“Waring type problem with prime variables” by Ivan Ermoshin

J. Friedlander and H. Iwaniec established a conditional asymptotic formula for the mean value of $\Lambda(n)r_0(n-2)r_0(n+2)$, where $\Lambda(n)$ is the von Mangoldt function and $\Lambda(n) = \log n$ for $m = p^k$ with p a prime and is zero otherwise, $r_0(n)$ is the number of ways to write n as a sum of two positive squares. This is equivalent to showing a strengthening of Lagrange’s four squares theorem, i.e. finding an asymptotic formula for $p = a^2 + b^2 + c^2 + d^2$ subject to hyperbolic condition $ad - bc = 1$. Here we aim at studying an analogous sum when one of the variables is restricted to prime, i.e. $\Lambda(n)r_1(n-2)r_0(n+2)$, where $r_1(n)$ is the number of ways to represent n as a sum of a square and a square of a prime.

Traditionally this kind of problems can be viewed via circle method, however here it fails to produce any upper or lower bound for the quantity in question as the sum in question corresponds to very sparse sequence, moreover the hyperbolic condition on the variables can be hardly implemented into the traditional tools. Instead, the proposed path includes half dimensional sieve technology, various estimates for the error term in prime number theorem in arithmetic progressions, splitting the divisors via the divisor trick by Hooley and so on. Note that due to the nature of the sequences in question the celebrated Bombieri-Vinogradov theorem can not be successfully applied here, thus one has to deal rather carefully with the remainder terms at each step, which is, indeed, one of the main features of such bounds.

As usual sieve would ideally allow to capture primes via $\Lambda(n)$ provided that one has enough information on the distribution of $a(n) = r_0(n-2)r_0(n+2)$ in an arithmetic progression $n \equiv 0(d)$ for square-free d. In doing that Ivan gets back to his work last year and follows the approach of S. Daniel to r_1^2 and A. Sedunova to r_0r_1. as well as applies certain variations of Hensel’s lemma. He obtains the “acceptable” level of the distribution that can be applied to get the upper bound, however the lower bounds are way trickier as usual and require more detailed analysis. I would like to emphasize that in the course of this work Ivan has successfully studied and learnt how to generalize and apply tools from three different rather complicated works: “Hyperbolic prime number theorem” by J. Friedlander and H. Iwaniec, “On the sum of a square and a square of a prime” by S. Daniel and “Intersections of the binary quadratic forms in primes and the paucity phenomenon” by A. Sedunova.

In my opinion the thesis deserves an excellent grade, and its author deserves a bachelor’s degree in mathematics.

Работа достойна оценки отлично, а её автор заслуживает присвоения степени бакалавра.

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