Supervisor's report
on a Bachelor's thesis by Semyon Petrov,
"Complexity of transforming two-way finite automata to unambiguous finite automata"

In his Bachelor's thesis, Semyon Petrov investigates the relative size of two-way finite automata and unambiguous one-way finite automata. Finite automata are mathematical models of computation with a fixed number of memory states, which does not depend upon the size of the input data. Several kinds of finite automata automata are known, which correspond to different fundamental notions of computations: deterministic and nondeterministic, one-way and two-way, etc. All these models can solve the same class of problems, yet the number of memory states needed to solve the same problem depends upon the particular automaton model. The *complexity of transforming* one model to another is characterized by an integer function f, so that an n-state automaton from one class can be transformed to an $f(n)$-state automaton from the other class, and this number of states is necessary in the worst case. This is an important point of comparison between the models.

For some pairs of automaton models, this complexity function is known precisely. The most well-known example is a transformation from one-way nondeterministic (1NFA) to one-way deterministic (1DFA): Rabin and Scott (1959) proved that $f(n) = 2^n$ states are sufficient, shortly followed by Lupanov (1962), who proved that this number of states is necessary in the worst case. In the case of two-way automata, the complexity of transforming nondeterministic automata (2NFA) to deterministic (2DFA) is a major open problem, connected to the *L vs. NL* question in the computational complexity theory. On the other hand, the complexity of transforming two-way automata to one-way is known: Kapoutsis (2005) has determined that the transformation of 2DFA to 1DFA takes $n(n^n - (n-1)^n)$ states in the worst case, whereas the transformation of 2DFA to 1NFA requires $\binom{2n}{n+1} \approx 4^n$ states in the worst case.

The problem investigated by Mr. Petrov is the complexity of transforming 2DFA to an intermediate model between 1DFA and 1NFA: the *one-way unambiguous finite automata* (1UFA), which may use nondeterminism, yet among all computations on the same input, at most one can be accepting. The complexity in question lies within the bounds $\binom{2n}{n+1} \leqslant \cdot \leqslant n(n^n - (n-1)^n)$, but nothing beyond that has been known previously.

In his thesis, Mr. Petrov presents two results. The first result is a transformation of an n-state 2DFA to a 1UFA with fewer than $2^n \cdot n!$ states; it is obtained by expanding the transformation of Kapoutsis (2005) with an additional data structure, which accounts for the increase in the number of states. The other result is a lower bound showing that for some n-state 2DFA, every 1UFA requires at least $9^{n-o(1)}$ states. The proof of the latter bound includes a sophisticated analysis of computations of two-way automata, reducing the problem to finding a rank of a certain matrix, and is followed by an estimation of the rank of that matrix using the methods of group representation theory. To readers unfamiliar with the subject, the bounds obtained in this thesis might look as if they are far apart, but, in fact, these results already took quite a lot of efforts to obtain, and nothing of this kind was known before.

The results have been prepared for publication, but have not been published yet. There are all reasons to expect a publication in a decent international venue.

Overall, the thesis contains a solution of a substantial research problem. The author has obtained all the principal ideas of his thesis independently. The results are well presented. Thus, as per the Final Examination rules, the thesis meets all the requirements for awarding the mark "Excellent".

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