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Cooperative Stock Market Game

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Abstract This study explores the game-theoretic model of the stock market, analyzing the behavior and strategic choices of market participants, and constructing a cooperative game model. The basic concepts of game theory are introduced, and the importance and practical application of game theory in the stock market are explained. A cooperative game-theoretic model is constructed, the characteristic function is established, and its superadditivity is proven. The model is analyzed, the Shapley value is derived. It is demonstrated that the Shapley value belongs to the core.

Keywords: cooperative game, core, Shapley value, stock market.

1. Introduction

In (Moorthy, 1993), the main developments in the understanding of competitive marketing strategies are reviewed. In (Karnani, 1984), a dynamic game theoretic model of marketing competition in an oligopoly is presented.Market microstructure models are also reviewed in (Allen and Morris, 2013), which applies a game theoretic lens to asset pricing. In (Sutton, 1996) and (Ullah, 2021), the recent literature on game-theoretic models is review of market structure and their empirical implementation. Agency theory and Aoki's cooperative game theory are employed to discuss differences in the governance structures of U.S. and Japanese firms and their implications for stock price reactions in (Lee, 1997). In (Shandilya, 2022), they have modeled, analyzed and compared various cost allocation methods of cooperative game theory specifically for the cost allocation in a transmission expansion planning problem. In (David WK and Petrosyan, 2012) and (Petrosyan and Sedakov, 2014), dynamic cooperative games, subgame consistent cooperation and multi-stage network games with perfect information are introduced. In (Agbo, Rousselière and Salanió, 2015), a theoretical model is constructed to study a market structure with a marketing cooperative and direct sales.

2. Several Concepts in the Stock Market

(1) Stock market. The stock market includes the primary market and the secondary market. The primary market is where stocks are issued and is the main way for listed companies to raise funds; the secondary market is also called the secondary market, where issued stocks are transferred, traded and circulated. The secondary market is built on the basis of the primary market, which provides the possibility for the issuance of stocks in the primary market. The two markets are interrelated,

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affect each other, and cannot be separated. The stock market is a place where both speculators and investors are active. Changes in the stock market are closely related to the development of the entire market economy, and it is a proxy for the economic and financial activities of a country or region.

(2) Institutions investors. Institutional investors are all kinds of legal person institutions, including financial institutions, enterprises and business legal persons, and various funds, such as securities investment funds, social security funds, and enterprise annuities.

(3) Individual investors. Individual investors refer to social natural persons engaged in securities investment. They are the most extensive investors in the stock market and the source of vitality of the market.

2.1. Information asymmetry between institutions and individuals

Although there are many individual investors in my country's stock market, they are a disadvantaged group in the market. Most individual investors cannot make money in the stock market. Usually, after a year or a cycle of bull and bear markets for several years, most individual investors lose money. There are many reasons, the main reason is that individual investors and institutional investors in the stock market are in a serious asymmetric competition, which is mainly manifested in the following aspects.

(1) Small capital. The small amount of scattered funds of individual investors cannot carry out effective investment portfolios, and they cannot compete with large-scale institutions in the game.

(2) The ability to collect and analyze information is weak. They have generally been involved in the stock market for a short time, do not have professional investment knowledge, and do not understand the characteristics of the market. It can be said with certainty that individual investors do not have the ability to comprehensively collect and analyze subject information and process it.

(3) Irrational investment. Individual investors always try to invest rationally when they enter the stock market. However, when they find that their own abilities cannot grasp the certainty of the future, they will seek psychological support from policies, the media, and experts, and then they will be affected by external influences and form understandings. The deviation on the investment will eventually lead to the irrationality of investment.

3. Construction of Cooperative Game Model

Every year, listed companies generate profits, which are distributed to shareholders holding company stocks as dividends. Major shareholders, company managers, and institutional investors hold more shares in the company and have access to a wider range of information channels, while individual investors hold fewer shares in the company and have less access to information. Therefore, major shareholders, company managers, and institutional investors can use their advantage of information asymmetry to cooperation and encroach on the interests of individual investors.

3.1. Variable description and assumptions

(1) Players (major shareholder, company manager, institutional investor).

(2) The total shareholding ratio is 1. The majority shareholder's shareholding ratio is q_1. The company manager's shareholding ratio is q_2. The institutional

investor's shareholding ratio is q_3. The individual investor's shareholding ratio is $(1-q_1-q_2-q_3)\,, 0<q_{\mathrm{i}}<1, i=1,2,3,\ q_1+q_2+q_3<1$.

(3) The company's annual dividend income is r(r>0). Major shareholders, company managers and institutional investors conduct tripartite cooperation. They cooperate to occupy dividend income m $(0<m\leq r)$. Major shareholders get $\theta_1 m$, company managers get $\theta_2 m$, and institutional investors get $(1-\theta_1-\theta_2)m,\ 0<\theta<1$.

(4) If there is no cooperation between the major shareholders, company managers and institutional investors, the major shareholders need to supervise the company managers, and the supervision cost is C_0. And the fixed income of the company managers is K_0.

(5) If the cooperation between major shareholders, corporate managers and institutional investors is found. The action is found to be punished with probability p $(0<p<1)$. Major shareholders are punished by C_1, corporate managers are punished by C_2, and institutional investors are punished by C_3.

3.2. Model building

(1) Players of the game

Assume that there are 3 independent brokers participating in the game among the relevant stakeholders of the company, and $N=\{1,2,3\}$ is the set of all players.

(2) The strategies of all players of the game

Player 1's strategy: (x_1,x_2), where x_1 is that player 1 chooses to occupy the interests of individual investors, and x_2 is that player 1 chooses not to occupy the interests of individual investors.

Player 2's strategy: (x_1,x_2), where x_1 is that player 2 chooses to occupy the interests of individual investors, and x_2 is that player 2 chooses not to occupy the interests of individual investors.

Player 3's strategy: (x_1,x_2), where x_1 is that player 3 chooses to occupy the interests of individual investors, and x_2 is that player 3 chooses not to occupy the interests of individual investors.

Assuming that among the three players, two or more players choose to occupy the interests of individual investors, the overall result is to choose to occupy the interests of individual investors. If two or more players choose not to occupy the interests of individual investors, then the overall result is not to occupy the interests of individual investors. Therefore, we can get the players' strategy profile.

Player's strategy profile and corresponding payoff function:

$$\begin{pmatrix} x_1,x_1,x_1 \\ x_1,x_1,x_2 \\ x_1,x_2,x_1 \\ x_1,x_2,x_2 \\ x_2,x_2,x_2 \\ x_2,x_2,x_1 \\ x_2,x_1,x_2 \\ x_2,x_1,x_1 \end{pmatrix} \Rightarrow \begin{pmatrix} K_i\left(x_1,x_1,x_1\right)=K_i\left(x_1\right) \\ K_i\left(x_1,x_1,x_2\right)=K_i\left(x_1\right) \\ K_i\left(x_1,x_2,x_1\right)=K_i\left(x_1\right) \\ K_i\left(x_1,x_2,x_2\right)=K_i\left(x_2\right) \\ K_i\left(x_2,x_2,x_2\right)=K_i\left(x_2\right) \\ K_i\left(x_2,x_2,x_1\right)=K_i\left(x_2\right) \\ K_i\left(x_2,x_1,x_2\right)=K_i\left(x_2\right) \\ K_i\left(x_2,x_1,x_1\right)=K_i\left(x_1\right) \end{pmatrix}, i=1,2,3.$$

(3) The payoff function of all players of the game

From the above assumptions and the player's strategy, the player's payoff function can be obtained. For convenience, we define x_1^1,x_1^2,x_1^3, it respectively means that player 1, player 2, and player 3 choose strategy 1. We define x_2^1,x_2^2,x_2^3, it respectively means that player 1, player 2, and player 3 choose strategy 2.

The payoff function of player 1 is defined as:

$$K_1\left(x_1^1, x_1^2, x_1^3\right) = q_1(r-m) + \theta_1 m - PC_1, \tag{1}$$

$$K_1\left(x_1^1, x_1^2, x_2^3\right) = q_1(r-m) + \theta_1 m - PC_1, \tag{2}$$

$$K_1\left(x_1^1, x_2^2, x_1^3\right) = q_1(r-m) + \theta_1 m - PC_1, \tag{3}$$

$$K_1\left(x_1^1, x_2^2, x_2^3\right) = q_1 r - C_0, \tag{4}$$

$$K_1\left(x_2^1, x_2^2, x_2^3\right) = q_1 r - C_0, \tag{5}$$

$$K_1\left(x_2^1, x_2^2, x_1^3\right) = q_1 r - C_0, \tag{6}$$

$$K_1\left(x_2^1, x_1^2, x_2^3\right) = q_1 r - C_0, \tag{7}$$

$$K_1\left(x_2^1, x_1^2, x_1^3\right) = q_1(r-m) + \theta_1 m - PC_1. \tag{8}$$

The payoff function of player 2 is defined as:

$$K_2\left(x_1^1, x_1^2, x_1^3\right) = q_2(r-m) + \theta_2 m - PC_2 + K_0, \tag{9}$$

$$K_2\left(x_1^1, x_1^2, x_2^3\right) = q_2(r-m) + \theta_2 m - PC_2 + K_0, \tag{10}$$

$$K_2\left(x_1^1, x_2^2, x_1^3\right) = q_2(r-m) + \theta_2 m - PC_2 + K_0, \tag{11}$$

$$K_2\left(x_1^1, x_2^2, x_2^3\right) = q_2 r + K_0, \tag{12}$$

$$K_2\left(x_2^1, x_2^2, x_2^3\right) = q_2 r + K_0, \tag{13}$$

$$K_2\left(x_2^1, x_2^2, x_1^3\right) = q_2 r + K_0, \tag{14}$$

$$K_2\left(x_2^1, x_1^2, x_2^3\right) = q_2 r + K_0, \tag{15}$$

$$K_2\left(x_2^1, x_1^2, x_1^3\right) = q_2(r-m) + \theta_2 m - PC_2 + K_0. \tag{16}$$

The payoff function of player 3 is defined as:

$$K_3\left(x_1^1, x_1^2, x_1^3\right) = q_3(r-m) + \left(1 - \theta_1 - \theta_2\right) m - PC_3, \tag{17}$$

$$K_3\left(x_1^1, x_1^2, x_2^3\right) = q_3(r-m) + \left(1 - \theta_1 - \theta_2\right) m - PC_3, \tag{18}$$

$$K_3\left(x_1^1, x_2^2, x_1^3\right) = q_3(r-m) + \left(1 - \theta_1 - \theta_2\right) m - PC_3, \tag{19}$$

$$K_3\left(x_1^1, x_2^2, x_2^3\right) = q_3 r, \tag{20}$$

$$K_3\left(x_2^1, x_2^2, x_2^3\right) = q_3 r, \tag{21}$$

$$K_3\left(x_2^1, x_2^2, x_1^3\right) = q_3 r, \tag{22}$$

$$K_3\left(x_2^1, x_1^2, x_2^3\right) = q_3 r, \tag{23}$$

$$K_3\left(x_2^1, x_1^2, x_1^3\right) = q_3(r-m) + \left(1 - \theta_1 - \theta_2\right) m - PC_3. \tag{24}$$

From the above function, we can see that we have 8 combinations of strategies, but the final result of each player is only two payoff functions.

4. Calculate the Core of the Cooperative Game

To calculate the core of the game, we need to first find $v(N)$ and $v(S),\ S \subset N$.

$v(N)$: the maximum total payoff of all players. In this case, we have three players and all possible coalitions are:

$\{1\},\ \{2\},\ \{3\}\,,$
$\{1,2\},\ \{1,3\},\ \{2,3\}\,,$
$\{1,2,3\}\,.$

The maximum payoff for each coalition is:

$\{1\}$: $q_1 r - C_0$,
$\{2\}$: $q_2 r + K_0$,
$\{3\}$: $q_3 r$,
$\{1,2\}$: $q_1(r-m) + \theta_1 m - PC_1 + q_2(r-m) + \theta_2 m - PC_2 + K_0$,
$\{1,3\}$: $q_1(r-m) + \theta_1 m - PC_1 + q_3(r-m) + (1-\theta_1-\theta_2)\, m - PC_3$,
$\{2,3\}$: $q_2(r-m) + \theta_2 m - PC_2 + K_0 + q_3(r-m) + (1-\theta_1-\theta_2)\, m - PC_3$,
$\{1,2,3\}$: $q_1(r-m) - PC_1 + q_2(r-m) - PC_2 + K_0 + q_3(r-m) + m - PC_3$.

Therefore,

$$v(N) = q_1(r-m) - PC_1 + q_2(r-m) - PC_2 + K_0 + q_3(r-m) + m - PC_3. \quad (25)$$

$v(S)$ represents the maximum payoff that can be obtained by any coalition S. In this case, we have 8 possible coalitions, which are:

$\{1\},\ \{2\},\ \{3\}\,,$
$\{1,2\},\ \{1,3\},\ \{2,3\}\,,$
$\{1,2,3\},\ \{\}\,.$

To find $v(1)$, we need to minimize the revenue of player 2 and player 3 and maximize the revenue of player 1, that is, when both player 2 and player 3 choose strategy x_2, the revenue of player 2 and player 3 is the smallest. And player 1 can choose strategy x_1 or strategy x_2.

$$\begin{cases} v(1) = \max\{\min_{2,3}\{1\}\} & \\ q_1 r - C_0, & (x_1, x_2, x_2) \\ q_1 r - C_0, & (x_2, x_2, x_2) \end{cases} \quad (26)$$

So, we can get $v(1) = q_1 r - C_0$.

To find $v(2)$, we need to minimize the revenue of player 1 and player 3 and maximize the revenue of player 2, that is, when both player 1 and player 3 choose strategy x_2, the revenue of player 1 and player 3 is the smallest. And player 2 can choose strategy x_1 or strategy x_2.

$$\begin{cases} v(2) = \max\{\min_{1,3}\{2\}\} & \\ q_2 r + K_0, & (x_2, x_1, x_2) \\ q_2 r + K_0, & (x_2, x_2, x_2) \end{cases} \quad (27)$$

So, we can get $v(2) = q_2 r + K_0$.

To find $v(3)$, we need to minimize the revenue of player 1 and player 2 and maximize the revenue of player 3, that is, when both player 1 and player 2 choose strategy x_2, the revenue of player 1 and player 2 is the smallest. And player 3 can choose strategy x_1 or strategy x_2.

$$\begin{cases} v(3) = \max\{\min_{1,2}\{3\}\} & \\ q_3 r, & (x_2, x_2, x_1) \\ q_3 r, & (x_2, x_2, x_2) \end{cases} \quad (28)$$

So, we can get $v(3) = q_3 r$.

To find $v(1,2)$, we need to minimize the revenue of player 3 and maximize the revenue of player 1 and player 2, that is, when player 3 choose strategy x_2, the revenue of player 3 is the smallest. Player 1 and player 2 can choose strategy x_1 or strategy x_2.

$$\begin{cases} v(1,2) = \max\{\min_3\{1,2\}\} \\ q_1(r-m) + \theta_1 m - PC_1 + q_2(r-m) + \theta_2 m - PC_2 + K_0, & (x_1,x_1,x_2) \\ q_1(r-m) + \theta_1 m - PC_1 + q_2 r + K_0, & (x_1,x_2,x_2) \\ q_1 r - C_0 + q_2(r-m) + \theta_2 m - PC_2 + K_0, & (x_2,x_1,x_2) \\ q_1 r - C_0 + q_2 r + K_0, & (x_2,x_2,x_2) \end{cases} \tag{29}$$

So, $v(1,2) = q_1(r-m) + \theta_1 m - PC_1 + q_2(r-m) + \theta_2 m - PC_2 + K_0$.

To find $v(1,3)$, we need to minimize the revenue of player 2 and maximize the revenue of player 1 and player 3, that is, when player 2 choose strategy x_2, the revenue of player 2 is the smallest. Player 1 and player 3 can choose strategy x_1 or strategy x_2.

$$\begin{cases} v(1,3) = \max\{\min_2\{1,3\}\} \\ q_1(r-m) + \theta_1 m - PC_1 + q_3(r-m) + (1-\theta_1-\theta_2)\, m - PC_3, & (x_1,x_2,x_1) \\ q_1(r-m) + \theta_1 m - PC_1 + q_3 r, & (x_1,x_2,x_2) \\ q_1 r - C_0 + q_3(r-m) + (1-\theta_1-\theta_2)\, m - PC_3, & (x_2,x_2,x_1) \\ q_1 r - C_0 + q_3 r, & (x_2,x_2,x_2) \end{cases} \tag{30}$$

So, $v(1,3) = q_1(r-m) + \theta_1 m - PC_1 + q_3(r-m) + (1-\theta_1-\theta_2)\, m - PC_3$.

To find $v(2,3)$, we need to minimize the revenue of player 1 and maximize the revenue of player 2 and player 3, that is, when player 1 choose strategy x_2, the revenue of player 1 is the smallest. Player 2 and player 3 can choose strategy x_1 or strategy x_2.

$$\begin{cases} v(2,3) = \max\{\min_1\{2,3\}\} \\ q_2(r-m) + \theta_2 m - PC_2 + K_0 + q_3(r-m) + (1-\theta_1-\theta_2)\, m - PC_3) \\ q_2(r-m) + \theta_2 m - PC_2 + K_0 + q_3 r) \\ q_2 r + K_0 + q_3(r-m) + (1-\theta_1-\theta_2)\, m - PC_3) \\ q_2 r + K_0 + q_3 r) \end{cases} \tag{31}$$

The strategy chosen by the player in the above formula is (x_2,x_1,x_1), (x_2,x_1,x_2), (x_2,x_2,x_1),(x_2,x_2,x_2).

So, $v(2,3) = q_2(r-m) + \theta_2 m - PC_2 + K_0 + q_3(r-m) + (1-\theta_1-\theta_2)\, m - PC_3$.

$$v(1,2,3) = v(N) \tag{32}$$

So, $v(1,2,3) = q_1(r-m) - PC_1 + q_2(r-m) - PC_2 + K_0 + q_3(r-m) + m - PC_3$.

Now let's prove superadditivity.

$$v(1,2,3) \geq v(1,2) + v(3), \tag{33}$$

$$(1-\theta_1-\theta_2-q_3)m - PC_3 \geq 0, \tag{34}$$

$$v(1,2) \geq v(1) + v(2), \tag{35}$$

$$(\theta_1 - q_1)m - PC_1 + (\theta_2 - q_2)m - PC_2 + C_0 \geq 0, \tag{36}$$

$$v(1,2,3) \geq v(1) + v(2,3), \tag{37}$$

$$(\theta_1 - q_1)m - PC_1 + C_0 \geq 0, \tag{38}$$

$$v(1,2,3) \geq v(2) + v(1,3), \tag{39}$$

$$(\theta_2 - q_2)m - PC_2 \geq 0. \tag{40}$$

Similarly we can get

$$v(1,3) \geq v(2) + v(3), \tag{41}$$

$$v(2,3) \geq v(1) + v(2). \tag{42}$$

Suppose the players in the cooperative game (N, v) have come to an agreement on distribution of a payoff to the whole coalition N (imputation α^*), under which none of the imputations dominates α^*. Then such a distribution is stable in that it is disadvantageous for any coalition S to separate from other players and distribute a payoff $v(S)$ among its members. This suggests that it may be wise to examine the set of nondominant imputations.

Definition 1. The set of nondominant imputations in the cooperative game (N, v) is called the core.

For the imputation α belong to the core, it is necessary and sufficient that

$$v(S) \leq \alpha(S) = \sum_{i \in S} \alpha_i \tag{43}$$

hold for all $S \subset N$.

The vector $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ in the cooperative game belongs to the core if and only if

$$\begin{cases} \alpha_1 \geqslant v(1), \\ \alpha_2 \geqslant v(2), \\ \alpha_3 \geqslant v(3), \\ \alpha_1 + \alpha_2 \geqslant v(1,2), \\ \alpha_1 + \alpha_3 \geqslant v(1,3), \\ \alpha_2 + \alpha_3 \geqslant v(2,3), \\ \alpha_1 + \alpha_2 + \alpha_3 = v(1,2,3). \end{cases} \tag{44}$$

Now, we know the payoff of each coalition. From this we can write the inequality and find the core.

$$\begin{cases} \alpha_1 \geqslant q_1 r - C_0, \\ \alpha_2 \geqslant q_2 r + K_0, \\ \alpha_3 \geqslant q_3 r, \\ \alpha_1 + \alpha_2 \geqslant (q_1 + q_2)(r - m) + (\theta_1 + \theta_2)m - P(C_1 + C_2) + K_0, \\ \alpha_1 + \alpha_3 \geqslant (q_1 + q_3)(r - m) + (1 - \theta_2)\, m - P(C_1 + C_3), \\ \alpha_2 + \alpha_3 \geqslant (q_2 + q_3)(r - m) + (1 - \theta_1)\, m - P(C_2 + C_3) + K_0, \\ \alpha_1 + \alpha_2 + \alpha_3 = (q_1 + q_2 + q_3)(r - m) - P(C_1 + C_2 + C_3) + m + K_0. \end{cases} \tag{45}$$

Above inequalities, we define the core. We can find one of imputation from one core. We get $\alpha^* = (q_1(r-m) + \theta_1 m - PC_1, q_2(r-m) + \theta_2 m - PC_2 + K_0, q_3(r-m) + (1 - \theta_1 - \theta_2)\, m - PC_3)$. The imputation α^* is a fair compromise from the interior of the core.

5. Computing Shapley Values for Cooperative Games

The Shapley value is a concept in cooperative game theory that measures the average marginal contribution of each player to the coalitions in the game. It assigns a unique payoff distribution to each player in the game, based on their contribution to every possible coalition that can be formed. Essentially, it determines the fair distribution of the total payoff among the players in a game where cooperation is necessary to achieve certain goals. It is calculated as follows:

$$Sh_i(v) = \sum_{S \subseteq N, i \in S} \frac{(|S|-1)!(n-|S|)!}{n!}[v(S) - v(S \backslash \{i\})], \tag{46}$$

where, Sh_i is the Shapley Value of player i, n is the total number of players in the game, S is a coalition of players excluding player i, $|S|$ is the number of players in the coalition S, $v(S)$ is the value of the coalition S, $v(S) - v(S\backslash\{i\})$ is contribution of member i in the coalition.

Now, we can calculate the Shapley values of Player 1, Player 2, and Player 3 respectively. We need to know the contribution of each player in each coalition. The coalition containing player 1 has $\{1\}, \{1,2\}, \{1,3\}, \{1,2,3\}$, the coalition containing player 2 has $\{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}$, and the coalition containing player 3 has $\{3\}, \{1,3\}, \{2,3\}, \{1,2,3\}$.

$$\begin{aligned}
Sh_1 = {} & \frac{2}{6}(q_1 r - C_0) + \frac{1}{6}[q_1(r-m) + \theta_1 m - PC_1 + q_2(r-m) + \theta_2 m \\
& -PC_2 + K_0 - q_2 r - K_0] + \frac{1}{6}[q_1(r-m) + \theta_1 m - PC_1 + q_3(r-m) \\
& + (1-\theta_1-\theta_2)m - PC_3 - q_3 r] + \frac{2}{6}[q_1(r-m) - PC_1 + q_2(r-m) \\
& -PC_2 + K_0 + q_3(r-m) + m - PC_3] - \frac{2}{6}[q_2(r-m) + \theta_2 m - PC_2 \\
& +K_0 + q_3(r-m) + (1-\theta_1-\theta_2)m - PC_3] \\
& = q_1 r + \frac{1}{6}m(1 - 4q_1 - q_2 - q_3 + 3\theta_1) - \frac{1}{6}P(4C_1 + C_2 + C_3) - \frac{2}{6}C_0,
\end{aligned} \tag{47}$$

$$\begin{aligned}
Sh_2 = {} & \frac{2}{6}(q_2 r + K_0) + \frac{1}{6}[q_1(r-m) + \theta_1 m - PC_1 + q_2(r-m) + \theta_2 m \\
& -PC_2 + K_0 - q_1 r + C_0] + \frac{1}{6}[q_2(r-m) + \theta_2 m - PC_2 + q_3(r-m) + K_0 \\
& + (1-\theta_1-\theta_2)m - PC_3 - q_3 r] + \frac{2}{6}[q_1(r-m) - PC_1 + q_2(r-m) \\
& -PC_2 + K_0 + q_3(r-m) + m - PC_3] - \frac{2}{6}[q_1(r-m) + \theta_1 m - PC_1 \\
& +q_3(r-m) + (1-\theta_1-\theta_2)m - PC_3] \\
& = q_2 r + \frac{1}{6}m(1 - q_1 - 4q_2 - q_3 + 3\theta_2) - \frac{1}{6}P(C_1 + 4C_2 + C_3) + K_0 + \frac{1}{6}C_0,
\end{aligned} \tag{48}$$

$$\begin{aligned} Sh_3 =& \frac{2}{6} q_3 r + \frac{1}{6} \left[q_1 (r-m) + \theta_1 m - PC_1 + q_3 (r-m) + (1-\theta_1-\theta_2)\, m \right. \\ & \left. -PC_3 - q_1 r + C_0 \right] + \frac{1}{6} \left[q_2 (r-m) + \theta_2 m - PC_2 + K_0 + q_3 (r-m) \right. \\ & \left. + (1-\theta_1-\theta_2)\, m - PC_3 - q_2 r - K_0 \right] + \frac{2}{6} \left[q_1 (r-m) - PC_1 + q_2 (r-m) \right. \\ & \left. -PC_2 + K_0 + q_3 (r-m) + m - PC_3 \right] - \frac{2}{6} \left[q_1 (r-m) + \theta_1 m - PC_1 \right. \\ & \left. + q_2 (r-m) + \theta_2 m - PC_2 + K_0 \right] \\ & = q_3 r + \frac{1}{6} m \left(4 - q_1 - q_2 - 4 q_3 - 3\theta_1 - 3\theta_2 \right) - \frac{1}{6} P \left(C_1 + C_2 + 4 C_3 \right) + \frac{1}{6} C_0. \end{aligned} \tag{49}$$

It is clear that

$$\sum_{i=1}^{3} Sh_i = v(1,2,3), \tag{50}$$

$$\begin{aligned} & q_1 r + \tfrac{1}{6} m \left(1 - 4q_1 - q_2 - q_3 + 3\theta_1\right) - \tfrac{1}{6} P \left(4C_1 + C_2 + C_3\right) - \tfrac{2}{6} C_0 + q_2 r \\ & + \tfrac{1}{6} m \left(1 - q_1 - 4q_2 - q_3 + 3\theta_2\right) - \tfrac{1}{6} P \left(C_1 + 4C_2 + C_3\right) + K_0 + \tfrac{1}{6} C_0 + q_3 r \\ & + \tfrac{1}{6} m \left(4 - q_1 - q_2 - 4q_3 - 3\theta_1 - 3\theta_2\right) - \tfrac{1}{6} P \left(C_1 + C_2 + 4C_3\right) + \tfrac{1}{6} C_0 = \\ & q_1 (r-m) - PC_1 + q_2 (r-m) - PC_2 + K_0 + q_3 (r-m) + m - PC_3. \end{aligned} \tag{51}$$

The Shapley value reflects the mutual restraint among major shareholders, company managers and institutional investors in the cooperation. By controlling the marginal utility of one party, it can affect the "fair" distribution of the other two parties in the cooperation. The purpose of supervision is to make one of the three withdraw from the cooperation considering the unfair distribution brought about by the cooperation.

5.1. Verify that the Shapley value belongs to the core

we obtained the Shapley value of each player by calculation and got the core. We need to judge whether the Shapley value belongs to the core, then we must prove that the following inequality holds.

$$Sh_1 + Sh_2 \geq v(1,2), \tag{52}$$

$$\begin{aligned} & q_1 r + \frac{1}{6} m \left(1 - 4q_1 - q_2 - q_3 + 3\theta_1\right) - \frac{1}{6} P \left(4C_1 + C_2 + C_3\right) - \frac{2}{6} C_0 + q_2 r \\ & \quad + \frac{1}{6} m \left(1 - q_1 - 4q_2 - q_3 + 3\theta_2\right) - \frac{1}{6} P \left(C_1 + 4C_2 + C_3\right) + K_0 + \frac{1}{6} C_0 \\ & \qquad \geq q_1 (r-m) + \theta_1 m - PC_1 + q_2 (r-m) + \theta_2 m - PC_2 + K_0 \end{aligned} \tag{53}$$

$$\frac{1}{6} m (2 + q_1 + q_2 - 2q_3 - 3\theta_1 - 3\theta_2) + \frac{1}{6} P (C_1 + C_2 - 2C_3) - \frac{1}{6} C_0 \geqslant 0, \tag{54}$$

$$Sh_2 + Sh_3 \geq v(2,3), \tag{55}$$

$$\begin{aligned} & q_2 r + \frac{1}{6} m \left(1 - q_1 - 4q_2 - q_3 + 3\theta_2\right) - \frac{1}{6} P \left(C_1 + 4C_2 + C_3\right) + K_0 + \frac{1}{6} C_0 + \\ & q_3 r + \frac{1}{6} m \left(4 - q_1 - q_2 - 4q_3 - 3\theta_1 - 3\theta_2\right) - \frac{1}{6} P \left(C_1 + C_2 + 4C_3\right) + \frac{1}{6} C_0 \\ & \geq q_2 (r-m) + \theta_2 m - PC_2 + K_0 + q_3 (r-m) + (1-\theta_1-\theta_2)\, m - PC_3 \end{aligned} \tag{56}$$

$$\frac{1}{6}m(q_2+q_3-1-2q_1+3\theta_1)+\frac{1}{6}P(C_2-2C_1+C_3)+\frac{2}{6}C_0\geqslant 0, \tag{57}$$

$$Sh_1+Sh_3\geq v(1,3) \tag{58}$$

$$\begin{aligned} q_1r+\frac{1}{6}m\left(1-4q_1-q_2-q_3+3\theta_1\right)-\frac{1}{6}P\left(4C_1+C_2+C_3\right)-\frac{2}{6}C_0+q_3r \\ +\frac{1}{6}m\left(4-q_1-q_2-4q_3-3\theta_1-3\theta_2\right)-\frac{1}{6}P\left(C_1+C_2+4C_3\right)+\frac{1}{6}C_0 \\ \geq q_1(r-m)+\theta_1 m-PC_1+q_3(r-m)+\left(1-\theta_1-\theta_2\right)m-PC_3 \end{aligned} \tag{59}$$

$$\frac{1}{6}m(q_1-5q_2+q_3-1+3\theta_2)+\frac{1}{6}P(C_1-2C_2+C_3)-\frac{1}{6}C_0\geqslant 0. \tag{60}$$

Through the proof of the above inequality, we can get the conclusion that the Shapley value belongs to the core.

6. Advice to Individual Investors

In Chapter 3, we established a cooperative game model among major shareholders, corporate managers, and institutional investors. From this model, we can see that major shareholders, corporate managers, and institutional investors have significant advantages in terms of both funding and information, while individual investors are at a disadvantage due to their smaller financial resources and information asymmetry. This can easily lead to the expropriation of the dividends that individual investors deserve. Based on the previous model analysis, this chapter proposes relevant recommendations for individual investors to better protect their earnings from being expropriated and to survive in the stock market in the long term.

(1) Learn to leverage the power of regulatory agencies

In the analysis of Chapter 3, we can see that the only way to constrain insider trading among major shareholders, corporate managers, and institutional investors is through the punishment of regulatory agencies for violations. We can make recommendations to regulatory agencies to strengthen their supervision of listed companies and make them more transparent. If a company experiences a major incident, it should promptly release a public announcement so that more investors can understand the changes in the company and make reasonable investment decisions. At the same time, we should increase the punishment for investors who violate regulations. If the punishment is not severe enough, they will still be able to obtain excess profits after paying the fine, which defeats the purpose of regulation. Individual investors should also have a certain level of legal awareness. If their rights are truly infringed upon, they should be able to report it to the regulatory agency to better protect their earnings.

(2) Improve the ability to collect and analyze information

If individual investors want to survive better in the stock market, they must have the ability to independently collect and analyze information.Listed companies generally announce major events and issue quarterly and annual reports on their operations. If you can identify companies with good business performance, high profitability, and growth potential from these financial reports and hold them for the long term, it will greatly help your investment decisions. However, the information provided by listed companies is not always true. If a company makes a significant mistake in its operations that leads to a lack of capital turnover, or even bankruptcy risk, but the listed company does not promptly disclose the news and instead tells investors that everything is normal, it can cause huge losses if individual investors lack the ability to analyze and discern the news. Therefore, improving the ability to collect and analyze information is very important, and it will help you survive better in the market.

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