, default correlation models are reviewed and finally evolution of papers about modeling Loss-Given-Default (LGD) and its correlation with default probability is presented.

1.1 Credit risk models

Credit risk has proved to be a debated topic particularly in the aftermath of 2007-2008 global financial crisis and the appearance of the default contagion phenomena. Recently it targets not only the so-called junk stocks but also the most credit worthy institutions like AIG and Lehman brothers after the crisis. However, despite the recently heated topic, it had been already a concern for policy

[3] Firm-value models and structural models are used interchangeably, option-based credit models is an alternative name as well

makers and regulatory institutions long before in banking industry respecting the regulatory issues and banks internal risk management policies.

Credit models involve estimating default probabilities and term structure of spreads as price of default risk. There are two major approaches in credit risk modelling, structural and intensity based-based models known as reduced-form as well. The former takes default as an endogenous event while the latter models default as an exogenous variable. Primary works on structural models originates from Merton 1976 in line with Black-Scholes options pricing model. Merton assumes a company with liabilities like Zero-Coupon-Bond (ZCB) and takes equity as European-option on the company assets where liabilities' par value is the strike price. Accordingly, the risk-neutral probability of default is simply when $V_T < D$ which is $N(-d_2)$ in Black-Scholes framework. Relying on Merton, default can only happens if assets fall below outstanding debt at the time of servicing or refinancing the debt. Other structural models such as Black and Cox 1976 is similar to the Merton model in that they use the firm's structural variables such as asset and debt values as basis for their modelling. However, Black and Cox states that default can occur at any time, not just at the expiration of the debt, this property puts it in the family of first passage time models. The model allows defaults to occur as soon as the firm's assets value falls below a certain threshold, which does not necessarily have to be debt value. This assumption in Merton model was contrary to bond safety covenants which allows bond holders to push a firm into bankruptcy under certain special situations even if the firm hast not explicitly defaulted on a payment; Furthermore, Delianedis and Geske 1977 account for more complex capital structures by creating two tranches of risky debt. At date T1 the firm is obliged to make the payment of F1 for short term liabilities. The firm cannot sell its assets to meet its obligation. Rather, the firm must go to capital markets and raise funds (equity or new debt) to finance the payments. Clearly, the ability to raise funds will depend on the amount of debt outstanding. If the present value of all debt outstanding, together with the required payment, F1, exceeds the value of the firm, then the shareholders will declare the firm bankrupt. Viewed from time zero, equity holders have a compound option on the assets of the firm. No default by T1, then they can exercise their claim, make the payment of F1 dollars and receive a call option on the assets of the firm. Hence, at date zero, they have an option on an option, or a compound option.

In continue, the first passage family was extended by Longstaff-Schwartz 1995 taking a stochastic process for interest rates rather than constant[4]. Leland and Toft (1996) took the next major step through incorporating bankruptcy costs and tax effects which allows a formal characterization of optimal capital structure, debt capacity, and credit spreads in a classic trade-off model. The Black-Cox model produces low credit spread because assets that begin above the barrier cannot reach the barrier immediately by diffusion only. To increase the spreads jumps came into the asset value process. Zhou 1997 introduced a jump component to the underlying continuous process, but the model is somehow intractable. In an alternative approach, Finkelstein et al 2002 CreditGrades model, allows the barrier to fluctuate randomly. The uncertainty in the barrier admits the possibility that the firm's asset value may be closer to the default point. This leads to higher short-term spreads than are produced without the barrier uncertainty. Moreover, Moody's KV 2003, came up with a modified structural model outputs Distance to Default (DD) to be mapped on an internally developed database of companies with real default probabilities historically complying with the DD calculated, hence, the outcome is regarded as a real world probability of default. The work is taken as a gist of main insights gleaned from Black-Cox 1976, Geske 1977 and Longstaff-Schwartz 1995. In their framework the option is a perpetual down-and-out that can be exercised at any time, repurchase or issue of debt is possible and restriction on asset sales exists[5]. Also, it accommodates five different types of liabilities: short-term liabilities, long-term liabilities, convertible debt, preferred equity and common equity[6]. Brigo and Tarenghi 2004 developed AT1P , on the ground of Black-Cox model, providing time dependency in both the volatility and the barrier hence non-constant business risk and debt level, and contrary to Zhou, still preserving closed form pricing formulas and a more flexible model comparing to Black-Cox in a sense that parameters are perfectly calibrated to CDS market data[7]. The most intriguing characteristic of the model belongs to its independency from the current asset value which is difficult to be estimated particularly for non-listed companies. In their framework it is possible to rescale the initial value of the firm's assets $A_0 = 1$ and express the (free) barrier parameter H as a fraction of it and hence, it is not necessary to know the real value of the firm. Moreover, Brigo 2009[8] comes up with Scenario Barrier Time-Varying AT1P model (SBTV) to reduce effect of uncertain

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[4] Reduced Form vs. Structural Models of Credit Risk: A Case Study of Three Models∗ Navneet Arora, Jeffrey R. Bohn, Fanlin Zhu Moody's KMV February 17, 2005

[5] Default forecasting in KMV, masters thesis, Yuqian Steve Lu, 2008, University of oxford

[6] Modeling default risk, modeling methodology , Crosbie and Bohn (2003)

[7] Although CDS market Is not available in Iran but any other suitable proxy give privilege to the model

[8] Credit Calibration with Structural Models: The Lehman case and Equity Swaps under Counterparty Risk Damiano Brigo∗ Massimo Morini Marco Tarenghi, December 22, 2009

accounting data accomplished by defining random barriers and calibrating probabilities and barriers. Here the market price is taken as the weighted average price of different scenarios by probabilities calibrated. The model outputs a smoother implied volatilities contrary to AT1P and efficiently complies with intensity models.

There are stylized facts that structural models are not able to generate positive short-term positive spreads. This was addressed by adding jumps in the process. Moreover, credit spreads implied from structural models are much lower than real data referred as credit spread puzzle. While empirical evidence is still scant, a few empirical researchers have begun to test these model extensions. Lyden and Saraniti (2000) compare the Merton and the Longstaff-Schwartz models and find that both models under-predicted spreads; the assumption of stochastic interest rates did not seem to change the qualitative nature of the finding. Eom, Helwege, and Huang (2003) find evidence contradicting conventional wisdom on the bias of structural model spreads. They find structural models that depart from the Merton framework tend to over-predict spreads for the debt of firms with high volatility or high leverage. For safer bonds, these models, with the exception of Leland-Toft 1996, under-predict spreads; following table summarizes literature evolution.

Row	Milestone	Description
1	Merton 1974	Option-based risky ZCB pricing
2	Black-Cox 1976	Came up with first passage default time model
3	Geske 1977	Introduce Short and Long Term debt
4	Longstaff-Schwartz 1995	Assuming interest rates mean-reverting stochastic process
5	Zhou 1997	Added jumps to the underlying process, it is not AT[9]
6	Leland 1998	Adding tax and bankruptcy measures to value risky debt
7	CreditGrades 2002	Modelled barrier as a continuous stochastic process
8	Moody's KMV 2003	Commercial model (DD), mixture of previous works
9	Brigo, Tarenghi 2004[10]	AT1P, introduced non-constant volatility, model independent of current asset value
10	Brigo, Tarenghi 2006	(SBTV) reducing effect of unreliable accounting data

Table 1: evolution of structural models through time

[9] Analytically Tractable
[10] Selected model since it does not rely on asset value

In the other category of credit models known as reduced-form[11] models, the random nature of defaults is typically characterized in terms of the first "arrival" of Poisson process. Intensity based models model the risk of default as an event that arrives exogenously. There are several types of reduced form models. Lando 1998, Duffie and Singleton 1999 showed in their work that price of a risky bond of a company can be calculated by a default adjusted discount rate. The extra rate is referred as intensity. The first model that actively used the concept of default intensity came from Robert Jarrow and Stuart Turnbull 1995. They constructed their model based on two classes of zero coupon bonds, a risk free ZCB and a risky one. The paper suggests that when default intensity was held constant, the risky debt's value is proportional to the risk free by[12], while δ is recovery rate and μ is market price of default risk (a positive constant less than 1).

Based on Jarrow and Philip Protterb 2004, the difference between these two models can be characterized in terms of the information assumed known by the modeler. Structural models assume that the modeler has the same information set as the firm's manager—complete knowledge of all the firm's assets and liabilities. In most situations, this knowledge leads to a predictable default time. In contrast, reduced form models assume that the modeler has the same information set as the market-incomplete knowledge of the firm's condition. Consequently, for pricing and hedging, reduced form models are the preferred methodology[13]. Jarrow concludes that if one is interested in pricing a firm's risky debt or related credit derivatives, then reduced form models are the preferred approach that have been constructed, purposefully, to be based on the information available to the market.

1.2 Loss given Default (LGD)

Loss given default is defined as the amount of funds that is lost by a bank when a borrower defaults on a loan. As defaults and credit events generally end up courts, there is considerable uncertainty as to what an accurate recovery would be if a company defaults. Based on IRB approach banks are able to design internal models to calculate capital reserve in light of common characteristics identified by studies of academics and industry, and apparently, LGD is not an exception in this regard.

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[11] Or intensity-based models

[12] $D_{risky}(t,T) = \left[e^{-\lambda\mu(T-t)} + \left(1 - e^{-\lambda\mu(T-t)}\right)\delta\right] \times D_{risky-free}$, Credit derivatives, A primer on credit risk, modelling and instruments, George Chaoko, Andera Sjoman et al.

[13] Structural verses reduced form models: a new information based perspective, Robert A. Jarrow and Philip Protterb, Journal of investment management, Vol. 2, No. 2, (2004), pp. 1–10

Historically, recoveries are in a range from 20% to 80% which depends to which definition for default we refer. BIS[14] defines four events as default. Schuermann 2004[15] mentions the debts' place in capital structure, seniority, general economy status and industry as the main determinants of LGD. Also, recovery rates differ depending on claiming in which stages of the bankruptcy process has been made[16]. Schuermann also argues the recoveries distribution with evidence of higher probability for lower recoveries empirically. According to Altman and Kishore 1996 one should know about seniority and collateral to predict the recovery rate. Likewise, Gupton, Gates and Carty report that syndicated loan recoveries for senior secured loans were 70% in average while the unsecured one fall to 52%. Moreover, the importance of monitoring reviewed by Carey 1998 through comparing investment grade and lower credit grade debts highlighted by attributing the difference in performance of higher risk instrument to the closer monitoring.

Fray 2000, shows that in recession, recovery is about a third lower than in an expansion. Altman, Brady, Resti and Seroni 2003, suggest when aggregate default rates are high, recovery rates are low.

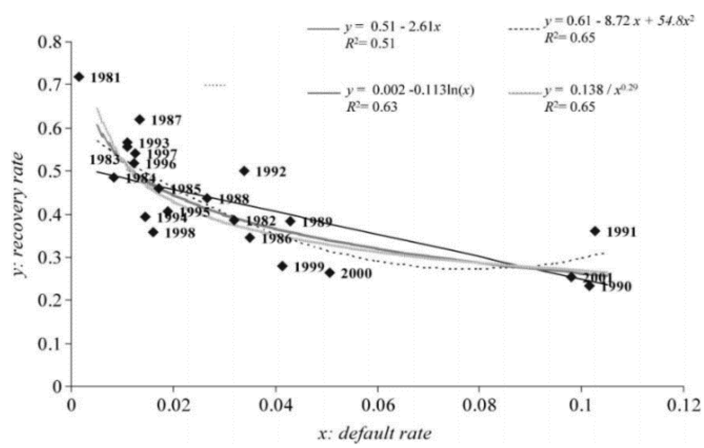

Figure 2: recoveries and default rate dependency, Altman 2003

Also, Hu and Peraudin 2002 presented that correlation between recoveries and aggregate defaults rates for the US are -20% on average and about -30% when considering only tails which implies higher correlations in recession. Moreover, Altman and Kishore 1996 revealed that some industries like utilities are more recession resisitent than others.

[14] Basel Committee on banking Supervision document
[15] What do we know about LGD? Federal reserve Bank of New York, By Til Schuermann 2004
[16] Last Cash Paid- default- Chapter 11- emergence due to liquidation or genuine emergence, it take on average 2.5 years

Industry	Avg. Recovery	Industry	Avg. Recovery
Utilities	70%	Communication	37%
Services	46%	Financial Institutions	36%
Food	45%	Construction, Real Estate	35%
Trade	44%	General Stores	33%
Manufacturing	42%	Textile	32%
Building	39%	Paper	30%
Transportation	38%	Lodging, Hospitals	26%

Table 2: Industry impact from Altman and Kishore 1996

A recent work by Archarya, Bharath and Srinivasan 2003 found that when industries are in distress, mean LGD is on average 10% to 20% higher than otherwise. In their work utilities is still the highest recovery industry sector. Apart from industry effect, to the size, contrary to its importance in modeling probability of default, based on literatures it seems to have no strong effect on losses ones default has occured. Asarnow and Edwards 1995 find no relation between LGD and loan sizes in their study of loan data in Citi bank middle market and large corporation lending. Likewise, Thornburn 2000 also found that firm size does not matter in determining LGD. Similar results obtained from Carty and Liberman 1996 and others as well.

There are various models that connect LGD rate to default rate. Fraye 2000 assumes recovery is a linear function of normal risk factor associated to the Vasicek distribution. Pykhtin 2003 parameterizes the amount, volatility and systematic risk of the loan collateral and infers the loan's LGD and brings up a closed form expression for expected loss and economic capital. Geise 2005 applies econometric estimates of correlations between default rates and loss given default rates and calculate their impacts on the credit risk capital. Fraye 2013 models LGD as a function of default rates. In his paper an asymptotic portfolio is assumed with entities all having the same expected loss and default correlation. In this work the model chiefly inspires from works in Fraye 2000 and Giese 2005 to fulfill the stylized facts about the concept. Accordingly, LGD is taken stochastic and modeled by Beta distribution which is calibrated to industry norms and correlated with default rates.

1.3 Default correlation

This part reviews the historical behavior of correlation through time and introduces models proposed in line with the stylized facts studied empirically.

The degree to which defaults occur together is critical for financial lenders such as commercial banks, credit unions as well as insurance companies etc. Default correlation is addressed in the literature from different points of view, some deal with empirical analysis of correlation behavior in time particularly business cycles and evidence the dynamic characteristic of correlation through time. Others address the industry sectors and find correlation clustering phenomena inter and intra sector and find that default correlation between industries is positive with the exception of energy sector as the recession resistant sector with a low or negative correlation with others. Moreover, default correlation within sectors is higher than between sectors and this suggests that systematic factors like recession, structural weakness such as general decline of a sector have a greater impacts on defaults than do idiosyncratic factors. Hence a lender is advised to have a sector-diversified loan portfolio to reduce default correlation risk. Systematic risk and correlation are highly dependent and historically, a systematic decline in stocks almost involves the entire stock market and correlation between stocks increase sharply.

	Correlation Level	Correlation Volatility
Expansionary period	27.46%	71.17%
Normal economic period	32.73%	83.40%
Recession	36.96%	80.48%

Table 3: correlation level and correlation volatility with respect to the state of economy, Meissner 2013

Meissner 2013 monitors correlation between stocks in Dow and Dow index and observes that correlation in Dow increases when Dow increases more strongly, however, there is this increase accelerates in time of severe decline in Dow during 2008 to Aug 2009 from a non-crisis average of 27% to over 50%. (The red triangle graph represents Dow)

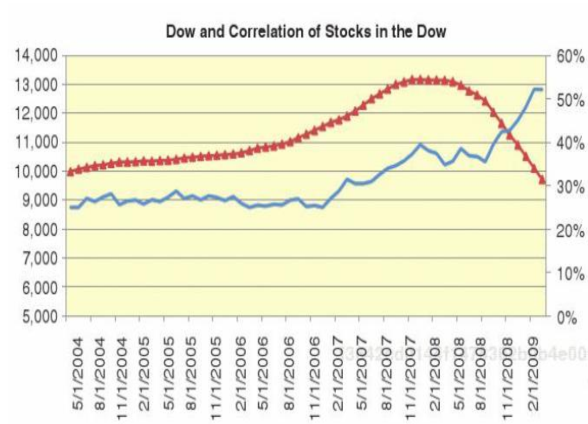

Figure 3: Dow and correlation of stocks in Dow, Meissner 2013

To model default correlations Lucas 1995 proposed the binomial model taking default as a binary variable. Furthermore, he shows that correlation levels and as well as correlation volatilities are higher in economic crisis. In the following figure a mean-reverting behavior of correlation through time is noticeable. It shows the monthly average correlation levels and depicts there is a low correlation in strong economic growth, while it increases during recession.

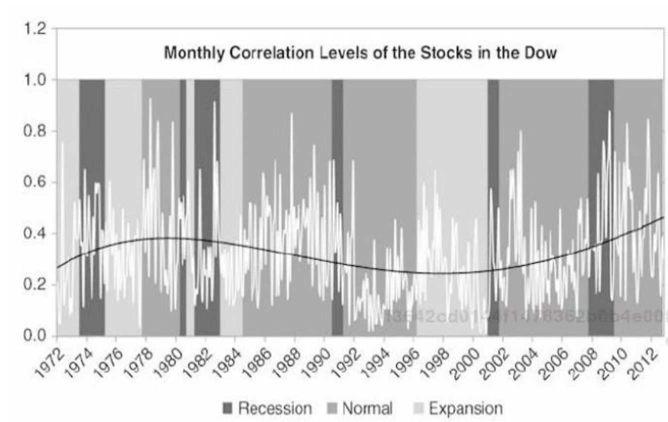

Figure 4: Monthly correlation levels of stocks in the Dow, Meissner 2013

This is also the case for correlation volatility where during economic decline tends to increase.

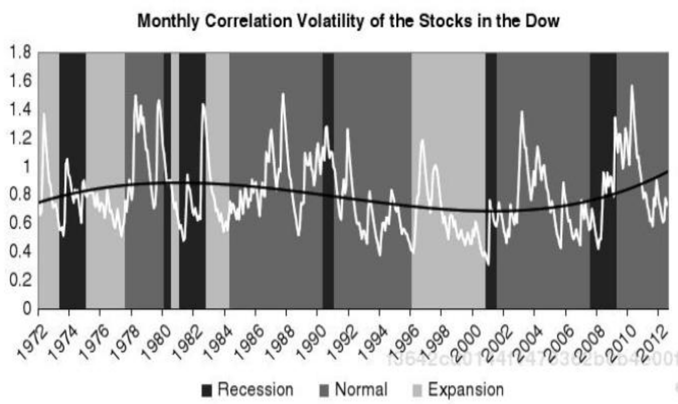

Figure 5: Monthly correlation volatility of the stocks in the Dow, Meissner 2013

Apparently, to the dynamic nature of correlation, using statistical methods for correlation in finance is not applicable since most financial dependencies are not linear. There are different models proposed in literature, Heston 1993 correlates two stochastic process of stocks and its volatility through the diffusion part. This method is widely used in finance due to its dynamic and versatile characteristics. Zhou 2001 applies Heston correlation to derive analytical expression for joint default distribution in Black-Cox first passage time framework. In this thesis the default correlation is modelled through asset-value approach which correlates defaults through the stochastic process of asset returns via Heston methodology as well. Brigo and Pallavicini 2008 apply two Heston correlation, the first correlates two factors that effects the interest rate process and the second correlates interest rate process with the default intensity process.

The other famous or infamous correlation model is Copula approach. One-factor copulas were introduced to finance by Oldrich Vasicek in 1987. More versatile, multivariate copulas were applied to finance by David Li 2000. There are lots of copula models and among all, one-factor Gaussian and from the Archimedean family Clayton, Gumble and Frank copulas are the most popular in finance industry. Moreover, some extension like t-copula originates from t-Student distribution which is categorized in two-factor copula models. Contrary to Gaussian copula, t-copula is capable of modelling tail dependence. Copulas found a place in modelling of correlation in finance, Meisser 2007 and Brigo and Chourdakis 2009 apply a bivariate Gaussian copula to model CDS seller and the reference asset with counterparty credit risk. Basic structural models assume that correlations are constant. Empirical evidence suggests that assets correlations are positively related to default rates.

From the reports by Meissner, correlations are not deterministic and show a mean-reverting behavior through time, more importantly, the default correlation of two firms tends to increase by time, hence, static models do not capture the entire features of default correlations. Hull White 2010 propose dynamic correlated model based on asset-value approach. In their work, the stochastic parts of asset processes are correlated by a one-factor Gaussian copula, where correlation and recovery rates are both Beta random variables correlated again to the market factor via Gaussian copula. In each time step a unique LGD and correlation is associated with the market factor, hence the higher the market factor, the lower correlations and LGD from Beta distribution.

This thesis steps forward and extends the basic Vasicek model of Basel through Hull White 2010 insights. However, contrary to Hull-White, here t-student copula is used to correlate defaults, furthermore, the LGD and correlations are random, and the correlation modeled by Clayton copula to capture the negative-negative tail dependence between these variables. This methodology is more consistent with empirical results.

CHAPTER 2: METHODOLOGY

This chapter concisely reviews regulatory issues and proceeds to the problem motivation and formulation. Subsequent to introduction of Basel II and III accords, bank balance sheet and lending procedure is reviewed, afterwards, each component of expected loss and the proposed models for probability of default, recovery rate and how to model their interactions is modeled and finally the analysis in portfolio level is performed.

2.1 Regulations in banking

Historically, investment opportunities were solely available to affluent people who were considered to afford losses by their wealth, however, as investment activities grew as all classes of people began to enjoy higher disposable income and finding new places to put their money, to avoid fraudulent activities, in theory these investors were protected by the Blue Sky laws (enacted in Kansas 1911). These state laws were meant to protect people from worthless securities; they are basic disclosure laws that require a company to provide prospectus in which the promoters can rely on. In this section the most recent and important regulations concerning credit risk analysis are presented.

2.1.1 Basel II & III

Basel II was initially published in 2004 as an international standard for banking regulators to control how much capital banks need to put aside in order to guard against the types of financial and operational risks they face through lending and investment practices. Basel II is based on three pillars. The first pillar deals with maintenance of regulatory capital for the three major components of risks that a bank faces including credit risk, operational risk, and market risk. The second pillar is a regulatory response to the first one. It also provides a framework for dealing with systematic risk, strategic risk, liquidity risk etc. It is the International capital Adequacy Assessment Process (ICAAP) which is the result of Pillar I and II accords. The third pillar aims to complement the minimum capital requirements and supervisory review process by developing a set of disclosure requirements which allows the market participants to gauge the capital adequacy of an institution. Under Basel II, the risk of counterparty default and credit migration risk were addressed but mark-to-market losses due to credit value adjustments were not. This problem was considered under development of Basel III[17].

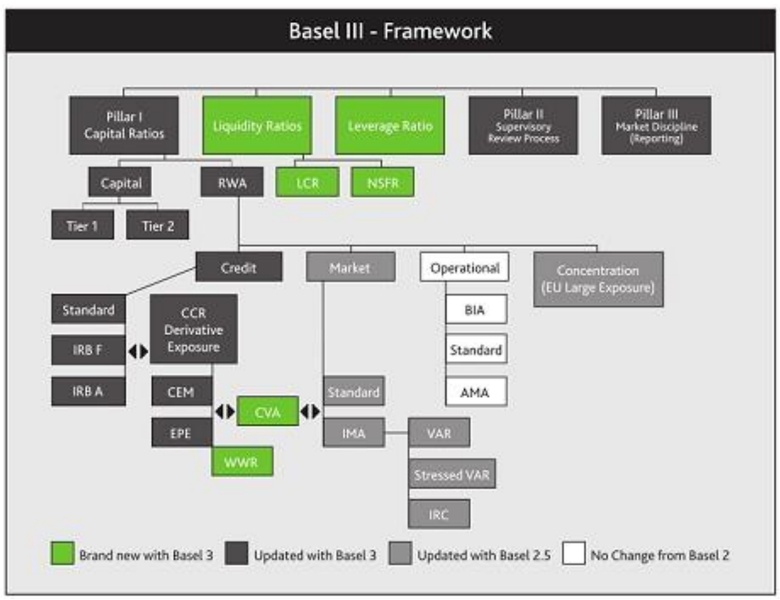

Figure 6: Basel framework, Moody's Analytics

Following 2007-2007 financial crisis, Basel committee considered a major overhaul to the former Basel accords. Although the committee had increased capital requirements, it continued to regulate extra reserves to cover credit risk in Basel III too. This followed by a tighter capital regulations to take liquidity risk into account as well. The first proposal for Base III issued in December 2009 and the final version released in a year after. The regulation consists of six parts

[17] KTH-Royal institute of technology, Masters 'Thesis, Dan Franzen, Otto Sjoholm

involving definition of capital and requirements, capital conservation buffer, countercyclical buffer, leverage ratio, liquidity risk and finally counterparty credit risk. Under Basel III bank total capital consists of Tier 1 that represents the equity capital like share capital and retained earnings and should be at least 4.5% of the risk-weighted assets at all times. Additional Tier 1 items include non-cumulative preferred stock and the total Tier capital should be at least 6% of the risk-weighted assets at all times. Tier 2 is the debt which is subordinated to depositors. The total Tier 1&2 must be 8% of the risk-weighted assets at all times. The required reserves were more than doubled comparing to Basel II. Common equity to the Basel committee is regarded as "going-concern capital"[18]. When a bank is going-concern common equity absorbs losses. Tier 2 capital is referred to as "gone-concern capital" when a bank is no longer a going-concern, then losses have to be absorbed by Tier 2 capital that ranks below depositors in liquidation.

Basel III is part of the continuous struggling effort to enhance the banking regulatory framework. It is built on Basel I and Basel II documents and seeks to improve the banking sectors' ability to deal with financial and economic stress, improve risk management and strengthen banks transparency. The Basel committee call for more capital for "systematically important" banks. This is not a standardized term across countries, however, in US it is considered banks with capital above $50 billion.[19]

2.1.2 Bank balance sheet and Basel

Each item in the balance sheet of a bank corresponds to an interest-related income or expense item, and the average yield for the period and the net-interest income highly depends on the shape of the yield curve. Banks usually try to overcome the undesirable impacts of yield curve flattening[20] phenomena, due to narrowing the difference rate between long term and short term borrowing, through charging more for their services. Moreover, the volume of bank fee generating activities may differ also based on interest rate expectations and demand behavior for loan. For instance, as interest rates rise, there is less demand for mortgages and on the other hand the prepayments happen less due to higher cost of borrowing again, and as a result fee income and associated economic value originating from mortgage services may increase or remain stable in these situations. The interest rate can jointly

[18] The concept is an assumption in accounting that entity will be able continue operating sufficient to carry out its commitments, obligations, and objectives etc.
[19] Basel Committee for Bank Supervision, Basel III, A global regulatory framework for more resilient banks and banking system
[20] Normally yield curves are upward slopping to stimulate spending in recession period, the flattened and downward slop yield curve happen in economic booms to slow down the economy and indicator of lower short rates in future

act with other risk factors facing a bank. In a rising interest rate period the payments of the loan decrease due to higher level of payment value or lower earnings, this exposes the bank, particularly for floating rate borrowings, to credit risk. For a bank with short term liabilities, rising interest rates will increase the likelihood of liquidity risk and credit quality problems as well.

To mitigate the credit risk banks develop certain internal credit analysis procedures next to national and international regulatory requirements. Evaluating the creditworthiness of a corporate client and a rough valuation is mainly done by credit department through reviewing the financial statements. This is usually supplemented with site visits to confirm the claims via direct observation and evaluation. The primary financial status criterion are related to the capital structure and the major financial ratios, cross-sectional as well as time series analysis. Besides, cash generation power and the strategic position of the company in the market is taken into account by the credit team as well. Finally, the credit team determines the approved amount of the loan in addition to the required collateral and other formalities to proceed for lending. Parallel to reviewing the creditworthiness of the company and after fulfilling the requirements, the risk management department is responsible to evaluate the extra risk the loan imposes on the bank and determines the appropriate interest rate to be charged for the counterparty corporation. The job is completed through evaluating the required capital reserves concerning the defaults in a single trade and portfolio level in regard with predefined credit limits.

The chart depicts the inner-link of the problem components, the model and the Basel regulatory capital.

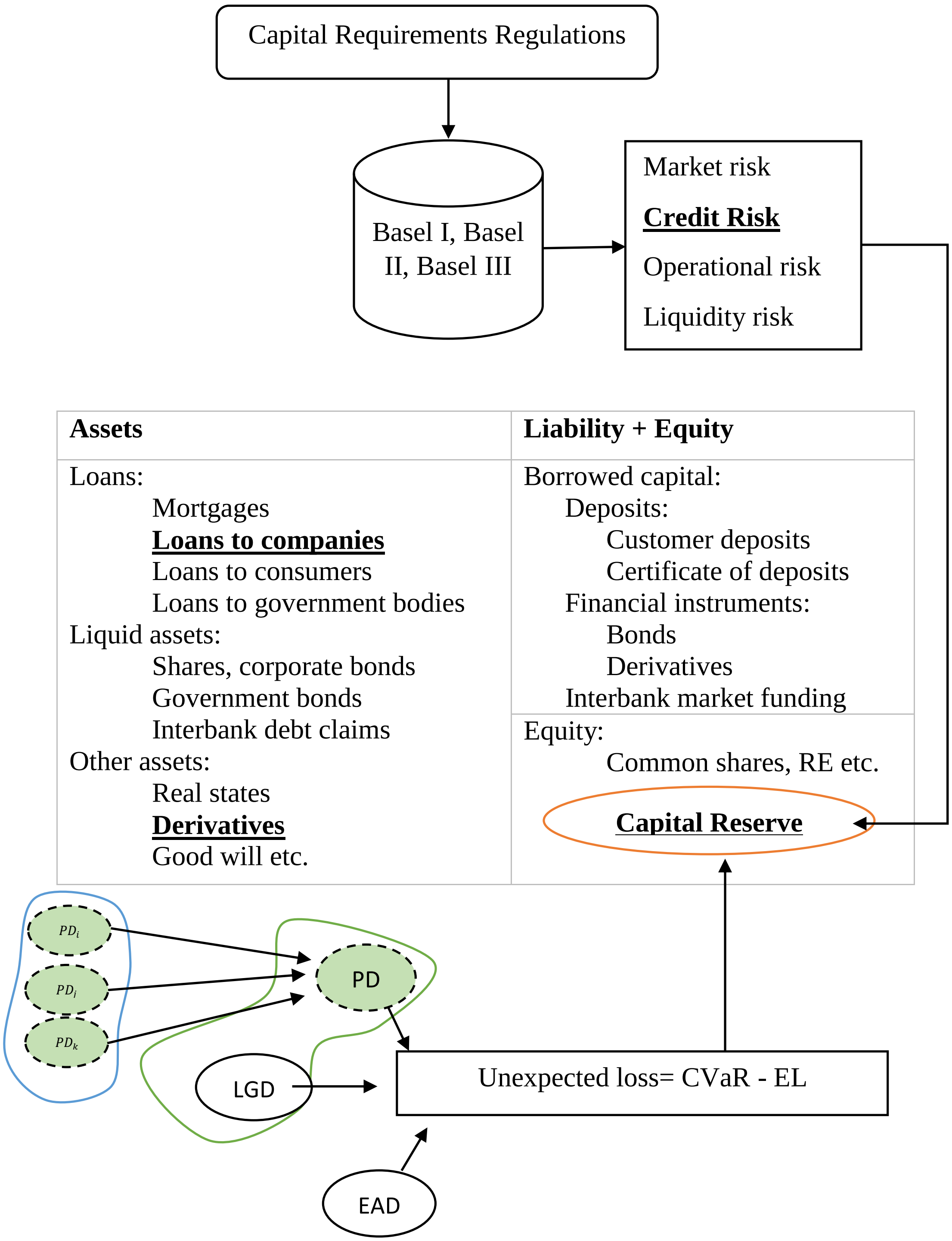

Figure 7: Bank balance sheet, Basel regulatory and model linkage

2.2 Problem formulation

The objective is to devise a generic model for loan portfolio loss distribution and evaluate the corresponding credit-VaR and the proposed economic capital consistent with downturn economy situations. To evaluate the performance of the model with Basel accord, a sample of bank loans is constructed and the required capital generated by the model, the proposed portfolio structure and the management strategy for lending is compared with the one suggested in Basel framework.

2.2.1 Inputs and assumptions

Following table illustrates the main inputs and the associated models for each variable. The main interactions exist between default rates themselves beside LGD and default rates. Moreover, the correlations are also modelled to be dependent on default rates as well.

Row	Inputs	Model	Remark	Basel
1	Probability of Default(PD)	Historical	Rating agencies	Merton
		Merton	-	
		Black-Cox	-	
		Brigo AT1P	-	
2	Loss Given Default	Beta dstr.	Stochastic	deterministic
3	Default correlation	Beta dstr.	J. Lopez study[21]	J. Lopez study
4	$\mathrm{Cor}(\text{default rate}_i,\ \text{default rate}_j)$	t-student copula	Vasicek extension	Gaussian-copula
5	$\mathrm{Cor}(\text{Market},\ \mathrm{LGD}_i)$	Clayton-copula	Historical data	independent
6	$\mathrm{Cor}(\rho_i, \mathrm{M})$	Gaussian copula	stochastic	deterministic
7	Exposure	Pure discount loan	Deterministic/loan principal	Deterministic/ loan principal

Table 4: problem inputs

2.2.1.1 Credit exposure

Literally the future credit exposure at time t is defined as the total positive exposure of the bank at time t if the counterparty corporation defaults assuming zero recovery rate. The current exposure simply is known at time $t = 0$ which is the loan principle in this case. The exposure normally is calculated in trade level and counterparty level for a single client. However, contrary to calculating exposure in derivative contracts that bears a level of complexity, to the nature of a loan, the exposure is nearly deterministic specifically for short term where there is a trivial probability of change or shift in the term structure of interest rates and particularly when the contract is fixed rate. In the bank under study, loans granted are mainly pure discount loans and the principle and corresponding interest accumulate to be paid at maturity, hence it could be treated as a ZCB. Conventionally the Exposure at Default (EAD) is assumed to be loan principal.

2.2.1.2 Probability of default

A default event is an event where the counterparty cannot face its obligations on the payments owed to the bank for a reason. There are several credit events that might lead to default, including bankruptcy when a company become insolvent, failure to pay after a reasonable amount of time after the due date (usually 90 days), significant downgrading of credit rating, and credit event after merger that the new merged entity financially is weaker than the original entity and finally government action or market disruptions typically confiscation of assets or effects of war. These events are often categorized as being driven by either market risk or company-specific risk[22].

There are three methods to extract term-structure of default rates for a risky entity, obtaining historical default information from rating agencies like Moody's, taking structural models like Merton, Black-Cox etc. and finally, taking the implied approach from current market data which resembles getting implied volatility from current market option prices and is considered as the most reliable source of constructing default term-structure[23], since current market information reflects market agreed perception about the evolution of the market in the future and default rates derived may be different from historical default rates.

The primary advantage of using rating agencies information is the ease and accessibility of determining ratings for issuers. However, ratings are not perfect specifically for new structural products that have been prone to severely inaccurate assessment[24]. Moreover, agencies do not have the capacity to constantly monitor and update their ratings in real-time and their assessment often lag behind the market. And the most serious issue is the applicability of such tables in the Iranian market regarding different market structure, country risk, recovery rates etc. Although the meaning of the assigned rate e.g. "B" is standard by definition, however, the default probability is not static and default intensities and spreads definitely change by time; Using diffusion process to describe changes in the value of the firm, Merton 1974 demonstrates it can be modeled based on Black-Scholes option

[22] Credit derivatives, a primer on credit risk, modelling and Instruments, CSMD, page 18
[23] David Li 1999, on default correlation, copula approach
[24] Financial simulation modelling, Keith A. Allman, Josh Laurito, Michael Loh, page 111

pricing technology, and Black-Cox came up with first passage time model, applying barrier option technology.

The alternative frequent method is implicit approach by using observable market information. Two general markets can be used for this purpose, credit default swaps (CDS) and bond market. The banking loan market is the other option, however, the less liquidity and difficulty of obtaining information is the main reason not to be practically appealing. For more frequently traded CDS with different tenures bootstrapping is done to calculate the implied default probability for each year. The process can be completed using bond prices as well. Bonds however have additional layers of complexity due to their variety, fixed/floating differences, optionality, different covenants and different payment schedule all make modelling bonds more difficult than CDS. Hence, probabilities obtained are higher than physical default probabilities encapsulating other risks than merely default. Li 1998 presents one approach to building the credit curve from market information based on Duffie and Singleton 1995 default treatment and obtains a yield spread curve over Treasury. The credit curve construction is then based on this spread yield curve and exogenous assumptions about the recovery rate based on the seniority and the rating of the bonds, and the industry of the corporation. Since there is neither a market for CDS nor for bonds in Iran, matching a comparable company overseas with one in the local market does not seem to be a reliable solution. However, the method is currently used by the bank. By and large, the most applicable method the bank can implement lies in the field of structural models which extract default probabilities from the information available in the financial statements of the counterparty corporation. As far as the bank has access to these data it makes privilege for these category of models to be more appealing in practice. However, the structural models assume a listed company with equity value easily observable from market, unfortunately it is not the case in the Iranian market where most counterparties' stocks are not traded in the exchange market and it demands some valuation analysis to be done in advance.

Brigo 2004 proposed a structural model independent of the current value of company assets which makes it appealing for the case of this study. However, in the framework of Basel, the Vasicek model is applied that is on the ground of Merton insight. To calculate probability of default consistent with Basel, Merton model and Vasicek process is introduced and the detailed technical review of the selected model, AT1P, is presented in Appendix B.

2.2.1.2.1 Merton model

Merton 1974[25] comes up with assumption of liabilities as ZCB, and interprets default if asset values hit ZCB face value at maturity. Despite its simplicity, it is a widely used model in industry. Following graph illustrates the model,

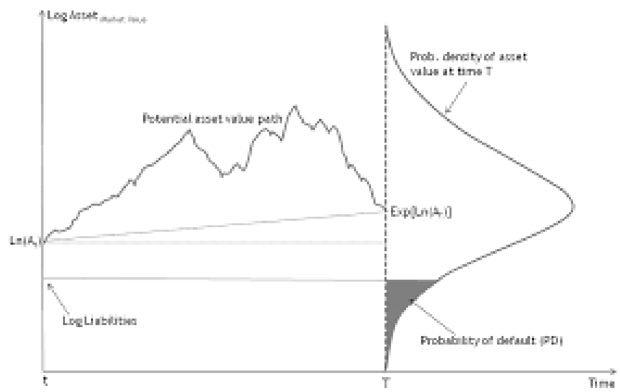

Figure 8: Merton structural model

Referring Black-Scholes options pricing framework, the payoff to creditors is,

$$D(V_T, T) = \min(V_T, D) = D - (D - V_T)^+ \qquad (1)$$

Hence creditors are short a put option written on the assets of the borrowing firm with a strike price of equal B, the face value of debt. Based on put-call parity, equity is a call option on the firm's assets,

$$D(V_t, t) = P(t, T) - Put_{BS}(V_t, D, r, T - t, \sigma) \ \ and$$

$$E(V_t, t) = Call_{BS}(V_t, D, r, T - t, \sigma) \qquad (2)$$

Merton insight suggests that spreads between the credit-risky debt and otherwise identical risk-free debt is simply value of this put option. Based on Merton's insight, the main determinants of credit spread are, maturity of debt, leverage D, and business risk of assets of the firm σ. following the model, the spread over the risk-free rate is obtainable by price of a defaultable ZCB which is,

$$P^{ZCB}_{defaultable} = (1 - PD)\ e^{-yT}\ F \qquad (3)$$ [26]

And credit spread is $s = \gamma - r_f$ while (1- PD) is survival probability. For further discussion please refer to appendix A.

[25] Merton 1974

[26] F is face value, y is yield and PD is probability of default

Practitioners apply the model with some adjustments in their interpretation of data from company's financial statement. For instance, since the credit-VaR conventionally targets one year horizon, the debt in the model is taken as the short term liabilities with half of the long term debt. Moreover, if there is any interim cash out flow like interest or dividend within a year, they will be accrued to the year end and added to the short term due debt. Besides, in case of covenants, this barrier could be defined consistent with the safety covenant. As another issue, to obtain assets value and volatility, the equality of,

$$\sigma_E E_0 = N(d1)\sigma_A V_0 \tag{4}$$

does hold only in small instances of time. This makes the formula practically unattractive. One approach to get asset volatility is to use an iterating method by running the model for a specific period of time on historical data of e.g. one year and then extract V_t for each time interval and finally calculate the standard deviation of the assets value in the period[27]. Apparently, the last V_t will be V_0 for the problem. The alternative method relies on calibration to CDS market data by matching survival probabilities from the model to the market prices of CDS. Although both solutions are challenging in the Iranian market to the reasons mentioned above, however, estimating the volatility from a comparable traded company in the corresponding local industry through iteration method is more reliable than calibrating to CDS data overseas.

2.2.1.2.2 Vasicek model

The Vasicek model basically comes from Merton, however, the difference is that in Vasicek model, instead of taking liabilities of a firm to get probability of default (PD), default probability is given and the debt level is inferred from the PD. To derive the probability of default for a firm taking into account systematic risk, from the Merton model asset values should follow,

$$dV_t = (\mu_t - k_{t^{28}})V_t dt + \sigma_V V_t dS_t + \beta_V V_t dB_t \tag{5}$$

Where σ_V is sensitivity of assets value to systematic risk and β_V is the sensitivity to the idiosyncratic risk. Moreover, dS_t is a Wiener process associated with systematic risk and follows $N(0, dt)$ and dB_t follows a Wiener process of $N(0, dt)$ associated with idiosyncratic risk. In this

[27] MODELING DEFAULT RISK, Crosbie and Bohn (2002), Moody's KMV
[28] Payout ratio

model σ_V, β_V and μ_t assumed to be constant and dB_t , dS_t independent Wiener processes. The value of assets at time t is,

$$V_t = V_0 \exp\left\{\mu_V t - \frac{t}{2}(\sigma_V^2 + \beta_V^2) + \begin{bmatrix} \sigma_V & \beta_V \end{bmatrix} \begin{bmatrix} dS_t \\ dB_t \end{bmatrix}\right\} \tag{6}$$

Taking time horizon of one year, $T = 1$, then B_1 *and* S_1 follow $N(0,1)$ distribution and above equation simplifies to,

$$V_1 = V_0 \exp\left\{\mu_V - \frac{1}{2}(\sigma_V^2 + \beta_V^2) + \begin{bmatrix} \sigma_V & \beta_V \end{bmatrix} \begin{bmatrix} S_1 \\ B_1 \end{bmatrix}\right\} \tag{7}$$

To model default event, a binary random variable D introduced as follow. Here $D = 1$ means default has occurred. It says, after a period of one year value of assets fell below debt level.

$$D = \begin{cases} 1, & with\ probability \quad PD \\ 0, & with\ probability \quad 1 - PD \end{cases} \tag{8}$$

Thus, PD could be expressed as,

$$PD = P\left(V_0 \exp\left\{\mu_V - \frac{1}{2}(\sigma_V^2 + \beta_V^2) + \begin{bmatrix} \sigma_V \beta_V \end{bmatrix} \begin{bmatrix} S_1 \\ B_1 \end{bmatrix}\right\} < L\right)$$

$$PD = P\left(\mu_V - \frac{1}{2}(\sigma_V^2 + \beta_V^2) + \begin{bmatrix} \sigma_V & \beta_V \end{bmatrix} \begin{bmatrix} S_1 \\ B_1 \end{bmatrix} < \ln\frac{L}{V_0}\right)$$

$$PD = P\left(\begin{bmatrix} \sigma_V & \beta_V \end{bmatrix} \begin{bmatrix} S_1 \\ B_1 \end{bmatrix} < \ln\frac{L}{V_0} - \mu_V + \frac{1}{2}(\sigma_V^2 + \beta_V^2)\right) \tag{9}$$

Since B_1 *and* S_1 follow $N(0,1)$ and are independent, then $\sigma_V S_1$, $\beta_V B_1$ are $N(0, \sigma_V{}^2)$ *and* $N(0, \beta_V{}^2)$ respectively. Hence $\beta_V B_1 + \sigma_V S_1$ is easily $N(0, \sigma_V{}^2 + \beta_V{}^2)$ distributed**.** So, by standardizing the random variable,

$$PD = P\left(\frac{\sigma_V S_1 + \beta_V B_1}{\sqrt{\sigma_V^2 + \beta_V^2}} < \frac{\ln\frac{L}{V_0} - \mu_V + \frac{1}{2}(\sigma_V^2 + \beta_V^2)}{\sqrt{\sigma_V^2 + \beta_V^2}}\right)$$

$$PD = \Phi\left(\frac{\ln\frac{L}{V_0} - \mu_V + \frac{1}{2}(\sigma_V^2 + \beta_V^2)}{\sqrt{\sigma_V^2 + \beta_V^2}}\right) \tag{10}$$

Now $\rho_V = \frac{\sigma_V{}^2}{\sigma_V{}^2 + \beta_V{}^2}$ is defined as the proportion of systematic risk, hence from above equation there is,

$$\frac{\sigma_V\,S_1 + \beta_V\,B_1}{\sqrt{\sigma_V^2 + \beta_V^2}} = \frac{\sigma_V\,S_1}{\sqrt{\sigma_V^2 + \beta_V^2}} + \frac{\beta_V\,B_1}{\sqrt{\sigma_V^2 + \beta_V^2}},$$

$$\frac{\beta_V^2}{\sigma_V^2 + \beta_V^2} = 1 - \rho_V,$$

$$\frac{\sigma_V\,S_1 + \beta_V\,B_1}{\sqrt{\sigma_V^2 + \beta_V^2}} = \sqrt{\rho_V}\,S_1 + \sqrt{1-\rho_V}\,B_1. \tag{11}$$

And probability of default can be rewritten as follow,

$$PD = P\left(\sqrt{\rho_V}\,S_1 + \sqrt{1-\rho_V}\,B_1 < \frac{\ln\frac{L}{V_0} - \mu_V + \frac{1}{2}\,(\sigma_V^2 + \beta_V^2)}{\sqrt{\sigma_V^2 + \beta_V^2}}\right) \tag{12}$$

Which is equivalent to,

$$\Phi^{-1}(PD) = \left(\frac{\ln\frac{L}{V_0} - \mu_V + \frac{1}{2}\,(\sigma_V^2 + \beta_V^2)}{\sqrt{\sigma_V^2 + \beta_V^2}}\right). \tag{13}$$

Hence, D gets value of 0 *or* 1 if,

$$\begin{cases} 0\,if\,\sqrt{\rho_V}\,S_1 + \sqrt{1-\rho_V}\,B_1 > \Phi^{-1}(PD) \\ 1\,if\,\sqrt{\rho_V}\,S_1 + \sqrt{1-\rho_V}\,B_1 \le \Phi^{-1}(PD) \end{cases} \tag{14}$$

And if an estimation of probability of default of the firm is available given the systematic factor, the conditional probability of default is,

$$P(D=1|S_1=y) = P\left(\sqrt{\rho_V}\,S_1 + \sqrt{1-\rho_V}\,B_1 \le \Phi^{-1}(PD)\,|\,S_1 = y\right),$$

$$P(D=1|S_1=y) = P\left(\sqrt{\rho_V}\,y + \sqrt{1-\rho_V}\,B_1 \le \Phi^{-1}(PD)\right),$$

$$P(D=1|S_1=y) = P\left(B_1 \le \frac{\Phi^{-1}(PD) - \sqrt{\rho_V}\,y}{\sqrt{1-\rho_V}}\right),$$

$$P(D=1|S_1=y) = P\left(B_1 \le \frac{\Phi^{-1}(PD) - \sqrt{\rho_V}\,y}{\sqrt{1-\rho_V}}\right). \tag{15}$$

And since B_1 follows $N(0{,}1)$,

$$P(D=1|S_1=-y)=\Phi\left(\frac{\Phi^{-1}(PD)+\sqrt{\rho_V}\,y}{\sqrt{1-\rho_V}}\right) \tag{16}$$

This is called conditional probability of default of the firm (CPD). In order to account for tail dependence, here the Vasicek model is extended to account for tail dependence in default rates and default correlations as follow.

2.2.1.3 Default correlation

There is general agreement that the state of the economy in a country has a direct impact on observed default rates. A report by Standard and Poor's stated that, "*a healthy economy in 1996 contributed to a significant decline in the total number of corporate defaults. Compared to 1995, defaults were reduced by one-half …*"[29] another report by Moody's Investors Service[30] stated that "*the sources of default rate volatility are many, but macroeconomic trends are certainly the most influential factors*".

The default correlation of two risky entities can be defined with respect to their survival time (or time-to default) T_A *and* T_B,

$$\rho_{AB} = \frac{Cov\,(T_A, T_B)}{\sqrt{Var(T_A)\,Var(T_B)}} \tag{17}$$

When studying the expected loss in a multi-name loan portfolio, the objective is to extract loss distribution. There are different methods to correlate default likelihood of two entities with each other such as correlating the stochastic processes of assets with each other by Heston 1993 method. Heston applied the method to negatively correlate stochastic stock returns and stochastic volatility. The defaults correlation is introduced by correlating the two Brownian motions dz_1 *and* dz_2. The instantaneous correlation between the Brownian motions is

$$Corr[dz_1(t), dz_2(t)] = \rho\, dt \tag{18}$$

The Heston correlation approach is a dynamic versatile, and mathematically rigorous correlation model. It allows to positively or negatively correlate stochastic processes and permits dynamic correlation modeling since $dz(t)$ is a function of time. Thus, it is an integral part of

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[29] Standard and Poor's rating performance 1996, February 1997

[30] Moody's Investors Service, corporate Bond defaults and default rates, January 1996

correlation modelling in finance[31]. Moreover, when applying reduced-form models with stochastic hazard rates, one can correlate the stochastic process of default intensities and generate Heston model correlated default probabilities as well. The alternative way is using copulas to obtain the jointly distribution of risky entities with desired correlation. The copula functions allow the joining of multiple uni-variate distributions to a single multivariate distribution. Numerous types of copula functions exist and among the most popular are Gaussian, t-Student from elliptical, and Clayton and Gumble from Archimedean families.

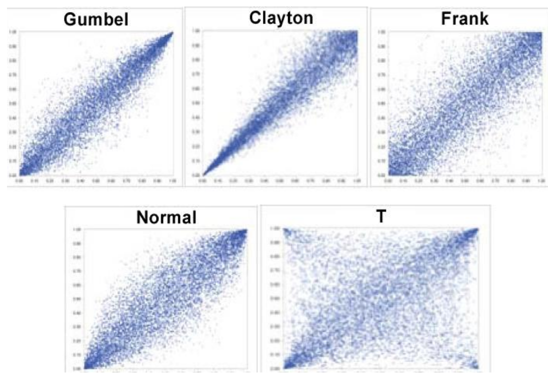

Figure 9: Copula models, source,
http://www.assetinsights.net/Glossary/G_Clayton_Copula.html

Following the above equation for correlation is cumbersome since it prompts to define $\binom{N}{2}$ pairs of correlation if there are N counterparty corporations. Moreover, to incorporate the systematic risk of default which usually happens in recession and economy downturn, taking the pair correlations is not enough. Basel II puts the ground work for capital adequacy on Vasicek Model to account for default correlations and Credit-VaR calculation. Vasicek 1987 proposed one-factor Gaussian copula which correlates default probabilities via asset-value. He assumes,

$$X_i = \rho_i M + \sqrt{1 - \rho_i^2}\ U_i \tag{19}$$

where M and U_i follow Wiener processes and by construction, the Wiener process of X_i has a common factor M and idiosyncratic factor U_i. To the one year horizon of the credit analysis, the Wiener process M, U_i and X_i with distribution of $N(0, dt)$ transform into standard Normal variables. The ρ_i is random (but determined in each period) weights between common factor and U_i while $U_i s$ are independent from each other and M as well. M can be modelled as a factor that defines defaulting environment. When M is low, there is a tendency for $X_i s$ to be low and the rate at which default occur

[31] Correlation risk modeling and management, by Gunter Miessner, 2013

is relatively high and the reverse is true when M is high. One possible proxy for M is a variable modelling evolution of a well-diversified stock index such as Tehran Exchange Index, TEPIX. Hence, in the one-factor copula framework, instead of defining $\binom{N}{2}$, the entities are correlated implicitly. The binary default variable is defined as,

$$\begin{aligned} D_i = 1 \quad & if \quad X_i \leq H_i^{*}{}^{32} \quad : \quad Default \\ D_i = 0 \quad & if \quad X_i > H_i^* \quad : \quad No\ default \end{aligned} \tag{20}$$

And the joint default probability is,

$$PD_{ij} = Prob(X_i \leq H_i^*, X_j \leq H_i^*) \tag{21}$$

Moreover, the correlation between assets of company i and j is formulated as,

$$\rho_{ij}^{asset} = \frac{cov(\rho_i M + \sqrt{(1-\rho_i{}^2)}\, U_i, \rho_j M + \sqrt{(1-\rho_j{}^2)}\, U_j}{\sigma(X_i)\, \sigma(X_j)} = \frac{Cov(\rho_i M, \rho_j M)}{dt} = \frac{\rho_i \rho_j Var(M)}{1\times1} = \rho_i \rho_j \tag{22}$$

The parameter ρ_i defines how sensitive is the probability of default of company i to the common factor. The higher ρ_i, the more the company i is influenced by the common factor M. Consequently, the joint probability of default is,

$$PD_{ij} = \Phi_2(H_i^*, H_j^*, \rho_{ij}^{asset}) \tag{23}$$

And Φ_2 is the bivariate accumulative Normal distribution and defaults are correlated by a Gaussian copula.

The Gaussian-copula was seriously blamed as of the fundamental causes of global financial crisis due to underestimating default correlations in such situations. It goes back to the nature of bivariate Normal distribution which cannot get tail dependence[33] at any value for correlation parameter, while there is such behavior in economy downturns called negative-negative tail dependence when companies tendency to default increase all to gather. Contrary to Gaussian-copula, t-Student[34] copula satisfies the tail dependence equation and it is more desirable for financial crisis

[32] H_i^* is default threshold (please refer to appendix A for further discussion)

[33] A bivariate copula has tail dependence if $\lim_{y_1 \downarrow 0, y_2 \downarrow 0} Prob[(\tau_1 < N^{-1}(y_1) | (\tau_1 < N^{-1}(y_1)] > 0$, τ_i is default times and y_i is cumulative distribution of τ_i.

[34] Student's t-distribution with df degrees of freedom can be defined as the distribution of the random variable T with $T = \frac{Z}{\sqrt{\frac{Y}{df}}}$ where Z is a standard normal with expected value 0 and variance 1; Y has a chi-squared distribution with df degrees of freedom and Z and V are independent.

modeling. The following graphs compare standard Normal and t-student distribution with various degree of freedom. Heavier tails in t-distribution is observable.

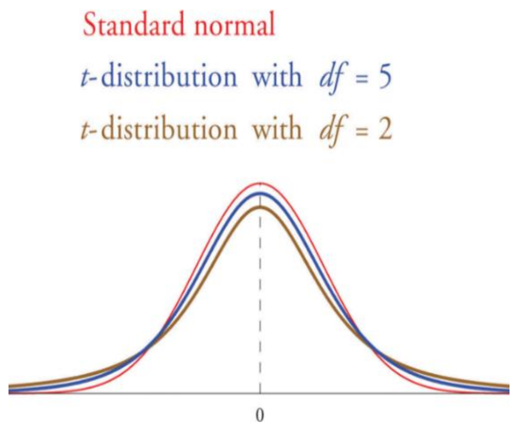

Figure 10: Standard Normal vs. t-student tails

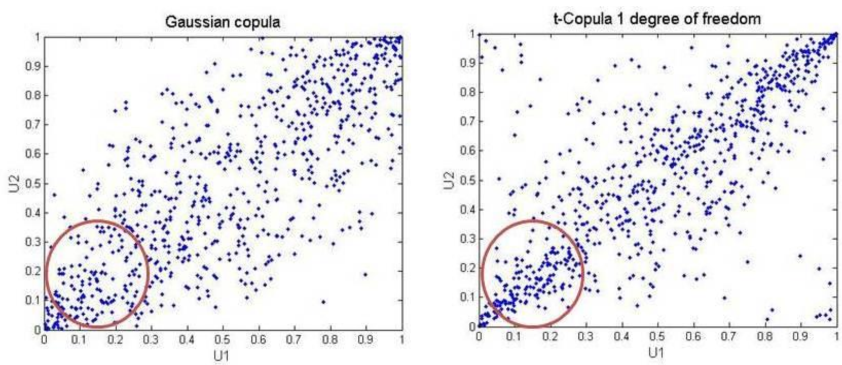

Figure 11: Gaussian Copula vs. t-Copula with *df*=1, source:
http://www.assetinsights.net/Glossary/G_Clayton_Copula.html

In the Vasicek model, in addition to its weakness in modeling tail dependence, assumptions go further of not only assuming constant and the same pairwise correlation among all entities, but also takes the same probability of default for all entities in the portfolio. To incorporate the tail dependence, here t-Student one-factor copula is used as a chosen preferred alternative. A multivariate t-Student distribution with df degree of freedom obtains when multivariate standard normal variables X_i are divided by Chi-squared variable Y with df degree of freedom.

$$t_i = \frac{dX_i}{\sqrt{\frac{Y}{df}}}, \qquad dX_i \sim N(0,1), \quad Y_i \sim \chi^2(df) \tag{24}$$

To implement the model, each X_i is determined according to one-factor model in equation 24, and then divided by $\sqrt{\frac{Y}{df}}$ to get t-student asset value variable. For small df it can dramatically increase default correlations. Default occurs once the assets (here t_i) falls below a threshold, and for instance, in case of $\frac{Y}{df}$ smaller than one, since each X_i is divided by the same $\sqrt{\frac{Y}{df}}$ this makes the assets value of all counterparties more extreme and thus increases the probability of observing more defaults. Besides, the new default thresholds transform from $\Phi^{-1}(PD_i)$ which is standard Normal cumulative invers to the t-Student inverse with df degree of freedom[35].

Moreover, contrary to Basel where correlation assumed to be the same between all pairs, the sensitivity to common factor, ρ, is modeled to be different for each entity. Hull-White 2004 suggest the correlation of equity returns of the counterparty to the market return as a proxy for ρ. This allows not only a specific correlation for each entity but also the pair correlation is the product of of $\rho_i \rho_j$. However, here the correlations are not treated as exogenous variables but come from the relationship between default rates and correlations studied by Lopez 2004.

$$\rho = 0.12 \frac{1 - \exp(-50 \times \text{PD})}{1 - \exp(-50)} + 0.24 \left[1 - \frac{1 - \exp(-50 \times \text{PD})}{1 - \exp(-50)}\right] \tag{25}$$

Hull approximated the expression with high level accuracy through,

$$\rho = 0.12 \left(1 + e^{-50 \times \text{PD}}\right) \tag{26}$$

To calculate the probability of default and default correlation the corresponding input data shall be collected,

Inputs to the model	**Source to collect**
Current market value of equity (E_0)	Current share price × total No. of shares or Equity valuation methods (e.g. residual Income)
Current market value of assets (V_0)	Iteration method (KMV)
Assets volatility (σ_A)	Comparable traded company analysis via KMV method
Default barrier[36]	Conventionally is $(STL + \frac{1}{2} \times LTL)$, covenant

[35] The code in R is : $\text{d}_\text{i} \ll \text{qt}(\text{PD}_\text{i}, \text{df})$ and $\text{rchisq}(\text{n}, \text{df})$ to generate n Chi − squared random variables in R
[36] In case of Merton model, the default barriers is the ST liabilities plus all interim cash flows (interests, dividends etc.) accrued to the end of the year.

Payout Ratio (if any)	Financial statements
LGD	Calibrated $Beta\ distrcibution$ to historical database or industry statistics
Maturity	Given from loan profile (here assumed 1year)
Assets growth rare	CAPM
Common factor sensitivity	$\rho_i = Cor(r_E, r_M) = \frac{Cov(r_E, r_M)}{\sigma(r_E)\,\sigma(r_M)}$ or from Lopez (2004)
Table 5: PD and asset correlation inputs and sources to collect	

Based on empirical evidence, asset correlations are stochastic and tend to increase when default rates are high. Servigny and Renault 2002 find that the correlations are higher in recession than in expansion periods. Similar results obtained by Das, Freed, Geng and Kapadia2004. Likewise, Ang and Chen 2002 find that the correlation between equity returns is higher during the market downturn. Hull-White 2010 suggests a Beta distribution for the correlation parameter in order to test the impact of stochastic correlation. The Beta distribution is the same for all i and the dependence is modeled by Gaussian copula through taking a sample from variable A that is standard Normal and correlated[37] with M, then ρ_i is set equal the same quintile of beta distribution in which the A comes from standard Normal. Hence, in case of economic downturn, M falls down and A will be very low as well, this associates with generating a higher sensitivity factor by construction drawn from Beta distribution. A negative correlation between M and ρ_i corresponds to a positive correlation between default rates and correlation

2.2.1.4 Loss given default (LGD)

A model for LGD (one minus recovery rate) should be able to capture general characteristics described in empirical studies and the idiosyncratic features of the specific debt in the bank. According to the historical recovery data distribution, the lower LGD rates are more likely than the higher rates and historically LGDs change by business cycles which implies they are contingent on the overall status of the economy as well.

[37] In Hull-White this correlation is set $-\sqrt{0.5}$ and it is the case here as well

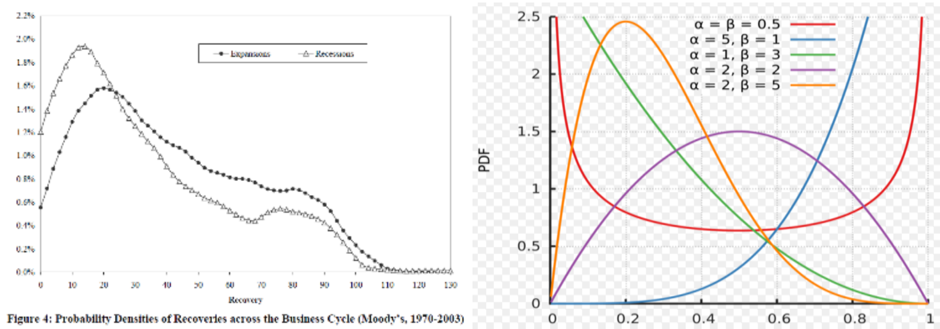

Figure 12: Beta distribution

Furthermore, knowing the industry of a company gives guidelines about the recovery ratios as well, for example, large industrial or consumer goods with lots of fixed assets to support the debt often have higher recoveries. On the other hand banks and financial institutions are assumed to have lower recoveries since they often are taken over by governments that insure depositors and policy holders to the detriment of creditors, Moreover, companies in the same industries usually have similar capital structure which can give guidance on what recoveries can be expected. Archarya, Bharath and Srinivasan 2003 found that when industries are in distress, mean LGD is on average 10% to 20% higher than otherwise.

Industry	Avg. Recovery (cents on dollar)	Industry	Avg. Recovery (cents on dollar)
Utilities	74	High Technology / Office Equipment	47
Insurance & Real Estate	37	Aerospace / Auto / Capital Goods	52
Telecommunications	53	Forest, Building Products / Homebuilders	54
Transportation	39	Consumer / Service	47
Financial Institutions	59	Leisure Time / Media	52
Healthcare / Chemicals	56	Energy & Natural Resources	60

Table 6: industry impact on recoveries, Archarya 2003

Geise 2005 suggests a conditional beta distribution to model loss given default. Although the recovery distribution domain goes beyond one in the historical density graph, however, it is a rare case which takes place solely for bonds. To make the model consistent with the empirical facts, LGD should be conditional on the common market factor, like the market index in case of default rates, and also captures each industry characteristics with regard to seniority and so on. To do so, the parameters of the Beta distribution should be calibrated to the industry/seniority statistics and then the newly calibrated model shall be applied in the copula for each specific counterparty corporation. To the results presented by Peraudin 2002, correlation between recoveries and aggregate defaults rates for the US are -20% on average and about -30% when considering only tails. Therefore, in order to model the negative-negative tail dependence of LGD and defaults, the Clayton copula is used to generate bivariate random variables with desired correlation. The copula will output standard Normal and the calibrated Beta random variables as marginal distributions with required correlation structure. Since the asset return is contingent on the common market factor, the LGD will be indirectly correlated to the systematic factor as well. For example, in case of weak market or recession, e.g. $M = -2$, the associated assets return will be very low by construction and the Clayton copula will generate a correlated (very low) LGD subsequently. The detail process is as described for assets correlation, but, here for recoveries Clayton copula is applied. Similarly, $A_i = \sqrt{0.5}\,M + \sqrt{0.5}\,U_i$ is the random variable correlated with M with an arbitrary weight of $\sqrt{0.5}$ the equally distributes weights between M and the idiosyncratic factor. Mapping A_i on its CDF generates the corresponding quintile, then a random uniform variable is generated $U(0,1)$ for partial derivative of Clayton copula to give v $(marginal\ distribution\ of\ recoveriy\ rate)$. This should be mapped on Beta distribution with the associated quintile. The result will be a random recovery rate from Beta distribution with the desired correlation pattern with market factor. A sample generated by $\alpha = 2$ based on Kendall $\tau = 0.5$[38]; this parameter can be estimated from regression on historical data corresponding to default rates and recovery rates. The sample simulation is as follow generated in R-studio; the LGDs are taken from $Beta(2,8)$.

[38] Empirical correlation

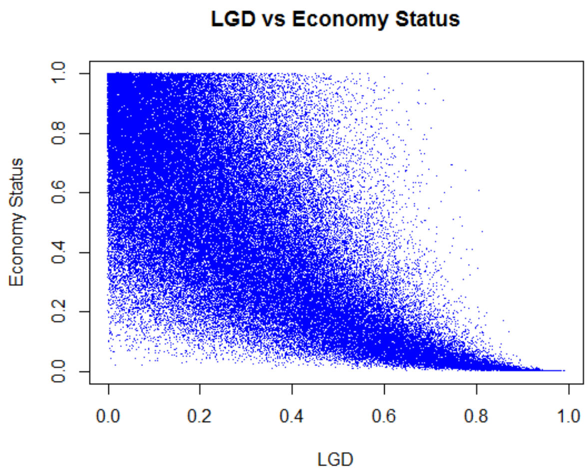

Figure 13: LGD and default rate dependence model by Clayton Copula

The model robustness originates from its consistency with empirical evidences to model negative-negative tail dependence and the calibrated Beta distribution to capture industry and seniority characteristics of the loan. In other words, all derivers of the recovery rates are appropriately modelled in this procedure.

2.2.2 Methodology

The bank has provided a sample of 197 financial statements and the corresponding ratings from counterparty corporations in five sectors including manufacturing, service, domestic trade and international trade (named "trade" in the thesis).

There are N counterparty corporations with assets V_i with $1 \leq i \leq N$. initially, probability of default is calculated based on historical, Merton, Black-Cox or AT1P models. The expected loss for each individual counterparty at time $T = 1$ is,

$$EL_i = LGD_i * EAD_i * D_i(T) \tag{27}$$

each asset i can have one of two states at given horizon, T, it can either be defaulted or not. As an indicator for the assets status, $D_i(T)$ which is binary variable, zero in case of survival and one if default happens. Hence, based on the model, if $X_i \leq H_i^*$ then $D_i(T) = 1$ *and* 0 otherwise, and, the total expected loss of the portfolio EL is,

$$EL(T) = \sum_{i=1}^{N} EL_i \tag{28}$$

An illustrative flow chart of the methodology is as follow,

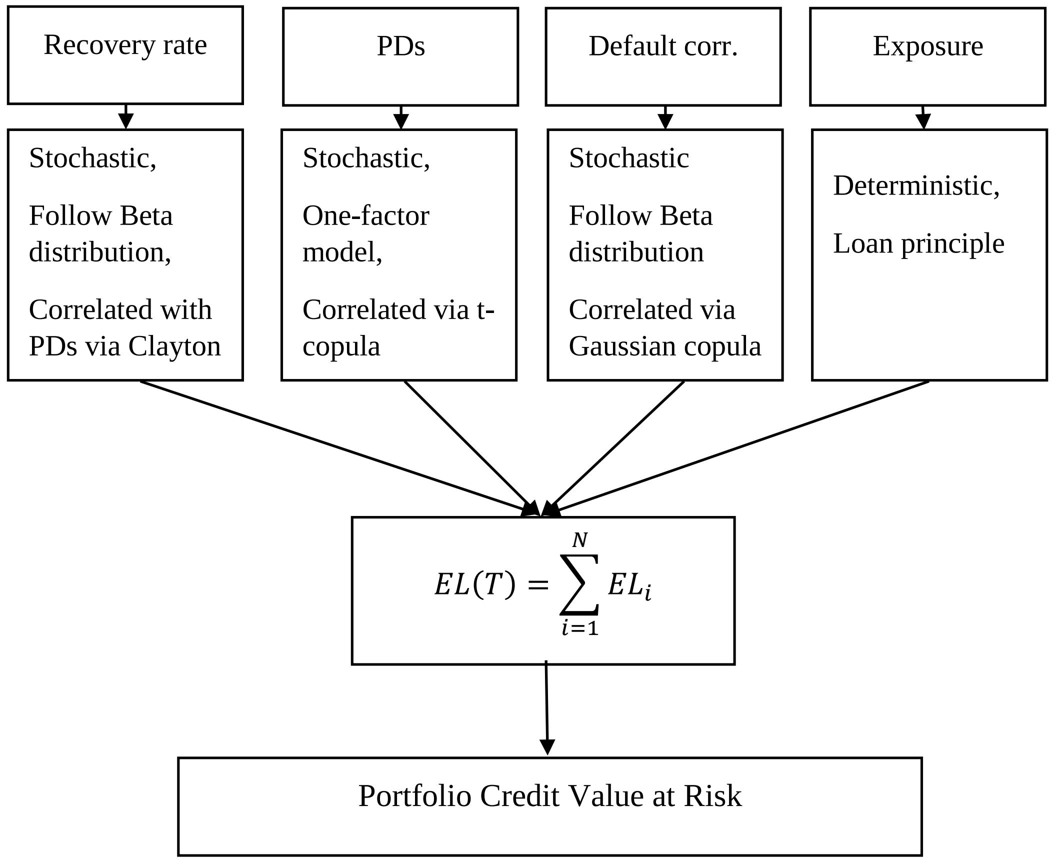

Figure 14: methodology flow chart

And the pseudo code for simulation process is presented in Appendix D. In the following, the Basel methodology for capital adequacy is reviewed with the corresponding assumptions and initial versions of extended models. In the next chapter the proposed model is implemented and outputs compared with the results from Basel.

2.3 Basel Asymptotic Risk Factor Approach (ARFA)

The ARFA approach is used by Basel framework to compute the capital needed to prevent the bank from bankruptcy under a one year period, with probability of more than $q = 0.999$. In the formula PD is probability of default and the same for all exposures, ρ_V is the firm's assets correlation with the systematic common factor and Φ^{-1} is the inverse of standard Normal distribution,

$$C_V = \Phi \left\{ \frac{\Phi^{-1}(PD) + \sqrt{\rho_V}\, \Phi^{-1}(q)}{\sqrt{1-\rho_V}} \right\} \tag{29}$$

To derive the formula Basel takes the following assumptions,

1. The portfolio is sufficiently fined grained so that the idiosyncratic risk is diversified away and only the systematic risk remains and it is the reason that is called a single factor model.
2. Firms' assets are correlated to the systematic risk factor which is $N(0,1)$ distributed.
3. The Loss Given Default is assumed constant and similar for each exposure
4. The loan generates no cash flows

In practice it is quite impossible to find a portfolio with LGDs, PDs, and correlation the same for all exposures. Hence to compute a more accurate 99.9% CVaR per unit of exposure, Gordy 2003, Pykhtin and Dev (2002) suggest,

$$C_p = \sum_{i=1}^{N} (w_i \, LGD_i) \times \Phi \left\{ \frac{\Phi^{-1}(PD_i) + \sqrt{\rho_i}\,\Phi^{-1}(q)}{\sqrt{1-\rho_i}} \right\} \qquad (30)$$

Where index i represents the corresponding PD, LGD and correlation of assets of firm i with the systematic risk and ω_i represents the exposure weight, $\omega_i = \frac{EAD_i}{\sum_{i=1}^{N} EAD_i}$. The formula still follows earlier assumptions in terms of well granularity and single systematic factor model.

Furthermore, Schonbucher (2002a) and Wehrspohn (2002) investigated that in case of one-factor model, by dividing the whole portfolio of credits in fine grained homogeneous portfolio clusters where all assets of the same cluster have the same PD, LGD , EAD, correlation and expiry date, the percentage of capital needed for the whole portfolio with only $q = 1 - \alpha$ of default probability becomes,

$$C_P = \sum_{k=1}^{N} EAD_k \times LGD_k \, \Phi \left\{ \frac{\Phi^{-1}(PD_k) + \sqrt{\rho_k}\,\Phi^{-1}(\alpha)}{\sqrt{1-\rho_k}} \right\} \qquad (31)$$

Accordingly, if a portfolio is constructed of homogenous sub portfolios then the value of regulatory capital to cover the entire portfolio is just the sum of the amounts required to cover each sub-portfolio. Here the index k represents each homogeneous sub-portfolio[39].

[39] This section mostly borrowed from: Lionel Martin , Analysis of IRB correlation coefficient with an application to credit portfolio, 2013, University of Uppsala, 2013

CHAPTER 3: IMPLEMENTATION AND RESULTS

In this part the proposed model is applied on a sample portfolio of loans from the bank. The chapter starts with the managerial issues, and afterwards, Basel Economic Capital is compared with the one suggested by the model. Finally, a mathematical model is proposed for the loan portfolio optimization and, the efficient frontiers plus the proposed portfolio structures in both the frameworks of Basel and the model is compared. The chapter investigates whether model and Basel suggest distinct lending strategy or not.

3.1 Managerial prelude

Study of likelihood of unexpected losses in a portfolio of exposures is fundamentally important for effective risk management. When default losses are modelled, it can be observed that the most frequent loss amount will be much lower than the average, because, occasionally, extremely large losses are suffered, which have the effect of increasing the average loss. Therefore, a credit provision is required as a means of protecting against distributing excess profits during the below average loss years.

To absorb the expected loss of an exposure portfolio the bank should take appropriate pricing methods to offer risk-adjusted rates for the loans granted. However, Economic Capital (EC) is required as a cushion for the risk of unexpected credit default losses in the bank, because the actual level of losses could be significantly higher than the expected loss.

Knowledge of credit default loss distribution arising from a portfolio of exposure provides a bank with management information of the amount of capital that the bank is putting at risk by holding that credit portfolio. Given the necessity of economic capital for unexpected losses, a percentile level provides a means of determining the level of economic capital for a required level of confidence. In order to capture a significant proportion of the tail of the credit default loss distribution, conventionally from the standard, the 99.9th percentile of loss level over a one-year time horizon is a suitable definition for credit risk economic capital[40].

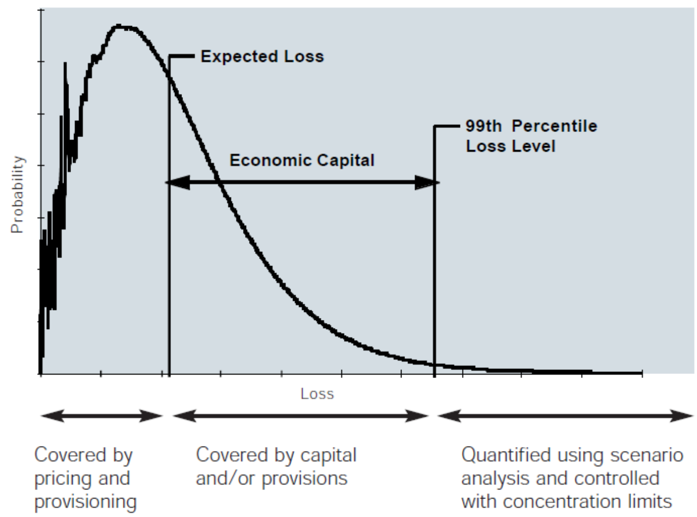

Figure 15: how banks treat their loan portfolio loss

It is possible to control the risk of losses that fall within each of the three parts of the loss distribution in the following ways,

Part of loss distribution	**Control mechanism**
Up to Expected Loss	Adequate pricing and provisioning
Expected Loss—99.9% Percentile Loss	Economic capital and/or provisioning
Greater than 99.9% Percentile Loss	Quantified using scenario analysis and controlled with concentration limits
Table 7: how banks treat loan portfolio loss	

[40] Credit Swiss, Credit Risk+ document

In the latest version of the Basel proposal for an Internal Ratings-Based ("IRB") approach (Basel Committee on Bank Supervision 2001), the bucketing system is required to partition instruments by internal borrower rating; by loan type (e.g., sovereign vs. corporate vs. project finance); by one or more proxies for seniority/collateral type, which determines loss severity in the event of default; and by maturity. More complex systems might further partition instruments by, for example, country and industry of borrower.

3.2 Descriptive analysis of loan portfolio

Loan data (of 197 companies) from bank is presented in Appendix E and in this part a descriptive statistics of data is presented. From the summary statistics of exposures, the minimum exposure of 2 mln belongs to a counterparty from "domestic trade" sector, while the maximum of 362,700 million is from this sector as well. Moreover, the average exposure is around 29,320 (mln) and 50% of the counterparties requested a loan below 10,000 million. The aggregate portfolio exposure is 5,776,872 mln.

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
      2    3000   10000   29320   30000  362700
```

A more illustrative distribution of exposures is depicted in the following pie chart. According to this graph, the major concentration of the bank loan portfolio lies in the "manufacturing" and "domestic trade" sectors, however, "domestic trade" sector dominants by 37% of total exposures and "manufacturing" places in the second rank by 29% of total exposures in the portfolio. "Real estates" 15%, "trade" 13%, and "service" by 6%, are the least exposures respectively.

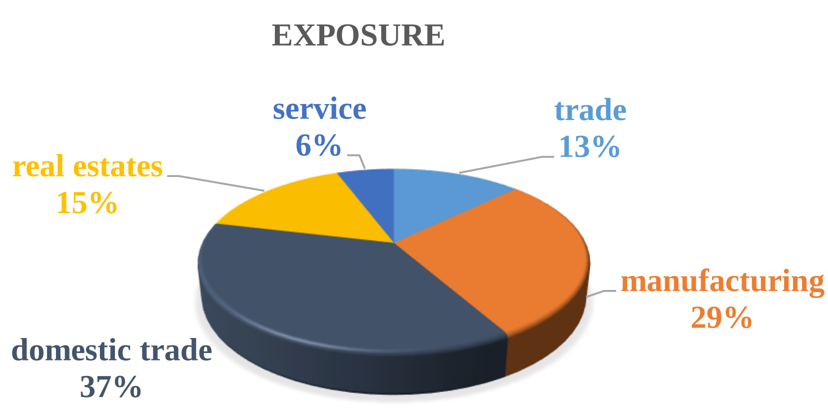

Figure 16: Exposure distribution among sectors

Summary of the default probabilities in the portfolio is as below. Apparently, depending on the credit rate assigned by the bank, PDs vary in a range from .02% to 17.7%.

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.00022 0.01166 0.04546 0.06780 0.17720 0.17720
```

The inner distribution of loan amounts in each sector is magnified in figure 17 and 18. Apparently, in all sectors the exposures are highly right skewed and hardy exceed 100 billion. However, there are some loans beyond 200 billion in “real estates” and “manufacturing” where the bank is more confident of collaterals and fixed assets in the corporation’s balance sheets. This is also the case for “domestic trade” and implicitly reflects banks closer business interaction with corporations in “domestic sector” rather companies active international “trade”.

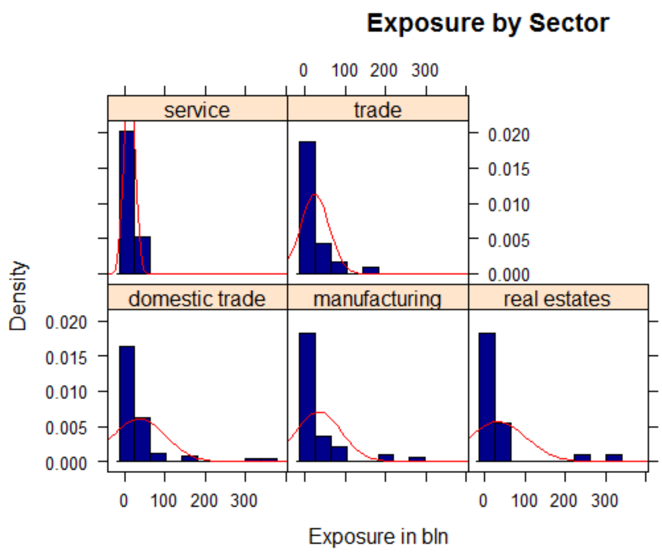

Figure 17: Exposure distribution in each sector

From the pie chart information and the illustrative charts below, bank is likely to grant loan to “trade” and “service” companies and attempts to interact with more credit worthy clients in these sectors. This is obvious from the credit grades distribution by sectors in the subsequent charts (figure 17, 18).

Moreover, in the frequency graph of credit rates by sector, below, bank portfolio chiefly is constructed by loans rated “Baa”, “Ba” and “B”. The charts suggest that distribution of loans with various rates differ in sectors. For instance, in “service” , “domestic trade” and “real estates”, the majority of loans are from rates “B” and “Ba”, “Baa” and “C”, however, in “trade” and “real estates”, grades of type “A” is more observable relatively. Moreover, “service” and “manufacturing” sectors carry the least creditworthy loans of rate “CCC”, but this is not the case for “real estates”.

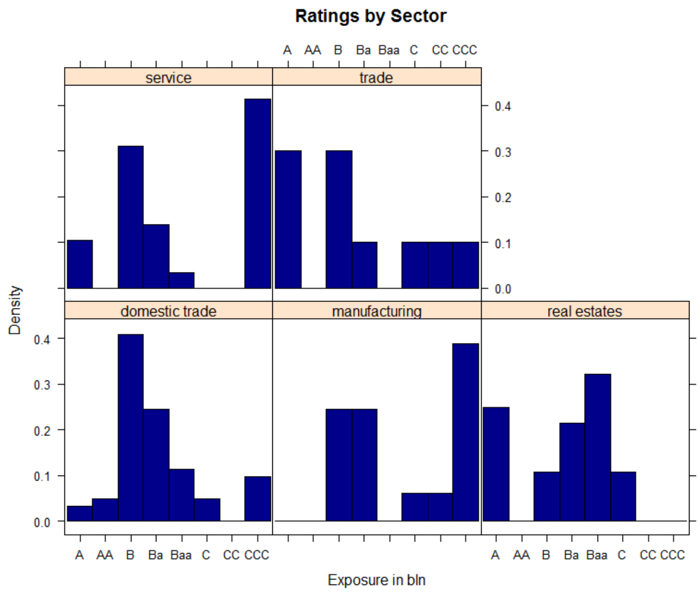

Figure 18: credit rate distribution in each sector

3.3 Model evaluation

In order to get ensured if the model fulfills all the expetations concerning the attributes of variables and their interactions, prior to evaluating the results with Basel a sample of outputs are reviewed. Follwing graphs illustrate the dependecy format of correlations and LGDs to the economy status. Obviously, from the right hand graph, there is a pinch in the down right revealing a higher level of dependency in bad economy situation, between LGD and economy, however, the likehood of co-movement declines as economy is experiencing normal or expansionary conditions. Applying Clayton copula enabled the model to capture such dependency pattern.

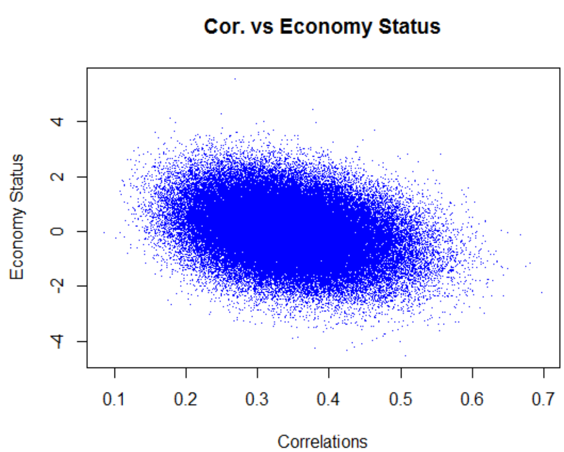

Figure 19: correlations vs. economy status

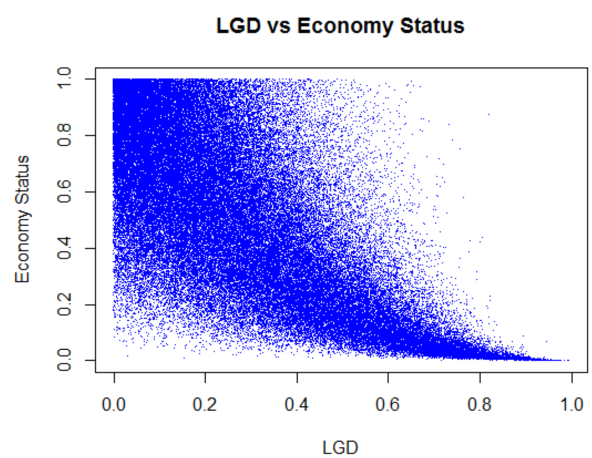

Figure 20: LGD vs. economy status

Moreover, the left scatter plot demonstrates the negative dependency of correlations and economy index which is created via Gaussian copula. Moreover, t-copula modeled tail dependence and intensified the default rates' co-movements in the extreme situations of economy booms and downturns. This is the case particularly in downturns where companies are likely to default together. In Gaussian copula the probability of this co-movement in extreme cases in zero for all range of correlation, however, this is not the case in t-copula.

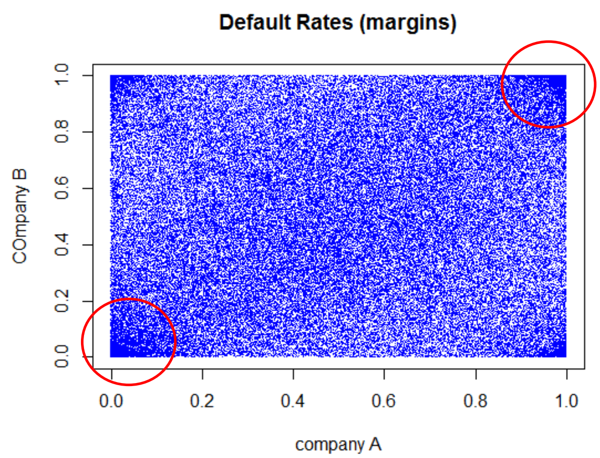

Figure 21: default rate correlation by t-copula

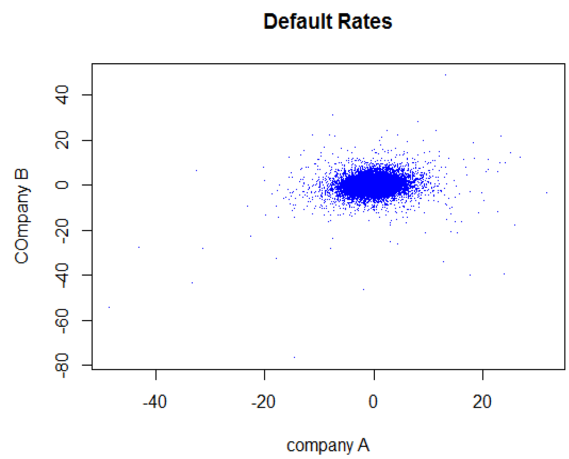

Figure 22: default rate of company A and B

The graph at the left depicts the margin distribution of default rates for two sample companies in the loan portfolio. The pinched areas at the corners is noticable representing higher correlation at extremes. It illustartes that when company A performs very god or bad, it is more likely to observe such behavior from company B, while it is not the case in normal situations. The pinched areas reveal a more intense tendency for co-movement at extremes.

3.4 Model implementation

The simulation of the model[41] on the bank laon portfolio is run for 100,000 times *(degree of freedom=3)* and Creit-VaR for various percentiles are compared with 99.9% percentile suggested by Basel. To make the portfolio consistent with Basel assumptions, the average of default probabilities is tabken as the common PD and the copula correlation is inserted on the ground of J. Lopez (2004) which investigates an empirical relationship between default probabilites and asset correlations.

[41] mybankBasel() in R

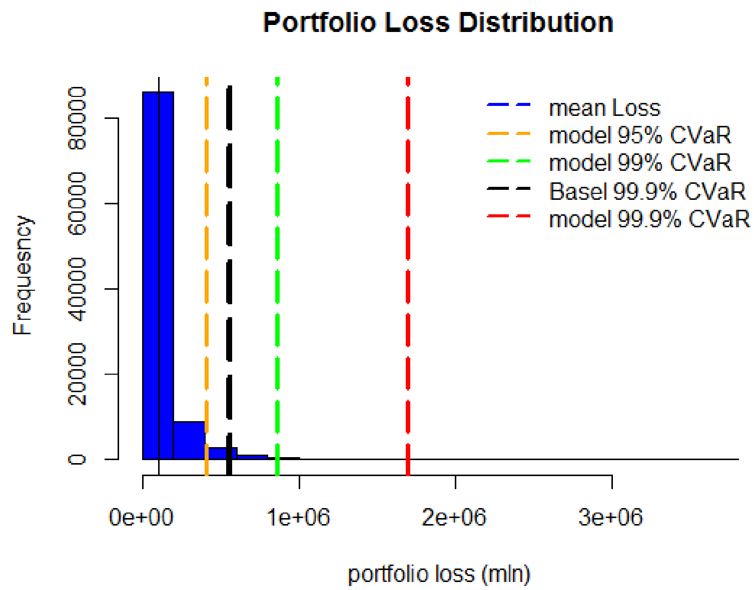

Figure 23: Model vs. Basel CVaR

The expected loss is 104,186 mln and the portfolio loss standard deviation is 179,317 mln. Apparently, the 99.9% CVaR from Basel lies between the model 95% and 99% CVaR. It implies that incorporating realities, such as stochastic recoveries and correlations as well as the interactions like dependency of recoveries and PDs and correlations with economy status, contribute to a higher level of WCDR and economic capital which goes beyond Basel figures.

```
          95%          99%        99.5%        99.9%   Basel_WCDR  Basel 99.9%
4.081149e+05 8.585043e+05 1.096985e+06 1.690528e+06 3.338937e-01 5.558981e+05
```

Moreover, the 99.9% percentile Worst case Default Rate (WCDR) for Basel is 33.38% which is close to the one for 99% percentile from the model. According to the model, the bank is 99.9% confident that the number of defaults in the portfolio will not exceed from 54% of the totol number of loans in the portfolio. In this case, since the portfolio consists of 197 counterparties, hence, with 99.9% confidence the bank will not experience more than 106 defaults within a year. This number is 41, 67 and 77 with 95%, 99% and 99.5% confidence respectively. Also, in Basel, the WCDR is 66 defaults in a year with 99.9% confidence.

```
                                   95%       99%     99.5%     99.9%
Percentile for number of defaults: 0.2131980 0.3451777 0.4060914 0.5431472
```

Comparing the outputs from both Basel and the model revelas that Basel underestimates possible default rates particularly in recessions when there is a tendency known as default contagion. The differnce for 99.9% percentile sometimes is more than three time of the figures Basel suggests.

Furthermore, to the simulation outputs, Basel implicitly account for solely 95% to 99% confidence of reality. It means, by applying Basel management is happy to be confident of his/ her strategies with 99.9% confidence, while incorporating real world attributes of risk factors and interactions in modeling, down grades this confidence to 95%- 99% and the manager has been lured by the unreliable confidence in his/ her strategic lending process.

The following histogram (figure 24) represents the distribution of number of defaults. The expected number of defualt for the portfolio is 13.7, implying approximately the bank will face with 14 defaults on average within the year. The bank is supposed to estoimated this expected loss and consider it in the amount of interests charges for loans granted. The remaining probable defaults shall be hedged against by reserves as economic capital.

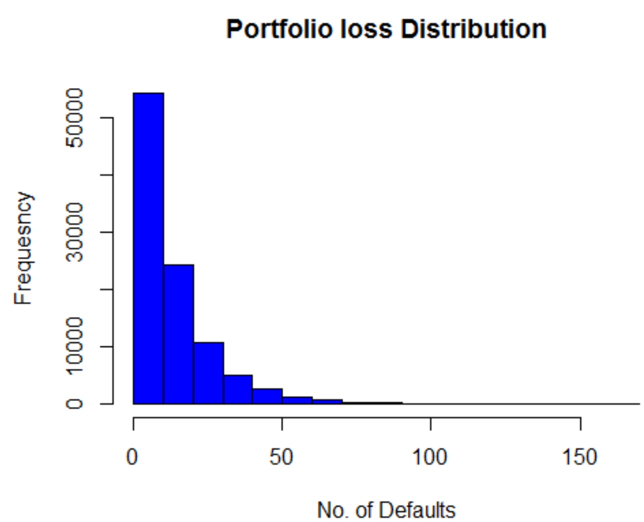

Figure 24: No. of defaults distribution

3.4.1 Expected shortfall (tail loss)

Although it is the standard most commonly applied, value-at-risk is not without shortcomings as a risk measure for defining economic capital. Because it is based on a single quintile of the loss distribution, VaR provides no information on the magnitude of loss incurred in the event that capital is exhausted. A more robust risk-measure is expected shortfall ("ES"), which is (loosely speaking) the expected loss conditional on being in the tail[42].

The following histogram (figure 25) demonstrates the tail distribution of portfolio losses beyond 99.9% percentile. It reveals what is going on at the tail above 99.9% percentile. The

[42] Gordy 2002

distribution is positively skewed showing the likelihood of extreme events is remarkably less comparing to the events close to the VaR at that level. Comparing CVaR of different percentiles with the corresponding expected shortfalls at that level demonstrates higher values for expected shortfalls.

One problem with VAR is that, when used in an attempt to limit the risks taken by a bank, it can lead to undesirable results. When a bank intends that the one-year 99.9% VAR of the loan portfolio must be kept at less than 1.6 million. There is a danger that the bank will construct a portfolio where there is a 99.9% chance that the loss is less than $1.6 million and a 0.1% chance that it is 2 million. The bank is satisfying the risk limits imposed, but is clearly taking unacceptable risks. Where VAR asks the question 'how bad can things get? expected shortfall asks 'if things do get bad, what is our expected loss?'. A risk measure that is used for specifying capital requirements can be thought of as the amount of cash (or capital) that must be added to a position to make its risk acceptable to regulators. Artzner, et al. (1999) have proposed a number of properties that such a risk measure should have such as,

- Monotonicity: if a portfolio has lower returns than another portfolio for every state of the world, its risk measure should be greater.
- Translation invariance: if we add an amount of cash K to a portfolio, its risk measure should go down by K.
- Homogeneity: changing the size of a portfolio by a factor (lambda) while keeping the relative amounts of different items in the portfolio the same should result in the risk measure being multiplied by (lambda).
- Sub-additive: the risk measure for two portfolios after they have been merged should be no greater than the sum of their risk measures before they were merged.

The first three conditions are straightforward given that the risk measure is the amount of cash needed to be added to the portfolio to make its risk acceptable. The fourth condition states that diversification helps reduce risks. When two risks are aggregated, the total of the risk measures corresponding to the risks should either decrease or stay the same. VAR satisfies the first three conditions, but it does not always satisfy the fourth, as will now be illustrated.

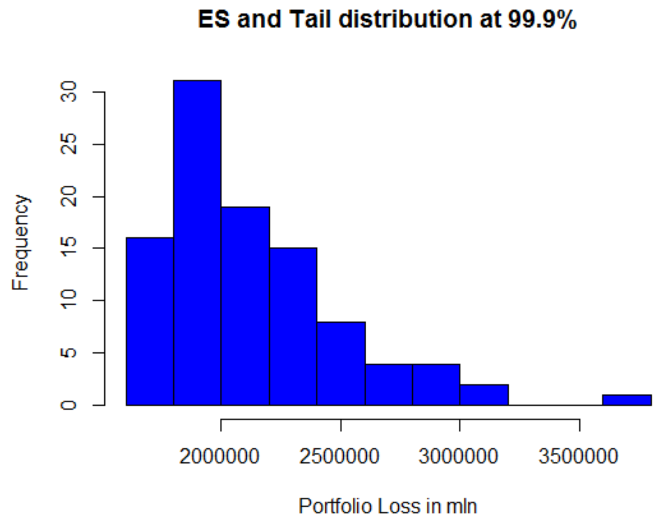

Figure 25: portfolio expected shortfall

Risk measures satisfying all four of the conditions are referred to as coherent. The example illustrates that VAR is not always coherent. It does not satisfy the sub-additivity condition. This is not just a theoretical issue. Risk managers sometimes find that, when the London portfolio is combined with that of New York to form a single portfolio for risk management purposes, the total VAR goes up rather than down. In contrast, it can be shown that the expected shortfall measure is coherent. A risk measure can be characterized by the weights it assigns to quintiles of the loss distribution. VAR gives a 100% weighting to the X^{th} quintile and zero to other quintiles. Expected shortfall gives equal weight to all quintiles greater than the X^{th} quintile and zero weight to all quintiles below the X^{th} quintile[43].

By and large, the bank expected loss beyond a certain percentile is greater than the CVaR at the corresponding confidence level. The figures are depicted as below.

```
Expected shortfall at confidence of:

99.9%: 2,131,408 mln
99.5%: 1,479,650 mln
99%:   1,221,897 mln
95%:     693,484 mln
```

[43] The talk about expected shortfall and VaR comparison borrowed mostly from www.risk.net and Risk management and financial institutions, John Hull

3.4.2 Sector credit risk analysis

This section involves analysis of credit risk for each sector to evaluate the risk contribution of each sector in the portfolio. The distribution of sectors in the portfolio is,

```
domestic trade   manufacturing    real estates         service           trade
            61              49              28              29              30
```

Following table depicts contribution of each sector in the total exposure of the portfolio. Apparently, "domestic trade" ranks top and "manufacturing" holds the second largest exposure. "Real estates", "trade" and "service" seat at the bottom respectively. An exercise of analyzing its

Row	Sector	Exposure (mln)
1	trade	740,000
2	manufacturing	1,670,000
3	domestic trade	2,137,000
4	real estates	900,000
5	service	331,000

Table 8: exposures by sector

portfolio credit risk for the bank is to evaluate the risk contribution of sectors and also the individual counterparties in order to come up with appropriate strategies[44].

Different percentiles of Credit-VaR is presented in the table below. EL is the expected loss of a particular sector. It is noticeable that again Basel 99.9% CVaR lies between 95% and 99% percentile of the model for each sector. Moreover, the largest CVaR percentile belongs to "manufacturing" sector, and "service" occupies the last position in term of CVaR.

Row	Sector	EL	95%	99%	99.5%	99.9%	Basel
1	manufacturing	44,239	174,143	355,262	443,608	688,585	203,298
2	dom. trade	28,803	141,177	346,613	437,949	664,277	171,487
3	real estates	5,749	32,089	58,308	75,069	152,865	55,932
4	services	6,972	31,362	61,850	76,538	110,338	36,909
5	trade	17,202	82,880	159,136	197,693	283,685	71,166

Table 9: CVaR percentile by sector (mln)

[44] The Bank can optimize its portfolio structure of loans or make decisions of best restructuring strategy in crisis

Figure 26 demonstrates a comparative analysis of CVaR percentile among sectors more illustrative. Obviously "manufacturing" and "domestic trade" dominate others while "service" and "trade" have the least amount of CVaR respectively.

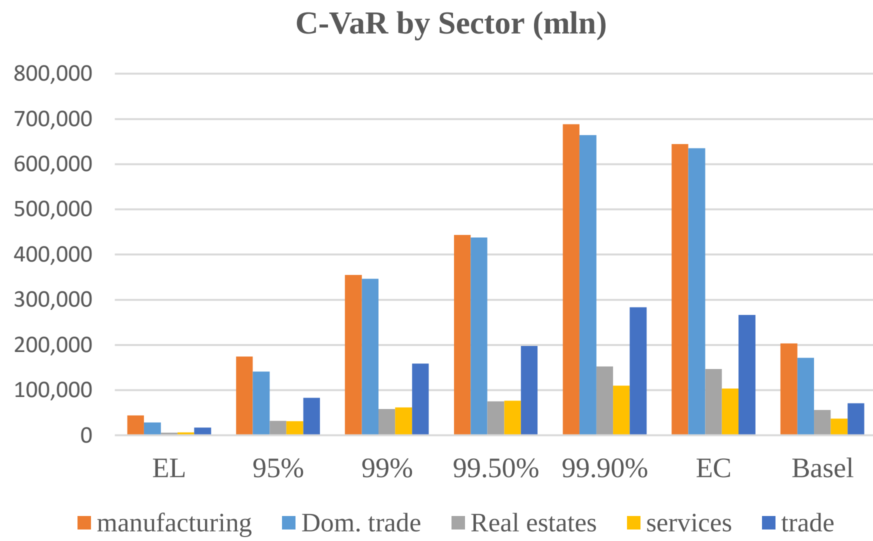

Figure 26: CVaR by sector, Basel vs. model

In order to acquire a sensible judgment of risk contribution of each sector in the portfolio, the corresponding amount of economic capital in each sector is divided by its total exposure. The output is an indicator of relative riskiness of the sector. Results are depicted in the following graph. It suggests, although the total exposure in "manufacturing" and "domestic trade" is quite close to each other, "manufacturing" exposure is almost 80% of "domestic trade" (respectively 1,670 and 2,137 bln) , however, "manufacturing" is much riskier comparing to "domestic trade". The ratio of required capital reserve to the exposure is 39% and 30% for these sectors respectively. This makes "domestic trade" more appealing to the bank with higher possibility of interest income by granting larger loans but imposing a lower amount of economic capital comparing to a sector with similar exposure.

Furthermore, "real estates" appear as the safe haven in the portfolio with the minimum required economic capital reflecting a considerable lower level of Credit-VaR comparing with others. Strategically, in case of two sectors with same level of exposures, the bank prefers to grant higher amount of loans to the one with lower EC contribution. This is the case for "Real Estates" and "Trade" where exposures are relatively close among other pairs in the portfolio, however, "Trade" imposes a risk[45] of above 35% while this is around 16% for "Real Estates". By and large, "domestic trade"

[45] Ratio of corresponding economic capital (EC) to the Exposure is a proxy of riskiness of a loan in portfolio

carries a reasonable combination of exposure and risk among other sectors and "trade" and "service" are the least attractive target for lending.

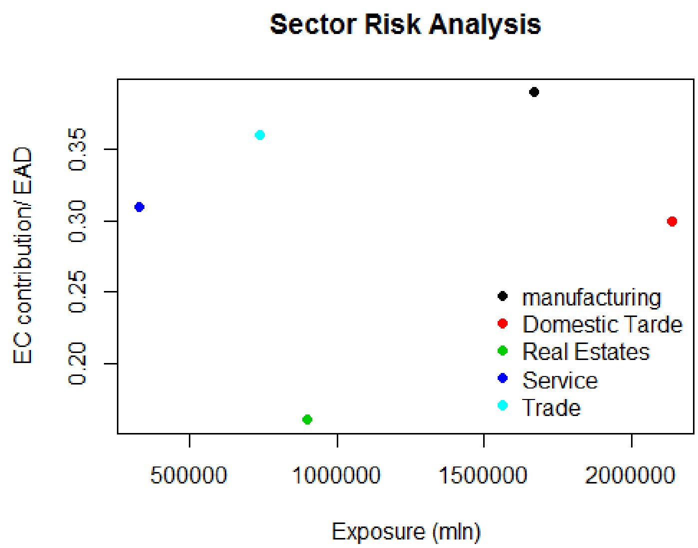

Figure 27: Sector EC contribution

Although Basel generates distinct inter-sector CVaR values, however, it slightly influence the risk order of sectors, in other words, different results does not necessarily lead to an completely different strategy for the bank to manage its portfolio credit risk, whether applying Basel or the model. According to the graph, the order of three first sectors is the same alike the model proposed, however, according to Basel, "service" is considered as the riskiest sector and "trade" ranks 4th comparing to its order suggested by the model. Moreover, contrary to rather similar riskiness order, the level of estimated EC contributions are far different with much lower variety among different sectors.

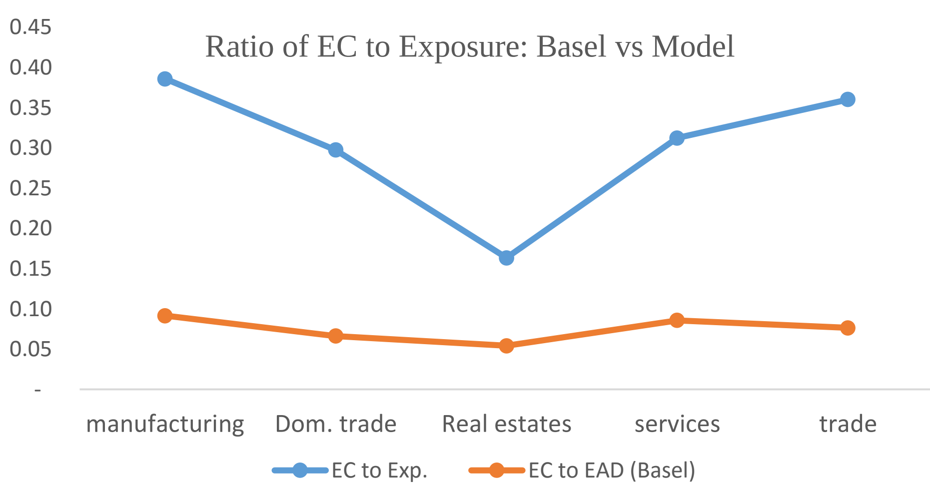

Figure 28: Sector EC contribution, Basel vs. model

Accordingly, Basel makes management somehow indifferent between sectors due to a rather close levels of EC contributions as a measure of riskiness, while the model distinguishes between sectors through allocating a wider range of risk contributions, this makes the manager more sensitive to formulate a more scrutinized lending strategy.

Obviously, banks look for a portfolio which prompts a lower capital reserve in order to boost their flexibility in generating interest income as much as possible from the available capital. Hence, they should be naturally more inclined to higher loan amounts with lower level of required regulatory capital which implies “the southern” and “the eastern” parts of the following graph. Tough, the optimum area of the chart locates in the bottom right with loans of having the maximum exposure and the minimum EC contribution.

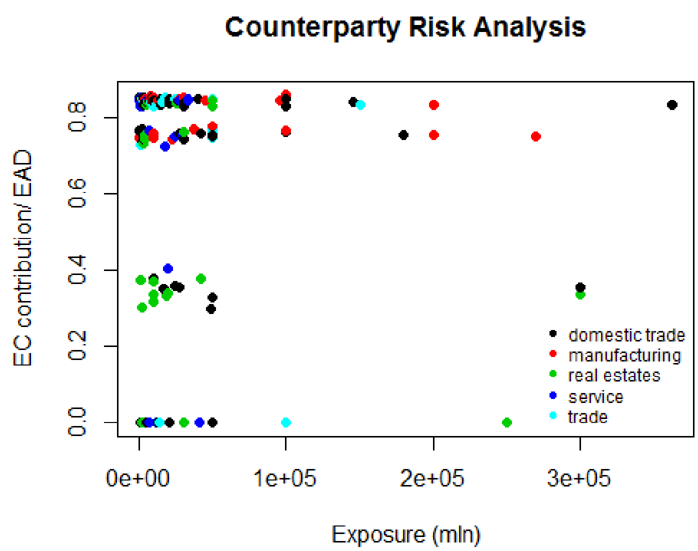

Figure 29: risk contribution at counterparty level

The graph depicts a dense of loans from different sectors at the top left, on the other hand, there are rarely optimum lending, such as the ones to “real estates” and “domestic trade” in southeast. It also reveals existence of loans from “domestic trade” with far large exposures and risk contribution. The bank is recommended to be cautious enough in treating with such borrowers.

3.4.3 Structuring the loan portfolio

In order to evaluate the effect of applying Basel in place of the model, the improved loan portfolio strategy of the bank is modeled in Markowitz framework. The main question is whether Basel has any influence on the construction of optimum portfolio or not and to what extend it impacts the optimum solution comparing to the results generated from the model.

According to the work of Harry Markowitz[46] in the early 1950s each portfolio can be classified along the axes of risk and return. Any portfolio that has a minimal amount of risk for a given amount of return is called efficient, and the line that connects these portfolios in a risk-return graph is called the efficient frontier. In 1993, Terri Gollinger and John Morgan, at the time working with Mellon Bank in Pittsburgh, published the pioneering article "Calculation of an Efficient Frontier for a Commercial Loan Portfolio" in the Journal of Portfolio Management[47]. This article takes Markowitz's portfolio theory to the banking sector and to the allocation and optimization of loan portfolios in particular. Like their approach, here the industry sectors take the place of securities in the Markowitz model, and the risk contribution as the ratio d economic capital to the total exposure is used as a proxy for risk. This ratio represents how risky is the company in a way that higher EC implies riskier loan. Just as an investor searches for an optimal combination of risk and return in creating a portfolio of securities, a bank extends loans to those industries that minimize risk (EC contribution) for a given level of return.

Spreads on loans are taken as returns for securities. According to the hazard rate model, having the PDs (and conclusively the average hazard rates) and recoveries in hand, the spreads are available from the triangle credit expression below,

$$\bar{h} = \frac{spread}{1 - Recovery} \tag{32}$$

These spreads are the additional rate that the bank charge each sector comparing the interest on deposit accounts. Considering loans of the senior secured class, the average recovery rate is 71.11% according to Moody's. Spreads are calculated accordingly and presented in the following table,

[46] Markowitz, H., Portfolio Selection, The Journal of Finance, March 1952, pp. 77-91.
[47] Gollinger, T.L. and J.B. Morgan, Efficient Frontier for a Commercial Loan Portfolio, The Journal of Portfolio Management, Winter 1993, pp. 39-46.

Sector	Risk contr. ($\boldsymbol{\sigma}$)	Spread($\boldsymbol{\mu}$)	Weights
manufacturing	.39	2.05%	W1
Domestic trade	.30	3.20%	W2
Real estates	.16	1.42%	W3
Service	.31	0.80%	W4
trade	.36	2.71%	W5
Table 10: loan portfolio risk and returns			

Moreover, the covariance matrix of loans is constructed based on the correlation structure in the t-copula where the pair wise correlation between loans defined as $\rho_i\rho_j$, hence the variance-covariance matrix of the loan portfolio is,

$$W^T\hat{\Sigma}W \tag{33}$$

Where $\hat{\Sigma}$ is the variance-covariance matrix. The best way to allocate the lending capacity of the bank across various industries basically, involves finding the industry weights that result in the most efficient solutions. So far, the weights of the industry sectors assumed constant by freezing them at 20 percent to construct an equivalently weighted portfolio. For the decision parameters, the model propose the optimal values considering the objectives, requirements, and constraints defined. In this case, the objective is to optimize the return on assets (loans) of the portfolio by deciding on the portfolio shares. Furthermore, the requirement that the EC contribution of the portfolio should not exceed a predefined threshold determined by bank. This limits the risk the bank is willing to take on. Hence, solutions with a higher returns, but an EC exceeding this ceiling, will be discarded.

$$\boldsymbol{Objective\ function}: \quad \boldsymbol{Maximize}\ \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} \boldsymbol{\mu_i * w_i}$$

$\boldsymbol{subject\ to}$:

$$\boldsymbol{\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \rho_{ij} \sigma_i \sigma_j}$$

$$\boldsymbol{w_i \geq 0.1 \quad \forall\ i}$$

$$\boldsymbol{\sum_{i=1}^{N} w_i = 1}$$

In addition, a constraint that the weights should add up to 100 percent is defined. Also, a minimum weight of 10 percent is considered, ensuring that the bank keeps a presence in all sectors. More constraints could be added concerning any national regulatory requirements.

The model is solved in Excel solver48 by GRG Nonlinear Solving Method for nonlinear optimization. The optimal solution found (in the following table) is valid for a risk ceiling that, in the case, was set at 12 percent. According to Markowitz portfolio theory, any portfolio is defined along the axes of risk and return. This implies that a different maximum standard deviation will result in a different optimum for the portfolio allocation. By varying the risk ceiling and running the optimization simulation multiple times, the efficient frontier is obtained as follow.

risk	**return**	**Portfolio weights**				
		manufacturing	**dom. trade**	**Real estates**	**service**	**trade**
0.12	0.0188	0.10	0.18	0.50	0.11	0.11
0.13	0.0211	0.10	0.28	0.37	0.10	0.16
0.14[49]	0.0224	0.10	0.33	0.29	0.10	0.18
0.15	0.0234	0.10	0.37	0.23	0.10	0.20
0.16	0.0243	0.10	0.41	0.17	0.10	0.21
0.17	0.0251	0.10	0.45	0.12	0.10	0.23
0.18	0.0258	0.10	0.522	0.10	0.10	0.178
0.19	0.0261	0.10	0.584	0.10	0.10	0.116
0.2	0.0262	0.10	0.60	0.10	0.10	0.10

Table 11: improved portfolio structure (weights)

From the efficient frontier the improved solution regarding the composition of the loan portfolio depends on the bank's risk appetite and obviously like the case in security investments case risk and return go hand in hand.

Comparing the efficient frontier from Basel to the one produced by the model, the range of maximum returns for the given risk appetite is the same, however, Basel presents a lower level of associated risk to a specific rate of return in comparison to the model. Besides, based on Basel, portfolios with EC contribution of less than 3.4% is not feasible while in case of model 12% is the minimum capital reserve ratio that is possible concerning the efficient frontier.

[48] The GRG Nonlinear Solving Method for nonlinear optimization uses the Generalized Reduced Gradient (GRG2) code, which was developed by Leon Lasdon, University of Texas at Austin, and Alan Waren, Cleveland State University, and enhanced by Frontline Systems, Inc. http://www.solver.com/excel-solver-algorithms-and-methods-used
[49] The equivalently weighted portfolio results in 2.03% return at 14% risk level

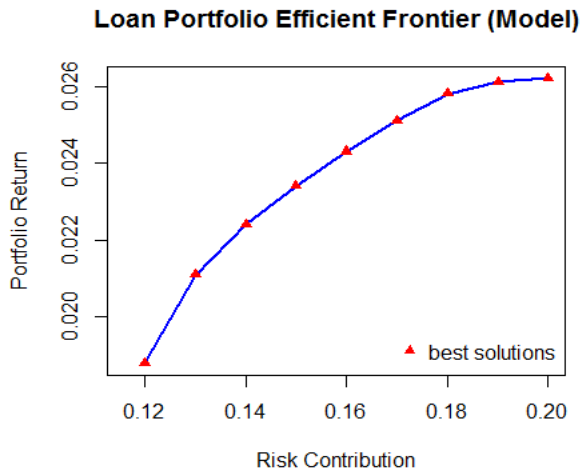

Figure 30: portfolio efficient frontier (model)

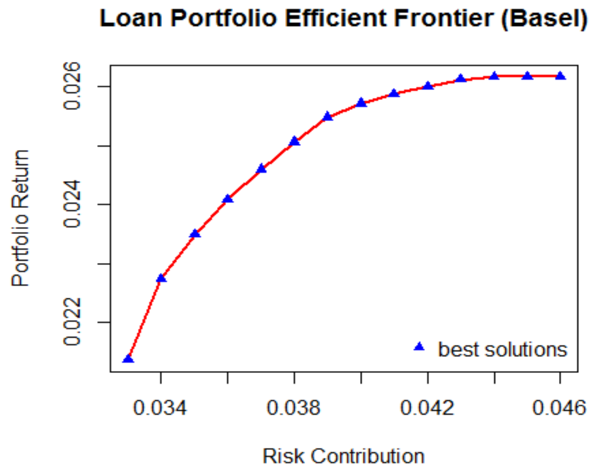

Figure 31: efficient frontier (Basel)

Furthermore, having different efficient frontiers obtained from two approaches, the average portfolio weights is compared in the following diagram. Although the average differences are not far from each other, however, if the bank relies on Basel and tries to extract the optimum portfolio based on the outputs of Basel model the improved portfolio structure that Basel suggests is not practically better than the one proposed by the model.

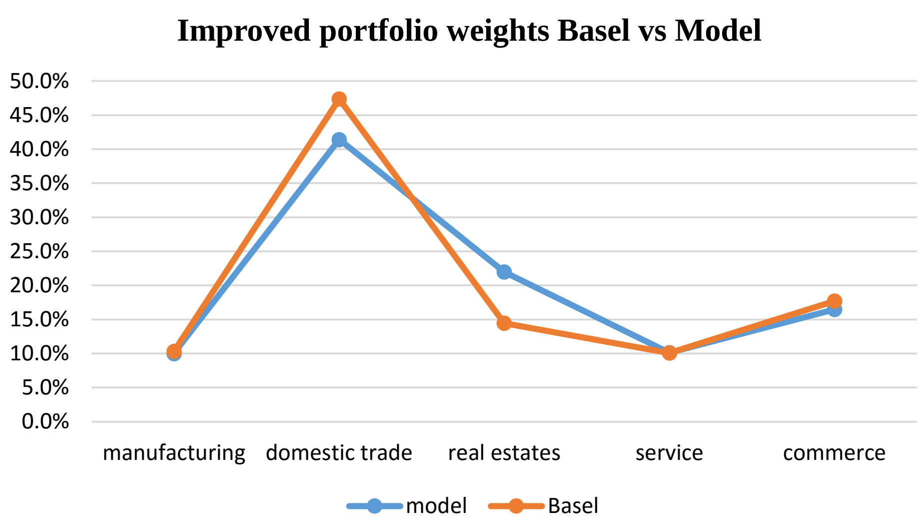

Figure 32: Improved portfolio weights Basel vs. Model

This is examined by comparing generated returns of the portfolio from Basel with the ones suggested by the model for a specific level of risk. For instance, based on the solution in Basel framework, the maximum return for the portfolio is 2.136% at 13.33% risk level, however, the maximum return proposed in the framework of the model is 2.158%. Conclusively, the model provides the bank with better solutions (returns) at a particular level of risk. This improvement is roughly .5% on average.

3.5 Conclusion

Basel results look quite acceptable in normal economic situations, however, it is not reliable for crisis or economic downturn periods when extreme values are more likely to take place. That is because applying a more sophisticated model increased the level of economic capital sometimes two times of the amount Basel suggests. This demonstrates the effect of simplification of the complex interactions and unrealistic assumptions for each of the individual risk factors in portfolio risk.

Moreover, the 99.9 % CVaR that Basel suggests lies between the 95% and 99% percentile of portfolio loss distribution proposed by the model. In other words, the 99.9% percentile Credit-VaR of Basel is approximately 95% and hardly up to 99% confidence of the real capital at risk, and not the 99.9% confidence. This leaves the bank with a considerable level of capital at risk which is not hedged by keeping as reserve or any other hedging strategy. Moreover it artificially lures management to take strategies with 99.9% confidence while in practice he/she is taking risks more than he/she assumed.

Applying Basel or the model does not have a significant impact on the risk contribution of sectors, however, the magnitudes and variance of riskiness differ remarkably between two models. Basel risk contribution ratios are much lower and less variant among sectors, however, the differences among sectors are more noticeable in the model. This implies that Basel makes management somehow indifferent between sectors to which grant more loans, while the model makes it more crystal-clear for manager with a quite distinguished risk contribution levels.

Furthermore, the efficient frontiers extracted from Basel and the model are different, however, the return varies in a similar range. The discrepancy mostly appears in the risk levels. e.g. manager expects 2.4% return in price of 3.8% risk, while the realistic risk level is around 15%; applying Basel gives managers an unrealistic confidence of portfolio risk-return profile

Last but not the least, if the manager relies on Basel, the improved loan portfolio structure will not be the better one in practice, there are some inefficiencies of roughly .5% away from better solution which is material in large portfolio values.

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Appendix A

Zero Coupon Bonds (ZCB) and spreads

Let $Z(t,T)$ denotes today's value of riskless ZCB[50] with payoff of 1$ at T. If $R(t,T)$ is the continuously compounding yield to maturity of this bond, then we have $Z(t,T) = e^{-R(t,T)(T-t)}$ as reflecting the time value of money or todays' time-t value of 1$. Now if we have a risky ZCB bond paying 1$ in good state with probability $P(t,T)$ and nothing (zero-recovery) at other state with probability $(1 - P(t,T))$, the expected payoff under this physical probability measure conditional on all information available at time-t (conditional on that the issuer has not defaulted by time t) is survival probability of the issuer. If $Z_0^d(T,T)$[51] be payoff at time T of this ZCB which is unknown at time-t, its expected value computed on knowledge of $P(t,T)$ is

$$E_t^P[Z_0^d(t,T)] = P(t,T) \times 1\$ + (1 - P(t,T)) \times 0\$ = P(t,T)$$

Where $E_t^P[.]$ denotes expectation formed on the basis of information available at time t, given the survival probability $P(t,T)$. Hence if we have default free and default able ZCB prices for continuum of maturities then we have survival probabilities for all maturities and also densities for all maturities. From this a term structure of survival probabilities can be derived.

Risk-Neutral Valuation and Probabilities

While the ZCB is risky and there is a chance of no payoff, one may want to discount the promised further when assessing the current value of the bond. There are two equivalent ways of thinking about this discounting, one can apply a higher discount rate over risk-free as

$$Z_t^d(t,T) = e^{-[R(t,T)+S(t,T)](T-t)}$$

Now the promised payment of the bond is discounted by a higher rate of $[R(t,T) + S(\mathrm{t,T})]$. Alternatively one can think of the "artificial" probability, $Q(t,T)$ conditional on information available at time- t is

$$Z_0^d(t,T) = e^{-R(t,T)(T-t)}\,[Q(t,T) \times 1 + (1 - Q(t,T)) \times 0] = Z_0(t,T)Q(t,T)^{52}$$

---

[50] ZCB is a bond with no coupon but the only payment at maturity

[51] $Z_0^d(T,T)$ denotes a defaultable ZCB with zero recovery

[52] The alternative way of writing price of a default able ZCB is $Z_t^d(t,T) = Z(t,T)E_t^{\mathbb{Q}}[1_{\tau>T}]$ in which the second part is the risk- neutral probability of default $Q(t,T)$ assuming independent τ and risk-free rate

While $\left(1 - Q(t,T)\right)$ is the probability attached to the default of the bond issuer. The physical probabilities does not coincide with risk-neutral ones since investors are risk averse, because they are ready to pay a higher amount for a riskless investment rather than a risky one, hence the today's price of a safe investment shall be higher therefore physical survival probabilities are larger than risk-neutral ones and the case is vice versa for default probabilities[53]. Assuming the τ (default time) be independent of risk free rate we have the equation which represents a prominent result of,

Price of a risky-bond = price of a risk-free bond × risk-neutral survival probability of risky bond issuer

In order to incorporate all premiums for the loan adjusted return the bank shall follow the general equation for the adjusted rate,

$$r_{adjusted} = r_{real} + inf_{premium} + default\ risk_{premium} + liquidity\ risk_{premium} + maturity\ risk_{premium}$$

The loan spread is: $Spread = default\ risk_{premium} + liquidity\ risk_{premium}$

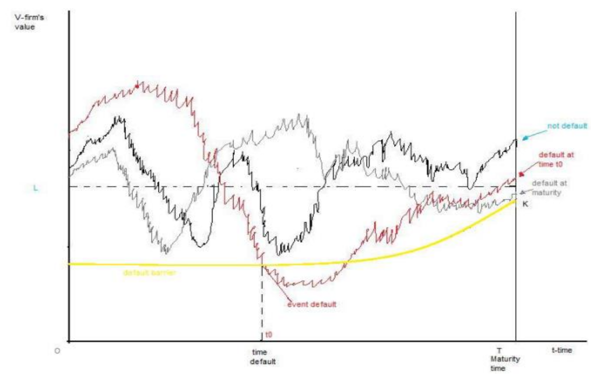
[53] For further discussion and the proof please refer to understanding credit derivatives and related instruments, p.148 and 149

Appendix B

From Merton to AT1P

According to Merton 1974[54] defaults happens at maturity date T and creditors take over the firm and realize an amount of V_T.

As reviewed in chapter 1, Black-Cox 1976 suggested the first model from family of first passage time, in addition, they take into account the safety covenants in loan contracts which enables creditors to take over the borrowing firm when its value A_t falls low enough "safety level"$H(t)$. Hitting this barrier is considered early default and this makes default time unpredictable, ex-ante.



Due to possibility of default at any time prior maturity the spreads generated by Black-Cox are higher than Merton.[55] The first candidate of the barrier is the face value of debt discounted to the present time, however, one may cut some slacks to the counterparty give it some time to recover even if the level goes below the barrier $LP(t,T)$ and the safety level can be chosen to be lower than $LP(t,T)$. Clearly, pricing this bond is solving a barrier option pricing problem, and first passage time models make use of barrier option techniques. Here τ, default time, can be defined as

$$\tau = \inf\{t \geq 0, ; V_t \leq H(t)$$

if this quantity is smaller than the debt final maturity T, and by T if further $V_T \leq L$, in all other cases there is no default. If $H(t)$ is the barrier depending on t and zero coupon bond maturity date T, for each counterparty corporation $i,$ Black and Cox assume a constant parameter Geometric Brownian Motion

[54] Merton 1974

[55] A review of Merton's model of firm's capital structure with its wide applications, Suresh Sundaresan 2013

$$dV_i = (\mu_i - k_i)V_i dt + \sigma_i V_i dX_i$$

So that

$$dlnV_i = \left(\mu_i - k_i - \frac{\sigma_i^2}{2}\right)dt + \sigma_i dX_i$$

In the equations μ_i is the expected growth rate of assets for company i, σ_i is business risk or assets volatility, k_i is payout ratio and X_i a random variable that follows Wiener process. Furthermore, μ_i and σ_i are assumed constant. Based on Merton, firm default when its assets value falls below the face value of liabilities. In Black-Cox framework, defaults takes place as soon as assets values hits the default barrier H_i, safety covenant, from above. The exponential barrier is defined as,

$$H_i(t,T) = \begin{cases} L & t = T \\ Ke^{-\gamma\,(T-t)} & t < T \end{cases}$$

where $K\ and\ \gamma$ are positive parameters. Black and Cox also assumed that $Ke^{-\gamma\,(T-t)} < Le^{-r\,(T-t)}$. This assumption means that safety covenant are lower than the final debt present value. If $\gamma = 0$ it's a special case of flat barrier. According to Hull White 2010, corresponding to H_i, there is barrier H_i^* such that company i when X_i falls below H_i^* for the first time. Assuming $X_i(0) = 0$ there is and zero payout ratio $(k = 0)$,

$$X_i(t) = \frac{lnV_i(t) - lnV_i(0) - \left(\mu_i - \frac{\sigma_i^2}{2}\right)t}{\sigma_i}$$

And

$$H_i^* = \frac{lnH_i - lnV_i(0) - \left(\mu_i - \frac{\sigma_i^2}{2}\right)t}{\sigma_i}$$

Where $a_1 = r - K - \gamma - \frac{\sigma_A{}^2}{2}$ and $a_1 = \frac{a_1}{\sigma_A{}^2}$.

Hence the default probability will be

$$PD_i = Prob(V_i \leq H_i) = Prob(X_i \leq H_i^*)$$

Harrison[56] 1990 showed that probability of first hitting the barrier between times t and $t + T$ is,

[56] Most parts from Hull White 2010

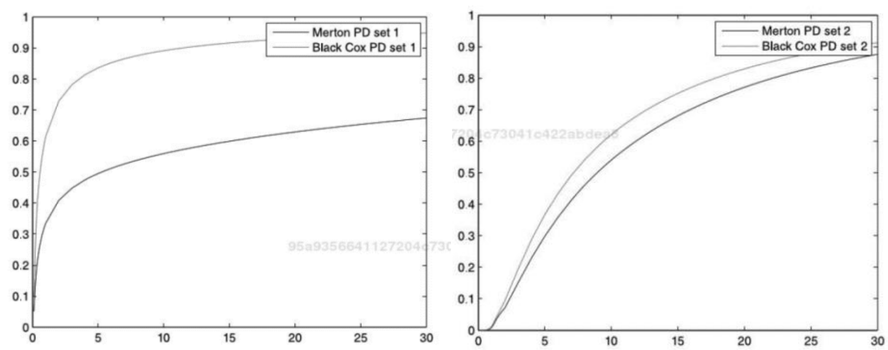
$$Prob = \Phi\left(\frac{\beta_i + \gamma_i(t+T) - X_i(t)}{\sqrt{T}}\right) + \exp\big(2(X_i(t) - \beta_i - \gamma_i t)\big)\,\Phi(\frac{\beta_i + \gamma_i(t-T) - X_i(t)}{\sqrt{T}})$$

While $\beta_i = \frac{lnH_i - lnV_i(0)}{\sigma_i}$, $\gamma_i = \frac{-\left(\mu_i - \frac{\sigma_i^2}{2}\right)}{\sigma_i}$.

Comparing results from Merton and Black-Cox for different scenarios reveal a relevant difference which originates from the early possibility of default in Black-Cox model. Brigo[57] compares two scenarios of,

$$Set\ 1 :\ \frac{L}{V_0} = 0.9\ ;\ \ \sigma_1 =\ 0.2; \qquad Set\ 2\text{:}\ \frac{L}{V_0} = 0.2\ ;\ \ \sigma_1 =\ 0.9;$$

and demonstrates the results in the graphs show a relevant difference.



Brigo and Tarenghi 2004, have extended Black-Cox first passage model first by means of time-varying volatility and curved barriers techniques and then further by random barrier and volatility scenarios. The AT1P model is selected for two reasons, first it's less complexity comparing to SBTV and secondly the idea of not requiring the current value of assets where the model suffices to insert the corresponding ratio of the barriers concerning the asset value level, this increases its applicability in Iranian market.

Analytically Tractable 1th- Passage Model (AT1P)

Analytically Tractable first Passage (AT1P) model assumes the risk neutral dynamics for the value of the firm characterized by r and payout ratios of k and instantaneous volatility of σ_t

$$dV_i = (\mu_i - k_i)V_i dt + \ \sigma_i V_i dX_i$$

[57] Brigo D. 2011, Credit risk management, Kings' college FM10 master course lecture notes

And assumes default barrier function depending on parameters H and B of the form

$$H(t) = H \exp(\int_0^t (r_u - k_u - B\sigma_u^2) du)$$

Letting τ be the first time firm value $V_t$hit the barrier $H(t,T)$ from above, starting from $V_0 \geq H$

$$\tau = \inf\{t \geq 0, ; V_t \leq H(t,T)$$

The survival probability is given analytically by

$$\mathbb{Q}\{\tau > T\} = \Phi\left[\frac{\ln\left(\frac{V_0}{H}\right) + \frac{2B-1}{2}\int_0^T \sigma_u^2 du}{\sqrt{\int_0^T \sigma_u^2 du}}\right] - \left(\frac{H}{V_0}\right)^{2B-1} \Phi\left[\frac{\ln\left(\frac{H}{V_0}\right) + \frac{2B-1}{2}\int_0^T \sigma_u^2 du}{\sqrt{\int_0^T \sigma_u^2 du}}\right]$$

Apparently the barrier varies in time, following the firm and market conditions

$$H(t) = H \exp\{\int_0^t (r_u - k_u - B\sigma_u^2) du\}$$

$$= \frac{H}{V_0} \mathrm{E}[V_t] \quad \times \quad \exp(-B \int_0^t \sigma_u^2 \, du)$$

First part is the backbone of the barrier while the second part cuts some slack in high volatility conditions controlling by B. H and V_0 always appear in formulas in ratios like $\frac{H}{V_0}$. Therefore, it is possible to rescale the initial value of the firm's assets V_0 = 1 and express the (free) barrier parameter H as a fraction of it. In this case, it is not necessary to know the real value of the firm. Here, H may depend on the level of liabilities, on safety covenants, and in general on the characteristics of the capital structure of the company.

Appendix C

Summary properties of Clayton copula[58]

- g: the generator function for the copula
- g^{-1}: inverse of the generator function
- C: the copula in terms of two CDFs, u and v
- C_1: the marginal CDF of the copula, $\frac{\partial c}{\partial u}$
- c: the copula's density function
- τ: Kendall's tau
- ρ: Spearman's rho

Clayton

$$g^{-1} = (1+t)^{-\frac{1}{\alpha}}$$

$$g = t^{-\alpha} - 1$$

$$C = (u^{-\alpha} + v^{-\alpha} - 1)^{-\frac{1}{\alpha}}$$

$$C_1 = \frac{\partial C}{\partial u} = u^{-(\alpha+1)}(u^{-\alpha} + v^{-\alpha} - 1)^{-\frac{1+\alpha}{\alpha}}$$

$$v = u\left(C_1^{-\frac{\alpha}{1+\alpha}} + u^{\alpha} - 1\right)^{-\frac{1}{\alpha}}$$

$$c = (1+\alpha)(uv)^{-(\alpha+1)}(u^{-\alpha} + v^{-\alpha} - 1)^{-\frac{1+2\alpha}{\alpha}}$$

$$\alpha > 0$$

$$\tau = \frac{\alpha}{\alpha+2}$$

[58] Mathematics and statistics for financial risk management, 2th edition, by Michael Miller

Appendix D

Simulation pseudo code

- Step#1 get number of counterparties in the portfolio
- Step#2 take the company information such as industry, balance sheet, credit grade…
- Step#3 get/calculate probability of default based on historical data, Merton or Brigo AT1P
- Step#4 extract the spread over locally-defined risk free rate
- Step#5 calculate average market sensitivity factors based on Lopez 2004
- Step#6 generate one $N(0,1)$ as market status
- Step#7 generate N sample from Beta for correlations correlated with M
- Step#8 generate N independent $N(0,1)$ for idiosyncratic risk
- Step#9 generate one Chi-square random variable with desired $1 \leq df \leq 3$
- Step#10 implement one-factor t-student copula to generate correlated binary defaults events
- Step#11 generate correlated LGD with PDs conditioned on Market status from Clayton copula
- Step#12 calculate Expected Loss
- Step#13 run step#4 to 12 for 100,000 times
- Step#14 aggregate ELs and generate histogram and 99.9% CVaR
- Step#15 subtract expected loss from 99.9% CVaR to get economic capital

Appendix E

Bank loan data (IRR)

Row	Sector	EAD	PD	Rec.[59]	Bank Rating	weight of EAD
1	Service	19,707,163,492	0.176%	50%	BBB	0.3%
2	domestic trade	30,000,000,000	4.546%	50%	B	0.5%
3	domestic trade	145,746,276,596	4.546%	50%	B	2.5%
4	domestic trade	3,500,000,000	4.546%	50%	B	0.1%
5	domestic trade	1,573,339	1.166%	50%	BB	0.0%
6	trade	1,568,910,286	17.723%	60%	CC	0.0%
7	trade	17,892,911,203	17.723%	60%	CC	0.3%
8	trade	100,000,000,000	17.723%	60%	CC	1.7%
9	trade	3,000,000,000	0.051%	60%	A-	0.1%
10	trade	5,000,000,000	0.051%	60%	A-	0.1%
11	trade	600,000,000	0.051%	60%	A-	0.0%
12	trade	50,000,000,000	4.546%	60%	B+	0.9%
13	trade	10,000,000,000	4.546%	60%	B+	0.2%
14	trade	10,000,000,000	4.546%	60%	B+	0.2%
15	trade	7,500,000,000	17.723%	60%	C	0.1%
16	trade	5,000,000,000	17.723%	60%	C	0.1%
17	trade	24,200,000,000	17.723%	60%	C	0.4%
18	trade	12,000,000,000	4.546%	60%	B+	0.2%
19	trade	10,000,000,000	4.546%	60%	B+	0.2%
20	trade	15,500,000,000	4.546%	60%	B+	0.3%
21	trade	1,000,000,000	0.051%	60%	A	0.0%
22	trade	13,400,000,000	0.051%	60%	A	0.2%
23	trade	5,000,000,000	0.051%	60%	A	0.1%
24	manufacturing	9,174,656,573	17.723%	70%	CCC-	0.2%
25	trade	30,000,000,000	0.051%	60%	A	0.5%
26	trade	3,000,000,000	0.051%	60%	A	0.1%
27	trade	100,000,000,000	0.051%	60%	A	1.7%
28	trade	2,000,000,000	17.723%	60%	CCC-	0.0%
29	trade	30,000,000,000	17.723%	60%	CCC-	0.5%
30	trade	2,460,000,000	17.723%	60%	CCC-	0.0%
31	trade	20,000,000,000	4.546%	60%	B+	0.3%
32	trade	150,000,000,000	4.546%	60%	B+	2.6%
33	trade	10,000,000,000	4.546%	60%	B+	0.2%
34	trade	50,000,000,000	1.166%	60%	BB	0.9%
35	trade	50,000,000,000	1.166%	60%	BB	0.9%
36	trade	527,605,810	1.166%	60%	BB	0.0%
37	manufacturing	9,673,910,588	17.723%	70%	CCC-	0.2%

[59] Average for the industry

38	manufacturing	5,500,000,000	17.723%	70%	CCC-	0.1%
39	manufacturing	15,000,000,000	17.723%	70%	CCC	0.3%
40	manufacturing	100,000,000,000	17.723%	70%	CCC	1.7%
41	manufacturing	20,000,000,000	17.723%	70%	CCC	0.3%
42	manufacturing	700,000,000	17.723%	70%	CCC	0.0%
43	manufacturing	2,000,000,000	1.166%	70%	BB	0.0%
44	manufacturing	4,000,000,000	1.166%	70%	BB	0.1%
45	manufacturing	9,632,332,506	1.166%	70%	BB	0.2%
46	manufacturing	3,000,000,000	4.546%	70%	B+	0.1%
47	manufacturing	10,000,000,000	4.546%	70%	B+	0.2%
48	manufacturing	100,000,000,000	4.546%	70%	B+	1.7%
49	manufacturing	15,000,000,000	4.546%	70%	B+	0.3%
50	manufacturing	96,000,000,000	17.723%	70%	CC	1.7%
51	manufacturing	20,000,000,000	17.723%	70%	CC	0.3%
52	manufacturing	15,000,000,000	17.723%	70%	CC	0.3%
53	manufacturing	3,929,506,560	17.723%	70%	C	0.1%
54	manufacturing	4,113,085,824	17.723%	70%	C	0.1%
55	manufacturing	5,811,514,594	17.723%	70%	C	0.1%
56	manufacturing	12,641,302,571	17.723%	70%	CCC+	0.2%
57	manufacturing	10,000,000,000	17.723%	70%	CCC+	0.2%
58	domestic trade	99,934,275,967	1.166%	50%	BB	1.7%
59	manufacturing	27,000,000,000	17.723%	70%	CCC+	0.5%
60	manufacturing	10,000,000,000	1.166%	70%	BB+	0.2%
61	manufacturing	270,000,000,000	1.166%	70%	BB+	4.7%
62	manufacturing	10,000,000,000	1.166%	70%	BB+	0.2%
63	manufacturing	2,000,000,000	4.546%	70%	B-	0.0%
64	manufacturing	20,000,000,000	4.546%	70%	B-	0.3%
65	manufacturing	8,500,000,000	4.546%	70%	B-	0.1%
66	manufacturing	10,000,000,000	17.723%	70%	CCC	0.2%
67	manufacturing	50,000,000,000	17.723%	70%	CCC	0.9%
68	manufacturing	30,000,000,000	17.723%	70%	CCC	0.5%
69	manufacturing	50,000,000,000	17.723%	70%	CCC+	0.9%
70	manufacturing	7,970,400,000	17.723%	70%	CCC+	0.1%
71	manufacturing	15,000,000,000	17.723%	70%	CCC+	0.3%
72	manufacturing	5,000,000,000	17.723%	70%	CCC+	0.1%
73	manufacturing	15,000,000,000	17.723%	70%	CCC+	0.3%
74	manufacturing	7,448,033,600	17.723%	70%	CCC+	0.1%
75	manufacturing	200,000,000,000	1.166%	70%	BB+	3.5%
76	manufacturing	100,000,000,000	1.166%	70%	BB+	1.7%
77	manufacturing	383,246,654	1.166%	70%	BB+	0.0%
78	manufacturing	648,615,762	4.546%	70%	B-	0.0%
79	manufacturing	200,000,000,000	4.546%	70%	B-	3.5%
80	manufacturing	45,000,000,000	4.546%	70%	B-	0.8%

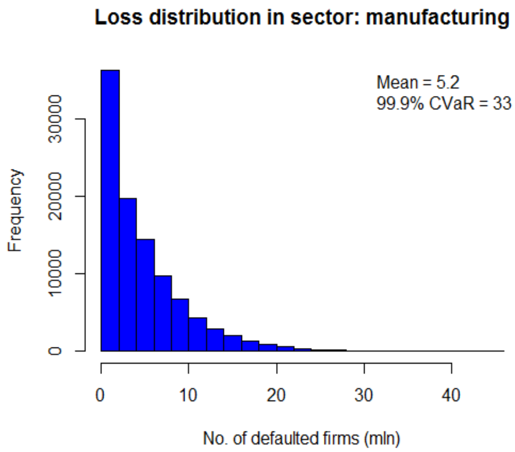
81	manufacturing	37,000,000,000	1.166%	70%	BB	0.6%
82	manufacturing	22,334,000,000	1.166%	70%	BB	0.4%
83	manufacturing	50,000,000,000	1.166%	70%	BB	0.9%
84	real estates	50,000,000,000	17.723%	70%	C	0.9%
85	real estates	5,000,000,000	17.723%	70%	C	0.1%
86	real estates	6,000,000,000	17.723%	70%	C	0.1%
87	real estates	4,200,000,000	1.166%	70%	BB	0.1%
88	real estates	3,000,000,000	1.166%	70%	BB	0.1%
89	real estates	3,800,000,000	1.166%	70%	BB	0.1%
90	real estates	3,000,000,000	0.051%	70%	A+	0.1%
91	real estates	30,000,000,000	0.051%	70%	A+	0.5%
92	real estates	1,800,000,000	0.051%	70%	A+	0.0%
93	real estates	5,000,000,000	0.051%	70%	A+	0.1%
94	real estates	50,000,000,000	4.546%	70%	B	0.9%
95	real estates	5,000,000,000	4.546%	70%	B	0.1%
96	real estates	25,000,000,000	4.546%	70%	B	0.4%
97	real estates	5,000,000,000	1.166%	70%	BB+	0.1%
98	real estates	30,000,000,000	1.166%	70%	BB+	0.5%
99	real estates	1,500,000,000	1.166%	70%	BB+	0.0%
100	real estates	19,000,000,000	0.176%	70%	BBB	0.3%
101	real estates	10,000,000,000	0.176%	70%	BBB	0.2%
102	real estates	300,000,000,000	0.176%	70%	BBB	5.2%
103	real estates	5,000,000,000	0.051%	70%	A	0.1%
104	real estates	5,000,000,000	0.051%	70%	A	0.1%
105	real estates	250,000,000,000	0.051%	70%	A	4.3%
106	real estates	10,000,000,000	0.176%	70%	BBB	0.2%
107	real estates	10,000,000,000	0.176%	70%	BBB	0.2%
108	real estates	18,120,000,000	0.176%	70%	BBB	0.3%
109	real estates	42,000,000,000	0.176%	70%	BBB	0.7%
110	service	2,000,000,000	17.723%	50%	CCC+	0.0%
111	service	5,000,000,000	17.723%	50%	CCC+	0.1%
112	service	500,000,000	17.723%	50%	CCC+	0.0%
113	service	3,000,000,000	4.546%	50%	B-	0.1%
114	service	840,000,000	4.546%	50%	B-	0.0%
115	service	2,000,000,000	4.546%	50%	B-	0.0%
116	service	17,915,000,000	1.166%	50%	BB+	0.3%
117	service	14,750,000,000	4.546%	50%	B-	0.3%
118	service	32,682,000,000	4.546%	50%	B-	0.6%
119	service	1,900,000,000	4.546%	50%	B-	0.0%
120	service	7,000,000,000	0.051%	50%	A	0.1%
121	service	41,000,000,000	0.051%	50%	A	0.7%
122	service	12,000,000,000	0.051%	50%	A	0.2%
123	service	7,000,000,000	1.166%	50%	BB	0.1%

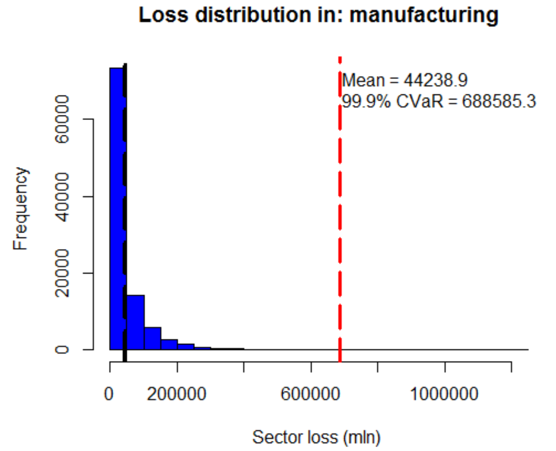
124	service	29,000,000,000	1.166%	50%	BB	0.5%
125	service	24,000,000,000	1.166%	50%	BB	0.4%
126	service	30,000,000,000	4.546%	50%	B	0.5%
127	service	7,000,000,000	4.546%	50%	B	0.1%
128	service	1,200,000,000	4.546%	50%	B	0.0%
129	service	27,000,000,000	17.723%	50%	CCC+	0.5%
130	service	1,000,000,000	17.723%	50%	CCC+	0.0%
131	service	33,000,000,000	17.723%	50%	CCC+	0.6%
132	service	10,000,000,000	17.723%	50%	CCC+	0.2%
133	domestic trade	29,942,009,227	1.166%	50%	BB	0.5%
134	domestic trade	50,000,000,000	0.176%	50%	BBB	0.9%
135	domestic trade	27,362,400,000	0.176%	50%	BBB	0.5%
136	domestic trade	16,499,315,040	0.176%	50%	BBB	0.3%
137	domestic trade	180,000,000,000	1.166%	50%	BB-	3.1%
138	domestic trade	500,000,000	1.166%	50%	BB-	0.0%
139	domestic trade	2,100,000,000	1.166%	50%	BB-	0.0%
140	domestic trade	5,000,000,000	4.546%	50%	B	0.1%
141	domestic trade	20,000,000,000	4.546%	50%	B	0.3%
142	domestic trade	6,661,679,675	4.546%	50%	B	0.1%
143	domestic trade	5,000,000,000	0.051%	50%	A	0.1%
144	domestic trade	12,500,000,000	0.051%	50%	A	0.2%
146	domestic trade	42,271,127,360	1.166%	50%	BB	0.7%
147	domestic trade	2,000,000,000	1.166%	50%	BB	0.0%
148	domestic trade	1,000,000,000	1.166%	50%	BB	0.0%
149	domestic trade	40,000,000,000	17.723%	50%	C	0.7%
150	domestic trade	100,000,000,000	17.723%	50%	C	1.7%
151	domestic trade	3,000,000,000	17.723%	50%	C	0.1%
152	domestic trade	362,735,145,342	4.546%	50%	B-	6.3%
153	domestic trade	573,680,000	4.546%	50%	B-	0.0%
154	domestic trade	22,637,600,000	4.546%	50%	B-	0.4%
155	domestic trade	3,353,174,720	4.546%	50%	B	0.1%
156	domestic trade	10,000,000,000	4.546%	50%	B	0.2%
157	domestic trade	20,000,000,000	4.546%	50%	B	0.3%
158	domestic trade	884,568,384	17.723%	50%	CCC-	0.0%
159	domestic trade	5,327,857,143	17.723%	50%	CCC-	0.1%
160	domestic trade	1,852,632,311	17.723%	50%	CCC-	0.0%
161	domestic trade	26,399,545,251	4.546%	50%	B	0.5%
162	domestic trade	200,000,000	4.546%	50%	B	0.0%
163	domestic trade	14,500,000,000	4.546%	50%	B	0.3%
164	domestic trade	1,000,000,000	0.022%	50%	AA	0.0%
165	domestic trade	20,000,000,000	0.022%	50%	AA	0.3%
166	domestic trade	50,000,000,000	0.022%	50%	AA	0.9%
167	domestic trade	1,778,063,249	1.166%	50%	BB	0.0%

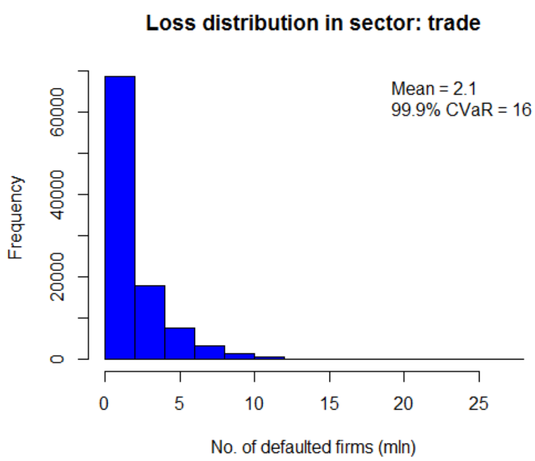
168	domestic trade	1,500,000,000	1.166%	50%	BB	0.0%
169	domestic trade	27,402,239,941	1.166%	50%	BB	0.5%
170	domestic trade	50,000,000,000	1.166%	50%	BB+	0.9%
171	domestic trade	50,000,000,000	1.166%	50%	BB+	0.9%
172	domestic trade	3,107,613,422	1.166%	50%	BB+	0.1%
173	domestic trade	4,475,150,000	4.546%	50%	B	0.1%
174	domestic trade	4,300,000,000	4.546%	50%	B	0.1%
175	domestic trade	30,000,000,000	4.546%	50%	B	0.5%
176	domestic trade	30,000,000,000	4.546%	50%	B+	0.5%
177	domestic trade	10,000,000,000	4.546%	50%	B+	0.2%
178	domestic trade	1,500,000,000	4.546%	50%	B+	0.0%
179	domestic trade	100,000,000,000	4.546%	50%	B	1.7%
180	domestic trade	30,000,000,000	4.546%	50%	B	0.5%
181	domestic trade	3,000,000,000	4.546%	50%	B	0.1%
182	domestic trade	20,000,000,000	4.546%	50%	B	0.3%
183	domestic trade	9,000,000,000	17.723%	50%	CCC+	0.2%
184	domestic trade	15,000,000,000	17.723%	50%	CCC+	0.3%
185	domestic trade	1,000,000,000	17.723%	50%	CCC+	0.0%
186	domestic trade	300,000,000,000	0.176%	50%	BBB	5.2%
187	domestic trade	9,280,508,181	0.176%	50%	BBB	0.2%
188	domestic trade	48,546,865,705	0.176%	50%	BBB	0.8%
189	domestic trade	24,244,773,352	0.176%	50%	BBB	0.4%
190	service	500,000,000	17.723%	50%	CCC	0.0%
191	manufacturing	800,000,000	4.546%	70%	B-	0.0%
192	manufacturing	4,500,000,000	4.546%	70%	B-	0.1%
193	real estates	1,670,000,000	0.176%	70%	BBB	0.0%
194	real estates	800,000,000	0.176%	70%	BBB	0.0%
195	service	120,000,000	17.723%	50%	CCC	0.0%
196	service	250,000,000	17.723%	50%	CCC	0.0%
197	service	100,000,000	17.723%	50%	CCC+	0.0%
198	service	490,000,000	17.723%	50%	CCC+	0.0%

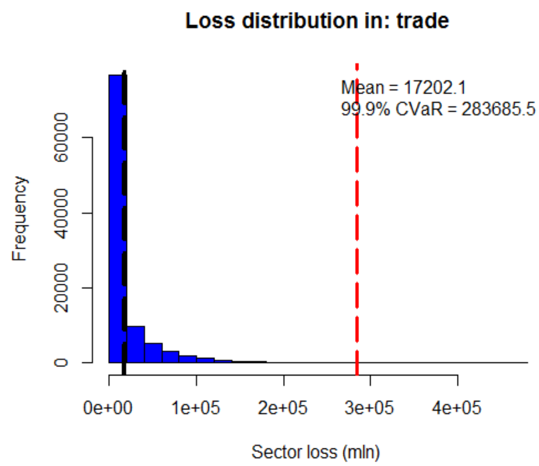
Appendix F

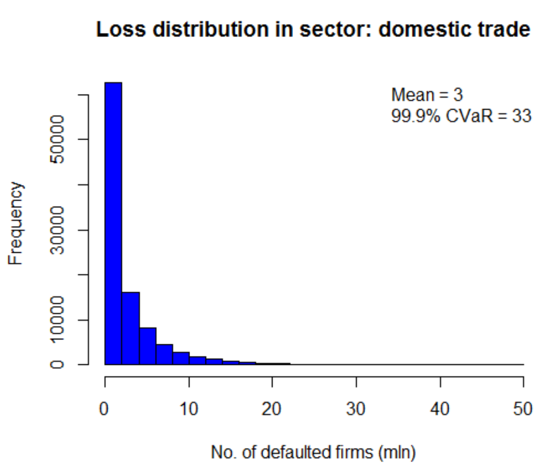
Sector analysis

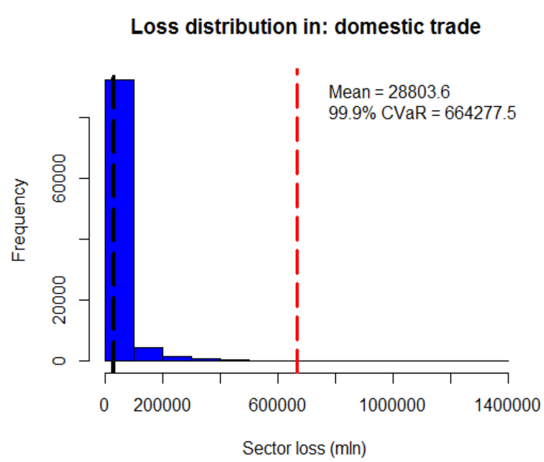


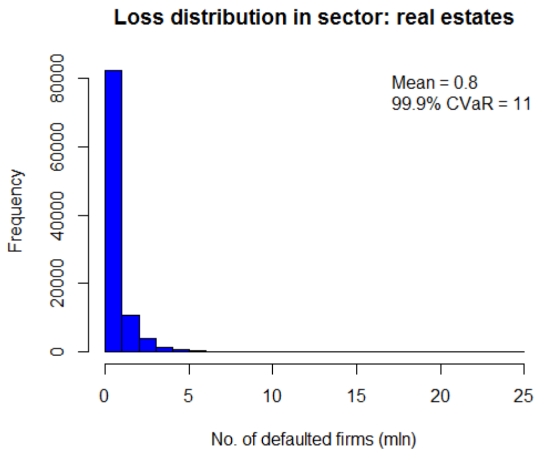


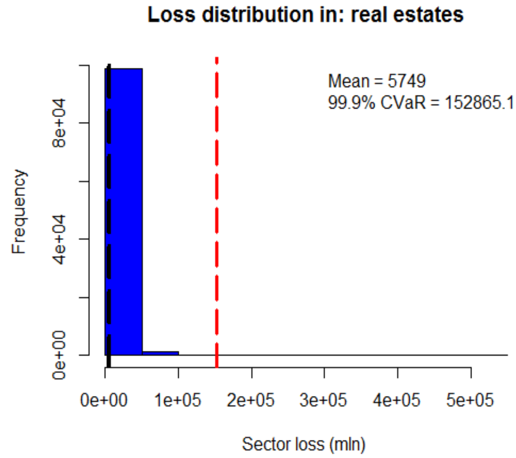


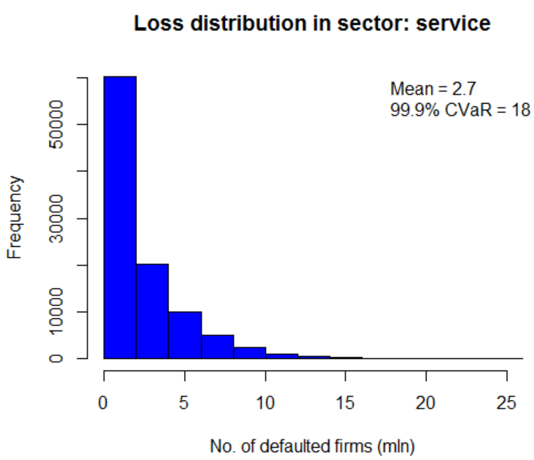


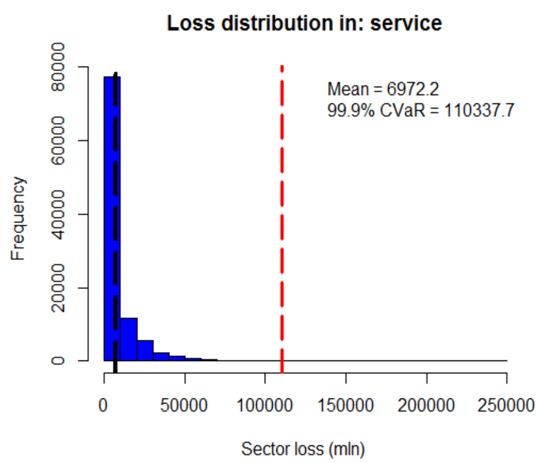





Loss distribution in sector: real estates
Mean = 0.8
99.9% CVaR = 11
Frequency
No. of defaulted firms (mln)


Loss distribution in: real estates
Mean = 5749
99.9% CVaR = 152865.1
Frequency
Sector loss (mln)


Loss distribution in sector: service
Mean = 2.7
99.9% CVaR = 18
Frequency
No. of defaulted firms (mln)


Loss distribution in: service
Mean = 6972.2
99.9% CVaR = 110337.7
Frequency
Sector loss (mln)

Appendix G

Code in R.

Loan Portfolio Loss Distribution and Optimization

Author: Amir Azamtarrahian

Date: April 2016

This function gets loans' data and outputs the corresponding credit spreads, optimum loan structure and lending,

Economic capital for each counterparty and portfolio. Moreover, it delivers CVaR at different confidence intervals

```
####################################################################################
plus sensitvity analysis on model parametrs.
myBaselBank<<- function( Nsim= 100000,

                # set as REcovery for senior secured loan , mean= 71.18% ,
                # Stdev= %21.09, source Moody's

                aR=rep(2.571, NC),
                bR=rep(1.041, NC),

                dof=3, # degree of freedom for t-student copula
                ttau=.5) # tau for Clayton Copula of recovery and Pd rates

{
    # Reading data of loans and exposures from Excel

    loanData <<- read.csv("D:/Temp Works/Thesis/R/loanData.csv")

    #loanData<<- loanData[197,]

    NC<<- length(loanData[,1])  # Number of loans in portfolio

    EAD<<- as.vector(loanData[,3])   # Exposures

    PD<<- loanData[,4]    # Probabikity of defaults

    rec<<- loanData[,5]    # Recovery rates


    # tHis function gets mean and stdev of recovery rates and calibrates to Beta distribution

    myRec<<- function(meanRec=.6, sigRec=.309){
```

```
        library(rootSolve)

        # this describes the system of equations for mean and variance of Beta
        model <<- function(x) c(F1 = (x[1]/ (x[1]+x[2])) - meanRec,
                    F2 = (x[1]*x[2]/(((x[1]+x[2])^2)*(x[1]+x[2]+1))) - (sigRec^2))

        ss <<- multiroot(f = model, start = c(.1, .1))

        # ss is vector of solutions, a & b pars. for Beta distribution
        ss$root

    }


    # This function produces the Basel rho and WCDR

    baselII<<- function(p){

        ro<<- .12*(1+exp(-50* p))
        wcdr<<- pnorm((qnorm(p) + sqrt(ro)*qnorm(.999))/ sqrt(1- ro))

        c_var<<- (1-.7118)*wcdr

        c(wcdr, c_var)
    }


    # Starts the clock!
    ptm <- proc.time()

    assetRho<<- sqrt(.12*(1+exp(-50* PD)))  # rho of exposyures to the Market factor


    # this loop gives each exposure rho based on Basel formula of rho and PDs and calibrate Beta
distr.

    aC<<- rep(0,NC); bC<<- rep(0, NC)

    for (k in 1:NC){

        myRec( assetRho[k] , .1)

        aC[k]<<-ss$root[1]
        bC[k]<<-ss$root[2]
       }

    # alpha is set based on Kendall Tau= sqrt(0.5)
```

```
set.seed(11111*runif(1))
M<<- rep(0, Nsim)
chi<<- rep(0, Nsim)
NC<<- NC
alfa<<- 2*ttau/(1-ttau)

rrho<<- matrix(0, nrow=Nsim, ncol=NC)
tPD<<- matrix(0, nrow=Nsim, ncol=NC)
LGD<<- matrix(0, nrow=Nsim, ncol=NC)
N<<- matrix(0, nrow=Nsim, ncol=NC)  # Binary variable if default  N[i]=1
aggEL<<- matrix(0, nrow=Nsim, ncol=NC)

v<<- matrix(0, nrow=Nsim, ncol=NC)
u<<- matrix(0, nrow=Nsim, ncol=NC)
z1<<- matrix(0, nrow=Nsim, ncol=NC)
u1<<- matrix(0, nrow=Nsim, ncol=NC)
v1<<- matrix(0, nrow=Nsim, ncol=NC)
Rec<<- matrix(0, nrow=Nsim, ncol=NC)
EAD<<- EAD
prD<<- mean(PD)
for( j in 1:Nsim){

    M[j]<<- rnorm(1,0,1)
    chi[j]<<- rchisq(1,dof)

    for (i in 1:NC){

        z1[j,i]<<-rnorm(1,0,1)
        # correlating correlations to default rates by copula

        rrho[j,i]<<- qbeta(pnorm(-sqrt(0.05)* M[j] + sqrt(0.5) *rnorm(1)),  aC[i], bC[i])


        # generating correlated default rates by t-copula
        tPD[j,i]<<- (((rrho[j,i])* M[j] +
                    sqrt(1-rrho[j,i]^2)*z1[j,i])/sqrt(chi[j]/dof))



        # Checking if defaults or not
        if (tPD[j,i] <= qt(PD[i], dof)) { N[j,i]<<- 1 } else {N[j,i]<<- 0}


        #Generating correlated recovery and default rate by Clayton copula
        u[j,i]<<-pnorm(sqrt(0.5)* M[j] + sqrt(0.5) *rnorm(1))

        v[j,i]<<- u[j,i]*(((runif(1))^(-alfa/(1+alfa))) + (u[j,i]^alfa) - 1 )^(-1/alfa);
```

```
            Rec[j,i]<<- qbeta(v[j,i],aR[i],bR[i] )
            LGD[j,i]<<- 1- Rec[j,i]

            aggEL[j,i]<<- aggEL[j,i] + (LGD[j,i]*N[j,i]*EAD[i])

        }
    }

    # Stop the clock
    proc.time() - ptm

    h<<-hist(apply(aggEL, 1, sum) , col="blue", main="Portfolio Loss Distribution",
                            xlab="portfolio loss (mln)",ylab="Frequesncy")

    #abline(v=mean(apply(aggEL, 1, sum)), col="red" , lwd=3, lty=5)

    abline(v=  NC* mean(EAD)*baselII(prD)[2], col="black" , lwd=4, lty=5)

    abline(v= quantile(apply(aggEL, 1, sum), c(.95)) , col="orange" , lwd=3, lty=5)

    abline(v= mean(apply(aggEL, 1, sum),  col="pink" , lwd=3, lty=5))


    abline(v= quantile(apply(aggEL, 1, sum), c(.99)) , col="green" , lwd=3, lty=5)


    abline(v= quantile(apply(aggEL, 1, sum), c(.999)) , col="red" , lwd=3, lty=5)

    legend("topright", legend=c("mean Loss","model 95% CVaR","model 99% CVaR","Basel
99.9% CVaR","model 99.9% CVaR"),
                        col=c("pink","orange","green","black", "red"), lty=5,lwd=2,
bty="n")

    # calculating EC of Portfolio, PEC

    PEC<<- quantile(apply(aggEL, 1, sum), c(.999)) - mean(apply(aggEL, 1, sum))

    mean(apply(aggEL, 1, sum))

    c(quantile(apply(aggEL, 1, sum), c(.95)),
       quantile(apply(aggEL, 1, sum), c(.99)),
          quantile(apply(aggEL, 1, sum), c(.995)),
             quantile(apply(aggEL, 1, sum), c(.999)),
                baselII(prD)[1], (NC* mean(EAD)*baselII(prD)[2]),
                                                    PEC)

}
# Histograms by sector
```

```
require(lattice)# plot by each group
histogram(~ (EAD/1000)|factor(Sector), data= loanData, nint = 10, main="
                              Exposure by Sector",xlab = "Exposure in bln", type = "density",
        panel = function(x, ...) {
              panel.histogram(x, col = "darkblue", ...)
              panel.mathdensity(dmath = dnorm, col = "red",
                                      args = list(mean=mean(x),sd=sd(x)))
        } )

require(lattice)# plot by each group
histogram(~ MEB.Rating|factor(Sector), data= loanData, nint = 10,
                        main="Ratings by Sector", xlab = "Exposure in bln", type = "density",
        panel = function(x, ...) {
              panel.histogram(x, col = "darkblue", ...)
              panel.mathdensity(dmath = dnorm, col = "red",
                                       args = list(mean=mean(x),sd=sd(x)))
        } )
##########################

plot(LGD[,10], u[,10], pch=".", main="LGD vs Economy Status", xlab="LGD", ylab="Economy
Status", col="blue")

mean(apply(aggEL,1,sum))

plot(rrho[,10], M, pch=".", main="Cor. vs Economy Status", xlab="Correlations", ylab="Economy
Status", col="blue")

plot(tPD[,10], tPD[,12], pch=".", main="Default Rates", xlab="company A", ylab="COmpany B",
col="blue")
plot(pt(tPD[,10],5), pt(tPD[,12],5), pch=".", main="Default Rates (margins)", xlab="company A",
                                                            ylab="COmpany B", col="blue")
hist(apply(N,1,sum), col="blue", main="Portfolio loss Distribution", xlab="No. of Defaults",
                                                                   ylab="Frequesncy")
quantile(apply(N,1,sum)/NC, c(.95,.99,  .995, .999))

# Expected Shortfall

mean(apply(aggEL,1,sum)[apply(aggEL,1,sum)> quantile(apply(aggEL,1,sum), c(.999))])

hist(apply(aggEL,1,sum)[apply(aggEL,1,sum)> quantile(apply(aggEL,1,sum), c(.999))],
   main="ES and Tail distribution at 99.9%", xlab="Portfolio Loss in mln", col="blue")

library(pastecs)
stat.desc(EAD)
# fubnction to generates each sector expected loss

sectorEL<<- function(secName){
```

```
#theSec<<- as.character(secName)

ina<<-which(loanData[,1] %in% loanData[loanData$Sector== secName,1])

hist(apply(aggEL[,ina],1,sum), main= paste("Loss distribution in:", secName), xlab="Sector loss
(mln) ", col="blue")

abline(v= mean(apply(aggEL[,ina],1,sum)),  col="black" , lwd=4, lty=5)

abline(v= quantile(apply(aggEL[,ina],1,sum), c(.999)) , col="red" , lwd=3, lty=5)

legend("topright", legend = c(paste("Mean =", round((mean(apply(aggEL[,ina],1,sum))), 1)),
                              paste("99.9% CVaR =", round((quantile(apply(aggEL[,ina],1,sum), c(.999))),
1))),
       bty = "n")

quantile(apply(aggEL[,ina],1,sum), c(.95,.99,  .995, .999))

SEC<<- quantile(apply(aggEL[,ina],1,sum), c(.999)) - mean(apply(aggEL[,ina],1,sum))

SRC<<- SEC/sum(EAD[ina])

plot(sum(EAD[ina]), SRC, col =c(1:5), bty="n", pch=19, cex=.75)

hist(apply(N[,ina],1,sum), main= paste("Loss distribution in sector:", secName),
                                             xlab="No. of defaulted firms (mln)", col="blue")

legend("topright", legend = c(paste("Mean =", round((mean(apply(N[,ina],1,sum))), 1)),
                              paste("99.9% CVaR =", round((quantile(apply(N[,ina],1,sum), c(.999))), 1))),
                                                                           bty = "n")
c(quantile(apply(aggEL[,ina],1,sum), c(.95,.99,  .995, .999))[1],
  quantile(apply(aggEL[,ina],1,sum), c(.95,.99,  .995, .999))[2],
  quantile(apply(aggEL[,ina],1,sum), c(.95,.99,  .995, .999))[3],
  quantile(apply(aggEL[,ina],1,sum), c(.95,.99,  .995, .999))[4],
                                     mean(apply(aggEL[,ina],1,sum)))

# sector analysisi of Basel mode

mean(PD[ina])

(1-.7118)*length(ina)* mean(EAD[ina])*  (pnorm((qnorm(mean(PD[ina])) +
sqrt(roS)*qnorm(.999))/sqrt(1- roS))- mean(PD[ina]))

roS<<- .12*(1+exp(-50* mean(PD[ina])))

roS
}
```

```
# CVaR for each company

EC<<- apply(aggEL,2 ,quantile,c(.999))- apply(aggEL,2,mean) #EC for each company at 99.9%

#ina2<<- which(EC %in% EC[EC<0])

EC<<-replace(EC, EC<0, 0)

RC<<- EC/EAD

plot(RC, col= loanData[,2], pch=19)

plot(EC, col= loanData[,2], pch=19 )

plot(EAD, RC, col= loanData[,2], pch=19,main="Counterparty Risk Analysis", xlab="Exposure (mln)", ylab="EC contribution/ EAD")

legend("bottomright", legend= levels(loanData[,2]), col =c(1:5), bty="n", pch=19, cex=.75)

#######################################################
#sectors RC to EAD
SEAD<<- c(1669761,2136620,  899890,330954 ,739650 ) # mfg, dom tr, real, service, trade
SRC<<- c( 0.39,
        0.30,
        0.16 ,
        0.31,
        0.36)
plot(SEAD, SRC, col= c(1:5), pch=19, main="Sector Risk Analysis", xlab="Exposure (mln)", ylab="EC contribution/ EAD")
legend("bottomright", legend= c("manufacturing", "Domestic Tarde", "Real Estates", "Service", "Trade"),
          col =c(1:5), bty="n", pch=19, cex=1)
,                                    legend=c("best solutions"), col=c("blue"), bty="n", pch=17)
```