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Solution of the Meeting Time Choice Problem for n Persons

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Abstract We consider a game-theoretic model of negotiations of n persons about a meeting time. The problem is to determine the time of the meeting, with the consensus of all players required to make a final decision. The solution is found by backward induction in the class of stationary strategies. Players' wins are represented by piecewise linear functions having one peak. An subgame perfect equilibrium for the problem in the case of $\delta \leqslant \frac{1}{2}$ is found in analytical form.

Keywords: optimal timing, linear utility functions, sequential bargaining, Rubinstein bargaining model, subgame perfect equilibrium, stationary strategies, backward induction.

1. Introduction

In article (Rubinstein, 1982) a method for solving the two-player bargaining problem with alternating bids on the infinite time axis was proposed. A discounting factor δ was introduced to limit the negotiation time, which reflected the impatience of the players. In (Baron and Ferejohn, 1989) the existence of a perfect subgame equilibrium in the model of sequential negotiation with majority rule and discounting was proved. The player who made the offer was chosen with equal probability. In (Eraslan, 2002), a model with linear utility functions was proposed in which the probabilities of choosing the participants and discounting coefficients differed. The uniqueness of the equilibrium perfect by subgames was proved. Quadratic utility functions are considered in (Cho and Duggan, 2003), and a multivariate model of sequential negotiation is considered in (Banks et al., 2006).

The n-person negotiation model with majority rule, discounting, and consensus is considered in (Mazalov and Nosalskaya, 2012) and (Mazalov et al., 2014). Offers are made at random and players either accept or refuse. An equilibrium in threshold strategies has been found.

In (Cardona and Ponsati, 2007) and (Cardona and Ponsati, 2011), Rubinstein's scheme was used to solve the problem of negotiating meeting times. For the general case with unimodal utility functions, the existence of an equilibrium perfect in subgames was proved.

In (Mazalov and Yashin, 2022) the solution of the meeting time negotiation for n players, small values of δ and values of δ close to 1 was given.

2. Problem Statement

The negotiation involves n players who agree on a meeting time $x \in [0;1]$. Each player has his own utility function $u_i(x), i = 1, 2, ..., n$, represented by a piecewise

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linear function with one peak. Players take turns proposing different solutions and need the agreement of all participants to accept them. Players take turns: $1 \to 2 \to 3 \to \ldots \to (n-1) \to n \to 1 \to \ldots$

In order to ensure that the negotiations do not last indefinitely, a discount factor $\delta < 1$ is introduced. After each negotiation session, the utility functions of all players decrease proportionally to δ.

If n is odd, then let's represent the number of players $n = 2k+1$. If n is even, then let's represent $n = 2k$. Assume that the utility functions of the players have the following form:

$$u_1(x) = x,$$

$$u_2(x) = \begin{cases} \frac{n-1}{n-2}x & 0 \leqslant x \leqslant \frac{n-2}{n-1}, \\ (n-1)(1-x) & \frac{n-2}{n-1} \leqslant x \leqslant 1, \end{cases}$$

$$u_3(x) = \begin{cases} \frac{n-1}{n-3}x & 0 \leqslant x \leqslant \frac{n-3}{n-1}, \\ \frac{n-1}{2}(1-x) & \frac{n-3}{n-1} \leqslant x \leqslant 1, \end{cases}$$

$$\ldots$$

$$u_{n-k}(x) = \begin{cases} \frac{n-1}{k}x & 0 \leqslant x \leqslant \frac{k}{n-1}, \\ \frac{n-1}{n-k-1}(1-x) & \frac{k}{n-1} \leqslant x \leqslant 1, \end{cases}$$

$$u_{n-k+1}(x) = \begin{cases} \frac{n-1}{k-1}x & 0 \leqslant x \leqslant \frac{k-1}{n-1}, \\ \frac{n-1}{n-k}(1-x) & \frac{k-1}{n-1} \leqslant x \leqslant 1, \end{cases}$$

$$\ldots$$

$$u_{n-1}(x) = \begin{cases} (n-1)x & 0 \leqslant x \leqslant \frac{1}{n-1}, \\ \frac{n-1}{n-2}(1-x) & \frac{1}{n-1} \leqslant x \leqslant 1, \end{cases}$$

$$u_n(x) = 1 - x.$$

Each player is interested in maximizing his winning function.

The solution x^* is sought in the class of stationary strategies. The players' decisions do not change during the negotiation time, i.e. player i will make the same offer at step i and at subsequent steps $n+i, 2n+i, \ldots$.

This will allow us to use the method of backward induction. To do this, assume that player 1 makes his offer. Then player n, looks for his best answer knowing player 1's solution, then player $(n-1)$ looks for his best answer knowing player n,'s solution, and so on. At the end the best answer of player 1, is found, and it must coincide with his proposal at the beginning of the previous negotiations.

The article (Mazalov and Yashin, 2022) found a solution for δ satisfying the condition $\delta(1 - \frac{\delta}{n-1}) \leqslant \frac{2}{n-1}$. The solution has the form $x^* = 1 - \frac{\delta}{n-1}$, where n is the number of players. The condition strongly restricts δ. For example, for 10 players it will be $\delta \leqslant 0.23$, and for 20 it will be $\delta \leqslant 0.11$. This article proves that this solution works for all $\delta \leqslant \frac{1}{2}$.

3. Solving the Negotiation Problem of n Persons in the General Case

Proposition 1. *If* $\delta \leqslant \frac{1}{2}$, *then* $x^* = 1 - \frac{\delta}{n-1}$

In the initial stages, the solution will be repeated from the article (Mazalov, Yashin, 2022). The sequence of player moves will be $3 \to 2 \to 1 \to n \to (n-1) \to \cdots \to 3$.

The last offer is **player 3**. His utility function has a maximum at $\frac{n-3}{n-1}$, assuming that this value is his offer: $x = \frac{n-3}{n-1}$.

Before him, **player 2** made a move. He knows what the previous player will propose and looks for his best answer $y \in [0,1]$, based on the following inequalities:

- In order for player 1 to accept proposition y, it is necessary that $u_1(y) \geqslant \delta u_1(x)$. This is equivalent to the inequality $y \geqslant \delta\frac{n-3}{n-1}$.
- In order for player 3 to accept the offer y, it is necessary that $u_3(y) \geqslant \delta u_3(x)$. This is equivalent to the inequality
  $\begin{cases} \frac{n-1}{n-3}y \geqslant \delta, \\ \frac{n-1}{2}(1-y) \geqslant \delta. \end{cases}$
- In order for player 4 to accept the offer y, it is necessary that $u_4(y) \geqslant \delta u_4(x)$. This is equivalent to the inequality
  $\begin{cases} \frac{n-1}{n-4}y \geqslant \frac{n-1}{3}\delta(1-\frac{n-3}{n-1}), \\ \frac{n-1}{3}(1-y) \geqslant \frac{n-1}{3}\delta(1-\frac{n-3}{n-1}). \end{cases}$

...

- In order for player $n-1$ to accept the offer y, $u_{n-1}(y) \geqslant \delta u_{n-1}(x)$. This is equivalent to the inequality.
  $\begin{cases} (n-1)y \geqslant \frac{2}{n-2}\delta, \\ \frac{n-1}{n-2}(1-y) \geqslant \frac{2}{n-2}\delta. \end{cases}$
- Finally, in order for player n to accept offer y, it is necessary that $u_n(y) \geqslant \delta u_n(x)$. This is equivalent to the inequality $1-y \geqslant \frac{2}{n-2}\delta$.

Combine the constraints:

$$\begin{cases} y \geqslant \delta\frac{n-3}{n-1}, \\ y \geqslant \delta\frac{2}{3}\frac{n-4}{n-1}, \\ y \geqslant \delta\frac{n-3}{2}, \qquad\qquad y \leqslant 1-\delta\frac{2}{n-1}. \\ \dots \\ y \geqslant \delta\frac{2}{(n-1)(n-2)}, \end{cases}$$

Comparing the inequalities for the left boundary, we see that

$$\delta\frac{n-3}{n-1} \geqslant \delta\frac{2}{3}\,\frac{n-4}{n-1} \geqslant \delta\frac{n-3}{2} \geqslant \ldots \geqslant \delta\frac{2}{(n-1)(n-2)}.$$

We get the segment $[\delta\frac{n-3}{n-1}, 1-\delta\frac{2}{n-1}]$, and any offer from this segment will be accepted by the other players. Since the maximal winnings of the 2 player are received at the point $\frac{n-2}{n-1}$, then suppose $\frac{n-2}{n-1} \in [\delta\frac{n-3}{n-1}, 1-\delta\frac{2}{n-1}]$ and obtain the condition $\delta \leqslant \frac{1}{2}$. Player 2 will offer $\hat{x}_2 = \frac{n-2}{n-1}$.

The move goes to **player 1**. His proposition y is found in the same way:

(2) $\begin{cases} \frac{n-1}{n-2}y \geqslant \delta, \\ (n-1)(1-y) \geqslant \delta, \end{cases}$

(3) $\begin{cases} \frac{n-1}{n-3}y \geqslant \delta\frac{n-1}{2}(1-\frac{n-2}{n-1}), \\ \frac{n-1}{2}(1-y) \geqslant \delta\frac{n-1}{2}(1-\frac{n-2}{n-1}), \end{cases}$

...

$$(n-1)\ \begin{cases} (n-1)y \geqslant \delta\frac{n-1}{n-2}(1-\frac{n-2}{n-1}), \\ \frac{n-1}{n-2}(1-y) \geqslant \delta\frac{n-1}{n-2}(1-\frac{n-2}{n-1}), \end{cases}$$

$(n)\ 1-y \geqslant \delta(1-\frac{n-2}{n-1})$.

Combine the constraints:

$$\begin{cases} y \geqslant \frac{n-2}{n-1}\delta, \\ y \geqslant \frac{n-3}{2(n-1)}\delta, \\ \dots \\ y \geqslant \frac{1}{(n-1)(n-2)}\delta, \end{cases} \qquad y \leqslant 1-\delta\frac{1}{n-1}.$$

Hence, any sentence from the segment $[\frac{n-2}{n-1}\delta; 1-\delta\frac{1}{n-1}]$ will be accepted by the other players. The best answer of player 1 is $x^* = 1-\frac{\delta}{n-1}$, which is the solution.

The move goes to **player n**. By analogy, we get a valid cutoff for $[\delta(1-\frac{\delta}{n-1}); 1-\delta^2\frac{1}{n-1}]$. The best proposition of player n is $\hat{x}_n = \delta(1-\frac{\delta}{n-1})$.

Move **player n-1**. His proposition y is found in the same way. The cutoff for the player's proposition $n-1$ is $[\delta^2(1-\frac{\delta}{n-1}); 1-\delta+\delta^2(1-\frac{\delta}{n-1})]$. Two cases arise at this point:

1. $\delta^2(1-\frac{\delta}{n-1}) \leqslant \frac{1}{n-1}$;
2. $\frac{1}{n-1} \leqslant \delta^2(1-\frac{\delta}{n-1})$.

Case 1 is described in more detail in (Mazalov and Yashin, 2022). Since $\delta^2(1-\frac{\delta}{n-1}) \leqslant \frac{1}{n-1}$ and the right boundary of the segment is always greater than $\frac{1}{2}$, the maximum value of this player's utility function will belong to this interval and the solution for player $n-1$ is $\hat{x}_{n-1} = \frac{1}{n-1}$.

The move goes to **player n - 2.** For this player the cutoff for the proposition y will be $[\delta\frac{1}{n-1}; 1-\delta\frac{n-2}{n-1}]$. Since $\delta\frac{1}{n-1} \leqslant \frac{2}{n-1} \leqslant 1-\delta\frac{n-2}{n-1}$, then $\hat{x}_{n-2} = \frac{2}{n-1}$.

Continuing the reasoning, we obtain that for players $(n-3), \dots, 3$ the best proposition are $\frac{3}{n-1}, \frac{4}{n-1}, \dots, \frac{n-3}{n-1}$. Thus, the proposition of player 3 at the beginning and at the end of backward induction coincide.

Case 2. The difference lies in the condition $\frac{1}{n-1} \leqslant \delta^2(1-\frac{\delta}{n-1})$, from which it follows that the best offer of player $(n-1)$ would be $\hat{x}_{n-1} = \delta^2(1-\frac{\delta}{n-1})$. Suppose $\hat{x}_{n-1} \in [\frac{1}{n-1}; \frac{3}{n-1}]$.

Move **player (n - 2)**. His proposition will be accepted by the other players if the following inequalities are satisfied:

$(1)\ y \geqslant \delta^3(1-\frac{\delta}{n-1})$,

$$(2)\ \begin{cases} \frac{n-1}{n-2}y \geqslant \delta^3\frac{n-1}{n-2}(1-\frac{\delta}{n-1}), \\ (n-1)(1-y) \geqslant \delta^3\frac{n-1}{n-2}(1-\frac{\delta}{n-1}), \end{cases}$$

$$(3)\ \begin{cases} \frac{n-1}{n-3}y \geqslant \delta^3\frac{n-1}{n-3}(1-\frac{\delta}{n-1}), \\ \frac{n-1}{2}(1-y) \geqslant \delta^3\frac{n-1}{n-3}(1-\frac{\delta}{n-1}), \end{cases}$$

$$\dots$$

$$(n-3)\ \begin{cases} \frac{n-1}{3}y \geqslant \delta^3\ \frac{n-1}{3}(1-\frac{\delta}{n-1}), \\ \frac{n-1}{n-4}(1-y) \geqslant \delta^3\ \frac{n-1}{3}(1-\frac{\delta}{n-1}), \end{cases}$$

$$(n-1)\ \begin{cases} (n-1)y \geqslant \frac{n-1}{n-2}\delta(1-\delta^2(1-\frac{\delta}{n-1}), \\ \frac{n-1}{n-2}(1-y) \geqslant \frac{n-1}{n-2}\delta(1-\delta^2(1-\frac{\delta}{n-1}), \end{cases}$$

$(n)\ 1 - y \geqslant \delta(1 - \delta^2(1 - \frac{\delta}{n-1}))$.

Combine the constraints:

$$\begin{cases} y \geqslant \delta^3(1 - \frac{\delta}{n-1}), \\ y \geqslant \frac{\delta}{n-2}(1 - \delta^2(1 - \frac{\delta}{n-1})), \end{cases} \qquad \begin{cases} y \leqslant 1 - \delta(1 - \delta^2(1 - \frac{\delta}{n-1})), \\ y \leqslant 1 - \frac{\delta^3}{n-2}(1 - \frac{\delta}{n-1}), \\ y \leqslant 1 - \frac{2\delta^3}{n-3}(1 - \frac{\delta}{n-1}), \\ \dots \\ y \leqslant 1 - \frac{n-4}{3}\delta^3(1 - \frac{\delta}{n-1}). \end{cases}$$

Keep the strongest ones, we come to the segment $[\delta^3(1-\frac{\delta}{n-1}); 1-\delta+\delta^3(1-\frac{\delta}{n-1})]$.

The right side of the segment is always greater than $\frac{1}{2}$, and cases are possible:

1. $\delta^3(1 - \frac{\delta}{n-1}) \leqslant \frac{2}{n-1}$;
2. $\frac{2}{n-1} \leqslant \delta^3(1 - \frac{\delta}{n-1})$.

Consider case 2-1. The player's $(n-2)$ proposition will be $\hat{x} = \frac{2}{n-1}$, so we can use the solution for case 1 above. This gives us the following lemma.

Lemma 1. *If* $\delta^2(1 - \frac{\delta}{n-1}) \geqslant \frac{1}{n-1}$, *and* $\delta^3(1 - \frac{\delta}{n-1}) \leqslant \frac{2}{n-1}$, *then* $x^* = 1 - \frac{\delta}{n-1}$

The lemma extends the segment on which the solution operates.

Consider case 2-2. The best offer of a player $(n-2)$ would be $\hat{x}_{n-2} = \delta^3(1 - \frac{\delta}{n-1})$. The turn comes **player (n - 3)**. His sentence is found in the same way. We get the segment $[\delta^4(1 - \frac{\delta}{n-1}); 1 - \delta + \delta^4(1 - \frac{\delta}{n-1})]$. Again the right bound of the segment is always greater than $\frac{1}{2}$ and 2 cases are possible:

1. $\delta^4(1 - \frac{\delta}{n-1}) \leqslant \frac{3}{n-1}$;
2. $\frac{3}{n-1} \leqslant \delta^4(1 - \frac{\delta}{n-1})$.

Case 2-2-1 is similar to the previous case, the player's $(n-3)$ offer will be $\hat{x}_{n-3} = \frac{3}{n-1}$, from which the lemma follows :

Lemma 2. *If* $\delta^3(1 - \frac{\delta}{n-1}) > \frac{2}{n-1}$ *and* $\delta^4(1 - \frac{\delta}{n-1}) \leqslant \frac{3}{n-1}$, *then* $x^* = 1 - \frac{\delta}{n-1}$.

Case 2-2-2 allows to continue the calculation by increasing the segment δ where the solution x^* will work. Continuing by analogy, there will come a moment when the condition required for case 2 is no longer satisfied, that is, only the case $\delta^{(i+1)}(1-\frac{\delta}{n-1}) \leqslant \frac{i}{n-1}$ remains, and it is at this point that we can claim that x^* works over the entire $0 \leqslant \delta \leqslant \frac{1}{2}$ segment.

For example, for 1500 players on a player $(n-7)$ move, the condition for case 2 will no longer hold.

We made the following suppositions when solving this problem: $\delta(1 - \frac{\delta}{n-1}) \in [0; \frac{2}{n-1}]$; $\delta^2(1 - \frac{\delta}{n-1}) \in [\frac{1}{n-1}; \frac{3}{n-1}]$; $\delta^3(1 - \frac{\delta}{n-1}) \in [\frac{2}{n-1}; \frac{4}{n-1}]$; $\dots$

Consider the first constraint $\delta(1 - \frac{\delta}{n-1})$, but on $[\frac{2}{n-1}; \frac{3}{n-1}]$ and perform the actions that were in the solution above: Move **player (n - 1)**. He makes a sentence that must satisfy the following inequalities:

(1) $y \geqslant \delta^2(1 - \frac{\delta}{n-1})$,

(2) $\begin{cases} \frac{n-1}{n-2}y \geqslant \delta^2\frac{n-1}{n-2}(1 - \frac{\delta}{n-1}), \\ (n-1)(1-y) \geqslant \delta^2\frac{n-1}{n-2}(1 - \frac{\delta}{n-1}), \end{cases}$

(3) $\begin{cases} \frac{n-1}{n-3}y \geqslant \delta^2\frac{n-1}{n-3}(1 - \frac{\delta}{n-1}), \\ \frac{n-1}{2}(1-y) \geqslant \delta^2\frac{n-1}{n-3}(1 - \frac{\delta}{n-1}), \end{cases}$

$$\dots$$

$(n-3)$ $\begin{cases} \frac{n-1}{3}y \geqslant \delta^2 \frac{n-1}{3}(1-\frac{\delta}{n-1}), \\ \frac{n-1}{n-4}(1-y) \geqslant \delta^2 \frac{n-1}{3}(1-\frac{\delta}{n-1}), \end{cases}$

For the player $(n-2)$ we see changes:

$(n-2)$ $\begin{cases} \frac{n-1}{2}y \geqslant \frac{n-1}{n-3}\delta(1-\delta(1-\frac{\delta}{n-1})), \\ \frac{n-1}{n-3}(1-y) \geqslant \frac{n-1}{n-3}\delta(1-\delta(1-\frac{\delta}{n-1})), \end{cases}$

(n) $1-y \geqslant \delta(1-\delta(1-\frac{\delta}{n-1}))$.

This gives us the following constraints:

$$\begin{cases} y \geqslant \delta^2(1-\frac{\delta}{n-1}), \\ y \geqslant \frac{2}{n-3}\delta(1-\delta(1-\frac{\delta}{n-1})), \end{cases} \qquad \begin{cases} y \leqslant 1-\delta(1-\delta(1-\frac{\delta}{n-1})), \\ y \leqslant 1-\frac{\delta^2}{n-2}(1-\frac{\delta}{n-1}), \\ y \leqslant 1-\frac{2\delta^2}{n-3}(1-\frac{\delta}{n-1}), \\ \dots \\ y \leqslant 1-\frac{n-4}{3}\delta^2(1-\frac{\delta}{n-1}). \end{cases}$$

We see that the inequality that was $y \leqslant 1-\frac{n-3}{2}\delta^2(1-\frac{\delta}{n-1})$, changed to $y \leqslant 1-\delta(1-\delta(1-\frac{\delta}{n-1}))$, and inequality appeared in the other part $y \geqslant \frac{2}{n-3}\delta(1-\delta(1-\frac{\delta}{n-1}))$. By virtue of the above condition $\delta(1-\frac{\delta}{n-1}) \in [\frac{2}{n-1}; \frac{3}{n-1}]$, get

$$1-\delta(1-\delta(1-\frac{\delta}{n-1})) \leqslant 1-\frac{n-4}{3}\delta^2(1-\frac{\delta}{n-1})$$

$$\delta^2(1-\frac{\delta}{n-1}) \geqslant \frac{2}{n-3}\delta(1-\delta(1-\frac{\delta}{n-1}))$$

This brings us to the segment for the player's $(n-1)$ offer : $[\delta^2(1-\frac{\delta}{n-1}); 1-\delta+\delta^2(1-\frac{\delta}{n-1})]$. It is the same as under the condition $\delta(1-\frac{\delta}{n-1}) \in [0; \frac{2}{n-1}]$
Thus, the above solution works for $\delta(1-\frac{\delta}{n-1}) \in [0; \frac{3}{n-1}]$. Now for 10 players $\delta \leqslant 0.347$, instead of $\delta \leqslant 0.23$, as it was before.

We will continue to consider the cases. Now suppose $\delta(1-\frac{\delta}{n-1}) \in [\frac{3}{n-1}; \frac{4}{n-1}]$. Again we find the conditions for **player (n - 1)**:

(1) $y \geqslant \delta^2(1-\frac{\delta}{n-1})$,

(2) $\begin{cases} \frac{n-1}{n-2}y \geqslant \delta^2 \frac{n-1}{n-2}(1-\frac{\delta}{n-1}), \\ (n-1)(1-y) \geqslant \delta^2 \frac{n-1}{n-2}(1-\frac{\delta}{n-1}), \end{cases}$

(3) $\begin{cases} \frac{n-1}{n-3}y \geqslant \delta^2 \frac{n-1}{n-3}(1-\frac{\delta}{n-1}), \\ \frac{n-1}{2}(1-y) \geqslant \delta^2 \frac{n-1}{n-3}(1-\frac{\delta}{n-1}), \end{cases}$

$$\dots$$

$(n-4)$ $\begin{cases} \frac{n-1}{4}y \geqslant \delta^2 \frac{n-1}{4}(1-\frac{\delta}{n-1}), \\ \frac{n-1}{n-5}(1-y) \geqslant \delta^2 \frac{n-1}{4}(1-\frac{\delta}{n-1}), \end{cases}$

$(n-3)$ $\begin{cases} \frac{n-1}{3}y \geqslant \frac{n-1}{n-4}\delta(1-\delta(1-\frac{\delta}{n-1})), \\ \frac{n-1}{n-4}(1-y) \geqslant \frac{n-1}{n-4}\delta(1-\delta(1-\frac{\delta}{n-1})), \end{cases}$

$(n-2)$ $\begin{cases} \frac{n-1}{2}y \geqslant \frac{n-1}{n-3}\delta(1-\delta(1-\frac{\delta}{n-1})), \\ \frac{n-1}{n-3}(1-y) \geqslant \frac{n-1}{n-3}\delta(1-\delta(1-\frac{\delta}{n-1})), \end{cases}$

(n) $1 - y \geqslant \delta(1-\delta(1-\frac{\delta}{n-1}))$.

This gives us the following constraints:

$$\begin{cases} y \geqslant \delta^2(1-\frac{\delta}{n-1}), \\ y \geqslant \frac{2}{n-3}\delta(1-\delta(1-\frac{\delta}{n-1})), \\ y \geqslant \frac{3}{n-4}\delta(1-\delta(1-\frac{\delta}{n-1})), \end{cases} \qquad \begin{cases} y \leqslant 1-\delta(1-\delta(1-\frac{\delta}{n-1})), \\ y \leqslant 1-\frac{\delta^2}{n-2}(1-\frac{\delta}{n-1}), \\ y \leqslant 1-\frac{2\delta^2}{n-3}(1-\frac{\delta}{n-1}), \\ \dots \\ y \leqslant 1-\frac{n-5}{4}\delta^2(1-\frac{\delta}{n-1}). \end{cases}$$

We observe that in the conditions for the right boundary $y \leqslant 1-\frac{n-4}{3}\delta^2(1-\frac{\delta}{n-1})$ replaced by $y \leqslant 1-\delta(1-\delta(1-\frac{\delta}{n-1}))$, and a condition was added for the left border $y \geqslant \frac{3}{n-4}\delta(1-\delta(1-\frac{\delta}{n-1}))$. Because of $\delta(1-\frac{\delta}{n-1}) \in [\frac{3}{n-1}; \frac{4}{n-1}]$ we get:

$$1-\delta(1-\delta(1-\frac{\delta}{n-1})) \leqslant 1-\frac{n-5}{4}\delta^2(1-\frac{\delta}{n-1}),$$

$$\delta^2(1-\frac{\delta}{n-1}) \geqslant \frac{3}{n-4}\delta(1-\delta(1-\frac{\delta}{n-1})),$$

which leads to the segment $[\delta^2(1-\frac{\delta}{n-1}); 1-\delta(1-\delta(1-\frac{\delta}{n-1}))]$. Again, the segment is equal to what was obtained above, which means we weaken the restriction to $\delta(1-\frac{\delta}{n-1}) \in [0; \frac{4}{n-1}]$.

...

We continue until the condition $\delta(1-\frac{\delta}{n-1}) \in [\frac{k-1}{n-1}; \frac{k}{n-1}]$ comes into consideration. Note also that for an even number of players $\frac{1}{2} \in [\frac{k-1}{n-1}; \frac{k}{n-1}]$, and for an odd number of players $\frac{k}{n-1} = \frac{1}{2}$. Checking the inequalities, we again obtain the segment $[\delta^2(1-\frac{\delta}{n-1}); 1-\delta(1-\delta(1-\frac{\delta}{n-1}))]$, but now the condition $\delta(1-\frac{\delta}{n-1})$ works on the whole segment $[0; \frac{k}{n-1}]$ and this gives us the desired constraint $\delta \leqslant \frac{1}{2}$, which tells us that we have no other constraints on δ than $\delta \leqslant \frac{1}{2}$.

Constraints $\delta^2(1-\frac{\delta}{n-1}) \in [\frac{1}{n-1}; \frac{3}{n-1}]$, $\delta^3(1-\frac{\delta}{n-1}) \in [\frac{2}{n-1}; \frac{4}{n-1}]$, ..., are checked similarly.

4. Conclusion

In the article, an equilibrium is found in an analytical form, perfect by subgames in the class of stationary strategies for the problem of meeting time. The arguments started in the article are continued (Mazalov and Yashin, 2022) and a complete solution of the problem for the case $\delta \leqslant \frac{1}{2}$.

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