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Collaborative Working Capital Optimization for Combined Supply Network

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Abstract This study introduces an enhanced theoretical model aimed at minimizing the total costs of working capital within Supply Chains by leveraging Reverse Factoring and Inventory Financing. A comprehensive model was then developed for financial costs in supply chains with a combined topology. This model was not only stated but also transformed into a practical tool for professionals. To validate its applicability, the model was implemented in real-world business scenarios. Further, this paper offers guidelines for future deployments of the developed tool.

Keywords: financial supply chain, working capital management, supply chain finance solutions, inventory financing, reverse factoring, non-convex optimization.

1. Introduction

In an increasingly interconnected global economy, supply chain financing has emerged as a critical aspect of contemporary business operations. Companies are now part of expansive supply chains, which are encompassing numerous entities, and playing a vital role in the smooth movement of goods, services, and information. The complexity of these networks necessitates management and effective collaboration among all members to ensure efficiency and profitability (Mentzer et al., 2001). However, while there has been substantial focus on the logistical aspects of supply chains, the financial side often receives less attention, creating a knowledge gap that this research aims to address.

The problem of the research is the lack of comprehensive approaches to manage the financial side of supply chain operations, specifically, the optimization of working capital in a collaborative manner. The subject of the study, therefore, is the working capital in the supply chain, considered from a holistic, network-based perspective rather than an individual, company-centric view.

The significance of this research lies in its potential to contribute both theoretically and practically. On the theoretical front, it offers broadening of current mathematical model for calculating financial costs on working capital, and the approach to its optimization. Practically, the research provides a guideline for intermediaries for managing financial costs on working capital in supply chains more effectively, leading to cost reduction and improved operational efficiency.

The primary goal of this study is to improve the theoretical model for reducing the total costs of working capital in the supply chain using financial instruments and implementing it on real-life supply networks. To achieve this goal, the following tasks are set:

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1. Create a generalized model for total financial costs in supply chains with combined topology; develop a tool for practical implementation of the model.
2. Implement the model in real-life business cases, provide guideline for future implementations.

This study aims to build and test a new generalized model for managing working capital in supply chains of arbitrary topology, intending to bridge the existing theoretical gap and provide a practical solution for businesses in diverse supply networks.

Each task, aligned with a specific chapter, contributes to the accomplishment of the overarching research goal. The tasks start with analysing existing methods in supply chain financing, followed by developing a comprehensive model for working capital management in combined supply chains.

The final stage of the study involves the practical application of the model in real-life business cases and the development of a software tool. This tool, designed to be user-friendly and flexible, aims to make the optimization process accessible to management of logistic service providers and other intermediaries.

By linking theoretical advancements with practical tool, this research aims to deliver a comprehensive solution that can potentially benefit supply chain finance management. The proposed model and software tool could help businesses streamline their financial operations, reduce costs, and enhance efficiency.

2. Supply Chain Financing: Tools and Approaches

2.1. Supply Chain Structure

According to Mentzer, supply chain can be defined as "a set of three of more entities (organizations and individuals) directly involved in the upstream and downstream flows of products, materials and/or information from a source to a customer" (Mentzer et al., 2001). For the purpose of this work, despite some disputes (Smirnova et al., 2021) in definitions of supply chains, this definition will be taken. The main set of defining principles for a supply chain will be:

1. more than 3 firms (later: members),
2. flow of goods and finance between members,
3. all members connected to any other member with a flow,
4. clear direction of flow of goods from source to customer,
5. clear definition of level of each member.

Supply chain management is a critical component of modern business operations. It is essential for companies to be able to manage their supply chains effectively in order to remain competitive in today's global marketplace. The goal of supply chain management is to integrate supply and demand management across and within companies. By doing so, companies can ensure that they are able to efficiently and effectively meet the needs of their customers while minimizing costs and maximizing profits.

The classification of supply chains is crucial for understanding the structure and function of a supply chain. The topology, types, and members of a supply chain can vary significantly based on the nature of the business and industry.

The members of a supply chain include the suppliers, manufacturers, distributors, retailers, and customers. The members of a supply chain can vary based on the nature of the product and the industry. In some industries, the supply chain may involve only a few members, while in others, it may involve multiple members.

The literature (Mentzer et al., 2001) offers 3 different types of supply chain structures, each with a different level of complexity. Moving from the least to the most complex and realistic, there could be "Direct supply chain" (supplier to manufacturer to customer), "Extended supply chain" (extra suppliers, customers at the ends of the chain) and the "Ultimate supply chain" (third-party organizations involved, Fig. 1). For simplicity, there are three types of third-party organizations in this case. A financial provider supports the relationship between two entities by providing financial solutions and bearing financial risks. A 3PL logistics provider (logistics service operator) is also present in the example, as he handles warehousing, transportation, and other logistics tasks. Finally, a market research agency gathers and analyzes data about the end client in order to offer the entire supply chain with information about their tastes and expectations.

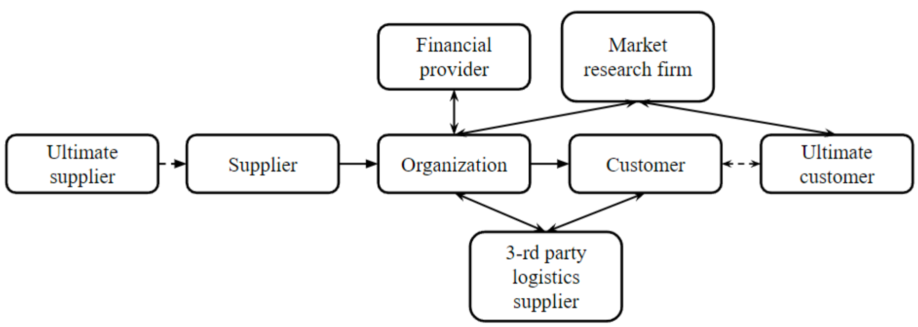

Fig. 1. Ultimate supply chain. Source: (Mentzer et al., 2001, p. 5)

Some authors (Gupta and Dutta, 2011) have classified the flows in a supply chain into three categories: downstream flow of goods, upstream flow of money, and information flow (which goes both ways). By investigating and improving these flows, businesses can optimize their supply chain and improve their overall performance.

The financial flows in a supply chain, also known as the financial supply chain, run parallel to the flow of goods through the supply chain (material supply chain). These financial flows can be viewed as the financial side shadowing the material side, as they follow each other with a certain level of simplicity. As practically all companies have suppliers and consumers, money flows connect physical operations. This highlights the importance of managing both the material and financial flows in a supply chain to optimize performance and improve financial outcomes.

Understanding the structure of the supply chain is crucial for effective supply chain financing. According to literature (Pfohl and Gomm, 2009), an effective supply chain financing strategy must take into account the different stages, partners, and transactions involved in the supply chain.

2.2. Supply Chain Finance

In the growing field of supply chain management, the concept of supply chain finance is beginning to draw significant attention. As an area of study and practice, supply chain finance centers on the various practices and transactions that enable the purchase, sale, and payment of goods and services in the supply chain (International Chamber of Commerce, 2016). While the concept has begun to form, scholars have yet to arrive at a universally accepted definition of supply chain finance (Blackman et al., 2017).

Historically, researchers have primarily focused on the flow of materials in the supply chain, analyzing technical aspects and strategies related to physical goods (Lee and Billington, 1995; Segev et al., 1998). This strong emphasis on material flows has left the financial aspects of the supply chain somewhat underexplored. Pfohl et al. (Pfohl and Gomm, 2009) were among the first to underline this gap, introducing the term "financial supply chain" to describe the upstream flow of cash within the supply chain.

Supply chain finance is not just about tracking the flow of money, though. It also includes operational and strategic management of financial flows, such as how to finance transactions (Hofmann, 2005), integrating financial data (Fairchild, 2005), and mitigating risks in supplier financing (Hofmann, 2011). It's also about the holistic coordination of these elements within and between companies in the supply chain (Popa, 2013). Therefore, a comprehensive definition of supply chain finance should also consider these aspects.

Over the years, the focus has gradually shifted towards the financial aspects of supply chain management. Companies have seen substantial benefits from managing the physical flows of goods in their supply chains and are now turning their attention to managing financial flows (Hofmann, 2011). The financial crisis emphasized the importance of financial supply chain management as companies faced challenges in securing loans and corporate borrowing costs increased (Weiss, 2012).

Despite the growing interest, the development of an encompassing theoretical framework for supply chain finance is still in its infancy (Hartley-Urquhart, 2006; Gupta and Dutta, 2011). There is a notable effort to enhance the integration of financial flows in the supply chain and synchronize it with the material and information flows (Wuttke et al., 2013a; Wuttke et al., 2013b). However, compared to the progress in managing downstream flows of goods and information, the upstream flow of cash remains a challenging area (Pfohl and Gomm, 2009).

The Institute for Supply Chain Management suggests that the realm of supply chain finance sits at the intersection of supply chain management and corporate finance. The introduction of various supply chain finance solutions has allowed companies to manage working capital more efficiently, indicating the promise that supply chain finance holds for future research and practice (Caniato et al., 2016). Despite the advances, the researchers' opinions on the concept and definition of supply chain finance still differ significantly, underlining the need for further exploration and consensus-building in this field.

2.3. Collaborative Working Capital Models

Working capital is a fundamental concept in business operations and finance, acting as a barometer of a company's operational and financial efficiency. Despite its importance, there is ongoing debate among researchers over its precise definition (Bhattacharya, 2009; Jones, 2006). Two main interpretations exist: one defines working capital as the difference between current assets and current liabilities, as per the traditional accounting perspective (Bhattacharya, 2009), expressed in the formula:

$$\text{Working capital} = \text{Current assets} - \text{Current liabilities}. \quad (1)$$

An alternative view, more aligned with the operational aspect of business, considers working capital as the sum of inventories and accounts receivable minus accounts payable (Jones, 2006), expressed as:

$$\text{Working capital} = \text{Inventory} + \text{Accounts Receivable} - \text{Accounts Payable}. \quad (2)$$

This research will keep to the second interpretation due to its stronger relevance to day-to-day operations of businesses.

The management of working capital is a difficult task involving careful handling of a company's current assets and liabilities (Hill et al., 2010). As, consequentially, working capital management is an essential facet of a company's financial management strategy, it can be viewed from two perspectives - an individual firm's perspective or from an inter-organizational level (Pirttila, 2014; Monto, 2013).

Traditionally, working capital management was considered from an individual firm's perspective, focusing on how a single company manages its short-term assets and liabilities (Kaur, 2010; Belt and Smith, 1991; Moss and Stine, 1993). However, recent research has started to consider working capital management from an inter-organizational perspective, recognizing the interconnected nature of today's business environment and the role of supply chains in working capital management (Hill et al., 2010; More and Basu, 2013; Wuttke et al., 2013b; Huff and Rogers, 2015; Hofmann and Zumsteg, 2015; Blackman et al., 2017).

The inter-organizational perspective emphasizes that a company's working capital position is influenced not only by its internal operations but also by the practices of other entities in its supply chain (Virolainen et al., 2019; Singh and Kumar, 2014; Kayani et al., 2019). This interconnected approach to working capital management presents a broader understanding and offers new opportunities for improving operational efficiency and financial performance.

One common metric for managing working capital from an individual perspective is the Cash Conversion Cycle (CCC). It measures the time it takes for a company to convert resource inputs into cash flows (Shin and Soenen, 1998; Deloof, 2003;

Hutchison et al., 2007). The CCC serves as an invaluable tool in understanding the effectiveness of working capital management, with shorter cycles generally implying better management and greater efficiency (Ulbrich et al., 2008; Hofmann and Kotzab, 2010; Viskari and Karri, 2012).

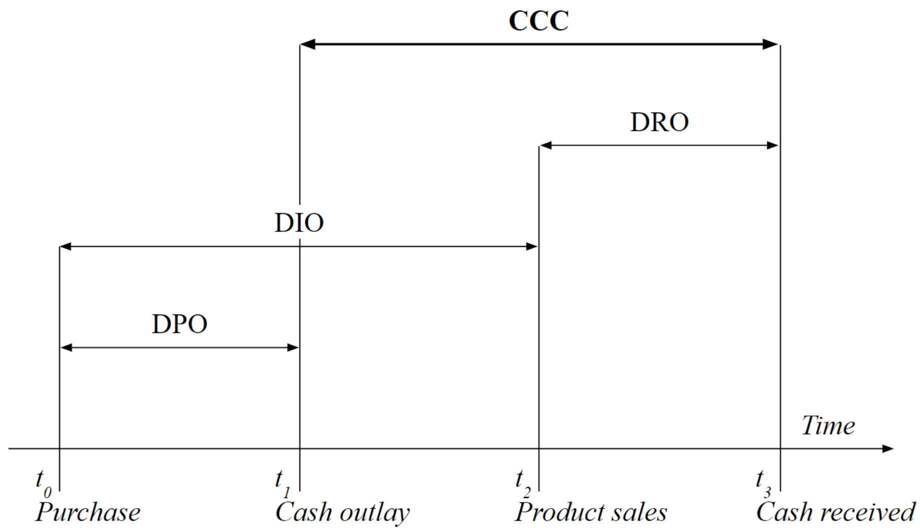

Fig. 2. Cash conversion cycle. Source: (Ivakina et al., 2019, p. 181)

Cash conversion cycle (CCC) consists of days inventory outstanding (DIO) — time that the organization needs to turn the inventory (Inv) into sales, days receivables outstanding (DRO) — time it takes the organization to collect Accounts Receivable (AR), and days payable outstanding (DPO) — time it takes the organization to pay off Accounts Payable (AP). So, cash conversion cycle is a time metric, which demonstrates the amount of time the organization has funds tied up in Working Capital, from paying for the inventory to the supplier to collecting accounts receivable from the customers, as demonstrated in Fig. 2 and by the formula below (Omesa et al., 2013; Ivakina et al., 2019):

$$\mathrm{CCC} = \mathrm{DIO} + \mathrm{DPO} - \mathrm{DRO}. \tag{3}$$

Nevertheless, there is no single opinion on how to calculate ratios mentioned above (Smirnova et al., 2021). For continuity of the research the same approach as in Smirnova's research will be taken:

$$\mathrm{DIO} = \frac{\mathrm{Inv} \times t}{\mathrm{CoGS}},\ \mathrm{DRO} = \frac{\mathrm{AR} \times t}{\mathrm{Net\ Sales}},\ \mathrm{DPO} = \frac{\mathrm{AP} \times t}{\mathrm{CoGS}}, \tag{4}$$

where t is the duration of measured cycle, CoGS is Cost of Goods Sold.

Cash conversion cycle can take negative as well as positive values. Important nuance to consider is that the metrics DIO, DPO and DRO are measured in days, but are not literally days, and are the financial metric. This is important for explanation of the results of the future model: negative CCC would mean that the firm manages to have low inventory and/or to collect payments from customers before it has to pay to suppliers. Though minimizing CCC is considered a positive thing for the business (Garanina and Petrova, 2015), it can vary from industry to industry and from context to context.

According to literature (Tangsucheeva and Prabhu, 2013), low CCC can mean low financial costs on Working Capital. Financial costs on working capital are determined by accounts receivable, accounts payable, and inventory, with weighted average cost of capital (Viskari and Karri, 2013). They determine the costs of the working capital being tied up for the period until the payment is received from the customer:

$$\begin{aligned} FC_{WC} = Inv \times \left((1+\text{WACC})^{\frac{\text{DIO}}{t}} - 1\right) + \\ + AR \times \left((1+\text{WACC})^{\frac{\text{DRO}}{t}} - 1\right) - \\ - AP \times \left((1+\text{WACC})^{\frac{\text{DPO}}{t}} - 1\right), \end{aligned} \tag{5}$$

where WACC is a weighted average cost of capital.

Despite the existence of the studies underlining importance of calculating financial costs on working capital in terms of connections on inter-organizational level (More and Basu, 2013; Wuttke et al., 2013b; Hofmann and Zumsteg, 2015; Blackman et al., 2017; Huff and Rogers, 2015; Virolainen et al., 2019), there is not a lot of studies covering it, with the focus mostly shifted towards inner-organizational level. Despite that, the interest in managing the financial costs on working capital at inter-organizational level is on the rise, with several studies emphasize that it can be beneficial for all members of supply chain (Hofmann and Kotzab, 2010; Talonpoika et al., 2016; Protopappa-Sieke and Seifert, 2017).

The main problem with optimizing individual cash conversion cycle is that in terms of the supply chain it does not affect the performance. This phenomenon was studied, for example, in the work of Hofmann and Kotzab (Hofmann and Kotzab, 2010), and can be easily demonstrated: optimizing CCC can be done through 3 parameters: DIO, DPO, DRO. Adjusting DIO would mean to send the goods to the customer quicker (increasing his DIO proportionately), adjusting DRO or DPO would harm DPO of customer or DPO of the supplier. To address this, the collaborative cash conversion cycle is introduced:

$$\text{CCCC} = \sum_{l=1}^{n} \sum_{k=1}^{m} \text{CCC}_l^k. \tag{6}$$

where l is the number of levels in supply chain and k is the number of members on the level. Same applies to total financial costs on working capital, which are calculated as follows (Viskari and Karri, 2012):

$$\text{TFC}_{WC} = \sum_{l=1}^{n} \sum_{k=1}^{m} \text{FC}_{WC}{}_l^k. \tag{7}$$

Since it is difficult to toggle the parameters in CCC individually, CCCC optimization is what is left. But still supply chain optimization on its own is just shifting the weight towards the ends of the chain: initial manufacturers or end consumers. Optimizing it for the chain would only mean that end customers would have to pay in advance or immediately, and initial manufacturers should wait on the payment for long periods of time. To actually affect the effectiveness of supply chain's performance, some works (Protopappa-Sieke and Seifert, 2017) suggest to turn to supply chain finance solutions.

In the traditional models, the most common model is Factoring or Reverse Factoring. This is an arrangement where a financial intermediary essentially acts as a bridge between supplier and buyer, fast-tracking payment to the supplier while allowing the buyer more time to pay. The tradeoff for this service is an interest cost. This method notably results in an increased Days Payable Outstanding (DPO) for the buyer and a decreased Days Receivable Outstanding (DRO) for the supplier, optimizing the cash conversion cycle and thus working capital for both parties (Mian and Smith, 1992; Klapper, 2006).

Turning towards the innovative models, Inventory Financing and Dynamic Discounting stand out. In Inventory Financing, a third-party logistics provider or similar entity holds onto inventories until reclaimed by the buyer, which reduces inventory for both the supplier and the buyer, thereby optimizing their working capital. This model also reduces the DRO for the supplier and increases the DPO for the buyer, leading to further optimization of the cash conversion cycle (Hofmann, 2009; Chen et al., 2011; Gelsomino et al., 2019). Dynamic Discounting involves the buyer offering a discount for early payment that is proportional to the decrease in DRO for the supplier, providing benefits in the form of a Cost of Goods Sold (CoGS) reduction (Nienhuis et al., 2013; Templar, Findlay, and Hofmann, 2016).

Lastly, for the collaborative models, Vendor-Managed Inventory and Consignment Stock are principal. In the Vendor-Managed Inventory model, the supplier assumes responsibility for inventory management and replenishment, leading to inventory reduction for buyers (Waller et al., 1999). Consignment Stock, on the other hand, delegates the management of inventory to the buyer while the supplier retains ownership until the goods are sold to end customers, thereby leading to inventory reduction for suppliers (Valentini and Zavanella, 2003).

For the purpose of this research 2 main models were chosen: Factoring / Reverse Factoring and Inventory Financing. The reasoning behind the choice is that they allow to affect the maximum number of metrics for all members of the potential chain (DIO, DRO, DPO). Also, while not requiring substantial levels of collaboration (almost integration) of members, which is necessary for collaborative types, they are the most popular and effective solutions available for the businesses worldwide (Gelsomino et al., 2019; Chen et al., 2019).

The model of Factoring / Reverse Factoring was described, among others, in the work of L. Klapper (Klapper, 2006), as financial agreement between supplier, buyer and an intermediary (factor), by which the supplier sells his accounts receivable with a discount to the intermediary for a faster acquiring of payment. The main difference between factoring and reverse factoring is the initiator of the financing solution — for factoring it is a supplier, while for reverse factoring it is a buyer with a stronger financial position. In general, reverse factoring is often applied in the supply chain when the buyer has stronger credit rating, and supplier is in need of short-term financing. In this case, when the buyer has strong position in terms of credit ratings, it is beneficial for small suppliers to use this approach, because the interest, based on buyer's score, will be lower, than for the situation when the supplier itself would have turned to loans.

Inventory Financing, according to Hofmann, 2009, is an agreement between supplier, buyer and 3PL operator (or other Logistics Service Provider — LSP), by which LSP buys inventories from the supplier and acquires temporary ownership over them until they are bought by the buyer. The buyer is required to buy the goods from LSP by the agreement. This can improve the short-term financial state of the participants of the supply chain by reducing Days Inventory Outstanding, Days Receivable Outstanding and Days Payable Outstanding. In this case LSP takes over the logistic functions (transport, warehousing) and finances the inventories of the supplier for the interest.

2.4. Working Capital Management Models

According to some authors (Hartley-Urquhart, 2006), financial supply chains should be controlled in the same way that physical supply chains are. Gupta and Dutta, representing academia, hold a similar viewpoint. They believe, in particular, that managing the upstream flow of cash is just as important as managing the downstream flow of commodities (Gupta and Dutta, 2011). As a result, numerous academics have attempted to provide a theoretical framework for financial supply chain management. In the supply chain, one of the most crucial parts has been working capital management.

Despite the interest in research papers to supply chain view on the optimization (Hutchison et al., 2007; Randall and Farris, 2009; Hofmann and Kotzab, 2010; Huff and Rogers, 2015; Lorentz et al., 2016), there are still almost no models in the field. Several of the few attempts to build such models were done by Viskari and Kärri (Viskari and Karri, 2013), Pirttilä (Pirttila, 2014), but none of them used multi-objective optimization. Considering the importance of multi-objective optimization even at individual level, it can be concluded that the research gap remains.

The most resent research (Ivakina et al., 2019; Smirnova et al., 2021), is the closest attempt at building such model, but it only covers one topology: 2-1-2 scheme with the emphasize on the middle member, as displayed in Fig. 3.

On the schematic (Fig. 3) the arrows represent the flow of goods, circles — participants. Smirnova suggested 2 goals for optimization:

1. Limiting the individual CCC of each participant of the supply chain to the recommended industry-specific stability interval;
2. Minimizing the total financial costs of the supply chain.

Suggested reasoning behind this is that many studies have shown a negative relationship between profitability and working capital levels (Shin and Soenen, 1998; Padachi, 2006; Sen and Oruc, 2009; Mathuva, 2010; Bhunia and Das, 2012;

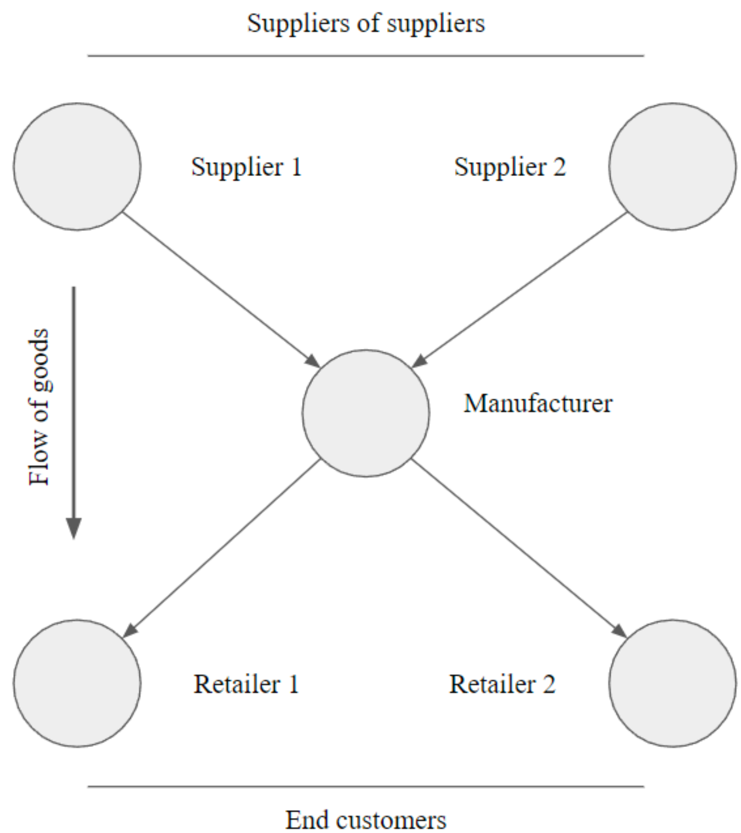

Fig. 3. 2-1-2 topology of supply chains. Source: (Smirnova, 2021, p. 319)

Garanina and Petrova, 2015; Singh, Kumar, and Colombage, 2017). Traditional theory also suggests that managing working capital should focus on both maximizing profitability and maintaining a necessary level of liquidity (Zeidan and Shapir, 2017; Chang, 2018). Maximizing profitability or liquidity should not be the only goal as there is an inverse relationship between a company's profitability and liquidity (Shin and Soenen, 1998). Companies should aim to balance these goals by adhering to a recommended industry/company-specific stability interval to maintain a favourable balance between return and liquidity (Giannoccaro and Pontrandolfo, 2004; Gao et al., 2018). Minimizing the total financial costs of the supply chain is also important as companies in the supply chain are motivated to maximize their individual profits rather than the total profit of the supply chain.

The goals set for the previous studies are well-suited for the task, and will be implemented here as well. Despite all of the above, the approach is not all-encompassing, and the research gap of this study is in lack of the model that can cover any possible topology of the supply chain. The change will be in unification of the approach to fit arbitrary topology.

3. Collaborative Working Capital Optimization Model for Combined Supply Chain

3.1. Mathematical Model Development

One approach to address the challenges of working capital optimization is to adopt a collaborative approach, where multiple parties in the supply chain network work together to achieve common goals. Collaborative optimization can help to reduce inventory levels, improve cash flow, and enhance supply chain resilience, among other benefits.

Multi-variate optimization, also known as mathematical optimization, is a critical branch of mathematical programming and operations research that is central to numerous scientific disciplines. This domain involves the identification of the best possible solution, often signified by the highest or lowest achievable outcome, subject to certain constraints or bounds, within a set of potential solutions defined by decision variables (Boyd and Vandenberghe, 2004).

Mathematically, a generic multi-variate optimization problem is formulated as follows:

$$\text{minimize } f(x) \text{ subject to } g(x) \leq 0, h(x) = 0, \tag{8}$$

where $f(x)$ is the objective function to be minimized, $g(x) \leq 0$ represents inequality constraints, $h(x) = 0$ represents equality constraints, and $x = (x_1, x_2, x_3, ..., x_n)$ is the vector of n decision variables. This technique is of high value when managing complex systems composed of multiple interconnected variables that influence each other, such as supply chains. Given that supply chains inherently involve multiple decision variables, multi-variate optimization becomes vital in managing these systems effectively. By leveraging this method, supply chain managers can simultaneously optimize various aspects of their operations, leading to more efficient and cost-effective outcomes (Ding et al., 2017). The optimization techniques, such as multivariate optimization, play a significant role in managing complex systems with multiple interrelated variables. It provides a framework to deal with various aspects of such systems in a unified manner, seeking to find the best possible solution within a set of potential alternatives. However, this traditional form of optimization has some limitations, particularly when dealing with complex, non-convex, and high-dimensional problems. For instance, the objective function could be non-convex, implying multiple local minima exist, which may lead to the optimization algorithm getting stuck in one of these local minima instead of finding the global minimum (Nocedal and Wright, 2006). Also, the function could be non-differentiable, making gradient-based methods inapplicable.

In this context, population-based stochastic search algorithms such as Differential Evolution (DE) may be of use. DE is known for its simplicity, robustness, and efficacy in handling global optimization problems, including those which are non-linear, non-convex, and high-dimensional. Unlike traditional optimization techniques that might get stuck in local minima, DE, by its very nature, continues to explore the search space, making it a powerful tool in finding global solutions in complex optimization scenarios such as the one displayed in this research.

Differential Evolution (DE) is a population-based stochastic search algorithm. First introduced by Storn and Price in 1997 (Storn and Price, 1997), the algorithm has been widely used for solving multidimensional optimization problems, particularly those which are non-linear and non-convex in nature.

The fundamental principle of DE involves initializing a population of candidate solutions and then evolving that population over time to improve the quality of the solutions. It uses mechanisms such as mutation, crossover, and selection to generate new candidate solutions.

In the context of SCF, the decision variables could correspond to factors such as payment terms, the share of early payment to supplier, and the share of goods not delivered by 3PL intermediary. The optimization task in this case is to minimize the Total Financial Cost (TFC) on Working Capital under several constraints and bounds, which could involve certain limitations on individual financial costs (FCs) and restrictions on Cash Conversion Cycles (CCCs).

The differential evolution algorithm operates by iteratively generating trial solutions through mutation, crossover, and selection operations. In each iteration, the algorithm randomly selects distinct solutions from the population, perturbs one of them by the weighted difference of the others to generate a mutant vector. This mutant vector then undergoes crossover with another randomly chosen solution to form a trial vector. If the trial vector provides a lower TFC than the current solution, it replaces the current solution in the population.

However, one limitation of the differential evolution algorithm as implemented in the 'scipy.optimize' package in Python is the lack of direct support for constraint handling. One possible workaround for this limitation is the use of penalty methods, where a penalty term is added to the objective function to discourage violation of the constraints. In this context, penalties could be imposed whenever the solutions do not achieve the desired reductions of individual FCs or when the CCCs do not fall within industry-recommended intervals.

Mathematically, this could be represented as:

$$f'(x) = f(x) + \sum_{i=1}^{m} P_i g_i(x) + \sum_{j=1}^{p} P_j h_j(x), \tag{9}$$

where $f(x)$ is the penalized objective function,$f(x)$ is the original objective function (TFC in this case), $g_i(x)$ and $h_j(x)$ are the inequality and equality constraints respectively, and P_i and P_j are the penalty factors. This method helps in maintaining the constraints within acceptable limits while continuing to optimize the objective function.

Through this approach, differential evolution can serve as an effective tool for solving the optimization problem uncovered in this research.

Returning to the mathematical model of the research, previous studies on supply chain optimization have mainly focused on up to the 2-1-2 topology, where there are two suppliers, one manufacturer, and two customers, with the manufacturer being the main member of the chain at the middle. However, this topology is just one of many possible configurations in a supply chain network. In reality, supply chains can be much more complex, with multiple tiers of suppliers, manufacturers, and distributors.

To address this limitation, this thesis proposes a collaborative working capital optimization model for combined supply chain networks that can be applied to any possible topology. The model aims to optimize the working capital of all members in the supply chain network by considering their interdependent relationships and collaborative efforts. For the purpose of generalization, it is assumed the topology of l levels with k members each (Fig. 4). Later, for the specific problem, the members or connections can be dismantled on some levels.
The object of the minimization is total costs on working capital: here and later

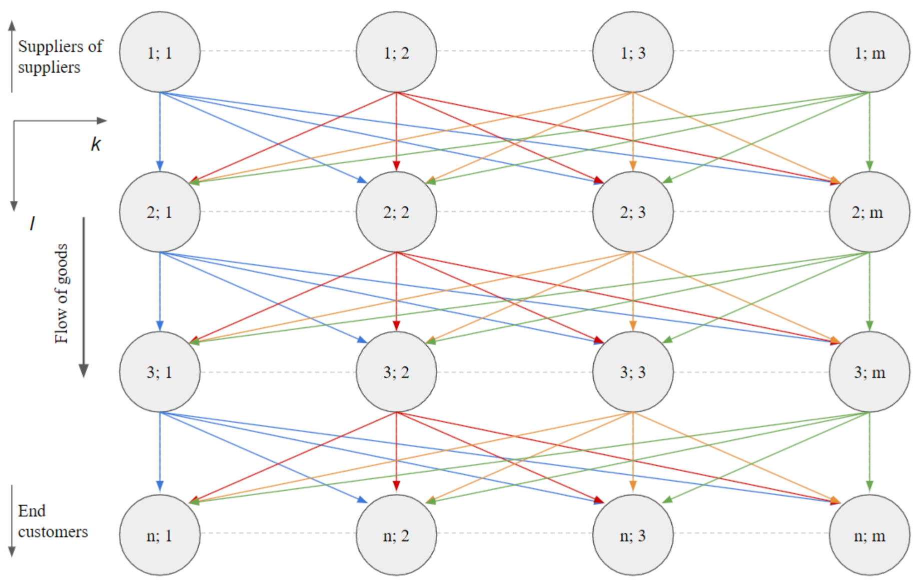

Fig. 4. Supply chain of an arbitrary topology. Source: made by authors

marked as TFC. As specified in previous chapter, sum of financial costs on working capital for the members of SC gives the total cost:

$$TFC = \sum_{l=1}^{n} \sum_{k=1}^{m} FC_l^k, \tag{10}$$

The approach to minimize the financial costs is through reduction of non-value-adding time. The time each member spends is being represented financially by DRO (Days of accounts receivable), DPO (days of accounts payable) and DIO (days of inventory outstanding. Cash Conversion Cycle (CCC) is:

$$CCC_l^k = DIO_l^k + DRO_l^k - DPO_l^k, \tag{11}$$

for the whole supply chain being:

$$CCCC = \sum_{l=1}^{n} \sum_{k=1}^{m} CCC_l^k. \tag{12}$$

The relations between DRO, DPO, DIO and corresponding values from financial statements: AR (Accounts Receivable), AP (Accounts Payable) and Inv (Inventory) are as follows:

$$\begin{aligned} Inv_l^k &= \frac{DIO_l^k * CoGS_l^k}{t}, \\ AR_l^k &= \frac{DRO_l^k * Revenue_l^k}{t}, \\ AP_l^k &= \frac{DPO_l^k * CoGS_l^k}{t}, \end{aligned} \tag{13}$$

where t is the period in question.

In a complete supply chain, the accounts receivable (AR) values of upper members will correspond with the accounts payable (AP) values of the members connected to them below. This is because the receiver of the goods will be paying money to the sender of the goods. This relationship can be expressed mathematically as follows:

$$DRO_l^k = DPO_{l-1}^k, l \in [1, n-1]. \tag{14}$$

Since the AP value of the members of the first level and the AR value of the members of the last level are connected to the sides that are not affected in the optimization model, these values should be considered constant:

$$DRO_n^k, AR_n^k, DPO_1^k, AP_1^k = const. \tag{15}$$

Each member has the costs on the working capital calculated by the following equation:

$$\begin{aligned} FC_l^k = Inv_l^k * ((1 + WACC_l^k)^{\frac{DIO_l^k}{t}} - 1) + \\ + AR_l^k * ((1 + WACC_l^k)^{\frac{DRO_l^k}{t}} - 1) - \\ - AP_l^k * ((1 + WACC_l^k)^{\frac{DPO_l^k}{t}} - 1), \end{aligned} \tag{16}$$

The formula is used to calculate the total financial cost of working capital (FC_l^k) for each member k in level l of the supply chain network. INV_l^k represents the inventory level of member k in level l of the supply chain network, DIO_l^k represents the Days of Inventory Outstanding for member k in level l of the supply chain network. It is the average number of days that inventory is held before being sold. $WACC_l^k$ represents the Weighted Average Cost of Capital for member k in level l of the supply chain network. It is the average cost of all the capital used in the business, weighted by the proportion of each type of capital. AR_l^k represents the accounts receivable for member k in level l of the supply chain network. It is the amount of money owed to the member by its customers for goods or services sold on credit.

DRO_l^k represents the Days of Accounts Receivable Outstanding for member k in level l of the supply chain network. It is the average number of days that accounts receivable are outstanding before being paid. AP_l^k represents the accounts payable for member k in level l of the supply chain network. It is the amount of money owed by the member to its suppliers for goods or services purchased on credit. DPO_l^k the Days of Accounts Payable Outstanding for member k in level l of the supply chain network. It is the average number of days that accounts payable are outstanding before being paid. t represents the time period in question, typically measured in days. The first term of the formula calculates the financial cost of holding inventory, based on the inventory level and the Days of Inventory Outstanding. The second term calculates the financial cost of extending credit to customers, based on the accounts receivable and the Days of Accounts Receivable Outstanding. The third term calculates the financial cost of paying suppliers, based on the accounts payable and the Days of Accounts Payable Outstanding. The formula considers the Weighted Average Cost of Capital for each member, which reflects the cost of financing the business. The result is the total financial cost of working capital for each member in the supply chain network.

The primary way to influence these values is through the instruments outlined in the first chapter. Specifically, it will be Inventory Financing and Reverse Factoring.

In the context of the collaborative working capital optimization model, Reverse Factoring can be used as a tool for managing supply chain finance and improving the financial stability and efficiency of the supply chain network. By incorporating Reverse Factoring into the optimization model, supply chain managers can improve cash flow, reduce financing costs, and enhance supplier relations, while ensuring the financial stability and supply-demand balance of the network. This works by implementing an intermediary (usually a bank) into the chain, as is shown in the schematic (Fig. 5).

On the Fig. 5 the order of operations is displayed. x is share of early payment, which supplier gets from bank immediately, q_{RF}^4 is number of days for the bank to make an early payment (constant), r is an interest rate, under which the bank works.

In this case it will affect the Days of Accounts Receivable Outstanding. If the intermediary (bank) would deliver part of the payment (x) to the supplier in q_{RF}^4 days, q_{RF}^4 being an average speed of bank operations, the initial DRO will split into 2 parts (considering the flow of goods being uniform and steady):

$$DRO_l^k = q_{RF}^4 * x_l^k + DPO_{l+1}^k(1 - x_l^k), l \in [1, n-1], \tag{17}$$

where q_{RF}^4 is the speed of operations of bank in days, x_l^k is part of the goods taken by the bank.

In the context of the collaborative working capital optimization model, Inventory financing is a process where the intermediary (usually 3PL aggregators or other logistic service providers — LSP) buys the inventories from the supplier and stores them until the buyer pays for them with interest. This lets businesses reduce the money tied up in Inventories, and provides agility in meeting the demand. The

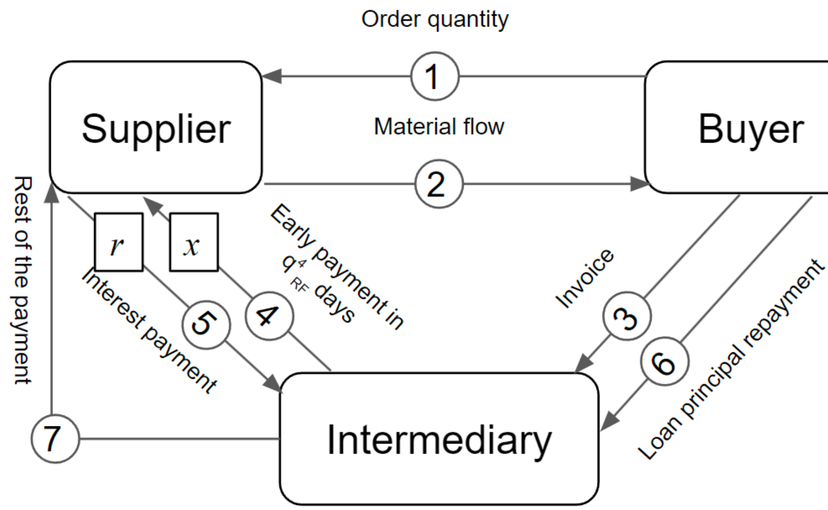

Fig. 5. Reverse Factoring with introduced parameters. Source: made by authors

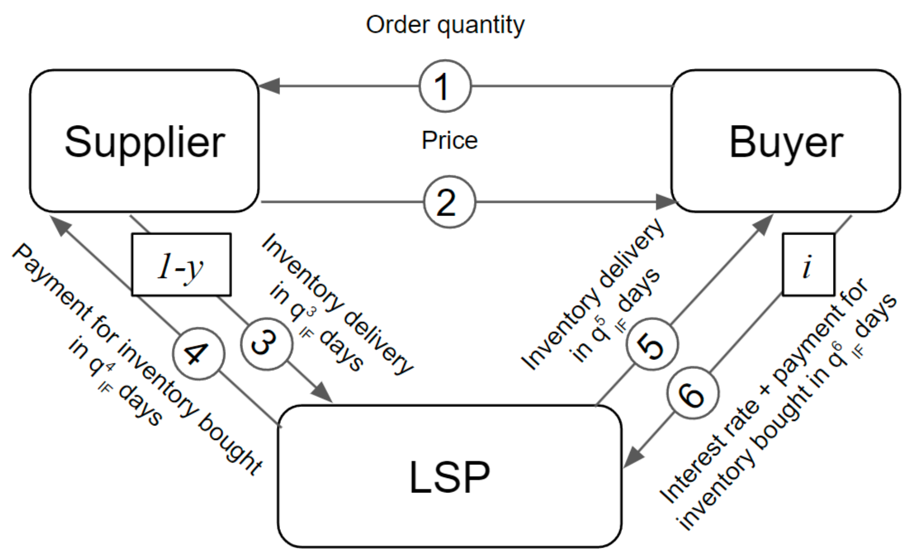

Fig. 6. Inventory Financing with introduced parameters. Source: made by authors

schematic for order of operations would look according to Fig. 6. On the picture the order of operations is displayed, with y being the share of inventories not financed and delivered as is (and $1 - y$ being the share financed), q^3_{IF} is number of days for the LSP to acquire the inventories from the supplier (constant), q^4_{IF} is number of days for the LSP to pay the supplier for the inventories taken (constant), q^5_{IF} is number of days for the buyer to acquire the inventories from the LSP (constant), q^6_{IF} is number of days for the buyer to pay for the inventories from the LSP (constant), i is an interest rate, under which the LSP works.

As the procedure is complex, in involves DIO, DRO and DPO:

$$\begin{aligned} DRO_l^k &= y_l^k * DRO_{l\,0}^k + (1-y_l^k) * q_{IF}^4, l \in [1, n-2], \\ DPO_l^k &= y_l^k * DPO_{l\,0}^k + (1-y_l^k) * q_{IF}^6, l \in [2, n-1], \\ DIO_l^k &= y_l^k * DIO_{l\,0}^k + (1-y_l^k) * q_{IF}^3, l \in [1, n-1]. \end{aligned} \tag{18}$$

As the model strives to implement both methods of managing supply chain financing, the following formulas will be used:

$$\begin{aligned} DIO_l^k = y_l^k * DIO_{l\,0}^k + (1-y_l^k) * q_{IF}^3, \\ l \in [1, n-1], \\ DRO_l^k = x_l^k * (q_{RF}^4 * x_l^k + DPO_{l+1}^k(1-x_l^k)) + \\ +(1-x_l^k) * (y_l^k * DRO_{l\,0}^k + (1-y_l^k) * q_{IF}^4), \\ l \in [1, n-2], \\ DPO_l^k = (1-y_l^k) * (y_l^k * DPO_{l\,0}^k + (1-y_l^k) * q_{IF}^6) + y_l^k * P_l^k, \\ l \in [2, n-1], \end{aligned} \tag{19}$$

where P_l^k is the new payment term for Reverse Factoring. Simply put, the formula above is the weightened average between Inventory Financing and Reverse Factoring approaches.

For the "bottom" of the chain - distributors or retailers - 3PLs can no longer participate in mediating the flow of goods due to their inability to control demand from potential end customers. Therefore, the following formulas will apply:

$$\begin{aligned} DIO_l^k = DIO_{l\,0}^k, l = n, \\ DRO_l^k = q_{RF}^4 * x_l^k + DPO_{l+1}^k(1-x_l^k), l = n-1, \\ DPO_l^k = P_l^k, l = n. \end{aligned} \tag{20}$$

Although, some connections will not be affected at all, as they extend beyond the region of control, and thus can't be altered:

$$\begin{aligned} DRO_l^k &= DRO_{l\,0}^k, l = n, \\ DPO_l^k &= DPO_{l\,0}^k, l = 1. \end{aligned} \tag{21}$$

Financial Costs on Reverse Factoring will be:

$$FC_{l\;RF}^k = AR_{l\,0}^k * x_l^k * \frac{P_{l+1}^k}{t} * r_l^k, l \in [1, n-1], \tag{22}$$

which is used to calculate the total financial cost of inventory financing for each member k in level l of the supply chain network. The financial cost is denoted as $FC_{l\;IF}^k$. The cost is a function of the accounts receivable for member k in level l of the supply chain network, denoted as $AR_{l\,0}^k$. The cost is also a function of the time period in question, t, and the interest rate for IF for member k in level l, denoted as r_l^k. The variable x_l^k represents the share of early payment which member k in level l of the supply chain network gets from the bank. The variable P_{l+1}^k represents the payment term for member k in level $l+1$ of the supply chain network. The formula calculates the total financial cost of RF for member k in level l of the supply chain network by multiplying the accounts receivable by the proportion of RF used, the payment term, and the interest rate. This formula only applies for levels $l \in [1, n-1]$, since RF is not used for the bottom level of the supply chain network.

Considering that literature (Hofmann, 2009) defined the premium which LSP receives proving the service of Inventory Financing as "the difference between the value of the financed goods and the present value of the financed inventory", Inventory Financing cost is calculated as follows:

$$FC_{l\,IF}^k = INV_l^k * (1 - x_l^k) - \frac{(1 - y_l^k) INV_l^k}{(1 + i_{l+1}^k)^t}, l \in [1, n-2], \tag{23}$$

which describes the calculation of the financial cost of inventory financing for member k in level l of the supply chain network. The financial cost is denoted as $FC_{l\,IF}^k$. The cost is a function of the inventory level of member k in level l of the supply chain network, denoted as INV_l^k. The formula also includes the time period in question, t. The variable i_{l+1}^k represents the interest rate for inventory financing for member k in level $l+1$ of the supply chain network. This gives the total financial cost of reverse factoring for member k in level l of the supply chain network. The formula only applies for levels $l \in [1, n-2]$ since inventory financing is not used for the bottom level of the supply chain network.

As the main objective is to reduce financial costs on working capital for the supply chain, TFC is the objective function:

$$\begin{aligned} TFC(x_l^k, y_l^k, P_l^k) = \sum_{l=1}^{n} \sum_{k=1}^{m} (Inv_l^k * ((1 + WACC_l^k)^{\frac{DIO_l^k}{t}} - 1) + \\ + AR_l^k * ((1 + WACC_l^k)^{\frac{DRO_l^k}{t}} - 1) - \\ - AP_l^k * ((1 + WACC_l^k)^{\frac{DPO_l^k}{t}} - 1) + FC_{l\,IF}^k + FC_{l\,RF}^k), \end{aligned} \tag{24}$$

As x_l^k, y_l^k, P_l^k aren't tied to any specific member, but more to a connection between members, it was decided to associate the number of parameters with the link rather than with the member. The distribution then will look as in Fig. 7.

The members can be marked with corresponding coordinates, while the connections will be marked with pairs of coordinates.

Another case to be addressed is the DPO-DRO situations in following scenarios, displayed on Fig. 8.

In this cases there is no possibility to implement $DRO_l^k = DPO_{l+1}^k$ formula, because the flow of goods from one member goes to 2 or more of them. To balance out this situation, the following formula could be implemented in the case of no

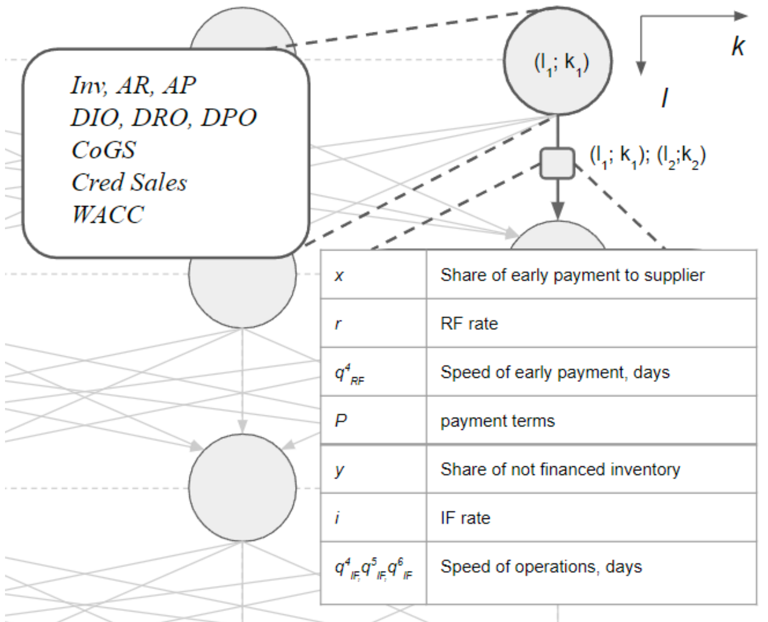

Fig. 7. Distribution of parameters and variables on the graph. Source: made by authors

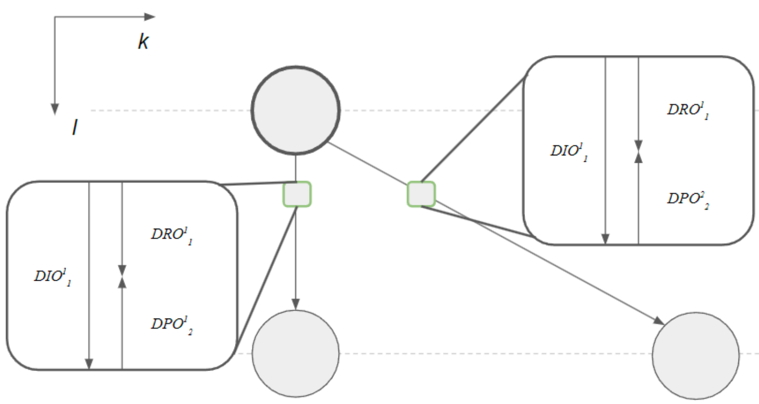

Fig. 8. Scenario of multiple connections to one element. Source: made by authors

intermediaries:

$$DRO_l^k = \frac{\sum_{k=1}^{m}(DPO_{l+1}^k * AP_{l+1}^k)}{\sum_{k=1}^{m} AP_{l+1}^k}. \tag{25}$$

basically, implementing weighted average of days of accounts payable outstanding by the weights of accounts payable. This will result in the following adjustments in the formulas:

$$DRO_l^k = \frac{\sum_{k=1}^m \left(x_l^k (q_{RF}^4 x_l^k + DPO_{l+1}^k (1 - x_l^k)\right)}{\sum_{k=1}^m AP_{l+1}^k}$$
$$+ \frac{\sum_{k=1}^m \left((1 - x_l^k)(y_l^k DRO_{l,0}^k + (1 - y_l^k) q_{IF}^4) AP_{l+1}^k \right)}{\sum_{k=1}^m AP_{l+1}^k}, \quad l \in [1, n-2] \tag{26}$$
$$DRO_l^k = \frac{\sum_{k=1}^m (q_{RF}^4 * x_l^k + DPO_{l+1}^k (1 - x_l^k) * AP_{l+1}^k)}{\sum_{k=1}^m AP_{l+1}^k}, l = n - 1.$$

The decision variables then would be x, y, P, all others being either constants or functions from them.

The main constraint for the model is the necessity of reduction of total financial costs:

$$TFC < TFC_0 \tag{27}$$

Another important constraint covers individual FCs of the members:

$$FC_l^k \leq FC_{l\,0}^k. \tag{28}$$

Individual CCCs of the members of the Supply Chain should fit into industry-specific intervals, as a second goal mentioned in the previous studies (Smirnova et al., 2021), and as an anchor for FC solution: the metric being an extendable one, CCCs behave as references to approximate industry levels, under which businesses can comfortably operate:

$$CCC_{industry\ low} \leq CCC_l^k \leq CCC_{industry\ high}. \tag{29}$$

where $CCC_{industry\ low}$ and $CCC_{industry\ high}$ can be either specified manually or taken from open sources. All of the variables in question are coefficients, except for P, which is measured in days. That means that:

$$0 \leq x \leq 1, 0 \leq y \leq 1, P > 0. \tag{30}$$

As the Collaborative Working Capital Optimization Model for Combined Supply Chain Network aims to optimize the working capital of multiple parties involved in the supply chain network by promoting collaboration and information sharing among them, the model assumes that each party's working capital is affected by the others' actions, and therefore, optimizing the overall working capital requires a collaborative approach.

The model incorporates collaboration among multiple parties in the supply chain network by formulating a joint optimization problem that considers the objectives and constraints of each party. Specifically, the model seeks to minimize the sum of the working capital of all parties subject to their individual constraints and a set of joint constraints that promote collaboration.

The model is based on several assumptions:

1. The model assumes that the enterprises involved are functioning continuously without interruption (ex.: seasonality, weekends);
2. The flow of goods through the Supply Chain is equally distributed in time;
3. The rates and ability of intermediaries to provide operations is constant through the whole supply chain;
4. Supply Chain is consistent and fully known for the levels covered in the model, with no extra suppliers/buyers not included in the model.

3.2. Tool for Collaborative Working Capital Optimization

The input of the model is customizable: levels and columns could be specified, members can be disabled, if needed, as shown in Fig. 9. Excel is used as input window, with 3 consecutive steps: topology, elements, connections. The order of input is as follows: the user starts with the topology page, specifying number of levels, number of columns (elements per level), timeframe (365 by default) and CCC requirements. For example, for the topology 3-2-1-3 number of levels will be 4, number of columns — 3 (it goes as a maximum number of members per level, as later the unnecessary elements from the less-occupied levels can be deactivated).

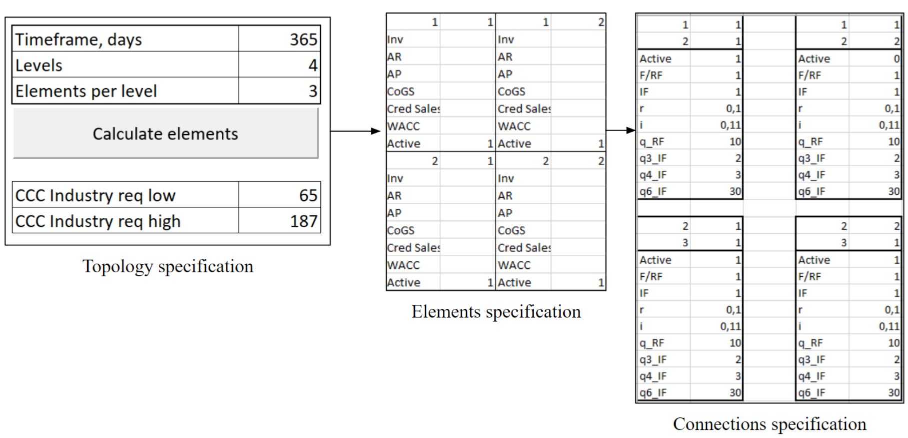

Fig. 9. Order of inputs. Source: made by authors

Transfer of information between pages is done with the buttons placed on each page. After the click on the first button ("Calculate elements"), the VBA script-builds the tables on the second page, where the information about the members can be inputted, as well as unnecessary elements can be deactivated (by putting 0 value in the corresponding "Active" field). For example, as the supply chain mentioned above is 3-2-1-3, elements (2,3), (3,2), (3,3) are unnecessary and "Active" value is 0. The transition to the next page is done again by the button, this time "Calculate connections".

Connections are built depending on number of active elements on the previous page. The VBA algorithm will assume that if 2 elements are both active and located on the neighboring levels, they are connected. The connections specification page demonstrates tables with metrics for each such connection, marked by sender-recipient coordinates at the top of the tables. Between all the active elements there are 10 possible connections, which can be seen on Fig. 9 on the right. The connections can be deactivated the same way as elements, and if so, won't be counted into the actual model. The algorithm will suggest default values for the connection parameters, as shown in the Tab. 1.

Table 1. Metrics for connections (source: created by authors)

Metric	Description	Default value	Source of the default value pick
Active	Whether or not the connection exists	1	-
F/RF	Whether or not Factoring / Reverse Factoring is applicable at this connection	1	-
IF	Whether or not Inventory Financing is applicable at this connection	1	-
r	Interest rate on Factoring / Reverse Factoring	0.1	Klapper, 2006
i	Interest rate on Inventory Financing	0.11	Berger and Udell, 2006
q_RF	Speed of early payment, days	10	Gelsomino and Steeman, 2017
q3_IF	Number of days for the LSP to acquire the inventories from the supplier	2	Gelsomino and Steeman, 2017
q4_IF	Number of days for the LSP to pay the supplier for the inventories taken	3	Klapper, 2006
q6_IF	Number of days for the buyer to pay for the inventories from the LSP	30	Klapper, 2006

As algorithm was meant to provide as universal approach as possible, a lot of the parameters are provided as an example, and all of them can be easily togglable according to business needs. If some or all instruments are unavailable for certain members, they can be switched off, the same applies to the interest percentages or speed of operations.

After the input is complete, Python script can be initiated. The algorithm by which it performs is depicted on Fig. 10, and basically consists of 3 main parts: inputs processing, optimization block and objective function block. The script transforms the initial data into set of dictionaries with coordinates as keys, where all the elements and connections parameters are stored. After that, optimization process sends values of the variables into objective function, getting back the value of total financial costs, until the minimum is found, which will satisfy the constraints.

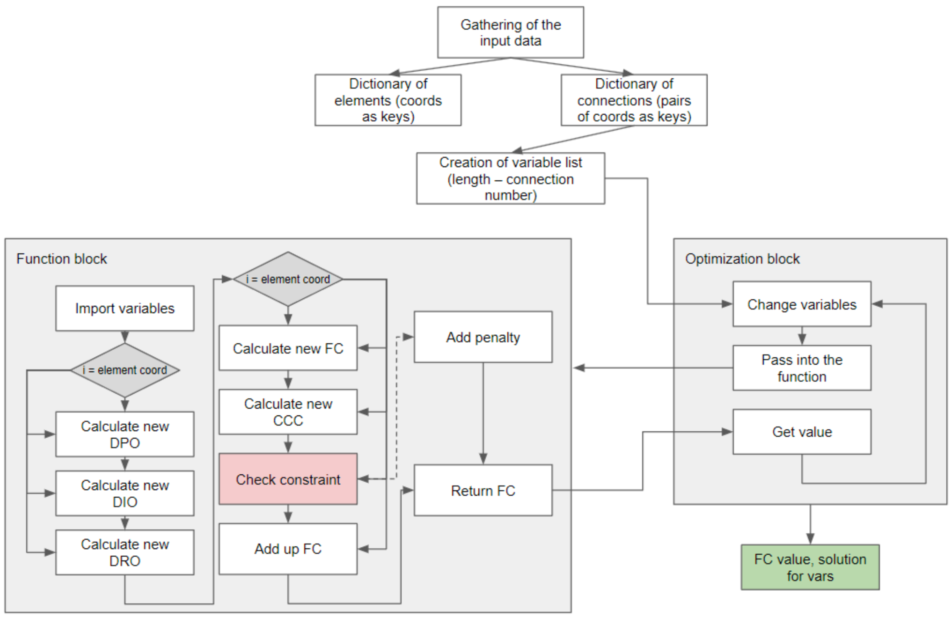

Fig. 10. Python algorithm blocks. Source: made by authors

As it was discussed in the beginning of the previous section, Differential Evolution method in scipy does not support constraints setting, and as a workaround it was decided to use penalties. Each time an objective function block finishes the calculation, it is checked on the constraint fulfilment, and if they are not met, a penalty to the result is added. The size of the penalty was designed to be dynamic, so the algorithm could get positive and negative feedback even while still being outside of constraint boundaries:

$$\begin{aligned} if\ FC_l^k > FC_{l\,0}^k: \\ Penalty_1 = (FC_l^k - FC_{l\,0}^k) * 1e7, \\ if\ CCC_l^k > CCC_{industry\ low}: \\ Penalty_2 = (CCC_l^k > CCC_{industry\ low}) * 1e7, \\ if\ CCC_l^k < CCC_{industry\ high}: \\ Penalty_3 = (CCC_{industry\ high} - CCC_l^k) * 1e7. \end{aligned} \tag{31}$$

This approach has a certain peculiarity, as the weight of the penalty is set experimentally. Current weight, 1e7, worked for test runs, but there could be potential situations where the potential decrease of the objective function could outweigh the penalty. If this happens, the weight can be toggled by the operator, setting it higher.

Although the model has several strengths, such as the ability to capture the interdependence between different stages of the complex supply chain and the ability to optimize financial costs on working capital and shares of intermediary participation while also providing a wide range of options for customization, it also has some limitations. One of the main limitations is that the flow of goods through the supply chain is assumed to be constant. In addition, the rates and ability of intermediaries to provide operations are also assumed to be constant throughout the supply chain. Finally, the supply chain is assumed to be consistent and fully known for the levels covered in the model, with no extra suppliers/buyers not included in the model.

To address these limitations, future research could focus on developing models that can capture the dynamics of the supply chain and incorporate uncertainties and disruptions. Additionally, future research could investigate the application of machine learning techniques to optimize supply chain networks that have ambiguity in the structure.

Based on the research findings of this section, it can be concluded that the mathematical model covering arbitrary topologies of the supply chain was developed successfully, and the chosen optimization approach performs satisfactorily, providing promising test results. The modernization of mathematical model provides broad horizons for future research covering supply chain financial costs on working capital, and developed optimization tool offers practitioners easy way to apply the model.

When it comes to applying the collaborative working capital optimization tool, Logistics Services Providers (LSPs) such as 3PL aggregators are ideally suited to implementing it. This is because unlike linear participants in the supply chain, LSPs have access to all members and information about them. By knowing the whole structure of the supply chain and basic financial information about the members, the LSP can calculate the best approach to minimize costs.

To implement the tool, the LSP must gather data on inventories, accounts receivable (AR), accounts payable (AP), cost of goods sold (CoGS), weighted average cost of capital (WACC), and net sales of all participants in the combined supply chain. This data will allow the LSP to calculate the optimal approach to minimizing financial costs while also trying to fit cash conversion cycles (CCCs) into the constraints defined by the industry (Fig. 11).

After gathering all the necessary data, the next step in implementing the collaborative working capital optimization tool is to put this data into the input of the model. This includes inputting data on inventories, AR, AP, CoGS, WACC, and net sales for all participants in the combined supply chain.

Once the data is inputted, the LSP can run the model to calculate the optimal approach to minimize financial costs while also fitting cash conversion cycles into the industry-defined constraints. Additionally, individual financial costs on working capital should be less or at least equal to the initial ones, otherwise communicating the value of collaboration to the participant who will suffer losses would become problematic.

After running the model, the LSP must check the results to ensure that they meet the constraints and basic adequacy. If the results do not meet the constraints, the LSP must redo the process by adjusting the input data or constraints to find a better solution. The approach to toggling the model is as follows: after getting

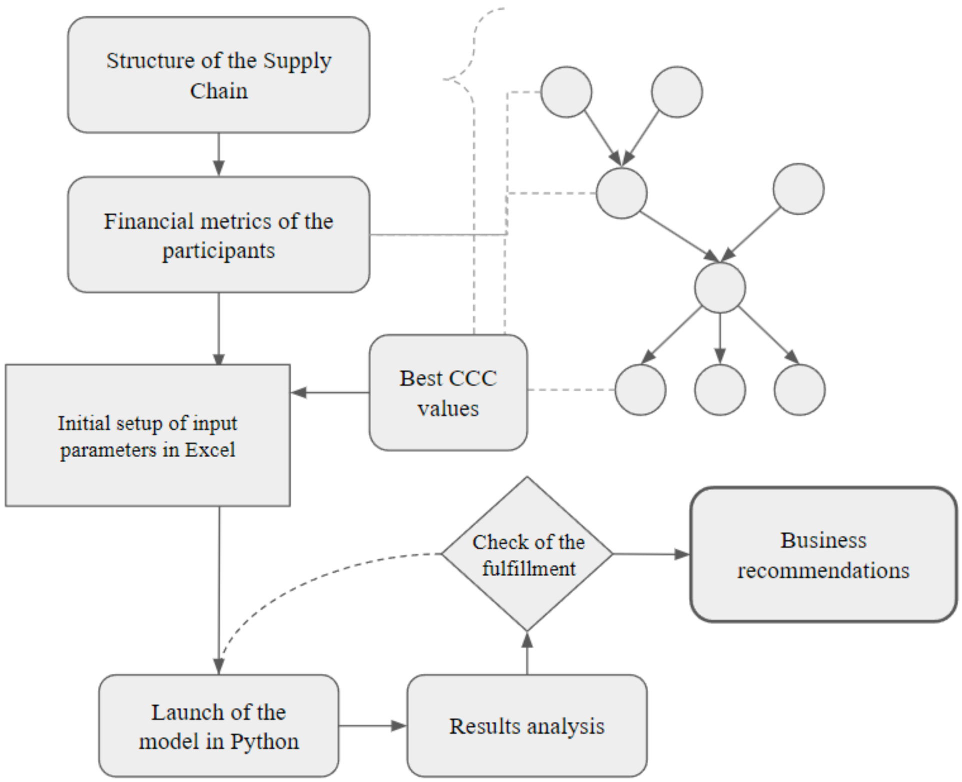

Fig. 11. Guideline for applying the tool. Source: made by authors

unsatisfactory results (model can fail to meet several constraints if they are too strict), operator can increase the penalty for not meeting the constraint. If it does not help, broadening the CCC range can be a solution as well. Another case could be illogical result of the objective function — that means that the result potentially can be mathematically correct, but hardly applicable. This case can be a result of penalty size set too high. Though it is important to point out that neither of these occasions were encountered in testing.

If the results do meet the constraints, the LSP can use the findings to make business conclusions and recommendations for businesses. These conclusions and recommendations should focus on which parts of their flows should be supported by intermediaries to optimize working capital and minimize financial costs.

4. Cases of Usage

4.1. Case 1. Supply chain from electronic technology industry

The first case study that will be discussed in order to implement the base model in practice pertains to a multi-level supply chain operating in the electronic technology industry. This complex supply chain includes multiple companies situated at various points in the chain, from suppliers to wholesaler and retailers (Fig. 12).

At the highest level of the supply chain (level 1), there are three supplier companies. The suppliers, located at coordinates (1,1), (1,2) and (1,3), provide crucial components for electronic devices.

The manufacturers, on the second level (level 2), at coordinates (2,1) and (2,2) assemble these components into finished goods.

At the third level (level 3), there is a wholesaler at coordinates (3,1) responsible for distribution to the final retailers.

Finally, on the fourth and last level of the supply chain (level 4), there are three retailers located at coordinates (4,1), (4,2), and (4,3). These retailers sell the electronic goods to the end consumers.

The working capital management within this supply chain could be characterized as collaborative but lacked supply chain-level coordination. Moreover, the internal payment periods among the supply chain members were identical and did not influence the collaborative cash conversion cycle at all. As a result, the collaborative cash conversion cycle at the end of 20XX was 2031,9 days, and the total financial costs for the supply chain were excessively high, amounting to 10,464 million rubles (Tab. 2).

According to J.P. Morgan Capital Index, recommended levels of CCC for electronic technology industry are from 65 to 187 days. In this research this range will be set as a goal for every participant.
The supply chain's intermediary, responsible for managing the collaborative working capital, is a logistics service provider (LSP). Based on the base model, the LSP formulates the optimization task as follows:

Goal: To reduce the total financial costs of the supply chain by at least 10% (considering the potential costs of restructuring the connections).

Constraints: To limit the cash conversion cycle of the affected members to the recommended industry-specific stability interval, with a lower limit of 65 days and an upper limit of 187 days.

The financial metrics of the companies involved are as shown in Tab. 2.

Following the implementation of the optimization plan, the collaborative cash conversion cycle was dramatically reduced to 1309,9 days (Tab. 3), and the total financial costs dropped to 5,885 million rubles. A significant reduction of total financial costs by 44% was achieved, which means that the main goal is fulfilled.

In comparison to the initial metrics the values have changed significantly, as depicted in Tab. 4.

Supply chain finance solutions were instrumental in realizing all of the collaborative working capital management goals. Based on the optimization results, the following recommendations are proposed regarding the involvement of intermediaries in the supply chain's processes (Tab. 5):

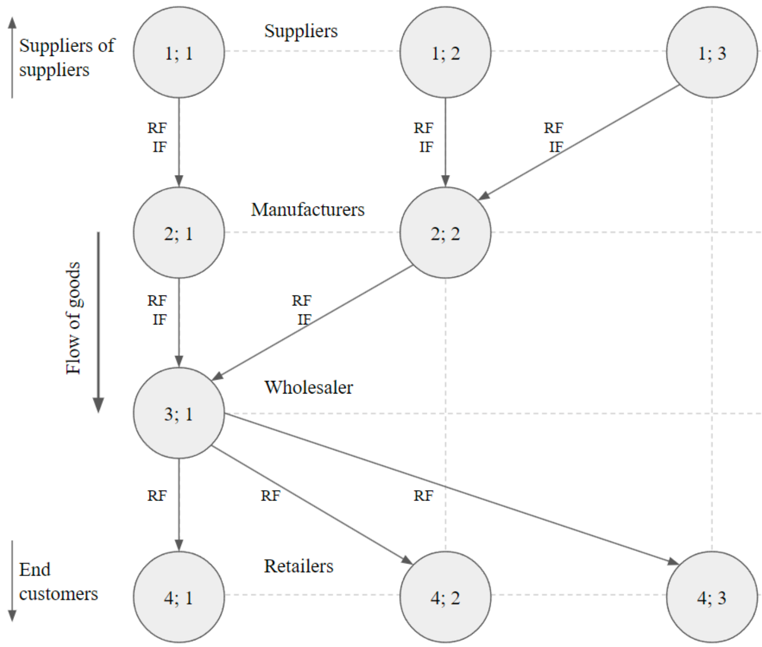

Fig. 12. Case from electronic industry: supply chain topology. Source: made by authors

Table 2. Initial data for Case 1 (source: created by authors)

Element coordinate	Inventories	AR	AP	FC	DIO	DRO	DPO	CCC
(1,1)	2621	919	509.2	190.2	315.6	38.5	61.3	292.8
(1,2)	3906	3339	1475	255.2	247.1	63.7	93.3	217.5
(1,3)	3341.6	2629.8	829.8	206.4	250.9	59.2	62.3	247.8
(2,1)	69398.9	69178.9	51685.6	4022.7	291.5	111.2	217.1	185.6
(2,2)	64541	75405	51168.7	3989	273.8	110.2	217.1	167
(3,1)	2998.2	1415.9	804	290.8	303.1	51.6	81.3	273.4
(4,1)	6071.5	2706.3	3454.8	554.8	416.3	51.1	236.9	230.6
(4,2)	5767.9	4322	3765.7	491.6	346.9	74.9	226.5	195.3
(4,3)	5585.8	3904	3765.7	463.3	420.9	84.8	283.8	221.9
Totals			10464			2031.9		

1. For the connection between members (1,1) and (2,1), it would be advantageous to have a share of early payment from the bank of approximately 47.4% and have 66.9% of the goods delivered via the Logistics Service Provider (LSP).
2. For the (1,2) to (2,2) member connection, a share of early payment from the bank of 45.9% combined with a 54.2% share of goods delivered via the LSP is recommended.
3. In the case of the (1,3) and (2,2) connection, the optimization showed that the most beneficial share of early payment from the bank would be 20.7% and the share of goods delivery through the LSP should be 34.6%.
4. For the pair (2,1) and (3,1), the algorithm recommends to not use the Reverse Factoring solution, with a 43.1% share of goods being delivered via the LSP.

Table 3. Results of the optimization for Case 1 (source: created by authors)

Element coordinate	Inventories	AR	AP	FC	DIO	DRO	DPO	CCC
(1,1)	879.4	1175	509.2	61.2	105.9	49.2	61.3	93.8
(1,2)	1808	4151.6	1475	222.3	114.4	79.2	93.3	100.3
(1,3)	2195.8	3194	829.9	170.2	164.8	71.9	62.3	174.4
(2,1)	39674.8	40272.4	37427.2	1659.1	166.7	64.8	157.2	74.2
(2,2)	53531.9	62922.5	57367.9	2054	227.1	92	243.4	75.7
(3,1)	2998.2	1415.9	2087.3	209.2	303.1	51.6	211	143.7
(4,1)	6071.5	2706.3	3454.8	554.8	416.3	51.1	236.9	230.6
(4,2)	5767.9	4322	3765.7	491.6	346.9	74.9	226.5	195.3
(4,3)	5585.8	3904	3765.7	463.3	420.9	84.8	283.8	221.9
Totals			5885.7			1309.9		

Table 4. Fulfilment of the goal and constraint fitting for Case 1 (source: created by authors)

Element coordinate	CCC in range	FC reduction
(1,1)	+	68%
(1,2)	+	13%
(1,3)	+	18%
(2,1)	+	59%
(2,2)	+	49%
(3,1)	+	28%
(4,1)	-	-
(4,2)	-	-
(4,3)	-	-
	TFC reduction:	44%

5. As for the pair (2,2) and (3,1), the optimization showed that Reverse Factoring is not profitable in this connection as well, and a 17.2% share of goods delivery through the LSP would be the most effective combination.
6. Regarding the connections from (3,1) to (4,1), (4,2), and (4,3), the recommended shares of early payment from the bank would be 35.8%, 24.0%, and 15.7% respectively.

Table 5. Recommendations for Case 1 (source: created by authors)

Coordinate pair	Share of early payment	Share of goods delivered via LSP
((1,1), (2,1))	47.4%	66.9%
((1,2), (2,2))	45.9%	54.2%
((1,3), (2,2))	20.7%	34.6%
((2,1), (3,1))	0.1%	43.1%
((2,2), (3,1))	0.1%	17.2%
((3,1), (4,1))	35.8%	-
((3,1), (4,2))	24.0%	-
((3,1), (4,3))	15.7%	-

The recommendations highlight that a mix of early payment options and involvement of LSPs in goods delivery could enhance the financial and operational efficiency of the supply chain.

In conclusion, the deployment of supply chain finance solutions in the multi-tiered supply chain of the electronic technology industry had an impressive and decidedly positive impact on the collaborative management of working capital. Substantial changes were made in reducing financial costs by 44%, which exceeded the revised target, and streamlining cash conversion cycles. Notably, all affected cash conversion cycles were successfully adjusted to fall within industry-specific stability intervals. Because of the algorithm's performance, these accomplishments highlight the significant improvements to the supply chain's financial efficiency. Looking ahead, the supply chain members are encouraged to use the recommendations provided by the algorithm.

The model suggests to not use any instruments at the retailer stage, but use both instruments heavily on the manufacturer-distributor ones. The mathematical model recommended avoiding using RF instrument on the manufacturer-wholesaler stage and relying substantially on the intermediaries for the flow of goods and finance for the rest of the chain.

4.2. Case 2. Supply chain from pharmaceutics industry

The second case study is dedicated to implementing the base model in practice within a multi-level supply chain operating in the pharmaceutics industry. This supply chain includes several companies located at different stages of the chain, from raw material suppliers to wholesalers and retailers.

At the top of the supply chain (Fig. 13), denoted as level 1, there are three supplier companies situated at coordinates (1,1), (1,2), and (1,3). These entities provide the essential ingredients and raw materials for pharmaceutical products.

At level 2, there are the manufacturers. These companies, found at coordinates (2,1), (2,2), and (2,3), are responsible for utilizing the received raw materials and formulating finished pharmaceutics goods.

On the third level, marked as level 3, there are two wholesalers, located at coordinates (3,1) and (3,2). Their role is to effectively distribute these pharmaceutical goods to the retailers. The final stage of the supply chain, level 4, is occupied by the retailers. Companies at coordinates (4,1), (4,2), and (4,3) sell the pharmaceutical products to the end consumers.

The management of working capital within this supply chain can be characterized as collaborative. The shared payment periods among the supply chain members were identical and did not influence the collaborative cash conversion cycle. Consequently, the collaborative cash conversion cycle at the end of the selected period was a concerning 893 days, with total financial costs being rather high at approximately 634,835 thousand rubles.

According to industry standards, recommended levels of the cash conversion cycle (CCC) for the pharmaceutical industry range from 100 to 200 days. This range is set as a performance goal for each participant in this study.

The supply chain's intermediary, tasked with managing the collaborative working capital, is a logistics service provider (LSP). The LSP formulates the optimization task using the base model as follows:

Goal: To reduce the total financial costs of the supply chain by 10% (considering the potential costs of restructuring the connections).

Constraints: To keep the cash conversion cycle of the affected members within the industry-specific stability interval, ranging from a minimum of 100 days to a maximum of 200 days.

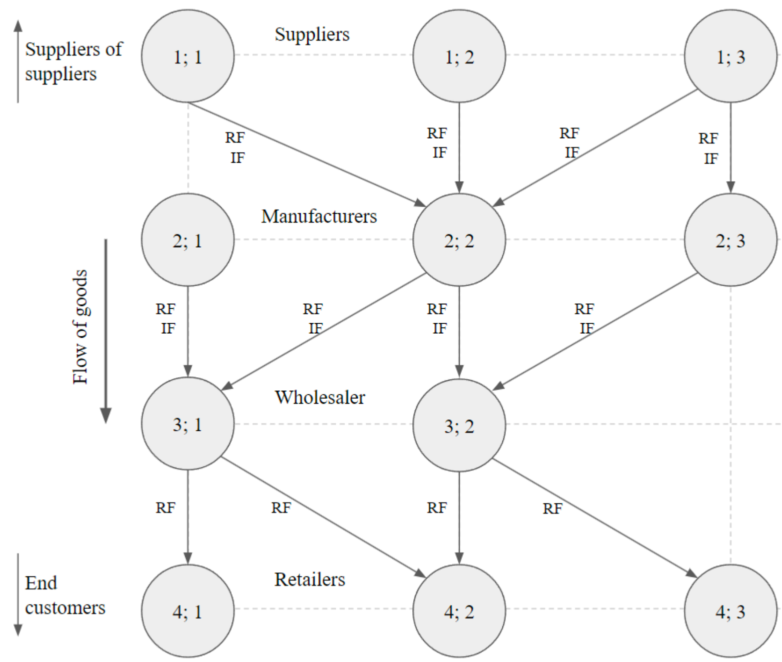

Fig. 13. Case from pharmaceutics industry: supply chain topology. Source: made by authors

The financial metrics of the companies involved are as shown in Tab. 6.

Following the execution of the optimization plan, the collaborative cash conversion cycle was substantially reduced to 750.7 days (Tab. 7), and the total financial costs declined to 564,487.3 thousand rubles. A noteworthy reduction in total financial costs of 11% was achieved, which meets the initial goal. As fitting into constraints fully turned out to be a struggle for the algorithm, it can be concluded that this model is much less flexible in terms of optimization. Nonetheless, algorithm substantially increased the financial situation of the members. It is also noticeable

Table 6. Initial data for Case 2 (source: created by authors)

Element coordinate	Inventories	AR	AP	FC	DIO	DRO	DPO	CCC
(1,1)	214719	68131	35467	68447.9	213.2	48.8	35.2	226.7
(1,2)	36625	123072	90047	5859.5	41.4	98.9	101.7	38.6
(1,3)	31131.2	121841.3	94549.4	7296.3	31.4	108.8	95.3	44.8
(2,1)	2489	3589	1536	507.4	43.8	63.9	27.1	80.7
(2,2)	951997	1900226	883800	278062.7	87.5	113.3	81.3	119.6
(2,3)	7536	11968	1703	1987.4	51.2	74.4	11.6	114
(3,1)	423638.7	883605.1	402129	135985.7	88.5	124	84	128.5
(3,2)	393984	918949.3	345830.9	139687.9	84	133	73.7	143.2
(4,1)	47250	88266	85841	183.9	28.1	36.7	51	13.8
(4,2)	16351	128791	114590	-1717.5	12.3	65.3	86.2	-8.6
(4,3)	15860.5	110760.3	99693.3	-1466.1	13.6	63.9	85.3	-7.8
Totals			634835.1					893.5

that the elements with which the algorithm struggled the most in terms of CCCs optimization are also the ones with the lowest FC change.

Table 7. Results of the optimization for Case 2 (source: created by authors)

Element coordinate	Inventories	AR	AP	FC	DIO	DRO	DPO	CCC
(1,1)	166751.2	56627	35467	40231.5	165.6	40.5	35.2	170.9
(1,2)	36436.7	122205.6	90047	5613.5	41.2	98.2	101.7	37.6
(1,3)	30975.2	119812.6	94549.4	6691.7	31.2	107	95.3	42.9
(2,1)	2439.7	3605.8	1536	507.4	43	64.2	27.1	80.1
(2,2)	812663.5	1513228.2	637311.3	256830.4	74.7	90.3	58.6	106.4
(2,3)	6984.4	10535.4	1907.8	1581.5	47.5	65.5	13	100
(3,1)	423638.7	883605.1	524046.7	122627.6	88.5	124	109.5	103
(3,2)	393984	918949.3	490619.9	133403.4	84	133	104.6	112.4
(4,1)	47250	88266	85841	183.9	28.1	36.7	51	13.8
(4,2)	16351	128791	114590	-1717.5	12.3	65.3	86.2	-8.6
(4,3)	15860.5	110760.3	99693.3	-1466.1	13.6	63.9	85.3	-7.8
Totals			564487.3					750.7

In comparison to the initial metrics the values have changed significantly, as shown in Tab. 8.

Supply chain finance solutions played a crucial role in moving towards the goals of collaborative working capital management. According to the optimization results, the following recommendations are made regarding the involvement of intermediaries in the supply chain's processes (Tab. 9):

1. For the connection between members (1,1) and (2,2), a share of early payment from the bank of 19.7% and a 22.6% share of goods delivered via the Logistics Service Provider (LSP) would be optimal.
2. For the (1,2) to (2,2), (1,3) to (2,2), (2,1) and (3,1) member connections, no instrument is suggested.

Table 8. Fulfillment of the goal and constraint fitting for Case 2 (source: created by authors)

Element coordinate	CCC in range	FC reduction
(1,1)	+	41%
(1,2)	-	4%
(1,3)	-	8%
(2,1)	-	0%
(2,2)	+	8%
(2,3)	-	20%
(3,1)	+	10%
(3,2)	+	4%
(4,1)	-	-
(4,2)	-	-
(4,3)	-	-
TFC reduction:		11%

3. As for the pair (2,2) and (3,1), the share of early payment is 13.7% with a 20.3% share of goods delivery through the LSP.
4. Regarding the connections from (3,1) to (4,1) and (4,2), the recommended shares of early payment from the bank are 42.2% and 21.9%, respectively.

Table 9. Recommendations for Case 2 (source: created by authors)

Coordinate pair	Share of early payment	Share of goods delivered via LSP
((1,1),(2,2))	19.7%	22.6%
((1,2),(2,2))	0.4%	0.5%
((1,3),(2,2))	3.2%	0.1%
((1,3),(2,3))	0.9%	0.9%
((2,1),(3,1))	3.6%	2.1%
((2,2),(3,1))	13.7%	20.3%
((2,2),(3,2))	39.2%	9.7%
((2,3),(3,2))	45.7%	7.6%
((3,1),(4,1))	42.2%	55.3%
((3,1),(4,2))	21.9%	-
((3,2),(4,2))	42.6%	-
((3,2),(4,3))	36.0%	-

These recommendations imply that a mix of early payment options and the involvement of LSPs in goods delivery can increase the financial and operational efficiency of the supply chain. The implementation of supply chain finance solutions in the multi-tiered supply chain of the pharmaceutical industry has had a positive effect on the collaborative management of working capital. The initial goal of reducing the financial costs was achieved, with a significant reduction realized. Almost all affected cash conversion cycles were successfully modified to fall within industry-specific stability intervals. These achievements underscore the significant improvements in the supply chain's financial efficiency resulting from the algorithm's recommendations. Looking forward, supply chain members are encouraged to utilize these recommendations to further improve efficiency. The model suggests using financial instruments heavily at the manufacturer-wholesaler stage, but sparingly or not at all for the beginning of the supply chain. This optimizes the financial costs of the supply chain and aligns it with industry-defined constraints.

5. Research Summary and Limitations

This study sought to create a multi-objective collaborative model for optimizing working capital management in combined supply chains. A mathematical model based on Differential Evolution optimization was developed, emphasizing collaborative optimization across supply chains, catering to arbitrary topologies and offering extensive customization options.

A tool and a guideline were also produced for practical implementation of the model, detailing the steps from information gathering to model output interpretation. To illustrate its real-world implications, two industry case studies were conducted, revealing substantial improvements in working capital management. The model's results highlight its benefits in reducing financial costs for all participating members.

This work contributes to the academic research of collaborative working capital optimization in supply chains of arbitrary topologies. On the practical side, the model, application guide for the tool based on the model, and case studies offer insights for 3PL-providers and industry specialists striving for more efficient supply chain operations through better working capital management.

While the results of this work are promising, several limitations should be noted. Firstly, the successful implementation of the proposed model requires a high level of collaboration and trust among the supply chain members. The accuracy of the financial metrics, which are crucial for the model's efficiency, can be compromised if the members pursue individualistic targets or engage in manipulation. Trust in the intermediary who implements the tool is essential for the integrity and success of the optimization process.

Secondly, it is recommended that supply chain members operate on the same financial platform, such as being financed by a single bank or partnering banks, and using the same financial services. This compatibility greatly influences the ability of an intermediary to implement the model's recommendations, which could be a barrier in heterogeneous financial environments.

The interest rates of banks and LSP intermediaries also present a limitation. While default values are provided within the tool, these should ideally be adjusted to reflect specific market conditions for the most accurate results.

Additionally, while industry recommendations for CCC ranges were used in the case studies, a more tailored result could be achieved if each member provided ranges applicable to their specific business operations.

Future research could focus on expanding the model to include more financial instruments, thereby increasing its robustness and applicability. Also, research could explore mechanisms to enhance trust and collaboration among supply chain members, including methods to verify the accuracy of shared financial data. Additionally, future work could explore solutions for supply chains operating in more heterogeneous financial environments. By addressing these areas, the model's real-world applicability and effectiveness could be further enhanced.

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