Supplementary material to the paper
"Network formation with asymmetric players and chance moves"
by Ping Sun and Elena Parilina
(Numerical example)

Let the set of players be $N \cup \{0\}$, where $N = \{1, 2, 3\}$, and player 0 is the Nature. The values of characteristic function are $v(\{1\}) = 1$, $v(\{2\}) = v(\{1, 2\}) = 2$, $v(\{3\}) = 1/2$, $v(\{2, 3\}) = 5/2$, $v(\{1, 3\}) = v(\{1, 2, 3\}) = 3$. The strategy set of the leader is $U_0 = \{\Gamma_1, \Gamma_2, \Gamma_3\}$, where $\Gamma_1 = \{12, 13\}$, $\Gamma_2 = \{12, 23\}$, $\Gamma_3 = \{13, 23\}$, the structures of which are shown in Fig. 1. And $T = \{I_1, I_2\}$. The payoff function for players of type I_1 is defined as

$$K_i(\Gamma, I_1) = Y_i(\Gamma) + |\Gamma(i)|, \tag{1}$$

where $Y_i(\Gamma)$ is a component of Myerson value (see Myerson (1977)), and for players of type I_2, it is defined as

$$K_i(\Gamma, I_2) = AT_i(\Gamma_{C^{\Gamma}(i)}) \tag{2}$$

where $AT_i(\Gamma_{C^{\Gamma}(i)})$ is the average tree solution value (see Herings et al. (2008)) of Player i in game $v_{C^{\Gamma}(i)}$ with structure $\Gamma_{C^{\Gamma}(i)}$.

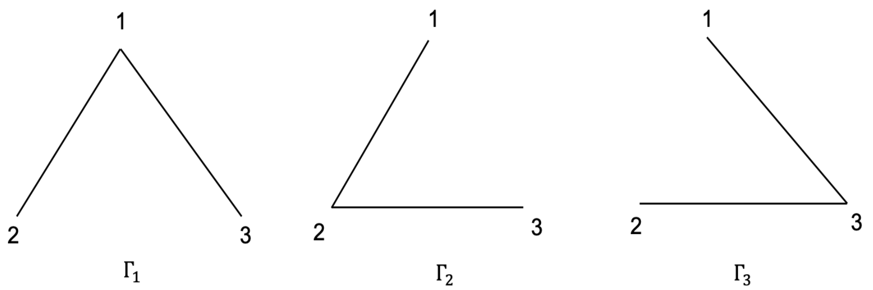

Figure 1: Network structures of Γ_1, Γ_2 and Γ_3.

The probability distribution p is given in Table 1. Let Player 1 be defined as the leader, and his fixed type, I' be I_2.

Types \ Players / Probability	Player 1	Player 2	Player 3
2/21	I_1	I_1	I_1
4/21	I_1	I_1	I_2
3/21	I_1	I_2	I_1
2/21	I_1	I_2	I_2
4/21	I_2	I_1	I_1
3/21	I_2	I_1	I_2
2/21	I_2	I_2	I_1
1/21	I_2	I_2	I_2

Table 1: Probability distribution defining the chance move.

Given Table 1, the conditional probability distribution with Player 1 being the leader is shown in Table 2.

t_2 \ t_3	I_1	I_2
I_1	2/5	3/10
I_2	1/5	1/10

Table 2: Conditional probability distribution $p(\cdot \mid t_1 = I_2)$.

Fig. 2-4 show the subgames $\Phi(x_{\Gamma_1})$, $\Phi(x_{\Gamma_2})$ and $\Phi(x_{\Gamma_3})$ respectively. Moreover, the black lines starting from the personal vertices of Player 2 and Player 3 show the unique Bayesian equilibrium in each subgame, and the black lines of different lengths starting from the vertices at which the Nature makes the chance move show the various probabilities with which the Nature selects different type profiles. Note that Fig. 2-4 are obtained by software Gambit (see McKelvey et al. (2006)). The program which is used to calculate the Bayesian equilibrium of subgame $\Phi(x_{\Gamma_1})$ is provided by the link: `http://hdl.handle.net/11701/27019`. The first column of the outcome is the probability vector with the first element of which Player 2 of type I_1 chooses action 'a' and chooses action 'r' with the second element. And the second column of the outcome is the probability vector with the first element of which Player 2 of type I_2 chooses action 'a' and chooses action 'r' with the second element. The third and the last columns are for Player 3 of types I_1 and I_2 respectively. Moreover, for subgames $\Phi(x_{\Gamma_2})$ and $\Phi(x_{\Gamma_3})$, the Bayesian equilibrium can also be obtained only by changing the payoffs of players in the program.

From Fig. 2-4, we can see that in subgame $\Phi(x_{\Gamma_1})$, under the Bayesian equilibrium, Player 2 of type I_1 will choose 'a' with probability 1, choose 'r' when he is of type I_2, and for Player 3 of type either I_1 or I_2, he will choose 'a' with probability 1. Similar conclusion can be obtained for both subgames $\Phi(x_{\Gamma_2})$ and $\Phi(x_{\Gamma_3})$.

Finally, three different expected payoff vectors given by various strategies of the leader can be calculated after we get the Bayesian equilibrium in each subgame. Particularly, we get $G(b^1) = (49/40, 127/60, 37/24)$, where $b_1^1 = (1, 0, 0)$ under which Γ_1 is chosen with probability 1, $G(b^2) = (53/60, 19/6, 61/60)$, where $b_1^2 = (0, 1, 0)$ under which Γ_2 is chosen with probability 1, and $G(b^3) = (133/120, 67/30, 223/120)$, where $b_1^3 = (0, 0, 1)$ under which Γ_1 is chosen with probability 1. Thus, behavior strategy profile b^1, where $(b^1)_{-1}^{\Gamma_1}$, $(b^1)_{-1}^{\Gamma_2}$ and $(b^1)_{-1}^{\Gamma_3}$ are the Bayesian equilibria in the corresponding subgames, is the unique stable partially Bayesian equilibrium, also a Nash equilibrium in the game. And $(49/40, 127/60, 37/24)$ is the expected payoff vector under the stable partially Bayesian equilibrium. Fig. 5 shows the whole extensive game Φ, and all red lines compose the unique stable partially Bayesian equilibrium.

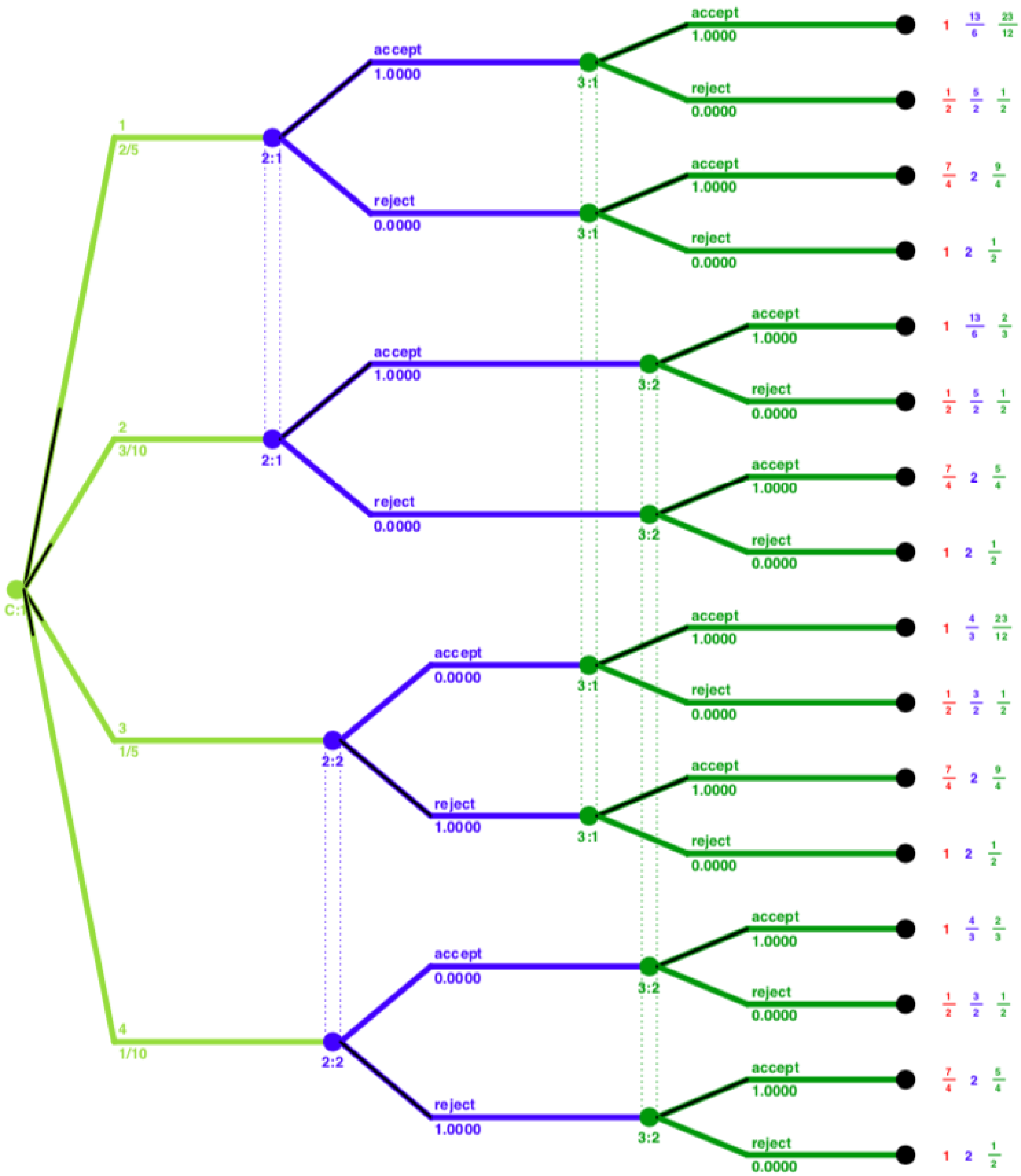

Figure 2: Subgame $\Phi(x_{\Gamma_1})$ with Player 1 as the leader.

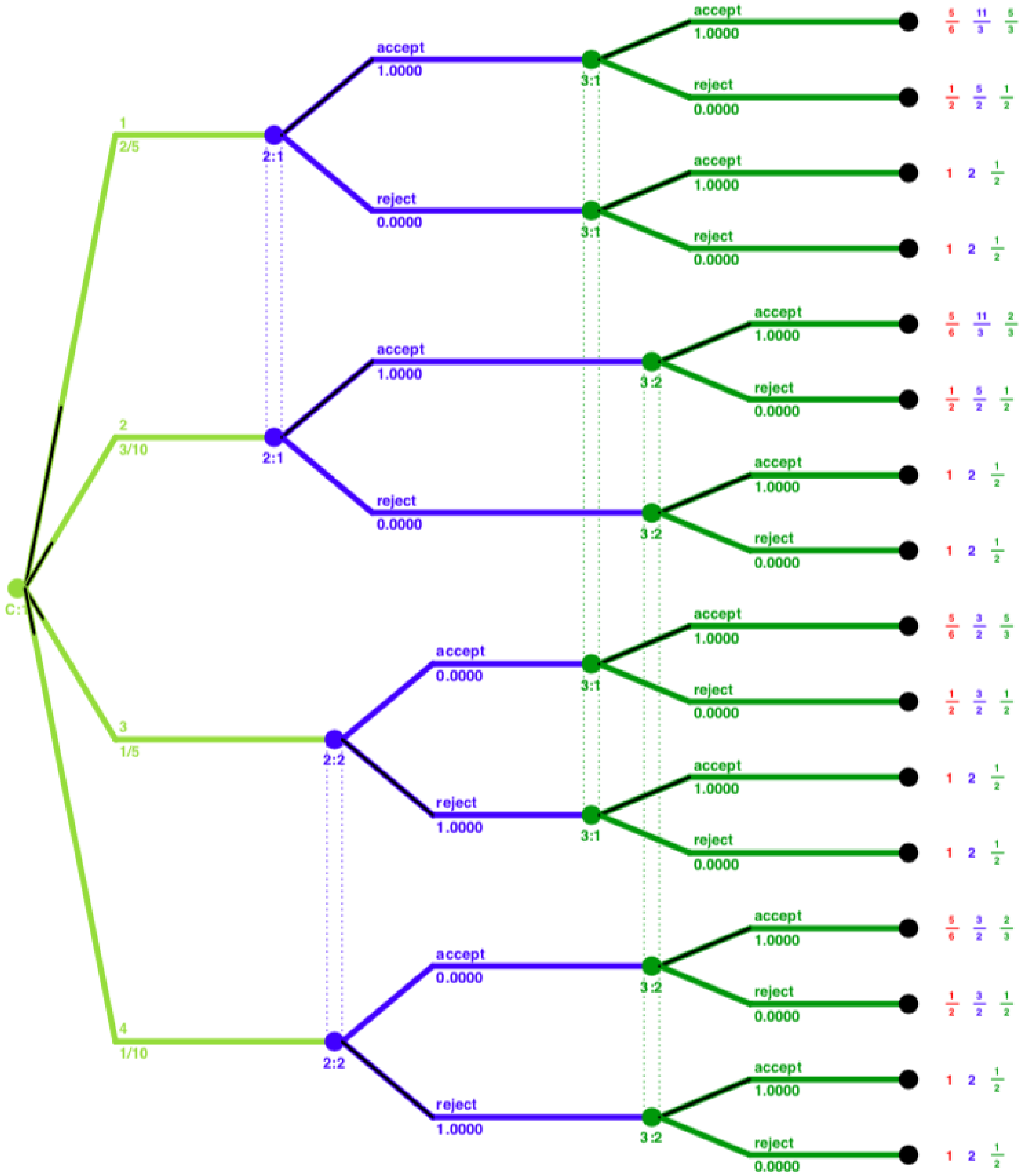

Figure 3: Subgame $\Phi(x_{\Gamma_2})$ with Player 1 as the leader.

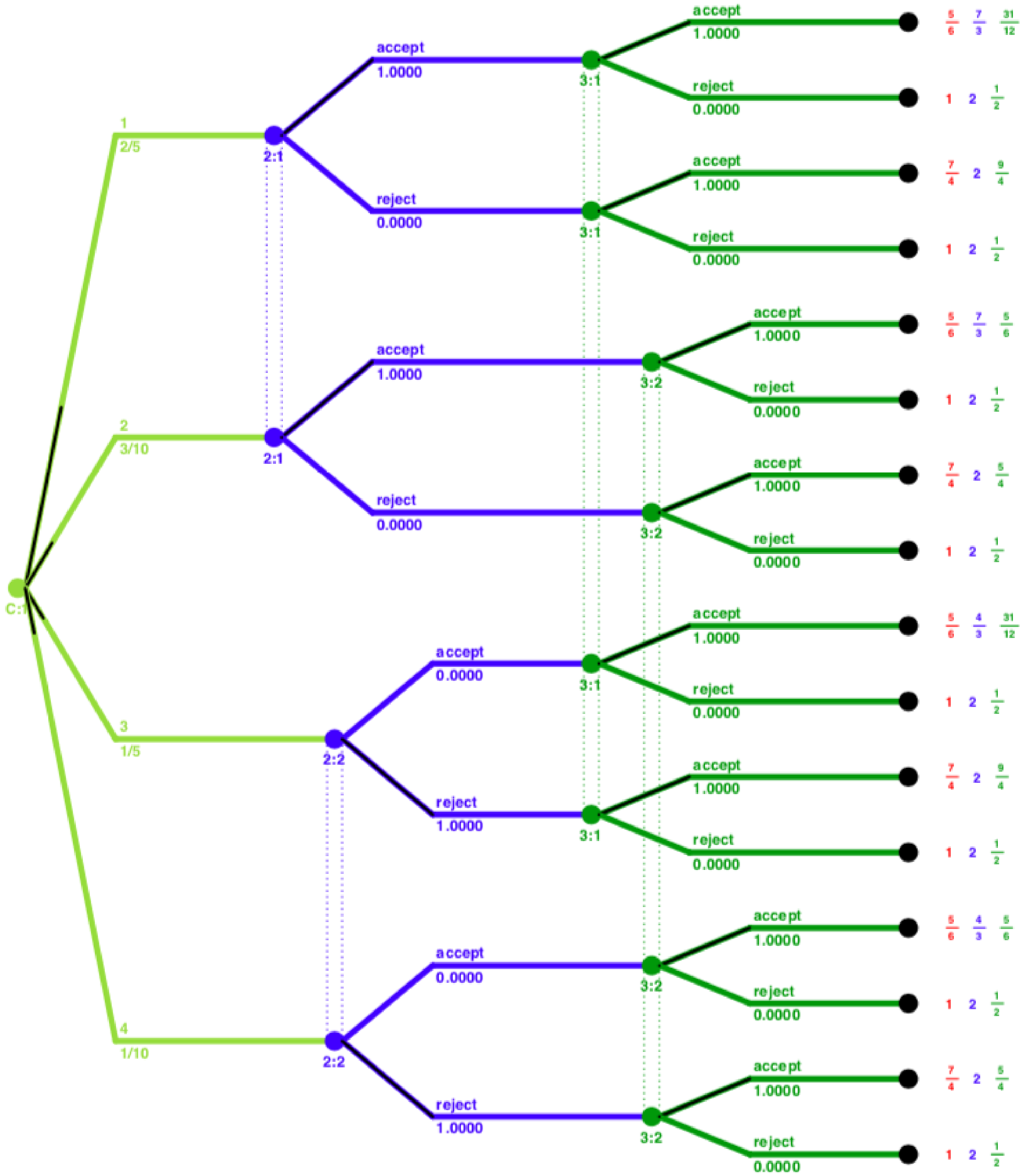

Figure 4: Subgame $\Phi(x_{\Gamma_3})$ with Player 1 as the leader.

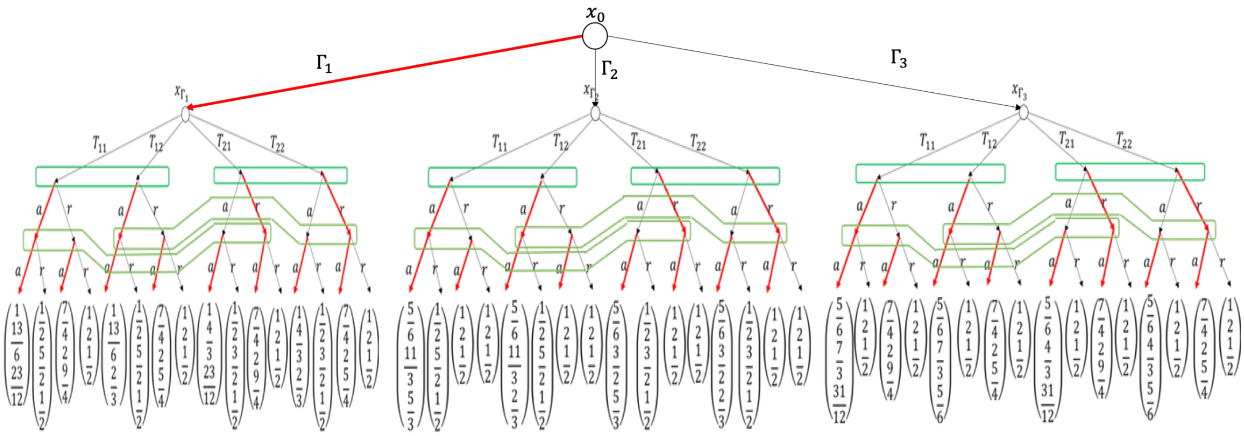

Figure 5: Extensive-form game Φ.

References

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