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Learning of Supply Chain Cost-Revenue Sharing Contract in the Form of Trade-off Mechanism in Pollution Control Problem [*]

Shimai Su

St.Petersburg State University,
7/9, Universitetskaya nab., St. Petersburg, 199034, Russia
E-mail: `st073379@student.spbu.ru`

Abstract A comprehensive comparison between supply chain cost-revenue sharing contract and trade-off mechanism is considered. Either of the two main members – manufacturer and retailers in the supply chain acts as the bellwether to initiate the green transition program through a cost-revenue sharing contract. While the trade-off mechanism which is applied in the pollution control problem is inspired by the sharing setting behind the contract, i.e., the player transfers part of her profit to another player in trade for her limited responsibility in reducing pollution. However, the power structure is not included in the trade-off mechanism where a third party should be designated to implement the details. The conclusion is given in the end.

Keywords: supply chain, trade-off mechanism, pollution control, differential game.

1. Introduction

A supply chain consists of all parties involved, directly or indirectly, in fulfilling a customer request (Chopra and Meindl, 2013). Typically, a supply chain consists of five primary members: supplies, manufactures, distributors, retailers and consumers. Depending on the frequency of each decision made by the members and functioning time, the supply chain decision are classified into three phases starting from the strategic decision which aims for the long term strategies to the tactical decision and lastly, the operational decision which is made weekly or daily. While the majority of researches with regard to the low-carbon supply chain take the operational decision approach because the pollution rate is measured over the course of days. And in general, the purpose of supply chain is to serve the need of customers and bring profit to itself. But the need of customers has been updated under the background of global climate change. The customer with awareness of low-carbon consumption prefers to buy the products from environment-friendly parties and this change motivates the rest of members in the supply chain to promote the green activity programs. We observe that lot of literature (Wang et al., 2019, De Giovanni and Zaccour, 2013, Ji et al., 2017, Li et al., 2022, Wu et al., 2022) are concentrating on the interaction between two prominent supply chain members – manufactures and retailers. One possible reason is that these two members are of larger importance in steering the economic activities.

Additionally, when it comes to the supply chain model, basically, there are two principal supply chain models – forward supply chain and closed-loop sup-

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ply chain. Forward supply chain indicates that the flow of the product is unidirectional through the chain. The manufactures have no place in reverse activities. Meanwhile, Closed-Loop Supply Chain (CLSC) in which the used product can be recycled and sold again after remanufacturing is an eco-friendly and profitable model compared with the former. There is also a description about duel channel supply chain (Ji et al., 2017) in which the products can be brought to the market through online and retailer two channels taking into account the current growth of e-commerce. Moreover, on the basis of these models, the constraints or policies which the supply chain is utilized to accomplish the green upgrade can be diverse, such as carbon tax (Wang et al., 2019, Wu et al., 2022), cap-and-trade (Ghosh et al., 2020, Li et al., 2022, Zhang et al., 2021), green supply chain management (Herrmann et al., 2021, Zhao et al., 2012), consumers' low-carbon preference (Ghosh et al., 2020, Ji et al., 2017, Wang et al., 2019, Ye et al., 2017), low-carbon subsidy (Wu et al., 2022, Zhu et al., 2011), green activity program and contract design (De Giovanni and Zaccour, 2013). The trade-off mechanism designed in (Su and Parilina, 2023) is exactly learning from contract design, in which the retailer share the manufacturer's sales profit and the cost of green activities.

The paper is well organized as follows. In Section 2, we introduce how the cost-revenue sharing contract works in supply chain and the trade-off mechanism is elaborately described in Section 3. In Section 4, we compare cost-revenue sharing contract with trade-off mechanism both in similarities and differences. Section 5 concludes the paper.

2. Cost-Revenue Sharing Contract in Supply Chain

The cost-revenue sharing contract (De Giovanni and Zaccour, 2013) in supply chain is proposed to kick off the green transition. This two-player dynamic game is played by one manufacturer and one retailer and the manufacturer can produce the product through the new purchased materials or the recycling goods. The recycling activity here represents efforts to effectively reduce the pollution generated during the manufacturing process and the manufacturer can increase the return rate by investing a green activity programs which contain advertising incentives, employees-training programs and etc. The general idea of the contract is that the manufacturer transfers part of her revenues to retailer, and the retailer takes part of the cost generated by the manufacturer's green activities. It's assumed that the retailer is the leader and declares the support rate for the green activities from the manufacturer and the manufacturer is indeed a follower in this case. More specifically, in this game-theoretic model, the level of green activities is presented as $A(t)$ and the cost for them is approximated by $C(A(t)) = (u_M A(t)^2)/2$. Besides, $\alpha - \beta p(t)$ indicates the demand for the manufacturer's product, where $\alpha, \beta > 0$ denote the market potential and marginal effect of pricing on current sales respectively. $p(t)$ tells the retail price which is manipulated by the retailer and ω is the constant wholesale price charged by the manufacturer. Since the green activities bring the recycling of used product, the return rate is denoted as $r(t)$. Correspondingly, $I(r(t))$ shows the state-dependent incentive offered by the manufacturer and $B(t)$ demonstrates the support rate given by retailer.

Integrating all the necessary parameters as mentioned, the cost-revenue sharing contract reflecting in the objective functions of the manufacturer M and retailer R gives,

$$J_M = \int_0^\infty e^{-\rho t}\Big((\alpha - \beta p(t))(\omega - C(r(t)) - I(r(t))) - \frac{u_M}{2}(1 - B(t))A(t)^2\Big)dt, \tag{1}$$

$$J_R = \int_0^\infty e^{-\rho t}\Big((\alpha - \beta p(t))(p(t) - \omega + I(r(t))) - \frac{u_M}{2}B(t)A(t)^2\Big)dt, \tag{2}$$

where $\rho > 0$ is the common discount rate.

Now let's take a good look at the objective functions (1) and (2). The cost-revenue sharing idea is right hidden in these equations. For the objection function (1) of manufacturer M, $\big(\alpha - \beta p(t)\big)I(r(t))$ is the part of her revenue that she transfers to the retailer over time. In return, the retailer covers $B(t)A(t)^2 u_m/2$ part of cost for implementing the green activities as shown in (2). What's more, this idea can be applied in environmental agreements, in which the players' behaviors resemble those of manufacturers and retailers.

3. Trade-Off Mechanism in Pollution Control Problem

The trade-off mechanism (Su and Parilina, 2023) is inspired by the cost-revenue sharing contract described in Section 2. However, before explaining how the trade-off mechanism was learnt from cost-revenue sharing contract, it is beneficial to compare with other two common approaches so that the role of trade-off mechanism can be understood within a whole picture.

In the two-player differential game of pollution control, two different types of countries – developed and developing countries can form in assorted ways. In a noncooperative scenario, both players individually seek to maximize their profits. Obviously, this kind of behavior is not environmentally friendly, i.e., does not solve the pollution issue. Meanwhile, in a cooperative scenario, players turn to maximize their joint profit, which provides the opportunity to address environmental problem, i.e. to reduce the pollution stock and to obtain the largest joint payoff. While, there are several problems regarding to the realization of a cooperative scenario, and among them, the first problem is how to allocate the joint profit fairly between two players, and the second one is how to achieve a full cooperative behavior, especially, when full coordination of the players' behavior is questionable in reality.

Therefore, we consider the third scenario in (Su and Parilina, 2023), in which the cooperation reached in a contract is different from a fully cooperative scenario, but it is carried through a trade-off mechanism, usually used in supply chain coordination (De Giovanni and Zaccour, 2022, Kuchesfehani et al, 2022). This mechanism is a form of cooperative behavior proposed to find an efficient solution to mitigate the pollution damage, but loosely, it does not require full coordination of players' behavior over time. As shown in the proposed trade-off mechanism, although two players are still acting by maximizing their own profits, which gives us a fake image that they are behaving in a noncooperative way, there is a trade of profit and pollution between them. A vulnerable player, i.e., developed country compensates an invulnerable player's costs on taking the jobs in the production reduction by transferring the share of her profits to the latter.

3.1. The Trade-Off Mechanism Model

In the pollution-control differential game with one developed and one developing countries, the dynamics of the pollution stock S also shown in (Fanokoa et al., 2010, Masoudi and Zaccour, 2013) are given by

$$\dot{S}(t) = \mu \sum_{i \in N} e_i(t) - \varepsilon S(t), \ S(0) = S_0, \tag{3}$$

where $e_i(t)$ denotes the quantity of emissions generated by player i, $\mu > 0$ is the marginal influence on pollution accumulation of the players' emissions, and $\varepsilon > 0$ is the nature's absorption rate.

The trade-off mechanism assumes that players agree on two parameters: (i) the compensation coefficient $0 < \tau < 1$ showing the part of profit given by a vulnerable player to an invulnerable player for appealing the latter to tackle the pollution issue, (ii) the cost coefficient $0 < \theta < 1$ indicating the magnitude of pollution amount that an invulnerable player should be responsible for. The parameters (τ, θ) can be interpreted as a contract between two players and can be negotiated. These parameters are exogenously given, but one can assume them as decision variables of the players in the process of negotiations. Obviously, the feedback-Nash equilibrium significantly depends on the values of (τ, θ). Under this condition, an invulnerable player's payoff function takes the form:

$$\max_{e_2 > 0} W_2 = \int_0^\infty e^{-\rho t} \Big(\alpha_2 e_2(t) + \tau \alpha_1 e_1(t) - \frac{1}{2} e_2^2(t) - \frac{1}{2} \beta_1 \theta S^2(t) \Big) dt, \tag{4}$$

while a vulnerable player's payoff function is

$$\max_{e_1 > 0} W_1 = \int_0^\infty e^{-\rho t} \Big((1-\tau) \alpha_1 e_1(t) - \frac{1}{2} e_1^2(t) - \frac{1}{2} \beta_1 (1-\theta) S^2(t) \Big) dt. \tag{5}$$

The part of revenue that the vulnerable player compensates to invulnerable player is $\tau \alpha_1 e_1(t)$ over time as shown in (4). At the same time, the invulnerable player is obligatory to accept part of the pollution reduction mission $\beta_1 \theta S^2(t)/2$. Similar operations are witnessed in the trade-off mechanism again.

Proposition 1. *In a trade-off mechanism scenario, the feedback-Nash equilibrium in a two-player differential game defined by objective functions (4) and (5) s.t. (3), is given by*

$$\begin{aligned} e_1^{ToM}(t) &= \alpha_1(1-\tau) + \mu(x_1 S^{ToM}(t) + y_1), \\ e_2^{ToM}(t) &= \alpha_2 + \mu(x_2 S^{ToM}(t) + y_2), \end{aligned}$$

where x_1, x_2, y_1, and y_2 are the solutions of the system of the equations (10) *given in the proof.*

The corresponding equilibrium state trajectory is

$$S^{ToM}(t) = \frac{\mu B + \mu^2 y_{12}}{\mu^2 x_{12} - \varepsilon} (e^{(\mu^2 x_{12} - \varepsilon)t} - 1) + e^{(\mu^2 x_{12} - \varepsilon)t} S_0,$$

where $x_{12} = x_1 + x_2$, $y_{12} = y_1 + y_2$, and $B = \alpha_1(1-\tau) + \alpha_2$.

The steady state stock of emissions is

$$S_\infty^{ToM} = \frac{\mu B + \mu^2 y_{12}}{\varepsilon - \mu^2 x_{12}},$$

which is globally asymptotically stable when $\mu^2 x_{12} - \varepsilon < 0$.

The Nash equilibrium players' payoffs are

$$V_1^{ToM} = \frac{1}{2}x_1 S_0^2 + y_1 S_0 + z_1,$$
$$V_2^{ToM} = \frac{1}{2}x_2 S_0^2 + y_2 S_0 + z_2,$$

where z_1 *and* z_2 *are defined in the proof.*

Proof. The optimization problem for each player is

$$W_1^{ToM} = \int_0^\infty e^{-\rho t}\Big(\alpha_1 e_1(t)(1-\tau) - \frac{1}{2}e_1^2 - \frac{1}{2}\beta_1(1-\theta)S^2(t)\Big)dt \to \max_{e_1 \geq 0}, \quad (6)$$

$$W_2^{ToM} = \int_0^\infty e^{-\rho t}\Big(\alpha_2 e_2(t) + \tau(\alpha_1 e_1(t)) - \frac{1}{2}e_2^2(t) - \frac{1}{2}\beta_1 \theta S^2(t)\Big)dt \to \max_{e_2 \geq 0}. \quad (7)$$

Assuming the linear-quadratic form of the value functions $V_1(S) = \frac{1}{2}x_1 S^2 + y_1 S + z_1$ and $V_2(S) = \frac{1}{2}x_2 S^2 + y_2 S + z_2$, we write down the HJB equations for (6) and (7):

$$\rho V_1(S) = \max_{e_1}\Big\{\alpha_1 e_1(1-\tau) - \frac{1}{2}e_1^2 - \frac{1}{2}\beta_1(1-\theta)S^2 + V_1'(S)[\mu(e_1+e_2) - \varepsilon S]\Big\}, \quad (8)$$

$$\rho V_2(S) = \max_{e_2}\Big\{\alpha_2 e_2 + \tau(\alpha_1 e_1) - \frac{1}{2}e_2^2 - \frac{1}{2}\beta_1 \theta S^2 + V_2'(S)[\mu(e_1+e_2) - \varepsilon S]\Big\}. \quad (9)$$

Maximizing the expression in RHS in (8), we obtain that $e_1 = \alpha_1 + \mu V_1'(S)$, and maximizing the expression in RHS in (9), we obtain that $e_2 = \alpha_2 + \mu V_2'(S)$. Taking into account the derivatives $V_1'(S) = x_1 S + y_1$, $V_2'(S) = x_2 S + y_2$, and substituting these expressions into (8), we obtain an equation:

$$\begin{aligned}\rho\Big(\frac{1}{2}x_1 S^2 + y_1 S + z_1\Big) &= \alpha_1(1-\tau)[\alpha_1(1-\tau) + \mu(x_1 S + y_1)] - \\ &- \frac{1}{2}[\alpha_1(1-\tau) + \mu(x_1 S + y_1)]^2 - \frac{1}{2}\beta_1(1-\theta)S^2 + \\ &+ (x_1 S + y_1)\Big(\mu[\alpha_1(1-\tau) + \alpha_2 + \mu(x_1 S + y_1 + x_2 S + y_2)] - \varepsilon S\Big).\end{aligned}$$

Taking into account the derivative $V_2'(S) = x_2 S + y_2$, and substituting the expressions into (9), we obtain an equation:

$$\begin{aligned}\rho\Big(\frac{1}{2}x_2 S^2 + y_2 S + z_2\Big) &= \alpha_2[\alpha_2 + \mu(x_2 S + y_2)] + \\ &+ \tau\alpha_1[\mu(x_1 S + y_1) + \alpha_1(1-\tau)] - \frac{1}{2}[\alpha_2 + \mu(x_2 S + y_2)]^2 - \frac{1}{2}\beta_1\theta S^2 + \\ &+ (x_2 S + y_2)\Big(\mu[\mu(x_1 S + y_1) + \alpha_1(1-\tau) + \alpha_2 + \mu(x_2 S + y_2)] - \varepsilon S]\Big).\end{aligned}$$

By identification, two linear quadratic equations containing x_1, x_2 can be written as

$$\begin{aligned} &\mu^2x_1^2 + 2\mu^2x_1x_2 - 2\varepsilon x_1 - \rho x_1 - \beta_1(1-\theta) = 0,\\ &\mu^2x_2^2 + 2\mu^2x_1x_2 - 2\varepsilon x_2 - \rho x_2 - \beta_1\theta = 0. \end{aligned}$$

Rewriting these equations which should be solved to find x_1 and x_2, and summarizing with the rest of equations, we obtain the system:

$$\begin{cases} 3\mu^4x_1^4 - 4\mu^2(2\varepsilon+\rho)x_1^3 + \big((2\varepsilon+\rho)^2 + 6\mu^2\beta_1\theta - 2\mu^2\beta_1\big)x_1^2 - (1-\theta)^2\beta_1^2 = 0,\\ 3\mu^4x_2^4 - 4\mu^2(2\varepsilon+\rho)x_2^3 + \big((2\varepsilon+\rho)^2 - 6\mu^2\beta_1\theta + 4\beta_1\mu^2\big)x_2^2 - \beta_1^2\theta^2 = 0,\\ y_1 = \dfrac{\mu^3x_1[(x_2B + \tau\alpha_1x_1)A - \mu^2x_1x_2B]}{A(A^2-\mu^4x_1x_2)} - \dfrac{\mu x_1B}{A},\\ y_2 = \dfrac{\mu^3x_1x_2B - \mu(x_2B + \tau\alpha_1x_1)A}{A^2-\mu^4x_1x_2},\\ z_1 = \dfrac{2\mu y_1B + \alpha_1^2(1-\tau)^2 + \mu^2y_1^2 + 2\mu^2y_1y_2}{2\rho},\\ z_2 = \dfrac{2\mu y_2B + \alpha_2^2 + 2\alpha_1^2\tau(1-\tau) + \mu^2y_2^2 + 2\mu^2y_1y_2 + \tau\alpha_1\mu y_1}{2\rho}, \end{cases} \tag{10}$$

where $A = \mu^2x_1 + \mu^2x_2 - \rho - \varepsilon$ and $B = \alpha_1(1-\tau) + \alpha_2$.

In the system (10), we need to solve the first two equations, then substituting x_1 and x_2 into the rest four equations we find y_1, y_2, z_1, and z_2. We should notice that we require that x_1, x_2 be negative to prove the stability of the steady state.

The expression of the equilibrium stock $S^{ToM}(t)$ is obtained as a solution of equation (3) and it is given by (1). If t tends to infinity in (3), we obtain the steady state of emission stock given by (1), which globally asymptotically stable when $\mu^2x_{12} - \varepsilon < 0$.

3.2. Numerical Example

In this section, we present a numerical example to illustrate the performance of a trade-off mechanism with respect to the values of (τ, θ). The parameters of the game are

$$\begin{aligned} &\beta_1 = 2,\ \alpha_1 = 7,\ \alpha_2 = 4,\\ &\varepsilon = 0.5,\ \mu = 0.3,\ \rho = 0.2,\ S_0 = 2. \end{aligned}$$

As shown in Table 1, we specially select three sets of (τ, θ) to track the change of the profit for both players. It is anticipated that the developing country obtains less profit when she is required to be responsible for massive pollution reduction works as indicated in the first column. When the cost and revenue are equally shared by two players and when the developed country gives more than half of her profit to incentivize the developing country to deal with more than half of the pollution reduction task, the developing country gets much more than developed country. One possible reason for this situation is that for developing country, the cost for reducing pollution is not comparable with the profit obtained from developed country. In addition, from the last row, the results tell us that the pollution stock declines with more compensation to the developing country. This also corroborates the fact that the develop country has a leading position in the climate change problem.

Table 1. Benefits from adopting trade-off mechanism for three sets of (τ, θ).

(τ, θ)	$(0.2, 0.8)$	$(0.5, 0.5)$	$(0.7, 0.6)$
Player 1	87.15	15.12	2.42
Player 2	3.85	64.54	60.20
Steady State	4.56	3.40	2.65

4. Comparison between Cost-Revenue Sharing Contract and Trade-Off Mechanism

The difference between cost-revenue sharing contract in supply chain and trade-off mechanism in environmental agreements may seem ambiguous from first glance if we pay attention to their objective functions. First of all, the most critical reason that the cost-revenue sharing method can be learnt from and reform in the trade-off mechanism is in both cases, the multi-objectives can be consolidated into one objective. For instance, in principle, the manufacturer and retailer should aim to maximize their individual profit. However, the green activity program, supported by the cost-revenue sharing contract, can benefit both of them by attracting more customers due to the production of low-carbon products. Similar logic applies to environmental agreements.

However, when we compare the role of members in both the previous investigated papers and Section 2, either the manufacture is acting as the bellwether or the retailer is dominating, this configuration is recognized as power structure (Li et al., 2022). Noticeably, there is no Stackelberg model used in the trade-off mechanism and two players choose their strategies simultaneously. This also differs the trade-off mechanism from the cost-revenue sharing contract because the former does not require coordination of players' order in decision making.

5. Conclusion

In this paper, we introduce the cost-revenue sharing contract in supply chain and the trade-off mechanism from the prospective of their underlying concepts and the direct reflection in their objection functions. The similarities in their objective functions are clearly expressed and the difference lies in the their settings of power structure. It's expected that knowledge from one area can be effectively applied in others once the common features are identified.

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