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Analysis and control of macroeconomic trends based on the Leontief model

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Contents

1 Abstract

This thesis uses R language to aggregate the Chinese input-output tables released by the National Bureau of Statistics for the years 2002, 2005, 2007, 2010, 2012, 2015, 2017, 2018, and 2020 into the Advanced manufacturing industry, Modern service industry, and other industries. The integration status among multi-sector is first discussed. Second, a time series is used to predict and compare the development trends of total output and GDP of China and the United States under various factors. This analysis is based on the OECD, which uses Total fixed capital formation as the investment share of net profits and the theory that the investment needed to increase output is proportional to the required capital density. In addition, addressing the linear control characteristics in the dynamic input-output model, classic program control is applied, and based on a linear non-homogeneous differential equation including three industries and GDP, control is exerted on the Advanced manufacturing industry to obtain an equation of controlled factors, and the effectiveness of this program control is verified by numerical calculation using Python. The dynamic input-output system is finally abstracted into saddle-point equilibrium theory based on the dynamic input-output model put forth by Leontief using the concept of game theory, and a new optimal control for solving the dynamic input-output problem is designed by using the saddle-point equilibrium strategy.

2 Introduction

Input-output analysis, proposed by Leontief in 1936, has been widely applied in economic analysis and continuously improved and developed in practice. As a crucial quantitative tool, input-output analysis can reflect the interdependence between input and output of multi-sector in the national economy and can be applied in economic analysis, policy simulation, economic forecasting and decision-making, and economic control.

Based on the input-output tables released by the National Bureau of Statistics of China in 2002, 2005, 2007, 2010, 2012, 2015, 2017, 2018, and 2020, the third part of this thesis establishes a static input-output model and provides relevant balance equations. One considers the direct consumption coefficient of m industries and analyzed the one-way integration degree, comprehensive integration degree, and interactive degree of integration. Through multidimensional quantification of China's "modern service industry" and "advanced manufacturing industry," one finds that the integration of the "two industries" has achieved some results, but the contribution of the modern service industry to the advanced manufacturing industry is still limited. The influence of the "two industries" is differentiated, and traditional advantageous industries still exist. To address this issue, it is necessary to construct a sound domestic demand system for the modern service industry, connect the "dual circulation" of the "two industries," strengthen the industrial connection, enhance the innovation connection between the "two industries," and thus achieve deep integration of the "two industries."

According to [Gao,2019], the modern service industry can drive technological progress in related industries to a certain extent and promote industrial structure upgrading. [Sun,2020] believes that the modern service industry includes modern logistics, information transportation, computer services and software industry, scientific research, technology services, and geological exploration. Based on the "Thirteenth Five-Year Plan" for the transformation of scientific and technological innovation in China's modern service industry in 2017, this paper divides the modern service industry in combination with Sun's viewpoint.

According to [Singhry,2016], [Dimitris,2018], and [Jin,2017], the advanced manufacturing industry refers to industries that primarily use emerging technologies as the primary means. [Li,2014] and [Hua,2017] believe that advanced manufacturing includes six industries: electronic equipment manufacturing, general and special-purpose equipment manufacturing, instrumentation and meter manufacturing, computer and electronic equipment manufacturing, and transportation equipment manufacturing. Based on the "End-of-year Reserve Policy for Advanced Manufacturing Industry (2021)" published by the Chinese Tax Bureau, this paper divides the advanced manufacturing industry along with Li and Hua's perspective.

In the research of industrial integration, [Xia,2020] measured the integration of the productive service industry and manufacturing industry by using the ratio of intermediate input to total output. [Ma,2011] proposed to measure the degree of integration by the proportion of the input to the total intermediate input. [Peng,2019] pointed out the comprehensive integration degree in industrial coupling. In this section, we combine the views of Xia and Ma and use the degree of service-oriented of the advanced manufacturing industry and the degree of manufacturing-oriented of the modern service industry as indicators of a one-way integration degree. One also considers Peng's view to measure the comprehensive integration degree.

As the economic structure is a constantly changing and developing dynamic process, static input-output models cannot accurately reflect the actual situation of expanding social product reproduction. Therefore, many scholars have explored the construction of dynamic input-output models. For example, [Leontief,1956] proposed the "dynamic inverse model", laying the foundation for dynamic input-output models. [Miller, 2009] incorporated the time factor into the input-output analysis. [Smirnov,2021] considered GDP as the $(n+1)$th variable in the input-output equilibrium equation, defining total output as the derivative for time.

In the fourth part of this paper, one combines Smirnov's views and establishes a dynamic input-output model based on data from the OECD for China and the United States. Through this model and the least squares method, one predicts the capital density for both countries under inflationary and non-inflationary scenarios. One uses fixed capital formation as the investment share of net profit. For China's input-output table, one considers the maximum aggregation of total output and uses time-series forecasting to predict the development trend of the total output's maximum aggregate and the GDP from 2019 to 2024. Similarly, for the United States input-output table, one first aggregate into three categories: advanced manufacturing industry, modern service industry, and others, and then used time-series forecasting to predict the development trend of the total output of each categorization and GDP. Finally, one compares the differences between the two countries.

According to my analysis, the United States requires a higher unit of fixed capital formation to increase one unit of GDP compared to China. However, since 2010, China's GDP growth rate has been higher than that of the United States. The modern service industry contributes more to the GDP of the United States than the advanced manufacturing industry, while in China, it is the opposite. Therefore, China needs to maintain an open and sharing attitude, actively promote digital transformation and upgrading, increase investment in modern services such as science and technology, and promote industrial transformation and upgrading. For the United States, it is crucial to maintain openness, relax trade controls, actively promote international economic and technological cooperation, and promote sound economic development.

The properties of linear control exist in dynamic input-output models. In the fifth part of this thesis, One combines the classical program control theory given by [Tamasyan, 2008]. Based on the linear non-homogeneous differential equation with three industries and GDP, One exerts control on the aggregated advanced manufacturing industry in the United States from 2017 to 2018, obtained the control equation of multi-factor influence, and used Python to verify the program control through numerical calculation effectiveness.

The sixth section of the research is based on Leontief's dynamic input-output model and involves the continuous optimal strategy design problem. The dynamic input-output system is abstracted into a saddle-point equilibrium using game theory. A new approach for solving the dynamic input-output problems is devised using the saddle point equilibrium strategy, which provides a foundation for macroeconomic decision-making.

3 Static input-output model

3.1 The basic structure of China's input-output table

3.1.1 The three quadrants of input-output table

The Chinese input-output table currently only includes three quadrants, as opposed to the four-quadrant input-output tables released by the OECD or WOLD, and there is very little research on the fourth quadrant, which is the income redistribution quadrant.

The first quadrant of the input-output table is the intermediate product matrix X, whose elements are denoted by x_{ij}. This matrix is given by the expression $x_{ij} = P_i a_{ij} In_j$. Here, P_i is the price of the i-th unit of consumption, a_{ij} is the direct consumption coefficient, and In_j is the annual production of the j-th sector. The value of the direct consumption coefficient a_{ij} determines the amount of i-th sector's product required for the j-th sector's production and also implies the level of technology used in each economic sector. Each element x_{ij} of the first quadrant represents the amount of i-th product consumed in the production of j-th product when viewed from columns and represents the amount of i-th product allocated to the production of j-th product when viewed from rows.

The second quadrant represents the use of products, such as final consumption expenditure (FCE), gross capital formation (GCF), and exports. The column-wise structure reflects the final use of each product (Y), where Y is a column vector defined as $Y = (Y_1, Y_2, \cdots, Y_n)$. FCE includes household consumption expenditure and government consumption expenditure. GCF includes gross fixed capital formation and an increase in inventories.

The third quadrant represents value-added E, in which elements E_j are the value-added of each sector j. It's determined by the difference between the expected total input $X_j = P_j In_j$ of sector j and the sum of internal consumption values x_{xj}, that is $E_j = X_j - x_{xj}$. Here, In_j is the total consumption of sector j. When viewed in columns, E_j represents the proportion of different types of initial inputs that make up the total input of the industry, including the proportion of value-added to total input. When viewed in rows, it reflects the distribution of a specific type of income in different sectors. For example, the labor remuneration row vector can be used to calculate the proportion of labor remuneration in each sector, that is, which sectors have a larger share. labor compensation (W_j) is the first of E's four components. Production tax (T_j) comes in second. The third is the fixed asset depreciation D_j. The fourth is operating surplus O_j, which is the source of economic investment and income. The direct labor compensation coefficient is defined as $rw_j = \frac{W_j}{X_j}$. The direct production tax coefficient is defined as $rt_j = \frac{T_j}{X_j}$. The direct fixed asset depreciation coefficient is defined as $rd_j = \frac{D_j}{X_j}$. The direct operating surplus coefficient is defined as $ro_j = \frac{O_j}{X_j}$. Here, rw_j, rt_j, rd_j, and ro_j represent the direct labor compensation, net direct production taxes, directly fixed asset depreciation, and direct operating surplus of the j-th industry unit output, respectively. The sum of these four coefficients equals the value-added coefficient rp_j, which represents the proportion of each department's value-added or initial input to the total output, i.e., $rp_j = rw_j + rt_j + rd_j + ro_j$.

3.1.2 Interrelation of value-added input-output table

Intermediate consumption + Final consumption = Total output. ○

Intermediate Input + Initial input = Total Input. •

○ indicates the distribution and consumption direction of a sector's products in the row direction, and the sum of each item is equal to the total output.

• represents the various inputs involved in the production of a product, and the sum of their values is the total input.

Here, imports' values are negative, and imports of each sector are combined with final consumption.

Row relationships:

$$\begin{aligned} x_{11} + x_{12} + \ldots + x_{1n} + y_1 &= X_1, \\ x_{21} + x_{22} + \ldots + x_{2n} + y_2 &= X_2, \\ &\vdots \\ x_{n1} + x_{n2} + \ldots + x_{nn} + y_n &= X_n, \end{aligned} \tag{1}$$

that is

$$\sum_{j=1}^{n} x_{ij} + y_i = X_i \quad (i = 1, 2, \ldots, n), \tag{2}$$

where, X_i represents the column sum of the input-output table, which is the total output of the i-th industry. x_{ij} represents the value of product i consumed by industry j in production.

If Q represents the intermediate flow matrix, then equation (2) can be written as:

$$Q + Y = X, \tag{3}$$

where, X and Y represent column vectors for total output and final consumption, respectively.

3.1.3 Direct consumption coefficient

The direct consumption coefficient refers to the consumption of one product by a production unit in the production of another product. It is defined as:

$$a_{ij} = \frac{x_{ij}}{X_j}, \tag{4}$$

X_j represents the column total in the value-added input-output table. a_{ij} represents the value of product i consumed per unit of output in sector j, reflecting an indicator of the technical and economic linkages between the two sectors.

The row model can be written as:

$$\sum_{j=1}^{n} a_{ij} X_j + y_i = X_i \quad (i = 1, 2, \ldots, n), \tag{5}$$

that is:

$$AX + Y = X, \tag{6}$$

where A is the direct consumption matrix, and X and Y are column vectors representing the total output and final demand, respectively.

We can rewrite the above equation as:

$$\begin{aligned} (I - A)X &= Y, \\ X &= (I - A)^{-1} Y. \end{aligned} \tag{7}$$

$(I - A)^{-1}$ is called the Leontief inverse matrix, where I is the identity matrix.

The economic meaning of $(I - A)$ is

$$(I-A)=\begin{bmatrix} 1-a_{11} & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & 1-a_{22} & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ -a_{n1} & -a_{n2} & \cdots & 1-a_{nn} \end{bmatrix}. \tag{8}$$

As $a_{ii} < 1, (1 - a_{ii}) > 0$, the diagonal elements of the matrix $(I - A)$ are higher than 0, while the off-diagonal those that are negative or zero. Each column of the matrix $(I - A)$ represents the production set of a sector, where the diagonal element represents the net output after deducting the self-consumption. In the output perspective, it represents the final product Y, whereas, it represents the newly created value or the value-added E.

GDP is defined as the sum of the value-added E_j created by all the j sectors, i.e.

$$GDP = \sum_{j=1}^{n} E_j. \tag{9}$$

According to [Jing, 2018], relative labor compensation, relative net production taxes, and relatively fixed asset depreciation refer to the relative levels of these factors in different industries, rather than the proportion they occupy in the value-added. This is because value-added does not include direct consumption, and these factors are all considered in total output. Therefore, it is necessary to multiply the total output by their relative levels to calculate their contribution to the total output.

[Ma and Zhong, 2016] presented a formula for calculating the operating surplus O_j. One has revised its formula as follows:

$$O_j = X_j \left(1 - rw_j - t_j - d_j - ap_j\right). \tag{10}$$

3.1.4 Complete consumption coefficient

The general equilibrium theory of Walras is reflected in input-output analysis through the concept of the complete consumption coefficient, which represents the interdependence among different products in the production process.

The complete consumption coefficient c_{ij} of a product j is defined as the total consumption of another product i in the production of a unit of final consumption j, including both direct and indirect consumption coefficients.

$$c_{ij} = a_{ij} + c_{i1}a_{1j} + c_{i2}a_{2j} + \cdots + c_{ii}a_{ij} + \cdots + c_{in}a_{nj}, \tag{11}$$

where, a_{ij} represents the direct consumption coefficient, and $c_{in}a_{nj}$ represents the total indirect consumption of product i by product j through product n.

Example 3.1. *The manufacture of a refrigerator directly consumes electricity, plastic, and electronic products. However, the production of plastic, steel, and electronic products all require the consumption of electricity. For the refrigerator, this is an indirect consumption of electricity through plastic, steel, and electronic products. In addition, the steel needed for the refrigerator comes from iron ore, which also requires the consumption of electricity. The electricity consumption of iron ore is a second-order indirect consumption for the refrigerator. This continues for other inputs as well. The direct consumption of electricity for refrigerator production plus all the indirect consumption of electricity is the complete consumption of electricity for the refrigerator.*

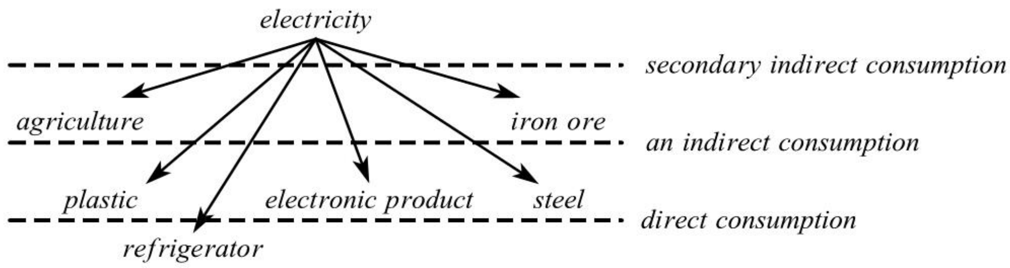

Figure 1: The complete consumption of electricity in the production of refrigerators

c_{ij} can be further expressed as:

$$c_{ij} = a_{ij} + \sum_{k=1}^{n} c_{ik} a_{kj},$$
$$C = A + CA,$$
$$C(I - A) = A,$$
$$C = A(I - A)^{-1},$$
$$C = [I - (I - A)](I - A)^{-1} = (I - A)^{-1} - I,$$

where, A is the matrix of direct consumption coefficient, C is the matrix of complete consumption coefficient, and I denotes the identity matrix.

The above equation shows that the Leontief inverse matrix and the complete consumption coefficient differ by 1 on the diagonal elements, i.e.,

$$\mathrm{L} = (I - A)^{-1} = (C + I) = \begin{bmatrix} c_{11}+1 & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22}+1 & \cdots & c_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn}+1 \end{bmatrix}. \quad (12)$$

3.1.5 Influence coefficient

In input-output analysis, the complete consumption coefficient reflects backward linkages, which, through the use of product inputs reflects the pull of other sector products in the production process. The influence coefficient

(IC) is determined by calculating the sum of the value-added increments of all sectors and dividing them by their average value.

$$r_j = \frac{\sum_{i=1}^{n} c_{ij}}{\frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{n} c_{ij}}, \tag{13}$$

where, c_{ij} is the coefficient of the Leontief inverse matrix, which represents the total value-added increase of all sectors brought by one unit of final consumption of sector j. The column sum of the complete consumption coefficients of different sectors reflects the size of their respective backward linkage.

3.2 Aggregation, reduction and inter-sectoral integration calculation of input-output table

3.2.1 Aggregation

According to the "Classification of National Economic Industries (2017)" in China, an industry can be divided into multi-sectors. Similarly, these sectors (elements) can be formed using these sectors. The category code is represented by a single Latin letter, i.e., different categories are represented by the letters A, B, C, ..., and T in order; the primary category code is represented by two Arabic numerals, starting from 01 and coded in sequence; the subcategory code is represented by three Arabic numerals, with the first two digits being the dominant category code and the third digit is the subcategory sequence code; the minor category code is represented by four Arabic numerals, with the first three digits being the subcategory code and the fourth digit is the primary category sequence code. For example:

30 - Non-metallic mineral products industry

301 - Cement, lime, and gypsum manufacturing

3011 - Cement manufacturing

3012 - Lime and gypsum manufacturing

34 - General equipment industry

341 - Lime and gypsum manufacturing

3411 - Boiler and auxiliary equipment manufacturing

3412 - Internal combustion engine and parts manufacturing

...

Based on the "Thirteenth Five-Year Plan for Modern Service Industry Technology Innovation and Transformation" in 2017, combined with Sun's perspective and the classification of the scope of China's input-output tables, the modern service industry (MSI) is defined. According to the classification of the advanced manufacturing industry in the Ministry of Finance's policy on VAT carry-forward for advanced manufacturing industries (2021) in China, and combined with the perspective of Li and Hua, the advanced manufacturing industry (AMI) is classified. Sub-sectors related to the "advanced manufacturing industry" and "modern service industry" in China's input-output tables are aggregated.

Category one consists of the advanced manufacturing industries including the Non-metallic mineral products industry (NMI), General and special equipment manufacturing (GSM), Transportation equipment manufacturing industry (TEI), Electrical machinery and equipment manufacturing (EME), Communication equipment, computer and other electronic equipment manufacturing (CEM), and Instrument Manufacturing (IMQ). Category two consists of the modern service industries including Transportation, warehousing, and postal industry (TWI), Information transmission, software and information technology service industry (ITS), Finance and insurance industry (FIS), Real estate industry (REI), Leasing and Business Services (LBS), and Scientific research and technical service industry (SRT).

Table 1: The classification of AMI and MSI

The classification of AMI	The classification of MSI
NMI	TWI
GSM	ITS
TEI	FIS
EME	REI
CEM	LBS
IMQ	SRT

One aggregated the subsets of the aggregated sectors into the sectors listed in Table 1 and provided the specific R code in reference[Dan, 2023].

3.2.2 Reduction

We typically analyze the development of m sectors for each sector in the input-output table. Since the direct consumption coefficient reflects the technical and economic relationships between industries, it is necessary to consider the technological level of the remaining $n - m$ sectors when calculating the direct consumption coefficients of these m sectors. In traditional research, only the analyzed sectors are selected. In this paper, we also consider the a_{ij} and corresponding final consumption expenditures of the m sectors, but this requires the technological levels of the $n - m$ sectors that have been removed. The process of reducing the number of industries is called reduction. That is, we need to retain m Reductions from the set of n

Reductions while maintaining the equality structure of the balance equation.

$$a_{11}X_1 + a_{12}X_2 + \ldots + a_{1n}X_n + y_1 = X_1,$$
$$a_{21}X_1 + a_{22}X_2 + \ldots + a_{2n}X_n + y_2 = X_2,$$
$$\vdots$$
$$a_{n1}X_1 + a_{n2}X_2 + \ldots + a_{nn}X_n + y_n = X_n.$$

Namely:

$$\begin{aligned} (1 - a_{11})\, X_1 - a_{12}X_2 - \ldots - a_{1n}X_n &= y_1, \\ -a_{21}X_1 + (1 - a_{22})\, X_2 - \ldots - a_{2n}X_n &= y_2, \\ &\vdots \\ -a_{n1} - a_{n2}X_2 - \ldots + (1 - a_{nn})\, X_n &= y_n. \end{aligned} \tag{14}$$

We divide the matrix Q into four blocks, denoted as Q_{11}, Q_{12}, Q_{21}, and Q_{22}. In this case, the vectors X and Y will be partitioned into two groups of proportionally-sized subvectors. The first group of vectors X_1 and Y_1 correspond to the remaining m sectors, and the second group of vectors X_2 and Y_2 correspond to the reduced $n - m$ sectors. The balance equations listed above can then be represented in block form as follows:

$$(I_1 - Q_{11})X_1 - Q_{12}X_2 = Y_1,$$
$$-Q_{21}X_1 + (I_2 - Q_{22})X_2 = Y_2,$$

where, I_1 is a $m \times m$ dimensional identity matrix, and I_2 is a $(n - m) \times (n - m)$ dimensional identity matrix.

In the first block Q_{11} of size $m \times m$, only the elements corresponding to the selected sectors for analysis are retained.

$$\begin{aligned}
X_2 = (I_2 - Q_{22})^{-1} Y_2 + (I_2 - Q_{22})^{-1} Q_{21} X_1, \\
(I_1 - Q_{11}) X_1 - Q_{12} \left[(I_2 - Q_{22})^{-1} Y_2 + (I_2 - Q_{22})^{-1} Q_{21} X_1 \right] = Y_1, \\
\left[(I_1 - Q_{11}) - Q_{12} (I_2 - Q_{22})^{-1} Q_{21} \right] = Y_1 + Q_{12} (I_2 - Q_{22})^{-1} Y_2.
\end{aligned}$$

Simplified vector:

$$R_v = Q_{11} + Q_{12} (I_2 - Q_{22})^{-1} Q_{21}. \tag{15}$$

Simplified vector for final consumption:

$$Y_v = Y_1 + Q_{12} (I_2 - Q_{22})^{-1} Y_2. \tag{16}$$

For the modified direct consumption coefficient matrix R_v, its elements are denoted as x_{ij}^r. The total output X_i^r corresponding to R_v is consistent with the initial total output X_i, at which point the equation reaches a balance. Y_v denotes the modified final consumption column vector taking into account the $n-m$ sectors. Thus, we have obtained the direct coefficient matrix R_v for m sectors considering the direct coefficient of the remaining $n-m$ sectors.

For the reduction of the annual Chinese input-output table, where the dimension of m is 12×12. A general program code was written in the R language in this paper. See [Dan, 2023] for details. It takes the reduction of the 2020 input-output table as an example.

3.2.3 Calculation of the degree of integration between advanced manufacturing industry and modern service industry

Due to the lack of detailed information in input-output tables, it is challenging to accurately calculate the specific output value of the modern service industry (MSI) within the advanced manufacturing industry (AMI). To address this issue, Ma et al. proposed measuring the degree of integration between industries using the ratio of the intermediate input of industry i to the total intermediate input of industry j as a proxy.

In this section, we consider using the servitization degree of the advanced manufacturing industry and the Manufacturing degree of the modern service industry as the indicators for one-way integration to characterize the contribution of each industry to the other.

Definition 3.1. *The servitization degree of the advanced manufacturing industry (SDAMI) refers to the extent to which modern service industry products are incorporated into the advanced manufacturing industry, i.e.,*

$$\begin{aligned} AMSD_j &= \frac{k_j}{\sum_{i=1}^{12}\sum_{j=1}^{6} x_{ij}^r} \\ k_j &= \sum_{i=1}^{6} x_{ij}^r, \\ AMSD &= \sum_{j=1}^{6} AMSD_j, \end{aligned} \tag{17}$$

where $AMSD_j$ represents the servicization degree of the advanced manufacturing industry sector j. $AMSD$ represents the overall servitization degree of the advanced manufacturing industry. i represents the row vector of the input-output table, j represents the column vector of the input-output table, and x_{ij}^r represents the coefficient of the intermediate input or intermediate consumption matrix about the advanced manufacturing industry and modern service industry, after the reduction to the dimension of 12×12.

The indicator reflects the level of integration of modern service industries into advanced manufacturing industries and the demand-driven role of advanced manufacturing industries for modern service industries. The larger the $AMSD$, the greater the degree to which modern service industries are integrated into the production process of advanced manufacturing industries. It also suggests that modern service sectors contribute more to advanced industrial sectors.

Definition 3.2. *The manufacturing degree of the modern service industry (MDMSI) is a measure of the degree to which the modern service industry is transformed into a manufacturing-oriented industry by absorbing advanced manufacturing technology and products and providing high value-added services with more manufacturing production elements to customers. The calculation formula is as follows:*

$$\begin{aligned} ASMD_i &= \frac{k_j}{\sum_{i=1}^{12} \sum_{j=7}^{12} x_{ij}^r} \\ k_j &= \sum_{i=7}^{12} x_{ij}^r, \\ ASMD &= \sum_{j=1}^{6} ASMD_i, \end{aligned} \tag{18}$$

$ASMD_i$ represents the manufacturing degree of the i-th sector in modern service industry, and $ASMD$ represents the overall manufacturing degree of modern service industry.

The larger the $ASMD$, the greater the degree of integration of the advanced manufacturing industry into the production process of the modern service industry, and the greater the dependence of the modern service industry on the advanced manufacturing industry.

Definition 3.3. *The degree of integration shown above only represents the degree of integration between two sectors moving in one direction; it does not represent the state of integration as a whole. To reflect the full amount of economic integration, one built a comprehensive integration degree.*

$$CID = \frac{AMSD}{ASMD}. \tag{19}$$

If $CID > 1$, it indicates that industrial integration is mainly reflected in the driving role of the modern service industry in the advanced manufacturing industry. If $CID < 1$, it means that the "reverse driving" of the advanced manufacturing industry on the modern service industry is firm and it is more dependent on the advanced manufacturing industry.

3.3 Analysis of the integration status between China's modern service industry and advanced manufacturing industry.

3.3.1 One-way integration degree

Based on the annual input-output table of China, combined with formulas (17) and (18), one calculated the $SDAMI$ and the $MDMSI$.

The specific results are shown in Figures 2 and 3. Firstly, the overall integration of advanced manufacturing and the modern service industry shows an unbalanced feature. The average degree of $SDAMI$ is 0.1435. The average degree of $MDMSI$ is 0.3207, indicating that the contribution of advanced manufacturing to the modern service industry is much greater than the contribution of the modern service industry to advanced manufacturing, and the latter is always higher than the former. The $SDAMI$ shows an overall upward trend, while $MDMSI$ has shown a downward trend since 2010, but it has always been higher than the $SDAMI$.

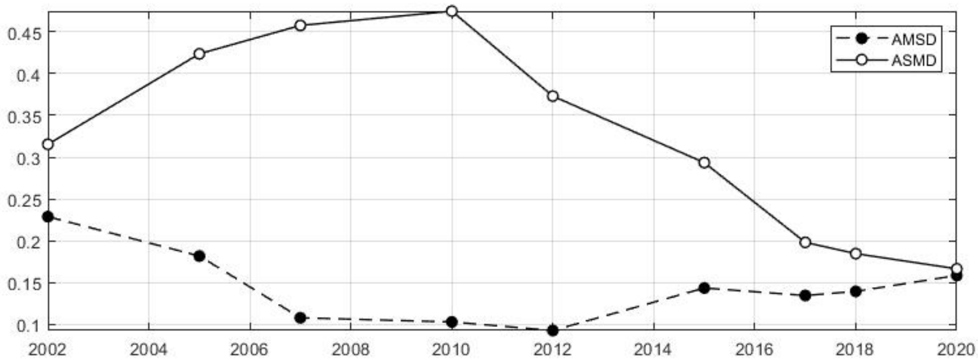

Figure 2: Trend of the $AMSD$ and the $ASMD$

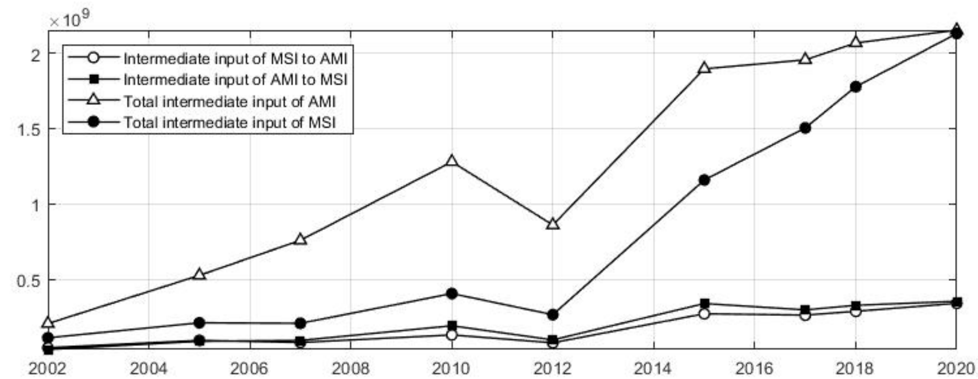

Figure 3: The changes in intermediate inputs

Secondly, the intermediate inputs of the advanced manufacturing industry have always been higher than those of the modern service industry. Under the influence of the economic crisis, the intermediate inputs of the advanced manufacturing industry and modern service industry had the largest decline in 2012, reaching 48.689% and 52.614% respectively, compared to 2010. Thirdly, since 2012, China's dependence on the advanced manufacturing industry on the modern service industry has gradually increased, but compared with developed countries, the contribution of the modern service industry is still insufficient.

Figures 4 and 5 show the servitization and manufacturing nation levels of each sector in advanced manufacturing and modern services. NMI have the highest serviceization level, with an average of 0.0453, EME has the lowest average at 0.006. This further confirms the typicality of intra-industry trade in NMI. LBS has the highest manufacturing nation level, followed by ITS. The manufacturing nation level of SRT is the lowest. The advanced manufacturing levels of the REI and FIS reached their lowest values in 2018, at 0.0003 and 0.0305, respectively, reflecting the characteristics of China's real estate bubble economy, with inflated values and insufficient contribution to the virtual economy.

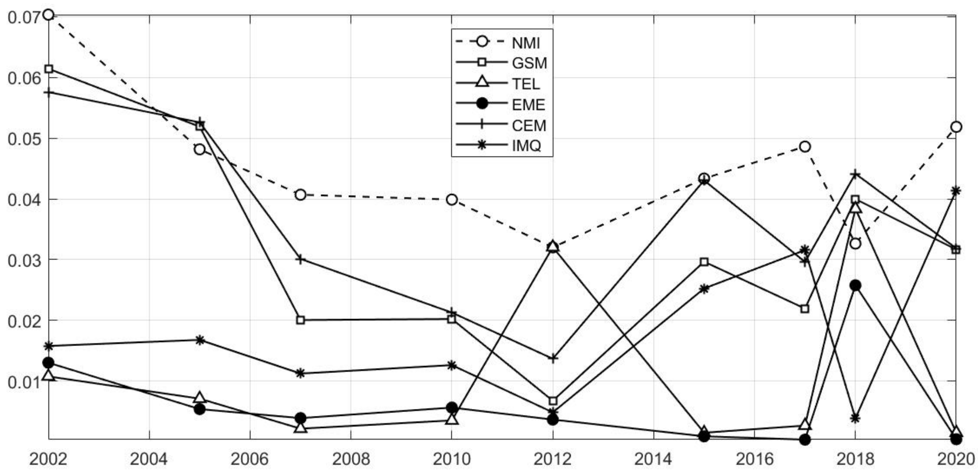

Figure 4: Serviceization degree of advanced manufacturing industry sectors

3.3.2 Comprehensive integration degree

According to the analysis of Figure 6, firstly, the comprehensive integration degree of the "two industries" had a turning point in 2010. It continued to decline from 2002 to 2010, with a decline of 70.05%. It reached its

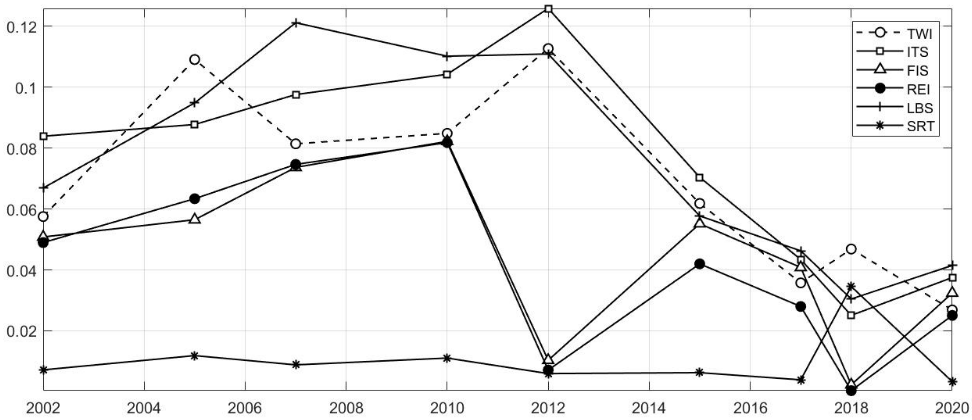

Figure 5: Manufacturing degree of modern service industry sectors

lowest value in 2010, only 0.2174. From 2012 to 2020, it continued to rise, with an increase of 281.89%. The integration effect of the "two industries" has significantly improved after the financial crisis. Secondly, although China's comprehensive degree of the "two industries" has been increasing year by year since 2010, it is lower than the average integration degree of developed countries, indicating that the development of integrated industries is relatively lagging. This phenomenon is in line with the current situation of China's manufacturing industry in the middle and low end of the international division of labor. International experience confirms the positive correlation between the degree of industry integration and economic development.

3.3.3 Interactive degree of integration

"Two Industries" Interdependence Description

According to the analysis of Figure 7, Table 2, and Table 4, firstly, the dependence of MSI on AMI is relatively stable, fluctuating around 0.6326. However, the dependence of AMI on MSI has undergone significant changes due to direct consumption under the influence of technological changes. In

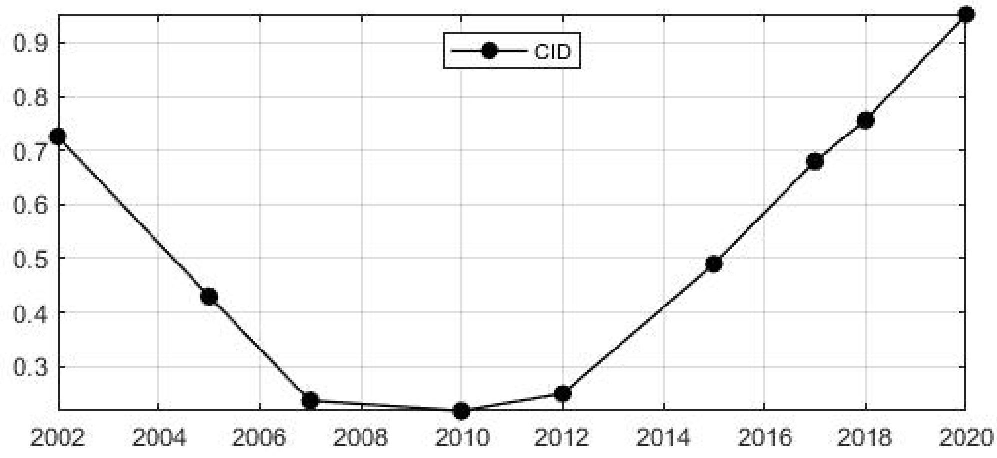

Figure 6: Comprehensive integration degree

2012, the direct consumption of MSI by AMI reached its lowest point due to the financial crisis. Since 2017, the dependence of MSI on the AMI industry has been higher than the dependence of AMI on MSI.

Second, all six AMI have a strong dependence on TWI, indicating the supporting role of the "modern circulation industry in promoting the transformation and upgrading of the manufacturing industry." However, their dependence on ITS, SRT is relatively weak, and it is necessary to continuously accelerate the transformation of scientific and technological achievements to effectively promote the innovation capacity of the manufacturing industry.

Third, most MSI have a trend of first decreasing and then increasing in their direct consumption coefficients on AMI. Some service industries have a strong dependence on AMI, such as SRT on GSM.

"Two Industries" Influence Relationship

According to Table 5, TWI have the biggest average influence on AMI, indicating that they have the highest demand-pull effect on the production of AMI and the strongest radiation effect. However, the influence of the

Table 2: Direct consumption coefficient of AMI on MSI

$Industry$	$Year$	NMI	GSM	TEI	EME	CEM	IMQ
	2002	0.0345	0.0551	0.0286	0.0330	0.0266	0.0036
	2005	0.0851	0.0652	0.0312	0.0438	0.0376	0.0076
	2007	0.0471	0.0600	0.0357	0.0444	0.0328	0.0036
	2010	0.0620	0.0746	0.0452	0.0538	0.0357	0.0059
TWI	2012	0.0142	0.0182	0.0011	0.0010	0.0087	0.0005
	2015	0.0620	0.0569	0.0438	0.0456	0.0276	0.0040
	2017	0.0394	0.0407	0.0401	0.0355	0.0233	0.0034
	2018	0.0393	0.0411	0.0417	0.0346	0.0239	0.0036
	2020	0.0340	0.0408	0.0377	0.0351	0.0228	0.0032
	2002	0.0176	0.0466	0.0284	0.0331	0.0237	0.0038
	2005	0.0479	0.0501	0.0296	0.0415	0.0311	0.0061
	2007	0.0136	0.0331	0.0176	0.0261	0.0413	0.0023
	2010	0.0141	0.0319	0.0177	0.0242	0.0359	0.0027
ITS	2012	0.0667	0.0485	0.0046	0.0056	0.0562	0.0043
	2015	0.0117	0.0142	0.0062	0.0089	0.0190	0.0012
	2017	0.0049	0.0079	0.0055	0.0061	0.0324	0.0009
	2018	0.0051	0.0078	0.0054	0.0062	0.0310	0.0010
	2020	0.0041	0.0076	0.0050	0.0062	0.0285	0.0009
	2002	0.0111	0.0234	0.0128	0.0137	0.0144	0.0013
	2005	0.0298	0.0266	0.0132	0.0173	0.0192	0.0031
	2007	0.0362	0.0820	0.0805	0.0595	0.0775	0.0071
	2010	0.0379	0.0855	0.0859	0.0611	0.0680	0.0087
SRT	2012	0.0021	0.0010	0.0002	0.0002	0.0007	0.0001
	2015	0.0279	0.0482	0.0490	0.0316	0.0405	0.0042
	2017	0.0109	0.0170	0.0222	0.0106	0.0170	0.0016
	2018	0.0095	0.0132	0.0193	0.0094	0.0156	0.0015
	2020	0.0080	0.0123	0.0157	0.0089	0.0148	0.0014

Table 3: Influence coefficient of of AMI on MSI

	$Year$	NMI	GSM	TEI	EME	CEM	IMQ
	2002	0.7033	1.5028	1.1044	0.8932	1.6833	0.1129
	2005	1.2802	1.1752	0.8672	0.8444	1.6517	0.1812
	2007	0.6850	1.2694	1.1764	0.9812	1.7771	0.1108
	2010	0.7693	1.3101	1.2461	1.0180	1.4975	0.1590
IC of AMI on MSI	2012	2.0661	2.1895	0.1666	0.1159	1.3884	0.0736
	2015	0.9984	1.3484	1.2349	0.9728	1.3291	0.1164
	2017	0.8336	1.1926	1.2832	0.8753	1.6915	0.1238
	2018	0.8404	1.1499	1.2740	0.8662	1.7383	0.1312
	2020	0.7827	1.2095	1.1947	0.9349	1.7519	0.1263
	Mean	0.9954	1.3719	1.0608	0.8336	1.6121	0.1261

Table 4: Direct consumption coefficient of MSI on AMI

$Industry$	$Year$	TWI	ITS	FIS	REI	LBS	SRT
NMI	2002	0.0137	0.0025	0.005	0.0217	0.0029	0.0054
	2005	0.0196	0.0052	0.0041	0.0077	0.0062	0.0071
	2007	0.0101	0.0008	0.0015	0.0026	0.0027	0.0017
	2010	0.0095	0.0009	0.0013	0.0047	0.0031	0.0024
	2012	0.0031	0.0065	0.0054	0.0016	0.0033	0.0016
	2015	0.007	0.0011	0.0024	0.0048	0.0046	0.0034
	2017	0.0033	0.0007	0.0019	0.0023	0.0031	0.0043
	2018	0.0034	0.0008	0.0019	0.0032	0.0031	0.0047
	2020	0.0038	0.001	0.0013	0.0037	0.0033	0.0057
GSM	2002	0.0381	0.0083	0.0069	0.0064	0.0107	0.0062
	2005	0.0326	0.0097	0.0056	0.0032	0.0139	0.0103
	2007	0.031	0.0027	0.003	0.0023	0.0057	0.0026
	2010	0.0317	0.0031	0.0035	0.0039	0.0067	0.0037
	2012	0.0391	0.0068	0.0279	0.0008	0.0114	0.0011
	2015	0.0193	0.001	0.0033	0.0017	0.0079	0.004
	2017	0.0168	0.0015	0.0055	0.0011	0.0066	0.0042
	2018	0.0184	0.0015	0.0046	0.0012	0.0064	0.0046
	2020	0.0185	0.0018	0.0029	0.0013	0.0063	0.0052
TEI	2002	0.1055	0.0144	0.0066	0.0074	0.0176	0.0033
	2005	0.0896	0.0154	0.0048	0.0036	0.0217	0.0058
	2007	0.0711	0.0028	0.003	0.0024	0.0152	0.0028
	2010	0.063	0.0027	0.0024	0.0028	0.0132	0.0034
	2012	0.0181	0.0048	0.1353	0.0075	0.014	0.0371
	2015	0.0851	0.0028	0.0012	0.0003	0.0348	0.0047
	2017	0.0917	0.0004	0.0012	0.0002	0.0335	0.0039
	2018	0.086	0.0005	0.0012	0.0002	0.0338	0.0044
	2020	0.1031	0.0006	0.0008	0.0003	0.0389	0.0058

Table 5: Influence coefficient of of MSI on AMI

	$Year$	TWI	ITS	FIS	REI	LBS	SRT
IC of MSI on AMI	2002	1.8825	1.5301	0.4911	0.4602	1.1564	0.4798
	2005	1.7408	1.4880	0.4828	0.2439	1.4138	0.6308
	2007	2.1794	0.9129	0.4182	0.3349	1.3097	0.8448
	2010	2.0241	0.9792	0.5622	0.4636	1.1331	0.8377
	2012	1.2070	0.9878	2.1763	0.2966	0.7911	0.5413
	2015	1.5523	0.9622	0.4188	0.2314	1.4007	1.4345
	2017	1.4413	0.8004	0.4095	0.1800	1.0275	2.1413
	2018	1.3507	0.8789	0.3997	0.1886	1.0054	2.1768
	2020	1.3721	0.9783	0.2544	0.1810	0.9327	2.2815
	$Mean$	1.6389	1.0575	0.6236	0.2866	1.1300	1.2631

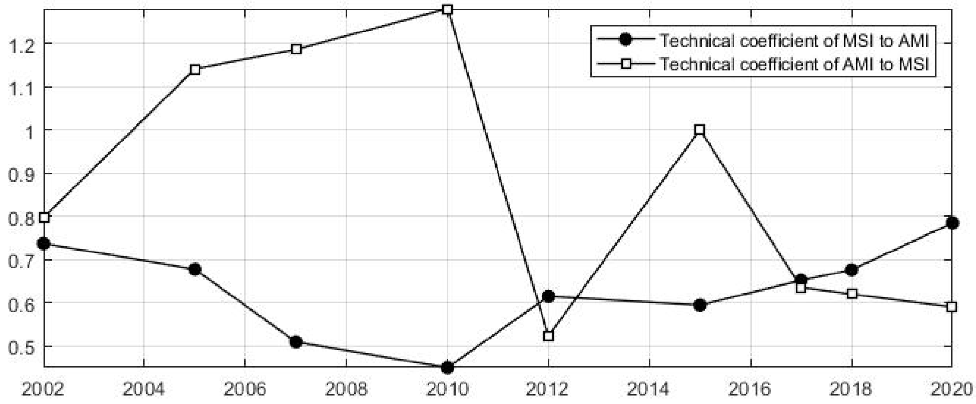

Figure 7: Direct consumption coefficients between “two industries”

FIS and REI on AMI is below the average level, indicating that the support of the virtual economy industry for the real manufacturing industry is insufficient, and there is still a "spinning wheel effect" in the development of the industry. That is, the rapid development of the virtual economy has a crowding-out effect on the real economy of the manufacturing industry to some extent.

Table 3 shows that the average influence of the GSM, TEI, and CEM is greater than 1, and the IMQ has the smallest influence on the MSI.

3.4 The conclusion of the static model

Firstly, the integration of the "two industries" has begun to show initial results. From the perspective of the one-way integration degree, the modern service industry is becoming less "manufacturing-oriented", while the servitization degree of the advanced manufacturing industry continues to increase steadily.

Secondly, the integration of the "two industries" is still dependent on inertia, but the role of high-tech is increasing. Based on the results of direct consumption coefficient calculations, since 2015, the advanced manufacturing industry has significantly increased its dependence on high-tech modern service industries, such as information transmission, software and information technology services, and scientific research and technical services industry. The advanced manufacturing industry has always been highly dependent on low-tech modern service industries, such as transportation, warehousing, and the postal industry.

Thirdly, the impact of the "two industries" has led to a differentiation in the industry landscape, where traditional advantage industries such as general and special-purpose equipment manufacturing and transportation equipment manufacturing still occupy a relatively large volume.

As a result, it is recommended that the home market's potential be utilized and that service-oriented, intelligent, and digital transformation processes be actively encouraged. At the same time, we must work to maintain a healthy cycle of domestic and global demand, take advantage of "Belt and Road" construction opportunities, loosen market access regulations in terms of systems, technology, and rules, and increase the area for international trade, foreign investment, and technological collaboration.

4 The dynamic input-output model of equilibrium economy

4.1 Introduction

The dynamic input-output model is constructed based on the static model with the addition of the time dimension and dynamic elements. It can accurately predict future development trends and changes in the economy, providing effective support and guidance for policy formulation and implementation. To this end, this section draws on Smirnov's dynamic differential equation construction method to establish a dynamic input-output model based on the models released by OECD for China and the United States, to predict their total output and GDP changes, compare the differences in data between the two countries, and provide corresponding conclusions and measures.

Considering that the fourth quadrant of the Chinese input-output table, which represents income redistribution, has not been thoroughly studied and the data released by the National Bureau of Statistics of China is non-continuous, this article mainly relies on the input-output tables of China and the United States published by the OECD to construct the dynamic input-output model for predictive analysis.

The OECD input-output table shows the relationship between 45 economic sectors, with all values expressed in current US dollars. The matrix $Q = \{x_{ij}\}_{ij=1}^{45}$ represents the intermediate consumption by the j-th sector of the production and services provided by the i-th sector. Row vector E_j represents the value added received by each sector, including W_j for labor costs, ONT_j for net production taxes, Prh_j for net profits, TF for Taxes fewer subsidies on intermediate and final imported products, and TD for Taxes fewer subsidies on intermediate and final domestic products. Column vector Y represents the final consumption of industrial sector output, including Gross Fixed Capital Formation($GFCF$), Changes in inventories(III), Direct purchases abroad by residents (imports), Direct purchases abroad, Exports (cross-border), and Imports (cross-border). Xj is the input of the jth sector.

	Industry	Final expenditure	
Industry	Q	Y	X
TXS_IMP_FNL	TF		
TXS_INT_FNL	TD		
	E		
	X		

Figure 8: $OECD$ Schematic diagram of OECD input-output table

Figure 8 has the following balance relationships:

$$X_j = \sum_{i=1}^{n} x_{ij} + TF_j + TD_j + E_j = \sum_{j=1}^{n} x_{ij} + Y_i, \tag{20}$$

where $i, j = 1, \ldots, 45$.

So considering the sum of all sectors, the total output is given by:

$$X = \sum_{j=1}^{n} X_j. \tag{21}$$

GDP is the sum of value added E_j and TXS_INT_FNL, that is:

$$GDP = \sum_{j=1}^{n} E_j + \sum_{j=1}^{m} TD_j = \sum_{i=1}^{n} Y_i + \sum_{j=m-n}^{m} TF_j + \sum_{j=m-n}^{m} TD_j, \tag{22}$$

where $n = 45, m = 54$.

Similarly, we can obtain:

$$a_{ij} = \frac{X_j - E_j - TD_j}{X_j}, \quad td_j = \frac{TD_j}{X_j}, \tag{23}$$

$$rw_j = \frac{W_j}{E_j}, \quad rt_j = \frac{ONT_j}{E_j}, \quad ap_j = \sum_{j=1}^{n} a_{ij}, \tag{24}$$

$$\begin{aligned} td = \textstyle\sum_{j=1}^{n} td_j, \quad rw = \textstyle\sum_{j=1}^{n} rw_j, \\ rt = \textstyle\sum_{j=1}^{n} rt_j, \quad ap = \textstyle\sum_{j=1}^{n} ap_j, \end{aligned} \tag{25}$$

a_{ij} represents the amount of intermediate inputs of sector i used in producing one unit of output of sector j, i.e. the direct consumption coefficient. ap_j is the share of intermediate inputs used by sector j in the total output X_i of sector i. ap represents the total share of intermediate inputs used in the total output X of all sectors. td_j represents the share of domestic taxes net of subsidies on production and imports in sector j's total output X_i, while rw_j represents the share of wages in sector j's value added. rt_j represents the share of production taxes in sector j's value-added.

Therefore, the net profits of each sector's annual output are:

$$\mathrm{Prh}_j = (1 - ap_j - td_j)\,(1 - rw_j - rt_j)\,X_j. \tag{26}$$

The sum of net profits of all sectors' annual output is:

$$\mathrm{Prh}_a = \sum_{j=1}^{n} Prh_j = (1 - ap - td)\,(1 - rw - rt). \tag{27}$$

Similarly,

$$Y_{r_i} = \frac{Y_i}{GDP}, \quad E_j = (1 - ap_j)\,X_j, \tag{28}$$

$$Y_r = \sum_{i=1}^{n} Y_{r_i}, \quad td_n = \frac{\sum_{j=m-n}^{m} TD_j}{GDP}, \quad \sum_{j=1}^{n} E_j = (1 - ap)\,X, \tag{29}$$

Y_{r_i} represents the share of final consumption Y_i in GDP for sector i. E_j represents the value-added of sector j. Y_r represents the share of final consumption in GDP, and td_n represents the share of domestic taxes less subsidies for the m sectors from $m - n$ in GDP.

Therefore, we have:

$$GDP = \sum_{j=1}^{n} E_j + \sum_{j=1}^{m} TD_j = (1 - ap_j)\,X + td_n \cdot GDP. \tag{30}$$

The traits of the necessary state variables—which serve as the foundation for creating a dynamic economic development model—are described above. The economic sectors are regarded as a group when the input-output table is created. We must first create a system of differential equations that accurately describes the economic development process to handle the economic sectors as a dynamic system.

Definition 4.1. *In the absence of technological progress, the total output X_t changes over time can be defined as its derivative at time t, $\dot{X}(t) = \frac{dX(t)}{dt}$, where $X_t = (X_1(t) \dots X_n(t))$ is the output vector of all economic sectors in terms of inputs and according to the principle of ownership and responsibility. The vector $\dot{X}(t)$ describes the production acceleration of all economic sectors.*

The relative growth of output $\frac{\triangle X}{X_0}$ under the same technology and initial output level X_0 requires an increase proportional to the total amount of fixed asset formation $\frac{\triangle GFCF}{GFCF_0}$ and inventory change $\frac{\triangle III}{III_0}$. The investment amount C_p required to expand output can be assumed to be proportional to the required acceleration. Therefore, in conjunction with Definition 4.1, the ratios determining the investment amounts of the various economic sectors will take the following form:

$$Cp_i(t) = Fe_i \cdot \dot{X}_i(t), \tag{31}$$

where Fe_i represents the capital density of each economic sector. It is the proportional coefficient between the output growth $\dot{X}i(t)$ and the investment $Cp_i(t)$ required to ensure it.

Fe_i indicates the production acceleration of goods and services per unit of time, $\frac{\Delta X_i}{\Delta t} = \frac{X_i(T_2)-I_i(T_1)}{T_2-T_1}$. Therefore, the value of Fe_i is determined by the ratio for each sector.

$$Fe_i = \frac{Cp_i(t)}{\frac{\Delta X_i}{\Delta_t}} = \frac{Cp_i(t)\,(T_2 - T_1)}{X_i\,(T_2) - X_i\,(T_1)} = \frac{Cp_i(t)\,(T_2 - T_1)}{X_i\,(T_1)\left(\frac{X_i(T_2)}{X_i(T_1)} - 1\right)}. \tag{32}$$

For the OECD input-output tables, capital density can be written as:

$$Fe_i(t) = \frac{Cp_j(t)}{X_j(t+1) - X_j(t)}. \tag{33}$$

$$Fe_{n+1}(t) = \frac{Cp(t)}{GDP(t+1) - GDP(t)}. \tag{34}$$

4.2 Construct the system of differential equation

In input-output modeling, the construction of differential equations is related to the sources of investment. Generally, the sources of investment include gross fixed capital formation ($GFCF$), changes in inventories (III), government expenditures, imports (IM), and exports (EX), among others. However, for investment purposes, we usually only consider $GFCF$ and III as they are investment sources based on internal demand.

Based on equation (31), we consider the investment to be a fraction of the net profit. It is written as:

$$Cp_i = rn_j \cdot Prh_j, \quad j = 1, \ldots, n, \tag{35}$$

where rn_j represents the share of net profits used for investment.

Equation (35) is written as:

$$Cp_i = GFCF_j + III_j, \tag{36}$$

$$rn_n = \frac{Cp}{GDP}, \tag{37}$$

rn_n represents the share of investment C_p in GDP.

Based on equations (23), (24), (25), and (31), we can obtain a dynamic system of differential equations that describes the economic development of each sector:

$$\dot{X} = \frac{rn_j \cdot (1 - ap_j - td_j)(1 - rw_j - rt_j)}{Fe_i} \left(\sum_{j=1}^{n} a_{ij} \cdot X_j + Y_{r_j} \cdot GDP \right), \tag{38}$$

$$G\dot{D}P = \frac{rn_n}{Fe_{n+1}} \left(\sum_{j=1}^{n} (1 - ap_j) X_j + td_n \cdot GDP \right). \tag{39}$$

Similarly, the dynamic system of differential equations for the overall economic development is as follows:

$$\dot{X} = \tfrac{rn \cdot (1 - ap - td)(1 - rw - rt)}{Fe_i} \left(ap \cdot X + Y_r \cdot GDP \right), \tag{40}$$

$$G\dot{D}P = \frac{rn_n}{Fe_{n+1}} \left((1 - ap) X + td_n \cdot GDP \right). \tag{41}$$

In vector form, the system $(38), (39)$ is shown below:

$$\dot{\mathbf{X}} = \mathbf{D}\mathbf{X}, \quad \mathbf{D} = \mathbf{M}\widetilde{\mathbf{Q}} \tag{42}$$

where,

$$
\widetilde{\mathbf{Q}} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} & Yr_1 \\ x_{21} & x_{22} & \cdots & x_{2n} & Yr_2 \\ \vdots & I & \ddots & \vdots & \vdots \\ x_{n1} & x_{n1} & \cdots & x_{nn} & Yr_n \\ 1-ap_1 & 1-ap_2 & \cdots & 1-ap_n & td_n \end{bmatrix}, \tag{43}
$$

$$
\mathbf{M} = \begin{bmatrix} \frac{\alpha_1}{Fe_1} & 0 & \cdots & 0 & 0 \\ 0 & \frac{\alpha_2}{Fe_2} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \frac{\alpha_n}{Fe_n} & 0 \\ 0 & 0 & \cdots & 0 & \frac{td_n}{Fe_{n+1}} \end{bmatrix}, \tag{44}
$$

$\alpha_j = rn_j\,(1 - rw_j - rt_j)\,(1 - td_j - ap_j),\ j = 1, \cdots, n.$

Considering the direct consumption coefficient a_{pj} can better estimate production costs and resource utilization, the wage rate r_{wj} can help analyze the relationship between production costs and output, and the net production tax rt_j can more accurately calculate production costs and profits. This model takes into account these factors, which can help us more accurately predict total output and GDP, and assist governments and businesses in formulating economic policies and strategic planning. For example, the government can adjust tax policies based on the predicted results of input-output tables, guide the rational allocation of resources, and promote the development of various sectors.

4.3 China's dynamic input-output analysis

Based on the input-output tables of China from 2000 to 2018 published by the OECD, and given that the detailed structure of value-added is not provided, I used equations (39) and (40) to analyze the overall economic development level of China. By using linear regression, I predicted the capital density Fe_i and Fe_{n+1} under non-inflationary and inflationary conditions, respectively. I also used time series forecasting to predict the sum of GDP and total output from 2019 to 2024.

Where GDP_n represents nominal GDP. GDP_d represents GDP deflator. GDP_r represents real GDP in RMB. I_r represents inflation rate. GDP_{ru} represents real GDP in USD.

Because the official inflation rate of China is not provided, it is calculated as follows:

Each year's real GDP is equal to the 1978 real GDP multiplied by the current 1978 index.

$$\begin{aligned} GDP \text{ index } &= \frac{\text{GDP (current prices)}}{\text{GDP deflator index of the base perioid}} \times 100\%, \\ \text{Inflation rate } &= \frac{\text{GDP (real) index of reporting period} - \text{GDP (real) index of base period}}{\text{GDP (real) index of base period}}. \end{aligned} \tag{45}$$

To calculate the real GDP in US dollars, we need to use the exchange rate between the US dollar and the Chinese yuan to convert the GDP in yuan to dollars. To do so, we obtain the data in Table 6.

4.3.1 Predict China's capital density

According to Figure 9, in 2015, the gross fixed capital formation per unit required to increase GDP is relatively high, and the unit capital expenditure required to increase total output is negative, which is related to China's

Table 6: China's Real GDP and Inflation Rates from 2000 to 2018

$Year$	GDP_n	GDP_d	GDP_r	I_r	GDP_{ru}
2000	10028010	3.586	2796547.7	0.02	369050.52
2001	11086310	3.659	3029777.3	0.02	366001.98
2002	12171740	3.681	3306415.6	0.01	398964.68
2003	13742200	3.777	3638234.3	0.03	439482.25
2004	16184020	4.04	4006104.3	0.07	484011.09
2005	18731890	4.197	4462631	0.04	549196.44
2006	21943850	4.362	5030254.4	0.04	630702.60
2007	27009230	4.7	5746129.4	0.08	766593.83
2008	31924460	5.067	6300877.4	0.08	906556.91
2009	34851770	5.056	6893148.1	0.00	1009114.8
2010	41211930	5.404	7626313	0.07	1149776.11
2011	48794020	5.84	8354695.6	0.08	1292664.97
2012	53858000	5.977	9011343.5	0.02	1431962.85
2013	59296320	6.106	9711400.1	0.02	1566924.53
2014	64356310	6.169	10432425.3	0.01	1697627.01
2015	68885820	6.169	11167061.7	0.00	1821029.52
2016	74639510	6.255	11931863.5	0.01	1795667.78
2017	83203590	6.52	12760674.6	0.04	1907792.58
2018	91928110	6.748	13622226.1	0.03	2058535.61

implementation of the policy of "reducing overcapacity and destocking" in 2015, and it is one of the key reasons why the total output denominated in USD in 2016 is less than the total output denominated in USD in 2015. According to the data of the input-output table in 2000-2018, the least square method is used to predict China's 2019-2025 annual total output and required investment. The results show that in the forecasted value of 2019-2025, the gross capital formation per unit required to increase GDP in 2020 is relatively low, but overall it is stable and positive and continues to develop.

Based on Figure 10, the predicted unit capital expenditure for real GDP growth under inflation shows a wave-like pattern, with a relatively high capital density in 2021 compared to 2019, indicating a higher unit capital expenditure required for GDP growth in 2020. Due to the influence of policies in 2015 and other reasons, the capital density in 2015 was significantly

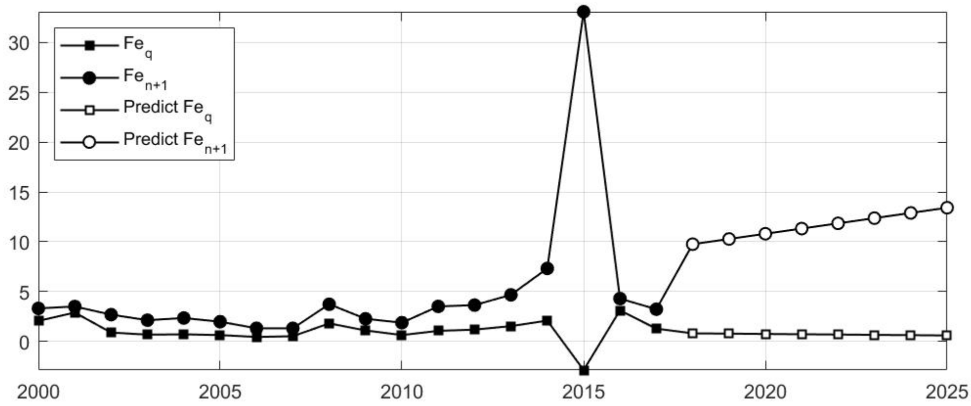

Figure 9: Predicted values of capital density Fe_q and Fe_{n+1} under nominal GDP

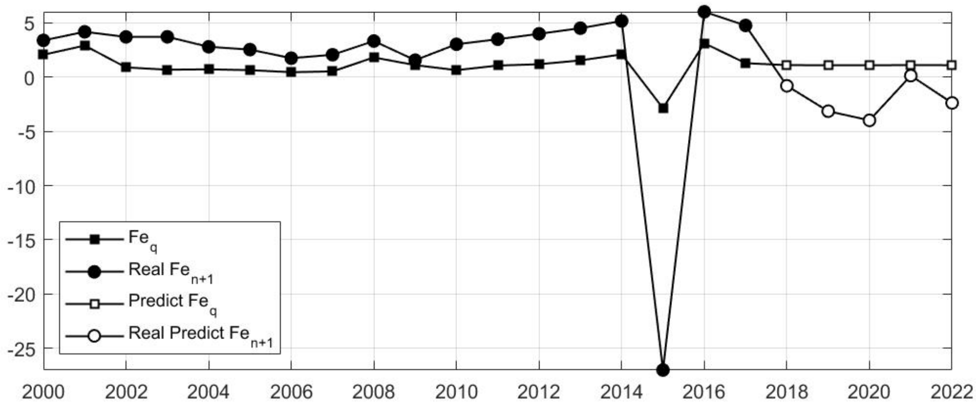

Figure 10: Predicted values of capital density Fe_q and Fe_{n+1} under real GDP

different from that of previous years. To make better predictions, we consider a more general form and exclude the capital density value in 2015.

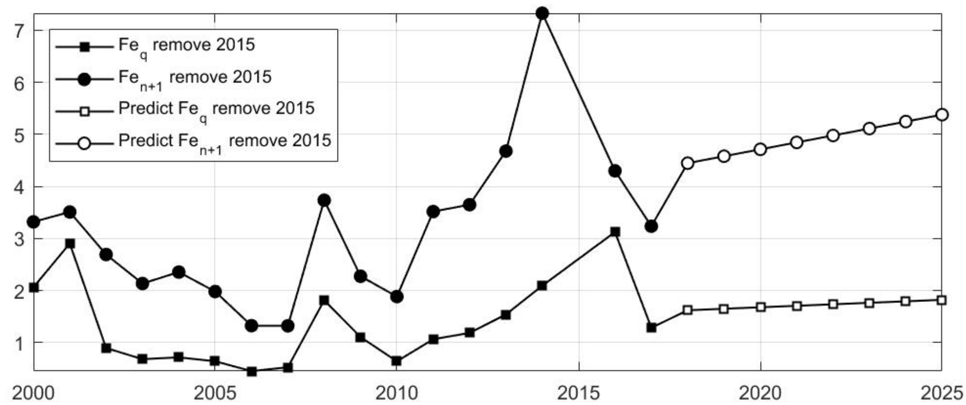

Figure 11: Predicted values of capital density Fe_q and Fe_{n+1} under nominal GDP (remove 2015)

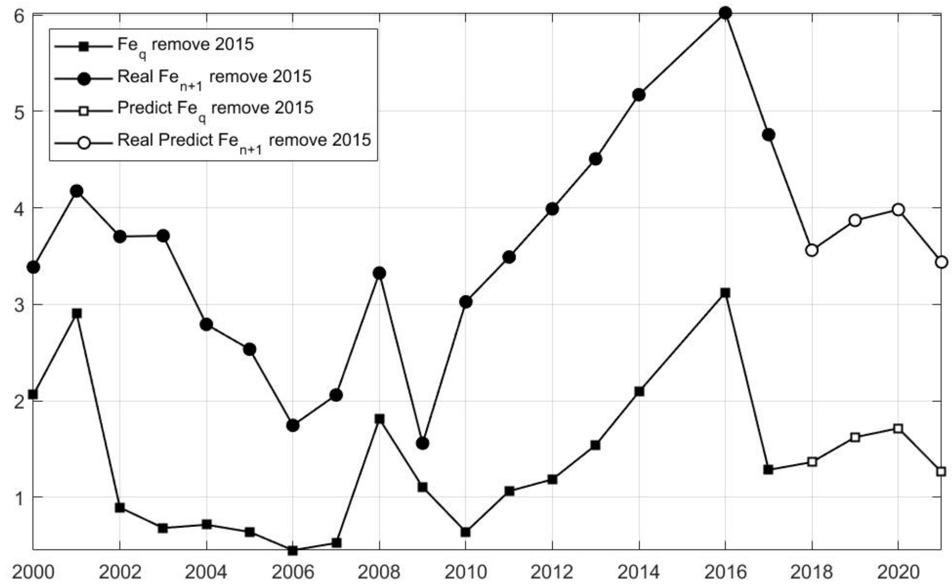

Figure 12: Predicted values of capital density Fe_q and Fe_{n+1} under real GDP (remove 2015)

According to Figure 11 and Figure 12, after removing the outliers, the overall trend of F_{e_i} and $F_{e_{n+1}}$ tends to be consistent for real GDP and nominal GDP. The capital density was the highest in 2020, which means that the total amount of fixed capital formation required to increase unit GDP output in 2020 was relatively large.

4.3.2 Predicting GDP and the total output of maximum aggregation

Based on equations (39) and (40) and time series analysis, one predicted the GDP and total output under inflation, with the year as the independent variable, and the inflation rate of each year as the independent variable. One used an autoregressive integrated moving average ($ARIMA$) model to predict GDP and total output from 2019 to 2025. The results show that, according to Figure 13 and Figure 14, the nominal GDP from 2000 to 2018 is consistent with the predicted GDP. However, there are differences between predicted and nominal GDP in 2019 and 2020, with the largest difference in

2020 reaching 1966152 million US dollars. One of the primary causes of this phenomenon is the impact of the COVID-19 pandemic in the fourth quarter of 2020. The total output and GDP declined slightly in 2016, partially due to the "de-capacity and de-stocking" policy at the end of 2015. However, the overall economic situation is showing an upward trend.

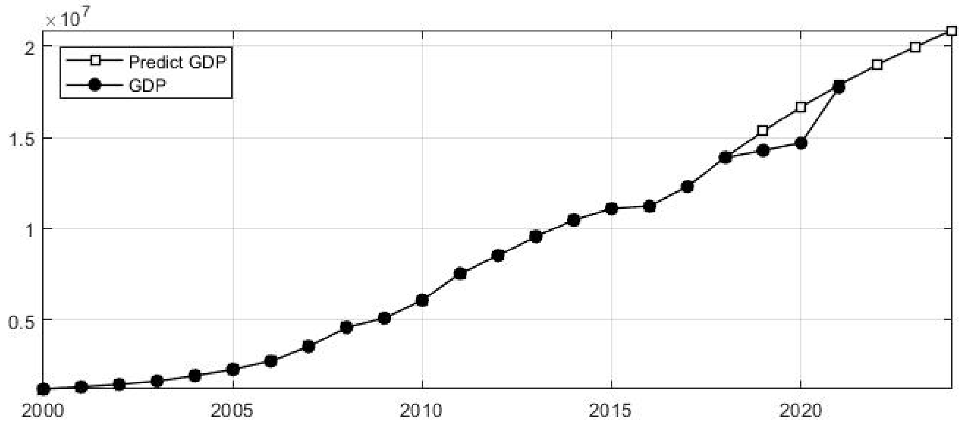

Figure 13: Nominal GDP and predicted GDP

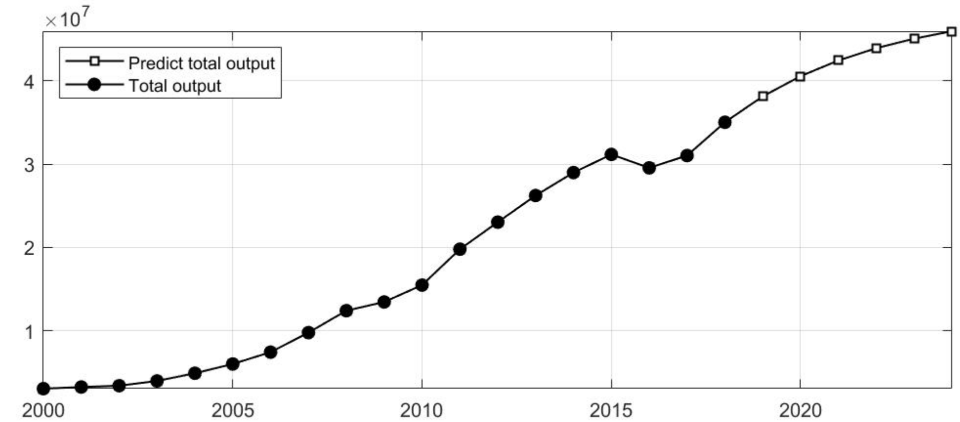

Figure 14: Total input and predicted total input

Figure 15 shows the relative growth of GDP and total output. The largest increase in total output occurred in 2010, reaching 4, 305, 444 million USD. The largest increase in GDP occurred in 2017, reaching 1, 584, 266 million USD.

According to the research, in nominal GDP in 2015, the total fixed capital

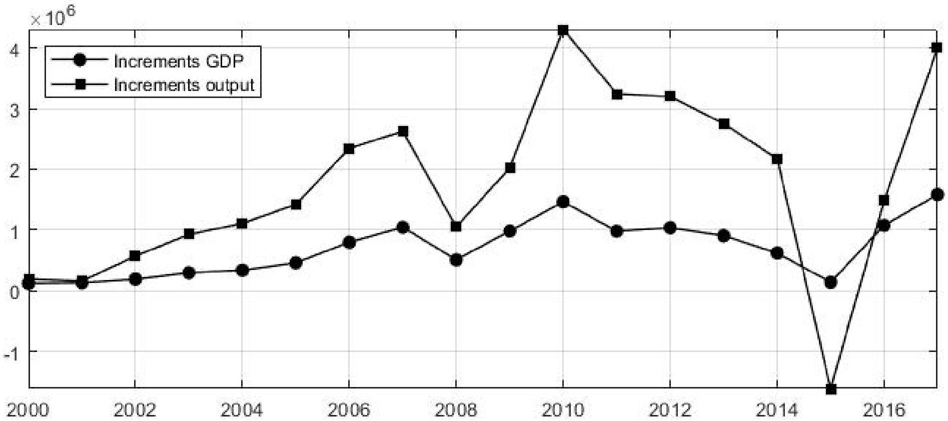

Figure 15: The incremental values of GDP and total output from 2000 to 2017

formation required to increase unit total output was the highest, while the total fixed capital formation required to increase unit GDP was the lowest. In actual GDP, the capital intensity measured by GDP in 2015 was negative, which implies deflation in 2015. Based on the capital intensity from 2000 to 2018, we predict that the capital intensity in 2020 will be higher than that in 2019 and 2021. The predicted nominal GDP from 2000 to 2018 is consistent with the published nominal GDP. The predicted GDP for 2019, 2020, 2021, and 2022 differs from the published nominal GDP by 7.4%, 13.3%, 0.6%, and 3.3%, respectively. The difference in 2020 is the largest, partly due to the impact of COVID-19. Overall, the economic situation is expected to improve. I predict that the predicted nominal GDP for 2023 and 2024 will reach US $199,494.50$ million and US $208,602.67$ million, respectively.

4.4 Dynamic input-output analysis of the United States

Based on the input-output data for the United States published by OECD and following the approach of Dimitris and Gao, 45 sectors were aggregated into three sectors, namely Advanced manufacturing industry (consisting of 12 sectors), Modern service industry (consisting of 12 sectors), and Other industries (consisting of 23 sectors), as detailed in the appendix, using the general code provided in Section 3.2.1. Formulas (37) and (38) were used to predict the economic performance of the MSI and AMI. Similarly, linear regression was employed to forecast the capital density Fe_i and Fe_{n+1} under inflation, and time series analysis was used to predict GDP from 2019 to 2024 and the total output of the three aggregated sectors.

4.4.1 Predicting US Capital density

Based on Figure 16, looking at historical years, in 2007, the AMI and MSI had the highest capital expenditures required to increase unit total output, while in 2001 and 2008, these sectors had the lowest capital expenditures required. The low capital expenditure required for AMI in 2001 was due to the sluggish development of the manufacturing industry. The global financial crisis of 2007-2008 was also an important factor that contributed to the lowest capital expenditures for the MSI in 2008. Looking at the predicted values, considering inflation, the capital density of the AMI is always higher than that of the MSI and other industries.

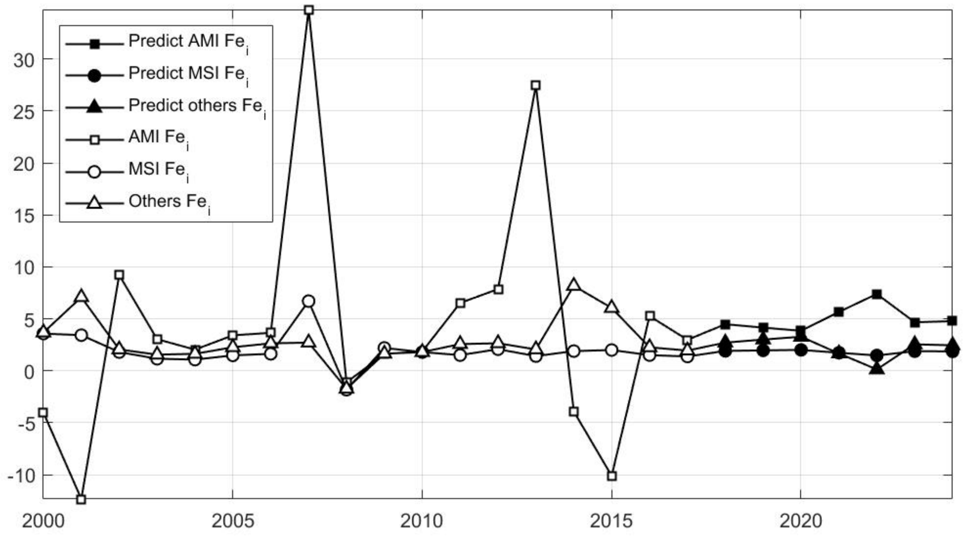

Figure 16: Predicted values of capital density Fe_i under real GDP

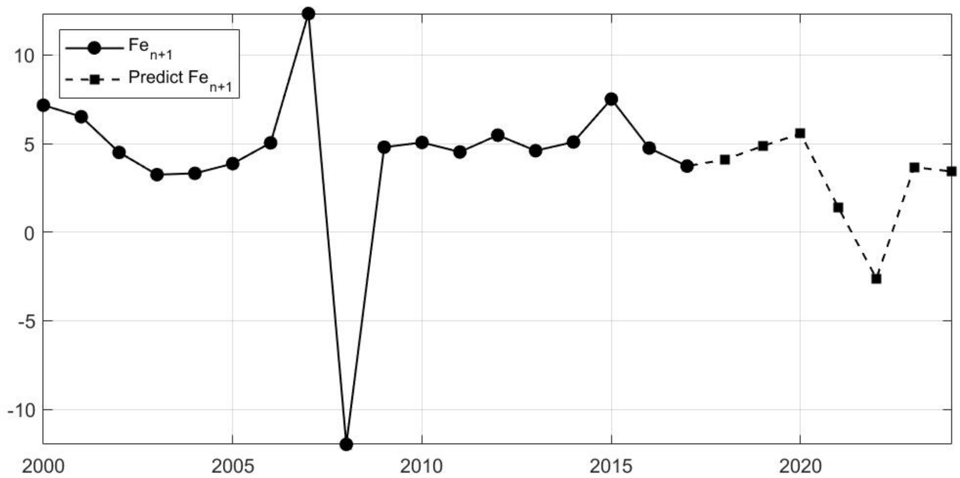

Figure 17: Predicted values of capital density Fe_{n+1} under real GDP

4.4.2 Predicting the US GDP and the total output among sectors

According to Figure 18, the nominal GDP from 2000 to 2018 is consistent with the predicted GDP. Currently, the US government has only released GDP data up to 2021. Our predicted values for 2019, 2020, 2021, and 2022, are expected to reach $215,906.93$ million, $224,878.63$ million, $233,095.49$ million, and $240,621.01$ million USD respectively, with percentage differences of 0.97%, 6.34%, 0.04%, and 1.84%. The predicted nominal GDP for 2023 and 2024 will be 247, 513.36 million and 253, 825.81 million USD, respectively.

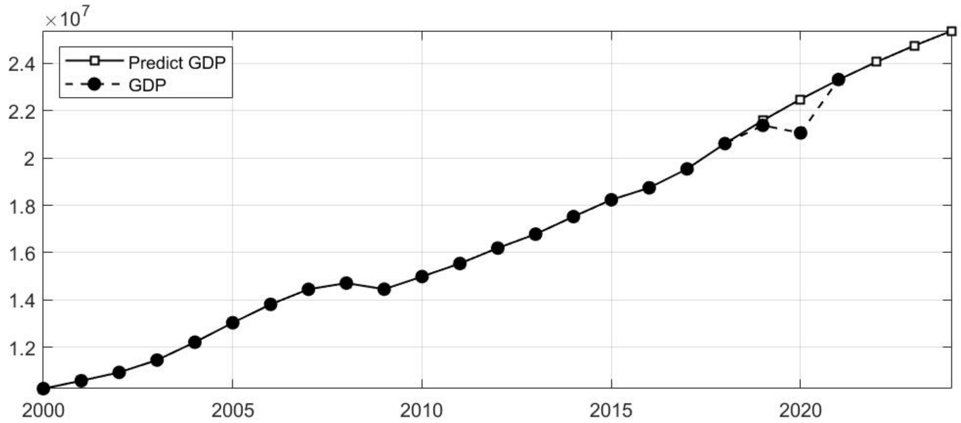

Figure 18: Nominal GDP and predicted GDP

Based on Figure 19, looking at the historical data from 2000 to 2018, the real and predicted values were consistent. Overall, the total output of the modern service industry is always higher than that of the advanced manufacturing industry. The year 2008 is a turning point for the two industries, and the obvious reason is the impact of the financial crisis. In the predicted values for 2019-2024, the total output of the modern service industry is predicted to increase, while the total output of the advanced manufacturing industry is predicted to decrease. This also confirms that the modern service industry contributes more to the economy of a developed country.

4.5 Comparison between China and the United States

Firstly, the dynamic input-output model is feasible and practical. Based on the largest aggregation result of input-output sectors in China, the maximum aggregation total output, GDP, and predicted total output, GDP are equal. The prediction errors are within an acceptable range, indicating that the

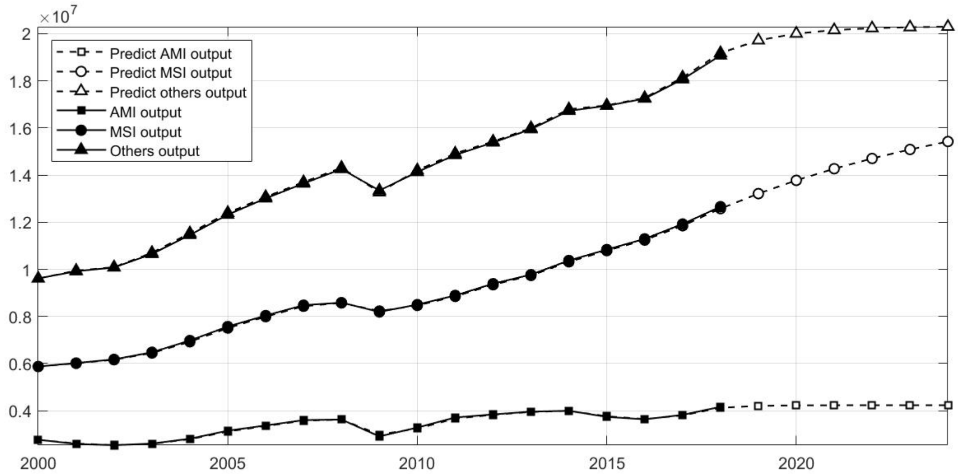

Figure 19: Total inputs and predicted total inputs of the 3 sectors

model is accurate.

Secondly, overall, the capital density of the United States, i.e., the total amount of fixed capital formation required to increase unit GDP, is higher than that of China, such as in 2007 and 2015.

Thirdly, China's GDP growth rate is higher than that of the United States, especially after 2010. In 2020, the economies of both countries declined compared to the previous year, partly due to the impact of the COVID-19 pandemic in the fourth quarter.

Fourthly, the contribution of the modern service industry to GDP in the United States is higher than that of the advanced manufacturing industry, while in China it is the opposite.

For China, it is essential to maintain an open and sharing attitude, actively promote digital transformation and upgrading, increase investment in modern service industries such as science and technology, and promote industrial transformation and upgrading.

For the United States, it is necessary to maintain an open attitude, relax trade regulations, actively promote international economic and technological cooperation, and promote stable and positive economic development.

5 Program control of equilibrium economy

For the linear control characteristics present in the dynamic input-output model, one uses classical program control and imposes control on advanced manufacturing based on linear non-homogeneous differential equations involving the three sectors and GDP. To this end, the control equations influenced by multiple factors are obtained, and the effectiveness of the program control is verified by numerical calculations using Python.

Given that the value added in China's input-output table released by the OECD does not list its detailed components. To this end, based on formula (42) and the U.S. input-output model constructed in Section 4.4, one exerts controls on advanced manufacturing, modern service industry, and other advanced manufacturing industries aggregated in these three industries from 2017 to 2018.

5.1 Program control problem statement

Control system:

$$\dot{X}(t) = \mathbf{D}X(t) + \mathbf{Q}u + f(t), \tag{46}$$

where

$\mathbf{X} = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$ - state vector, T- transposition

$\mathbf{u} = (u_1, \ldots, u_r)^T \in \mathbb{R}^r$- control vector,

$\mathbf{P}(t)$, $\mathbf{Q}(t)$ are $(n \times n)$ and $(n \times r)$ matrices with continuous components,

$\mathbf{f}(t)$ - n-dimensional continuous vector function - a disturbance.

This is a particular case of continuous one when $\mathbf{D}$ = constant, $\mathbf{Q}$.

The general solution of the Cauchy form of system (46) is:

$$x\left(t,0,x_0\right)=Y(t)\left[x_0+\int_0^t Y^{-1}(\tau)(Q(\tau)u(\tau)+f(\tau))d\tau\right]. \qquad (47)$$

Let two points x_0, x_1 and interval $t\in[0,T]$ are given. It is necessary to find an admissible control $\mathbf{u}(t)$ such that

$$\mathbf{x}\left(T,\mathbf{x}_0,\mathbf{u}(\cdot)\right)=\mathbf{x}_1. \qquad (48)$$

The pair $\mathbf{x}_0$, $\mathbf{x}_1$ is said to be controllable on interval $t\in[0,T]$ if there exists an admissible control that is a solution of equation (47).

5.2 Program control algorithm

① Check the complete controllable of the system.

Stationary systems: the Kalman criterion. rang $\left[Q,DQ,\ldots,D^{n-1}Q\right]=n$.

Nonstationary systems: Sufficient conditions for complete controllability. $\operatorname{rang} S\left(t_*\right)=n$.

② Calculate the fundamental matrix $\mathbf{Y}(t)$ of the homogeneous system $\dot{\mathbf{x}}=\mathbf{D}(t)\mathbf{x}$.

③ Construct the matrix $\mathbf{B}(t)=\mathbf{Y}^{-1}(t)\mathbf{Q}(t)$.

④ Calculate the Gramian $\mathbf{A}(T)=\int_0^T\mathbf{B}(\tau)\mathbf{B}^T(\tau)d\tau$.

⑤ Calculate the vector $\eta=\mathbf{Y}^{-1}(T)\mathbf{x}_1-\mathbf{x}_0-\int_0^T\mathbf{Y}^{-1}(\tau)\mathbf{f}(\tau)d\tau$.

⑥ Solve the system $\mathbf{A}(T)\mathbf{c}=\eta$.

⑦ Solve the integral equation $\int_0^T\mathbf{B}(\tau)\mathbf{v}(\tau)d\tau=\mathbf{0}$

or take $\mathbf{v}(t)\equiv\mathbf{0}$.

⑧ Form the program control $\mathbf{u}(t)=\mathbf{B}^T(t)\mathbf{c}+\mathbf{v}(t)$.

5.3 The united states input-output program control

Based on Section 4.4, according to the calculation results of Python, we know the data ($n = 4$) of the United States in 2017-2018. We set control for the advanced manufacturing industry to obtain the following non-homogeneous equation.

$$\dot{X} = \begin{bmatrix} 0.0195 & 0.0013 & 0.0031 & 0.0047 \\ 0.0081 & 0.0155 & 0.0098 & 0.0151 \\ 0.0240 & 0.0067 & 0.0114 & 0.0336 \\ 0.0375 & 0.0379 & 0.0315 & 0.0011 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u,$$

$$x_0 = \begin{bmatrix} 3823518.3 \\ 11922242.2 \\ 18063252.8 \\ 19541523.1 \end{bmatrix}, x_1 = \begin{bmatrix} 4164850 \\ 12656899.5 \\ 19074639.5 \\ 20611103.4 \end{bmatrix}.$$

One investigates the system's complete controllability by establishing program control for a given point $\{x_0, x_1\}$ in the segment $[0, 1]$. In this case, the elements x_0 and x_1 indicate the total output of the advanced manufacturing industry, the total output of the modern service industry, others, and GDP in 2017 and 2018.

$$D = \begin{bmatrix} 0.0195 & 0.0013 & 0.0031 & 0.0047 \\ 0.0081 & 0.0155 & 0.0098 & 0.0151 \\ 0.0240 & 0.0067 & 0.0114 & 0.0336 \\ 0.0375 & 0.0379 & 0.0315 & 0.0011 \end{bmatrix}, Q = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, f(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$T = 1.\ x_0 = \begin{bmatrix} 3823518.3 \\ 11922242.2 \\ 18063252.8 \\ 19541523.1 \end{bmatrix},\ x_1 = \begin{bmatrix} 4164850 \\ 12656899.5 \\ 19074639.5 \\ 20611103.4 \end{bmatrix}.$$

First, the complete controllable of the system needs to be checked. Since it is stationary, we use the Kalman criterion ($det\ D \neq 0$).

$$Rank(Q, DQ, D^2Q, D^3Q) = \begin{bmatrix} 1 & 0.0195 & 0.00064143 & 0.00002 \\ 0 & 0.0081 & 0.00108495 & 0.00006 \\ 0 & 0.024 & 0.00108495 & 0.00010 \\ 0 & 0.0375 & 0.00182829 & 0.00013 \end{bmatrix} = 4.$$

Therefore, it is completely controllable.

Because of the complexity of the calculation equation, we omit the intermediate steps and directly give the control u(t). Here, we substitute the exponent e for 2.7183 and provide a numerical solution to the problem. The code shows the entire computation procedure, see [Dan, 2023].

$u(t) = [[1.61183421082146e + 18 \times ((-0.7006 \times 2.7183^{0.003t} + 0.0113 \times 2.7183^{0.015t} + 0.3318 \times 2.7183^{0.059t} + 0.3574/2.7183^{0.03t}) \times (-0.2937 \times 2.7183^{0.003t} \cdots) + \cdots + (\cdots + 0.6445/2.7183^{0.03t}) \times (0.4216 \times 2.7183^{0.003t} + 0.0018 \times 2.7183^{0.015t} + 0.3076 \times 2.7183^{0.059t} + 0.269/2.7183^{0.03t}) \times (0.4881 \times 2.7183^{0.003t} + 0.218 \times 2.7183^{0.015t} + 0.217 \times 2.7183^{0.059t} + 0.076/2.7183^{0.03t})))]].$

Now, we need to verify the effectiveness of this program control. Table 7 shows the errors between the aggregated advanced manufacturing industry, modern service industry, others, and the GDP of the United States in 2018 and the controlled items.

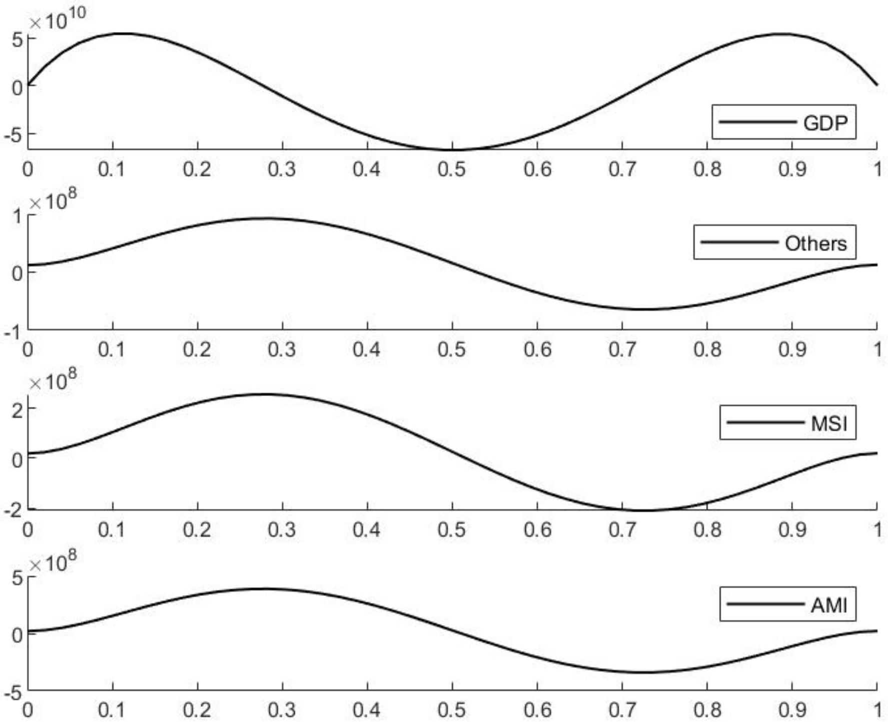

Figure 20: The control process of total output and GDP among 3 sectors in 2017-2018

Table 7: The error rate between the control total output and the actual total output

Year	2018 actual total output	2018 controlled total output	Error rate
AMI	4164850	4165722.87	0.02%
MSI	12656899.5	12667558.54	0.08%
Others	19074639.5	19085804.38	0.05%
GDP	20611103.4	20620479.56	0.04%

For the trend of this control, we will conduct further research in the future.

6 Input-output model of saddle point equilibrium strategy

Based on the[Qu, 1999], [Kang, 1992] and [Mao, 1992]'s opinion, this part discusses the continuous dynamic input-output model under the worst interference of unknown factors using the concept of game theory. It also considers the random uncertain components of the national economy in great detail. Based on formula (6), we can write the general dynamic input-output model of the Leontief. The input-output process of each sector is a dynamic economic activity process that defines the output level of each sector and the final consumption products as a function of time t.

$$X(t) = AX(t) + B\dot{X}(t) + Y(t), \tag{49}$$

where, $t \in [t_*, t^*]$(product planning period), n dimension vectors $X(t)$ and $Y(t)$ respectively is the output vector and the final consumption product vector (excluding the investment part). The n time-invariant matrix A is the direct consumption coefficient matrix, whose elements satisfy $\sum a_{ij} < 1$ and $0 \leqslant a_{ij} < 1, i, j = 1, 2, \cdots, n$. $\dot{X}(t)$ is the rate of change of investment in each sector. The n time-invariant matrix B is the investment coefficient matrix.

First, one considers $\dot{X}(t)$ as the control strategy $u(t)$ of player 1, and takes the uncertain factor, the random variable $z(t)$ as the control strategy of player 2, thus forming a linear quadratic differential game problem. Then the optimal control strategy of the dynamic input-output system is obtained by solving the saddle point equilibrium strategy.

6.1 The game model of dynamic input-output system

Due to the complexity of real economic activities, formula (49) cannot describe real economic laws very well. Therefore, we amend the above model and introduce a random factor used to represent uncertain factors in social and economic activities variable $z(t)$, thus obtaining a formula (50).

$$X(t) = AX(t) + B\dot{X}(t) + Y(t) + z(t). \tag{50}$$

The investment change rate $\dot{X}(t)$ of each sector determines the change in the output capacity of each sector, and the final consumption product vector $Y(t)$ can be controlled by adjusting it. Therefore, the above continuous type dynamic input-output model (50) is transformed into the following form of state space description:

$$\begin{cases} \dot{X}(t) = u(t) \\ Y(t) = (I - A)X(t) - Bu(t) - z(t) \end{cases} \tag{51}$$

Among them, $t \in [t_*, t^*]$. I is the identity matrix of order n. $u(t) \in R^n$ is the decision-making control variable of player 1, $z(t) \in R^n$ is the "natural" decision-making control variable of player 2, output level vector $X(t)$ is the state variable of the system, and final consumption product vector $Y(t)$ is the output variable of the system.

We use the n dimensional column vector $G(t)$ to represent the social demand vector for the product, assuming it is a known continuous function vector. When the national economy is in a dynamic equilibrium, the social demand vector $G(t)$ is equal to the system output vector $Y(t)$. However, it is difficult to eliminate the supply-demand imbalance. At this time, on the one hand, we hope to make the performance index $J(u, z)$ obtain a minimum value by adjusting the control variable $u(t)$, and on the other hand, it is hoped that under the worst interference of the random factor

$z(t)$, the performance index $J(u,z)$ takes the maximum value, where

$$J(u,z)=\tfrac{1}{2}\int_{t_*}^{t^*}[Y(t)-G(t)]^{\mathrm{T}}P[Y(t)-G(t)]+u^{\mathrm{T}}(t)Ru(t)dt, \qquad (52)$$

where, P and R are positive definite matrices of order n respectively.

Their practical significance is to distinguish between the difference between the number of final consumer products provided by each sector and the social demand, and the difference in the primary and secondary degrees required by the changes in the output capacity of each sector. At this point, a complete dynamic input-output system game model is formed. In the following chapters, we will use the method of solving the saddle point equilibrium strategy to solve this dynamic input-output problem.

6.2 Optimal control construction

The above model is a normal linear quadratic differential game model. Next, we use the result of Deissenberg to solve it, and construct the Hamilton function of the system (52) as follows:

$$H(X,u,Z,\theta,t)=\frac{1}{2}[Y(t)-G(t)]^{\mathrm{T}}P[Y(t)-G(t)]+\frac{1}{2}u^{\mathrm{T}}Ru+\theta^{\mathrm{T}}u. \qquad (53)$$

Then,

$$\begin{aligned}
\tfrac{\partial H}{\partial u}&=\left(R+B^{\mathrm{T}}PB\right)u-B^{\mathrm{T}}P(I-A)X+\lambda+B^{\mathrm{T}}PG(t)+B^{\mathrm{T}}Pz,\\
\tfrac{\partial H}{\partial z}&=Pz-P(I-A)X+PG(t)+PBu,\\
\tfrac{\partial H}{\partial X}&=(I-A)^{\mathrm{T}}P(I-A)x-(I-A)^{\mathrm{T}}PBu-(I-A)^{\mathrm{T}}Pz-(I-A)^{\mathrm{T}}PG(t).
\end{aligned}$$

According to the minimum principle, the optimal control of the system (52) $u^*(t)$, $z^*(t)$ and the optimal trajectory $X^*(t)$ and the corresponding co-state variable $\theta^*(t)$ satisfy:

$$\begin{aligned}\dot{X} &= u, X(t_0) = X_0,\\ \dot{\theta} &= -(I-A)^{\mathrm{T}}P(I-A)X + (I-A)^{\mathrm{T}}PBu + (I-A)^{\mathrm{T}}Pz+\\ &\quad (I-A)^{\mathrm{T}}PG(t), \theta(t^*) = 0,\\ u &= -R^{-1}\theta,\\ z &= (I-A)X - BR^{-1}\theta - G(t).\end{aligned} \tag{54}$$

Let:

$$\theta(t) = S(t)X(t) + v(t). \tag{55}$$

Then after proper calculation, $S(t)$ and $v(t)$ respectively satisfy the following matrix Riccati differential equation:

$$\begin{aligned}&\dot{S}(t) + (I-A)^{\mathrm{T}}PBR^{-1}S(t) + S(t)R^{-1}B^{\mathrm{T}}P^{\mathrm{T}}(I-A) - S(t)R^{-1}S(t) = 0,\\ &S(t^*) = 0.\\ &\dot{v}(t) + (I-A)^{\mathrm{T}}PBR^{-1}r(t) + v(t)R^{-1}B^{\mathrm{T}}P^{\mathrm{T}}(I-A) - S(t)R^{-1}v(t) = 0,\\ &v(t^*) = 0.\end{aligned} \tag{56}$$

Substituting equation (55) into equation (54), the optimal control law of the system is:

$$\begin{aligned}u^*(t) &= -R^{-1}S(t)X(t) - R^{-1}v(t),\\ z^*(t) &= \left[I - BR^{-1}S(t)\right]X(t) - BR^{-1}v(t) - G(t),\end{aligned} \tag{57}$$

where, $S(t)$ and $v(t)$ are uniquely determined by equation (56).

Hence, this section uses the saddle point equilibrium theory in the differential game to study the multi-sector dynamic input-output problem in macroeconomic decision-making and designs the realization method of the control strategy.

7 Conclusion

This master thesis first establishes a static input-output model in the third part based on the input-output table issued by the National Bureau of Statistics of China. I considered the direct consumption coefficients of m departments and analyzed the one-way fusion degree, fusion interaction degree, and comprehensive fusion degree. Through the multi-dimensional quantification of China's "modern service industry" and "advanced manufacturing industry", I found that the integration of "two industries" has achieved some results, but the contribution of the modern service industry to the advanced manufacturing industry is still limited; traditional advantageous industries are still promising. In this regard, a modern service industry demand system should be built and improved to open up the double cycle of "two industries"; through strengthening industrial linkages and strengthening the innovation linkages of "two industries", the deep integration of "two industries" should be realized.

The fourth part of this article establishes the dynamic input-output model of China and the United States based on the OECD. Through this model and the least squares method, the capital density of the two countries underinflation and non-inflation is predicted. For China's input-output table, Considering the maximum aggregated total output, use the time series to predict the maximum aggregated total output and the development trend of GDP from 2019 to 2024; similarly, the same is true for the US input-output table, and finally compare the differences between the two countries. Through my research, I found that the contribution of the modern service industry in the United States to GDP is higher than that of advanced manufacturing, and the opposite is true in China. For China, it is even more necessary to maintain an open and shared attitude and actively promote digital transformation and upgrading, increase investment in modern service industries such as science and technology, and promote industrial transformation and upgrading. For the United States, it should relax trade controls, actively promote international economic and technological cooperation, and promote sound economic development.

The classical program control theory is employed in the fifth section of this study. Based on the linear non-homogeneous differential equation including three sectors and GDP, I exerted control on the aggregated advanced manufacturing industry in the United States from 2017 to 2018, obtained the control equation of multi-factor influence, and used Python to verify the program control through numerical calculation effectiveness.

Finally, the continuous optimal strategy design problem is presented based on Leontief's dynamic input-output model. The dynamic input-output system is abstracted as a saddle point equilibrium theory, and a new approach for addressing the dynamic input-output problem is created utilizing the saddle point equilibrium strategy.

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