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COORDINATING BUYBACK CONTRACTS WITH LIMITED FUNDING

Master's Thesis by 2nd year student

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ЗАЯВЛЕНИЕ О САМОСТОЯТЕЛЬНОМ ХАРАКТЕРЕ ВЫПОЛНЕНИЯ ВЫПУСКНОЙ КВАЛИФИКАЦИОННОЙ РАБОТЫ

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АННОТАЦИЯ

Автор	Сон Ирина Олеговна
Название ВКР	Координирующие контракты с обратным выкупом при ограниченном финансировании
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Описание цели, задач и основных результатов	Целью данной работы является построение модели контракта с обратным выкупом при ограниченном финансировании и разработка алгоритма выбора параметров данного контракта, которые обеспечивают условную координацию цепи с двумя участниками. Для достижения поставленной цели были сформулированы следующие задачи: • Проанализировать и представить обзор современных исследований по применению контрактов для координации цепей поставок, в особенности контракта с обратным выкупом, а также по проблеме ограниченного финансирования в координации цепей поставок с помощью контрактов; • Построить модель контракта с обратным выкупом при ограниченном финансировании и проанализировать координационные свойства данного контракта; • Разработать алгоритм выбора параметров условно-координирующего контракта с обратным выкупом при ограниченном финансировании; • Применить разработанные модель и алгоритм для определения параметров данного контракта на кейсе компании. В работе были получены следующие результаты: • Установлено, что контракт с обратным выкупом при ограниченном финансировании не обеспечивает координацию, однако позволяет достичь условной координации цепи согласно определениям координирующего и условно-координирующего контрактов, используемым в работе; • Доказано, что в условиях ограниченного финансирования торговый кредит является более предпочтительной формой кредитования ритейлера, чем банковский кредит, т.к. он обеспечивает более высокие прибыли обоим участникам цепи поставок как при контракте с обратным выкупом, так и при контракте по оптовой цене; • Установлено, что более высокая прибыль цепи поставок, достигаемая при торговом кредите за счет роста прибыли обоих участников, совпадает с уровнем прибыли цепи в случае, когда средства ритейлера не ограничены;

	• Разработан алгоритм выбора параметров контракта с обратным выкупом при ограниченном финансировании, который обеспечивает условную координацию цепи с двумя участниками и максимизирует прибыль цепи с учетом источника кредитования, выбранного ритейлером (торговый или банковский кредит); • Данный алгоритм также позволяет выбрать параметры контракта по оптовой цене, которые выгодны обоим участникам цепи – также с учетом выбранного источника кредитования.
Ключевые слова	Цепь поставок, координация, обратный выкуп, контракт, ограниченное финансирование, торговый кредит

ABSTRACT

Author	Irina Son
Master Thesis Title	Coordinating Buyback Contracts with Limited Funding
Educational Program	38.04.02 «Management»
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Description of the Goal, Methodology and Results	The goal of this study is to build a model of the buyback contract with limited funding that allows for two forms of borrowing – bank loan and trade credit – and develop an algorithm for selecting parameters of the buyback contract with limited funding that provide conditional coordination of a supplier-retailer supply chain. To achieve the research goal, the following tasks were formulated: • To analyze existing approaches to supply chain coordination with contracts, and with buyback contract in particular, and approaches to limited funding in order to justify the scope and approach to modeling the buyback contract with limited funding and studying its coordinating properties; • To construct the model of the buyback contract with limited funding and analyze coordinating properties of this contract; • To develop an algorithm for selecting the parameters of conditionally coordinating buyback contract with limited funding; and • To apply the constructed model and developed algorithm to a company case. Results obtained in the study are as follows: • It is found that the buyback contract with limited funding does not coordinate the supply chain, but allows to achieve its conditional coordination according to the definitions of coordinating and conditionally coordinating contracts applied in the paper; • It is proved that in the case of the retailer's limited funding trade credit is preferable to the bank loan as it provides the supply chain members with higher profits relative to those earned by them with

	the bank loan, this conclusion is fair for both wholesale-price and buyback contract types; • It is shown that the supply chain profit reached with the trade credit coincides with that achieved in the case where retailer's funds are sufficient, this profit level is higher relative to the supply chain profit with the bank loan; • The algorithm for selecting the parameters of the buyback contract with limited funding is proposed, it allows to achieve conditional coordination in the supplier-retailer supply chain and maximizes its total profit considering the type of credit chosen by the retailer (trade credit or bank loan); • The algorithm also allows to select the parameters of the wholesale-price contract that are beneficial for both supplier and retailer – with the type of credit taken into account as well.
Keywords	Supply chain, coordination, buyback, contract, limited funding, trade credit

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INTRODUCTION

Optimal performance of supply chains is a hot topic these days. It commonly implies an optimal set of actions taken by the supply chain members so as to maximize supply chain's total profit while maximizing their own profits. To ensure that the supply chain members act in accordance with the optimal set of actions, their relationships and interdependencies should be managed through cooperation, with coordination being its main component and the subject of this study.

To coordinate the actions of supply chain parties several mechanisms can be applied, including contracts. The ubiquity of the contracts' application makes the problem of setting optimal contract parameters that ensure the maximum of supply chain's profit highly relevant today.

Nonetheless, practice shows that the process of managing contractual relationships often fails to provide the contracting parties with the best possible results. Despite the fact that companies can save up to 80% on transaction costs with generally accepted and clearly regulated rules and procedures related to contracting, only 48% of business entities actually run an organized process of contract management (Berezinets, Meshkova & Nikolchenko, 2019).

As the field of research, supply chain contracting is focused on the profit maximization by the companies involved in the supply chain, so it is straightforward to confirm that the problem studied in this body of research is essential not only for management in general, but also for financial management.

Besides addressing general management issues, the study touches upon such particular aspect of financial resources management as credit used by the retailer to pay the supplier in exchange for the goods retailer wants to purchase for further retailing. This issue is indicated by the term "limited funding", which refers to a situation where the retailer's holdings of cash are constrained and might be insufficient to pay the supplier. The paper considers two types of credit – bank loan and trade credit.

The issue of limited funding has grown since the start of the COVID pandemic. Recent research on the state of US supply chains carried out by SAP (2022), one of the world's largest enterprise software companies, found that more than half of businesses (that took part in the survey) suffered a decrease in revenue and faced the need for taking new financing measures, such as business loans.

Consequently, researchers have been showing interest in the supply chain coordination with contracts with limited funding considered. However, the majority of those studies focus on the contract types other than the buyback contract (Kouvelis, Zhao, 2012; Chen, 2015; Zhan, Chen, Hu, 2018). Besides, the papers dedicated to the buyback contract do not exhaustively consider the problem of limited funding – they analyze coordinating properties of the contract with only one form of borrowing applied and focus on different aspects of limited funding, such as bankruptcy risk tolerance or working capital management (Xiao, Sethi, Liu, Ma, 2017; Fu, Liu, 2019).

Therefore, the goal of this study is to build a model of the buyback contract with limited funding that allows for two forms of borrowing – bank loan and trade credit – and develop an algorithm for selecting parameters of the buyback contract with limited funding that provide conditional coordination of a supplier-retailer supply chain.

Although there exist various approaches to defining it, in this paper the term 'supply chain coordination' refers to the situation where the contract parameters agreed by supply chain members allow both supplier and retailer to maximize their profits, thus achieving the maximum of supply chain profit. Unfortunately, supply chain coordination by this definition fails virtually for all contract types studied in the field of supply chain contracting, which is why the term 'conditional coordination' is used.

Conditional coordination of a supplier-retailer supply chain indicates the situation where the set of the contract parameters ensures maximal retailer's profit, improves supplier's profit relative to the benchmark contract instead of maximizing it, and thus maximizes supply chain's profit. The benchmark contract is usually the wholesale-price contract as this is the simplest and most popular contract type that is generally considered to be non-coordinating. The definition of conditional coordination and, hence, the framework for modeling the buyback contract have been proposed by Berezinets et al. (2020).

To achieve the research goal, the following tasks were formulated:

- To analyze existing approaches to supply chain coordination with contracts, and with buyback contract in particular, and approaches to limited funding in order to justify the scope and approach to modeling the buyback contract with limited funding and studying its coordinating properties;
- To construct the model of the buyback contract with limited funding and analyze coordinating properties of this contract;
- To develop an algorithm for selecting the parameters of conditionally coordinating buyback contract with limited funding; and
- To apply the constructed model and developed algorithm to a company case.

The object of the study is the buyback contract with limited funding, and also the wholesale-price contract with limited funding as required by the definition of conditional coordination. The subject of the study is conditional coordination, and also the procedure for selecting the parameters of the conditionally coordinating buyback contract with limited funding fixed in the algorithm that can support the decision-making process when negotiating the contract parameters.

The paper is organized as follows. Chapter 1 deals with the contract as the coordination mechanism. Section 1.1 gives an overview of research on supply chain coordination with contracts. Section 1.2 focuses on the buyback contract, provides an overview of existing research on supply chain coordination with this contract type. Section 1.3 gives an overview of research dedicated to limited funding and the ways of dealing with it studied by researchers in supply chain contracting.

Chapter 2 focuses on modeling the buyback contract with limited funding and presents the solution of the problem of supply chain coordination with this contract. Section 2.1 describes the model in detail. Section 2.2 presents the solution of the coordination problem with the modeled contract when retailer raises the bank loan. Section 2.3 is dedicated to solving the coordination problem with the modeled contract when retailer uses trade credit. Section 2.4 considers the model of conditionally coordinating buyback contract with limited funding.

Chapter 3 examines conditionally coordinating buyback contract with limited funding in the case of uniform distribution of demand and presents the algorithm for selecting its parameters based on the constructed model. Section 3.1 describes the model with the demand distributed as uniformly in detail. Section 3.2 presents the results of comparative analysis of the modeled contract with the bank loan against that with trade credit. Comparative analysis is carried out in order to justify the supply chain members' motivation that drives their choice of the type of credit out of the two alternatives examined in the paper. Section 3.3 focuses on the algorithm that allows to select the parameters of conditionally coordinating buyback contract with limited funding based on a wholesale price they have agreed prior to the start of the selection process. The algorithm also allows to select the parameters of the wholesale-price contract that improve the profits of supply chain members in the case where their negotiation does not arrive at the conditionally coordinating buyback contract. Section 3.4 examines the numeric example of the contract written by the members of the supply chain involved in manufacturing and retailing of the perishable product with short shelf life.

Conclusion summarizes the results obtained in the study. Details on modeling the buyback and wholesale-price contracts both with bank loan and trade credit, and problem solving can be found in the Appendices.

CHAPTER 1. BUYBACK CONTRACT WITH LIMITED FUNDING

1.1. Contracts as a Mechanism of Supply Chain Coordination

Supply chain can be defined as a network of interconnected firms that are involved in different processes and activities producing value in the form of products and services, which are delivered to the ultimate consumer (Giannoccaro, 2018; Christopher, 2011:13).

Management of a supply chain can adopt different levels of centralization, depending on the concentration of the decision-making power exercised by one supply chain member. High centralization (centralized supply chain) implies that only one supply chain member fully exercises the decision-making authority for all the partners in the supply chain. Low centralization (decentralized supply chain) is characterized by each firm making decisions independently from the rest of supply chain members (Giannoccaro, 2018).

The construct of centralization originates from the field of strategic management and is referred to as a dimension of organizational structure that reflects the degree to which the decision-making autonomy is spread or concentrated among the layers of organizational hierarchy (Davis-Sramek, Germain & Krotov, 2015). In supply chain management centralization can be defined as a degree to which a single firm makes decisions concerning the supply chain operations for all the firms belonging to this supply chain to comply with.

The choice of the level of centralization greatly affects the supply chain's performance and, hence, its financial result, such as profit. Researchers in supply chain management and supply chain contracting agree that high level of centralization is preferable to its low level. Centralization brings such benefits to the supply chain as improved efficiency and reduced distortions of demand information moving up the supply chain from the retail end to the manufacturing end (i.e., the bullwhip effect) (Lee, Padmanabhan & Whang, 1997), better management of complex problems and better performance in uncertain environment. It also allows the partners along the supply chain to resolve their conflicting goals (Giannoccaro, 2018).

Decentralization, on the other hand, can increase flexibility and communication between supply chain members, and improve innovation. However, it implies that the decision-making authority is distributed among the members. As each firm in the supply chain is primarily concerned with her own performance and does not align her objectives with those of the supply chain, this creates an incentive conflict in the supply chain and results in its poor performance.

One example of implementation of the centralized approach to the decision-making is the vendor-managed inventory (VMI), where one firm in the supply chain (the supplier) manages the inventory belonging both to her and to the other entities involved in that supply chain. VMI is discussed in more detail further in Section 1.1. The centralized approach allows to achieve the optimal supply chain performance since all decisions in the supply chain are made and controlled by a single firm, thus coordination of the centralized supply chain is naturally assured.

The majority of papers on centralization implicitly assume that whatever level of centralization is adopted in the supply chain, its members are fully rational and informed (Giannoccaro, 2018). Real supply chains, however, frequently show the features that are different from those assumed. According to Liu, Parlar and Zhu (2007), it is common that supply chain parties act in an independent fashion as they have different access to information on demand and internal operations and focus on different aspects of performance (e.g., retailer focuses on marketing while supplier focuses on internal operations).

This results in suboptimal financial result of the supply chain as compared to the centralized one. Consequently, coordination mechanisms are required to motivate the supply chain parties to align their goals and ensure the optimal performance and financial result of the supply chain.

As introduced by Malone and Crowston (1994), coordination can be defined as the process of managing dependencies between activities. Research on coordination emerged in late 1980s as an attempt to resolve issues connected with complex systems in diverse spheres. By that time there had already been extensive body of research on the issues related to coordination in various disciplines, which shaped the interdisciplinary approach to the coordination theory that draws upon such scientific areas of study as computer science, organization theory, management science, economics, and psychology. Based on the definition proposed by Malone and Crowston (1994), coordination theory aims to identify and analyze different kinds of dependencies and develop coordination processes that help manage them.

Regarding the subject of this study, two research areas investigating the problem of coordination are especially important in the context of this study, namely economics and operations research. The former field focuses on the ways incentives and information flows impact the allocation of resources between actors; the latter concerns the mechanisms of coordination, their properties, and optimal techniques for making coordinated decisions; together they provide a framework for studying supply chain coordination.

One example here is classical microeconomics with one of its major results, which is the formal proof that maximization of consumers' individual "utilities" and maximization of firms' individual profits results in the globally "optimal" allocation of resources. This implies that no actor can increase their utility without decreasing utility of another actor (Debreu, 1959 cited in Malone & Crowston, 1994).

Another example is agency theory that, among other things, touches upon the limitations of contracts for allocating resources (Ross, 1973 cited in Malone & Crowston, 1994). Agency theory investigates ways to incentivize agents in order to act in the interest of principals, though

Jensen and Meckling (1976) cited in Malone and Crowston (1994) found that there exist situations where no incentives can encourage the agent's performance that is optimal from the principal's point of view.

Lee (2000) cited in (Arshinder, Deshmukh, 2008) suggests to view supply chain coordination as a vehicle for redesigning decision rights, workflow, and resources between supply chain actors to achieve higher profit margins, improve customer service and reduce response time. Supply chain coordination counteracts the negative effects of decentralization – inaccurate forecasts, low capacity utilization, excessive inventory, higher inventory costs, delayed schedules and longer order fulfillment response, poor quality, inadequate customer service, and low customer satisfaction – and suggests the way to allocate its total profit so that each partner is better off (than he could be without deploying coordination mechanisms). The most common mechanisms are information technology, information sharing, joint decision-making, and contracts (Arshinder & Deshmukh, 2008).

Information technology suggests diverse tools that assist supply chain firms to plan, track and estimate lead times, thus enabling a faster exchange of products, information and money between them. Examples of such tools include electronic data interchange (EDI) and enterprise resource planning (ERP).

Sharing information on demand and inventory between supply chain members is another mechanism that brings such benefits as reduction in inventory and cost savings up to 12.1%, as found by Cachon and Fisher (2000) in their comparative study of full information sharing opposed to none. The authors note, though, that it is essential not to simply expand the information flow in the supply chain, but to share information on the lead times and batch sizes that really helps accelerate and smooth the flow of products through the supply chain.

Joint decision-making regarding replenishment, planning and other aspects of supply chain performance is another means for supply chain coordination. Examples of its implementation include vendor-managed inventory (VMI) and collaborative planning, forecasting and replenishment (CPFR). In the case of VMI, the supplier takes full responsibility for controlling and maintaining inventory levels and determining order quantities for his customers, which requires sharing data about demand and costs in a timely manner. With CPFR supply chain partners are supposed to put joint efforts into planning and synchronize their forecasts as the basis for future replenishment. Benefits of CPFR include higher sales, higher service levels, shorter response time, and lower inventory levels.

Another mechanism for achieving supply chain coordination is contracting. Applied in decentralized supply chains, they allow to manage the actions of supply chain partners so that total profit of the supply chain increases and is arbitrarily divided among its members.

Contract, by legal definition, is an agreement between parties that creates mutual rights and obligations enforceable by law (www.law.cornell.edu). The making of a contract requires the mutual agreement of the contracting parties, one of them making an offer and another one accepting (www.britannica.com). Contracts are widely applied in commercial law and form the legal foundation for transactions between entities across the world.

From the economic point of view, contract is "an agreement between a buyer and a supplier in which the terms of an exchange are determined by three factors: price, asset specificity, and guarantees" (Williamson, 1985 cited in Berezinets, Meshkova & Nikolchenko, 2019). The conclusion of the contract results from a negotiation, which serves the purpose of determining the terms of exchange (the contract parameters) acceptable to each party involved. The contract parameters are "acceptable" when the party's expected profit from participation in the contract is not lower than a certain profit level, which is called reservation profit. Reservation profit is expected profit that the company can derive without participating in the contract (Bernstein & Marx, 2006 cited in Berezinets, Meshkova & Nikolchenko, 2019). The contract is concluded if the parties reach an agreement on these conditions; otherwise, the contract is not concluded.

With supply chain contracting the firms manage their incentive conflicts and align their objectives with the supply chain's objective, through which optimal supply chain performance is achieved. Tsay, Nahmias and Agrawal (1999) distinguish the following positive effects of supply chain contracting – improvement of supply chain performance, sharing the risks between contracting parties, and facilitation of long-term partnerships.

Researchers in supply chain contracting deal with several contract types. These contract types have common parameters of the wholesale price and the order quantity, but differ in the incentives created in order for the supply chain parties to implement optimal actions required for supply chain coordination. Based on those incentives, the types of the contracts are as follows:

- revenue-sharing contract, based on which the supplier charges a wholesale price that is lower than the supplier's costs plus a percentage of the retailer's revenue earned by the end of the selling season (Giannocaro & Pontrandolfo,2004; Zenkevich & Gladkova, 2018);
- quantity-flexibility contract, which suggests that the supplier charges a wholesale price, but compensates the retailer's losses on units that are unsold by the end of the selling season; the supplier's payment to the retailer at the end of the selling season is equal to the amount of unsold inventory, but not higher than the threshold set in the contract (can be absolute quantity or a percentage of the retailer's order) (Tsay, 1999);
- sales-rebate contract, which implies that the supplier sets a wholesale price and a threshold for the order quantity, and for each unit sold by the retailer above this threshold, the supplier pays a bonus (rebate rate), thus incentivizing the retailer to order and put more effort

in selling larger volumes of the product (Taylor, 2002; Berezinets, Meshkova & Nikolchenko, 2019);

- quantity-discount contract, which has many types itself and basically allows to adjust the retailer's marginal cost curve so that the supplier's earnings decline progressively with each additional unit sold to the retailer (Cachon, 2003); and
- buyback contract, which is of particular interest in this study and discussed in detail further.

Based on different approaches to interpreting coordination in general, coordinating contract can be defined in different ways. One condition commonly included in definitions of the coordinating contract is maximization of the expected total profit earned by the supply chain. But this definition does not take into account the profit of each participant, which leaves room for avoiding compliance with the contract terms if individual profit does not seem acceptable.

Cachon (2003) defined coordinating contract as a contract which has the parameters that provide a unique Nash equilibrium (so that the partners have no incentive to deviate from the contract terms) and allow each partner to maximize their individual expected profit so as to maximize the expected supply chain profit. However, scholars develop alternative definitions of coordinating contract as their approaches to coordination may differ from the one suggested by Cachon (2003).

One of the alternative approaches to defining the coordinating contract suggests that it must maximize supply chain profit and provide supplier and retailer with a "win-win" situation, which implies that their individual profits with a coordinating contract should be higher than without it (Taylor, 2002), (Saha, 2013).

Another definition proposed by Heydari, Choi and Radkhah (2017) enhances the definition introduced by Cachon (2003) and states that a coordinating contract should maximize supply chain profit and provide a Pareto-optimal solution. Pareto-optimality implies that the set of coordinating contract parameters must provide at least one contracting party with the profit that is at least not worse compared to the non-coordinating contract.

One more approach to defining coordinating contract based on the definition by Cachon (2003) is suggested by Berezinets, Meshkova and Nikolcheko (2019) and Berezinets et al. (2020). The authors assume that under the assumption of risk neutrality of supply chain parties coordinating contract motivates each party to make decisions that provide maximum of the expected supply chain profit, i.e., each party intends to maximize individual profit and arrives at the contract parameters that maximize the supply chain profit. The authors also emphasize the equivalence of the problem of supply chain profit maximization and supply chain coordination as the latter is achieved with maximal supply chain profit.

Applying this approach to studying such contract types as revenue sharing (Zenkevich, Gladkova, 2018), sales rebate (Berezinets, Meshkova & Nikolchenko, 2019) and buyback contract (Berezinets et al., 2020), the authors come to a conclusion that these contracts do not allow to achieve supply chain coordination for their application maximizes only the retailer's and the supply chain profit, and not the supplier's. They introduce the definition of a conditionally coordinating contract that allows to achieve maximal retailer's profit, improve the supplier's profit compared to that earned with the wholesale-price contract, and, thus, maximizes the supply chain profit, which is the sum of the supplier's and retailer's profits. All the three types of supply contracts are proven to be conditionally coordinating.

1.2. Supply Chain Coordination with Buyback Contract

Parameters of the buyback contract include – wholesale price ω, quantity of goods q ordered by retailer, and buyback price b which supplier will pay to retailer for all the items left unsold at the end of the selling season. The supplier's promise of buying unsold goods back from the retailer gives the latter an incentive to order more as typically retailer is reluctant to order large amounts of the product and run the risk of overstocking caused by demand uncertainty.

Buyback contracts are commonly applied when demand is difficult to predict and the risk of obsolescence of the product may be high (Tsay, 2001). Examples of such industries are publishing and book selling business, fashion and beauty products, seasonal products and those with the short life cycle, such as FMCG and grocery retail (Pasternack, 1985; Tsay, 2001; Bose & Anand, 2007; Berezinets et al., 2020).

For instance, in fashion supply chains it is common for retailers to hold up to 40% of unsold goods by the end of the selling season. Examples of fashion supply chains include fashion department stores in the U.S. like Saks, Kohl's and J.C. Penney that operate using the buyback agreement written in their contracts with brands like Tommy Hilfiger and Ralph Lauren (Shen & Li, 2015).

The paper applies the game-theoretic approach as virtually all recent literature on supply contracts draws upon classic works like (Cachon, 2003). The author analyzed the newsvendor model transformed into a game, which is relatively simple, but sufficiently rich to study coordinating properties of supply contracts, and then scrutinized six types of contracts, including buyback contract and wholesale-price contract, which are explored in this body of research. The study applied the game-theoretic approach, which now plays an essential part in the field (Cachon, Netessine, 2004).

The newsvendor model considers a supplier-retailer supply chain during one selling season. The supplier manufactures a single type of product and sells it to the retailer, which then sells it to the end consumers in the market.

The relationship between supplier and retailer is regulated by their contract. According to this contract, supplier offers certain terms that vary depending on the contract type, and retailer reacts with an amount of goods he decides to order from the supplier. Retailer can place his order only in advance of the selling season and thus is faced with the newsvendor problem – during the selling season he is supposed to satisfy the stochastic market demand. As supplier receives the retailer's order quantity, she produces and fulfills the order before the start of the selling season. At the end of the selling season, payments for the goods are transferred in accordance with the agreed contract terms. The model assumes that both parties are risk-neutral, have full access to the information related to the contract and aim to maximize their profits.

Regarding the buyback contract, Cachon (2003) extended the model introduced by Pasternack (1985), one of the pioneering studies of buyback contracts that proved its coordinating properties provided that total supply chain profit was maximized. The author came to the same conclusion that supply chain coordination could be achieved under this contract type with any profit allocation, including extreme situations where the supply chain profit is completely earned by the retailer or the supplier. Nonetheless, the author highlighted the ambiguity of these cases since the optimal set of the contract parameters providing coordination is not a unique Nash equilibrium, which means that there also exists a suboptimal set of the contract parameters that may also show coordinating properties.

Cachon (2003) also showed evidence that profit allocation between supplier and retailer would depend on the simultaneous adjustment of the wholesale price and the buyback rate by presenting the wholesale price as a function of the buyback rate. In practice it means that these parameters should be negotiated simultaneously and not one after another, which is another reason to apply the game-theoretic approach, as it reflects the real process of negotiating the contract terms in the best way.

Another revision of the buyback contract was conducted by Cachon and Lariviere (2005). The authors primarily studied coordinating properties of the revenue-sharing contract and came to the conclusion that this contract type genuinely coordinates the supply chain in its classical one-to-one setting; then compared it to other supply contracts, such as buyback contracts, price-discount contracts, quantity-flexibility contracts, franchise contracts, and sales-rebate contracts.

Using the game-theoretic approach, the authors created a general model that was applicable for both stochastic and deterministic demand and did not depend on the revenue function. Their findings showed that the revenue-sharing contract was equivalent to the buyback contract in the newsvendor model and coordinated the supply chain with a fixed retail price. It was shown that for any buyback contract there exists a revenue-sharing contract that allows to generate the same cash flows regardless of the product demand. But this conclusion could not be generalized as the authors provided evidence that the revenue-sharing contract in some cases could coordinate the supply chain while the buyback contract could not.

Pasternack (1985), Cachon (2003), and Cachon and Lariviere (2005) provided a solid framework for further research on the topic of supply chain coordination with contracts, and there is an extensive body of research that further develops their approach in various directions, including:

- expansion of the supply chain by increasing the number of its partners, like (Wu, 2013);
- extension of the timespan in the model, as in (Zou, Pokharel, Piplani, 2008);
- coordination with the price-dependent demand, like (Granot, Yin, 2005) and (Zhao et al., 2014);
- information asymmetry, for example (Babich, Ritchken & Wang, 2012);
- relaxing the risk-neutrality assumption, like (Tsay, 2002); and
- capital constraints faced by the supply chain partners.

The first direction of research from the list above is dedicated to the extension of the number of contracting parties. For example, Wu (2013) investigates the buyback contract under the framework of competing supply chains applying it to the case of duopoly of two supply chains with a single supplier and retailer. The author comes to a conclusion that the buyback contracts help to increase the order quantity and decrease the wholesale and retail prices. The supply chain coordination with the buyback contract is achieved through its implementation in both supply chains thus leading to higher profits compared to the case with no buyback contract.

Another extension of the traditional buyback contract introduced by Pasternack (1985) considers a contract setting for multiple periods, such as (Zou, Pokharel, Piplani, 2008). The authors examine an assembly system that consists of one assembler (retailer) and two suppliers. The two-period model where two consecutive transactions take place is considered. By simultaneously adjusting the wholesale and buyback prices, the supply chain achieves coordination.

There is also the direction of research dedicated to the investigation of supply chain coordination with price-dependent demand. For example, Zhao et al. (2014) suggest some extensions of the results presented in (Granot, Yin, 2005). The paper focuses on how the demand uncertainty level affects the applicability of buyback contracts in supply chain coordination. Using the Stackelberg framework, the authors derive the Stackelberg equilibrium in a game with two players (supplier and retailer) and the additive form of demand and come to a conclusion that the demand uncertainty level is a critical factor in terms of applicability of buyback contracts.

A large number of studies explore the supply chain coordination problem with the relaxed assumption about the complete information available to all the supply chain partners. Information asymmetries may concern demand forecast and information about the costs run by the retailer or the supplier. For example, Babich, Ritchken and Wang (2012) solve the problem of designing a buyback contract for a two-echelon supply chain where the retailer has private information about the distribution of demand. The retailer's private information is modeled as the space of either discrete or continuous demand states. Assuming that the buyback price is strictly less than the wholesale price, the authors find an optimal buyback contract for the less informed supplier that can almost coordinate the supply chain. Such a contract does not satisfy the coordinating condition proposed by Pasternack (1985), but does generate the supplier's profit that is higher relative to traditional buyback contract.

One of the first studies analyzing the consequences of relaxing the risk-neutrality assumption is (Tsay, 2002). The author examines how risk sensitivity affects behavior of the contracting parties and their decisions concerning the buyback policy in different market conditions. The results of the study show that risk sensitivity leads to behaviors that differ from those predicted by the model with the risk-neutrality assumed. Accordingly, allocation of the expected profit between the contracting parties and chosen course of action depend on their risk sensitivity and market power.

Another stream of research in supply chain contracting is dedicated to financial constraints faced by the firms in the supply chain and is discussed in more detail in Section 1.3.

1.3. Limited Funding of Supply Chain Parties and Its Impact on Supply Chain Coordination

In this paper the term "limited funding" indicates a situation where the retailer's holdings of cash are constrained and might be insufficient to pay the supplier for the amount of goods that retailer decides to purchase from her.

Similar restrictions are widely explored in the existing literature on supply chain contracting. One example is the paper by Chen (2015). The author examined a supplier-retailer supply chain that is capital-constrained, and analyzed two options of funding the retailer's business – taking the bank loan and borrowing from the supplier (trade credit). The paper states that there exists a competition between the bank and the supplier as the retailer can choose how to finance the short-term operations and the contract conditions differ depending on that decision. The cost of the trade credit is supposed to be lower than the cost of the bank loan, which confirms the practice of the trade credit usage among firms.

The author compared the two financing options under a simple wholesale-price contract and a revenue-sharing contract, and came to a conclusion that under the wholesale-price contract trade credit is more beneficial to both partners and is a unique financing equilibrium. This result is explained by the fact that under bank financing of retailer's activity the supply chain acts as if it is not capital-constrained. Retailer's marginal cost is equal to the wholesale price; his default risks are shared with the bank. However, with trade credit financing, these risks are shared between the supplier and the retailer, thus decreasing the retailer's marginal cost and increasing the supplier's profit and, hence, the supply chain profit.

Examination of the bank and trade credit under the revenue-sharing contract shows different results, though. Bank credit results in the problem setting similar to that of the non-constrained supply chain, and, thus, coordination is achieved. Using the trade credit policy, however, does not allow to directly implement such a coordinating contract in its traditional setting used for the non-constrained supply chain. A feasible region exists in the model of the revenue-sharing contract in both cases, but the authors show that it is larger under the bank credit policy than under the trade credit policy. The profits expected by the supplier and the retailer are also higher under bank financing than under trade credit. However, the author suggests that under the game-theoretic framework coordinating revenue-sharing contract allows the supplier to negotiate such terms that provide her with the same profit under both types of financing.

Kouvelis and Zhao (2012) also study the supply chain consisting of the supplier and the retailer that both are capital-constrained. The authors include working capital and collateral into their model to allow for the bankruptcy risk as they consider two alternatives for borrowing – bank loan and trade credit. They state that when the retailer is offered an optimally structured trade credit, it is always preferable as the supply chain profit improves, implying that the supplier's and retailer's profits also should improve. The optimally structured trade credit has the interest rate between 0 and risk-free rate, which confirms the authors' prediction based on the empirical data that the trade credit rate should be lower than the bank's rate to improve the supply chain performance in the presence of financial constraints. Risk-free rate is preferable as it motivates the retailer to use all his working capital to pay up front for the order quantity. The authors show that despite the profit improvement, supply chain coordination is not fully achieved.

Later Kouvelis and Zhao studied the case where both supplier and retailer are capital-constrained in the short term (2015). They focused on the bank loans, namely secured loans with current assets as a collateral. Their model included default costs that can occur in case a partner fails to repay the loan, and gradually added these factors into the model of the revenue-sharing contract. The paper aimed to answer how the structure of default costs would affect the design and performance of the contract, and whether it would be possible to allocate expected default loss among the supplier and the retailer and meet the coordination requirement of the supply chain's maximal expected profit.

Three types of supplier-retailer supply chain were considered: two firms without coordinated working capital management, two firms with coordinated working capital management, and the case where one firm had control of all decisions related to the supply chain's activity, including working capital management. The term 'coordinated working capital management' can be explained as the alignment of funds borrowed by the supplier and the retailer with their revenue shares. It is assumed that the modeled supply chain operates under a general contract which subsumes quantity-discount, revenue-sharing and buyback contract types and can be reduced to any of these by setting proper values of the contract parameters.

Analyzing the cash flow constraints and bankruptcy risks, the authors highlight that default risk does not affect the properties of coordinating contracts when bank credit is used, which coincides with the conclusion made by Chen (2015).

The authors concluded that only revenue-sharing contract preserves coordinating properties in the presence of both variable and fixed bankruptcy costs. They show evidence that buyback contract coordinates the supply chain in the presence of only variable costs, but is Pareto-dominated by revenue-sharing contract when there are fixed default costs because profits under buyback contract are lower than with the revenue-sharing contract. Quantity-discount contract does not coordinate the supply chain in any setting of default costs.

As a result, new revenue-sharing contract with adjusted wholesale price was proposed as an instrument of working capital coordination in the supply chain with certain limitations.

Moon, Feng and Ryu (2015) examined the effect of budget constraints on the coordinating properties of revenue-sharing contract and extended basic supplier-retailer model to a multistage model. The multistage model takes into account that each partner can deal with several upstream partners, so the supply chain has a tree structure.

Discussing the issue of budget constraints, the authors distinguish absolute budget constraints, costs connected with default risk taken into account in the terms of the bank loan, and administrative costs. An absolute budget constraint means a limited order quantity that retailer can really afford to meet the budget when there is no opportunity to raise a loan. Since the order quantity cannot be increased, profit maximization cannot be achieved if an optimal order quantity is larger than the quantity affordable to the retailer. Default risk being one of the factors affecting the bank's interest rate requires that all partners agree with the loan terms, which can add costs related to the negotiation process. Administrative costs increase as the borrower's and, hence, the supply chain's flows are monitored in case the loan is raised in the financial market. The paper focuses on the case of absolute budget constraints when the supply chain partners have no access to the financial market.

The authors prove that with traditional revenue-sharing contract a budget-constrained supply chain does not achieve coordination when partners exceed the budget matching the contract terms. A modified revenue-sharing contract is proposed, with new wholesale price settings and a penalty factor to improve the revenue sharing mechanism under budget constraints. These contract terms allow the partners with insufficient budget to negotiate a lower wholesale price and transfer a share of their revenues, so the supply chain profit is still higher than without a coordinating contract. However, limitations to this approach include the position of the budget-constrained partners. If a partner at the end of the supply chain is faced with insufficient budget, the proposed contract cannot be implemented and the profit maximum is not achieved.

Xiao, Sethi, Liu and Ma (2017) investigate the problem similar to that studied by Kouvelis and Zhao (2012). They consider a supplier-retailer supply chain where the retailer receives a trade credit from the supplier due to lack of access to bank financing. This restriction implies a variable default cost that is included in the model.

The comparison of the revenue-sharing, quantity-discount and buyback contracts is carried out and leads to the following conclusions. Firstly, quantity-discount contract fails to coordinate a capital-constrained supply chain. Secondly, revenue-sharing and buyback contracts equally coordinate a supply chain under financing restrictions, however, only as long as total working capital is sufficient and above a specific threshold.

To overcome the limitation related to the level of total working capital, the authors propose a generalized revenue-sharing contract that provides flexible profit allocation between the supplier and the retailer and outperforms traditional revenue-sharing and buyback contracts. Under the generalized contract, the wholesale price depends on the order quantity, and the ratio of revenue sharing depends on the retailer's sales as his revenue is used to repay the bank loan before sharing.

Discussing possible ways of further research, the authors mention that in real supply chains the retailer may be more creditworthy whereas the supplier faces capital constraints and has no access to bank financing. This problem setting has not been widely examined and gives a novel perspective to the research on coordination of the capital-constrained supply chains.

Zhan, Chen and Hu (2018) examine the case similar to (Xiao, Sethi, Liu, Ma, 2017) with the sales-rebate contract. The authors compare the contract conditions with traditional trade credit financing and come to a conclusion that even though unique equilibrium exists in both cases, trade credit financing under the sales-rebate contract brings better opportunities for the supplier's pricing policy and improves financial results of the supply chain partners relative to traditional trade credit policy.

Research carried out by Fu and Liu (2019) focuses on the buyback contracts. The authors consider the same setting as (Xiao, Sethi, Liu, Ma, 2017) and (Zhan, Chen, Hu, 2018) and relax the assumption of risk-neutrality by considering a risk-averse retailer. In this setting it is assumed that in the presence of capital constraints supplier provides trade credit to the retailer and raises a bank loan.

The authors prove that with these assumptions held traditional revenue-sharing and buy-back contracts do not coordinate the supply chain since the risk-averse retailer is willing to choose an order quantity that is lower than optimal. To achieve coordination in this setting, the supplier, which is considered to be risk-neutral, should share default risks with the retailer. The authors propose a new risk-sharing contract that addresses this issue, motivates the retailer to increase the order quantity and, thus, allows to achieve coordination of the supply chain.

The terms of new contract suggest that when the order quantity is lower than the optimal quantity, the contract is reduced to traditional buyback contract, whereas for a larger order quantity additional risk-sharing terms arise. The authors demonstrate that under these terms both supplier and retailer reach the profit levels satisfying their capital constraints, properly manage risk constraints and achieve coordination, meaning that the supply chain profit is maximized. However, the algorithm proposed by the authors is to some extent unclear as the ways of its possible extension discussed in the paper do not address possible ways to generalize the model.

Berezinets et al. (2022) study coordinating properties of the revenue-sharing contract when the retailer has short-term financing necessity and needs to borrow either from the supplier or the bank. Financial constraint faced by the retailer is modeled as limited holdings of cash that retailer can use to pay the supplier for the purchase of goods. The solution proposed by the authors draws upon the definitions of coordinating and conditionally coordinating contracts discussed in Section 1.2 and proves that revenue-sharing contract does not allow to coordinate the supply chain with the retailer's financial constraint neither with trade credit, nor with a bank loan. However, the contract can be called conditionally coordinating in both cases. The authors also prove that trade credit allows the supplier to achieve higher profit than with the bank loan.

The summary of approaches and main results derived from the academic papers discussed in the overview of the literature dedicated to the problem of capital constraints in the supply chain is given in Table 1.1.

Table 1.1. Overview of the literature on limited funding in supply chain coordination with contracts

Research paper	**Contract type examined in the paper**	**Approach and main result**
(Chen, 2015), *A model of trade credit in a capital-*	Revenue-sharing contract	Compared the revenue-sharing contract to the wholesale-price contract in two settings – trade

Research paper	Contract type examined in the paper	Approach and main result
constrained distribution channel		and bank credit given to the retailer; proved that trade credit is more beneficial to both partners
(Kouvelis, Zhao, 2012), *Financing the Newsvendor: Supplier vs. Bank, and the Structure of Optimal Trade Credit Contracts*	Wholesale-price contract	Examined the supply chain with both supplier and retailer being capital constrained and considered two forms of credit – trade credit and bank loan; showed that with optimal terms of the trade credit the retailer always prefers it to the bank loan; with such trade credit, the supplier's and supply chain profits improve whereas the retailer's profit improves relative to the bank loan depending on his working capital and collateral
(Kouvelis, Zhao, 2015), *Supply Chain Contract Design under Financial Constraints and Bankruptcy Costs*	Revenue-sharing and quantity-discount contracts	Considered the case with both partners under capital constraint and included bankruptcy costs in the model; only revenue-sharing contract preserves coordinating properties in this setting
(Moon, Feng, Ryu, 2015), *Channel coordination for multi-stage supply chains with revenue-sharing contracts under budget constraints*	Revenue-sharing contract	Extended the number of partners and included the budget constraints in the model; proved that revenue-sharing contract with such constraints fails to coordinate the supply chain and proposed a new design to solve this problem
(Xiao, Sethi, Liu, Ma, 2017), *Coordinating Contracts for a Financially Constrained Supply Chain*	Revenue-sharing, quantity-discount and buyback contracts	Examined the case of trade credit given to the retailer in one-retailer-one-supplier model; proved that the revenue-sharing and buyback contracts coordinate the supply chain only when having a sufficient total working capital; proposed a generalized revenue-sharing contract that coordinates the supply chain

Research paper	Contract type examined in the paper	Approach and main result
(Zhan, Chen, Hu, 2018), *The value of trade credit with rebate contract in a capital-constrained supply chain*	Sales-rebate contract	Analyzed the effects of trade credit offered under the sales-rebate contract to the retailer in the supplier-retailer supply chain; showed evidence that such contract outperforms the traditional trade credit scheme
(Fu, Liu, 2019), *The coordination of a capital-constrained supply chain with a risk-averse retailer*	Revenue-sharing and buy-back contracts	Relaxed the assumption of risk-neutrality for the retailer and considered the effects of trade credit that was provided under his limited bankruptcy risk tolerance; proved that both contracts do not coordinate the supply chain on these assumptions and proposed a new risk-sharing contract
(Berezinets et al., 2022), *Coordinating Contracts as an Instrument of Supply Chain Profit Maximization under Short-Term Financing Necessity*	Revenue-sharing contract	Studied the effect of trade credit and bank loan on coordinating properties of the revenue-sharing contract for a supply chain with the retailer's short-term financing constraint

From Table 1.1 it is straightforward to conclude that the issue of limited funding has been widely considered by the scholars in the field of supply chain coordination, however with primary focus on the revenue-sharing and not the buyback type of contract. It is also clear that the scholars are not in agreement about the effect of financing constraints on the coordinating properties of the contracts as:

- controversial conclusions have been made regarding the revenue-sharing contract (Chen, 2015; Berezinets et al., 2022), and
- the model of the buyback contract has not been exhaustively extended with respect to the limited funding (Xiao, Sethi, Liu, Ma, 2017; Fu, Liu, 2019).

Even though there is substantial literature on the buyback contracts, it yields different results and does not give a clear answer whether the buyback contract fails to coordinate the supply chain or not. Together with the lack of research dealing with the aspect of limited funding, this outlines the gap in the knowledge of supply chain coordination with buyback contracts which this body of research aims to fill in.

Summary of Chapter 1

This chapter gives an overview of existing approaches to supply chain coordination with contracts, and focuses on the buyback contract as the object of the study. Existing approaches to limited funding are also considered in the context of supply chain coordination so as to justify the author's approach to modeling and studying the buyback contract with limited funding in Chapter 2.

The overview of the literature on supply chain coordination with contracts shows that virtually all the studies use game theory as an essential tool for analyzing supply chains as the networks with multiple agents and conflicting objectives. It also shows that game theorists are not completely in agreement about the possibility of coordination with different types of contracts.

A deeper overview of the buyback contract has also demonstrated the controversial conclusions about its coordinating properties due to different approaches to the definition of coordinating contract, which results in different conclusions about the coordination with the buyback contract. The approach to defining the coordinating contract based on (Berezinets et al., 2020) has been chosen for the convenience of exposition as the authors introduce the definition of conditionally coordinating contract and application of this approach to the buyback contract demonstrates that it provides conditional coordination of the supply chain with two players.

Additionally, the ways of extending the model of the buyback contract to allow for limited funding were considered. The overview of existing literature dedicated to this aspect has shown that the majority of papers focus on other contract types (Kouvelis, Zhao, 2012; Chen, 2015), and coordination with the buyback contract with limited funding taken into account has not been exhaustively examined by the scholars in supply chain contracting (Xiao, Sethi, Liu, Ma, 2017; Fu, Liu, 2019). This body of research differs in the way that it studies two forms of credit commonly applied in business practice. Thus, the contribution of the paper to the existing research covers not only the general aspects of financial management, but also the practical aspects of the application of the buyback contracts.

The approach chosen for presenting limited funding in the model of the buyback contract is based on the framework by proposed by (Berezinets et al., 2022). This framework allows for the limited funding by considering two forms of credit – bank loan and trade credit. Two interest rates – the bank's and the supplier's – are incorporated in the model along with the retailer's constrained holdings of cash. This does not add complexity to the model yet models the situation without loss of generality.

CHAPTER 2. THE MODEL OF CONDITIONALLY COORDINATING BUYBACK CONTRACT WITH LIMITED FUNDING

2.1. Model Description

In this study a decentralized supply chain consisting of one retailer [R, he] and one supplier [S, she] is considered within a single period. The product manufactured by the supplier is sold by retailer to end consumers. The retailer is to order a single type of product from the supplier once prior to the start of the selling season and no additional orders or amendments to the order are allowed.

Market demand for the product is a stochastic variable ξ, which follows a strictly increasing distribution. ξ is a continuous random variable with probability density function $f_\xi(x)$ and differentiable distribution function $F_\xi(x)$ with $x \geq 0$. Let τ denote sales volume, where

$$\tau = g(\xi) = \begin{cases} \xi, & 0 \leq \xi < q \\ q, & \xi \geq q \end{cases} \tag{2.1}$$

Then expected sales volume will be $E[\tau] = E[g(\xi)] = q - \int_0^q F_\xi(x)\,dx$, and its first derivative will be $\frac{d}{dq}E[\tau] = 1 - F_\xi(q)$.

Under a buyback contract supplier offers the retailer the following terms: a wholesale price ω per unit and a buyback price b per unit. In response to the supplier's offer the retailer chooses the order volume q that he will sell at the retail price p during the selling season. At the end of the selling season, supplier will pay up the buyback price for unsold units of the product; then retailer will be able to sell unsold goods at the salvage value ν. The notations used in the model are listed in Table 2.1.

Table 2.1. List of notations

ω	Wholesale price per unit (c. u.)
b	Buyback price per unit (c. u.)
q	Order quantity (u.)
τ	Sales volume (u.)
p	Retail price per unit (c. u.)
ν	Salvage value per unit (c. u.)
c_S	Supplier's costs per unit (c. u.)
c_R	Retailer's costs per unit (c. u.)
$c = c_S + c_R$	Supply chain total costs (c. u.)
K_R	Cash available to retailer for financing his procurement decision (c. u.)

r_B	Interest rate of the bank loan
r_S	Interest rate of the trade credit
$Prof_S$	Supplier's profit per transaction (c. u.)
$Prof_R$	Retailer's profit per transaction (c. u.)
$Prof_{SC} = Prof_S + Prof_R$	Supply chain profit per transaction (c. u.)
$\pi_S = E[Prof_S]$	Supplier's expected profit per transaction (c. u.)
$\pi_R = E[Prof_R]$	Retailer's expected profit per transaction (c. u.)
$\pi_{SC} = E[Prof_{SC}]$	Expected total profit of the supply chain per transaction (c. u.)

Approach to modeling the limited funding is based on the framework proposed by Berezinets et al. (2022). The retailer is faced with the limited funding, i.e., has insufficient volume of cash available for paying to the supplier. He considers two options for the short-term credit to pay for the order quantity he chooses. The first option is bank loan; the second option is trade credit or supplier financing. Regardless of the chosen source of financing, the retailer is supposed to repay the borrowed money with interest at the end of the selling season. Size of interest payment depends on the interest rate, which is r_B if retailer takes bank loan or r_S if the retailer decides to use supplier financing.

The set of the model assumptions is based on (Berezinets et al., 2022) and includes the following:

1. Both supplier and retailer are risk neutral;
2. Both supplier and retailer are rational and aim to maximize their profits;
3. There is no information asymmetry between supplier and retailer, they both know their costs, cash available, distribution of demand, interest rate on the bank loan, interest rate on the trade credit and retail price;
4. Retailer may choose between bank and supplier financing; bankruptcy risk is not considered as it is assumed that retailer can cover his loan obligation in full at the end of the selling season;
5. No moral hazard issues are considered as both players have no ex ante (i.e., before the event) intention to breach the contract;
6. Capital market is considered without taxes and transaction costs.

The following conditions also should be met:

7. It is assumed that the wholesale price is lower than retail price and higher than the supplier's costs. Salvage value is assumed to be lower than supplier's costs: $0 < \nu < c_S < \omega < p$;

8. Buyback price should not exceed wholesale price, but should be higher than the salvage value: $\nu < b < \omega$;
9. Interest rates on both sources of borrowing are given parameters that should belong to the interval between 0 and 1: $0 < r_B < 1, 0 < r_S < 1$.

The process of negotiating parameters of the buyback contract is modeled as a two-step game with two players. At the first step supplier chooses two contract parameters ω and b from the available set of supplier's strategies X_S. At the second step, in response to supplier's offer retailer chooses the order volume q from the available set of retailer's strategies X_R:

$$X_S = \{(\omega, b) \mid \nu < b < \omega, 0 < c_S < \omega\}$$
$$X_R = \{q(b, \omega) \mid q \geq 0\} \tag{2.2}$$

In other words, the model assumes that the supplier is the leader in decision making and chooses her strategy first, while the retailer is the follower and chooses his strategy in accordance with the one chosen by the supplier. In this setting supplier as a leader has the advantage of optimizing her profit function based on retailer's response and knowing his profit function.

Thus, buyback contract with limited funding can be defined as a combination of three parameters (b, ω, q), which is the same way as usual buyback contract with no additional conditions is defined. Definition of coordinating buyback contract with limited funding follows Berezinets et al. (2020).

Definition 2.1. A buyback contract with limited funding (b^*, ω^*, q^*) will coordinate the supply chain if the following conditions are met:

(1) $\max_q \pi_R = \pi_R\left(b, \omega(b), q_R^*\big(b, \omega(b)\big)\right), \forall b \in X_S$

(2) $\exists\ \omega^*(b)$: $q_R^*\big(b, \omega^*(b)\big) = q_{SC}^* = q^*, \forall b \in X_S$

(3) $\max_b \pi_S(b, \omega^*(b), q^*) = \pi_S(b^*, \omega^*, q^*), \omega^* = \omega(b^*)$.

According to Definition 2.1, the following steps need to be taken in order to determine parameters of a coordinating buyback contract for a supply chain with limited funding:

1. Determine retailer's optimal order volume q_R^* that allows retailer to maximize his individual profit π_R;
2. Determine supply chain's optimal order volume q_{SC}^* that allows to maximize supply chain profit π_{SC};
3. Determine a wholesale price ω^* for which retailer's optimal order volume q_R^* will coincide with the supply chain's optimal order volume q_{SC}^*: $q_R^* = q_{SC}^* = q^*$;

4. Determine a buyback price b^* for which supplier's expected profit $\pi_S(b^*, \omega^*, q^*)$ is going to be maximized when retailer orders the volume q^* that allows to maximize both retailer's and supply chain profits.

Depending on the lender (bank or supplier), expressions for profit and expected profit of each player and of the supply chain will be different. Nevertheless, the size of the loan in both cases will be determined by the amount of cash at retailer's disposal K_R and the order volume that retailer chooses in response to the buyback and wholesale prices suggested by the supplier. The size of the loan is expressed as $max\{(\omega + c_R)q - K_R , 0\}$, which is the amount of money he lacks to make an order q. In case retailer's amount of cash is sufficient to make the order, loan size $max\{(\omega + c_R)q - K_R , 0\}$ will be equal to zero. Otherwise, it will be equal to $\big((\omega + c_R)q - K_R\big)$, which is the difference between the amount of retailer's cash and his total costs associated with the order he is going to make, namely wholesale price and retailer's costs.

2.2. Coordinating Buyback Contract with the Bank Loan

As retailer can have insufficient funds to finance his procurement decision, he has an opportunity to take a bank loan with the rate r_B. The principal and interest charged by the bank will be repaid at the end of the selling season.

In this case profit functions depend on the order volume q that is to be chosen by retailer, volume of retailer's sales τ, and parameters related to his limited funding – amount of cash at disposal K_R and rate charged for the bank loan r_B. Expressions for supplier's, retailer's and supply chain profits are given in Table 2.2; further details are given in Appendix A1.

Table 2.2. Expressions for profits for buyback contract with bank loan

	Supplier	**Retailer**	**Supply Chain**
Profit	$Prof_S^{BB} = b\tau + (\omega - c_S - b)q$	$Prof_R^{BB} = (p - b - v)\tau + (b + v - \omega - c_R)q - r_B * \max\{(\omega + c_R)q - K_R , 0\}$	$Prof_{SC}^{BB} = (p - v)\tau + (v - c)q - r_B * \max\{(\omega + c_R)q - K_R , 0\}$
Expected Profit	$\pi_S^{BB} = (\omega - c_S)q - b\int_0^q F_\xi(x)\,dx$	$\pi_R^{BB} = (p - \omega - c_R)q + (b + v - p)\int_0^q F_\xi(x)\,dx - r_B * \max\{(\omega + c_R)q - K_R , 0\}$	$\pi_{SC}^{BB} = (p - c)q + (v - p)\int_0^q F_\xi(x)\,dx - r_B * max\{(\omega + c_R)q - K_R , 0\}$

Since retailer's funds can be either sufficient or insufficient, the problem of determining parameters of coordinating contract by Definition 2.1 is solved for these two cases – the case where the retailer has sufficient funds and does not need any credit and the case where the retailer takes the bank loan.

2.2.1. Retailer Has Sufficient Funds. In this case retailer's cash K_R is enough to pay up for the order he decides to make, so he does not need to take a bank loan and expressions for supplier's, retailer's and supply chain expected profit look as follows:

$$\pi_S^{BB} = (\omega - c_S)q - b\int_0^q F_\xi(x)\,dx,$$
$$\pi_R^{BB} = (p - \omega - c_R)q + (b + v - p)\int_0^q F_\xi(x)\,dx, \qquad (2.3)$$
$$\pi_{SC}^{BB} = (p - c)q + (v - p)\int_0^q F_\xi(x)\,dx,$$

where π_S^{BB} denotes supplier's expected profit with buyback contract, π_R^{BB} denotes retailer's expected profit with buyback contract and π_{SC}^{BB} supply chain's expected profit with the same contract.

Then the case becomes identical to that examined by Berezinets et al. (2020) – coordination cannot be achieved as the third condition stated in Definition 2.1 is not met. The results of the solution obtained in this case are summarized in Table 2.3.

Table 2.3. Solution to the case of sufficient funds

Retailer's optimal order volume	$q_R^* = F_\xi^{-1}\left(\frac{p-\omega-c_R}{p-b-v}\right)$
Supply chain optimal order volume	$q_{SC}^* = F_\xi^{-1}\left(\frac{p-c}{p-v}\right)$
Wholesale price	$\omega^* = c_S + b\,\frac{p-c}{p-v}$
Conditions	$b < (p - v)\,\frac{c_S}{c-v}$

As shown in Table 2.3, optimal order volumes q_R^* and q_{SC}^* that maximize retailer's and supply chain expected profits are expressed through the inverse function of the distribution function $F_\xi(x)$ that describes uncertain demand; wholesale price ω^* for which optimal order volumes coincide (i.e., $q_R^* = q_{SC}^*$) depends on buyback price; and buyback price should meet the constraint given in the last row of Table 2.3.

2.2.2. Retailer Takes the Bank Loan. In this case retailer's funds are insufficient to finance his decision on the order quantity, so he borrows from the bank. This condition is expressed as $(\omega + c_R)q > K_R$, where retailer's amount of cash is not enough to pay the supplier for ordering at the wholesale price ω and bearing costs c_R of carrying and retailing the ordered goods during the selling season.

The model does not include the bank as a player as its only function is to provide the retailer with the interest rate on the loan that retailer intends to take. This information is supposed to be shared between supplier and retailer in accordance with the assumption A3 as the interest rate r_B charged by the bank affects the values of retailer's and supply chain profit, as well as the amount of retailer's cash K_R.

Compared to the formulas (2.3), the expression for supplier's expected profit remains unchanged while the expressions for retailer's and supply chain expected profit have modifications – they include retailer's cash available K_R and interest rate on the loan r_B – reflecting the bank loan taken by the retailer. The formulas for these profits look as follows:

$$\pi_S^{BB} = (\omega - c_S)q - b\int_0^q F_\xi(x)\,dx,$$
$$\pi_R^{BB} = (p - \omega - c_R)q + (b + \nu - p)\int_0^q F_\xi(x)\,dx - r_B\big((\omega + c_R)q - K_R\big) =$$
$$\big(p - (\omega + c_R)(1 + r_B)\big)q + (b + \nu - p)\int_0^q F_\xi(x)\,dx + r_B K_R\,, \qquad (2.4)$$
$$\pi_{SC}^{BB} = (p - c)q + (\nu - p)\int_0^q F_\xi(x)\,dx - r_B\big((\omega + c_R)q - K_R\big) =$$
$$\big(p - c - r_B(\omega + c_R)\big)q + (\nu - p)\int_0^q F_\xi(x)\,dx + r_B K_R\,.$$

As shown in (2.4), when retailer takes a bank loan supply chain profit π_{SC}^{BB} is lower compared to the situation when retailer's funds are sufficient (2.3); this is reflected by the loan principal and interest subtracted from the retailer's profit and consequently from the supply chain profit.

The contract parameters that meet the conditions of Definition 2.1 are determined with the use of the 4-step approach given in Section 2.1.

The first step of the approach, which is to determine an optimal order volume that maximizes the retailer's expected profit $\pi_R^{BB}(q)$, requires finding the stationary point of this function. The stationary point of the function $\pi_R^{BB}(q)$ exists where its first derivative at q is zero, i.e. $\frac{\partial \pi_R^{BB}}{\partial q} = 0$. Solving this equation allows to find the stationary point q_R^0. Then the second-derivative test needs to be done in order to determine whether the found stationary point q_R^0 is a local maximum of the profit function $\pi_R^{BB}(q)$. To use this test, the second derivative $\frac{\partial^2 \pi_{SC}^{BB}}{\partial q^2}$ must be derived. If $\frac{\partial^2 \pi_{SC}^{BB}}{\partial q^2} < 0$, then retailer's profit function has a local maximum at q_R^0, i.e. $q_R^0 = q_R^*$, where q_R^* denotes the found local maximum of retailer's profit function. By choosing the order volume q_R^* retailer will be able to achieve the highest possible profit.

At the second step the same solution is found for supply chain profit π_{SC}^{BB} in order to determine supply chain's optimal order volume q_{SC}^* that is a local maximum of the function π_{SC}^{BB}. With the order volume q_{SC}^* the supply chain profit achieves its maximum, which is to be divided between supplier and retailer.

As the optimal order volume q_{SC}^* is to be ordered and then sold to end consumers by the retailer, at the third step the following equation is solved:

$$q_R^* = q_{SC}^* \qquad (2.5)$$

Solution of the equation (2.5) brings the formula for the wholesale price ω^* that depends on the buyback price b. This relationship between ω^* and b allows to calculate such buyback price that would ensure maximal supply chain profit for any value of the wholesale price that supplier and retailer agree upon during their negotiation.

At the final step an optimal buyback price b^* needs to be determined as supplier's profit is maximized with regard to this parameter of the buyback contract. The function of supplier's expected profit is expressed through the wholesale price ω^* determined at the previous step:

$$\pi_S^{BB} = b\left(\frac{p - c - cr_B}{p - v + br_B} q^* - \int_0^{q^*} F_\xi(x)\,dx\right) \tag{2.6}$$

Determining the buyback price b^* requires finding the stationary point of supplier's profit function π_S^{BB} and performing the second-derivative test in order to prove that the stationary point is a local maximum of the function π_S^{BB}.

Solution shows that the function $\pi_S^{BB}(b)$ does have a stationary point that is a local maximum b^*. The results of the solution are summarized in Table 2.4; detailed solution is given in Appendix A2.

Table 2.4. Parameters of coordinating buyback contract with the bank loan

Retailer's optimal order volume	$q_R^* = F_\xi^{-1}\left(\frac{p - (\omega + c_R)(1 + r_B)}{p - b - v}\right)$
Supply chain optimal order volume	$q_{SC}^* = F_\xi^{-1}\left(\frac{p - c - r_B(\omega + c_R)}{p - v}\right)$
Wholesale price	$\omega^* = b\frac{p - c - cr_B}{p - v + br_B} + c_S$
Buyback price	$b^* = \frac{\sqrt{\frac{q^*(p - c - cr_B)(p - v)}{\int_0^{q^*} F_\xi(x)\,dx}} + v - p}{r_B}$
Conditions	$b < p - v$ $p > c(1 + r_B)$

As shown in Table 2.4, coordination can be achieved by setting the optimal buyback price b^*. However, the determined parameter b^* does not satisfy the conditions set in Section 2.1, and therefore does not belong to the set of coordinating buyback contracts that is defined by the sets of the players' strategies X_S and X_R given in (2.2). This leads to a conclusion that the found optimal buyback price b^* does not allow to build a coordinating buyback contract with limited funding by Definition 2.1 for the case with the bank loan.

2.3. Coordinating Buyback Contract with Trade Credit

Trade credit is a business-to-business agreement in which a customer can buy goods without paying up front, and pay the supplier at a later scheduled date. This type of financing is usually encouraged globally by regulators as it can potentially free up cash flow and finance short-term growth. However, trade credit can put suppliers at a disadvantage as they receive deferred payment (Investopedia, 2022). Trade credit is usually cheaper than the bank loan (Kouvelis, Zhao, 2012), and this condition is met in the model: $0 < r_S < r_B < 1$. In the model of the buyback contract with the trade credit rate r_S is a given parameter.

Based on the information about the retailer's volume of funds, supplier can offer him a trade credit to cover the difference between the retailer's volume of cash and costs associated with the order he is going to make. In case retailer accepts supplier's offer and decides to use trade credit, supplier agrees to postpone the payment for retailer's order ωq until the end of the selling season and to bear retailer's costs $c_R q$ of carrying and retailing the ordered goods. In return supplier charges r_S, which denotes the interest rate on the trade credit and retailer's cost of borrowing from the supplier.

At the end of the selling season retailer will have to repay the principal $max\{(\omega + c_R)q - K_R , 0\}$, which is equal to the difference between the retailer's cash and costs, plus interest charged by the supplier. In this way, trade credit acts exactly like a bank loan, though with a different interest rate, in the model. As a result, a coordinating buyback contract with trade credit can be determined based on Definition 2.1.

As shown in Table 2.5, expressions for the profits with the trade credit are different from those with the bank loan (Table 2.2). Alterations made to the expressions include:

- applying a different interest rate r_S;
- additional term in the supplier's profit, which shows the interest supplier receives according to the trade credit agreement;
- supply chain profit does not depend on r_S.

Detailed expressions of the profit functions are given in Appendix A3.

Table 2.5. Expressions for profits for buyback contract with trade credit

	Supplier	Retailer	Supply Chain
Profit	$Prof_S^{BB} = b\tau + (\omega - c_S - b)q + r_S * \max\{(\omega + c_R)q - K_R, 0\}$	$Prof_R^{BB} = (p - b - \nu)\tau + (b + \nu - \omega - c_R)q - r_S * \max\{(\omega + c_R)q - K_R, 0\}$	$Prof_{SC}^{BB} = (p - \nu)\tau + (\nu - c)q$
Expected Profit	$\pi_S^{BB} = (\omega - c_S)q - b\int_0^q F_\xi(x)\,dx + r_S * \max\{(\omega + c_R)q - K_R, 0\}$	$\pi_R^{BB} = (p - \omega - c_R)q + (b + \nu - p)\int_0^q F_\xi(x)\,dx - r_B * \max\{(\omega + c_R)q - K_R, 0\}$	$\pi_{SC}^{BB} = (p - c)q + (\nu - p)\int_0^q F_\xi(x)\,dx$

Like with the bank loan, in negotiations over the buyback contract with trade credit retailer's funds can be either sufficient or insufficient, which is why the problem of determining the contract parameters with the trade credit by Definition 2.1 is solved for two cases – when the retailer has sufficient funds and does not borrow; and when retailer uses trade credit.

2.3.1. Retailer Has Sufficient Funds. In case retailer has enough cash and can pay for the order without trade credit, $\max\{(\omega + c_R)q - K_R, 0\}$ will be equal to zero, and the problem will be identical to that described in Section 2.2.1, where the retailer does not need a bank loan.

2.3.2. Retailer Uses Trade Credit. In case retailer's funds are insufficient to pay for the order quantity, he takes a trade credit, which allows him to postpone the order payment ωq and have supplier run his costs $c_R q$. Retailer's limited funding is expressed the same way as in Section 2.2.2: $(\omega + c_R)q > K_R$. Expressions for supplier's, retailer's and supply chain expected profits look as follows:

$$\begin{aligned}
\pi_S^{BB} &= \big(\omega - c_S + r_S(\omega + c_R)\big)q - b\int_0^q F_\xi(x)\,dx - r_S K_R \\
\pi_R^{BB} &= \big(p - (1 + r_S)(\omega + c_R)\big)q + (b + \nu - p)\int_0^q F_\xi(x)\,dx + r_S K_R \qquad (2.7) \\
\pi_{SC}^{BB} &= (p - c)q + (\nu - p)\int_0^q F_\xi(x)\,dx
\end{aligned}$$

As shown in (2.7), with trade credit supplier's expected profit increases by the interest paid by the retailer compared to (2.4) (retailer takes the bank loan). The interest added to the supplier's profit is subtracted from the retailer's profit as he must repay the borrowed funds in full; hence, the supply chain profit, which is the sum of individual profits earned by the two players, remains the same as in (2.3) (when retailer's funds are sufficient) and is higher than the total profit from (2.4) (when retailer takes the bank loan).

The process of determining the parameters of coordinating buyback contract with trade credit is the same as the solution described in Section 2.2.2 (contract with the bank loan).

At the first step of the solution an optimal order volume that maximizes the function of retailer's expected profit $\pi_R^{BB}(q)$ is determined through finding the stationary point of this function. The stationary point of the function $\pi_R^{BB}(q)$ is proved to be a local maximum of the profit function $\pi_R^{BB}(q)$, and thus the first condition of Definition 2.1 is met.

At the second step a local maximum of the function π_{SC}^{BB} is found, and the second condition of Definition 2.1 is met as well.

At the third step, by solving the same equation as (2.5) for the expected profits from (2.7), the formula for the wholesale price ω^* is derived and then, at the fourth step an optimal buyback price b^* can be determined through finding the stationary point of supplier's profit function π_S^{BB} and performing the second-derivative test in order to prove that the stationary point is a local maximum of the function π_S^{BB}. This function expressed through the optimal wholesale price ω^* looks as follows:

$$\pi_S^{BB} = b\left(\frac{p-c}{p-\nu}q^* - \int_0^{q^*} F_\xi(x)\,dx\right) - r_S K_R \tag{2.8}$$

Solution shows that the first derivative of the function π_S^{BB} at b is always positive, i.e., $\frac{\partial \pi_S^{BB}}{\partial b} > 0$, and its second derivative is equal to zero, i.e. $\frac{\partial^2 \pi_S^{BB}}{\partial b^2} = 0$. This implies that supplier's profit function is increasing at b, and thus does not have any local maximum. Therefore, the third condition of Definition 2.1 is not met and buyback contract with trade credit does not coordinate supply chain with the retailer's limited funding. The results of the solution are summarized in Table 2.6; detailed solution is given in Appendix A4.

Table 2.6. Parameters of coordinating buyback contract with trade credit

Retailer's optimal order volume	$q_R^* = F_\xi^{-1}\left(\frac{p-(\omega+c_R)(1+r_S)}{p-b-\nu}\right)$
Supply chain optimal order volume	$q_{SC}^* = F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)$
Wholesale price	$\omega^* = b\frac{p-c}{(p-\nu)(1+r_S)} + \frac{c}{1+r_S} - c_R$
Conditions	$b < p-\nu$ $p > c(1+r_S)$ $\frac{\nu-c_R}{p-\nu+c_R} < r_S < \frac{c_S}{c_R}$

2.4. Conditionally Coordinating Buyback Contract with Limited Funding

The results obtained in Sections 2.2 and 2.3 coincide with those presented in the existing body of research, according to which coordination with the buyback contract as defined in Definition 2.1 cannot be achieved. Nonetheless, if the found parameters (see Table 2.4, 2.5 and 2.6) can provide the supplier with the expected profit higher than the profit she could get by entering into a wholesale-price contract with the retailer, the supply chain can reach conditional coordination as defined by Berezinets et al. (2020). Let us define the conditionally coordinating buyback contract with limited funding.

Definition 2.2. A buyback contract with limited funding (b^*, ω^*, q^*) that complies with the following conditions:

(1) $\max_q \pi_R = \pi_R\left(b, \omega(b), q_R^*\big(b, \omega(b)\big)\right), \forall b \in X_S$

(2) $\exists\ \omega^*(b)$: $q_R^*\big(b, \omega^*(b)\big) = q_{SC}^* = q^*, \forall b \in X_S$

(3) $\pi_S^{BB}(b, \omega^*, q^*) > \pi_S^{WP}(\omega^0, q^0), q^* = q_{SC}^*: \max_q \pi_{SC}^{BB} = \pi_{SC}^{BB}\big(\omega^*, q_{SC}^*(\omega^*)\big), q^0 = q_R^*: \max_q \pi_R^{WP} = \pi_R^{WP}\big(\omega, q_R^*(\omega)\big),$

can be called conditionally coordinating.

The first two conditions are those from Definition 2.1, the third condition shows that conditionally coordinating buyback contract improves supplier's profit compared to the profit she can earn with the wholesale-price contract.

Wholesale-price contract is the simplest type of contract, defined as a set of two parameters (ω, q). Researchers in supply chain contracting agree that wholesale-price contract fails to coordinate the supply chain and is always dominated by other contract types, including the buyback contract (Cachon, 2003).

To determine the parameters of a conditionally coordinating buyback contract, parameters of a wholesale-price contract that maximize retailer's profit need to be found, then supplier's expected profit with the buyback contract can be compared with her profit with the wholesale-price contract. Solution for the wholesale-price contract that maximize retailer's profit accords with the 4-step approach used to determine coordinating properties of buyback contract in Section 2.1.

Detailed solution for the wholesale-price contract with the bank loan is given in Appendix A5; solution for the wholesale-price contract with trade credit is given in Appendix A6. These solutions confirm that neither source of borrowing allows to achieve coordination with wholesale-price contract.

In the case of the bank loan raised by the retailer, the wholesale-price contract coordinates the supply chain only if the wholesale price is equal to the supplier's cost, i.e.

$$\omega^* = c_S. \tag{2.9}$$

If so, the retailer succeeds in maximizing his profit, but the supplier's profit is equal to 0. Zero profit obviously makes the supplier prefer a higher wholesale price and the write a non-coordinating wholesale-price contract. The condition (2.9) for the supply chain coordination with the wholesale-price contract also holds in the case where the retailer's funds are sufficient and he does not use any credit.

With the trade credit, the wholesale-price contract coordinates the supply chain if the wholesale price is expressed as

$$\omega^* = \frac{c}{1+r_S} - c_R = c_S - c\frac{r_S}{1+r_S}. \tag{2.10}$$

It is straightforward to confirm that in this case the wholesale price is lower than the supplier's cost, i.e. $\omega^* < c_S$. Consequently, the supplier's expected profit function that depends on the interest rate r_S charged by the supplier and is below zero, which implies that with the coordinating wholesale-price contract with trade credit the supplier suffers losses when the retailer chooses the order quantity that maximizes his profit. So the supplier would also prefer to charge a higher wholesale price, and the contract would not be coordinating.

Determining the set of conditionally coordinating buyback contracts requires the values of the supplier's expected profit to be compared as in Definition 2.2; precisely the following values of the supplier's expected profit are compared:

- profit that is achieved when retailer orders the volume that maximizes supply chain profit with buyback contract, and
- profit that is achieved when retailer orders the volume that maximizes his own profit with wholesale-price contract.

Expressions for supply chain parties' individual and total profit for the two types of contract (wholesale price and buyback contracts) are given in Table 2.7.

Table 2.7. Expected profits with wholesale price and buyback contracts

	Supplier	**Retailer**	**Supply chain**
Retailer has sufficient funds			
Wholesale-price contract	$\pi_S^{WP} = (\omega - c_S)q$	$\pi_R^{WP} = (p - \omega - c_R)q + (v - p)\int_0^q F_\xi(x)\,dx$	$\pi_{SC}^{WP} = (p - c)q + (v - p)\int_0^q F_\xi(x)\,dx$
Buyback contract	$\pi_S^{BB} = (\omega - c_S)q - b\int_0^q F_\xi(x)\,dx$	$\pi_R^{BB} = (p - \omega - c_R)q + (b + v - p)\int_0^q F_\xi(x)\,dx$	$\pi_{SC}^{BB} = (p - c)q + (v - p)\int_0^q F_\xi(x)\,dx$
Retailer takes the bank loan			

	Supplier	Retailer	Supply chain
Wholesale-price contract	$\pi_S^{WP} = (\omega - c_S)q$	$\pi_R^{WP} = \big(p - (1 + r_B)(\omega + c_R)\big)q + (\nu - p)\int_0^q F_\xi(x)\,dx + r_B K_R$	$\pi_{SC}^{WP} = \big(p - c - r_B(\omega + c_R)\big)q + (\nu - p)\int_0^q F_\xi(x)\,dx + r_B K_R$
Buyback contract	$\pi_S^{BB} = (\omega - c_S)q - b\int_0^q F_\xi(x)\,dx$	$\pi_R^{BB} = \big(p - (\omega + c_R)(1 + r_B)\big)q + (b + \nu - p)\int_0^q F_\xi(x)\,dx + r_B K_R$	$\pi_{SC}^{BB} = \big(p - c - r_B(\omega + c_R)\big)q + (\nu - p)\int_0^q F_\xi(x)\,dx + r_B K_R$
Retailer uses trade credit			
Wholesale-price contract	$\pi_S^{WP} = \big(\omega - c_S + r_s(\omega + c_R)\big)q - r_S K_R$	$\pi_R^{WP} = \big(p - (1 + r_S)(\omega + c_R)\big)q + (\nu - p)\int_0^q F_\xi(x)\,dx + r_S K_R$	$\pi_{SC}^{WP} = (p - c)q + (\nu - p)\int_0^q F_\xi(x)\,dx$
Buyback contract	$\pi_S^{BB} = \big(\omega - c_S + r_S(\omega + c_R)\big)q - b\int_0^q F_\xi(x)\,dx - r_S K_R$	$\pi_R^{BB} = \big(p - (1 + r_S)(\omega + c_R)\big)q + (b + \nu - p)\int_0^q F_\xi(x)\,dx + r_S K_R$	$\pi_{SC}^{BB} = (p - c)q + (\nu - p)\int_0^q F_\xi(x)\,dx$

As the two profit functions are to be compared with respect to the same wholesale price and supply chain profit with buyback contract is maximized with regard to the wholesale price ω^*, supplier's profit function with the wholesale-price contract needs to be expressed through this wholesale price ω^*.

As shown in Sections 2.2.1 and 2.3.1, when retailer's funds are sufficient and he does not use credit, the formulas for supplier's, retailer's and supply chain profits look alike (see (2.3)). Thus, the two cases can be merged into one case of retailer's sufficient funds, and then three cases are examined below: the case where the retailer's funds are sufficient and he does not use any type of borrowing (Section 2.4.1), the case where the retailer's funds are limited and he takes the bank loan (Section 2.4.2), and the case where the retailer borrows from the supplier (Section 2.4.3). The formulas for parameters of the two contracts to be inserted in expressions for supplier's profit are given in Table 2.8. Detailed solution to all three cases is given in Appendix A7.

Table 2.8. Wholesale price and order volumes that maximize retailer's profit

Wholesale price	**Retailer's optimal order volume with buyback contract**	**Retailer's optimal order volume with wholesale-price contract**
Retailer has sufficient funds		
$\omega^* = c_S + b\frac{p-c}{p-v}$	$q_{SC}^* = F_\xi^{-1}\left(\frac{p-c}{p-v}\right)$	$q_R^* = F_\xi^{-1}\left(\frac{p-(1+r_S)(\omega^*+c_R)}{p-v}\right)$
Retailer takes the bank loan		
$\omega^* = c_S + b\frac{p-c-cr_B}{p-v+br_B}$	$q_{SC}^* = F_\xi^{-1}\left(\frac{p-c-r_B(\omega^*+c_R)}{p-v}\right)$	$q_R^* = F_\xi^{-1}\left(\frac{p-(1+r_B)(\omega^*+c_R)}{p-v}\right)$
Retailer uses trade credit		
$\omega^* = b\frac{p-c}{(p-v)(1+r_S)} + \frac{c}{1+r_S} - c_R$	$q_{SC}^* = F_\xi^{-1}\left(\frac{p-c}{p-v}\right)$	$q_R^* = F_\xi^{-1}\left(\frac{p-(1+r_S)(\omega^*+c_R)}{p-v}\right)$

2.4.1. Retailer Has Sufficient Funds. Expressions for supplier's profit with respect to the optimal wholesale price $\omega^* = c_S + b\frac{p-c}{p-v}$ and the order volumes that maximize supply chain profit with buyback contract and retailer's expected profit with wholesale-price contract look as follows:

$$\pi_S^{BB}(\omega^*, q^*) = b\left(\frac{p-c}{p-v}F_\xi^{-1}\left(\frac{p-c}{p-v}\right) - \int_0^{F_\xi^{-1}\left(\frac{p-c}{p-v}\right)} F_\xi(x)\,dx\right)$$
$$\pi_S^{WP}(\omega^*, q^0) = b\frac{p-c}{p-v}F_\xi^{-1}\left(\frac{(p-c)(p-b-v)}{(p-v)^2}\right) \quad (2.11)$$

To compare these values, the difference between the two profit functions is considered:

$$\pi_S^{BB}(\omega^*, q^*) - \pi_S^{WP}(\omega^*, q^0) =$$
$$b\left(\frac{p-c}{p-v}\left(F_\xi^{-1}\left(\frac{p-c}{p-v}\right) - F_\xi^{-1}\left(\frac{(p-c)(p-b-v)}{(p-v)^2}\right)\right) - \int_0^{F_\xi^{-1}\left(\frac{p-c}{p-v}\right)} F_\xi(x)\,dx\right) \quad (2.12)$$

As shown in (2.12), the difference between the supplier's profits is the product with the buyback price as one of the factors. As $b > 0$, the supplier's profit with the buyback contract exceeds her profit with wholesale-price contract if the second factor is also positive, i.e.

$$\frac{p-c}{p-\nu}\left(F_{\xi}^{-1}\left(\frac{p-c}{p-\nu}\right)-F_{\xi}^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)\right)-\int_0^{F_{\xi}^{-1}\left(\frac{p-c}{p-\nu}\right)}F_{\xi}(x)\,dx>0 \tag{2.13}$$

From (2.13) $\frac{p-c}{p-\nu}>0$ based on the condition 7 given in Section 2.1; the difference of the order quantities $F_{\xi}^{-1}\left(\frac{p-c}{p-\nu}\right)-F_{\xi}^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)$ given as the values of inverse function of the demand's distribution function $F_{\xi}(x)$ is positive since the difference of its arguments is positive and $F_{\xi}(x)$ is increasing.

If the condition (2.13) is met, supplier's profit with buyback contract is higher than her profit with wholesale-price contract, and thus a conditionally coordinating buyback contract can be determined.

2.4.2. Retailer Takes the Bank Loan. In this case, expressions for supplier's profit for the two contracts look as follows:

$$\begin{aligned}
\pi_S^{BB}(\omega^*,q^*) &= b\left(\frac{p-c-cr_B}{p-\nu+br_B}F_{\xi}^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)-\int_0^{F_{\xi}^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)}F_{\xi}(x)\,dx\right)\\
\pi_S^{WP}(\omega^*,q^0) &= b\frac{p-c-cr_B}{p-\nu+br_B}F_{\xi}^{-1}\left(\frac{(p-c-cr_B)(p-b-\nu)}{(p-\nu)(p-\nu+br_B)}\right)
\end{aligned} \tag{2.14}$$

and the difference between the supplier's profit functions is expressed in the following way:

$$\pi_S^{BB}(\omega^*,q^*)-\pi_S^{WP}(\omega^*,q^0)=$$

$$b\left(\frac{p-c-cr_B}{p-\nu+br_B}\left(F_{\xi}^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)-F_{\xi}^{-1}\left(\frac{(p-c-cr_B)(p-b-\nu)}{(p-\nu)(p-\nu+br_B)}\right)\right)-\int_0^{F_{\xi}^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)}F_{\xi}(x)\,dx\right) \tag{2.14}$$

As shown in (2.14), the difference is positive if the second factor is greater than zero, i.e.

$$\frac{p-c-cr_B}{p-\nu+br_B}\left(F_{\xi}^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)-F_{\xi}^{-1}\left(\frac{(p-c-cr_B)(p-b-\nu)}{(p-\nu)(p-\nu+br_B)}\right)\right)-\int_0^{F_{\xi}^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)}F_{\xi}(x)\,dx>0. \tag{2.15}$$

The difference of the order volumes $F_{\xi}^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)-F_{\xi}^{-1}\left(\frac{(p-c-cr_B)(p-b-\nu)}{(p-\nu)(p-\nu+br_B)}\right)$ given as values of inverse function of the demand's distribution function is positive as the difference of its arguments is positive and $F_{\xi}(x)$ is increasing; $\frac{p-c-cr_B}{p-\nu+br_B}>0$ according to the results obtained from the solution to the coordination problem (see Table 2.5).

If the condition (2.15) holds, supplier's profit with buyback contract exceeds that with the wholesale-price contract and all three conditions of Definition 2.2 are met; therefore, there exists a conditionally coordinating buyback contract with the bank loan.

2.4.3. Retailer Uses Trade Credit. In this case, the supplier's profit with buyback and wholesale-price contracts look can be written as:

$$\pi_S^{BB}(\omega^*, q^*) = b\left(\frac{p-c}{p-\nu}F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right) - \int_0^{F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)} F_\xi(x)\,dx\right) - r_S K_R$$
$$\pi_S^{WP}(\omega^*, q^0) = b\,\frac{p-c}{p-\nu}F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right) - r_S K_R \tag{2.16}$$

whereas the difference between the supplier's profit with the two contract types is:

$$\pi_S^{BB}(\omega^*, q^*) - \pi_S^{WP}(\omega^*, q^0) =$$
$$b\left(\frac{p-c}{p-\nu}\left(F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right) - F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)\right) - \int_0^{F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)} F_\xi(x)\,dx\right) \tag{2.17}$$

As shown in (2.17), the difference between the supplier's profits is positive if the second factor satisfies the condition:

$$\frac{p-c}{p-\nu}\left(F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right) - F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)\right) - \int_0^{F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)} F_\xi(x)\,dx > 0 \tag{2.18}$$

In (2.18) the difference of the order volumes $F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right) - F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)$ is positive as the difference of its arguments is positive and $F_\xi(x)$ is increasing; $\frac{p-c}{p-\nu} > 0$ based on the condition 7 given in Section 2.1.

If the condition (2.18) is met, supplier's profit with buyback contract is higher than her profit with wholesale-price contract, and thus a conditionally coordinating contract with trade credit can be determined.

Analysis of the two models of the buyback contract built for the cases of the bank loan and the trade credit given in Appendix A7 and Sections 2.4.1, 2.4.2 and 2.4.3 shows that conditional coordination with the buyback contract can be achieved with both bank loan and trade credit. It is proved that with both credit alternatives the supplier's expected profit with buyback contract improves relative to the wholesale-price contract, while the retailer's and supply chain's profit are maximized. However, these results only tell about supplier's potential benefits when dealing with the bank loan and the trade credit, and do not answer the question whether one source of financing is preferable to the other. To justify the choice of the credit type and supplier's motivation to comply with it, further comparison of buyback contract with bank loan and trade credit needs to be made.

Summary of Chapter 2

Based on the approach outlined in Chapter 1, this chapter focuses on the analysis of the coordinating buyback contract with limited funding in a supply chain with one retailer and one supplier and discusses theoretical results obtained in the study. For this purpose, the process of negotiating the contract presented as the newsvendor model transformed into the two-step game has been extended to allow for the two credit types – bank loan and trade credit. Each of the two models includes the case where the retailer's credit size is zero as his funds are enough to pay for the order quantity chosen during the negotiation.

Two contract types have been applied to the model – the buyback contract and the wholesale-price contract. Application of the latter was especially necessary for the analysis of conditionally coordinating buyback contract (Section 2.4).

Analysis of the model with the buyback contract has shown that it does not coordinate the two-echelon supply chain neither with the bank loan raised by the retailer, nor with the trade credit provided by the supplier. Coordination fails as the profits earned by the supply chain members do not achieve their maximum and, hence, the supply chain's profit as their sum is not maximized either.

As it has been mathematically justified that the buyback contract with limited funding does not have coordinating properties, the concept of conditional coordination proposed by Berezinets et al. (2020) has been applied to the models. Conditionally coordinating buyback contract with limited funding is defined as the one that maximizes retailer's profit and increases supplier's profit relative to the wholesale-price contract, thus maximizing the supply chain's profit. So the model analysis included the step dedicated to building the models of the wholesale-price contract with limited funding with the bank loan and trade credit.

Then the buyback contracts with bank loan and trade credit were examined to answer the question whether they can be conditionally coordinating in the presence of retailer's limited funds and credit necessity. Analysis of the model with bank loan showed that there exists a set of conditionally coordinating buyback contracts that provides the retailer with maximal profit and improves the supplier's profit; conditions that are to be satisfied in order to achieve conditional coordination have been identified. The model with the trade credit was analyzed with the same purpose, and the same conclusion was made – there exists a set of conditionally coordinating buyback contracts with trade credit that allow to achieve supply chain profit maximization if certain conditions are met; those conditions were identified as well.

Nonetheless, the supplier's motivation to offer the trade credit and possible retailer's preference for one type of credit over another is still unclear and requires further examination, which is done in Chapter 3.

CHAPTER 3. SELECTING PARAMETERS OF CONDITIONALLY COORDINATING BUYBACK CONTRACT WITH LIMITED FUNDING

3.1. Conditionally Coordinating Buyback Contract with Demand Distributed as Uniformly

Justification of the retailer's choice of the credit type and supplier's motivation to offer the trade credit is based on the comparative analysis of the contract with the bank loan and the contract with trade credit. The contract models analyzed further consider the supply chain producing and selling the product that is assumed to follow a uniform distribution of demand.

3.1.1. The Model with Uniform Distribution of Demand. Uniform distribution of demand has the following properties:

- quantity demanded lies within the closed interval with the minimum value equal to 0 and maximal value equal to β, i.e., $\xi \in [0, \beta]$
- distribution function of demand is $F_\xi(x) = \begin{cases} 0, x < 0 \\ \frac{x}{\beta}, 0 \le x \le \beta \\ 1, x > \beta \end{cases}$
- expected sales volume is $E[\tau] = q - \int_0^q \frac{x}{\beta} dx = q - \frac{q^2}{2\beta}$

With the uniform distribution of demand, the models of the conditionally coordinating buy-back contract with limited funding that determine its parameters with the bank loan, trade credit and none credit taken ("Retailer has sufficient funds") can be written as in Table 3.1. This table displays the order quantity that maximizes supply chain profit and the supplier's, retailer's and supply chain's profit earned with the corresponding order quantity. Detailed derivation of the expressions in Table 3.1 is given in Appendix A8.

Table 3.1. Coordinating buyback contract with limited funding with demand distributed as uniformly

Order quantity	**Supplier**	**Retailer**	**Supply chain**
Retailer has sufficient funds			
$q_{SC}^* = \frac{p-c}{p-v}\beta$	$\pi_S^{BB} = b\beta\left(\frac{p-c}{p-v}\right)^2\left(\frac{1}{2} - \frac{p-b-v}{p-v}\right)$	$\pi_R^{BB} = \frac{1}{2}b\beta\left(\frac{p-c}{p-v}\right)^2\frac{p-b-v}{p-v}$	$\pi_{SC}^{BB} = \frac{1}{2}\beta(p-v)\left(\frac{p-c}{p-v}\right)^2$
Retailer takes the bank loan			
$q_{SC}^* = \frac{p-c-cr_B}{p-v+br_B}\beta$	$\pi_S^{BB} = \frac{1}{2}b\beta\left(\frac{p-c-cr_B}{p-v+br_B}\right)^2$	$\pi_R^{BB} = \frac{1}{2}\beta(p-b-v)\left(\frac{p-c-cr_B}{p-v+br_B}\right)^2 + r_BK_R$	$\pi_{SC}^{BB} = \frac{1}{2}\beta(p-v)\left(\frac{p-c-cr_B}{p-v+br_B}\right)^2 + r_BK_R$

Order quantity	Supplier	Retailer	Supply chain
Retailer uses trade credit			
$q_{SC}^{*}=\frac{p-c}{p-v}\beta$	$\pi_{S}^{BB}=\frac{1}{2}b\beta\left(\frac{p-c}{p-v}\right)^{2}-r_{S}K_{R}$	$\pi_{R}^{BB}=\frac{1}{2}\beta(p-b-v)\left(\frac{p-c}{p-v}\right)^{2}+r_{S}K_{R}$	$\pi_{SC}^{BB}=\frac{1}{2}\beta(p-v)\left(\frac{p-c}{p-v}\right)^{2}$

Table 3.1 shows that the order volume q_{SC}^{*} that maximizes supply chain profit in the case with the retailer's sufficient funds is the same as the order volume that maximizes total profit with trade credit. It also shows that maximal supply chain profit with the retailer's sufficient funds is equal to its profit with trade credit.

Explanation for the latter observation is that in the model with the trade credit the credit payment is transferred between supplier and retailer and not to the third party as in the model with the bank loan (the bank). Therefore, the costs of borrowing are shared between the supply chain members and no additional costs associated with external sources of borrowing incur. As a result, the supply chain's profit with the trade does not decline due to these costs and reaches its maximum equal to the case where no credit is used.

3.1.2. Supply Chain Profit Allocation. As coordination allows to arbitrarily allocate supply chain profit between supplier and retailer, a parameter that allows for profit allocation has to be included into the model. This step is done by Berezinets et al. (2020) in their study of the buyback contracts that represents the case of retailer's sufficient funds in this body of research.

The authors find that buyback contract is conditionally coordinating when $b>\frac{p-v}{2}$, and express the buyback price through a parameter λ that belongs to the open interval between 0 and 1, i.e. $\lambda\in(0,1)$: $b=\lambda(p-v)$. The authors show that with λ supply chain profit can be arbitrarily split between supplier and retailer in proportion λ: $(1-\lambda)$ and, thus, λ determines supplier's share of total profit. The authors state that parameter λ can be considered as supplier's leading power in negotiation, or "negotiation leverage". In the case when retailer's funds are sufficient, the buyback price should satisfy the condition $b>\frac{p-v}{2}$, so it is clear that in this case λ should be greater than 0.5, i.e. $0.5<\lambda<1$.

Berezinets et al. (2020) also state that dependence of buyback price on negotiation leverage $b=\lambda(p-v)$ should always hold and arbitrarily divide the expected profit of the two-echelon supply chain between its members. Based on this property of the buyback price, the models of the conditionally coordinating buyback contract with demand distributed as uniformly can be written as in Table 3.2, which displays the supplier's, retailer's and supply chain profit expressed through the supplier's share of total profit, i.e. parameter λ. Derivation of the profit expressions is shown in detail in Appendix A8.

Table 3.2. Profit allocation with the conditionally coordinating buyback contract with limited funding

Wholesale price	**Supplier**	**Retailer**	**Supply chain**
		Retailer has sufficient funds	
$\omega^*(\lambda) = c_S + \lambda(p-c)$	$\pi_S^{BB}(\omega^*(\lambda), q^*) = \lambda \frac{\beta}{2}\frac{(p-c)^2}{p-v}$	$\pi_R^{BB}(\omega^*(\lambda), q^*) = (1-\lambda)\frac{\beta}{2}\frac{(p-c)^2}{p-v}$	$\pi_{SC}^{BB}(\omega^*(\lambda), q^*) = \frac{\beta}{2}\frac{(p-c)^2}{p-v}$
		Retailer takes the bank loan	
$\omega^*(\lambda) = \frac{c+\lambda(p-c)}{1+\lambda r_B} - c_R$	$\pi_S^{BB}(\omega^*(\lambda), q^*) = \lambda \frac{\beta}{2(p-v)}\left(\frac{p-c-cr_B}{1+\lambda r_B}\right)^2$	$\pi_R^{BB}(\omega^*(\lambda), q^*) = (1-\lambda)\frac{\beta}{2(p-v)}\left(\frac{p-c-cr_B}{1+\lambda r_B}\right)^2 + r_B K_R$	$\pi_{SC}^{BB}(\omega^*(\lambda), q^*) = \frac{\beta}{2(p-v)}\left(\frac{p-c-cr_B}{1+\lambda r_B}\right)^2 + r_B K_R$
		Retailer uses trade credit	
$\omega^*(\lambda) = \frac{c+\lambda(p-c)}{1+r_S} - c_R$	$\pi_S^{BB}(\omega^*(\lambda), q^*) = \lambda \frac{\beta}{2}\frac{(p-c)^2}{p-v} - r_S K_R$	$\pi_R^{BB}(\omega^*(\lambda), q^*) = (1-\lambda)\frac{\beta}{2}\frac{(p-c)^2}{p-v} + r_S K_R$	$\pi_{SC}^{BB}(\omega^*(\lambda), q^*) = \frac{\beta}{2}\frac{(p-c)^2}{p-v}$

With the bank loan the buyback contract is conditionally coordinating if the buyback price meets the condition $b > \frac{p-v}{2}$ (see Appendix A8). With $b = \lambda(p-v)$ inserted into the formulas of the optimal wholesale price and profit expressions it is straightforward to confirm that the parameter λ splits not entire supply chain profit, but the part of it that is dependent on the order quantity (see Table 3.2). Parts of the profits that are dependent on the order quantity are divided between the players in proportion λ: $(1-\lambda)$. The second term in the formula of retailer's profit is the product of bank interest rate and retailer's holdings of cash. As the retailer raises the loan in the amount that he lacks to pay for the order quantity and adds this sum to the cash at his disposal K_R, this term in the formula of retailer's profit can be considered as interest that retailer gets at the selling season for the cash available. The condition $b > \frac{p-v}{2}$ shows that to achieve conditional coordination supplier's negotiation leverage should be greater than 0.5, then her share of profit will be between 0.5 and 1, i.e. $0.5 < \lambda < 1$.

With the trade credit the buyback contract conditionally coordinates the supply chain when the same constraint as with the bank loan is met: $b > \frac{p-v}{2}$ (see Appendix A8).

As shown in Table 3.2, with the trade credit parameter λ splits total supply chain profit the same way it does with the bank loan – only parts of the profits that are dependent on the order quantity are arbitrarily divided between the players in proportion λ: $(1-\lambda)$.

To achieve conditional coordination, the buyback contract with the trade credit also should satisfy the condition $b > \frac{p-v}{2}$. With this condition met the supplier's share of total profit will be greater than 0.5, i.e. $0.5 < \lambda < 1$.

Therefore, conditionally coordinating buyback contract with limited funding allows to allocate the supply chain's profit so as to provide the supplier with more than its half. This is mathematically justified by the condition $0.5 < \lambda < 1$ imposed on the parameter λ that represents the supplier's negotiation leverage as well as her share of total profit. This condition holds in all three cases considered in the model – when the retailer has sufficient funds and does not use any credit, when retailer takes the bank loan, and when retailer uses the trade credit offered by the supplier. But to justify the supplier's motivation to offer the trade credit instead of letting him raise the bank loan and possible retailer's preference for one type of credit over another comparative analysis of the contracts with the two types of credit is carried out further.

3.2. Comparing the Contracts with the Trade Credit and Bank Loan

As the decision to offer the trade credit to the retailer is made by the supplier, it is important to justify the supplier's motivation that drives this decision. The supplier aims to improve her own profit, taking into account the goal of maximizing the supply chain profit. So the supplier will offer trade credit to the retailer only if her own profit increases with this credit type relative to that with the bank loan.

Comparative analysis of the models with the trade credit and bank loan has been conducted both for the conditionally coordinating buyback contract and wholesale-price contract. Analysis of the buyback contract showed that the supplier's profit with trade credit is higher than with bank loan if the trade credit rate meets the following constraint:

$$r_S < \frac{1}{\frac{2(p-c)(1+\lambda r_B)^2}{\lambda r_B\big((p-c)(2+\lambda r_B)-c r_B\big)}-1} \tag{3.1}$$

Detailed solution is given in Appendix A9. First, the difference between supply chain profits with trade credit and bank loan is considered, and it is shown that supply chain profit is always higher with trade credit than with bank loan as the difference between them is always positive for any value of λ that belongs to the interval between 0.5 and 1. Then supplier's profit is expressed as a share of supply chain profit through parameter λ for each case, and the difference between the supplier's profits with two sources of borrowing is considered. The solution shows that this difference is positive when the rate on the trade credit meets the condition (3.1). With the condition (3.1) satisfied, the supplier's profit with trade credit is higher than with the bank loan, so it is in her interest to offer the trade credit to the retailer in order to increase individual and supply chain's profit.

These findings are in line with the observations made by Kouvelis and Zhao (2012). The authors state that optimally structured contract with trade credit improves the supply chain profit thus increasing supplier's profit compared to optimally structured contract with the bank loan.

Besides, from Table 3.2 it is straightforward to confirm that for the same value of the wholesale price the trade credit rate can be expressed as $r_S = \lambda r_B$. The conclusion whether the trade credit from this expression always satisfies the condition (3.1) is ambiguous, hence the constraint imposed on the rate of trade credit offered by the supplier with buyback contract should meet the following constraint:

$$r_S \leq \ min \left\{ \lambda r_B; \frac{1}{\frac{2(p-c)(1+\lambda r_B)^2}{\lambda r_B\big((p-c)(2+\lambda r_B)-cr_B\big)} - 1} \right\} \tag{3.2}$$

To justify supplier's motivation to offer trade credit with the wholesale-price contract, the models with trade credit and bank loan are also compared. Detailed solution is given in Appendix A10. To prove that during the negotiation over the wholesale-price contract supplier is motivated to offer the trade credit, supplier's profit with trade credit is compared with that with the bank loan using the formulas for the wholesale price that maximize supply chain profit with buyback contract. Those formulas are inserted into the expressions for the order volume that maximizes retailer's profit with wholesale-price contract. Then modified formulas of the wholesale price and retailer's optimal order volume are inserted into the supplier's profit expressions. The formulas used in the comparative analysis of the wholesale-price contract formulas are given in Table 3.3.

Table 3.3. The contract parameters and supplier's profit with the wholesale-price contract

Wholesale price	**Order quantity**	**Supplier**
Retailer has sufficient funds		
$\omega(\lambda) = c_S + \lambda(p-c)$	$q_R^* = (1-\lambda)\frac{p-c}{p-\nu}\beta$	$\pi_S^{WP} = \lambda(1-\lambda)\frac{(p-c)^2}{p-\nu}\beta$
Retailer takes the bank loan		
$\omega(\lambda) = \frac{c+\lambda(p-c)}{1+\lambda r_B} - c_R$	$q_R^* = \frac{1-\lambda}{1+\lambda r_B}\frac{p-c-cr_B}{p-\nu}\beta$	$\pi_S^{WP} = \frac{\lambda(1-\lambda)}{(1+\lambda r_B)^2} * \frac{(p-c-cr_B)^2}{p-\nu}\beta$
Retailer uses trade credit		

Wholesale price	Order quantity	Supplier
$\omega(\lambda) = \frac{c+\lambda(p-c)}{1+r_S} - c_R$	$q_R^* = (1-\lambda)\frac{p-c}{p-\nu}\beta$	$\pi_S^{WP} = \lambda(1-\lambda)\frac{(p-c)^2}{p-\nu}\beta - r_S K_R$

Comparative analysis of the wholesale-price contract shows the same result as the conditionally coordinating buyback contract. It shows that, in this case, the supplier also obtains greater profit when offering the trade credit to the retailer instead of letting him borrow from the bank. The supplier's profit with trade credit is higher than with the bank loan if

$$r_S < \frac{1}{\frac{(p-c)(1+\lambda r_B)^2}{\lambda r_B\big((p-c)(2+\lambda r_B)-cr_B\big)} - 1} \tag{3.3}$$

Considering the equation $r_S = \lambda r_B$ derived from the expressions for the wholesale price from Table 3.2 and Table 3.3, the condition (3.3) imposed on the trade credit rate for the wholesale-price contract looks as follows:

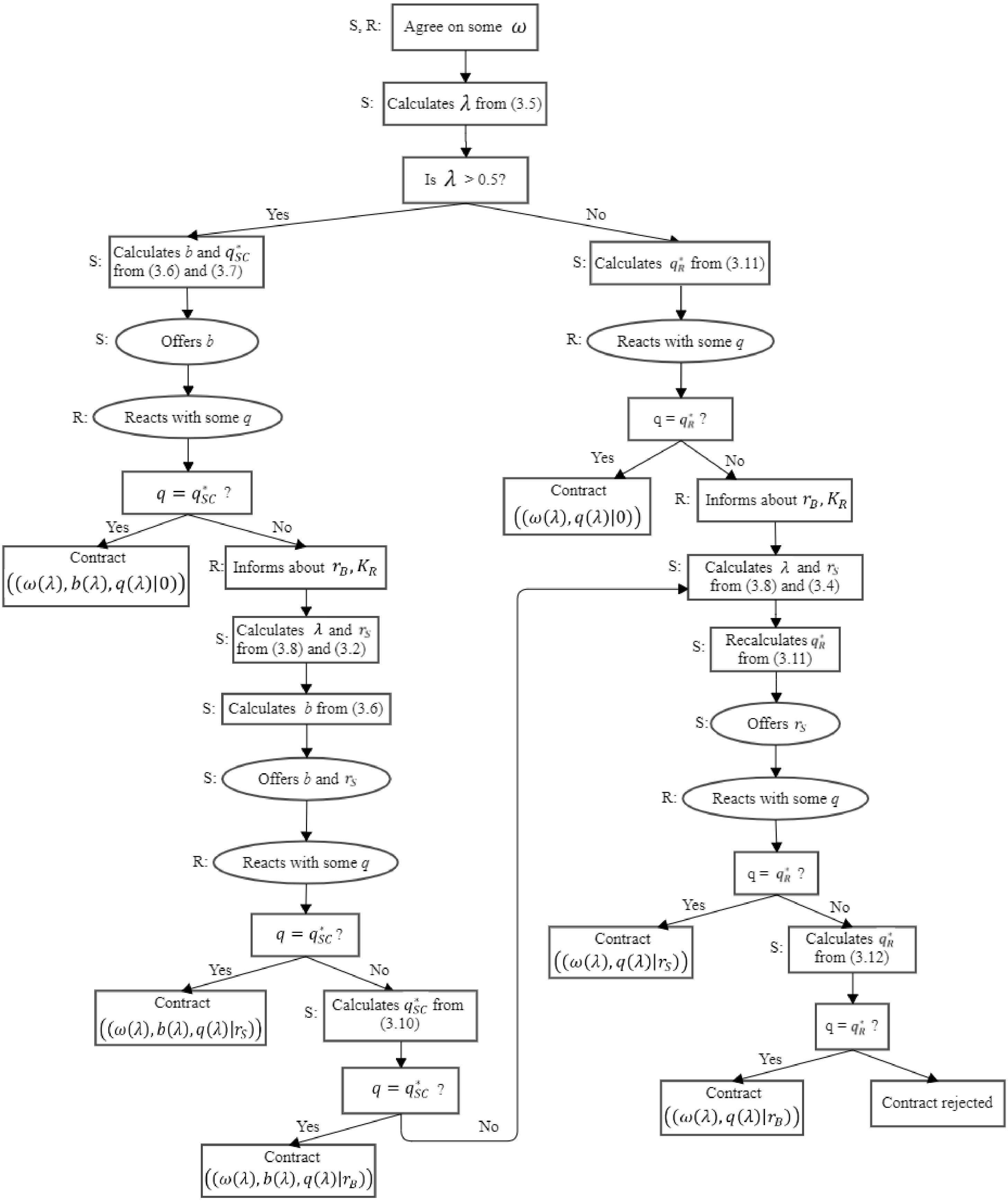
$$r_S \leq min\left\{\lambda r_B; \frac{1}{\frac{(p-c)(1+\lambda r_B)^2}{\lambda r_B\big((p-c)(2+\lambda r_B)-cr_B\big)} - 1}\right\} \tag{3.4}$$

3.3. Selecting Conditionally Coordinating Buyback Contract with Limited Funding

As a result, a framework for negotiating the terms of a contract with limited funding can be proposed. It allows:

- to choose the contract type out of two options – buyback contract when supplier's potential share of profit for a given wholesale price is higher than 0.5, and wholesale-price contract when her share of total profit is lower than 0.5;
- to decide which form of borrowing – bank loan or trade credit – to use when retailer's funds are not enough to pay for the order quantity;
- to determine the contract terms so that each supply chain party is better off considering the type of credit chosen by the retailer.

The framework is presented as the flowchart of the negotiating process in Picture 3.1.



Picture 3.1. Algorithm for selecting conditionally coordinating buyback contract with limited funding

Source: author’s own

Application of the framework starts from the point where supplier and retailer come to an agreement about some value of the wholesale price ω. Using this value supplier can calculate her share of supply chain profit with buyback contract (λ) from the formula:

$$\omega(\lambda) = c_S + \lambda(p - c) \tag{3.5}$$

Expression (3.5) for the case of sufficient funds is used as at this stage of negotiation supplier is not aware of retailer's possible limited funding; for a given value of the wholesale price (3.5) allows to find the value of λ. If the value of λ is between 0.5 and 1, buyback contract is chosen and further negotiation draws upon the left part of the flowchart; otherwise, when $0 < \lambda < 0.5$, it is more beneficial for supplier to suggest the wholesale-price contract (right side of the flowchart).

3.3.1. Supplier's profit share $\mathbf{0,5 < \lambda < 1}$. If for a given value of the wholesale price supplier's share of total profit is greater than 0.5, i.e. $0.5 < \lambda < 1$, the supplier will be able to offer the following contracts:

- conditionally coordinating buyback contract $\big((\omega(\lambda), b(\lambda), q(\lambda)|0)\big)$,
- conditionally coordinating buyback contract with trade credit $\big((\omega(\lambda), b(\lambda), q(\lambda)|r_S)\big)$,
- conditionally coordinating buyback contract with a bank loan $\big((\omega(\lambda), b(\lambda), q(\lambda)|r_B)\big)$,
- wholesale-price contract with trade credit $\big((\omega(\lambda), q(\lambda)|r_S)\big)$, or
- wholesale-price contract with a bank loan $\big((\omega(\lambda), q(\lambda)|r_B)\big)$.

The first three contract types relate to the conditionally coordinating buyback with limited funding and take into account the case where the retailer's funds are sufficient, so no credit is used. If the retailer does not accept the contract terms offered by supplier in those three cases, supplier and retailer switch to the wholesale-price contract either with the bank loan or the trade credit. If the agreement about the contract terms is not reached either, the contract is rejected.

The algorithm suggests that having calculated the value of λ supplier further calculates the buyback price b for the determined λ and optimal order quantity q_{SC}^* from the formulas

$$b = \lambda(p - v) \tag{3.6}$$

$$q_{SC}^* = \frac{p - c}{p - v}\beta \tag{3.7}$$

and offers this buyback price to the retailer.

Retailer responds with some order quantity q, and it is straightforward to conclude that q will be either equal or less than the optimal order quantity from (3.7), as the retailer is always reluctant to order large amounts and the design of coordinating contracts aims to incentivize him to increase the order quantity. If the retailer's order quantity is equal to the optimal one from (3.7), supplier and retailer sign a conditionally coordinating buyback contract $\left((\omega(\lambda), b(\lambda), q(\lambda)|0)\right)$, where $q(\lambda) = q_{SC}^* = \frac{p-c}{p-v}\beta$, and zero stands for the interest rate, which is zero in the case with sufficient funds as no borrowing takes place.

If the order quantity q chosen by retailer is lower than the one from (3.7), retailer is supposed to share information about the interest rate on the bank loan r_B and the amount of cash K_R he has.

With newly obtained information on the retailer's limited funding supplier recalculates her share of the total profit that can be earned with the retailer's bank loan. To obtain a new value of λ that takes the bank loan into account, the following formula of the wholesale price is used:

$$\omega^*(\lambda) = \frac{c + \lambda(p - c)}{1 + \lambda r_B} - c_R \tag{3.8}$$

As justified by the mathematical solution in Section 3.2, under the buyback contract that conditionally coordinates the supply chain, its total profit is always higher with trade credit than with a bank loan, provided the condition (3.2) is satisfied. So the supplier is better off when offering the trade credit to the retailer.

Based on the updated value of λ, the supplier can recalculate the buyback price (3.6) and determine the interest rate for the trade credit that satisfies the condition (3.2). Optimal order volume for a conditionally coordinating buyback contract with trade credit remains the same as in the case of retailer's sufficient funds, which is equal to (3.7).

After supplier offers a new buyback price and trade credit rate, retailer responds with a new order volume q. If the retailer's reaction q increases to the optimal one, they sign a conditionally coordinating buyback contract with trade credit $\left((\omega(\lambda), b(\lambda), q(\lambda)|r_S)\right)$, where $q(\lambda) = q_{SC}^* = \frac{p-c}{p-v}\beta$.

Otherwise, supplier turns down her offer of the trade credit and allows the retailer to take the bank loan in case the retailer's latest response is equal to (3.10):

$$q_{SC}^* = \frac{p - c - cr_B}{(p - v)(1 + \lambda r_B)}\beta \tag{3.10}$$

This order quantity is lower than (3.7), but provides conditional coordination under the buyback contract with the bank loan. Thus, supplier and retailer enter into a conditionally coordinating buyback contract with a bank loan $\left((\omega(\lambda), b(\lambda), q(\lambda)|r_B)\right)$, where $q(\lambda) = q_{SC}^* = \frac{p-c-cr_B}{(p-v)(1+\lambda r_B)}\beta$.

However, if the order quantity chosen by the retailer is lower than (3.10), buyback contract is not considered anymore and supplier continues with negotiating the wholesale-price contract, namely the wholesale-price contract with trade credit. For the share of supply chain profit λ previously determined from (3.8) supplier determines the order quantity q_R^* that provides her with this profit share and the trade credit rate that meets the condition (3.4) for this contract type. To calculate the order quantity at this step, the formula (3.11) is used:

$$q_R^* = (1-\lambda)\frac{p-c}{p-\nu}\beta \tag{3.11}$$

Then supplier offers the determined trade credit rate to the retailer.

Retailer responds with some order quantity q, which he decides to purchase for the given wholesale price ω. If the order quantity chosen by retailer is equal to (3.11), supplier and retailer enter into a wholesale-price contract with trade credit $\big((\omega(\lambda), q(\lambda)|r_S)\big)$, where $q(\lambda) = q_R^* = (1-\lambda)\frac{p-c}{p-\nu}\beta$. Wholesale-price contract with trade credit maximizes retailer's profit and provides the supplier with the same share of supply chain profit as under the conditionally coordinating buyback contract with limited funding, which is greater than 0.5.

If the retailer's response is lower than (3.11), supplier compares it with the order quantity q_R^* that provides her with the profit share λ and maximizes retailer's profit with the bank loan. To calculate this order quantity, the following expression is used:

$$q_R^* = \frac{1-\lambda}{1+\lambda r_B}\frac{p-c-cr_B}{p-\nu}\beta \tag{3.12}$$

If retailer's order quantity q is equal to (3.12), supplier lets retailer raise the bank loan, and they sign a wholesale-price contract with a bank loan $\big((\omega(\lambda), q(\lambda)|r_B)\big)$, where $q(\lambda) = q_R^* = \frac{1-\lambda}{1+\lambda r_B}\frac{p-c-cr_B}{p-\nu}\beta$.

Otherwise, if the retailer's order quantity is lower than (3.12), the wholesale-price contract is rejected and no contract is selected out of two alternatives (buyback and wholesale-price contract).

3.3.2. Supplier's profit share $0 < \lambda < 0,5$. If for a given value of the wholesale price supplier's share of total profit is going to be lower than 0.5, i.e. $0 < \lambda < 0,5$, supplier prefers the wholesale-price contract and may offer:

- wholesale-price contract $\big((\omega(\lambda), q(\lambda)|0)\big)$,
- wholesale-price contract with trade credit $\big((\omega(\lambda), q(\lambda)|r_S)\big)$, or
- wholesale-price contract with a bank loan $\big((\omega(\lambda), q(\lambda)|r_B)\big)$.

Having the value of λ calculated at the previous step, supplier calculates the value of the order quantity q_R^* using the formula (3.11). When the retailer responds with some order quantity

q, supplier compares it with the order quantity from (3.11). If q is equal to (3.11), the wholesale-price contract $\left((\omega(\lambda), q(\lambda)|0)\right)$, where $q(\lambda) = q_R^* = (1-\lambda)\frac{p-c}{p-\nu}\beta$ and zero stands for the cost of borrowing as it is not needed with sufficient funds.

If retailer's response is lower than (3.11), retailer is to explain his choice, so he shares information about his need for borrowing, interest rate on the bank loan r_B and the amount of cash K_R at his disposal.

With updated information on the retailer's funds, supplier recalculates her share of supply chain profit using the formula (3.8). As proved in Section 3.1.5, supplier's profit with the bank loan is lower than the profit she can get with the trade credit, so the supplier determines the rate on the trade credit using the formula (3.4) and offers it to the retailer.

Retailer reacts with a new order quantity. If new order quantity is higher than the previous one and equal to (3.11), the wholesale-price contract with trade credit $\left((\omega(\lambda), q(\lambda)|r_S)\right)$ is signed.

Otherwise, supplier has to compare the latest retailer's response with q_R^* from (3.12). In case the retailer's response is equal to (3.12), supplier refuses to provide the retailer with trade credit and allows him to take the bank loan instead. The wholesale-price contract with the bank loan $\left((\omega(\lambda), q(\lambda)|r_B)\right)$ is selected. However, if the retailer's response is lower than (3.12), the wholesale-price contract is completely rejected.

3.4. The Case of the Supply Chain for a Perishable Product

To demonstrate the applicability of the developed framework, a numerical case based on the example studied by Berezinets et al. (2020) has been examined. The authors considered the case of the supply chain with supplier and retailer involved in manufacturing and retailing of the perishable product with short shelf life. The data used in the contract negotiation included:

- retail price $p = \$8$,
- salvage value $\nu = \$1$,
- supplier's cost $c_S = \$3$,
- retailer's cost $c_R = \$0{,}3$,
- so total costs $c = \$3{,}3$,
- $\beta = 200$ as the product demand is distributed as uniformly between 0 and 200.

Assume that retailer's holdings of cash $K_R = \$30{,}65$ and is not enough to pay for any order volume in the setting given. The cost of borrowing from the bank r_B is equal to 10%.

3.4.1. Supplier's profit share $0,5 < \lambda < 1$. Suggest supplier and retailer agree on a wholesale price $\omega = \$5{,}5$. Supplier calculates her share of the total profit with the buyback contract using the formula (3.5) and obtains $\lambda = 0{,}53$, which meets the condition for a conditionally coordinating buyback contract.

As supplier wants to earn the highest profit possible, she is interested in arriving at a conditionally coordinating buyback contract that allows her to improve individual profit compared to the profit she could earn with a wholesale-price contract with the same wholesale price. For $\lambda = 0{,}53$ supplier's profit with the conditionally coordinating buyback contract can be \$167.25, retailer profit can be \$148.32, total expected profit can be \$315,57.

Getting this profit values is possible if the order quantity chosen by retailer is large enough, precisely $q^*_{SC} = 134$ units (from (3.7)), so the supplier is motivated to offer the buyback condition to incentivize the retailer to increase his order volume. Supplier calculates the buyback price for the determined profit share using the formula (3.6) and determines the buyback price $b = \$3{,}71$, which she offers to the retailer.

Retailer responds with some order volume q. The optimal order volume that provides conditional coordination by Definition 2.2 is 134 units. If q is equal to 134 units, conditionally coordinating buyback contract by Definition 2.2 is selected. Thus, supplier and retailer arrive at conditionally coordinating buyback contract $\big((\omega(\lambda) = \$5.5, b(\lambda) = \$3.71, q(\lambda) = 134|0)\big)$.

However, if the retailer responds with an order volume that is lower than 134 units, he is supposed to explain why he wants to order so little and to share information about his need for borrowing. Retailer informs supplier that the interest rate charged by the bank $r_B = 10\%$ and the amount of cash he has $K_R = \$30{,}65$.

Since the bank loan taken by the retailer will affect the allocation of total profit and supplier's share of it, supplier recalculates λ using (3.8) and obtains an updated profit share $\lambda = 0{,}6$. New profit share is greater than the initial $\lambda = 0.53$, however total supply chain profit with bank loan is lower than with trade credit [or equally with sufficient funds]. Thus, the supplier needs to incentivize retailer by offering the trade credit so that retailer decided to increase the order quantity and the supply chain profit improved.

To do so, supplier calculates the rate she should charge for the trade credit using (3.2) and a new buyback price as in (3.6). Trade credit rate $r_S \leq min\{0.06; 0.056\}$, which means that the supplier should offer the rate $r_S = 5.6\%$. The updated buyback price should be $b = \$4.2$. These terms are offered to the retailer.

Retailer responds with a new order quantity q, which should be 134 units for the retailer to get the trade credit and buyback price offered by the supplier. If retailer arrives at the decision to order 134 units, they sign the conditionally coordinating contract with trade credit $\left((\omega(\lambda) = \$5{,}5, b(\lambda) = \$4{,}2, q(\lambda) = 134 | r_S = 5{,}6\%)\right)$. With this contract, the supply chain profit is \$315,57; the supplier's portion of total profit ($\lambda = 0{,}6$) is equal to \$187,62; the retailer's profit is \$127,95.

If the retailer's response does not match the order quantity $q_{SC}^{*} = 134$, offering him the trade credit with the given wholesale price $\omega = \$5{,}5$ is not beneficial for the supplier; and the supplier turns down her offer of the trade credit.

Nonetheless, the supplier still can come to a conditionally coordinating buyback contract with the bank loan. To determine the order quantity for this contract to be conditionally coordinating, supplier calculates it with (3.8): $q_{SC}^{*} = 118$ units.

If retailer's latest response was 118 units, supplier signs the conditionally coordinating buyback contract with the bank loan $\left((\omega(\lambda) = \$5{,}5, b(\lambda) = \$4{,}2, q(\lambda) = 118 | r_B = 10\%)\right)$. According to its terms, total supply chain profit is \$245,87; supplier's share of it is \$147,52; and retailer earns \$98,35. Otherwise, if the retailer chooses to order less than 118 units, supplier turns down the buyback offer and continues with negotiating over the wholesale-price contract with trade credit.

As the retailer has already informed the supplier about the limited funding he is faced with, based on that information the supplier can determine the trade credit rate – it should meet the condition (3.4): $r_S \leq \ min\{0.06; 0.119\}$, so the supplier offers the trade credit with $r_S = 6\%$.

Retailer reacts with an updated order quantity. For the supplier to obtain 60% of supply chain profit ($\lambda = 0{,}6$) with the wholesale-price contract with trade credit, retailer's new choice of the order quantity has to be equal to (3.11). Supplier calculates this order quantity: $q_R^{*} = 54$. If retailer's latest response coincides with $q_R^{*} = 54$, the wholesale-price contract with trade credit $\left((\omega(\lambda) = \$5.5, q(\lambda) = 54 | r_S = 6\%)\right)$ is selected. With this contract, the supplier obtains \$149.64; retailer obtains \$52.33; supply chain profit is \$201.97.

But if the retailer's updated order quantity is lower than 54 units, the supplier should compare it with (3.12): for the bank's rate $r_B = 10\%$ the order quantity (3.12) should be $q_R^{*} = 47$. In case the retailer's order quantity is 47 units, the wholesale-price contract $\left((\omega(\lambda) = \$5.5, q(\lambda) = 47 | r_B)\right)$ is selected. With this contract, the supplier's profit share of 60% equals to \$116.55; the retailer's profit is \$41.91; the supply chain profit is \$158.46. If the retailer chooses to order less than 47 units, the contract is rejected.

Details on the profit allocation in the case considered in Section 3.4.1 are summarized in Table 3.4.

Table 3.4. The contracts and profit allocation when $0.5 < \lambda < 1$

Value of the wholesale price		$5,5	
Contract type		Buyback	Wholesale-price
Sufficient funds	b	$3.71	-
	π_S	$167.25	-
	π_R	$148.32	-
	π_{SC}	$315,57	-
Bank loan ($r_B = 10\%$)	b	$4.2	-
	π_S	$147,52	$116.55
	π_R	$98,35	$41.91
	π_{SC}	$245,87	$158.46
Trade credit	b	$4.2	-
	r_S	5.6%	6%
	π_S	$187,62	$149.64
	π_R	$127,95	$52.33
	π_{SC}	$315,57	$201.97

As shown in Table 3.4, the supplier's highest profit is earned with the conditionally coordinating buyback contract with trade credit, $\pi_S = \$187.62$. The retailer obtains the highest profit with the conditionally coordinating contract with sufficient funds, $\pi_R = \$148.32$. In both cases the supply chain profit is $\pi_{SC} = \$315.57$. With the bank loan, the supply chain profit decreases under the buyback contract; with the wholesale-price contracts both with trade credit and bank loan the supply chain profit declines more significantly – in both cases the supply chain is lower compared to that with the buyback contract with the bank loan. The lowest supply chain profit is achieved with the wholesale-price with the bank loan, $\pi_{SC} = \$158.46$. It corresponds with the lowest values of supplier's and retailer's profits given in Table 3.4, $\pi_S = \$116.55$ and $\pi_R = \$41.91$, respectively.

3.4.2. Supplier's profit share $\mathbf{0 < \lambda < 0,5}$. To demonstrate how the framework can be applied for the wholesale-price contract, suggest supplier and retailer agree on a wholesale price $\omega = \$4{,}65$.

Supplier calculates her share of the total profit with the buyback contract using the formula (3.5) and obtains $\lambda = 0{,}35$, which is less than 0,5 and does not meet the condition for a conditionally coordinating buyback contract, so the supplier considers the wholesale-price contract.

Retailer responds with some order volume q, which the supplier needs to compare with the order quantity given in (3.11): $q_R^* = 87$ units. If the retailer responds with this order quantity, the negotiation has come to an end and the wholesale-price contract $\big((\omega(\lambda) = \$4{,}65, q(\lambda) = 87|0)\big)$ is selected. According to its terms, the supplier's profit is \$143.59, the retailer's profit is \$133.33, and the supply chain profit is \$276.91.

But if the retailer responds with an order quantity that is lower than 87 units, he is supposed to explain the reason for such choice and share information about his need for borrowing. At this stage the retailer informs the supplier that the interest rate charged by the bank $r_B = 10\%$ and the amount of cash he has $K_R = \$30{,}65$.

Since the bank loan taken by retailer will change the profit levels of both supplier and retailer, supplier recalculates λ using (3.8) and obtains an updated value $\lambda = 0.39$ [round up to 0,4]. New value of λ is greater than the initial $\lambda = 0.35$, so the order quantity q_R^* will change as it depends on λ. With a wholesale-price contract supplier's profit with bank loan is lower than with trade credit, and thus supplier needs to incentivize retailer by offering a trade credit so that the retailer increased the amount of goods he wants to order and improved supply chain profit.

To do so, supplier calculates the rate she should charge for the trade credit using (3.4): $r_S \leq min\{0.04; 0.079\}$. So, supplier offers the trade credit with $r_S = 4\%$ to retailer.

Retailer responds with new order quantity q; supplier updates the order quantity q_R^* from (3.11) – now it should be 81 units for the retailer to get the trade credit. If retailer arrives at the decision to order 81 units, they sign the wholesale-price contract with trade credit $\big((\omega(\lambda) = \$4{,}65, q(\lambda) = 81|r_S = 4\%)\big)$. With this contract, the profits achieve the following levels: the supply chain profit is \$265,1; the supplier's profit is \$150,3; the retailer's profit is \$114,8.

But if retailer's response is lower than 81 units, supplier is not motivated to provide retailer with a trade credit, so she turns down her offer of the trade credit and lets the retailer to take the bank loan instead provided the retailer's order quantity is equal to (3.12): supplier's calculation results in $q_R^* = 72$ units.

If retailer's latest response is 72 units, the wholesale-price contract with the bank loan $\big((\omega(\lambda) = \$4{,}65, q(\lambda) = 72|r_B = 10\%)\big)$ is selected. According to its terms, total supply chain profit is \$214.94; the supplier's profit is \$121.07; and the retailer's profit is \$93.87. Otherwise, if the retailer chooses to order less than 72 units, the contract is completely rejected.

Details on the contracts and profit values achieved in the case described in Section 3.4.2 are given in Table 3.5.

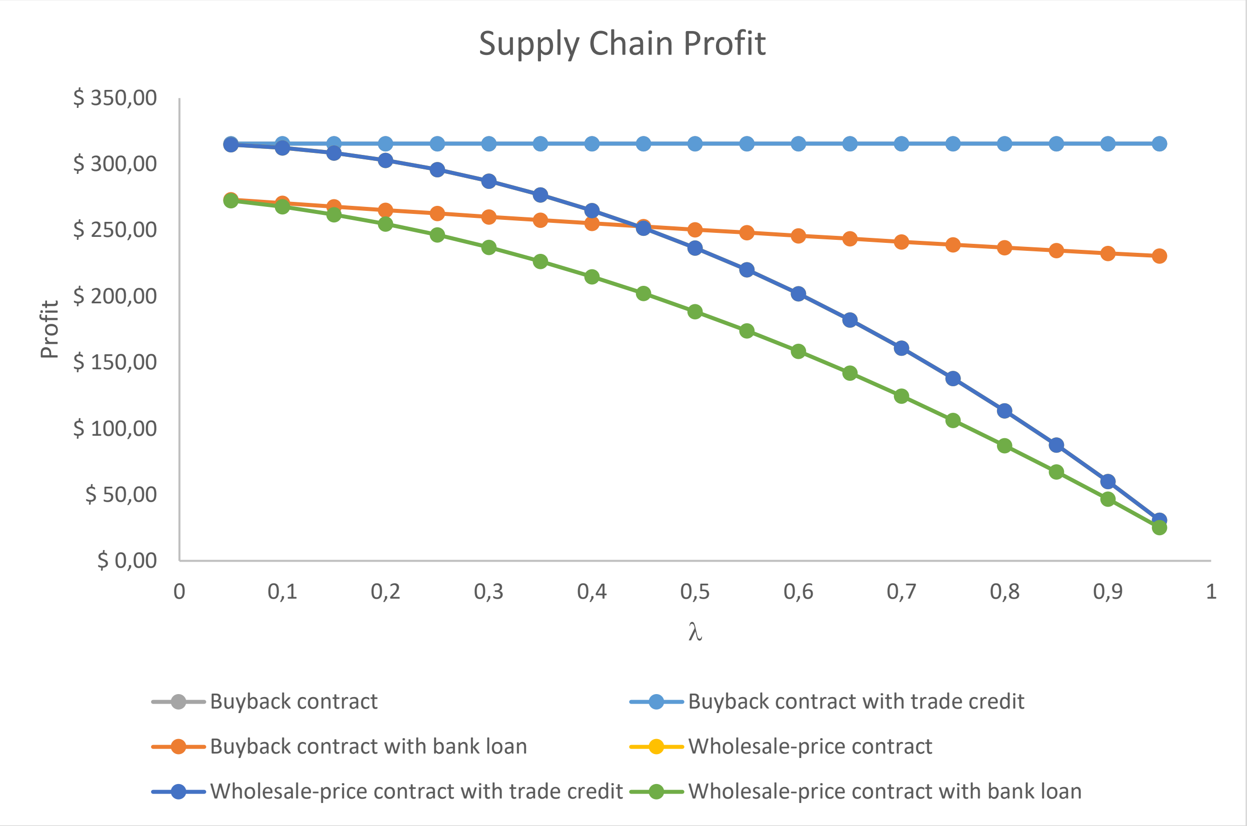
Table 3.5. The contracts and profit allocation when $0 < \lambda < 0.5$

Value of the wholesale price		$4,65
Contract type		Wholesale price
Sufficient funds	π_S	$143.59
	π_R	$133.33
	π_{SC}	$276.91
Bank loan ($r_B = 10\%$)	π_S	$121.07
	π_R	$93.87
	π_{SC}	$214.94
Trade credit	r_S	4%
	π_S	$150.25
	π_R	$114.83
	π_{SC}	$265.08

As shown in Table 3.5, the highest supplier's profit is earned with trade credit, $\pi_S = \$150.25$. The highest retailer's profit is achieved in the case where retailer has sufficient funds, $\pi_R = \$133.33$; in this case the supply chain profit is also the highest in Table 3.5, $\pi_{SC} = \$276.91$. Supplier and retailer both earn their lowest profits with the wholesale-price contract with the bank loan, $\pi_S = \$121.07$ and $\pi_R = \$93.87$; therefore the supply chain profit achieved in this case is also the lowest in Table 3.5, $\pi_{SC} = \$214.94$.

The case study confirms that with both with buyback and wholesale-price contract supplier's profit is the highest with the trade credit and the lowest with the bank loan. This conclusion is in line with (Kouvelis, Zhao, 2012). The authors state that in business practice it is very common for suppliers to provide retailers with trade credit even when retailers' funds are sufficient as it is beneficial for supplier and positively affects her profit. The highest retailer's profit with both contract types is achieved when his funds are sufficient, which is reasonable since in this case the retailer does not run any additional cost of borrowing. Supply chain profit with trade credit is the same as with sufficient funds with buyback contract since its profit function does not depend on λ and remains constant.

Rationale for the approach to choosing the type and terms of the contract can be illustrated with Picture 3.2 showing the supply chain profit with both the wholesale-price and buyback contracts with two forms of credit – bank loan and trade credit.



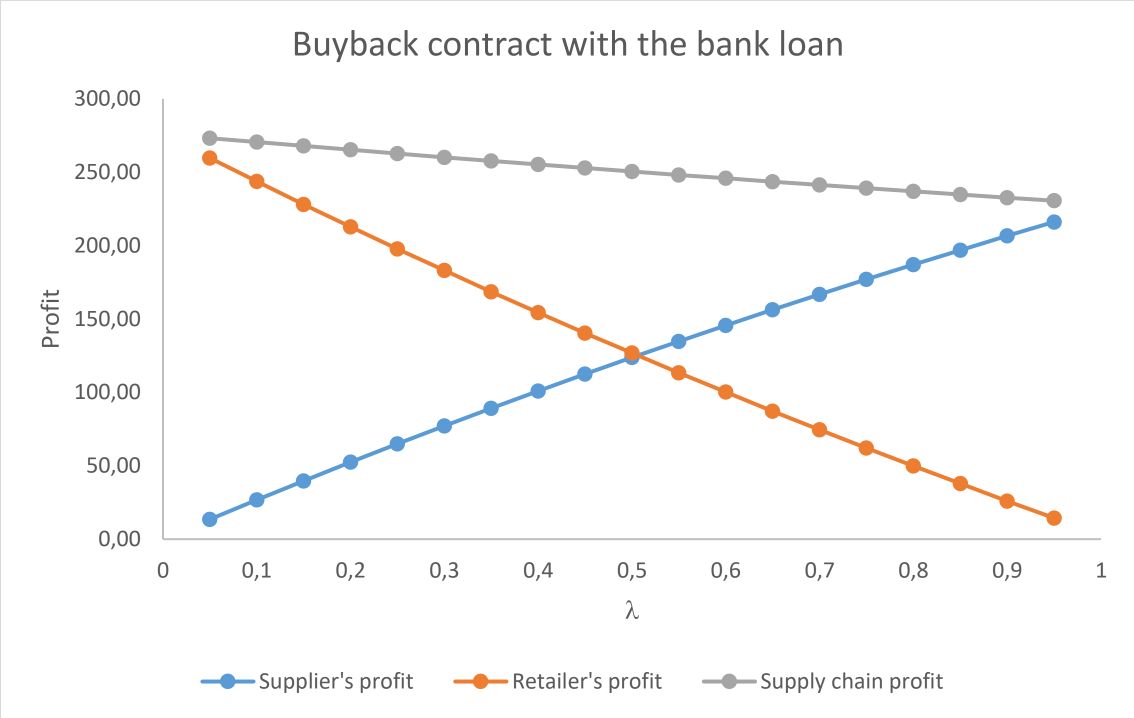
Picture 3.2. Supply chain profit with the wholesale-price and buyback contracts with trade credit and bank loan

Source: author's own

As shown in Picture 3.2, with the buyback contract both with sufficient funds and trade credit total profit remains steady at any λ, while with a bank loan it slightly declines. With both buyback and wholesale-price contracts, supply chain profits achieve similar values with the case of the retailer's sufficient funds and trade credit, thus making this type of credit more attractive in order to improve supply chain profit. Even though with buyback contract supply chain profit achieves higher levels in all three cases relative to the wholesale-price contract, it is important to mention that when supplier's share of supply chain profit is lower than 0.5, i.e. $0 < \lambda < 0{,}5$, the buyback contract does not provide conditional coordination, hence the supplier's profit is not improved compared to the wholesale-price contract and the supplier loses profit while the retailer's profit achieves its maximal values.

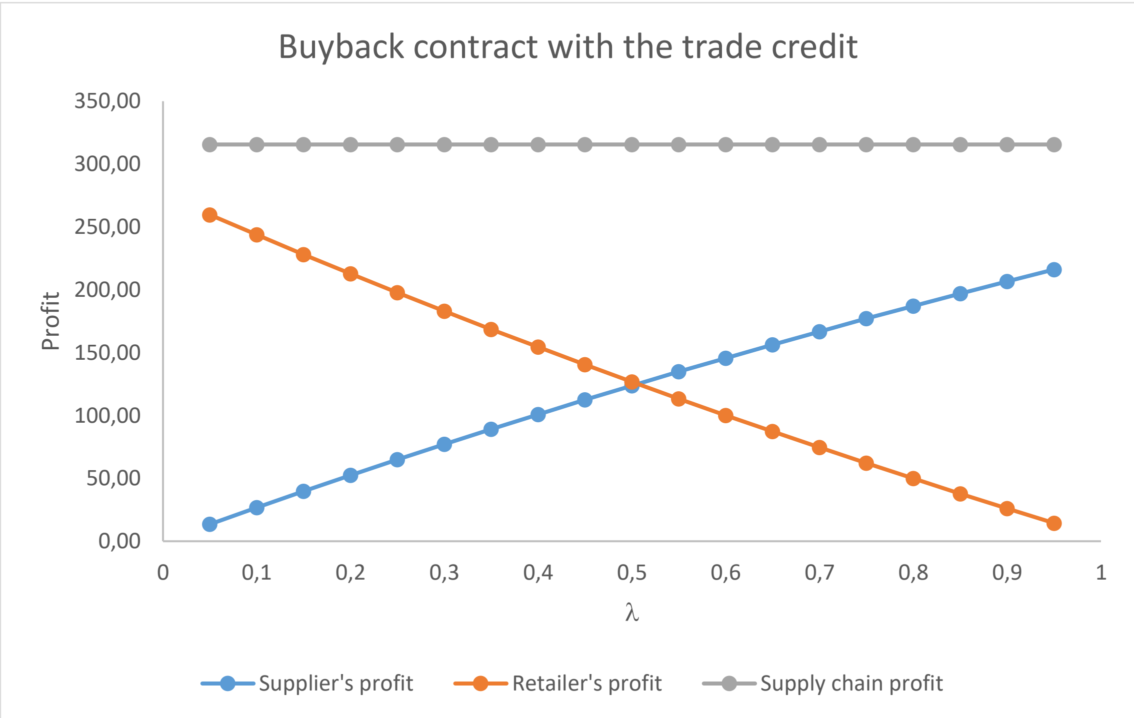
On the contrary, with the wholesale-price contract supply chain profit significantly goes down as λ grows, confirming that before λ reaches the threshold 0.5 supply chain parties should turn to the wholesale-price contract and after λ passes the threshold they should choose the buyback contract.

The preference of the trade credit over the bank loan is also shown in Pictures 3.3-3.6, where supply chain profit reaches higher levels with trade credit against the bank loan.



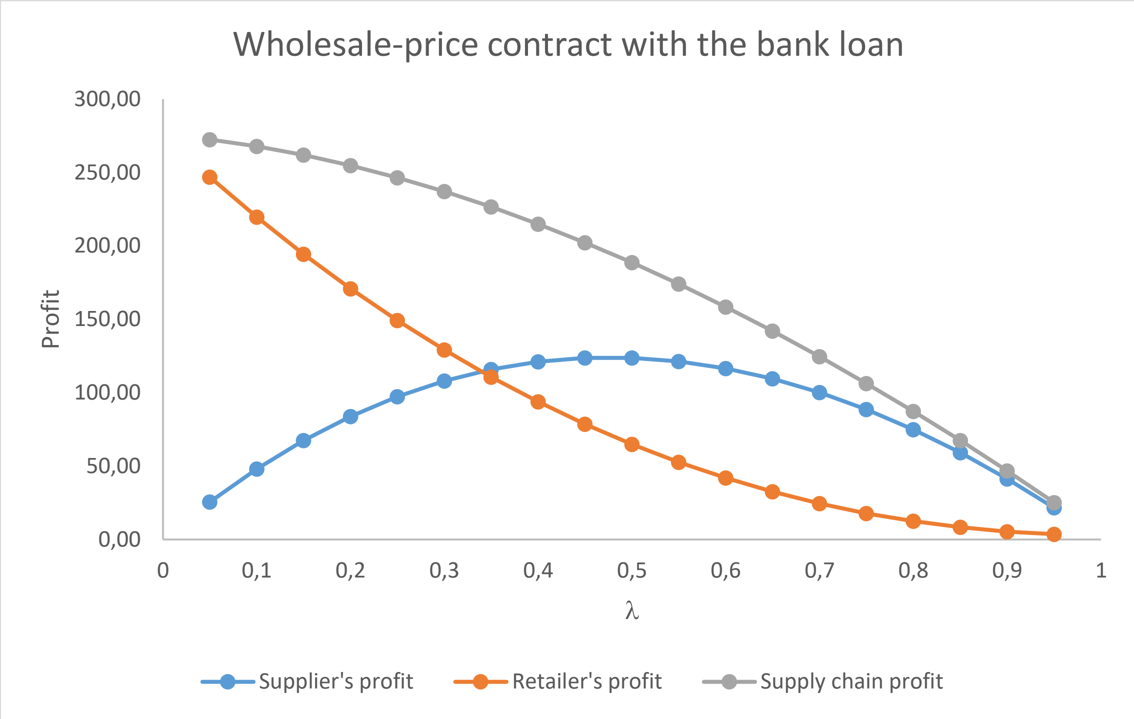
Picture 3.3. Profit allocation with the buyback contract with the bank loan

Source: author's own



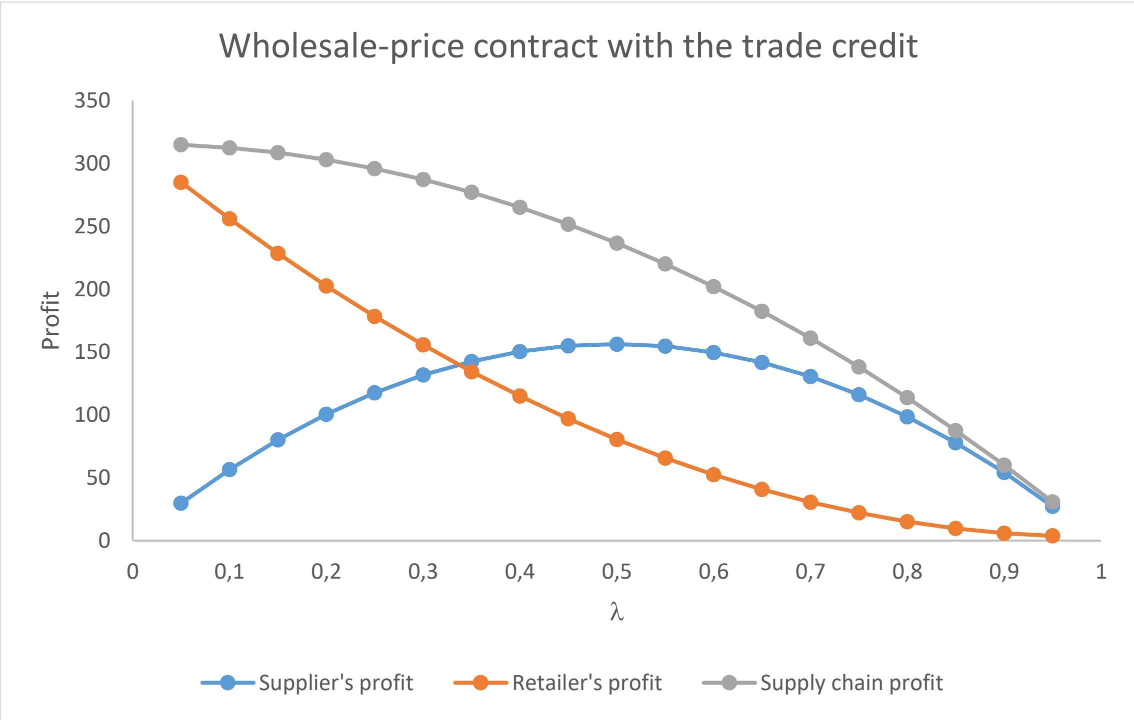
Picture 3.4. Profit allocation with the buyback contract with the trade credit

Source: author's own



Picture 3.5. Profit allocation with the wholesale-price contract with the bank loan

Source: author's own



Picture 3.6. Profit allocation with the wholesale-price contract with the trade credit

Source: author's own

Pictures 3.3-3.6 show the supplier's, retailer's and supply chain profits for any value of the supplier's profit share λ between 0 and 1 from the case study examined in Section 3.4.

As the supplier's share of total profit λ grows, her profit increases in all four charts. However, as λ reaches the threshold 0.5, supplier's profit with wholesale-price contract begins to decrease while her profit with buyback contract continues to go up. This confirms the mathematical solution proving that a conditionally coordinating buyback contract improves supplier's profit compared to the wholesale-price contract and justifies her choice of the contract type based on the value of parameter λ. It is also straightforward to confirm that trade credit allows the supplier to achieve the level of profit close to that with the retailer's sufficient funds, provided the interest rate on the trade credit meets the constraint – (3.2) for the buyback and (3.4) for the wholesale-price contract.

On the contrary, as the supplier profit share grows, retailer's profit declines with both contract types, although more dramatically with the wholesale-price contract. This provides motivation for retailer to agree upon a wholesale-price contract when λ is lower than the threshold 0.5 and enter into a buyback contract when λ reaches and then exceeds the threshold. As the retailer's profit line with sufficient funds gets close to the line related to trade credit, the conclusion made for the supplier's profit is also true for that of retailer's – with trade credit the retailer's profit improves compared with the bank loan and achieves almost the same level as in the case with the retailer's sufficient funds.

Summary of Chapter 3

This chapter is dedicated to the practical aspects of the application of the buyback contract with limited funding so as to achieve conditional coordination in the two-echelon supply chain. It focuses on the case when the demand for the product is distributed as uniformly, which is described in Section 3.1.

As shown in Chapter 2, the buyback contract that conditionally coordinates supply chain with limited funding and allows to improve the supplier's and retailer's profits can be determined both when the retailer borrows from the bank and from the supplier. But the results presented in Section 2.4 only tell about potential benefits provided to the supplier in case she offers a buyback condition to the retailer, whereas the choice of the type of credit is also crucial in the decision-making process.

To decide whether it is beneficial for the supplier to offer trade credit and what form of financing the retailer should choose, further comparison of the buyback contract with the bank loan and with the trade credit has been carried out in Section 3.2. The wholesale-price contract also has been examined in this section as supply chain parties negotiate about the wholesale-price contract when the parameter λ that is responsible for the supply chain profit allocation between them is lower than 0.5.

Comparative analysis has shown that under both contract types – wholesale-price and conditionally coordinating buyback contract – supplier's profit with trade credit is higher than with the bank loan if the trade credit rate is lower than the bank's interest rate. It is straightforward to confirm that with trade credit that the retailer's and supply chain's profits also improve relative to the bank loan. These findings are in line with the conclusions made by Kouvelis and Zhao (2012). Thus, trade credit is always preferable to the bank loan for all supply chain members – both when they enter into a conditionally coordinating buyback contract and wholesale-price contract. This conclusion is fair under certain conditions imposed on the trade credit rate, which is derived for both contract types.

Based on the mathematically justified solutions presented in Sections 2.4 and 3.2 the algorithm for selecting parameters of the buyback contract with limited funding that provide conditional coordination of a supplier-retailer supply chain is proposed in Section 3.3. This algorithm allows to select the parameters of conditionally coordinating buyback contract with limited funding based on a wholesale price they have agreed prior to the start of the selection process. The algorithm also helps to determine the parameters of the wholesale-price contract that improve the profits of supply chain members in the case when their negotiation does not arrive at the conditionally coordinating contract (when $\lambda < 0.5$).

To verify the applicability of the proposed model and algorithm, Section 3.4 examines the case of the supply chain that is engaged in manufacturing and retailing a perishable product with short shelf life. The case study demonstrates the possibility of achieving conditional coordination with the buyback contract with limited funding based on the models of the contract built for the cases with the bank loan and trade credit. The solution shows that with trade credit the supply chain profit achieves its maximum; this profit level coincides with maximal supply chain profit achieved when retailer's funds are sufficient and the issue of limited funding does not occur. It is also shown that the profits earned by the supplier and retailer in the case where the retailer accepts the offer of the trade credit exceed those earned by them in the case where the retailer takes the bank loan. Thus, it has been confirmed that trade credit is preferable to the bank loan as it allows the supply chain members to obtain higher total profit relative to the supply chain profit with the bank loan.

Application of the algorithm described in Section 3.3 showed that based on any value of the wholesale price agreed by the supplier and the retailer they can arrive at the following contracts:

- conditionally coordinating buyback contract $\big((\omega(\lambda), b(\lambda), q(\lambda)|0)\big)$,
- conditionally coordinating buyback contract with trade credit $\big((\omega(\lambda), b(\lambda), q(\lambda)|r_S)\big)$,

- conditionally coordinating buyback contract with a bank loan $\left((\omega(\lambda), b(\lambda), q(\lambda)|r_B)\right)$,
- wholesale-price contract $\left((\omega(\lambda), q(\lambda)|0)\right)$,
- wholesale-price contract with trade credit $\left((\omega(\lambda), q(\lambda)|r_S)\right)$, and
- wholesale-price contract with a bank loan $\left((\omega(\lambda), q(\lambda)|r_B)\right)$.

Thus, it has been verified that the proposed model and algorithm can support the decision-making during negotiations over the terms of the buyback contract to ensure the profit improvement achieved by both supply chain members.

CONCLUSION

The paper considers the buyback contract with limited funding. It investigates coordinating properties of this contract and aspects of its practical implementation in the case where the retailer's funds are limited and he is faced with the credit necessity.

As the study aims to build a model of the buyback contract with limited funding that allows for two forms of borrowing – bank loan and trade credit – and develop the algorithm for selecting parameters of the buyback contract with limited funding that provide conditional coordination of a supplier-retailer supply chain, the model for the supply chain with the retailer faced with limited funding is established. It suggests two types of credit to deal with the retailer's limited funds – bank loan and trade credit. Then two contract types – buyback and wholesale-price – are applied to it.

The model analysis shows that the buyback contract does not coordinate the two-echelon supply chain neither with the bank loan raised by the retailer, nor with the trade credit provided by the supplier. Coordination fails as the profits earned by the supply chain members do not achieve their maximum and, hence, the supply chain's profit as their sum is not maximized either.

As it has been mathematically justified that the buyback contract with limited funding does not have coordinating properties, the concept of conditional coordination proposed by Berezinets et al. (2020) has been applied to the model. Under conditionally coordinating buyback contract with limited funding retailer's and supply chain profits are to be maximized whereas supplier's profit must exceed her profit with the wholesale-price contract. So the model analysis included the step dedicated to building the model of the wholesale-price contract with limited funding for the cases with bank loan and trade credit.

Then the buyback contracts with bank loan and trade credit were examined to answer the question whether they can be conditionally coordinating in the presence of retailer's limited funds and credit necessity. Analysis of the model with the bank loan showed that there exists a set of conditionally coordinating buyback contracts that provides the retailer with maximal profit and improves the supplier's profit, and the conditions that are to be satisfied in order to achieve conditional coordination were identified. The model with the trade credit was analyzed with the same purpose, and the same conclusion was made – there exists a set of conditionally coordinating buyback contracts with trade credit that allow to achieve supply chain profit maximization if certain conditions are met; those conditions were identified as well. The solution also showed that with trade credit the supply chain profit achieves its maximum that coincides with maximal supply chain profit in the case where the retailer's funds are sufficient and the issue of limited funding does not occur.

Further, comparative analysis of the contract with trade credit and that with the bank loan was carried out in order to justify supplier's motivation to offer trade credit and retailer's choice between trade credit and bank loan. The wholesale-price contract also has been examined in this section as it is essential part of negotiating a conditionally coordinating buyback contract with limited funding – supplier and retailer negotiate about the conditionally coordinating buyback contract when the parameter λ that is responsible for the supply chain profit allocation between them is greater than 0.5, or about the wholesale-price contract when this parameter is lower than 0.5.

Comparative analysis has shown that under both contract types – wholesale-price and conditionally coordinating buyback contract – supplier's profit with trade credit is higher than with the bank loan if the trade credit rate is lower than the rate charged by the bank. Retailer's and supply chain's profits in this case also improve relative to the bank loan. Thus, rational supplier and retailer should always prefer the trade credit to the bank loan both when they enter into a conditionally coordinating buyback contract and wholesale-price contract. Analysis allowed to derive specific conditions for the trade credit rate that make this type of credit more attractive than the bank loan; those conditions were derived for both contract types.

Based on the mathematically justified solutions presented in the paper, the algorithm for selecting parameters of the buyback contract with limited funding was proposed. This algorithm allows to select the contract parameters based on a wholesale price they have agreed prior to the start of the selection process. The parameters of the buyback contract with limited funding that are determined with the use of the proposed algorithm provide conditional coordination of a supplier-retailer supply chain. The algorithm also helps to determine the parameters of the wholesale-price contract that improve the profits of supply chain members. Selection of the wholesale-price contract with limited funding concerns the case when the negotiation does not arrive at the conditionally coordinating contract (when $\lambda < 0.5$).

Consequently, the proposed model and algorithm were applied to the case of the supply chain that is engaged in manufacturing and retailing a perishable product with short shelf life. Application of the model and algorithm to a practical case aimed to verify their applicability.

The case study considered the models of the buyback contract built for the cases with the bank loan and trade credit, and demonstrated that conditional coordination with the buyback contract with limited funding could be achieved. It also showed that profits earned by the supplier and the retailer in the case where the retailer accepts the offer of the trade credit exceed those earned by them in the case where the retailer takes the bank loan. Thus, it was confirmed that trade credit is preferable to the bank loan as it allows the supply chain members to obtain higher total profit relative to the supply chain profit with the bank loan.

Application of the algorithm for selecting the contract parameters to the case illustrated how supplier and retailer can arrive at the contracts that are beneficial for both of them and improve total gain after they have agreed on some value of the wholesale price. Using the algorithm supplier and retailer can ensure the profit improvement and result in the following contracts depending on the mathematical justification and decisions made by both of them:

- conditionally coordinating buyback contract $\left((\omega(\lambda), b(\lambda), q(\lambda)|0)\right)$,
- conditionally coordinating buyback contract with trade credit $\left((\omega(\lambda), b(\lambda), q(\lambda)|r_S)\right)$,
- conditionally coordinating buyback contract with the bank loan $\left((\omega(\lambda), b(\lambda), q(\lambda)|r_B)\right)$,
- wholesale-price contract $\left((\omega(\lambda), q(\lambda)|0)\right)$,
- wholesale-price contract with trade credit $\left((\omega(\lambda), q(\lambda)|r_S)\right)$, and
- wholesale-price contract with the bank loan $\left((\omega(\lambda), q(\lambda)|r_B)\right)$.

To sum up, results obtained in the study provide both theoretical contribution and practical implication for practitioners in the field of finance. Theoretical contribution includes the following results:

- The models of the buyback and wholesale-price contracts with limited funding are constructed for a supplier-retailer supply chain;
- It is found that the buyback contract with limited funding does not coordinate the supply chain, but allows to achieve its conditional coordination according to the definitions of coordinating and conditionally coordinating contracts applied in the paper;
- With both wholesale-price and buyback contract, trade credit proves to be preferable to the bank loan in the case of retailer's limited funding as it provides the supply chain members with higher profits relative to those earned by them with the bank loan.

Practical implications of the study are as follows:

- The models of the conditionally coordinating buyback contract and wholesale-price contract with limited funding constructed for the case of demand distributed as uniformly;

- The algorithm for selecting the parameters of the buyback contract with limited funding is proposed, it allows to achieve conditional coordination with the buyback contract and improves total profit with the wholesale-price contract in the supplier-retailer supply chain considering the type of credit chosen by the retailer (trade credit or bank loan).

Thus, the tasks formulated in Introduction have been accomplished and, hence, the established goal of the study has been achieved.

Results obtained in the study are primarily addressed to specialists of finance departments engaged in the company's contract management. This is particularly relevant to the companies involved in the supply chains for the short life cycle products, such as fashion and grocery retail, publishing, seasonal products, and consumer electronics and corresponding accessories.

However, short life cycle of the product and complexity of the supply chains often connected with cross-border activities requires timely and efficient logistics, quality control and sharing information apart from coordination efforts. So the supply chains often include 3PL (third-party logistics) providers that nowadays seek to provide comprehensive services to their clients and not just logistics services like transportation and warehousing.

3PL providers' services include financial services that concern the financing of supply chain members faced with limited funding. So the algorithm proposed in this body of research can be addressed to them as a tool to support the decision-making during the negotiation between their clients – the supply chain members.

Besides, today 3PL providers may act as mediators in the interaction between the supply chain members and the bank or as creditors by providing the trade credit to the supply chain members. Mediating activity of 3PL providers includes consulting and control services as they often have better risk assessment, risk monitoring and organization capabilities, and also provide guarantee for the supply chain parties to raise the bank loan (Chakuu, Masi, Godsell, 2020; Wang et al., 2022). So investigating the buyback contract with limited funding with a 3PL provider involved in the supply chain represents an exciting opportunity for future research that will deal with the limitations of the results obtained in this paper. A brief overview of this direction of research in supply chain contracting shows that buyback contract has not been investigated, while there are examples considering revenue-sharing and wholesale-price contracts (Cai et al., 2013; Shen, Xu, Guo, 2019; Cao et al., 2023).

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REFERENCES

1. 'Contract' (n. d.) *Legal Information Institute*. Available from: https://www.law.cornell.edu/wex/contract [Accessed on 26 April 2023]
2. 'New Research Forecasts the State of U.S. Supply Chains in 2023' (2022, 24 Oct.) *SAP News*. Available from: https://news.sap.com/2022/10/us-supply-chains-in-2023-new-research-forecast/ [Accessed on 27 March 2023]
3. Arshinder A. K. & S.G. Deshmukh (2008) 'Supply Chain Coordination: Perspectives, Empirical Studies and Research Directions'. *International Journal of Production Economics*, 115 (2): 316-335. Retrieved from: Elsevier [Accessed on 22 May 2020]
4. Babich V., Li H., Ritchken P., Wang Y. (2012) 'Contracting with Asymmetric Demand Information in Supply Chains'. *European Journal of Operational Research*, 217 (2): 333-341. Retrieved from: Elsevier [Accessed on 22 May 2020]
5. Berezinets I. V., Zenkevich N. A., Rucheva A. S. & N. K. Nikolchenko (2020). 'Do the Buyback Contracts of Undistributed Stocks Coordinate the Supply Chain?' (in Russian). *Vestnik of Saint Petersburg University. Management* [Online], 19 (4): 461–492.
6. Berezinets, I., Meshkova, M. & N. Nikolchenko (2019) 'The Problem of Supply Chain Profit Maximization Using Sales Rebate Contract'. *Contributions to Game Theory and Management* [Online], 12: 70-99.
7. Berezinets, I., Voronova, T., Zenkevich, N., Nikolchenko, N. (2022) 'Coordinating Contracts as an Instrument of Supply Chain Profit Maximization under Short-Term Financing Necessity'. *Contributions to Game Theory and Management* [Online], 13: 57-94.
8. Bose, I., & P. Anand (2007). 'On Returns Policies with Exogenous Price'. *European Journal of Operational Research*, 178(3), 782–788. Retrieved from: Elsevier [Accessed on 28 May 2023]
9. Cachon G. P., Lariviere M. (2005) 'Supply Chain Coordination with Revenue-sharing Contracts: Strengths and Limitations'. *Management Science*, 51: 30–44. Available from: https://repository.upenn.edu/cgi/viewcontent.cgi?article=1129&context=oid_papers [Accessed on 17 April 2023]
10. Cachon, G. P. & M. Fisher (2000) 'Supply Chain Inventory Management and the Value of Shared Information'. *Management science*, 46 (8): 1032-1048. Available from: https://faculty.wharton.upenn.edu/wp-content/uploads/2012/04/Cachon_fisher_ms.pdf [Accessed on 25 May 2023]
11. Cachon, G. P. (2003) 'Supply Chain Coordination with Contracts'. In: Graves, S. & T. de Kok (Eds.) *Supply Chain Management: Design, Coordination and Operation*, 11: 227-339. Retrieved from: Elsevier [Accessed on 21 May 2020]
12. Cachon, G.P., Netessine, S. (2004). 'Game Theory in Supply Chain Analysis'. In: Simchi-Levi, D., Wu, S.D., Shen, ZJ. (eds.) *Handbook of Quantitative Supply Chain Analysis*. *International Series in Operations Research & Management Science*, 74. Springer, Boston, MA. Available from:

https://studylib.net/doc/8697096/game-theory-in-supply-chain-analysis [Accessed on 22 May 2023]

13. Cai, X., Chen, J., Xiao, Y., Xu, X., & Yu, G. (2013) 'Fresh-product Supply Chain Management with Logistics Outsourcing'. *Omega*, 41 (4): 752-765. Retrieved from: Elsevier [Accessed on 26 May 2023]
14. Cao, G., Wang, Y., Gao, H., Liu, H., Liu, H., Song, Z., & Fan, Y. (2023) 'Coordination Decision-Making for Intelligent Transformation of Logistics Services under Capital Constraint'. *Sustainability*, 15 (6), 5421. Retrieved from: MDPI [Accessed on 26 May 2023]
15. Chakuu, S., Masi, D., & Godsell, J. (2020) 'Towards a Framework on the Factors Conditioning the Role of Logistics Service Providers in the Provision of Inventory Financing'. *International Journal of Operations & Production Management*, 40 (7/8): 1225-1241. Retrieved from: Emerald [Accessed on 26 May 2023]
16. Chen X. (2015) 'A Model of Trade Credit in a Capital-constrained Distribution Channel'. *International Journal of Production Economics*, 159: 347-357. Retrieved from: Elsevier [Accessed on 31 May 2020]
17. Christopher, M. (2011) *Logistics & Supply Chain Management* (4th Edition). Pearson Education Limited (2011): p. 13. Harlow: Pearson Education.
18. Davis-Sramek, B., Germain, R. & K. Krotov (2015) 'Examining The Process R&D Investment–Performance Chain in Supply Chain Operations: The Effect of Centralization'. *International Journal of Production Economics*, 167: 246–256. Retrieved from: Elsevier [Accessed on 24 May 2023]
19. Fu Y., Liu F. (2019) 'The Coordination of a Capital-Constrained Supply Chain with a Risk-Averse Retailer'. In: *2019 8th International Conference on Industrial Technology and Management (ICITM)*. Cambridge, United Kingdom: pp. 1-6. Retrieved from: IEEE Xplore [Accessed on 1 June 2020]
20. Giannoccaro, I. & P. Pontrandolfo (2004). 'Supply Chain Coordination by Revenue Sharing Contracts'. *International Journal of Production Economics*, 89: 131-139. Retrieved from: Elsevier [Accessed on 22 November 2020]
21. Giannoccaro, I. (2018) 'Centralized vs. Decentralized Supply Chains: The Importance of Decision Maker's Cognitive Ability and Resistance to Change'. *Industrial Marketing Management*, 73: 59-69. Retrieved from: Elsevier [Accessed on 19 November 2020]
22. Granot, D., Yin, S. (2005) 'On the Effectiveness of Returns Policies in the Price Dependent Newsvendor Model'. *Naval Research Logistics*, 52 (8): 765–778. Available from: https://escholarship.org/uc/item/327118sw [Accessed on 28 May 2023]
23. Heydari, J., Choi, T. M., & Radkhah, S. (2017). 'Pareto Improving Supply Chain Coordination under a Money-back Guarantee Service Program'. *Service Science* [Online], 9(2), 91-105.

24. Kagan, J. (2022, 17 July) 'Trade Credit'. *Investopedia.* Available from: https://www.investopedia.com/terms/t/trade-credit.asp [Accessed on 27 March 2023]
25. Kouvelis P., Zhao W. (2012). 'Financing the Newsvendor: Supplier vs. Bank, and the Structure of Optimal Trade Credit Contracts'. *Operations Research*, 60 (3): 566-580. Retrieved from: Elsevier [Accessed on 29 November 2022]
26. Kouvelis P., Zhao W. (2015) 'Supply Chain Contract Design Under Financial Constraints and Bankruptcy Costs'. *Management Science* [Online], 62 (8): 2341-2357.
27. Lee, H. L., Padmanabhan, V. & S. Whang (1997) 'The Bullwhip Effect in Supply Chains'. *MIT Sloan Management Review* [Online], 38 (3): 92-102.
28. Liu L., Parlar M. & S.X. Zhu (2007) 'Pricing and Lead Time Decisions in Decentralized Supply Chains' *Management Science* [Online], Volume 53(5): 713–725.
29. Malone T., Crowston K. (1994) 'The Interdisciplinary Study of Coordination'. *ACM Computing Surveys*, 26 (1) (1994): 87-119. Available from: https://dspace.mit.edu/bitstream/handle/1721.1/2500/SWP-3630-31204001-CCS-157.pdf?sequence=1 [Accessed on 23 November 2020]
30. Mehren, A. T. von (2022, 30 Nov.) 'Contract'. *Encyclopedia Britannica*. Available from: https://www.britannica.com/money/topic/contract-law [Accessed on 26 April 2023]
31. Moon I., Feng X.-H., Ryu K.-Y. (2015) 'Channel Coordination for Multi-stage Supply Chains with Revenue-sharing Contracts under Budget Constraints'. *International Journal of Production Research*, 53 (16): 4819-4836. Available from: http://scm.snu.ac.kr/publication/paper/80.pdf [Accessed on 28 May 2023]
32. Pasternack B. A. (1985) 'Optimal Pricing and Return Policies for Perishable Commodities'. *Marketing Science*, 4 (2): 166-176. Available from: https://courses.ie.bilkent.edu.tr/ie460/wp-content/uploads/sites/12/2019/02/Pasternack-1985-MS.pdf [Accessed on 28 May 2023]
33. Saha, S. (2013). Supply Chain Coordination through Rebate Induced Contracts. Transportation Research Part E: Logistics and Transportation Review, 50 (1), 120-137. Retrieved from: Elsevier [Accessed on 28 May 2023]
34. Shen, B., Li, Q. (2015) 'Impacts of Returning Unsold Products in Retail Outsourcing Fashion Supply Chain: A Sustainability Analysis'. *Sustainability*, 7: 1172-1185. Retrieved from: MDPI [Accessed on 26 May 2023]
35. Shen, B., Xu, X., & Guo, S. (2019) 'The Impacts of Logistics Services on Short Life Cycle Products in a Global Supply Chain'. *Transportation Research Part E: Logistics and Transportation Review*, 131: 153-167. Retrieved from: Elsevier [Accessed on 26 May 2023]
36. Taylor, T. A. (2002) 'Supply Chain Coordination under Channel Rebates with Sales Effort Effects'. *Management science*, 48(8): 992-1007.

37. Tsay A. A. (2002) ‘Risk Sensitivity in Distribution Channel Partnerships: Implications for Manufacturer Return Policies’. *Journal of Retailing*, 78 (2): 147-160. Retrieved from: Elsevier [Accessed on 22 May 2020]
38. Tsay, A. (1999) ‘The Quantity Flexibility Contract and Supplier-Customer Incentives’. *Management Science*, 45(10): 1339-1358.
39. Tsay, A. A. (2001). ‘Managing Retail Channel Overstock: Markdown Money and Return Policies’. *Journal of Retailing*, 77(4), 457–492. Retrieved from: ResearchGate [Accessed on 28 May 2023]
40. Tsay, A. A., Nahmias, S., & N. Agrawal (1999). ‘Modeling Supply Chain Contracts: A Review’. In: Tayur, S., Ganeshan, R. & M. Magazine (Eds.) *Quantitative models for supply chain management*. MA: Kluwer Academic Publishers. pp. 299-336.
41. Wang, X., Zhao, J., Zhang, H., Tang, X. (2022) ‘The Impact of the Scale of Third-Party Logistics Guaranteeing Firms on Bank Credit Willingness in Supply Chain Finance: An ERP Study’. *Frontiers in Psychology* [Online], 13. Available from: https://www.frontiersin.org/articles/10.3389/fpsyg.2022.853888/full#ref10 [Accessed on 26 May 2023]
42. Wu D. (2013) ‘Coordination of Competing Supply Chains with Newsvendor and Buyback Contract’. *International Journal of Production Economics*, 144 (1): 1-13. Retrieved from: Elsevier [Accessed on 22 May 2020]
43. Xiao S., Sethi S.P., Liu M., Ma S. (2017) ‘Coordinating Contracts for a Financially Constrained Supply Chain’. *Omega*, 72: 71-86. Retrieved from: Elsevier [Accessed on 1 June 2020]
44. Zenkevich, N. & M. Gladkova (2018) ‘Revenue Sharing Contracts for Value Chain Coordination: The Case of Motion Picture Industry in the USA’ (in Russian). *Vestnik of Saint Petersburg University. Management* [Online], 17 (1): 26-45.
45. Zhan J., Chen X., Hu Q. (2018) ‘The Value of Trade Credit with Rebate Contract in a Capital-constrained Supply Chain’. *International Journal of Production Research* 57 (2): 1-18. Retrieved from: ResearchGate [Accessed on 1 June 2020]
46. Zhao Y., Choi T.-M., Cheng T.C.E., Sethi S. P., Wang S. (2014) ‘Buyback Contracts with Price-dependent Demands: Effects of Demand Uncertainty’. *European Journal of Operational Research*, 239 (3): 663-673. Retrieved from: Elsevier [Accessed on 28 May 2023]
47. Zou X., Pokharel S., Piplani R. (2008) ‘A Two-period Supply Contract Model for a Decentralized Assembly System’. *European Journal of Operational Research*, 187: 257-274. Retrieved from: Elsevier [Accessed on 22 May 2020]

APPENDICES

A1. Supplier's, retailer's and supply chain's profits and expected profits with the bank loan

$$Prof_S^{BB} = (\omega - c_S)q - b(q-\tau) = b\tau + (\omega - c_S - b)q$$

$$\begin{aligned} Prof_R^{BB} &= p\tau + (b+\nu)(q-\tau) + \max\{(\omega + c_R)q - K_R\,,0\} - (\omega + c_R)q \\ &\quad - \max\{(\omega + c_R)q - K_R\,,0\} * (1 + r_B) \\ &= p\tau + (b+\nu)(q-\tau) - (\omega + c_R)q - r_B * \max\{(\omega + c_R)q - K_R\,,0\} \\ &= p\tau + (b+\nu)q - (b+\nu)\tau - (\omega + c_R)q - r_B * \max\{(\omega + c_R)q - K_R\,,0\} \\ &= (p-b-\nu)\tau + (b+\nu-\omega-c_R)q - r_B * \max\{(\omega + c_R)q - K_R\,,0\} \end{aligned}$$

$$\begin{aligned} Prof_{SC}^{BB} &= Prof_S + Prof_R \\ &= b\tau + (\omega - c_S - b)q + (p-b-\nu)\tau + (b+\nu-\omega-c_R)q - r_B \\ &\quad * max\{(\omega + c_R)q - K_R\,,0\} \\ &= (p-\nu)\tau + (\nu - c)q - r_B * max\{(\omega + c_R)q - K_R\,,0\} \end{aligned}$$

$$\begin{aligned} \pi_S^{BB} &= E[Prof_S^{BB}] = b * E[\tau] + (\omega - c_S - b)q = b\left(q - \int_0^q F_\xi(x)\,dx\right) + (\omega - c_S - b)q \\ &= (\omega - c_S)q - b\int_0^q F_\xi(x)\,dx \end{aligned}$$

$$\begin{aligned} \pi_R^{BB} &= E[Prof_R^{BB}] = (p-b-\nu)E[\tau] + (b+\nu-\omega-c_R)q - r_B * \max\{(\omega + c_R)q - K_R\,,0\} \\ &= (p-b-\nu)\left(q - \int_0^q F_\xi(x)\,dx\right) + (b+\nu-\omega-c_R)q - r_B \\ &\quad * \max\{(\omega + c_R)q - K_R\,,0\} \\ &= (p-b-\nu)q + (b+\nu-\omega-c_R)q - (p-b-\nu)\int_0^q F_\xi(x)\,dx - r_B \\ &\quad * \max\{(\omega + c_R)q - K_R\,,0\} \\ &= (p-\omega-c_R)q + (b+\nu-p)\int_0^q F_\xi(x)\,dx - r_B * \max\{(\omega + c_R)q - K_R\,,0\} \end{aligned}$$

$$\pi_{SC}^{BB} = E[Prof_{SC}^{BB}] = (p-\nu)E[\tau] + (\nu - c)q - r_B * max\{(\omega + c_R)q - K_R\ , 0\}$$

$$= (p-\nu)\left(q - \int_0^q F_\xi(x)\,dx\right) + (\nu - c)q - r_B * max\{(\omega + c_R)q - K_R\ , 0\}$$

$$= (p-c)q + (\nu - p)\int_0^q F_\xi(x)\,dx - r_B * max\{(\omega + c_R)q - K_R\ , 0\}$$

A2. Determining a coordinating buyback contract with the bank loan

Step 1:

$$\frac{\partial \pi_R^{BB}}{\partial q} = p - (\omega + c_R)(1 + r_B) + (b + \nu - p)F_\xi(q) = 0 \Rightarrow F_\xi(q) = \frac{p - (\omega + c_R)(1 + r_B)}{p - b - \nu}$$

$$\Rightarrow\ q_R^0 = F_\xi^{-1}\left(\frac{p - (\omega + c_R)(1 + r_B)}{p - b - \nu}\right)$$

$$\frac{\partial^2 \pi_R^{BB}}{\partial q^2} = -(p - b - \nu)f_\xi(q) < 0\ if\ b < p - \nu \Rightarrow\ q_R^0 = q_R^*$$

Step 2:

$$\frac{\partial \pi_{SC}^{BB}}{\partial q} = p - c - r_B(\omega + c_R) + (\nu - p)F_\xi(q) = 0 \Rightarrow F_\xi(q) = \frac{p - c - r_B(\omega + c_R)}{p - \nu} \Rightarrow\ q_{SC}^0$$

$$= F_\xi^{-1}\left(\frac{p - c - r_B(\omega + c_R)}{p - \nu}\right)$$

$$\frac{\partial^2 \pi_{SC}^{BB}}{\partial q^2} = -(p - \nu)f_\xi(q) < 0 \Rightarrow q_{SC}^0 = q_{SC}^*$$

Step 3:

Consider $q_{SC}^* = q_R^*$: $F_\xi^{-1}\left(\frac{p-c-r_B(\omega+c_R)}{p-\nu}\right) = F_\xi^{-1}\left(\frac{p-(\omega+c_R)(1+r_B)}{p-b-\nu}\right)$

Distribution function $F_\xi(x)$ is increasing and values of its inverse function are equal, so their arguments are also equal:

$$\frac{p - c - r_B(\omega + c_R)}{p - \nu} = \frac{p - (\omega + c_R)(1 + r_B)}{p - b - \nu}$$

$$\big(p - c - r_B(\omega + c_R)\big)(p - b - \nu) = \big(p - (\omega + c_R)(1 + r_B)\big)(p - \nu)$$

$$\big(p - c - r_B(\omega + c_R)\big)(p - \nu) - b\big(p - c - r_B(\omega + c_R)\big) = \big(p - (\omega + c_R)(1 + r_B)\big)(p - \nu)$$

$$\big(p - c - r_B(\omega + c_R) - p + (\omega + c_R)(1 + r_B)\big)(p - \nu) = b\big(p - c - r_B(\omega + c_R)\big)$$

$$(\omega - c - c_R)(p - \nu) = b\big(p - c - r_B(\omega + c_R)\big)$$

$$(\omega - c_S)(p - \nu) = b\big(p - c - r_B(\omega + c - c_S)\big)$$

$$(\omega - c_S)(p - \nu) = b(p - c - r_B(\omega - c_S) - cr_B)$$

$$(\omega - c_S)(p - \nu + br_B) = b(p - c - cr_B)$$

$$\omega - c_S = b\frac{p - c - cr_B}{p - \nu + br_B}$$

$$\omega^* = b\frac{p - c - cr_B}{p - \nu + br_B} + c_S = \frac{b(p - c) + c(p - \nu)}{p - \nu + br_B} - c_R$$

Step 4:

$$\pi_S^{BB} = (\omega^* - c_S)q^* - b\int_0^{q^*} F_\xi(x)\,dx = \left(b\frac{p - c - cr_B}{p - \nu + br_B} + c_S - c_S\right)q^* - b\int_0^{q^*} F_\xi(x)\,dx$$

$$= b\frac{p - c - cr_B}{p - \nu + br_B}q^* - b\int_0^{q^*} F_\xi(x)\,dx$$

$$\frac{\partial\pi_S^{BB}}{\partial b} = -\int_0^{q^*} F_\xi(x)\,dx + \frac{q^*(p - c - cr_B)(p - \nu + br_B) - bq^*(p - c - cr_B)r_B}{(p - \nu + br_B)^2} =$$

$$= -\int_0^{q^*} F_\xi(x)\,dx + \frac{q^*(p - c - cr_B)(p - \nu + br_B - br_B)}{(p - \nu + br_B)^2}$$

$$= -\int_0^{q^*} F_\xi(x)\,dx + \frac{q^*(p - c - cr_B)(p - \nu)}{(p - \nu + br_B)^2} = 0$$

The model's assumptions state that $q > 0$ and its conditions state that $p - \nu > 0$ and $p - c - cr_B > 0$, and $(p - \nu + br_B)^2 > 0$ with any values of the parameters, hence the second term of the formula above is positive. Assume the first derivative of the function of the supplier's expected profit with respect to b is equal to $\frac{q^*(p-c-cr_B)(p-\nu)}{(p-\nu+br_B)^2}$, then the stationary point b^0 exists and can be found:

$$\int_0^{q^*} F_\xi(x)\,dx = \frac{q^*(p - c - cr_B)(p - \nu)}{(p - \nu + br_B)^2}$$

$$(p - \nu + br_B)^2 = \frac{q^*(p - c - cr_B)(p - \nu)}{\int_0^{q^*} F_\xi(x)\,dx}$$

$$p - \nu + br_B = \sqrt{\frac{q^*(p - c - cr_B)(p - \nu)}{\int_0^{q^*} F_\xi(x)\,dx}} \Rightarrow\ b^0 = \frac{\sqrt{\frac{q^*(p - c - cr_B)(p - \nu)}{\int_0^{q^*} F_\xi(x)\,dx}} + \nu - p}{r_B}$$

$$\frac{\partial^2\pi_S^{BB}}{\partial b^2}=\frac{-2q^*(p-c-cr_B)(p-v)(p-v+br_B)r_B}{(p-v+br_B)^3}=-\frac{2q^*(p-c-cr_B)(p-v)r_B}{(p-v+br_B)^2}<0$$

$$\Rightarrow b^0=b^*=\frac{\sqrt{\frac{q^*(p-c-cr_B)(p-v)}{\int_0^{q^*}F_\xi(x)\,dx}}+v-p}{r_B}$$

Since the second derivative of the function of supplier's expected profit with respect to b is negative at the stationary point b^0 (all factors in the numerator and denominator are positive), the stationary point b^0 is the local point for maximum point b^* for the function π_S^{BB}.

A3. Supplier's, retailer's and supply chain's profits and expected profits with the trade credit

$$\begin{aligned}Prof_S^{BB}&=(\omega-c_S)q-b(q-\tau)+r_S*\max\{(\omega+c_R)q-K_R\,,0\}\\&=b\tau+(\omega-c_S-b)q+r_S*\max\{(\omega+c_R)q-K_R\,,0\}\end{aligned}$$

$$\begin{aligned}Prof_R^{BB}&=p\tau+(b+v)(q-\tau)-(\omega+c_R)q+\max\{(\omega+c_R)q-K_R\,,0\}\\&\quad-\max\{(\omega+c_R)q-K_R\,,0\}*(1+r_S)\\&=p\tau+(b+v)(q-\tau)-(\omega+c_R)q-r_S*\max\{(\omega+c_R)q-K_R\,,0\}\\&=p\tau+(b+v)q-(b+v)\tau-(\omega+c_R)q-r_S*\max\{(\omega+c_R)q-K_R\,,0\}\\&=(p-b-v)\tau+(b+v-\omega-c_R)q-r_S*\max\{(\omega+c_R)q-K_R\,,0\}\end{aligned}$$

$$\begin{aligned}Prof_{SC}^{BB}&=Prof_S+Prof_R\\&=b\tau+(\omega-c_S-b)q+r_S*max\{(\omega+c_R)q-K_R\,,0\}+(p-b-v)\tau\\&\quad+(b+v-\omega-c_R)q-r_S*\max\{(\omega+c_R)q-K_R\,,0\}=(p-v)\tau+(v-c)q\end{aligned}$$

$$\begin{aligned}\pi_S^{BB}&=E[Prof_S^{BB}]=b*E[\tau]+(\omega-c_S-b)q+r_S*\max\{(\omega+c_R)q-K_R\,,0\}\\&=b\left(q-\int_0^q F_\xi(x)\,dx\right)+(\omega-c_S-b)q+r_S*\max\{(\omega+c_R)q-K_R\,,0\}\\&=(\omega-c_S)q-b\int_0^q F_\xi(x)\,dx+r_S*\max\{(\omega+c_R)q-K_R\,,0\}\end{aligned}$$

$$\pi_R^{BB} = E[Prof_R^{BB}] = (p-b-\nu)E[\tau] + (b+\nu-\omega-c_R)q - r_S * \max\{(\omega+c_R)q - K_R\,, 0\}$$

$$= (p-b-\nu)\left(q - \int_0^q F_\xi(x)\,dx\right) + (b+\nu-\omega-c_R)q - r_S$$

$$* \max\{(\omega+c_R)q - K_R\,, 0\}$$

$$= (p-b-\nu)q + (b+\nu-\omega-c_R)q - (p-b-\nu)\int_0^q F_\xi(x)\,dx - r_S$$

$$* \max\{(\omega+c_R)q - K_R\,, 0\}$$

$$= (p-\omega-c_R)q + (b+\nu-p)\int_0^q F_\xi(x)\,dx - r_B * \max\{(\omega+c_R)q - K_R\,, 0\}$$

$$\pi_{SC}^{BB} = E[Prof_{SC}^{BB}] = (p-\nu)E[\tau] + (\nu-c)q = (p-\nu)\left(q - \int_0^q F_\xi(x)\,dx\right) + (\nu-c)q$$

$$= (p-c)q + (\nu-p)\int_0^q F_\xi(x)\,dx$$

A4. Determining a coordinating buyback contract with trade credit

$$\pi_S^{BB} = (\omega-c_S)q - b\int_0^q F_\xi(x)\,dx + r_S\big((\omega+c_R)q - K_R\big)$$

$$= \big(\omega - c_S + r_S(\omega+c_R)\big)q - b\int_0^q F_\xi(x)\,dx - r_S K_R$$

$$\pi_R^{BB} = (p-\omega-c_R)q + (b+\nu-p)\int_0^q F_\xi(x)\,dx - r_S\big((\omega+c_R)q - K_R\big)$$

$$= \big(p-\omega-c_R - r_S(\omega+c_R)\big)q + (b+\nu-p)\int_0^q F_\xi(x)\,dx + r_S K_R$$

$$= \big(p-(1+r_S)(\omega+c_R)\big)q + (b+\nu-p)\int_0^q F_\xi(x)\,dx + r_S K_R$$

$$\pi_{SC}^{BB} = (p-c)q + (\nu-p)\int_0^q F_\xi(x)\,dx$$

Step 1:

$$\frac{\partial \pi_R^{BB}}{\partial q} = p - (\omega + c_R)(1 + r_S) + (b + \nu - p)F_\xi(q) = 0 \Rightarrow F_\xi(q) = \frac{p - (\omega + c_R)(1 + r_S)}{p - b - \nu}$$

$$\Rightarrow\ q_R^0 = F_\xi^{-1}\left(\frac{p - (\omega + c_R)(1 + r_S)}{p - b - \nu}\right)$$

$$\frac{\partial^2 \pi_R^{BB}}{\partial q^2} = -(p - b - \nu)f_\xi(q) < 0\ if\ b < p - \nu \Rightarrow\ q_R^0 = q_R^*$$

Step 2:

$$\frac{\partial \pi_{SC}^{BB}}{\partial q} = p - c + (\nu - p)F_\xi(q) = 0 \Rightarrow F_\xi(q) = \frac{p - c}{p - \nu} \Rightarrow\ q_{SC}^0 = F_\xi^{-1}\left(\frac{p - c}{p - \nu}\right)$$

$$\frac{\partial^2 \pi_{SC}^{BB}}{\partial q^2} = -(p - \nu)f_\xi(q) < 0 \overset{\nu<p}{\Longrightarrow} q_{SC}^0 = q_{SC}^*$$

Step 3:

Consider $q_{SC}^* = q_R^*$: $F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right) = F_\xi^{-1}\left(\frac{p-(\omega+c_R)(1+r_S)}{p-b-\nu}\right)$

Distribution function $F_\xi(x)$ is increasing and values of its inverse function are equal, hence their arguments are also equal:

$$\frac{p - c}{p - \nu} = \frac{p - (\omega + c_R)(1 + r_S)}{p - b - \nu}$$

$$(p - c)(p - b - \nu) = \big(p - (\omega + c_R)(1 + r_S)\big)(p - \nu)$$

$$(p - \nu)(p - c) - b(p - c) = \big(p - (\omega + c_R)(1 + r_S)\big)(p - \nu)$$

$$(p - \nu)(p - c) - (p - \nu)\big(p - (\omega + c_R)(1 + r_S)\big) = b(p - c)$$

$$(p - \nu)\big(p - c - p + (\omega + c_R)(1 + r_S)\big) = b(p - c)$$

$$(p - \nu)\big((\omega + c_R)(1 + r_S) - c\big) = b(p - c)$$

$$(\omega + c_R)(1 + r_S) - c = b\frac{p - c}{p - \nu}$$

$$(\omega + c_R)(1 + r_S) = b\frac{p - c}{p - \nu} + c$$

$$\omega + c_R = \frac{b(p - c) + c(p - \nu)}{(p - \nu)(1 + r_S)}$$

$$\omega^* = \frac{b(p - c) + c(p - \nu)}{(p - \nu)(1 + r_S)} - c_R = b\frac{p - c}{(p - \nu)(1 + r_S)} + \frac{c}{1 + r_S} - c_R$$

Step 4:

$$\pi_S^{BB} = \big(\omega^* - c_S + r_S(\omega^* + c_R)\big)q^* - b\int_0^{q^*} F_\xi(x)\,dx - r_S K_R$$

$$= \left(b\frac{p-c}{(p-\nu)(1+r_S)} + \frac{c}{1+r_S} - c_R - c_S\right.$$

$$\left. + r_S\left(b\frac{p-c}{(p-\nu)(1+r_S)} + \frac{c}{1+r_S} - c_R + c_R\right)\right)q^* - b\int_0^{q^*} F_\xi(x)\,dx - r_S K_R$$

$$= \left(b\frac{p-c}{(p-\nu)(1+r_S)} + \frac{c}{1+r_S} - c + r_S\left(b\frac{p-c}{(p-\nu)(1+r_S)} + \frac{c}{1+r_S}\right)\right)q^*$$

$$- b\int_0^{q^*} F_\xi(x)\,dx - r_S K_R$$

$$= \left((1+r_S)b\frac{p-c}{(p-\nu)(1+r_S)} + (1+r_S)\frac{c}{1+r_S} - c\right)q^* - b\int_0^{q^*} F_\xi(x)\,dx - r_S K_R$$

$$= \left(b\frac{p-c}{p-\nu} + c - c\right)q^* - b\int_0^{q^*} F_\xi(x)\,dx - r_S K_R$$

$$= b\left(\frac{p-c}{p-\nu}q^* - \int_0^{q^*} F_\xi(x)\,dx\right) - r_S K_R$$

$$\frac{\partial \pi_S^{BB}}{\partial b} = \frac{p-c}{p-\nu}q^* - \int_0^{q^*} F_\xi(x)\,dx > 0 \; as \; E[\tau] = q - \int_0^{q} F_\xi(x)\,dx > 0, p > c, p-\nu > 0$$

$$\frac{\partial^2 \pi_S^{BB}}{\partial b^2} = 0$$

Second derivative of supplier's expected profit π_S^{BB} with respect to b is equal to zero, hence no local point for maximum point b^* for the function π_S^{BB} exists.

A5. Determining a coordinating wholesale-price contract with the bank loan

$$Prof_S^{WP} = (\omega - c_S)q$$

$$Prof_R^{WP} = p\tau + \nu(q-\tau) + \max\{(\omega + c_R)q - K_R\,, 0\} - (\omega + c_R)q$$

$$- \max\{(\omega + c_R)q - K_R\,, 0\} * (1 + r_B)$$

$$= (p-\nu)\tau + (\nu - \omega - c_R)q - r_B * \max\{(\omega + c_R)q - K_R\,, 0\}$$

$$Prof_{SC}^{WP} = Prof_S + Prof_R$$
$$= (\omega - c_S)q + (p-\nu)\tau + (\nu - \omega - c_R)q - r_B * \max\{(\omega + c_R)q - K_R , 0\}$$
$$= (p-\nu)\tau + (\nu - c)q - r_B * \max\{(\omega + c_R)q - K_R , 0\}$$

$$\pi_S^{WP} = E[Prof_S^{WP}] = (\omega - c_S)q$$

$$\pi_R^{WP} = E[Prof_R^{WP}] = (p-\nu)E[\tau] + (\nu - \omega - c_R)q - r_B * \max\{(\omega + c_R)q - K_R , 0\}$$
$$= (p-\nu)\left(q - \int_0^q F_\xi(x)\,dx\right) + (\nu - \omega - c_R)q - r_B * \max\{(\omega + c_R)q - K_R , 0\}$$
$$= (p - \nu + \nu - \omega - c_R)q - (p-\nu)\int_0^q F_\xi(x)\,dx - r_B$$
$$* \max\{(\omega + c_R)q - K_R , 0\}$$
$$= (p - \omega - c_R)q + (\nu - p)\int_0^q F_\xi(x)\,dx - r_B * \max\{(\omega + c_R)q - K_R , 0\}$$

$$\pi_{SC}^{WP} = E[Prof_{SC}^{WP}] = (p-\nu)E[\tau] + (\nu - c)q - r_B * max\{(\omega + c_R)q - K_R , 0\}$$
$$= (p-\nu)\left(q - \int_0^q F_\xi(x)\,dx\right) + (\nu - c)q - r_B * max\{(\omega + c_R)q - K_R , 0\}$$
$$= (p-c)q + (\nu - p)\int_0^q F_\xi(x)\,dx - r_B * max\{(\omega + c_R)q - K_R , 0\}$$

a. Retailer has sufficient funds

$$\pi_S^{WP} = (\omega - c_S)q$$

$$\pi_R^{WP} = (p - \omega - c_R)q + (\nu - p)\int_0^q F_\xi(x)\,dx$$

$$\pi_{SC}^{WP} = (p-c)q + (\nu - p)\int_0^q F_\xi(x)\,dx$$

Step 1:

$$\frac{\partial \pi_R^{WP}}{\partial q} = p - \omega - c_R + (\nu - p)F_\xi(q) = 0 \Rightarrow F_\xi(q) = \frac{p - \omega - c_R}{p - \nu} \Rightarrow q_R^0 = F_\xi^{-1}\left(\frac{p - \omega - c_R}{p - \nu}\right)$$

$$\frac{\partial^2 \pi_R^{WP}}{\partial q^2} = -(p-\nu) f_\xi(q) < 0 \overset{\nu<p}{\Longrightarrow} q_R^0 = q_R^*$$

Step 2:

$$\frac{\partial \pi_{SC}^{WP}}{\partial q} = p - c + (\nu - p) F_\xi(q) = 0 \Rightarrow F_\xi(q) = \frac{p-c}{p-\nu} \Rightarrow \; q_{SC}^0 = F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)$$

$$\frac{\partial^2 \pi_{SC}^{BB}}{\partial q^2} = -(p-\nu) f_\xi(q) < 0 \overset{\nu<p}{\Longrightarrow} q_{SC}^0 = q_{SC}^*$$

Step 3:

Consider $q_{SC}^* = q_R^*$: $F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right) = F_\xi^{-1}\left(\frac{p-\omega-c_R}{p-\nu}\right)$

Distribution function $F_\xi(x)$ is increasing and values of its inverse function are equal, so their arguments are also equal:

$$\frac{p-c}{p-\nu} = \frac{p-\omega-c_R}{p-\nu}$$

$$p - c = p - \omega - c_R$$

$$\omega^* = p - c_R - p + c$$

$$\omega^* = c_S$$

Step 4:

$$\pi_S^{WP} = (\omega^* - c_S) q = (c_S - c_S) q = 0$$

b. Retailer takes the bank loan

$$\pi_S^{WP} = (\omega - c_S) q$$

$$\pi_R^{WP} = (p - \omega - c_R) q + (\nu - p) \int_0^q F_\xi(x)\,dx - r_B\big((\omega + c_R) q - K_R\big)$$

$$= \big(p - (1 + r_B)(\omega + c_R)\big) q + (\nu - p) \int_0^q F_\xi(x)\,dx + r_B K_R$$

$$\pi_{SC}^{WP} = (\omega - c_S)q + \big(p - (1 + r_B)(\omega + c_R)\big)q + (\nu - p)\int_0^q F_\xi(x)\,dx + r_B K_R$$

$$= \big(\omega - c_S + p - (1 + r_B)(\omega + c_R)\big)q + (\nu - p)\int_0^q F_\xi(x)\,dx + r_B K_R$$

$$= \big(\omega - c_S + p - \omega - c_R - r_B(\omega + c_R)\big)q + (\nu - p)\int_0^q F_\xi(x)\,dx + r_B K_R$$

$$= \big(p - c - r_B(\omega + c_R)\big)q + (\nu - p)\int_0^q F_\xi(x)\,dx + r_B K_R$$

Step 1:

$$\frac{\partial \pi_R^{WP}}{\partial q} = p - (1 + r_B)(\omega + c_R) + (\nu - p)F_\xi(q) = 0 \Rightarrow F_\xi(q) = \frac{p - (1 + r_B)(\omega + c_R)}{p - \nu} \Rightarrow q_R^0$$

$$= F_\xi^{-1}\left(\frac{p - (1 + r_B)(\omega + c_R)}{p - \nu}\right)$$

$$\frac{\partial^2 \pi_R^{WP}}{\partial q^2} = -(p - \nu)f_\xi(q) < 0 \overset{\nu<p}{\Longrightarrow} q_R^0 = q_R^*$$

Step 2:

$$\frac{\partial \pi_{SC}^{WP}}{\partial q} = p - c - r_B(\omega + c_R) + (\nu - p)F_\xi(q) = 0 \Rightarrow F_\xi(q) = \frac{p - c - r_B(\omega + c_R)}{p - \nu} \Rightarrow q_{SC}^0$$

$$= F_\xi^{-1}\left(\frac{p - c - r_B(\omega + c_R)}{p - \nu}\right)$$

$$\frac{\partial^2 \pi_{SC}^{BB}}{\partial q^2} = -(p - \nu)f_\xi(q) < 0 \overset{\nu<p}{\Longrightarrow} q_{SC}^0 = q_{SC}^*$$

Step 3:

Consider $q_{SC}^* = q_R^*$: $F_\xi^{-1}\left(\frac{p-c-r_B(\omega+c_R)}{p-\nu}\right) = F_\xi^{-1}\left(\frac{p-(1+r_B)(\omega+c_R)}{p-\nu}\right)$

Distribution function $F_\xi(x)$ is increasing and values of its inverse function are equal, hence their arguments are also equal:

$$\frac{p - c - r_B(\omega + c_R)}{p - \nu} = \frac{p - (1 + r_B)(\omega + c_R)}{p - \nu}$$

$$p - c - r_B(\omega + c_R) = p - \omega - c_R - r_B(\omega + c_R)$$

$$\omega^* = p - c_R - p + c$$

$$\omega^* = c_S$$

Step 4:

$$\pi_S^{WP} = (\omega^* - c_S)q = (c_S - c_S)q = 0$$

A6. Determining a coordinating wholesale-price contract with trade credit

$$Prof_S^{WP} = (\omega - c_S)q + r_s * \max\{(\omega + c_R)q - K_R\,, 0\}$$

$$\begin{aligned} Prof_R^{WP} &= p\tau + \nu(q - \tau) + \max\{(\omega + c_R)q - K_R\,, 0\} - (\omega + c_R)q \\ &\quad - \max\{(\omega + c_R)q - K_R\,, 0\} * (1 + r_B) \\ &= (p - \nu)\tau + (\nu - \omega - c_R)q - r_s * \max\{(\omega + c_R)q - K_R\,, 0\} \end{aligned}$$

$$\begin{aligned} Prof_{SC}^{WP} &= Prof_S + Prof_R \\ &= (\omega - c_S)q + r_s * \max\{(\omega + c_R)q - K_R\,, 0\} + (p - \nu)\tau + (\nu - \omega - c_R)q - r_s \\ &\quad * \max\{(\omega + c_R)q - K_R\,, 0\} = (p - \nu)\tau + (\nu - c)q \end{aligned}$$

$$\pi_S^{WP} = E[Prof_S^{WP}] = (\omega - c_S)q + r_s * \max\{(\omega + c_R)q - K_R\,, 0\}$$

$$\begin{aligned} \pi_R^{WP} &= E[Prof_R^{WP}] = (p - \nu)E[\tau] + (\nu - \omega - c_R)q - r_S * \max\{(\omega + c_R)q - K_R\,, 0\} \\ &= (p - \nu)\left(q - \int_0^q F_\xi(x)\,dx\right) + (\nu - \omega - c_R)q - r_S * \max\{(\omega + c_R)q - K_R\,, 0\} \\ &= (p - \nu + \nu - \omega - c_R)q - (p - \nu)\int_0^q F_\xi(x)\,dx - r_S * \max\{(\omega + c_R)q - K_R\,, 0\} \\ &= (p - \omega - c_R)q + (\nu - p)\int_0^q F_\xi(x)\,dx - r_S * \max\{(\omega + c_R)q - K_R\,, 0\} \end{aligned}$$

$$\begin{aligned} \pi_{SC}^{WP} &= E[Prof_{SC}^{WP}] = (p - \nu)E[\tau] + (\nu - c)q = (p - \nu)\left(q - \int_0^q F_\xi(x)\,dx\right) + (\nu - c)q \\ &= (p - c)q + (\nu - p)\int_0^q F_\xi(x)\,dx \end{aligned}$$

a. Retailer has sufficient funds

$$\pi_S^{WP} = (\omega - c_S)q$$

$$\pi_R^{WP} = (p - \omega - c_R)q + (\nu - p)\int_0^q F_\xi(x)\,dx$$

$$\pi_{SC}^{WP} = (p-c)q + (\nu - p)\int_0^q F_\xi(x)\,dx$$

Solution is the same as that for the case of sufficient funds in Appendix A5.

b. Retailer uses trade credit

$$\pi_S^{WP} = (\omega - c_S)q + r_s\big((\omega + c_R)q - K_R\big) = \big(\omega - c_S + r_s(\omega + c_R)\big)q - r_S K_R$$

$$\pi_R^{WP} = (p - \omega - c_R)q + (\nu - p)\int_0^q F_\xi(x)\,dx - r_S\big((\omega + c_R)q - K_R\big)$$

$$= \big(p - (1 + r_S)(\omega + c_R)\big)q + (\nu - p)\int_0^q F_\xi(x)\,dx + r_S K_R$$

$$\pi_{SC}^{WP} = \big(\omega - c_S + r_s(\omega + c_R)\big)q - r_S K_R + \big(p - (1 + r_S)(\omega + c_R)\big)q + (\nu - p)\int_0^q F_\xi(x)\,dx + r_S K_R$$

$$= \big(\omega - c_S + r_s(\omega + c_R) + p - (1 + r_S)(\omega + c_R)\big)q + (\nu - p)\int_0^q F_\xi(x)\,dx$$

$$= (\omega - c_S + p - \omega - c_R)q + (\nu - p)\int_0^q F_\xi(x)\,dx$$

$$= (p - c)q + (\nu - p)\int_0^q F_\xi(x)\,dx$$

Step 1:

$$\frac{\partial \pi_R^{WP}}{\partial q} = p - (1 + r_S)(\omega + c_R) + (\nu - p)F_\xi(q) = 0 \Rightarrow F_\xi(q) = \frac{p - (1 + r_S)(\omega + c_R)}{p - \nu} \Rightarrow q_R^0 = F_\xi^{-1}\left(\frac{p - (1 + r_S)(\omega + c_R)}{p - \nu}\right)$$

$$\frac{\partial^2 \pi_R^{WP}}{\partial q^2} = -(p - \nu)f_\xi(q) < 0 \overset{\nu<p}{\Longrightarrow} q_R^0 = q_R^*$$

Step 2:

$$\frac{\partial \pi_{SC}^{WP}}{\partial q} = p - c + (\nu - p)F_\xi(q) = 0 \Rightarrow F_\xi(q) = \frac{p - c}{p - \nu} \Rightarrow q_{SC}^0 = F_\xi^{-1}\left(\frac{p - c}{p - \nu}\right)$$

$$\frac{\partial^2 \pi_{SC}^{BB}}{\partial q^2} = -(p-\nu) f_\xi(q) < 0 \overset{\nu<p}{\Longrightarrow} q_{SC}^0 = q_{SC}^*$$

Step 3:

Consider $q_{SC}^* = q_R^*$: $F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right) = F_\xi^{-1}\left(\frac{p-(1+r_S)(\omega+c_R)}{p-\nu}\right)$

Distribution function $F_\xi(x)$ is increasing and values of its inverse function are equal, hence their arguments are also equal:

$$\frac{p-c}{p-\nu} = \frac{p-(1+r_S)(\omega+c_R)}{p-\nu}$$

$$p-c = p-(1+r_S)(\omega+c_R)$$

$$\omega^* + c_R = \frac{p-p+c}{1+r_S}$$

$$\omega^* = \frac{c}{1+r_S} - c_R$$

Step 4:

$$\begin{aligned}\pi_S^{WP} &= \big(\omega^* - c_S + r_s(\omega^* + c_R)\big)q - r_S K_R \\ &= \left(\frac{c}{1+r_S} - c_R - c_S + r_s\left(\frac{c}{1+r_S} - c_R + c_R\right)\right)q - r_S K_R \\ &= \frac{c - c(1+r_S) + c r_s}{1+r_S} q - r_S K_R = \frac{0}{1+r_S} q - r_S K_R = -r_S K_R < 0\end{aligned}$$

A7. Determining a conditionally coordinating buyback contract with limited funding

(!) $\pi_S^{BB}(b,\omega^*,q^*) > \pi_S^{WP}(\omega^0,q^0), q^* = q_{SC}^*: \max_q \pi_{SC}^{BB} = \pi_{SC}^{BB}\big(\omega^*, q_{SC}^*(\omega^*)\big), q^0$

$$= q_R^*: \max_q \pi_R^{WP} = \pi_R^{WP}\big(\omega, q_R^*(\omega)\big)$$

a. Retailer has sufficient funds

$$\omega^* = c_S + b\frac{p-c}{p-\nu}$$

$$q^* = q_{SC}^* = F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)$$

$$\pi_S^{BB}(\omega^*, q^*) = (\omega^* - c_S)q^* - b\int_0^{q^*} F_\xi(x)\,dx$$

$$= \left(c_S + b\frac{p-c}{p-\nu} - c_S\right) F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right) - b\int_0^{F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)} F_\xi(x)\,dx$$

$$= b\left(\frac{p-c}{p-\nu} F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right) - \int_0^{F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)} F_\xi(x)\,dx\right)$$

$$q^0 = q_R^* = F_\xi^{-1}\left(\frac{p-\omega^*-c_R}{p-\nu}\right) = F_\xi^{-1}\left(\frac{p-c_S-b\frac{p-c}{p-\nu}-c_R}{p-\nu}\right) = F_\xi^{-1}\left(\frac{p-c-b\frac{p-c}{p-\nu}}{p-\nu}\right)$$

$$= F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)$$

$$\pi_S^{WP}(\omega^*, q^0) = (\omega^* - c_S)q^0 = \left(c_S + b\frac{p-c}{p-\nu} - c_S\right) F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)$$

$$= b\frac{p-c}{p-\nu} F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)$$

$$\pi_S^{BB}(\omega^*, q^*) - \pi_S^{WP}(\omega^*, q^0)$$

$$= b\left(\frac{p-c}{p-\nu} F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right) - \int_0^{F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)} F_\xi(x)\,dx\right)$$

$$- b\frac{p-c}{p-\nu} F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)$$

$$= b\left(\frac{p-c}{p-\nu}\left(F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right) - F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)\right)\right.$$

$$\left. - \int_0^{F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)} F_\xi(x)\,dx\right)$$

$$\frac{p-c}{p-\nu}\left(F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right) - F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)\right) - \int_0^{F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)} F_\xi(x)\,dx > 0$$

(!) $F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right) - F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right) > 0$

Distribution function $F_\xi(x)$ is increasing, so the arguments of its inverse function also can be compared:

$$\frac{p-c}{p-\nu} - \frac{(p-c)(p-b-\nu)}{(p-\nu)^2} = \frac{p-c}{p-\nu} * \frac{b}{p-\nu} > 0$$

b. Retailer takes the bank loan

$$\omega^* = b\frac{p-c-cr_B}{p-\nu+br_B} + c_S = \frac{b(p-c)+c(p-\nu)}{p-\nu+br_B} - c_R$$

$$q^* = q_{SC}^* = F_\xi^{-1}\left(\frac{p-c-r_B(\omega^*+c_R)}{p-\nu}\right)$$

$$= F_\xi^{-1}\left(\frac{p-c-r_B\left(\frac{b(p-c)+c(p-\nu)}{p-\nu+br_B}-c_R+c_R\right)}{p-\nu}\right) = F_\xi^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)$$

$$\pi_S^{BB}(\omega^*,q^*) = (\omega^*-c_S)q^* - b\int_0^{q^*} F_\xi(x)\,dx$$

$$= \left(b\frac{p-c-cr_B}{p-\nu+br_B}+c_S-c_S\right)F_\xi^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right) - b\int_0^{F_\xi^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)} F_\xi(x)\,dx$$

$$= b\left(\frac{p-c-cr_B}{p-\nu+br_B}F_\xi^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right) - \int_0^{F_\xi^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)} F_\xi(x)\,dx\right)$$

$$q^0 = q_R^* = F_\xi^{-1}\left(\frac{p-(1+r_B)(\omega^*+c_R)}{p-\nu}\right)$$

$$= F_\xi^{-1}\left(\frac{p-(1+r_B)\left(\frac{b(p-c)+c(p-\nu)}{p-\nu+br_B}-c_R+c_R\right)}{p-\nu}\right)$$

$$= F_\xi^{-1}\left(\frac{(p-c-cr_B)(p-b-\nu)}{(p-\nu)(p-\nu+br_B)}\right)$$

$$\pi_S^{WP}(\omega^*,q^0) = (\omega^*-c_S)q^0 = \left(b\frac{p-c-cr_B}{p-\nu+br_B}+c_S-c_S\right)F_\xi^{-1}\left(\frac{(p-c-cr_B)(p-b-\nu)}{(p-\nu)(p-\nu+br_B)}\right)$$

$$= b\frac{p-c-cr_B}{p-\nu+br_B}F_\xi^{-1}\left(\frac{(p-c-cr_B)(p-b-\nu)}{(p-\nu)(p-\nu+br_B)}\right)$$

$$\pi_S^{BB}(\omega^*,q^*) - \pi_S^{WP}(\omega^*,q^0)$$

$$= b\left(\frac{p-c-cr_B}{p-\nu+br_B}F_\xi^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right) + \int_0^{F_\xi^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)} F_\xi(x)\,dx\right)$$

$$- b\frac{p-c-cr_B}{p-\nu+br_B}F_\xi^{-1}\left(\frac{(p-c-cr_B)(p-b-\nu)}{(p-\nu)(p-\nu+br_B)}\right)$$

$$= b\left(\frac{p-c-cr_B}{p-\nu+br_B}\left(F_\xi^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right) - F_\xi^{-1}\left(\frac{(p-c-cr_B)(p-b-\nu)}{(p-\nu)(p-\nu+br_B)}\right)\right)\right.$$

$$\left. - \int_0^{F_\xi^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)} F_\xi(x)\,dx\right)$$

$$\frac{p-c-cr_B}{p-\nu+br_B}\left(F_\xi^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)-F_\xi^{-1}\left(\frac{(p-c-cr_B)(p-b-\nu)}{(p-\nu)(p-\nu+br_B)}\right)\right)$$

$$-\int_0^{F_\xi^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)} F_\xi(x)\,dx > 0$$

$$(!)\ F_\xi^{-1}\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)-F_\xi^{-1}\left(\frac{(p-c-cr_B)(p-b-\nu)}{(p-\nu)(p-\nu+br_B)}\right) > 0$$

$$\frac{p-c-cr_B}{p-\nu+br_B}-\frac{(p-c-cr_B)(p-b-\nu)}{(p-\nu)(p-\nu+br_B)}=\frac{p-c-cr_B}{p-\nu+br_B}\left(1-\frac{p-b-\nu}{p-\nu}\right)$$

$$=\frac{p-c-cr_B}{p-\nu+br_B}*\frac{b}{p-\nu}>0$$

$$b<p-\nu \implies \frac{b}{p-\nu}<1 \Rightarrow 0<1-\frac{b}{p-\nu}<1 \implies (p-\nu+br_B)\left(1-\frac{b}{p-\nu}\right)$$

$$<1\ if\ p-\nu+br_B<1 \Rightarrow r_B<\frac{1-p+\nu}{b}$$

c. Retailer uses trade credit

$$\omega^*=b\frac{p-c}{(p-\nu)(1+r_S)}+\frac{c}{1+r_S}-c_R$$

$$q^*=q_{SC}^*=F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)$$

$$\pi_S^{BB}(\omega^*,q^*)=\big(\omega^*-c_S+r_S(\omega^*+c_R)\big)q^*-b\int_0^{q^*}F_\xi(x)\,dx-r_SK_R$$

$$=\left(b\frac{p-c}{(p-\nu)(1+r_S)}+\frac{c}{1+r_S}-c_R-c_S\right.$$

$$\left.+r_S\left(b\frac{p-c}{(p-\nu)(1+r_S)}+\frac{c}{1+r_S}-c_R+c_R\right)\right)F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)$$

$$-b\int_0^{F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)}F_\xi(x)\,dx-r_SK_R$$

$$=b\left(\frac{p-c}{p-\nu}F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)-\int_0^{F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)}F_\xi(x)\,dx\right)-r_SK_R$$

$$q^0 = q_R^* = F_\xi^{-1}\left(\frac{p-(1+r_S)(\omega^*+c_R)}{p-\nu}\right)$$

$$= F_\xi^{-1}\left(\frac{p-(1+r_S)\left(b\frac{p-c}{(p-\nu)(1+r_S)}+\frac{c}{1+r_S}-c_R+c_R\right)}{p-\nu}\right)$$

$$= F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)$$

$$\pi_S^{WP}(\omega^*,q^0) = \left(\omega^*-c_S+r_s(\omega^*+c_R)\right)-r_SK_R$$

$$= \left(b\frac{p-c}{(p-\nu)(1+r_S)}+\frac{c}{1+r_S}-c_R-c_S\right.$$

$$\left.+r_s\left(b\frac{p-c}{(p-\nu)(1+r_S)}+\frac{c}{1+r_S}-c_R+c_R\right)\right)F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)-r_SK_R$$

$$= b\frac{p-c}{p-\nu}F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)-r_SK_R$$

$$\pi_S^{BB}(\omega^*,q^*)-\pi_S^{WP}(\omega^*,q^0)$$

$$= b\left(\frac{p-c}{p-\nu}F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)-\int_0^{F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)}F_\xi(x)\,dx\right)-r_SK_R$$

$$-b\frac{p-c}{p-\nu}F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)+r_SK_R$$

$$= b\left(\frac{p-c}{p-\nu}\left(F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)-F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)\right)\right.$$

$$\left.-\int_0^{F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)}F_\xi(x)\,dx\right)$$

$$\frac{p-c}{p-\nu}\left(F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)-F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)\right)-\int_0^{F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)}F_\xi(x)\,dx>0$$

$$(!)\ F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)-F_\xi^{-1}\left(\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}\right)>0$$

$$\frac{p-c}{p-\nu}-\frac{(p-c)(p-b-\nu)}{(p-\nu)^2}=\frac{p-c}{p-\nu}*\frac{b}{p-\nu}>0$$

A8. Conditionally coordinating buyback contract with limited funding with demand distributed as uniformly

a. Retailer takes the bank loan

$$\omega^* = b\frac{p-c-cr_B}{p-\nu+br_B} + c_S = \frac{b(p-c)+c(p-\nu)}{p-\nu+br_B} - c_R$$

$$q^* = q^*_{SC} = \beta\frac{p-c-cr_B}{p-\nu+br_B}$$

$$\pi_S^{BB}(\omega^*, q^*) = (\omega^* - c_S)q^* - b\int_0^{q^*} F_\xi(x)\,dx = (\omega^* - c_S)q^* - b\frac{{q^*}^2}{2\beta}$$

$$= \left(b\frac{p-c-cr_B}{p-\nu+br_B} + c_S - c_S\right)\beta\frac{p-c-cr_B}{p-\nu+br_B} - b\frac{\beta^2(p-c-cr_B)}{2\beta(p-\nu+br_B)}$$

$$= \frac{1}{2}b\beta\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)^2 > 0$$

$$q^0 = q^*_R = \beta\frac{(p-c-cr_B)(p-b-\nu)}{(p-\nu)(p-\nu+br_B)}$$

$$\pi_S^{WP}(\omega^*, q^0) = (\omega^* - c_S)q^0 = \left(b\frac{p-c-cr_B}{p-\nu+br_B} + c_S - c_S\right)\beta\frac{(p-c-cr_B)(p-b-\nu)}{(p-\nu)(p-\nu+br_B)}$$

$$= b\beta\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)^2\frac{p-b-\nu}{p-\nu}$$

$$\pi_S^{BB}(\omega^*, q^*) - \pi_S^{WP}(\omega^*, q^0) = \frac{1}{2}b\beta\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)^2 - b\beta\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)^2\frac{p-b-\nu}{p-\nu}$$

$$= b\beta\left(\frac{p-c-cr_B}{p-\nu+br_B}\right)^2\left(\frac{1}{2} - \frac{p-b-\nu}{p-\nu}\right) > 0 \; if \; \frac{1}{2} - \frac{p-b-\nu}{p-\nu} > 0$$

$$\frac{1}{2} - \frac{p-b-\nu}{p-\nu} > 0 \Leftrightarrow b > \frac{p-\nu}{2}$$

$$\omega^*(\lambda) = \frac{\lambda(p-\nu)(p-c)+c(p-\nu)}{p-\nu+\lambda(p-\nu)r_B} - c_R = \frac{c+\lambda(p-c)}{1+\lambda r_B} - c_R$$

$$\pi_S^{BB}(\omega^*(\lambda), q^*) = \beta\lambda\frac{p-\nu}{2}\left(\frac{p-c-cr_B}{p-\nu+\lambda(p-\nu)r_B}\right)^2 = \lambda\frac{\beta}{2(p-\nu)}\left(\frac{p-c-cr_B}{1+\lambda r_B}\right)^2$$

$$\pi_R^{BB}(\omega^*(\lambda), q^*) = \beta\frac{p-\nu-\lambda(p-\nu)}{2}\left(\frac{p-c-cr_B}{p-\nu+\lambda(p-\nu)r_B}\right)^2 + r_B K_R$$

$$= (1-\lambda)\frac{\beta}{2(p-\nu)}\left(\frac{p-c-cr_B}{1+\lambda r_B}\right)^2 + r_B K_R$$

$$\pi_{SC}^{BB}(\omega^*(\lambda), q^*) = \beta\frac{p-\nu}{2}\left(\frac{p-c-cr_B}{p-\nu+\lambda(p-\nu)r_B}\right)^2 + r_B K_R = \frac{\beta}{2(p-\nu)}\left(\frac{p-c-cr_B}{1+\lambda r_B}\right)^2 + r_B K_R$$

$$= \pi_S^{BB}(\omega^*(\lambda), q^*) + \pi_R^{BB}(\omega^*(\lambda), q^*)$$

b. Retailer uses trade credit

$$\omega^* = b\frac{p-c}{(p-\nu)(1+r_S)} + \frac{c}{1+r_S} - c_R$$

$$q^* = q_{SC}^* = \beta \frac{p-c}{p-\nu}$$

$$\pi_S^{BB}(\omega^*, q^*) = \big(\omega^* - c_S + r_S(\omega^* + c_R)\big)q^* - b\int_0^{q^*} F_\xi(x)\,dx - r_S K_R$$

$$= \Bigg(b\frac{p-c}{(p-\nu)(1+r_S)} + \frac{c}{1+r_S} - c_R - c_S$$

$$+ r_S\left(b\frac{p-c}{(p-\nu)(1+r_S)} + \frac{c}{1+r_S} - c_R + c_R\right)\Bigg)\beta\frac{p-c}{p-\nu} - b\frac{\left(\beta\frac{p-c}{p-\nu}\right)^2}{2\beta} - r_S K_R$$

$$= \frac{1}{2}b\beta\left(\frac{p-c}{p-\nu}\right)^2 - r_S K_R > 0 \; if \; b > \frac{2cr_S}{(1-r_S)(p-c)}(p-\nu) > \frac{cr_S}{p-c}(p-\nu)$$

$$1 - r_S < 1 \Longrightarrow \frac{1}{1-r_S} > 1 \Longrightarrow \frac{2cr_S}{(1-r_S)(p-c)} > \frac{cr_S}{p-c} \Longrightarrow both\; \pi_S^{BB}(\omega^*, q^*) \; and \; \pi_S^{WP}(\omega^*, q^0)$$

$$> 0 \; if \; b > \frac{2cr_S}{(1-r_S)(p-c)}(p-\nu)$$

$$q^0 = q_R^* = \beta\frac{p-(1+r_S)(\omega^*+c_R)}{p-\nu} = \beta\frac{p-c}{p-\nu} * \frac{p-b-\nu}{p-\nu}$$

$$\pi_S^{WP}(\omega^*, q^0) = \big(\omega^* - c_S + r_S(\omega^* + c_R)\big) - r_S K_R$$

$$= \Bigg(b\frac{p-c}{(p-\nu)(1+r_S)} + \frac{c}{1+r_S} - c_R - c_S$$

$$+ r_s\left(b\frac{p-c}{(p-\nu)(1+r_S)} + \frac{c}{1+r_S} - c_R + c_R\right)\Bigg)\beta\frac{p-c}{p-\nu} * \frac{p-b-\nu}{p-\nu} - r_S K_R$$

$$= b\beta\left(\frac{p-c}{p-\nu}\right)^2 * \frac{p-b-\nu}{p-\nu} - r_S K_R > 0 \; if \; b > \frac{cr_S}{p-c}(p-\nu)$$

$$\pi_S^{BB}(\omega^*, q^*) - \pi_S^{WP}(\omega^*, q^0) = \frac{1}{2}b\beta\left(\frac{p-c}{p-\nu}\right)^2 - r_S K_R - b\beta\left(\frac{p-c}{p-\nu}\right)^2 * \frac{p-b-\nu}{p-\nu} + r_S K_R$$

$$= b\beta\left(\frac{p-c}{p-\nu}\right)^2\left(\frac{1}{2} - \frac{p-b-\nu}{p-\nu}\right) > 0 \; if \; b > \frac{p-\nu}{2}$$

$$\omega^*(\lambda) = \lambda(p-\nu)\frac{p-c}{(p-\nu)(1+r_S)} + \frac{c}{1+r_S} - c_R = \frac{\lambda(p-c)+c}{1+r_S} - c_R$$

$$\pi_S^{BB}(\omega^*(\lambda), q^*) = \frac{1}{2}\lambda(p-\nu)\beta\left(\frac{p-c}{p-\nu}\right)^2 - r_S K_R = \lambda\frac{\beta}{2}\frac{(p-c)^2}{p-v} - r_S K_R$$

$$\pi_R^{BB}(\omega^*(\lambda), q^*) = \frac{1}{2}\beta(p-\lambda(p-\nu)-\nu)\left(\frac{p-c}{p-\nu}\right)^2 + r_S K_R = (1-\lambda)\frac{\beta}{2}\frac{(p-c)^2}{p-v} + r_S K_R$$

$$\pi_{SC}^{BB}(\omega^*(\lambda), q^*) = \frac{1}{2}\beta(p-\nu)\left(\frac{p-c}{p-\nu}\right)^2 = \pi_S^{BB}(\omega^*(\lambda), q^*) + \pi_R^{BB}(\omega^*(\lambda), q^*)$$

$$\pi_S^{BB}(\omega^*, q^*) - \pi_S^{WP}(\omega^*, q^0) > 0 \; if \; b > \frac{p-\nu}{2}$$

$$\pi_S^{BB}(\omega^*, q^*) > 0 \; if \; b > \frac{2cr_S}{(1-r_S)(p-c)}(p-\nu)$$

$$\pi_S^{WP}(\omega^*, q^0) > 0 \; if \; b > \frac{cr_S}{p-c}(p-\nu)$$

$$b > \frac{2}{1-r_S} * \frac{cr_S}{(p-c)}(p-\nu) > \frac{cr_S}{p-c}(p-\nu) \; as \; \frac{2}{1-r_S} > 2 \; \left(1 - r_S < 1 \Rightarrow \frac{1}{1-r_S} > 1\right)$$

Let us consider the difference $\frac{p-\nu}{2} - \frac{2cr_S}{(1-r_S)(p-c)}(p-\nu)$:

$$\frac{p-\nu}{2} - \frac{2cr_S}{(1-r_S)(p-c)}(p-\nu) = (p-\nu)\left(\frac{1}{2} - \frac{2cr_S}{(1-r_S)(p-c)}\right)$$

$$= (p-\nu) * \frac{(1-r_S)(p-c) - 2cr_S}{2(1-r_S)(p-c)} = (p-\nu) * \frac{(1-r_S)(p-c) - 2cr_S}{2(1-r_S)(p-c)}$$

$$= (p-\nu) * \frac{p(1-r_S) - c(1+r_S)}{2(1-r_S)(p-c)} > 0$$

Thus $\pi_S^{BB}(\omega^*, q^*) > 0, \pi_S^{WP}(\omega^*, q^0) > 0$ and $\pi_S^{BB}(\omega^*, q^*) - \pi_S^{WP}(\omega^*, q^0) > 0$ if $b > \frac{p-\nu}{2}$

A9. Comparison of the buyback contract with trade credit (TC) and bank loan (BL)

Let us consider the difference $\pi_S^{TC}(\lambda) - \pi_S^{BL}(\lambda)$. To do so, let us consider the difference $\pi_{SC}^{TC}(\lambda) - \pi_{SC}^{BL}(\lambda)$, express $\pi_S^{TC}(\lambda)$ and $\pi_S^{BL}(\lambda)$ through $\pi_{SC}^{TC}(\lambda)$ and $\pi_{SC}^{BL}(\lambda)$, and then consider the difference $\pi_S^{TC}(\lambda) - \pi_S^{BL}(\lambda)$.

$$\pi_{SC}^{TC}(\lambda) = \frac{1}{2}\beta(p-\nu)\left(\frac{p-c}{p-\nu}\right)^2$$

$$\pi_{SC}^{BL}(\lambda) = \frac{\beta}{2(p-\nu)}\left(\frac{p-c-cr_B}{1+\lambda r_B}\right)^2 + r_B K_R$$

$$\pi_{SC}^{TC}(\lambda) - \pi_{SC}^{BL}(\lambda) = \frac{1}{2}\beta(p-\nu)\left(\frac{p-c}{p-\nu}\right)^2 - \left(\frac{\beta}{2(p-\nu)}\left(\frac{p-c-cr_B}{1+\lambda r_B}\right)^2 + r_B K_R\right)$$

$$= \frac{1}{2}\beta(p-\nu)\left(\frac{p-c}{p-\nu}\right)^2 - \frac{\beta}{2(p-\nu)}\left(\frac{p-c-cr_B}{1+\lambda r_B}\right)^2 - r_B K_R$$

$$= \frac{\beta}{2(p-\nu)}\left((p-c)^2 - \left(\frac{p-c-cr_B}{1+\lambda r_B}\right)^2\right) - r_B K_R$$

$$= \frac{\beta}{2(p-\nu)}\left(p-c-\frac{p-c-cr_B}{1+\lambda r_B}\right)\left(p-c+\frac{p-c-cr_B}{1+\lambda r_B}\right) - r_B K_R$$

$$= \frac{\beta}{2(p-\nu)} * r_B * \frac{c+\lambda(p-c)}{1+\lambda r_B} * \frac{(p-c)(2+\lambda r_B) - cr_B}{1+\lambda r_B} - r_B K_R$$

$$\beta > 0, p-\nu > 0 \Rightarrow \frac{\beta}{2(p-\nu)} > 0$$

$$r_B > 0$$

$r_B > 0, \lambda > 0, p - c > 0 \Rightarrow \frac{c + \lambda(p - c)}{1 + \lambda r_B} > 0$

$p - c > cr_B, 2 + \lambda r_B > 2 \Rightarrow (p - c)(2 + \lambda r_B) - cr_B > \ p - c - cr_B > 0$

Thus $\frac{\beta}{2(p-\nu)} * r_B * \frac{c+\lambda(p-c)}{1+\lambda r_B} * \frac{(p-c)(2+\lambda r_B)-cr_B}{1+\lambda r_B} > 0$

$(\omega + c_R)q - K_R > 0$

$\omega_{BL}^{BB} = \frac{c + \lambda(p - c)}{1 + \lambda r_B} - c_R$

$q_{SC}^{*} = \beta \frac{p - c - cr_B}{p - \nu + br_B} = \beta \frac{p - c - cr_B}{(p - \nu)(1 + \lambda r_B)}$

Let us insert ω_{BL}^{BB} and q_{SC}^{*} into $(\omega + c_R)q - K_R > 0$:

$$\left(\frac{c + \lambda(p - c)}{1 + \lambda r_B} - c_R + c_R\right)\beta \frac{p - c - cr_B}{(p - \nu)(1 + \lambda r_B)} - K_R > 0$$

$$\beta \frac{\big(c + \lambda(p - c)\big)(p - c - cr_B)}{(p - \nu)(1 + \lambda r_B)^2} > K_R$$

$$r_B * \beta \frac{\big(c + \lambda(p - c)\big)(p - c - cr_B)}{(p - \nu)(1 + \lambda r_B)^2} > r_B * K_R$$

Let us consider the difference between the left part of the inequation above and the first term of the difference $\frac{\beta}{2(p-\nu)} * r_B * \frac{c+\lambda(p-c)}{1+\lambda r_B} * \frac{(p-c)(2+\lambda r_B)-cr_B}{1+\lambda r_B} - r_B K_R$:

$$\frac{\beta}{2(p - \nu)} * r_B * \frac{c + \lambda(p - c)}{1 + \lambda r_B} * \frac{(p - c)(2 + \lambda r_B) - cr_B}{1 + \lambda r_B} - r_B * \beta \frac{\big(c + \lambda(p - c)\big)(p - c - cr_B)}{(p - \nu)(1 + \lambda r_B)^2}$$

$$= \frac{\beta r_B\big(c + \lambda(p - c)\big)}{(p - \nu)(1 + \lambda r_B)^2} * \left(\frac{(p - c)(2 + \lambda r_B) - cr_B}{2} - (p - c - cr_B)\right)$$

$$= \frac{\beta r_B\big(c + \lambda(p - c)\big)}{(p - \nu)(1 + \lambda r_B)^2} * \frac{(p - c)(2 + \lambda r_B) - cr_B - 2(p - c - cr_B)}{2}$$

$$= \frac{\beta r_B\big(c + \lambda(p - c)\big)}{(p - \nu)(1 + \lambda r_B)^2} * \frac{r_B\big(c + \lambda(p - c)\big)}{2} > 0$$

$$\frac{\beta}{2(p - \nu)} * r_B * \frac{c + \lambda(p - c)}{1 + \lambda r_B} * \frac{(p - c)(2 + \lambda r_B) - cr_B}{1 + \lambda r_B} > r_B * \beta \frac{\big(c + \lambda(p - c)\big)(p - c - cr_B)}{(p - \nu)(1 + \lambda r_B)^2}$$

$$> r_B K_R > 0 \Rightarrow \pi_{SC}^{TC}(\lambda) - \pi_{SC}^{BL}(\lambda) > 0$$

$\pi_S^{TC}(\lambda) = \lambda * \pi_{SC}^{TC}(\lambda) - r_S K_R$

$\pi_S^{BL}(\lambda) = \lambda * (\pi_{SC}^{BL}(\lambda) - r_B K_R)$

Then the difference between the supplier’s profits with the trade credit and bank loan will be

$$\pi_S^{TC}(\lambda) - \pi_S^{BL}(\lambda) = \lambda * \pi_{SC}^{TC}(\lambda) - r_S K_R - \lambda * (\pi_{SC}^{BL}(\lambda) - r_B K_R)$$

$$= \lambda\left(\pi_S^{TC}(\lambda) - \pi_{SC}^{BL}(\lambda)\right) + K_R(\lambda r_B - r_S)$$

$$= \lambda\left(\frac{\beta r_B}{2(p-\nu)} * \frac{c+\lambda(p-c)}{1+\lambda r_B} * \frac{(p-c)(2+\lambda r_B) - c r_B}{1+\lambda r_B} - r_B K_R\right)$$

$$+ K_R(\lambda r_B - r_S) = \frac{\lambda}{2} * \frac{\beta r_B}{p-\nu} * \frac{c+\lambda(p-c)}{1+\lambda r_B} * \frac{(p-c)(2+\lambda r_B) - c r_B}{1+\lambda r_B} - r_S K_R$$

$$r_S * \beta \frac{p-c}{p-\nu} * \frac{c+\lambda(p-c)}{1+r_S} > r_S K_R$$

Hence the difference $\pi_S^{TC}(\lambda) - \pi_S^{BL}(\lambda)$ will be positive if $\frac{\lambda}{2} * \frac{\beta r_B}{p-\nu} * \frac{c+\lambda(p-c)}{1+\lambda r_B} * \frac{(p-c)(2+\lambda r_B)-c r_B}{1+\lambda r_B} - r_S * \beta \frac{p-c}{p-\nu} * \frac{c+\lambda(p-c)}{1+r_S} > 0$.

$$\frac{\lambda}{2} * \frac{\beta r_B}{p-\nu} * \frac{c+\lambda(p-c)}{1+\lambda r_B} * \frac{(p-c)(2+\lambda r_B) - c r_B}{1+\lambda r_B} - r_S * \beta \frac{p-c}{p-\nu} * \frac{c+\lambda(p-c)}{1+r_S}$$

$$= \beta \frac{c+\lambda(p-c)}{p-\nu}\left(\frac{\lambda}{2} * r_B * \frac{(p-c)(2+\lambda r_B) - c r_B}{(1+\lambda r_B)^2} - r_S * \frac{p-c}{1+r_S}\right)$$

$$\beta \frac{c+\lambda(p-c)}{p-\nu} > 0 \Rightarrow \beta \frac{c+\lambda(p-c)}{p-\nu}\left(\frac{\lambda}{2} * r_B * \frac{(p-c)(2+\lambda r_B) - c r_B}{(1+\lambda r_B)^2} - r_S * \frac{p-c}{1+r_S}\right) > 0 \; if$$

$$\frac{\lambda}{2} * r_B * \frac{(p-c)(2+\lambda r_B) - c r_B}{(1+\lambda r_B)^2} - r_S * \frac{p-c}{1+r_S} > 0$$

$$\frac{\lambda r_B\big((p-c)(2+\lambda r_B) - c r_B\big)}{(p-c)(1+\lambda r_B)^2} > \frac{r_S}{1+r_S}$$

$$r_S < \frac{1}{\frac{2(p-c)(1+\lambda r_B)^2}{\lambda r_B\big((p-c)(2+\lambda r_B) - c r_B\big)} - 1}$$

Thus $\pi_S^{TC}(\lambda) - \pi_S^{BL}(\lambda) > 0$ when $r_S < \frac{1}{\frac{2(p-c)(1+\lambda r_B)^2}{\lambda r_B\left((p-c)(2+\lambda r_B)-c r_B\right)} - 1}$.

A10. Comparison of wholesale-price contract with trade credit (TC) and bank loan (BL)

Let us consider the difference $\pi_S^{TC}(\lambda) - \pi_S^{BL}(\lambda)$:

$$\omega_{BL}^{BB} = \frac{c+\lambda(p-c)}{1+\lambda r_B} - c_R$$

$$q_R^{BL} = \frac{(p-c-c r_B)(p-b-\nu)}{(p-\nu)(p-\nu+b r_B)}$$

$$b = \lambda(p-\nu) \Rightarrow q_R^* = \beta \frac{(p-c-c r_B)\big(p-\nu-\lambda(p-\nu)\big)}{(p-\nu)(p-\nu+\lambda(p-\nu) * r_B)} = \beta \frac{(p-c-c r_B)(1-\lambda)}{(p-\nu)(1+\lambda r_B)}$$

$$\pi_S^{BL} = (\omega_{BL}^{BB} - c_S)q_R^* = \left(b\frac{p-c-cr_B}{p-v+br_B} + c_S - c_S\right)\beta\frac{(p-c-cr_B)(p-b-v)}{(p-v)(p-v+br_B)}$$

$$= b\beta\left(\frac{p-c-cr_B}{p-v+br_B}\right)^2\frac{p-b-v}{p-v}$$

$$b = \lambda(p-v) \Rightarrow \pi_S^{BL}(\lambda) = \lambda(p-v)*\beta\left(\frac{p-c-cr_B}{p-v+\lambda(p-v)*r_B}\right)^2\frac{p-v-\lambda(p-v)}{p-v}$$

$$= \lambda(p-v)*\beta\left(\frac{p-c-cr_B}{(p-v)(1+\lambda r_B)}\right)^2*(1-\lambda) = \beta\frac{\lambda(1-\lambda)}{p-v}\left(\frac{p-c-cr_B}{1+\lambda r_B}\right)^2$$

$$\omega_{TC}^{BB} = \frac{\lambda(p-c)+c}{1+r_S} - c_R$$

$$q_R^{TC} = \beta\frac{p-c}{p-v}*\frac{p-b-v}{p-v}$$

$$b = \lambda(p-v) \Rightarrow q_R^* = \beta\frac{p-c}{p-v}*\frac{p-v-\lambda(p-v)}{p-v} = \beta(1-\lambda)\frac{p-c}{p-v}$$

$$\pi_S^{TC}(\lambda) = b\beta\left(\frac{p-c}{p-v}\right)^2*\frac{p-b-v}{p-v} - r_SK_R = \lambda(p-v)*\beta\left(\frac{p-c}{p-v}\right)^2*(1-\lambda) - r_SK_R$$

$$= \lambda(1-\lambda)\beta\frac{(p-c)^2}{p-v} - r_SK_R$$

$$\pi_S^{TC}(\lambda) - \pi_S^{BL}(\lambda) = \lambda(1-\lambda)\beta\frac{(p-c)^2}{p-v} - r_SK_R - \beta\frac{\lambda(1-\lambda)}{p-v}\left(\frac{p-c-cr_B}{1+\lambda r_B}\right)^2$$

$$= \beta\frac{\lambda(1-\lambda)}{p-v}\left((p-c)^2 - \left(\frac{p-c-cr_B}{1+\lambda r_B}\right)^2\right) - r_SK_R$$

$$= \beta\frac{\lambda(1-\lambda)}{p-v}*r_B\frac{c+\lambda(p-c)}{1+\lambda r_B}*\frac{(p-c)(2+\lambda r_B)-cr_B}{1+\lambda r_B} - r_SK_R$$

Let us insert ω_{TC}^{BB} and q_R^{TC} into $(\omega + c_R)q - K_R > 0$:

$$(\omega_{TC}^{BB} + c_R)q_R^{TC} - K_R > 0$$

$$\left(\frac{\lambda(p-c)+c}{1+r_S} - c_R + c_R\right)\beta(1-\lambda)\frac{p-c}{p-v} > K_R$$

$$\beta\frac{(\lambda(p-c)+c)(1-\lambda)(p-c)}{(1+r_S)(p-v)}*r_S > r_SK_R$$

Let us consider the difference between the left part of the inequation above and the first term of the difference $\beta\frac{\lambda(1-\lambda)}{p-v}*\frac{c+\lambda(p-c)}{1+\lambda r_B}*\frac{(p-c)(2+\lambda r_B)-cr_B}{1+\lambda r_B} - r_SK_R$:

$$\beta\frac{\lambda(1-\lambda)}{p-v}*r_B\frac{c+\lambda(p-c)}{1+\lambda r_B}*\frac{(p-c)(2+\lambda r_B)-cr_B}{1+\lambda r_B} - \beta\frac{(\lambda(p-c)+c)(1-\lambda)(p-c)}{(1+r_S)(p-v)}*r_S$$

$$= \beta\frac{(\lambda(p-c)+c)(1-\lambda)(p-c)}{p-v}\left(\frac{\lambda r_B\big((p-c)(2+\lambda r_B)-cr_B\big)}{(1+\lambda r_B)^2} - \frac{r_S(p-c)}{1+r_S}\right)$$

$$\beta \frac{(\lambda(p-c)+c)(1-\lambda)(p-c)}{p-\nu} > 0$$

$$\Rightarrow \beta \frac{(\lambda(p-c)+c)(1-\lambda)(p-c)}{p-\nu}\left(\frac{\lambda r_B\big((p-c)(2+\lambda r_B)-cr_B\big)}{(1+\lambda r_B)^2} - \frac{r_S(p-c)}{1+r_S}\right)$$

$$> 0 \; if \; \frac{\lambda r_B\big((p-c)(2+\lambda r_B)-cr_B\big)}{(1+\lambda r_B)^2} - \frac{r_S(p-c)}{1+r_S}$$

$$r_S < \frac{1}{\frac{(p-c)(1+\lambda r_B)^2}{\lambda r_B\big((p-c)(2+\lambda r_B)-cr_B\big)} - 1}$$

Thus $\pi_S^{TC}(\lambda) - \pi_S^{BL}(\lambda) > 0$ when $r_S < \frac{1}{\frac{(p-c)(1+\lambda r_B)^2}{\lambda r_B\left((p-c)(2+\lambda r_B)-cr_B\right)}-1}$.