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Master in Corporate Finance

EQUILIBRIUM RELATIONSHIP BETWEEN INDEX DERIVATIVES

Master Thesis by the 2nd year student
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АННОТАЦИЯ

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Название ВКР	Равновесное соотношение между деривативами на индекс
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Описание цели, задач и основных результатов	Цель: исследовать кросс-эффективность рынка фьючерсов и опционов и внутреннюю эффективность рынка опционов Варшавской фондовой биржи. Задачи: а) провести обзор литературы в поле эффективности рынка, производных ценных бумаг и их роли на финансовых рынках; б) выделить основные подходы к оценке внутренней эффективности рынка опционов и кросс-эффективности рынков опционов и фьючерсов; в) применить выбранные методологии к данным Варшавской фондовой биржи; г) сформулировать вывод на основании проведенного эмпирического исследования. Результаты: рынки деривативов Варшавской фондовой биржи внутренне неэффективны (возможен арбитраж в опционах), но присутствует кросс-эффективность (невозможен арбитраж между фьючерсами и опционами).
Ключевые слова	Индексные производные, опционы, фьючерсы, эффективность рынка.

ABSTRACT

Master Student's Name	Islamov Ruslan Ilgarovich
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Description of the goal, tasks and main results	Goal: to research the cross-market efficiency of options-futures markets and intra-market efficiency of options markets in Warsaw Stock Exchange. Tasks: a) provide an overview of academic literature in the field of market efficiency, derivative securities and their role in financial markets; b) summarize the existing approaches to options intra-market and options/futures cross- market efficiency; c) implement the chosen methodology to the data collected from WSE; d) draw conclusions on top of the conducted empirical research. Results: there is cross-market efficiency and there is no intra-market efficiency in WSE derivatives markets.
Keywords	Index derivatives, options, futures, market efficiency.

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INTRODUCTION

Efficient markets are vital for capital allocation, price discovery and risk management. Growth of financial markets can be determined by the degree of how efficiently they operate (Capelle-Blancard, Chaudhury, 2001). Derivatives market efficiency, in turn, to a big extent affects the general financial market efficiency, because derivatives provide liquidity and information to spot markets in terms of what the market anticipates towards these or those assets, also allowing for risk management and hedging. If derivatives market does not operate efficiently, risk management strategies may be hindered.

Efficient market hypothesis (EMH), coined by Fama (1970), states that in an efficient market prices fully reflect all available information. Available information that the author refers to depends upon the form of market efficiency. E.g., in the weak form of market efficiency past trading information of securities is reflected in market prices. There are three forms of market efficiency in total, which are discussed in more detail later in the paper. An efficient market can also be referred to as one where a single investor is not able to make substantially larger profits without having to bear additional risk. Therefore, arbitrage opportunities must be rare, insignificant and quickly eliminated in an efficient market.

The basis for no-arbitrage conditions is that investment strategies with the same future cashflows should be priced the same. In other words, there is no 'free lunch' in an efficient market. Investors prefer more to less, and take advantage of any riskless profits earning opportunity. Since no-arbitrage tests use only the observed daily market prices, these tests can be considered as tests of the weak form of market efficiency (Capelle-Blancard and Chaudhury, 2001).

Two distinct directions concerning the option market efficiency can be observed in the academic literature. The first approach is the model-based approach (Black-Scholes, Binomial, other models), which involves deriving a theoretical price of the option and then comparing it with the observed market price, thus calculating the mispricing, which is checked for statistical significance. The problem with this approach is that it tests two hypotheses simultaneously: first, that the model is valid itself, and second, that the market is efficient, while the test is unable to distinguish between the two (Galai, 1977). The second approach tests cross-market efficiency between spot and option prices (put-call parity) and internal market efficiency (box spread, etc.). The second approach is less restrictive in that it is not based upon assumptions about the normal distribution of the price of the underlying and estimation of its volatility (Bhat & Arekar, 2015). That is why the model-free approach is selected in this paper.

One framework for research in this field is provided by Stoll (1969), who was the first in the academic literature to describe the principle of put-call parity. His approach was later generalized and extended to employ futures prices instead of spot prices (Tucker, 1991). The main specificity of put-call parity and of put-call-futures parity concepts is that they can be utilized for European-type options only. It is shown in the academic literature that the discrepancies in the pricing of American-type options (i.e., situations when the put-call parity (PCP) does not hold) can be explained by the additional value of early exercise opportunity (Zivney, 1991). The general model has an obvious limitation in that it exists in a frictionless world, that is, no transaction costs are considered. So, for the sake of a realistic revision of the model, transaction costs have to be added into consideration.

According to the efficient market hypothesis, an efficient market is an equilibrium market. It is a market, where an agent is unable to make a riskless profit on the basis of the available information. If the market deviates from the equilibrium, and arbitrage opportunity appears, and the investors immediately take advantage of this discrepancy by buying low and selling high, thus obtaining a risk-free profit. Therefore, an equilibrium relationship between put and call options for the same underlying asset should be reflected in an efficient market. If numerous significant arbitrage opportunities appear (that are not explained away by the level of transaction costs) and so riskless profits can be made, then it can be concluded that there is no equilibrium relationship, and the derivatives' market efficiency has to be increased. It is important from investor's point of view, because an inefficient market exposes an investor to non-diversifiable risks. The practical implementation of an existing inefficiency in a market is in two main areas: first, economic agents can make riskless profits in such inefficient market. This is beneficial for economic agents like brokerage houses, investment banks and other investors, such as, for example, retail investors (even though they usually encounter higher levels of transaction costs), because they all can make profits without bearing additional risk. Second, a clear signal to increase market efficiency appears. This is useful for the agent that organizes trading of securities (stock exchange), who is signaled to increase market efficiency. As shown in Chen, Chin and Chung (2020), arbitrageurs may also decrease liquidity in the markets. In addition, as mentioned above, most investors are in search of effective markets, where risks can be effectively diversified or hedged. Therefore, the stock exchange is better off in case it manages to attract more investors with an efficient market.

In this paper, market efficiency in derivatives markets is examined. A great deal of attention is given to the analysis of transaction costs. More precisely, options' market intra-market efficiency and cross-market efficiency between options and futures markets are researched. A specific type of derivatives is chosen: index derivatives. This type of derivatives is, essentially, one of the most liquid derivatives in any financial market, together with forex (currency) derivatives. Index derivatives let investors track the performance of the broad market with lower transaction costs. Investors do not need to construct a basket of shares that constitute an index, instead they may purchase an index futures security. May an investor need to hedge against the performance of the market, she can purchase index options, which oftentimes have even lower trading costs than futures. Derivatives also add liquidity into the markets, because investors obtain an additional source of information about market prices, as well as market expectations about the future movement of these prices.

The issue of derivatives markets' efficiency has been given a lot of attention in the academic literature. Nevertheless, young and emerging derivatives markets lack coverage in this regard. In particular, to the best knowledge of the author, little to no academic literature addresses the options market equilibrium relationship and the put-call-futures parity in Warsaw Stock Exchange, while WSE is considered to be the largest stock exchange in Central and Eastern Europe. It is believed to be the leading exchange in Central and Eastern Europe in terms of liquidity, and it also attracts a substantial quantity of retail investors, being at the same time the leader in derivatives trading (Vozianov, 2015). In addition, one of the most liquid and demanded derivatives in WSE, together with forex derivatives, are index derivatives. These securities' popularity is based upon the fact that they allow to track and bet on the performance of the broad market with relatively low transaction cost. Instead of buying a basket of shares' derivatives, one can bet on or against the market, or even structure sophisticated trading strategies with up to three securities: index put and call options and futures.

The aim of the research is articulated as follows: to research the cross-market efficiency of options-futures markets and intra-market efficiency of options markets in Warsaw Stock Exchange. The criterion of market efficiency is the existence of arbitrage opportunities. A clear picture of the efficiency of the WSE derivatives market is important in the understanding of the whole WSE financial market.

The paper aims to accomplish the following goals:

1) Provide an overview of academic literature in the field of market efficiency, derivative securities and their role in financial markets, as well as cover the existing empirical research in the field in developed and developing countries.
2) Summarize the existing approaches to options intra-market and options/futures cross-market efficiency, outline the pros and cons of these approaches, choose and advocate the methodology for further research.
3) Implement the chosen methodology to the data collected from WSE, and to differentiate the results according to investor type, time to maturity and trader strategy, as well as option moneyness, from the standpoint of market efficiency and arbitrage profits.

4) Draw conclusions on top of the conducted empirical research and give recommendations for agents that trade in derivatives markets of WSE, as well as for WSE.

The object of the research is options and futures with WIG20 index as the underlying asset. The time period covered in the research is from December 2017 till December 2019. This yields a full two-year sample of daily closing prices, which is in coherence with Ackert and Tian (2001), as well as Zhang and Watada (2019). Because the author aims to avoid the unfavorable effect of the global market turmoil starting in early 2020, data after December 2019 is not included in the collection. Data is cross-sectional, collected from the official website of Warsaw Stock Exchange. For the means of the research, the following databases are employed: JSTOR, EBSCO, Scopus, Science Direct and others. Microsoft Excel and Stata were used to conduct the calculations on the basis of the gathered data and to further conduct the statistical tests of hypotheses.

The results of the research are to be beneficial for two groups of stakeholders: derivative instruments traders in WSE and the WSE itself. The managerial implementation of research findings for traders is somewhat straightforward: if the markets are proven inefficient, then economic agents can make profits without having to bear any additional risks (execute arbitrage transactions, in other words). Among the users of such information may be investment banks, mutual funds and other agents.

The implications of the research for Warsaw Stock Exchange may seem vague at first sight. Let us elaborate on that shortly. If the markets are proven inefficient, WSE obtains a clear signal to increase market efficiency. The reasons for increasing it are clear: inefficient markets may spook away investors, because if the market does not reflect the available information in market prices, then investors are exposed to additional, non-diversifiable risks. The exchange may therefore try to increase the efficiency of the market in numerous ways, such as increasing the transparency of market data, changing the regulations regime, etc.

The paper is organized as follows: the first chapter provides a review of existing academic research in the field of derivatives markets efficiency. It sets the conceptual field for further research. The concept of market efficiency is given more detail. In the first chapter the role of derivative securities in financial markets is discussed. The chapter covers main approaches to examining market efficiency in derivatives markets. Those are divided from the standpoints of model-based approaches and model-free approaches. It also provides a review of existing empirical research in developing and developed countries.

The second chapter is devoted to conducting empirical research: data collection and description of data, methodology of research, hypotheses testing procedure and its results are described. The context of the research is discussed, as well as Warsaw Stock Exchange is characterized in terms of its brief history and current state of affairs in the derivatives markets. We look deeper into the details of contracts traded on the floor WSE. Financial calculations are described, and the approaches covered in the first chapter are supplemented with a realistic revision. Hypotheses of the research are stated and tested with the use of relevant statistic tests. The corollary to the conducted empiric research is drawn in the final section of the chapter. The second chapter is followed by the conclusion to the whole paper.

CHAPTER 1. MARKET EFFICIENCY AND APPROACHES TO EFFICIENCY EVALUATION

1.1. Market efficiency

In this paper, as mentioned in the introduction, the criterion for market efficiency is the no-arbitrage condition. Arbitrage, in essence, is buying an asset low and selling it high. Arbitrage is possible when the market prices of assets exhibit discrepancies, which depend upon the parameters of the relation of interest (in this paper, two relations are considered: put-call-futures parity and box spread). In addition, arbitrage is possible only if the absolute mispricing exceeds the full transaction cost that will be incurred to close such an arbitrage transaction. It is shown in Bukhvalov (2010, 65) that transaction costs reduce room for arbitrage. Arbitrage transactions can be considered risk-free, because to lock in a profit the arbitrageur has to do as much as to hold the contracts until maturity. There is no uncertainty about the future fluctuations in the assets' prices, because the positions in contracts are offsetting. Therefore, in this paper a market is to be considered efficient, if no risk-free inter-temporary arbitrage is possible (accounted for transaction costs), that is, if the net arbitrage profits are persistently lower than zero.

As referred to by Fama (1970), an efficient market is one where market prices fully reflect all available information. There are three forms of market efficiency hypothesis (EMH), that is, weak form of EMH, semi-strong form of EMH and strong form of EMH. These are distinguished in relation to the set of available information, which is to be fully reflected in the market prices. Forms of market efficiency are effectively summarized by Brealey et al (2011, 335). In particular, weak form of market efficiency states that prices efficiently reflect all the information contained in the past series of asset prices. In this case it is impossible to earn superior returns simply by looking for patterns in asset prices. The semi-strong form of the hypothesis states that prices reflect all published information. That means it is impossible to make consistently superior returns just with the help of publicly available pieces of information. The strong form of the hypothesis states that market prices effectively impound all available information.

Two distinct directions concerning the option market efficiency can be observed in the academic literature. The first approach is the model-based approach (Black-Scholes, Binomial, other models), which involves deriving a theoretical price of the option and then comparing it with the observed market price, thus calculating the mispricing, which is checked for statistical significance. The second approach tests cross-market efficiency between spot and option prices (put-call parity) and internal market efficiency (box-spread, etc.). The second approach is less restrictive in that it is not based upon assumptions about the normal distribution of the price of the underlying and estimation of its volatility (Bhat and Arekar, 2015).

The framework for research in the field of model-free approaches is provided by Stoll (1969), who pioneered in the academic literature to describe the principle of put-call parity. His approach was later generalized and extended to plug futures prices instead of spot prices (Tucker, 1991). The main feature of this model is that it can be utilized for European-type options only. It is shown in the academic literature that any discrepancies in the pricing of American-type options (e.g., situations when the PCP does not hold) may be explained by the additional value of early exercise opportunity (Zivney, 1991). The general model has an obvious limitation in that it exists in a frictionless world, that is, no transaction costs are considered. So, for the sake of a realistic revision of the model, transaction costs have to be considered. Model-based and model-free approaches are provided with more detail later in the paper.

1.2. Derivative securities in financial markets

A derivative can be defined as a financial instrument whose value depends on the values of underlying variables (e.g., prices of traded assets). This definition is rather general, though there are a lot of different types of derivative contracts: forwards, futures, options, swaps, etc. Derivatives, like other financial securities, are found in exchange-traded markets and over-the-counter markets. For example, forwards are generally traded in OTC markets, and futures are generally traded on an exchange, their features are standardized by the exchange and the execution of the contract is also guaranteed by the exchange.

Derivatives are important for financial markets in several ways. First of all, they provide traders with risk management strategies. A single investor may want to hedge some risks. Derivative instruments, such as, above all, options, serve as means for creating multiple hedging strategies. An investor can hedge the downside risk of a specific exposure, or an upside risk, as well as risk of fluctuation. In this manner a wheat producer in the USA can sell a forward contract in order to lock in the price of the wheat she is going to sell in the future.

Derivatives also serve for price discovery in a sense that looking, for example, at futures prices one can determine the market sentiment towards the possible future price movements of the underlying asset, thus derivatives add to the information at one's disposal. In this regard, futures contracts add liquidity to the spot market of the underlying instrument.

Derivatives are also sometimes easier to trade than the underlying asset, oftentimes because of lower transaction costs. For example, most of the oil traded in exchanges is traded with the help of futures contracts. It also may be easier to bet on a change in the price of an asset with the help of derivatives. Derivatives allow for sophisticated trading strategies. Having said that, derivatives markets are significant for efficient functioning of broader financial markets.

There are three categories of traders in derivatives markets, as summarized in Hull (2006, 8): hedgers, speculators and arbitrageurs. Hedgers use derivatives to reduce the risk from potential price movements in the underlying asset. Speculators use derivatives to bet on the future direction of a market variable. Arbitrageurs take offsetting positions in several instruments to lock in a profit.

Let us now discuss the definitions of the main derivative contracts. Probably one of the simplest derivatives is a forward contract. Forward is a contract between two counterparties to buy or sell an asset on a specific date in the future at a price that is specified in the contract. There is also another contract type, which is very close to forward contract: futures. Futures is an agreement between two parties to buy or sell an asset at a certain time in the future for a certain price. Futures are different from forwards in that futures are standardized contracts traded on an exchange, and the contract execution is guaranteed by the exchange. Therefore, it reduces the counterparty risk that agents in a forward contract are exposed to. Futures contracts can be divided into two types: cash-settled and physically-settled. The former means that there is a transfer of the net cash position (according to the underlying spot price) between parties in the contract, while the latter results in a physical delivery of the underlying asset from the seller to the buyer. One of the most important cash-settled futures types is index futures, as stated in Burenin (2005, 81).

Let us now switch to option contracts. Options are financial instruments whose value depends upon the value of the underlying asset. An option contract gives the holder a right (but not an obligation) to buy or sell an asset on (or until) a specific date for a specified price. Options, therefore, are divided into put options (gives the holder the right to sell the underlying asset on (or until) a certain date in the future for a certain price) and call options (gives the holder the right to buy the underlying asset on (or until) a certain date in the future for a certain price). Options are therefore defined by a strike price and a maturity. Options are further divided into European options (can be exercised only *on* the expiration date) and American options (can be exercised at any time *until* the expiration date).

1.3. Model-based approaches to testing market efficiency

We start to consider the approaches to testing derivatives market efficiency with model-based approaches. The main of all model-based approaches to assessing market efficiency is the celebrated Black-Scholes model. According to Galai (1977, 172) the formula consists of four observable inputs: the price of the underlying asset *(V)*, the strike price *(K)*, the time to maturity *($\tau = T - t$)* and the riskless interest rate *(r)*, and one unobservable variable: the variance of the asset's distribution of rates of return *(σ^2)*. The model premium C is

$$C = VN(d_1) - Ke^{-r\tau}N(d_2), \quad (1)$$

where

$$d_1 = \frac{\ln\left(\frac{V}{K}\right) + (r + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}$$

and

$$d_2 = \frac{\ln\left(\frac{V}{K}\right) + (r - \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}} = d_1 - \sigma\sqrt{\tau}$$

and $N(\cdot)$ is the cumulative standard normal distribution.

Black-Scholes model is based on the following assumptions (Black and Scholes, 1973, 640):

1. The short-term interest rate is known and is constant through time.
2. The asset price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. Thus, the distribution of possible asset prices at the end of any finite interval is log-normal. The variance rate of the return on the asset is constant.
3. The asset pays no dividends or other distributions.
4. The option is European, that is, it can only be exercised at maturity.
5. There are no transaction costs in buying or selling the asset or the option.
6. It is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short-term interest rate.
7. There are no penalties to short selling. A seller who does not own a security will simply accept the price of the security from a buyer, and will agree to settle with the buyer on some future date by paying him an amount equal to the price of the security on that date.

In spite of all its advantages, this approach is somewhat complicated to implement in practice in that it tests two hypotheses simultaneously: first, that the model is valid itself, and second, that the market is efficient, while the test is unable to distinguish between the two (Galai, 1977). Therefore, in this paper model-free approaches are implemented instead.

1.4. Model-free approaches to testing market efficiency

1.4.1. Put-call-parity

Model-free approaches take the underlying and derivative market prices as given and simply looks at possible violations of no-arbitrage relationships among the prices. This is an advantage, because these approaches rely on no assumptions like those Black-Scholes model involves.

The first approach that is considered is the put-call parity, coined by Stoll (1969). The concept of put-call parity states that the put option, call option and the underlying asset are inter-related so that any two of these can be combined so as to yield the pay-off of the third instrument. The mathematical form of the parity is as follows:

$$c + Xe^{-r(T-t)} = p + S \qquad (2)$$

Where *c* – European call option price,

X – option strike price,

r – risk-free interest rate,

p – European put option price,

(T – t) – time to expiration of the options,

S – spot price of the underlying.

If put-call parity is violated, one can make risk-free arbitrage profit by pursuing a long arbitrage strategy or a short arbitrage strategy. The short strategy involves selling the put option and short selling the underlying asset and simultaneously buying the call option and lending at the risk-free rate. The long strategy involves buying the underlying asset and the put option and simultaneously selling the call option and borrowing at the risk-free rate. The name 'short' or 'long' means to indicate the arbitrage position in the underlying asset (Capelle-Blancard, Chaudhury, 2001).

1.4.2. Put-call-futures parity

The relationship in Equation 2 is further developed by Tucker (1991), substituting spot price with futures price, based on the following assumptions:

1. futures and options contracts have the same expiration date;
2. the options are all European options, and the futures position is held until its expiration;
3. lending and borrowing rates are the same;
4. there is no risk of daily liquidation;
5. transaction costs, such as taxes and commissions, can be ignored.

In the following manner spot price of the underlying is substituted with the discounted futures' price:

$$c + Xe^{-r(T-t)} = p + Fe^{-r(T-t)} \qquad (3)$$

where F – price of the futures.

If the put-call-futures parity condition holds, then the left-hand side and the right-hand side of Equation 3 should be equal. Having said that, it is now pertinent to derive the following from Equation 3 (adapted from Bhat and Arekar, 2015):

$$\varepsilon' = c - p - (F - X)e^{-r(T-t)} \quad (4)$$

where ε' – the pricing error, which should be equal to zero if the put-call-futures markets constitute an integrated market.

If ε'>0, then the arbitrageur can benefit from making a riskless profit by following a short arbitrage strategy.

If ε'<0, then the arbitrageur is able to make a riskless profit by following a long arbitrage strategy. These strategies were discussed in detail above (in section 1.4.1).

For the arbitrage to be profitable, absolute value of ε' has to exceed any explicit and implicit costs of trading.

1.4.3. Box spread

Box spread strategy is an arbitrage strategy which involves using four European options. A short and a long synthetic positions are built in the underlying assets. The conditions for executing box arbitrage follow from put-call parity condition. The equation is derived in Burenin (2015, 454), adapted for a continuous rate of interest as follows:

$$(c_1 - p_1) - (c_2 - p_2) = (X_2 - X_1)e^{-r(T-t)} \quad (5)$$

where c_1 and c_2 – call options' prices,
p_1 and p_2 – put options' prices,
X_1 and X_2 – options' strike prices,
r – risk-free rate.

Therefore, the mispricing ε' is the following:

$$\varepsilon' = (c_1 - p_1) - (c_2 - p_2) - (X_2 - X_1)e^{-r(T-t)} \quad (6)$$

If Equation 5 holds for the selected pair of put/call options ($\varepsilon' = 0$), then there are no internal arbitrage opportunities in the options market for the selected options. The equation states that the difference between options premiums should be equal to the discounted difference between the strike prices of contracts.

If $\varepsilon' < 0$, then the arbitrageur is interested in buying c_1 and selling p_1 (thus synthetically purchasing the underlying for the strike of X_1), and at the same time selling c_2 and buying p_2 (thus synthetically selling the underlying for the strike of X_2.

If $\varepsilon' > 0$, then the arbitrageur is interested in selling c_1 and buying p_1 (thus synthetically selling the underlying for the strike of X_1), and at the same time buying c_2 and selling p_2 (thus synthetically purchasing the underlying for the strike of X_2).

1.5. Review of empirical research in developed and developing countries

This paper examines the efficiency of Warsaw Stock Exchange financial derivatives markets. Poland is on its way to becoming a universally-recognized developed market. For example, it has been recognized as a developed market by FTSE rating in 2017. It is also a member of the OECD club. Nevertheless, its derivatives market is relatively young as compared to developed countries. The first derivatives were floated in WSE in 1998 (those were WIG20 futures, later in the paper more details will be provided). In addition, Poland is referred to as a developing country by the IMF. Therefore, the research of Polish derivatives market is more logical from a developing market perspective. That is why the take-aways from the literature review are focused more on the existing research in the field of developing countries. Developing countries are specific in that their somewhat immature financial markets exhibit different patterns than developed markets, and then change as the market matures (e.g., McMillan and Ülkü, 2010). Developed countries' markets are considered too, because they provide the basis for further research of developing countries' markets.

The concept of put-call parity has been widely adopted in the academic literature. Sternberg (1994) examines the put-call parity and market efficiency in CME traded S&P500 index futures and futures' options contracts and finds numerous violations of PCP that cannot be explained away by transaction costs. Vipul (2008) studies put-call parity and put-call-futures parity of the Indian Nifty index and finds that the parity is frequently violated. Wang, Kang, Sia and Li (2018) find that significant profits can be made from put-call-futures parity violations in Shanghai Stock Exchange as well, while at the same time examining how regulations influence market efficiency. Regulations' role in market efficiency is also researched by Wang (2010) in case of Singapore Exchange Limited (SGX) and Taiwan Futures Exchange (TAIFEX). Bhat and Arekar (2015) examine exchange-traded currency options market efficiency in India. They use daily closing prices data and find that there are many arbitrage opportunities in the market, and they vary when examined for time to maturity and other parameters. Jongadsayakul (2018) employs put-call parity and box spread methodologies on Thailand SET50 derivatives market and finds that violations' sizes depend on the liquidity of SET50 Index Options market.

There are numerous academic works that found that inefficiencies in the options markets are negligible after accounting for transaction costs. Lee and Nayar (1993) research the S&P 500 index futures and options and find that they constitute an integrated market. Fung, Cheng and Chan (1997) find that trading based on mispricing strategies is not practically attractive. This study suggests that the index futures and index options markets in Hong Kong are practically efficient during the 1993-1994 sample period. Draper and Fung (2002) examine London FTSE futures and options cross-market efficiency and find that arbitrage opportunities for traders facing transaction costs are small in number and confirm the efficiency of trading on the London International Financial Futures and Options Exchange. Brunetti and Torricelli (2007) examine the intra-market efficiency in Italian Mib30 index options market, and conclude that the market is efficient. Capelle-Blancard and Chaudhury (2001) find that although the French CAC40 index options market shows traces of inefficiency, in totality French index options market is efficient. Such conclusion is made as the majority of violations are unexploitable after incorporating transaction costs. The same is concluded by Mohanti and Priyan (2015) in relation to the Indian S&P CNX Nifty index options market. Zhang and Watada (2019) analyze the Chinese SSE 50 ETF options market and conclude that arbitrage opportunities are existent but infrequent when transaction costs are considered.

1.6. Corollary

An efficient market is one where market prices fully reflect available information. There are three forms of market efficiency: weak, semi-strong and strong. These differ by the criterion of the amount of information available to an investor. Weak form of market efficiency involves past trading information. Semi-strong form of market efficiency involves all publicly available information. Finally, strong form of market efficiency involves all available information, including inside information.

Derivative securities are very important for functioning of financial markets in several ways. The most widespread derivatives are futures and options. The definitions of these securities are covered in the respective section of the chapter. Derivatives serve for numerous purposes. First of all, they provide traders with risk management strategies. An investor can build numerous strategies with futures, and put or call options of different strike prices. Derivative securities also serve for price discovery as an additional source of market information. Ultimately, derivatives are oftentimes easier to trade than the underlying asset.

There are model-based and model-free approaches to assessing options market efficiency. The main of model-based approaches is the Black-Scholes model, which may be complicated to implement in practice, as described in the respective section of the chapter. Therefore, the model-free approaches are chosen to be implemented for examining cross-market and intra-market efficiency in this paper.

Model-free approaches to testing options markets efficiency are based upon the arbitrage opportunities criterion. The market is considered efficient, if arbitrage opportunities are rare and insignificant.

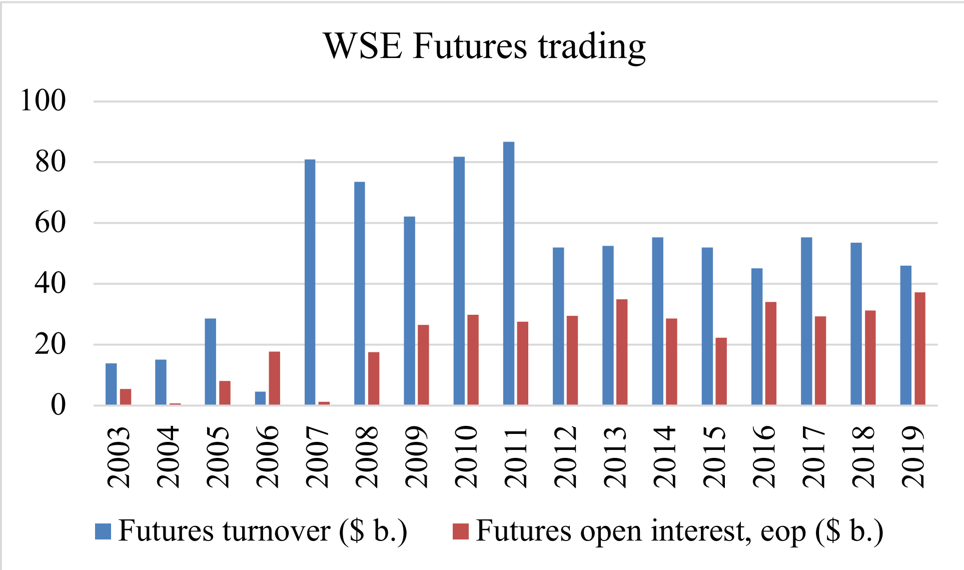
The approaches chosen for testing market efficiency are put-call-futures parity (the approach is basically an extension of the put-call parity approach) for cross-market efficiency and box spread for intra-market efficiency. The further research is based on respective existing empirical research in the field, which is extensively covered in the literature review section of the paper.

CHAPTER 2. DATA DESCRIPTION AND EMPIRICAL RESEARCH RESULTS

2.1. Warsaw Stock Exchange, derivatives and WIG20 index

Warsaw Stock Exchange has a long history. Polish financial market traditions go back to 1817, when the Warsaw Mercantile Exchange was established. Following the overthrow of Poland's former communist regime in 1989, WSE was created as a joint-stock company on April 12, 1991 (WSE website, History section). As per WSE website, there are around 3000 financial instruments listed on WSE.

A summary of futures and options trading dynamics throughout the period since 2003 is presented below in Graph 1 and Graph 2, respectively (based on WSE Main Statistics[1]).

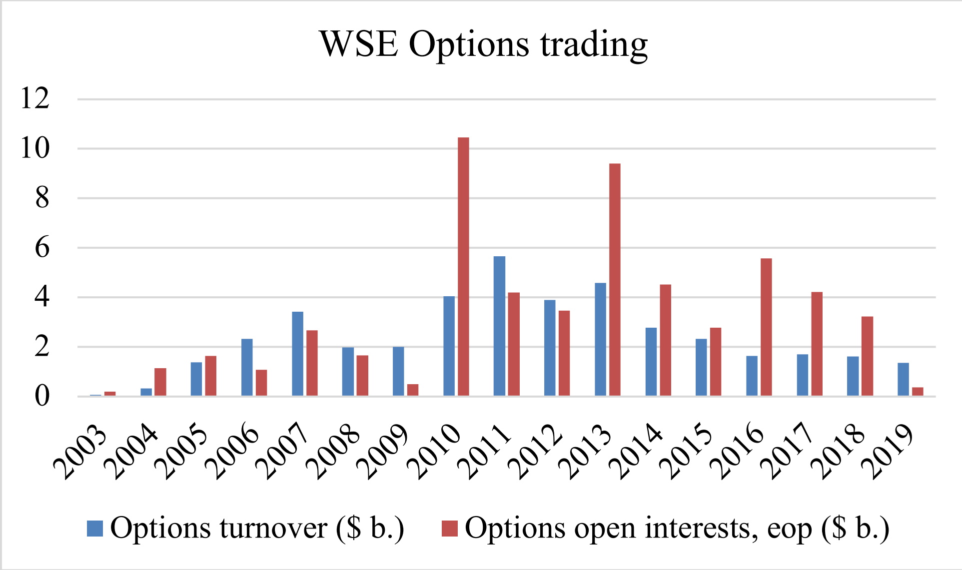


Graph 1. *Summary of futures trading in WSE.*

Source: [adapted by author from WSE website]

The amount of futures traded in WSE is substantial, as well as the open interest, as it can be seen form the graph above. Options trading on the floor of WSE, in turn, is summarized in Graph 2 below.

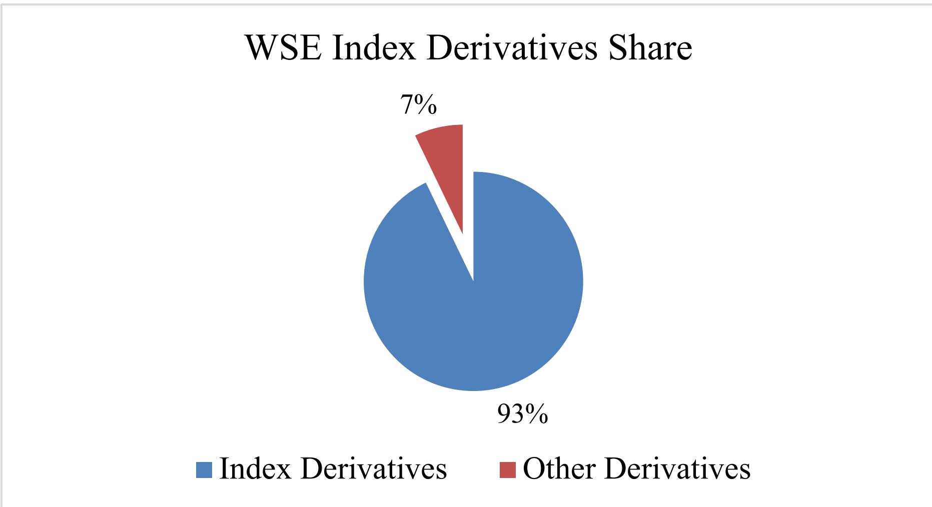
[1] 'eop' in the graphs stands for 'end of period', while $ b. stands for billions of USD.



Graph 2. *Summary of options trading in WSE.*

Source: [adapted by author from WSE website]

The WIG20 index was first computed by Warsaw Stock Exchange on April 16, 1994, and is calculated since then. It is constituted out of the 20 largest and most liquid firms traded on the floor of WSE. Considering only 20 blue-chip companies for index calculation is not a common practice globally, but the traded volume of these shares accounts for 80% of the entire stock market trade volume, as stated in Marcinkiewicz (2016, 3). It can also be seen from Graph 3 below that most of the derivatives traded in WSE are index derivatives.



Graph 3. *Index derivatives share in total WSE derivatives trade, as of 2019.*

Source: [adapted by author from WSE website]

WIG20 futures were the first derivatives introduced to the WSE (in 1998), they also are one of the most traded derivatives in WSE. Nowadays the exchange also offers options with WIG20 index as the underlying.

2.2. Data collection

2.2.1. Contract specification

Warsaw Stock Exchange (WSE) provides a platform for securities trading under its specific rules. To fully understand how trading is conducted in WSE, the WSE Rules are considered. The Rules prove useful, in particular, to point out the commissions and contracts' specification. WIG20 futures are issued on a quarterly basis and have a two-year maturity in the months of March, June, September and December of each year. The date of expiry is the third Friday of the expiry month. In case such day is not a trading day, the expiry date falls on the last trading day before the third Friday of the expiry month.

WIG20 options traded on the floor of WSE are put and call European options. The selected options have the same maturity as the selected WIG20 futures contracts. It is crucial to employ futures and options with the same maturity, for the put-call-futures parity assumptions (described in section 1.4.2) to hold.

The following maturities of WIG20 derivative securities are employed in this paper: December 2017; March, June, September, December of 2018; March, June, September, December of 2019. As stated above, options that are picked have the same maturities, as well as the expiration date: third Friday of each month of maturity.

2.2.2. Data collection procedure, market conventions

Daily closing prices data on trading of WIG20 futures and put and call options contracts are gathered from WSE website. After that contracts are matched for put-call-futures parity condition testing, which means that put/call option contracts are matched with a futures contract with a corresponding maturity. Contracts that were not traded on a particular day are not considered and are not included in the sample of triplets or options pairs.

A total of 2098 triplets of futures, call and put options were collected for put-call-futures parity testing. These consist of closing prices of respective traded contracts.

Options are matched by the same criteria for the means of box spread arbitrage condition testing. A total of 3051 pairs of call and put options were collected.

As for the risk-free rate to use in the model, it is pertinent to employ the three-month WIBOR rate (Warsaw Interbank Offer Rate), as it is used as a risk-free rate in the Polish market, according to Białkowski and Jakubowski (2008). WIBOR is the average of interest rates on the interbank deposit market in Poland, calculated as the arithmetic mean of the rates quoted by the WIBOR participants, after rejecting two outliers (the highest and lowest quote). The three-month maturity also corresponds to the maturity of futures contracts, since the nearest WIG20 futures contract are used in the model as the most liquid, and those contracts are quarterly.

It is also important to mention that the index calculation is constituted in a way that it does not account for the dividends. Therefore, the dividend yield needs to be combined with the risk-free rate in order to compensate for that. This matter is discussed further in section 2.4.

The daily values of WIBOR rate are extracted from WSE website.

In order to plug the WIBOR rates into Equation 4 (put-call-futures parity) and Equation 6 (box-spread), one needs to convert the rate to a continuously compounded rate using the following formula:

$$r_{cc} = \ln{(1 + WIBOR)} \tag{7}$$

The study assumes an equal cost of funds for all market participants, in accordance with Wang et al (2018) and others.

The sample reveals multiple violations of put-call-futures parity and box spread conditions (assuming a frictionless market, initially). Namely, without adding transaction costs into the equations, it is found that in 100% of put-call-futures parity triplets the equation does not hold, so these are all mispriced. For the box spread, there is the same tendency: 100% of put and call options pairs are mispriced. Nevertheless, investors have to encounter transaction costs in order to trade, so it would be incomplete to try assessing the room for arbitrage without having considered transaction costs. The next section sheds some light on transaction costs that investors encounter when trading on WSE floor.

2.3. Transaction costs

We further distinguish between two types of agents in this paper: retail investors that are not WSE members (therefore they have to pay brokerage in addition to other costs) and WSE members who are brokers themselves and hence pay WSE commissions instead of brokerage costs, which are in their case inapplicable. Retail investors occupy a significant part of the trading volume in WIG20 futures market according to Bohl, Salm, Schuppli (2011, 287), so market segmentation is accounted for in this paper.

There are two types of costs that an investor has to encounter in order to execute an arbitrage transaction: setup costs and cost of carry. Setup costs are further divided into direct and indirect. Direct costs are stock exchange fees (for member investors) or brokerage fees (in case of retail investors). Indirect costs are the costs that are determined by the liquidity of securities of interest, in other words, are derived from the bid-ask spread.

The setup cost for exchange members is as follows: 1,6 PLN[2] per futures contract and 0,6% per put/call option contract (with a 'collar': minimum 0,2 PLN per option and maximum 1,2 PLN per option), as per WSE Trading Rules.

Białkowski and Jakubowski (2008, 369) suggest a setup cost of 0,9% of the transaction value for retail investors in WSE derivatives market. They do not consider the indirect cost, though. Therefore, the cost of 0,9% is further added to the indirect cost of trading and cost of carry to obtain total transaction cost for retail investors.

Cost of carry, in essence, is the opportunity cost of the capital allocated for contract provision in order to sustain the position. This includes the initial margin (applicable to short options positions and retail long futures positions, because as per WSE Rules WSE members impose the margin on their clients themselves, and do not provide a margin for futures contracts for proprietary trading). The cost of carry is determined by the following three factors: the initial margin, the opportunity cost of capital (which is assumed to be the risk-free rate of return for the period until maturity) and transaction value. The mark-to-market provision is ignored, which is in line with Wang et al (2018). The initial margin for long futures position is assumed to be 10%. This margin applies to retail investors only, since, as mentioned above, according to the WSE Trading Rules exchange member agents are not obliged to post a margin for long futures positions. The initial margin on short option positions in WSE is determined by KDPW_CCP, which is a central clearing house for transactions in Poland. The exact amount of margin is calculated in accordance with SPAN methodology, which is quite common globally (for example, it is used in Chicago Mercantile Exchange). According to it the overall portfolio risk is evaluated by calculating the worst possible loss that might reasonably be incurred over a specified period of time. According to KDPW_CCP calculator, the initial margin on short option positions is equal to 100% of the option value.

As mentioned above in section 1.4.2, short arbitrage strategy in put-call-futures parity involves borrowing the futures from a broker or another market participant. Therefore, the trader has to pay interest on what she borrows. This cost is calculated on the basis of futures contract value and the respective period of time until maturity cost of funds (which, as mentioned above, is assumed to be the same interest rate per annum for all market participants).

[2] PLN stands for Polish zloty, the national Polish currency. As of 30.04.2020, 1 PLN=$0,24.

Since the research is based upon daily closing data, which does not include bid/ask spreads, indirect trading costs cannot be calculated directly. Different option strike prices would most likely imply different bid/ask spreads, in accordance with their liquidity. Nevertheless, indirect costs have to be accounted for, because it is crucial to incorporate all possible costs in examination of arbitrage in a market. In real trading conditions arbitrage only exists in the context of transaction costs. Foregoing indirect costs of trading would make the research simplistic.

Ackert and Tian (2001) suggest assuming a ‘usual spread’ in prices, and with its help deriving bid and ask prices from daily closing prices. Therefore, in line with this logic, in this paper the ‘usual spread’ is computed on the basis of current bid and ask prices of futures and options currently traded on WSE. In absence of other reliable sources of information, this would be the best approximation to adopt. The current trading information is extracted from Eikon Refinitiv. It is important to note that the main concern of the research in this paper is not the exact amount of arbitrage profits, but the general efficiency of derivatives markets.

Wang et al (2018, 2560) employ the following formula for calculation of indirect costs:

$$Ratio\ of\ the\ spread\ cost = \frac{Ask\ price}{(Bid\ price + Ask\ price)/2} - 1 \qquad (8)$$

$$Cost\ of\ the\ spread = Ratio\ of\ the\ spread\ cost * Transaction\ price \qquad (9)$$

So, the formula in (9) is used for the calculation of indirect costs of trading and is further incorporated into total setup cost.

2.4. Realistic revision of equations

In order to make the relations of put-call-futures parity and box spread in (4) and (6) in sections 1.4.2 and 1.4.3, respectively, there is a necessity to incorporate the dividend yield and the total transaction cost into them. As it was mentioned earlier, the absolute amount of mispricing has to be greater than any transaction costs encountered by the agent. Dividend yield is incorporated into the equations in accordance with Białkowski and Jakubowski (2008, 368). Thus, these two variables are plugged into the equations in the following manner for the put-call-futures parity:

$$\varepsilon = \left| c - p - (F - X)e^{-(r-d)(T-t)} \right| - TC, \qquad (10)$$

and for the box spread:

$$\varepsilon = \left|(c_1 - p_1) - (c_2 - p_2) - (X_2 - X_1)e^{-(r-d)(T-t)}\right| - TC, \qquad (11)$$

where d – tax-adjusted dividend yield for the WIG20 index,
TC – total transaction cost,
ε – arbitrage profit (having accounted for transaction costs).

According to Eikon Refinitiv, the dividend yield for WIG20 index is equal to 2,03%. This figure has to be adjusted for the dividend tax, which is equal to 19% in Poland. Therefore, the tax-adjusted dividend yield for WIG20 amounts to 1,64%.

A relevant remark would be to note that the effect of the dividend yield is somewhat marginal for arbitrage calculations. Contracts that are investigated for arbitrage opportunities in this paper are mostly considered within the time horizons up to 3 months (as the most liquid ones), so the effect of the discount rate is almost nil as compared with transaction costs. Nevertheless, the dividend yield is important to incorporate into formulas for the sake of exhaustiveness of factors in the research.

2.5. Data analysis and hypotheses testing

Now, having obtained all necessary inputs for the computations, it is possible to proceed to assessment of arbitrage opportunities, as a criterion of cross-market efficiency (put-call-futures parity) and of intra-market efficiency (box spread).

Let us also introduce a disambiguation concerning the terms of 'intra-market efficiency' and 'cross-market efficiency'. Intra-market efficiency is referred to as the market efficiency as reflected in put and call options prices only. The criterion of market efficiency in this case is no-arbitrage condition between put and call options of different maturities and strike prices. Cross-market efficiency refers to the market efficiency as reflected in market prices of put and call options and futures prices all simultaneously, with no-arbitrage condition between put and call options and futures markets as the criterion.

On the basis of all transaction costs having been incorporated into the equations in the manner described above in section 2.4, put-call-futures parity calculations exhibit 0,48% arbitrage opportunities for exchange members and 0,05% arbitrage opportunities for retail investors in the total of 2098 futures-options triplets.

As for box spread, there are 47,76% and 47,9% arbitrage opportunities for member and retail investors, respectively, in the total of 3051 options pairs.

The descriptive statistics for put-call-futures parity member investor arbitrage profits are presented below in Table 1.

Table 1. Put-call-futures parity member investor profits.

Member arbitrage profits summary	
Mean	-534,93
Standard error	5,83
Standard deviation	267,12
Variance	71355,41
Excess	0,84
Asymmetry	-0,59
Interval	2463,90
Minimum	-1591,05
Maximum	872,85
Sum	-1122292,32
Count	2098

Source: [Based on author's analysis]

The descriptive statistics for put-call-futures parity retail investor arbitrage profits are presented below in Table 2.

Table 2. Put-call-futures parity retail investor profits.

Retail arbitrage profits summary	
Mean	-748,47
Standard error	5,97
Median	-719,71
Standard deviation	273,51
Variance	74808,83
Excess	0,79
Asymmetry	-0,62
Interval	2469,60
Minimum	-1846,43
Maximum	623,17
Sum	-1570299,27
Count	2098

Source: [Based on author's analysis]

It can be suggested that, based on sample mean, retail investors' arbitrage profits are feasibly lower than those of member investors. This can be explained away by a lower level of transaction costs that member investors encounter as compared with retail investors. We also see a greater variance in retail arbitrage profits than in member arbitrage profits (as measured by the standard deviation).

Now let us turn to box spread condition descriptive statistics. The descriptive statistics for box spread member investor arbitrage profits are presented below in Table 3.

Table 3. Box spread member investor profits.

Member arbitrage profits summary	
Mean	39,09
Standard error	10,63
Median	-16,3
Standard deviation	586,96
Variance	344517,68
Excess	0,1
Asymmetry	0,31
Interval	4176,64
Minimum	-2101,14
Maximum	2075,5
Sum	119260,25
Count	3051

Source: [Based on author's analysis]

The descriptive statistics for box spread retail investor arbitrage profits are presented below in Table 4.

Table 4. Box spread retail investor profits.

Retail arbitrage profits summary	
Mean	41,88
Standard error	10,61
Median	-12,25
Standard deviation	586,28
Variance	343727,51

Table 4 (continued). Box spread retail investor profits.

Excess	0,10
Asymmetry	0,31
Interval	4173,80
Minimum	-2097,95
Maximum	2075,85
Sum	127776,30
Count	3051

Source: [Based on author's analysis]

From the tables 3 and 4 one may suppose that retail arbitrage profits are greater than member arbitrage profits. This can be explained by the fact that member option transaction costs are 'collared' (there is a minimum level of commission), while retail option transaction cost are a fixed amount of 0,9% of the transaction value. Therefore, options with low prices are cheaper to trade for retail investors.

Having taken note of the descriptive statistics, it is necessary now to proceed to hypotheses formulation. Any attempt of inference has to be checked for statistical significance. Let us now return to the goals of the research and once again re-state the issues that are investigated in this paper.

First of all, the paper aims to investigate cross-market efficiency between WIG20 options and futures in WSE from the standpoint of put-call-futures parity. This is further specified for the two types of investors considered: exchange members and retail investors.

Second, the paper aims to investigate internal market efficiency in options market from the standpoint of box spread condition. This is specified for member and retail investors as well.

Finally, the paper aims to provide a clear picture of how the room for arbitrage differs when accounted for trader strategy, time to maturity and option moneyness.

The hypotheses for further investigation are summarized below in Table 5, linked to the respective research questions.

Table 5. Hypotheses summary.

Hypothesis №	Research question	Ho	Ha
1	Is there cross-market efficiency between WIG20 options and futures markets from the standpoint of *member* investors?	Mean *member* put-call-futures parity arbitrage profits are equal to zero.	Mean *member* put-call-futures parity arbitrage profits are less than zero.
2	Is there cross-market efficiency between WIG20 options and futures markets from the standpoint of *retail* investors?	Mean *retail* put-call-futures parity arbitrage profits are equal to zero.	Mean *retail* put-call-futures parity arbitrage profits are less than zero.
3	Is there internal market efficiency in WIG20 options market from the standpoint of *member* investors?	Mean *member* box spread arbitrage profits are equal to zero.	Mean *member* box spread arbitrage profits are more than zero.
4	Is there internal market efficiency in WIG20 options market from the standpoint of *retail* investors?	Mean *retail* box spread arbitrage profits are equal to zero.	Mean *retail* box spread arbitrage profits are more than zero.
5	Do put-call-futures parity arbitrage profits of member investors significantly differ when accounted for *investment strategy* (short vs. long arbitrage)?	Mean short arbitrage and long arbitrage profits in put-call-futures parity are the same.	Mean short arbitrage and long arbitrage profits in put-call-futures parity are significantly different.
6	Do box spread arbitrage profits of member investors significantly differ when accounted for *time to maturity?*	There is no difference in arbitrage between contracts of different maturities.	There is a significant difference in arbitrage profits between contracts of different maturities.
7	Do box spread arbitrage profits of member investors significantly differ when accounted for *option moneyness*?	There is no difference in arbitrage between contracts of different moneyness.	There is a significant difference in arbitrage profits between contracts of different moneyness.

Statistical tests of significance of difference in the mean were run in Stata. Since the population variance is unknown, Student's one-sample T-tests are employed. In addition, since the subsamples (derived from put-call-futures parity data based on trading strategy and box-spread option pairs' time to maturity) are not independent, it would be inappropriate to employ Student's paired T-test for Hypotheses 5, 6 and 7. Instead, Kruskal-Wallis paired samples test is implemented, in accordance with Bhat and Arekar (2015, 11).

We start hypotheses testing with put-call-futures parity condition testing for member and retail investors.

2.5.1. Hypothesis 1

Mean member put-call-futures parity arbitrage profits are equal to zero.

Ho: $\mu = 0$

Ha: $\mu < 0$

The summary of a one-sample T-test conducted in Stata is presented in Figure 1 below.

```
. ttest Member_PCP=0

One-sample t test
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
Member~P |    2098   -534.9344    5.831909    267.1243   -546.3713   -523.4974
------------------------------------------------------------------------------
    mean = mean(Member_PCP)                                       t = -91.7254
Ho: mean = 0                                     degrees of freedom =     2097

    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 0.0000         Pr(|T| > |t|) = 0.0000          Pr(T > t) = 1.0000
```

Figure 1. *Member put-call-futures parity arbitrage profits.*

Source: [Based on author's analysis]

With a 5% level of significance, the null hypothesis is rejected. This means that the arbitrage profits of stock exchange members in WSE are significantly lower than zero, therefore arbitrage is not possible, and there is cross-market efficiency between futures and options markets for WSE member investors.

2.5.2. Hypothesis 2

Mean retail put-call-futures parity arbitrage profits are equal to zero.

Ho: $\mu = 0$

Ha: $\mu < 0$

The summary of a one-sample T-test conducted in Stata is presented in Figure 2 below.

```
. ttest Retail_PCP=0

One-sample t test
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
Retail~P |    2098   -748.4744    5.971366     273.512   -760.1848    -736.764
------------------------------------------------------------------------------
    mean = mean(Retail_PCP)                                       t = -1.3e+02
Ho: mean = 0                                     degrees of freedom =     2097

    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 0.0000         Pr(|T| > |t|) = 0.0000          Pr(T > t) = 1.0000
```

Figure 2. *Retail put-call-futures parity arbitrage profits.*

Source: [Based on author's analysis]

With a 5% level of significance, the null hypothesis is rejected. This means that the arbitrage profits of retail investors in WSE are significantly lower than zero, therefore arbitrage is not possible, and there is cross-market efficiency between futures and options markets for WSE retail investors.

Now we proceed to box spread condition arbitrage profits significance testing with the same methodology.

2.5.3. Hypothesis 3

Mean exchange member box spread arbitrage profits are equal to zero.

From descriptive statistics we see that box spread exhibits many more arbitrage opportunities than put-call-futures parity (when having accounted for transaction costs), so the alternative hypothesis is formulated in a different way.

Ho: $\mu = 0$

Ha: $\mu > 0$

The summary of a one-sample T-test conducted in Stata is presented in Figure 3 below.

```
. ttest Member_BS=0

One-sample t test
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
Member~S |    3051     39.0889    10.62636    586.9563    18.25335    59.92446
------------------------------------------------------------------------------
    mean = mean(Member_BS)                                        t =   3.6785
Ho: mean = 0                                     degrees of freedom =     3050

    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 0.9999         Pr(|T| > |t|) = 0.0002          Pr(T > t) = 0.0001
```

Figure 3. *Member box spread arbitrage profits.*

Source: [Based on author's analysis]

With a 5% level of significance, the null hypothesis is rejected. This means that the arbitrage profits of retail investors in WSE are significantly greater than zero, therefore arbitrage is possible, and there is no intra-market efficiency in options markets for WSE member investors.

2.5.4. Hypothesis 4

Mean retail investor box spread arbitrage profits are equal to zero.

Ho: $\mu = 0$

Ha: $\mu > 0$

The summary of a one-sample T-test conducted in Stata is presented in Figure 4 below.

```
. ttest Retail_BS=0

One-sample t test
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
Retail~S |    3051    41.88014    10.61417    586.2828    21.06849    62.69179
------------------------------------------------------------------------------
    mean = mean(Retail_BS)                                        t =   3.9457
Ho: mean = 0                                     degrees of freedom =     3050

    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 1.0000         Pr(|T| > |t|) = 0.0001          Pr(T > t) = 0.0000
```

Figure 4. *Retail box spread arbitrage profits.*

Source: [Based on author's analysis]

With a 5% level of significance, the null hypothesis is rejected. This means that the arbitrage profits of retail investors in WSE are significantly greater than zero, therefore arbitrage is possible, and there is no intra-market efficiency in options markets for WSE retail investors.

2.5.5. Hypothesis 5

Short arbitrage and long arbitrage strategies' profits within put-call-futures parity do not significantly differ.

Ho: $\mu_{short} = \mu_{long}$

Ha: $\mu_{short} \neq \mu_{long}$

The descriptive statistics of arbitrage profits in short arbitrage subsample are presented below in Table 6.

Table 6. Short arbitrage profits descriptive statistics.

Short arbitrage profits summary	
Mean	-559,62
Standard error	8,85
Median	-529,13
Standard deviation	280,99
Variance	78954,11
Excess	-0,4
Asymmetry	-0,49
Interval	1541,16
Minimum	-1458,18
Maximum	82,97
Sum	-564095,22
Count	1008

Source: [Based on author's analysis]

The descriptive statistics of arbitrage profits in long arbitrage subsample are presented below in Table 7.

Table 7. Long arbitrage profits descriptive statistics.

Mean	-512,11
Standard error	7,62
Median	-485,08
Standard deviation	251,61
Variance	63308,88
Excess	2,04
Asymmetry	-0,66
Interval	2463,90

Table 7 (continued). Long arbitrage profits descriptive statistics

Minimum	-1591,05
Maximum	872,85
Sum	-558197,09
Count	1090

Source: [Based on author's analysis]

The summary of a Kruskal-Wallis test conducted in Stata in relation to trader strategy subsamples of put-call-futures parity member arbitrage profits is presented in Figure 5 below.

```
. kwallis Profit , by(Strategy)

Kruskal-Wallis equality-of-populations rank test

  Strategy |  Obs  | Rank Sum
-----------+-------+----------
      long |  1090 | 1.20e+06
     short |  1008 | 1.00e+06

chi-squared =    14.915 with 1 d.f.
probability =     0.0001

chi-squared with ties =    14.915 with 1 d.f.
probability =     0.0001
```

Figure 5. *Long vs. Short PCP arbitrage profits.*

Source: [Based on author's analysis]

With a 5% level of significance, the null hypothesis is rejected. This means that there is a statistically significant difference in mean put-call-futures parity short and long arbitrage. This can be explained by the fact that in case of short arbitrage strategy, there is an additional transaction cost associated with borrowing the futures.

2.5.6. Hypothesis 6

Mean member box spread arbitrage profits do not significantly differ for contracts with up to 10 days until maturity (group 2) and for contracts with more than 10 days till maturity (group 1).

Ho: $\mu_2 = \mu_1$

Ha: $\mu_2 \neq \mu_1$

The descriptive statistics of box spread profits from contracts with more than 10 days to maturity (group 1) are presented below in Table 8.

Table 8. Descriptive statistics of box spread profits from contracts with more than 10 days to maturity (group 1).

Group 1 profits summary	
Mean	-33,34
Standard error	13,65
Median	-102,05
Standard deviation	608,66
Variance	370469,7
Excess	0,09
Asymmetry	0,41
Interval	4176,64
Minimum	-2101,14
Maximum	2075,5
Sum	-66273,93
Count	1988

Source: [Based on author's analysis]

The descriptive statistics of box spread profits from contracts with up to 10 days to maturity (group 2) are presented below in Table 9.

Table 9. Descriptive statistics of box spread profits from contracts with up to 10 days to maturity (group 2).

Group 2 profits summary	
Mean	174,54
Standard error	15,88
Median	117,65
Standard deviation	517,79
Variance	268102,81
Excess	0,33
Asymmetry	0,33
Interval	3565,21
Minimum	-1538,76
Maximum	2026,46

Table 9 (continued). Descriptive statistics of box spread profits from contracts with up to 10 days to maturity (group 2).

Sum	185534,18
Count	1063

Source: [Based on author's analysis]

The summary of a Kruskal-Wallis test conducted in Stata in relation to contract subsamples of box spread member arbitrage profits is presented in Figure 6 below.

```
. kwallis Profit, by(Maturity_group)

Kruskal-Wallis equality-of-populations rank test

  +----------------------------+
  | Maturi~p |  Obs | Rank Sum |
  |----------+------+----------|
  |        1 | 1988 | 2.79e+06 |
  |        2 | 1063 | 1.86e+06 |
  +----------------------------+

chi-squared =   109.473 with 1 d.f.
probability =     0.0001

chi-squared with ties =   109.473 with 1 d.f.
probability =     0.0001

.
```

Figure 6. *Box spread arbitrage profits with respect to contracts' maturity.*

Source: [Based on author's analysis]

With a 5% level of significance, the null hypothesis is rejected. This means that there is a statistically significant difference in mean box spread arbitrage profits for contracts of different maturities.

Such conclusion may seem very interesting, having considered that contracts with longer maturities are likely to have even higher indirect costs of trading (as they are less liquid) than the average level of indirect costs implemented in this research. This study assumes the same indirect cost for all contracts, so assuming even higher indirect costs for contracts with maturities of over than 10 days would further decrease room for arbitrage.

2.5.7. Hypothesis 7

Mean member box spread arbitrage profits do not significantly differ for contracts at the money (ATM) and for contracts in or out of the money (not ATM).

Ho: $\mu_2 = \mu_1$

Ha: $\mu_2 \neq \mu_1$

The descriptive statistics of box spread profits for at-the-money options are presented below in Table 10.

Table 10. Summary of at-the-money contracts' BS arbitrage profits.

ATM BS profits summary	
Mean	-116,44
Standard error	12,35
Median	-142,10
Standard deviation	419,16
Variance	175695,28
Excess	0,27
Asymmetry	0,39
Interval	3175,75
Minimum	-1208,01
Maximum	1967,73
Sum	-134139,78
Count	1152

Source: [Based on author's analysis]

The descriptive statistics of box spread profits for non-ATM (out-of-the-money and in-the-money) options are presented below in Table 11.

Table 11. Summary of not-at-the-money contracts' BS arbitrage profits.

Non-ATM BS profits summary	
Mean	133,44
Standard error	14,93
Median	113,08
Standard deviation	650,76
Variance	423489,30
Excess	-0,23
Asymmetry	0,05
Interval	4176,64
Minimum	-2101,14

Table 11 (continued). Summary of not-at-the-money contracts' BS arbitrage profits.

Maximum	2075,50
Sum	253400,03
Count	1899

Source: [Based on author's analysis]

The summary of a Kruskal-Wallis test conducted in Stata in relation to contract subsamples of box spread member arbitrage profits is presented in Figure 7 below.

```
. kwallis Profit , by(Moneyness)

Kruskal-Wallis equality-of-populations rank test
```

Moneyn~s	Obs	Rank Sum
ATM	1152	1.49e+06
notATM	1899	3.17e+06

```
chi-squared =    131.527 with 1 d.f.
probability =     0.0001

chi-squared with ties =   131.527 with 1 d.f.
probability =     0.0001

.
```

Figure 7. *Box spread arbitrage profits with respect to contracts' moneyness.*

Source: [Based on author's analysis]

With a 5% level of significance, the null hypothesis is rejected. This means that there is a statistically significant difference in mean box spread arbitrage profits for contracts of different moneyness. More precisely, non-ATM contracts exhibit a greater level of arbitrage profits.

2.6. Corollary

This chapter provided details for the Warsaw Stock Exchange as an institution, its financial markets and brief history. It was shown the WSE represents a financial market, which is the biggest one in terms of derivatives trading in Central and Eastern Europe. Amounts of derivatives traded on the floor of WSE are also substantial.

The data collection methodology is extensively described. Light is shed upon the details of contract specification in WSE, the process of data collection and relevant market conventions in WSE are described.

As mentioned in the paper before, examination of arbitrage opportunities in a specific market is only feasible and realistic in the context of transaction costs. Therefore, transaction costs that an investor encounters while trading in WSE are exhaustively specified. The investors are divided into two groups: exchange members and retail investors. Transaction costs differ for these two types of investors. Arbitrage opportunities are examined later in the paper. The information about transaction costs is included in initial equations in a manner described in Section 2.4.

Finally, Section 2.5 re-states the hypotheses of the research, describes the instruments used for hypotheses testing, as well as the results of their implementation.

As a result, we see that arbitrage profits of exchange members and retail investors following put-call-futures parity arbitrage strategies are statistically insignificant. For arbitrage profits significance testing one-sample Student's T-tests are employed. Therefore, arbitrage is impossible in Warsaw Stock Exchange WIG20 derivatives trading markets, neither for exchange members, nor for retail investors, and these markets are, therefore, efficient.

On the other hand, we see a different picture when examining the intra-market efficiency with the box-spread condition. We find that both for exchange members and retail investors there are significant arbitrage opportunities. Based on that we conclude that there is no intra-market efficiency in WSE WIG20 options markets.

The results of the study are compared with other studies in the field of derivatives' markets efficiency in Poland and other developing markets in Table 12 below.

Table 12. Overview of academic research in the field of developing countries' market efficiency.

Country	**Authors**	**Year**	**Derivatives markets**	**Underlying**	**Result**
Poland	Białkowski, Jakubowski	2008	Futures	Warsaw Stock Exchange WIG20 index	Market is inefficient for long futures arbitrage. Long futures arbitrage is profitable, short futures arbitrage is not profitable.

Table 12 (continued). Overview of academic research in the field of developing countries' market efficiency.

Poland	Marcinkiewicz	2016	Futures	Warsaw Stock Exchange WIG20 index	After WSE lifted short sale restrictions, the market efficiency increased, and arbitrage became unprofitable for traders with highest transaction costs.
India	Bhat, Arekar	2015	Options and futures	Currency	Market is inefficient. Arbitrage is profitable.
India	Vipul	2007	Options and futures	S&P CNX Nifty index	Market is inefficient. Arbitrage opportunities show persistent returns.
India	Mohanti, Priyan	2015	Futures and options	S&P CNX Nifty index	Market is efficient. Arbitrage is not profitable.
Thailand	Jongadsayakul	2018	Futures and options	Thailand SET50 index futures	Market is inefficient. Arbitrage is profitable. Box spread arbitrage profits are greater than put-call-futures parity arbitrage profits.
China	Zhang, Watada	2019	Options	Shanghai 50 ETF (tracks Shanghai 50 stock index)	Market is efficient. Arbitrage opportunities are infrequent.
China	Wang, Kang, Xia, Li	2018	Futures and options	Shanghai 50 stock index	Market is inefficient. Arbitrage is profitable.

Source: [Based on author's research]

On the basis of Table 12, it can be concluded that the general results of the research are in line with most of the existing academic research in the field of developing markets, while it is shown earlier in the paper (literature review, section 1.5) that in most developed markets arbitrage is impossible.

Later in the paper we examine the significance of difference in the mean of short and long arbitrage strategies in put-call-futures parity and find that long futures arbitrage has different mean arbitrage profits than short futures arbitrage. The significance of difference in the mean arbitrage profits for contracts with different maturities within box-spread strategies is tested. We find that contracts with up to 10 days maturities exhibit a greater amount of arbitrage profits than contracts with maturities of over than 10 days. It is also found that non-ATM contracts exhibit a greater level of arbitrage profits than ATM contracts.

CONCLUSION AND MANAGERIAL IMPLICATIONS

Conclusion

In this paper the theoretical framework for research in the field of market efficiency and arbitrage opportunities examination was covered. Market efficiency was, in coherence with the classics of the academic literature, divided into three forms: the weak form, the semi-strong form, and the strong form of market efficiency. These forms differ with respect to the amount of information considered. The form of market efficiency further tested in the paper corresponds to the weak form of market efficiency.

The role of derivative securities in financial markets was discussed in detail. They serve for three main purposes: risk management, price discovery and oftentimes easier trading than the underlying asset. A sufficient literature review was carried out, and a research gap in derivatives' markets of Poland was found and stated. In addition, the main approaches to examining options and futures market efficiency were considered, having been divided into model-based approaches and model-free approaches. Model-free approaches were chosen for further research in this paper, since they prove easier to implement in practice. Among the model-free approaches, put-call parity was considered, which was further supplemented and substituted by the put-call-futures parity approach, and box spread.

Warsaw Stock Exchange was described as an institution, and a summary of trading rules in relevant relations was provided, as well as a summary of market conventions concerning the inputs for equations of put-call-futures parity and box spread. WSE boasts significant levels of trading in derivative securities for different types of investors. Data collection methodology included constructing triplets of futures and options (put and call) for put-call-futures parity condition testing, and constructing pairs of options for the box spread condition. These two equations were further supplemented with relevant transaction costs, so a realistic revision was conducted in order to conform to the market realm. It is shown that arbitrage cannot be considered without bearing in mind the respective transaction costs.

After all of the relevant inputs to the financial models were considered, progress was made towards testing the hypotheses on the basis of realistically re-evaluated models. As a result, the study showed that there is cross-market efficiency for retail investors and exchange members in relation to put-call-futures parity. Arbitrage profits are insignificant, and the market is efficient with a 5% level of significance. Different results were found for the box spread condition. It is shown that for both retail investors and exchange members the mean arbitrage profits are significantly greater than zero with a 5% level of significance. Therefore, there is no intra-market efficiency in WSE traded WIG20 options.

As we proceed further, we found that there is a statistically significant difference in the mean of different arbitrage strategies within put-call-futures parity. Namely, long arbitrage strategy exhibits a different level of arbitrage profits than the short arbitrage strategy. This can be explained by the inherently lower level of transaction costs that a trader encounters in a long arbitrage strategy, because in order to pursue the short arbitrage strategy, she has to borrow the futures contract elsewhere, and therefore, to pay interest on this borrowing, which adds up to other transaction costs.

It was also found that within the box spread arbitrage strategies, contracts with lower maturities (of up to 10 days) exhibit a significantly different level of arbitrage profits than contracts with higher maturities (of over than 10 days). This result may seem interesting given the fact that a common level of indirect transaction costs is adopted in the research, and contracts with longer maturities usually exhibit a greater level of indirect transaction costs, because they are less liquid (nearby contracts are the most traded ones). Incorporating an even higher level of indirect transaction costs for contracts with longer maturities would imply an even further decrease in the room for arbitrage in these contracts.

In addition, it was found that non-ATM contracts represent greater arbitrage profits than ATM contracts.

Managerial implications

It is now pertinent to turn to how the results of the research are useful to the stakeholders pointed out in the introduction. First, it is useful both for retail investors and exchange members to pay their attention to internal options market strategies, since statistically significant arbitrage profits can be made there. According to Białkowski and Jakubowski (2008, 369), there is at least one mutual fund in Poland that is concentrating on possible arbitrage on the floor of WSE. The level of transaction costs for retail investors (greater than those of exchange members), allows them to pursue arbitrage strategies, too. It is also suggested to pay attention to nearby options contracts with maturities of up to 10 days, since they exhibit greater mean arbitrage profits.

As it can be seen from Graph 2 (section 2.1), the trading volume as well as open interest in WIG20 options in WSE has been declining over the period of 2017-2019, while futures trading volume and open interest (Graph 1) remain somewhat stable. This fact may be connected further to a poor internal market efficiency in options markets that has been found in this paper for the same time period, though the causality and other factors will most probably have to be considered. It would be useful to further examine the reasons for the decline in options trading volume and open interest, and therefore this is an area for further research. Decreasing levels of trading in any security are harmful for a stock exchange, since it is an important source of revenue (in the form of stock exchange commissions from brokers). It may also harm the trading activity in the underlying instruments, since, as discussed in the paper, derivative instruments, among other functions, serve as price discovery instruments for the underlying asset.

If the derivatives market is efficient, then companies will use it to hedge against their risks. As shown in Bukhvalov (2010, 65) in the context of market frictions like transaction costs, information asymmetry and others, companies that use hedging are more valuable than those who do not. Therefore, given that in an efficient derivatives market these companies hedge more, they increase their value, and the exchange is better off, too, since the broad market grows and becomes more attractive to investors. Ultimately, the exchange is motivated to increase the efficiency of the derivatives market.

Introducing changes in the exchange regulation regime, indeed, proves useful in increasing market efficiency. For example, as per Marcinkiewicz (2016), lifting the short-selling restriction in Warsaw Stock Exchange led to an increase in the respective WIG20 futures market efficiency. Nevertheless, there is a possible side-effect of this change in regulations. Short-sellers may make the falling market fall with yet greater pace, when they anticipate the security to further decrease in price.

There is also evidence from the markets of Taiwan and Singapore, as shown in Wang (2010), showing that relaxing the uptick rule[3] should improve market efficiency. Therefore, changing the exchange regulatory regime does influence market efficiency. Among other possible measures of changing the exchange regulations may be changing the daily limits of change in the price of an asset, or others, whose effectiveness can be tested afterwards.

[3] Uptick rule requires short sales to be conducted at a higher price than the previous trade.

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