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Optimization approach to the design of nonlinear control system controllers

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The optimization approach is applied to the synthesis and optimization of nonlinear real-time feedback optimal control system of a certain Maglev platform. To optimize the nonlinear control law, the integral functional criteria is minimized, which evaluates the quality of the dynamics of not one trajectory, but an ensemble of nonlinear trajectories of the system. The considered ensemble of trajectories covers the entire area of the engineering gap between the platform and the guide rails. In this area the magnetic forces provide highly nonlinear effects due to the considered design features of the object. At the same time, it is required to provide the stabilization within the entire engineering gap. It makes this statement to be a multi-input nonlinear control problem. The components of the feedback control law vector have a polynomial form of the state-space variables. As a result of computational optimization of trajectories ensemble, a class of Pareto-optimal polynomial regulators is constructed for considered control object. In the presented set, each Pareto-optimal point corresponds to a specific designed controller and investigated functional criteria which evaluates the entire ensemble of perturbed nonlinear trajectories. This allows a research engineer to choose various nonlinear regulators and achieve a compromise between stabilization accuracy and energy costs.

Keywords: nonlinear system, stabilization, nonlinear regulators, Maglev, real-time feedback, ensemble of trajectories, optimization.

1. Introduction. The development of nonlinear controller design for nonlinear systems is a relevant problem for various theoretical and practical studies. The paper develops a parametric optimization approach that extends the capabilities of existing tools and methods for the regulators constructing. Previously, this approach has been successfully used in solving problems of analysis and synthesis of stabilizing regulators for plasma shape and current in tokamaks [1, 2]. In that researches the linear systems have been investigated. A variety of examples of objects can be covered by parametric optimization [3, 4]. It should be noted that the optimization approach based on the use of an ensemble of trajectories and the quality functional specified on these trajectories is widely used in solving other problems. In particular, these are problems of electrodynamics [5–7] and image processing in the reconstruction of the velocity field [8–10].

Magnetic levitation or Maglev is a modern technology that uses directed upwards magnetic forces to balance the dominant downward force of gravity. It was applied to a wide range of fields in science and technology starting with hanging a small laboratory-scale object and finishing with large-scale transportation applications like Maglev vehicles suited to carry huge weights at speeds up to several hundred kilometers per hour.

There are two common suspension system designs widely used for Maglev vehicle applications. The first one is named an electrodynamic suspension (EDS) system and uses the repulsive force created by magnets moving relative to electrical conductors. The other one is named an electromagnetic suspension (EMS) system and uses the force of attraction between magnets and ferromagnetic materials. The gravity force is balanced differently in these cases due to the various nature of magnetic forces. The EDS force nature bases on the air gap, the speed of the vehicle, the electroconductivity of the material in the track and the source of the magnetic field. Therefore levitation has place as long as vehicle moves at significant speed (e. g. Inductrack [11]).

Opposing to EDS systems, the EMS force relies only upon the air gap between the track and the magnet and the magnetic field value created by the electromagnet. This system is capable of providing levitation both in static conditions and in the motion (e. g. Transrapid [12]).

Hanging frame of the rolling stock above the track with EMS faces inherent instability. It takes exact balance between magnetic attraction forces and the force of gravity for an air gap between track structure and the magnet to exist. However, with a small deviation of the air gap size, the attractive force between the short-circuited track coil and the onboard magnetic system rises or weakens accordingly. Therefore the object either grips or falls. The single way to provide stable levitation, in the EMS case, is automatic active control by adjusting the current supplied to an electromagnet's windings in response to signals that are fed back to the magnet's power supply from a sensor that consistently reads the air gap.

The mentioned features make providing stability to a Maglev system to be a difficult nonlinear control task. Different methods have yet been proposed to control magnetic levitation. Primarily, a linearized model about a nominal operating point, that is a quite common technique. It was demonstrated in lots of realistic applications [2]. However, in our case high nonlinearity of Maglev systems makes the regulator performance degrade swiftly with rising deviations from the nominal operating point.

Nonlinear techniques have been used by some authors to design stabilizing control laws. Nonetheless, in most studies methods are designed either for small levitating objects [13] or for inappropriate models in respect of physics and magnetic features of the system. There are markable proposed methods of nonlinear control for stabilizing levitating objects such as nonlinear model predictive control [14, 15], backstepping-technique [16] feedback linearization [13, 17], and sliding mode control strategies [18, 19].

The results of this work extends the research in [20]. In paper [20] the control law has a polynomial form. To achieve such a polynomial form the stable manifold method was used. The authors applied the analysis of the dynamics of a nonlinear ensemble of trajectories, but it was only to compare qualitative and quantitative characteristics, and ensemble optimization has not been applied yet. The novelty of the current work is that the functional criterion is investigated analytically and a variation of the functional criterion by parameters is obtained. The polynomial control law is optimized to minimize the integral functional criterion, which evaluates the dynamics quality of not the one trajectory, but an ensemble of nonlinear perturbed trajectories of the system. The considered ensemble of trajectories covers the entire area of the engineering gap between the platform and the guide rails. The presented analytical expressions of the variation of the functional criterion by parameters allow to implement various gradient and directed optimization methods. As a result of computational optimization of trajectories ensemble, a class of Pareto-optimal polynomial regulators is constructed for considered control object. In the presented set, each Pareto-optimal point corresponds to a specific controller and investigated functional criteria which evaluates the entire ensemble of perturbed nonlinear trajectories.

Two vertical cross-sections of the considered Maglev platform are shown by Figure 1. JSC NIIEFA proposed [21] suspension design of the object that contains the so-called reference magnets, which can be made on the basis of permanent and/or superconducting magnets, in addition to conventional electromagnets. The main lifting force necessary for levitation is created by reference magnets while electromagnets maintain the required value of the air gap — perform stabilizing function. The real experiments validated proposed construction and allowed to measure the dependence between magnet forces and the air gap. JSC NIIEFA proposes analogical approach in the another area of application as well [21].

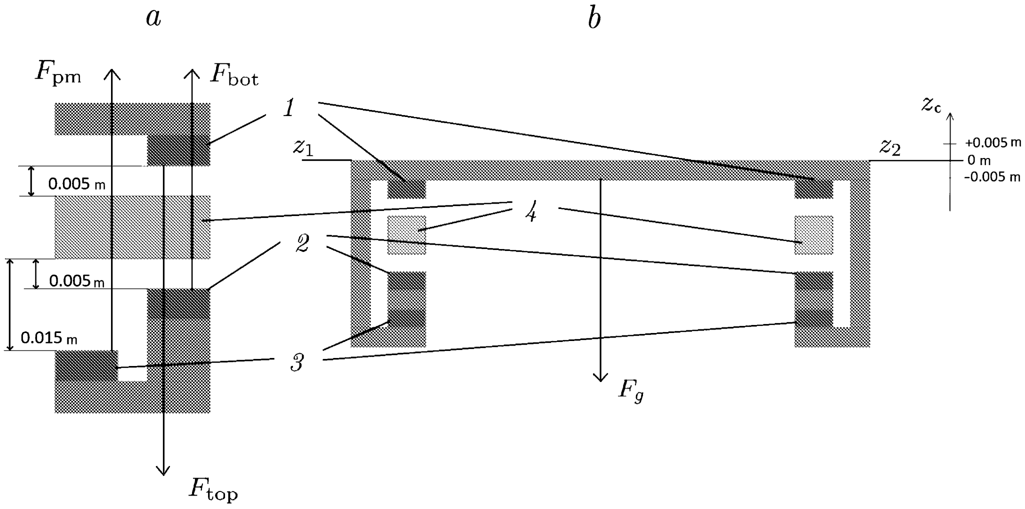

Figure 1. Vertical sections of the Maglev platform with a combined suspensions
a — side view; b — front view: *1* — upper electromagnet; *2* — lower electromagnet; *3* — permanent magnet; *4* — ferromagnetic guideway; z_1, z_2 — vertical positions of combined suspensions; F_{top} — electromagnetic force of the upper magnet; F_{bot} — electromagnetic force of the lower magnet; F_{pm} — magnetic force of the permanent magnet; F_g — gravity force.

The paper is organized as follows. Section 2 describes a physical model of the real levitating platform and introduces its state-space representation in deviations with the assumption that the platform moves only in a single plane. Thus, its position in space is defined by the offset of vertical z coordinate and roll angle. In Section 3 the problem of parametric optimization of nonlinear trajectories ensemble is solved. Essential analytical expressions are presented. Section 4 presents the computational results of optimization of nonlinear trajectories ensemble, includes a comparative case, and illustrates the new class of optimized polynomial controllers.

2. Maglev platform dynamics model. The model of dynamics of the object on the Figure 1 was proposed and described in [22, 23]. In these papers the mass m [kg] and width w [m] of the platform are specified. The vertical position characteristics of the electromagnetic suspensions are z_1 [m] and z_2 [m]. The forces provided by two electromagnetic suspensions depend on the supplied control signal $u = (u_1, u_2)^* \in \mathbb{R}^2$ respectively, the symbol $*$ is labeling the matrix transpose operation.

The considered Maglev platform dynamics is described by its state space vector $x \in \mathbb{R}^4$ consisting of Euler angle θ [rad] and vertical position of the platform center z_c [m], angular velocity $p = \dot{\theta}$ [rad/s] and vertical velocity $v_c = \dot{z}_c$ [m/s],

$$x = (x_1, x_2, x_3, x_4)^* = (\theta, z_c, p, v_c)^*.$$

The dynamics model of the object in deviations can be written in general form by the system

$$\begin{gathered} \dot{x} = f(x) + g(x)u, \\ y = D(x) \end{gathered} \tag{1}$$

with initial condition

$$x(0) = x_0. \tag{2}$$

In formulas (1), (2) x_0 is the initial deviation, $u = (u_1, u_2)^* \in \mathbb{R}^2$ is the control vector, $y = (z_1, z_2)^* \in \mathbb{R}^2$ is the output vector of observation equations with $D(x)$ defined by

$$D(x) = \begin{pmatrix} z_c + \frac{w \tan\theta}{2} \\ z_c - \frac{w \tan\theta}{2} \end{pmatrix},$$

matrix $g(x)$ has the form

$$g(x) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{F_{em}(z_c + \frac{w \tan\theta}{2})}{2mw} & -\frac{F_{em}(z_c - \frac{w \tan\theta}{2})}{2mw} \\ \frac{F_{em}(z_c + \frac{w \tan\theta}{2})}{2m} & \frac{F_{em}(z_c - \frac{w \tan\theta}{2})}{2m} \end{pmatrix},$$

vector function $f(x)$ is denoted by

$$f(x) = \begin{pmatrix} p \\ v_c \\ \frac{F_{pm}(z_c + \frac{w \tan\theta}{2}) - F_{pm}(z_c - \frac{w \tan\theta}{2})}{2mw} \\ \frac{F_{pm}(z_c + \frac{w \tan\theta}{2}) + F_{pm}(z_c - \frac{w \tan\theta}{2})}{2m} \end{pmatrix},$$

where F_{pm} is the permanent magnet force, and F_{em} is the component of electromagnet force which are derived in [24, 25] in the form

$$\begin{aligned} F_{\mathrm{em}}(z) &= \frac{k_1}{(z^2 + k_2 z + k_3)^2}, \\ F_{\mathrm{pm}}(z) &= \frac{p_1}{(z^2 + p_2 z + p_3)^2}, \end{aligned}$$

and coefficients p_j, $j = \overline{1,3}$, k_i, $i = \overline{1,3}$ were based on the technical documentations in [24] and [26]. Vector functions $f(x)$, $g(x)$, $D(x)$ are defined and continuous over the entire modeling interval $[0, T_{end}]$ and smooth enough. The end point T_{end} [s] is specified by the engineering requirements as the time of fading of all transition processes in the system (1).

We look for the stabilizing control law that provides stability for the system (1) and has polynomial form

$$u = f_{u_{poly}}(x) \equiv \begin{pmatrix} f^{(1)}_{u_{poly}}(x_1, x_2, x_3, x_4) \\ f^{(2)}_{u_{poly}}(x_1, x_2, x_3, x_4) \end{pmatrix}$$

with polynomials $f^{(1)}_{u_{poly}}, f^{(2)}_{u_{poly}}$.

3. Parametric optimization of ensemble of nonlinear trajectories. To provide stability of the system (1), (2) we parameterize the designed polynomial control law

$$u = f_{u_{poly}}(x) = \begin{pmatrix} f^{(1)}_{u_{poly}}(x_1, x_2, x_3, x_4) \\ f^{(2)}_{u_{poly}}(x_1, x_2, x_3, x_4) \end{pmatrix}, \tag{3}$$

and polynomial functions can be denoted as

$$f^{(i)}_{u_{poly}}(x_1, x_2, x_3, x_4) = \sum c^{(i)}_{m_1,m_2,m_3,m_4} x_1^{m_1} x_2^{m_2} x_3^{m_3} x_4^{m_4},$$
$$i = \overline{1,2},$$
$$m_1, m_2, m_3, m_4 \geqslant 0,$$
$$m_1 + m_2 + m_3 + m_4 \leqslant 3,$$

numbers $\{m_1, m_2, m_3, m_4\} \in M$, and M is a set of all possible combinations of monomials degrees, $c^{(i)}_{m_1,m_2,m_3,m_4} \in \mathbb{R}^1$ are coefficient of term with index (m_1, m_2, m_3, m_4) of i^{th} polynomial, $i = \overline{1,2}$. That set of coefficients $\{c^{(i)}_{m_1,m_2,m_3,m_4}\}$, $(m_1, m_2, m_3, m_4) \in M$, $i = \overline{1,2}$, are taken as varying parameters.

Then consider for system (1), (2) some set of initial deviations $\{x_0^{(j)}\}$, $j = \overline{1,N}$, where N is a number of ensemble trajectories which is perturbed by this initial set. System (1) can be represented in the form

$$\begin{gathered} \dot{x} = F(x) \equiv f(x) + g(x) f_{u_{poly}}(x), \\ y = D(x), \end{gathered} \tag{4}$$

with initial conditions

$$x(0) = \{x_0^{(j)}\}_{j=1}^N. \tag{5}$$

Then solutions of (4) compose an ensemble of trajectories $\{x^{(j)}(t)\}$, where $x^{(j)} \equiv x^{(j)}(t, x_0^{(j)})$, $j = \overline{1,N}$.

We introduce an integral quality criteria on the trajectories of the ensemble:

$$J_{ens} = \int\limits_0^{T_{end}} \frac{1}{N} \sum_{j=1}^N (y^{(j)^*} Q y^{(j)} + u^{(j)^*} R u^{(j)}) dt, \tag{6}$$

$y^{(j)} = y^{(j)}(t, x^{(j)})(t)$, $u^{(j)} = u^{(j)}(t, x^{(j)})(t)$, Q and R are weight matrices. Let us introduce the function $\varphi(x(t))$ as

$$\varphi(x^{(j)}(t)) \equiv D^*(x^{(j)}(t)) Q D(x^{(j)}(t)) + f^*_{u_{poly}}(x^{(j)}(t)) R f_{u_{poly}}(x^{(j)}(t)). \tag{7}$$

Thus, the functional integral criterion (6) can be represented in the form

$$J_{ens} = \int\limits_{0}^{T_{end}} \frac{1}{N}\sum_{j=1}^{N}\varphi(x^{(j)}(t))dt. \tag{8}$$

In order to minimize $J_{ens} \longrightarrow \min$ we are considering the set of polynomial coefficients $\{c_{m_1,m_2,m_3,m_4}\}$ as varied parameters, thus $J_{ens} = J_{ens}(\{c_{m_1,m_2,m_3,m_4}\})$.

In the books [6, 7, 27] it is presented a technique to obtaining the representation of the first variation for the integral functional criteria. Following this technique, we got a variation of functional (6) by the parameters when coefficient c_{m_1,m_2,m_3,m_4} is incremented by some Δc as $c_{m_1,m_2,m_3,m_4} + \Delta c$. We denote $\Delta_c F(x^{(j)}(t), \{c_{m_1,m_2,m_3,m_4}\})$ as $\Delta_c F(x^{(j)}(t), \{c_{m_1,m_2,m_3,m_4}\}) \equiv F(x^{(j)}(t), c_{m_1,m_2,m_3,m_4} + \Delta c) - F(x^{(j)}(t), c_{m_1,m_2,m_3,m_4})$, and $\Delta_c\varphi(x^{(j)}(t), \{c_{m_1,m_2,m_3,m_4}\})$, $\Delta_c f_{u_{poly}}(x^{(j)}(t), \{c_{m_1,m_2,m_3,m_4}\})$ are denoted analogically. Then the variation of functional (3) by the parameter can be represented in the form

$$\begin{gathered}\delta J_{ens}(\{c_{m_1,m_2,m_3,m_4}\}, \Delta c) = \int\limits_{0}^{T_{end}} \frac{1}{N}\sum_{j=1}^{N}\Big(-\psi^{(j)^*}(t, x^{(j)}(t)) \times \\ \times \Delta_c F(x^{(j)}(t), \{c_{m_1,m_2,m_3,m_4}\}) + \Delta_c\varphi(x^{(j)}(t), \{c_{m_1,m_2,m_3,m_4}\})\Big)\,dt,\end{gathered} \tag{9}$$

where

$$\begin{gathered}\Delta_c F(x^{(j)}(t), \{c_{m_1,m_2,m_3,m_4}\}) = g(x^{(j)}(t)) \times \Delta_c f_{u_{poly}}(x^{(j)}(t), \{c_{m_1,m_2,m_3,m_4}\}), \\ \Delta_c\varphi(x^{(j)}(t), \{c_{m_1,m_2,m_3,m_4}\}) = \Delta_c f^*_{u_{poly}}(x^{(j)}(t)) \cdot R \cdot \Delta_c f_{u_{poly}}(x^{(j)}(t)),\end{gathered} \tag{10}$$

and $\psi^{(j)}(t)$ are solutions of the conjugate differential system which are represented on the ensemble of trajectories as

$$\dot{\psi}^{(j)} = -\frac{\partial F}{\partial x}(x^{(j)}(t))\psi^{(j)} + \frac{\partial\varphi}{\partial x}(x^{(j)}(t)), \tag{11}$$

with terminal condition

$$\psi^{(j)^*}(T_{end}) = 0.$$

Taking into account the right part of system (4), it is possible to represent

$$\begin{gathered}\frac{\partial F(x)}{\partial x}(x^{(j)}(t)) = \frac{\partial f(x)}{\partial x}(x^{(j)}(t)) + \frac{\partial g(x)}{\partial x} f_{u_{poly}}(x^{(j)}(t)) + \\ + g(x^{(j)}(t))\frac{\partial f_{u_{poly}}(x)}{\partial x}(x^{(j)}(t))\end{gathered} \tag{12}$$

and

$$\begin{gathered}\frac{\partial\varphi}{\partial x}(x^{(j)}(t)) = \frac{1}{N}\sum_{j=1}^{N}(2D^*(x^{(j)}(t))Q\frac{\partial D(x)}{\partial x}(x^{(j)}(t)) + \\ + 2f^*_{u_{poly}}(x^{(j)}(t))R\frac{\partial f_{u_{poly}}(x)}{\partial x}(x^{(j)}(t)).\end{gathered} \tag{13}$$

Considering that the right parts of the above expressions (7)–(13) met the all necessary conditions [6, 27], thus, we have a representation for the criteria gradient:

$$\frac{\partial J_{ens}(\{c_{m_1,m_2,m_3,m_4}\})}{\partial c_{m_1,m_2,m_3,m_4}} = \int\limits_0^{T_{end}} \sum_{j=1}^{N} \Bigg(-\psi^{(j)*}(x^{(j)}(t)) \frac{\partial F}{\partial c_{m_1,m_2,m_3,m_4}}(x^{(j)}(t)) + \\ + \frac{\partial \varphi}{\partial c_{m_1,m_2,m_3,m_4}}(x^{(j)}(t)) \Bigg) dt. \tag{14}$$

This representation (14) allows one to apply various computational optimization methods, including gradient optimization and other directed optimization methods.

4. Numerical optimization results. Let $\{x_0^{(j)}\}_{j=1}^N$ be a set of initial deviations in (5) for the ensemble of N system's trajectories $\{x^{(j)}(t)\}$, where $x^{(j)} \equiv x^{(j)}(t, x_0^{(j)})$. The ensemble is simulated as a response of nonlinear system (4) closed with (3) to the set (5) of initial deviations from the *equilibriium position*. It seems interesting to compare the ensembles with the same initial conditions for various polynomial control laws. The initial approximation for the optimization process can be obtained with various ways. These can be various random search methods, for example [8], or it can be the polynomials obtained using the approach presented in [20, 28], as well as some other nonlinear methods.

The set of initial deviations is uniformly distributed in such a way that the deviations of air gaps in the output vector y fall in the range ± 4 mm: $|y_i^j(0)| \leqslant 0.004$ m, $i = \overline{1,2}, j = \overline{1,N}$, $N = 20$. The end of the simulation interval $T_{end} = 0.250$ s.

We compare the closed control system performance according to the quality criteria (6) formulated for the ensemble of transient processes perturbed by initial deviations (5) from the *equilibrium position*:

$$J_{\text{ens}} = \int\limits_0^{T_{end}} \frac{1}{N} \sum_{j=1}^{N} \Bigg(\underbrace{y^{(j)^*} Q y^{(j)}}_{\text{accuracy}} + \underbrace{u^{(j)^*} Q u^{(j)}}_{\text{energy costs}} \Bigg) dt, \tag{15}$$

where according to (4), $y^{(j)} \equiv y^{(j)}(t, x_0^{(j)}) \equiv D(x^{(j)}(t, x_0^{(j)}))$, Q and R are the weight matrices.

Practically, underbraced terms in (15) can be interpreted as a numerical estimation of the regulator accuracy J_{ac} and energy costs J_{ec} of entire ensemble. Thus, $J_{ens} = J_{ac} + J_{ec}$. Proposed optimization approach allows the an engineer to construct and choose the controllers whose trajectory ensembles are Pareto-optimal in terms of accuracy cost J_{ac} and energy cost J_{ec}. To find the points of such a set of Pareto, the researcher can vary the weight coefficients in (15).

Note, that interesting results were obtained in [20] and a numerical evaluation of the ensemble of nonlinear trajectories was carried out. But for the same initial data, these results are located above the Pareto curve of optimized controllers in the current work. This case is represented in Figure 2 as a *comparative case*.

Figures 3 and 4 illustrate the results of a numerical simulation with optimized regulator to stabilize the platform roll motion. Also, the simulation of the dynamics of current values in the coils of electromagnets has a practical interest during the process of stabilization. The dynamics of these values $I_{coils_1}^{(j)}(t)$, $I_{coils_2}^{(j)}(t)$ [A] has a complex dependence on the vertical position and supplied control values $(I_{coils_1}^{(j)}(t), I_{coils_2}^{(j)}(t))^* = V_{coils}(z^{(j)}(t), u^{(j)}(t))$

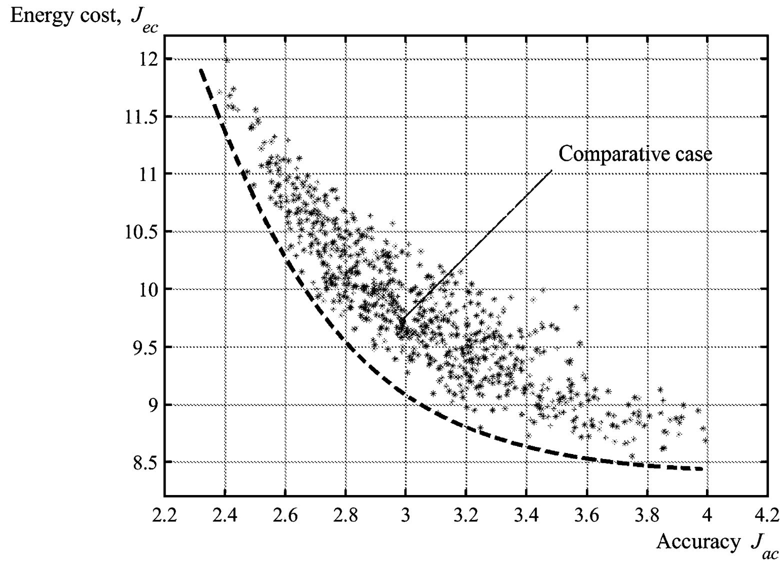

Figure 2. Visualization of the Pareto-optimal set of polynomial controllers

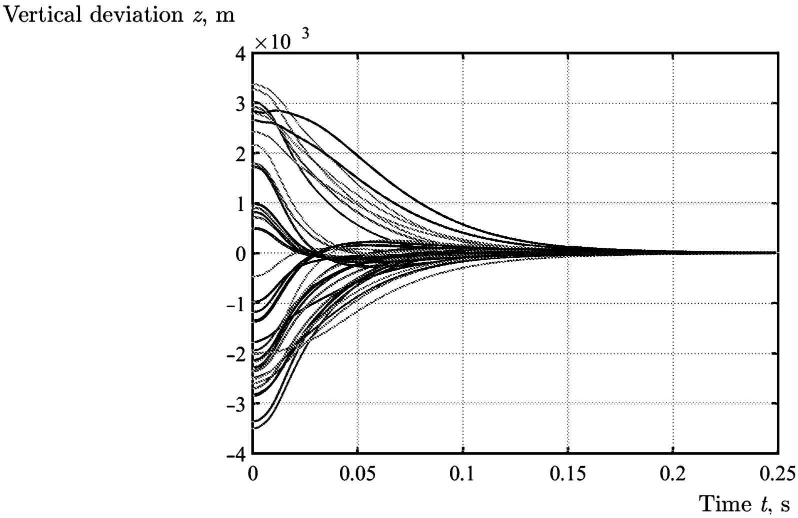

Figure 3. Ensemble of the vertical position trajectories $z_1^{(j)}, z_2^{(j)}$ for both suspensions with optimized controller

and illustrated on the Figure 4, where the form of function $V_{coils}(z^{(j)}(t), u^{(j)}(t))$ derived in [24, 25].

Thus, for considered object it was received a new class of stabilizing Pareto-optimal polynomial controllers.

5. Conclusion. The parametric optimization of trajectories ensemble is applied to the design and optimization of the polynomial control laws (3) to stabilize the Maglev nonlinear feedback control system (4). The advantage of the presented nonlinear approach is that the minimized integral functional criteria (6) evaluates the dynamics quality of not

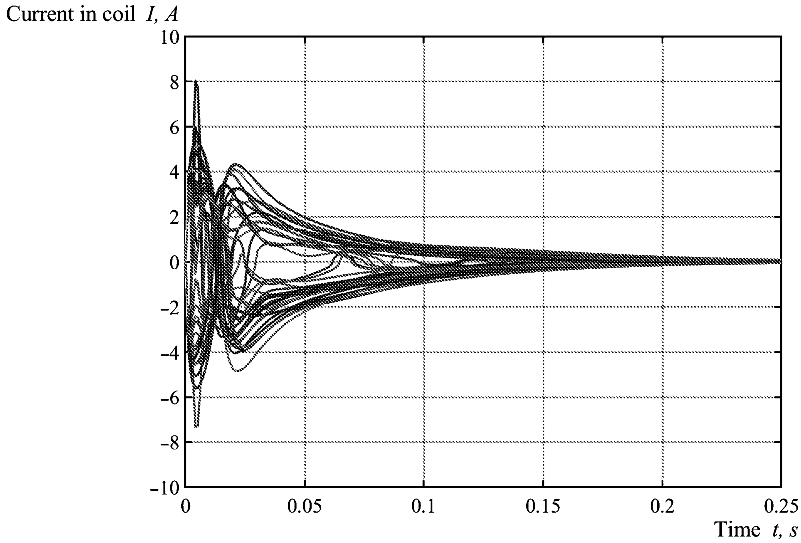

Figure 4. Trajectories ensemble of electromagnet coil currents $I^{(j)}_{coils_1}, I^{(j)}_{coils_2}$ for both suspensions with optimized controller

the one trajectory, but an ensemble of nonlinear perturbed trajectories. This ensemble of trajectories are perturbed by initial disturbances (5), which are distributed within entire engineering gap between platform and the guide rails. Presented optimization approach based on the consideration the entire ensemble of perturbed trajectories as well as the analytical representation of the first variation of functional criteria (14) allowed to get a new Pareto optimal set of polynomial controllers.

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Оптимизационный подход к проектированию нелинейных контроллеров систем управления

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Оптимизационный подход применяется к синтезу и оптимизации нелинейной системы оптимального управления с обратной связью в реальном времени для маглев платформы на магнитной подвеске. Для оптимизации нелинейного закона управления минимизируется интегральный функционал, который оценивает качество динамики не одной траектории, а ансамбля нелинейных траекторий системы и охватывает всю область инженерного зазора между платформой и направляющими рельсами. В этой области магнитные силы обеспечивают сильно нелинейные эффекты из-за рассмотренных конструктивных особенностей объекта, что делает задачу нелинейной задачей управления с несколькими входами и несколькими выходами. Компоненты вектора закона управления с обратной связью имеют полиномиальную форму от переменных пространства состояний. Для изучаемого объекта управления построен класс Парето-оптимальных полиномиальных регуляторов в результате вычислительной оптимизации траекторий ансамблей. В представленном множестве каждая Парето-оптимальная точка соответствует конкретному контроллеру и исследуемому функционалу, оценивающему весь ансамбль возмущенных траекторий. Это позволяет инженеру-исследователю выбирать различные нелинейные регуляторы и добиваться компромисса между точностью и энергетическими затратами.

Ключевые слова: нелинейные системы, стабилизация, нелинейные регуляторы, Маглев, реальное время, обратная связь, ансамбль траекторий, оптимизация.

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