REVIEW by the scientific supervisor of the postgraduate thesis
"Cubic normal forms, their classification and phase portraits"
by Aleksandr Chermnykh.

The work of A. Chermnykh is devoted to issues related to the classification of two-dimensional homogeneous cubic systems of ODE with respect to linear non-singular substitutions of variables. In the course of classification, in each equivalence class, according to the developed structural and normalization principles, the simplest system or 2 by 4 matrix, called the canonical form (CF), is distinguished.

Such a classification is relevant both in itself and is absolutely necessary for reducing two-dimensional systems with homogeneous cubic polynomials in the unperturbed part to generalized normal forms (GNF). GNF are the simplest systems that can be obtained from the original system by reversible substitutions. The fact is that all possible structures of the GNF depend solely on the type of unperturbed part, and the simpler it is, the more likely it is to constructively distinguish these structures. Therefore, V. Basov, in a number of works with various co-authors, obtained the ONF of systems with pre-allocated CF in unperturbed parts having linear-quadratic, linear-cubic, quadratic-cubic, quadratic and cubic structures.

The implementation of the latter classification, carried out for two-dimensional real systems $\dot{x} = P(x)$, where $P(x)$ is a vector homogeneous cubic polynomial, encountered great technical difficulties both with the volume – about a hundred CF were allocated, and with the maximum task set – to provide each CF with: 1) conditions on the coefficients of the original cubic system, under which it is linearly equivalent to its CF, 2) a linear substitution that implements this equivalence, 3) the resulting values of the CF elements.

This work is related to the addition, refinement or improvement of a number of results obtained earlier in solving the problem, and the application of the created classification for finding topological invariants in the Poincare circle. At the same time, the postgraduate thesis is a direct continuation of the bachelor's and master's ones.

The student understood the theory developed earlier, it is briefly given in the Introduction, and updated the package of programs written for its practical implementation in the Maple system, improving some of them and adding detailed explanations to all of them, allowing anyone to use them if they want.

Further, the student fully solved the maximum problem set above for cases where the right-hand side of the system has a common multiplier of the second or first order. In the case where there is no common factor, he obtained all the most important cubic normal forms that can have real practical applications. It is impossible to fully investigate this case due to the necessity to solve algebraic equations with parameters of high degrees.

The obtained classification, in particular, allowed us to draw topological portraits of trajectories in the Poincare circle for all cubic normal forms whose right-hand sides have no more than three terms and have no common factor. I would like to note that if this work was continued and normal forms were considered with a common multiplier and with at least four terms on the right side, which, in my opinion, does not seem difficult enough, then this would be enough for a graduate student to defend a PhD thesis. I hope that the missing results will be obtained soon.

A. Chermnykh worked independently, regularly, accurately and thoughtfully, which made it possible to record and verify the results without the usual pre-defense haste, which is especially important, since many of them are tabular in nature.

I believe that the postgraduate thesis of A. Chermnykh deserves an excellent rating.

Scientific supervisor,
professor / V. Basov /