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The collection contains papers from different fields of game theory and it's applications to management, pollution control, alliance stability, financial markets and so on.

The volume may be recommended for researches and post-graduate students of management, economic and applied mathematics departments.

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Успехи теории игр и менеджмента. Вып. 9. Сборник статей, представленных по результатам "Европейской конференции по теории игр" (SING11-GTM2015) / Под ред. Л.А. Петросяна и Н.А. Зенкевича. – СПб.: Санкт-Петербургский государственный университет, 2016. – 388 с.

Выпуск содержит работы участников "Европейской конференции по теории игр" и GTM2016. Представленные статьи относятся к теории игр и ее приложениям в менеджменте, охране окружающей среды, устойчивости альянсов, финансовым рынкам и др.

Издание представляет интерес для научных работников, аспирантов и студентов старших курсов университетов, специализирующихся по менеджменту, экономике и прикладной математике.

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Preface

This edited volume contains a selection of papers that are an outgrowth of the SING11-GTM2015 "European Conference on Game Theory" with additional contributed papers. These papers present an outlook of the current development of theory of games and its applications to various domains, such as management, economics and environment.

SING11- GTM2015 "European Conference on Game theory", a three day conference, was held in St. Petersburg, Russia in July 08-10, 2015. The conference was organized by Saint-Petersburg State University (SPbSU) in collaboration with The International Society of Dynamic Games (Russian Chapter). More than 180 participants from 35 countries had an opportunity to hear state-of-the-art presentations on a wide range of game-theoretic models, both theory and management applications.

Plenary lectures covered different areas of games and management applications. They had been delivered by Professor Hans Peters (University of Maastricht), Professor David Schmeidler (Tel Aviv University), Professor Georges Zaccour (HEC Montreal), Professor Alexander Vasin (Lomonosov Moscow State University).

The importance of strategic behavior in the human and social world is increasingly recognized in theory and practice. As a result, game theory has emerged as a fundamental instrument in pure and applied research. The discipline of game theory studies decision making in an interactive environment. It draws on mathematics, statistics, operations research, engineering, biology, economics, political science and other subjects. In canonical form, a game takes place when an individual pursues an objective(s) in a situation in which other individuals concurrently pursue other (possibly conflicting, possibly overlapping) objectives and in the same time the objectives cannot be reached by individual actions of one decision maker. The problem is then to determine each individual's optimal decision, how these decisions interact to produce equilibrium, and the properties of such outcomes. The foundations of game theory were laid more than seventy years ago by von Neumann and Morgenstern (1944).

Theoretical research and applications in games are proceeding apace, in areas ranging from aircraft and missile control to inventory management, market development, natural resources extraction, competition policy, negotiation techniques, macroeconomic and environmental planning, capital accumulation and investment.

In all these areas, game theory is perhaps the most sophisticated and fertile paradigm applied mathematics can offer to study and analyze decision making under real world conditions. The papers presented at "European Meeting on Game Theory" certainly reflect both the maturity and the vitality of modern day game theory and management science in general, and of dynamic games, in particular. The maturity can be seen from the sophistication of the theorems, proofs, methods and numerical algorithms contained in the most of the papers. The vitality is manifested by the range of new ideas, new applications, the growing number of young researchers and the expanding world wide coverage of research centers and institutes from whence the contributions originated.

SING11- GTM2015 offered an interactive program on wide range of latest developments in game theory and management. It includes recent advances in topics with high future potential and exiting developments in classical fields.

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Coordinating Contracts in Cooperative Supply Networks*

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Abstract It is widely accepted that the decentralization process exerts negative influence on the supply chain economic performance relatively to the case of an integrated supply chain in terms of total supply chain profit. In other words, a decentralized supply chain is less efficient than a centralized one, as in a decentralized supply chain each separate member tends to maximize his own benefits and pursue his private objectives, even if it harms the system wide performance. Coordination, in turn, helps to mitigate these negative effects of a decentralized decision-making. Nevertheless, coordination may be hard to achieve if some of the supply chain members are competing with each other, which leads to a new line of research on such systems, referred to as supply networks. Supply chain contract can be an effective coordination mechanism to motivate supply network members to be a part of entire system, in order to improve individual and system wide performance. There are different types of contracts, such as revenue-sharing, quantity-discount and other. The objective of the paper is methodology improvement of contract selection in cooperative supply networks for achieving better supply network economic performance. The research was focused on a two-level standard newsvendor model, which was adapted in order to reflect the situation of competing retailers. The methodology of coordination contracts decision-making was developed by devising a mechanism for contract selection for the case of multi-echelon supply network with two competing retailers enabling coordination at a system-wide level. The proposed model is a novel approach in applying coordination theory at systems with inside competition.

Keywords: Supply Chain Management, Supply Chain Coordination, Coordinating Contract, Supply Network, Bargaining Power in Contract Decision-Making

1. Introduction

In modern economy the most important features determining market competitiveness include product quality, company's flexibility, costs optimization, logistic accuracy, high service level and responsiveness to the ever-changing consumer needs. Companies which are not able to adapt in time to changing market environment should expect serious troubles in their long-term competitiveness. In this regard,

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the concept of supply chain management is becoming more and more important over the years, as it is seen as a strategic factor to balance customer orientation and profitable growth (Procenko, 2006).

According to (Fedotov, 2010), the research in the field of supply chain management is currently at the stage of its conceptual development, characterized by predominance of papers devoted to practical business needs. Main directions of the studies include (Fedotov, 2010):

– Strategic aspects of supply chain management;
– Detailed examination of specific functions;
– Engineering and IT support of supply chain management;
– Contract relationships in supply chains.

Current article is focused on the last of the listed lines of research, namely, contract relationships. Studies in this particular field emerged from the notion of supply chain coordination, introduced by Williamson (1986) as a part of a broader science of supply chain management. Managerial implication here lies in the necessity to improve supply chain economic performance.

In the ideal situation, all the processes throughout the supply chain would be managed by a single company, as, stated by Anupindi and Bassok (1999) , a single decision-maker optimizes the network performance with the union of information and resources available. Such a supply chain is usually referred to as an integrated or a centralized one. Hence, supply chain economic performance is at risk as soon as there are multiple decision-makers, who may have different private information and their own incentives, which are at odds with the supply chain as a whole. Unfortunately, current trends, such as globalization, application of outsourcing activities and spread of information technologies worldwide lead to further fragmentation and decentralization of supply chain operations.

This decentralization process exerts negative influence on the supply chain economic performance relatively to the case of an integrated supply chain in terms of total supply chain profit. In other words, a decentralized supply chain is less efficient than a centralized one, as in a decentralized supply chain each separate member tends to maximize his own benefits and pursue his private objectives, even if it harms the system wide performance. Coordination, in turn, helps to mitigate these negative effects of a decentralized decision-making.

Despite of wide literature devoted to both theoretical and practical analysis of contract coordination mechanisms in a supply chain, as well as their modeling and application, there is a gap in literature in what relates to researches devoted to coordination mechanisms in a different setting of supply chain - supply network - and to modeling the application of those mechanisms on real life cases and examples. Supply network is understood here as a set of three or more organizations directly involved in the upstream and downstream flows of products, services, finances and information from a source to a consumer, provided that two or more of them are direct competitors. In other words, supply network is a set of distinct supply chains connected into a system with existing competition between its members.

Therefore, supply network coordination can be defined as identifying interdependent activities between supply network members and devising mechanisms to manage those interdependencies for improving the supply network economic performance in the best interests of participating members Arishinder, 2011.

Thus, a supply network, as a set of supply chains, is coordinated by a set of supply chain contracts. Here, a supply chain contract stands for a set of rules, rights and obligations regulating relationships between supply chain or network members. Typically, a supply chain contract should capture the three types of flows encountered between the companies, i.e. material, information, and financial flows Hohn, 2010. Moreover, contracts are considered to be one of the most powerful mechanisms to achieve coordination, as they address directly the nature of relationships evolving within the supply system.

The goal of the present research is methodology improvement of contract selection in cooperative supply networks for achieving higher supply network economic performance, where economic performance stands for total supply network profit. The research was focused on a two-level one-period newsvendor model, which was adapted to reflect the situation of competing retailers and applied as a basis to describe the decision-making process in a given supply network. Then, the established framework was used to model the supply network economic performance in market dynamics under the implementation of different supply chain coordinating contracts.

2. Cooperation, Coordination and Collaboration

Following Mentzer et al. (2001), Supply Chain Management can be defined as a systemic, strategic coordination of the traditional business functions and the tactics across these business functions within a particular company and across businesses within the supply chain, for the purposes of improving the long-term performance of the individual companies and the supply chain as a whole. Therefore, as noted by Simatupang et al. (2002), the main concern of supply chain management is how to coordinate independent companies to work together as a whole to pursue the common goal of improving individual and overall supply chain economic performance in changing market conditions. This has been a major issue of early economic theory that differentiated between the firm and its hierarchies and price mechanisms as forms of coordination (Williamson, 1986). Following Coase (1937), if separate companies coordinate, it is referred as combination or integration.

In the context of industrial engineering research and in particular SCM research, the related terms cooperation, coordination, and collaboration are often used interchangeably without clearly distinguishing them from each other (Hammer, 2006). At the same moment, some authors Arishinder, 2011 assume that integration, collaboration and cooperation are just the elements of coordination. For the purposes of the current paper, terms cooperation, coordination and collaboration are assumed to be different levels of supply chain integration. Therefore, it is necessary to introduce clear distinction between the related terms.

Cooperation is defined as acting or working together for a shared purpose (Cambridge Dictionaries Online), working or acting together toward a common end or purpose, acquiescing willingly and being compliant (American Heritage Dictionary of the English Language), or as the act of working with someone toward a common goal (Heinle's Newbury House Dictionary). In the context of supply chain management, Quiett (2002) referred to cooperation as little more than toleration of each other. While this view might be a bit drastic, the other definitions imply that cooperation emphasizes mainly the alignment towards a common goal and a shared purpose. Hammer (2006) highlights that the notion of working together in the context of cooperation does not suggest a close operational working relationship, but rather a positive attitude towards each other. Therefore, for the purposes of the current paper cooperation is understood as *an existing willingness to work together towards a shared goal or purpose and openness towards negotiations.*

Coordination, in turn, refers to a more direct, active cooperation. It is defined as the activity of organizing separate things so that they work together (Cambridge Dictionaries Online), the act of making arrangements for a purpose, the harmony of various elements (American Heritage Dictionary of the English Language), and harmonious adjustment or interaction (Heinle's Newbury House Dictionary). Following Moharana et al. (2012), compared to cooperation, coordination indicates an interactive, joint decision making process, where separate entities influence each other's' decisions more directly. Besides horizontal coordination, i.e. coordination within a supply chain tier, and vertical coordination, i.e. coordination across supply chain tiers, for example between supplier and customer, coordination can also be distinguished from mechanism of coordination. According to Williamson (1991), the fundamental mechanisms are markets and hierarchies. Market structures refer mainly to incentive-driven coordination between separate, legally independent companies whereas hierarchical structures indicate either a high unilateral dependency or those companies are not legally independent or equity is shared. Hence, coordination is defined *as a set of incentives and direct actions making companies work together towards a common goal, as well as joint decision-making.*

Collaboration, therefore, can be defined as working together or with someone else for a special purpose (Cambridge Dictionaries Online), or simply as working with someone (American Heritage Dictionary of the English Language) or working together (Heinle's Newbury House Dictionary). Following Stank et al. (1999), whereas coordination is a joint, interactive process that results in joint decisions and activities, collaboration depends on the ability to trust each other, and to appreciate one another's knowledge and emphasizes the building of meaningful relationships. By that, it also indicates a higher degree of joint implementation and can be thought of as a teamwork effort. Then, collaboration can be defined as a *superstructure evolving between separate entities in form of shared vision, culture, mission, etc. that facilitates the processes of working together towards a common goal.*

3. Supply Chain Integration

According to Anupindi and Bassok, 1999, supply chain management deals with the management of material, information, and financial flows in a network consisting of vendors, manufacturers, distributors, and customers. Exchange of flows can be regarded as a routine transaction, occurring between any pair of suppliers and buyers in the system. Ideally, the quantity and pricing decisions in the supply chain would be made by a single decision maker who has all information at hand Hohn, 2010. Such a situation is generally referred to as a fully integrated, or centralized, supply chain. Respectively, a supply chain is called decentralized if the network consists of multiple decision-makers having different information and incentives.

Following Anupindi and Bassok, 1999, a single decision-maker optimizes the network with the union of information that otherwise various decision-makers have. Hence, supply chain performance is at risk as soon as there are multiple decision-makers in the network who may have different private information and incentives.

For instance, as it was highlighted by Corbett et al. (2004), decision-makers are often reluctant to share private information regarding cost and demand, which may lead to suboptimal supply chain decisions and economic performance.

This was first described in literature by Spengler (1950) as a problem of double marginalization. It can be shown that when operating independently, supplier and buyer will produce less than a vertically integrated monopolist, because they receive less than the total contribution margin at any given quantity. This clearly is a case, where locally optimal decisions of supplier and buyer do not optimize the global supply chain problem, or, in other words, the decentralized supply chain is inefficient, since the total expected profit of the decentralized supply chain is lower than the total expected profit of the fully integrated supply chain Hohn, 2010. Thus, the centralized, fully integrated system can be taken as a benchmark situation, while integration itself can be viewed as a tool for a decentralized supply chain to achieve or approach the economic performance of a centralized chain in terms of total profit.

For the research purposes cooperation, coordination and collaboration are assumed to be stages of supply chain integration process. Notable, that in SCM research, integration usually enhances two elements: interaction and collaboration. Both elements were introduced as separate philosophies and combined as integration. Following Hammer (2006), the interaction philosophy emphasizes exchange of information, while the collaboration philosophy highlights strategic alignment through a shared vision, collective goals, and joint rewards, along with an informal structure of managing relationships. Mentzer and Kahn (1996) stated that integration, therefore, is viewed as comprising interaction and collaboration activities.

Thus, dividing supply chain integration into distinct levels means recognition of specific stages in inter-firm relationships development, ranging from decentralized decision-making with poor interactions and no shared vision, goals or rewards to fully centralized decision-making with a single decision-maker having all available information and one unified goal and vision. It is necessary to note that, for the research purposes, moving along these stages towards increased supply chain integration is assumed to improve overall supply chain performance. Hence, the hierarchy of supply chain integration levels can be presented as follows (Fig. 1).

Fig. 1: Levels of supply chain integration

In the suggested framework, it is expected that all firms when establishing relationships in the supply chain start with decentralized decision-making. Following Jarillo (1998), cooperation is a little step further from decentralized decision-making, when the participants of the supply chain adapt their behaviors to that of other partners and create informal links between companies. Therefore, cooperation is an acknowledgement of the common goal and willingness to pursue supply chain profit maximization function instead of individual profit maximization functions by members of the supply chain.

While cooperation refers to creating informal links, coordination and collaboration are both aimed at devising formal mechanisms to manage supply chain interdependencies (Arishinder, 2011). In terms of coordinating intensity, collaboration can be seen as more intensive than coordination because most of the time it subsumes all characteristics of coordination as well. Therefore, in a hierarchy of different levels of integration, collaboration would be positioned above coordination. In his research Hammer (2006) agrees that, in the context of SCM, coordination aims at achieving global optimization within a defined supply chain network. Meanwhile collaboration aims to exploit hidden potential and consequently expand the optimization potential, i.e. shifting the efficient performance frontier upwards.

This view is also supported by Shaw (2000), who has differentiated between three types of coordination in terms of level of involvement, in ascending order: simple information exchange, formulated information sharing, and modeled collaboration. Simple information exchange is quite straightforward as it refers to information exchange without additional interpretation or rules. In formulated information sharing, such policies as restocking policies are shared together with operational information. In modeled collaboration, operational models are also shared, together with capabilities, factory load, inventories, and orders (Shaw, 2000). The importance of information exchange was confirmed by Swaminathen et al. (2003), who highlighted that information sharing is of central importance for coordination, as it allows for coordinated forecasts and forecasts based on richer information. Extending this idea, Sahin and Robinson (2002) have stated that a lack of coordination occurs when decision makers have incomplete information or incentives, which are not compatible with system-wide objectives.

This understanding can be directly linked to the three levels of collaboration that Quiett (2002) has identified, which are data exchange, cooperative collaboration and cognitive collaboration. These views, however, indicate a more extensive information sharing scheme on the highest level instead of a close, teamwork-like working relationship (Hammer, 2006).

As opposed to that, in a Deloitte study (Koudal, 2003) conducted in 2003, collaboration has been characterized by internal and external teamwork in the context of manufacturing companies, i.e. with customers and suppliers. As differentiating factors, strong cross-functional teams, stronger commitments to these teams, design for quality, and design for manufacturability techniques have been identified. Necessary elements were cited to be joint-working with suppliers and customers on production planning, inventory management, replenishment, forecasting, and demand planning.

An understanding in line with this interpretation of collaboration is provided by Liedtka (1996), who has defined collaboration as a process of decision making among interdependent parties, which involves joint ownership of decisions and collective responsibility for outcomes. Liedtka (1996) has emphasized the cross-functional teamwork aspect of collaboration with a clear focus on processes instead of functions. Because processes rarely stop at company boundaries, this includes external organizations as well. Therefore, the term partnership is also used to include external collaboration. Success factors identified in Liedtka's study (1996) are quite independent from legal forms of partnerships. The components of successful partnering comprise a partnering mindset, a partnering skillset, and a supporting organizational architecture.

Barratth (2004) has identified yet another, however closely related, set of elements that define collaboration. These are cross-functional activities, process alignment, joint decision making, and supply chain metrics. The elements that support a collaborative culture are trust, mutuality, information exchange, openness, and communication, which, in turn, is necessary for successful collaboration. It is important to note, that a rather close proximity to team working exists. As Christopher (2005) remarked, the closer the relationship between buyer and supplier the more likely it is that the expertise of both parties can be applied to mutual benefit. Consequently, higher levels of internal and external collaboration are expected to improve performances in the areas of collaboration (Stank, 2001).

Therefore, summarizing different approaches presented in literature, cooperation is referred to as willingness to participate in supply chain performance improvement, coordination encompasses joint decision-making, process alignment, information exchange and other active steps for supply chain performance improvement, while collaboration is a superstructure in form of creating a unite supply chain culture, mindset and architecture. Thus, in order to proceed to the next stage a given supply chain should fully embrace characteristics of the previous step(s). For example, in order to start working on activities to achieve coordination, a given supply chain should be already cooperative and embrace the characteristics of this stage.

Spekman et al., 1998 have drawn a similar conclusion. In their view, cooperation refers to rudimentary information exchange with little interaction and is seen as a necessary but not sufficient condition for managing business relationships. The next level would then be coordination. Just-in-time (JIT) and electronic data interchange (EDI) linkages can reflect such coordinated relationships. Again, though companies cooperate and coordinate, they still might not behave as true partners. According to (Spekman et al., 1998) in order to achieve collaboration, a level of trust and commitment beyond the one found in cooperation and coordination is required. Thus, supply chain partners may cooperate and coordinate, but still not collaborate.

4. Supply Network

Up to this point, the paper was focused on the relationships in a traditional supply chain, no matter what level of complexity was assumed. Notably, while increasing supply chain complexity, Mentzer et al. (2001) were only talking about the number of tiers a supply chain might have. Nevertheless, apart from the number of tiers, supply chain complexity may be increased further by the number of firms at a given tier as shown in Figure 2. According to Mentzer et al. (2001), Figures 2a and 2b are both representations of a direct supply chain, although it is clear that the supply chain 2b is more complex in both functional and managerial terms.

While direct supply chain in the Figure 2b consists of a supplier, an organization and three distinct customers, it can be argued that this is just a unite representation of three distinct supply chains. Nevertheless, following Mentzer et al. (2001), supply chain members are defined by their involvement in the upstream and downstream flows of products, services, finances and (or) information from an initial source to a consumer. Thus, supply chain in the Figure 2b is defined as a single supply chain if it serves one unique flow of products, services, finances and information from the ultimate supplier to the ultimate customer. Notably, the term unique flow here refers to the non-competitive nature of inter-firm relationships within the supply chain.

Absence of rivalry is of vital importance when it comes to supply chain management as it reassures that all the members of the given supply chain would have

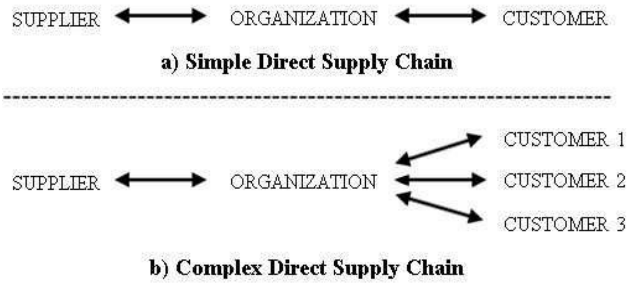

Fig. 2: Levels of direct supply chain complexity

incentives for mutual performance improvement, which, in terms of SCM, means working together to achieve coordination. A traditional example of such a supply chain is automotive industry, with one car assembler and multiple dealers, which are not owned by the manufacturer, but do not compete with each other as they either cover different segments and (or) regions (Fig. 3a).

At the same time, if companies within one supply chain tier compete with each other, it may be assumed that they are all serving different flows of products, services, finances and information and, therefore, are members of different supply chains. However, in a situation when all these flows go alongside the supply chain from one unite source to the same end consumer, despite the fact of existing competition, it can be argued that this system is close to supply chain in terms of management and optimization. In the case of automotive industry that would mean that one car assembler sells its cars through multiple dealers, who are competing with each other in the open market using both price and quantity (Fig. 3b).

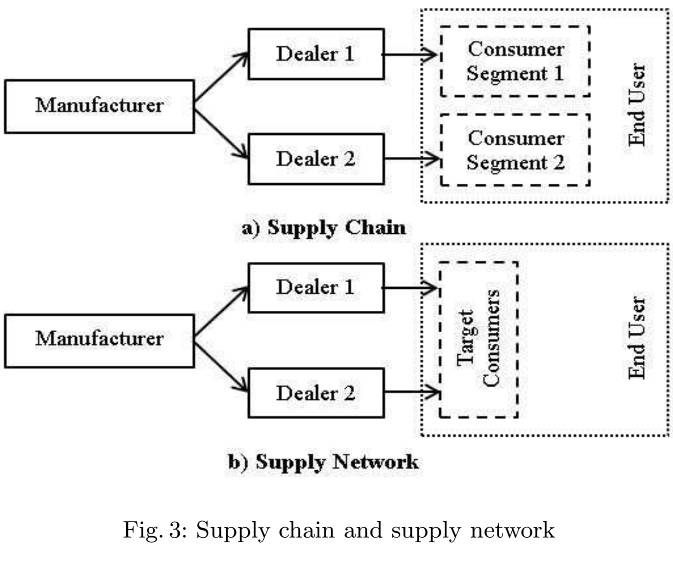

Fig. 3: Supply chain and supply network

The concept of supply chain assumes Mentzer et al. (2001), that all supply chain members are interconnected one after another, comprising a single line of relationships. Introducing competition inside one of the supply chain tiers means that these relationships no longer form a direct line, but rather a system of interconnected companies.

Therefore, for the research purposes, such system would be called supply network and defined as *a set of three or more organizations directly involved in the upstream and downstream flows of products, services, finances and (or) information from a source to a consumer provided that two or more of them are direct competitors*. In other words, supply network is a set of distinct supply chains connected into a system with existing competition at one or more of its tiers. Consequently, the main concern of supply network management is coordination of independent companies in order to improve the economic performance of the individual companies and the supply network as a whole.

Bryant, 1980 appears to be the first published paper to address the supply network setting, including into a supply chain a competitive oligopoly model with stochastic demands, which arise from a finite customer population. Another such model is Deneckere, Marvel and Peck (1997), addressing a market with a continuum of identical retailers offering a completely homogenous product. Most directly related to the current research are papers of Birge et al. (1998) Carr et al. (1999) and van Mieghem and Dada (1999), who consider the special case of the supply network model with two competing retailers.

5. Cooperative Supply Network

Birge et al. (1998) have shown that pricing and capacity decisions, those directly influencing economic performance, are affected greatly by the actual parameters that the decision makers can control as well as whether decision makers are optimizing system-wide or individual channel profits. This raises a question of integration in a supply network as opposed to that of its individual channels, e.g. separate supply chains.

As supply network is a system comprised of individual supply chains united by an integrated flow of products, services, finances and information, it can be claimed that supply network as a phenomenon shares some characteristics with a supply chain, level of integration being one of those.

Therefore, in terms of integration supply network follows the same steps as supply chain (Fig. 1), from being completely decentralized to fully integrated. Nevertheless, due to competition between its members, full integration here refers to achieving the same economic performance as if it was managed under a single decision-maker. Thus, in terms of supply network, cooperation is referred to as *willingness to participate in supply network performance improvement*, when its members understand that they can achieve better results and they are ready to invest in that. Coordination embraces any *activities aimed at supply network performance improvement*, while collaboration is *a superstructure in form of creating a unite culture, mindset and architecture*. Similarly to a supply chain, in order to proceed to the next stage, a given supply network should fully embrace characteristics of the previous step(s).

Hence, the process of coordination can be only initiated in a cooperative supply network, meaning that all its members are open towards negotiations and ready to invest in system-wide performance improvement.

6. Coordinating Contracts

Although contracts have been studied in law, economics, and marketing disciplines, their study in SCM takes a rather different approach. Following Tsay, 1999, what distinguishes SCM contract analysis may be its focus on operational details, requiring more explicit modeling of material flows and complicating factors such as uncertainty in the supply or demand of products, forecasting and the possibility of revising those forecasts, constrained production capacity, and penalties for overtime and expediting.

By viewing a supply chain as nexus-of-contracts (Wang and Sarkis, 2013), meaning a group of rational agents interacting with each other according to pre-specified set of rules, an improved supply chain management is achieved by designing appropriate contracts coordinating the agents' decisions. Typically, a supply chain contract should capture the three types of flows encountered between the members of supply chain, i.e. material, information, and financial flows Hohn, 2010. Nevertheless, to date there is no commonly accepted classification of the rules, parameters and dimensions fixed in those supply chain contracts.

One of the first classifications of supply chain contracts was suggested by Anupindi and Bassok (1999) and consisted of eight parameters: horizon length, pricing, periodicity of ordering, quantity commitment, flexibility, delivery commitment, quality and information sharing. In contrast, Tsay, 1999 classified supply chain contracts by eight contract clauses, including specification of decision rights, pricing, minimum purchase commitments, quantity flexibility, buy-back or returns policies, allocation rules, lead time, and quality.

Those two classifications were synthesized and developed further by Hohn, 2010. Integrated framework comprised eleven dimensions: specifications of decision rights, pricing, minimum purchase commitments, quantity-flexibility, buy-back or return policies, allocation rules, lead time, quality, horizon length, periodicity of ordering and information sharing.

Notably, supply chain contracts are not always required to be legal. Several papers in the literature consider contracts among independent agents that are divisions of the same company and a higher level manager can verify the execution of lateral promises (Lee and Whang 1999, Zhang 2006). Nevertheless, the process of contract design should explicitly point out the verifying ability of the enforcing agent. Two approaches to verification are presented in literature: direct and indirect. In direct verification, the conditions regarding the fulfillment of contract terms can be observed. In indirect verification, studied by Hezarkhani and Kubiak, 2010, the conditions may be only inferred. For example, in case of direct verification a retailer can observe and count the number of products received from a supplier, while indirect verifications require self-enforcing, e.g. manufacturer can verify that if the market selling price is greater than the total production cost and salvage value, the retailer would satisfy market demand as much as it can.

If the contract parameters are well defined, contract enforces coordination in the supply chain. First studies devoted to supply chain contracts and their coordination capabilities appeared in the scientific literature in 1980s. However, only in 1990s the systemic integrated research of this mechanism emerged, which summarized fragmented findings of previous papers and build on that. The earliest overviews focused on coordinating contracts included papers of Wang and Sarkis, 2013, Tsay, 1999, Cachon, 2003 and Lariviere, 2001.

Among the recent papers in this field it is necessary to mention an extensive overview of different types of contracts by Cachon, 2003 and his joint study with Lariviere (Cachon and Lariviere, 2005) on the interchangeability of contracts of different types. Most recent research includes subsequent comprehensive reviews of the topic by Hohn, 2010, Govindan and Popiuc, 2011 and Arishinder, 2011.

6.1. Coordinating Contract Definition

Following Tsay, 1999, from the point of view of supply chain coordination, a contract can be defined as *a coordination mechanism that provides incentives to all of its members so that the decentralized supply chain behaves nearly or exactly the same as the integrated one.*

This definition emphasizes the capability of supply chain contracts to integrate a supply chain in terms of centralizing decision making in a way and turning supply chain processes into optimal for the whole channel. However, not every coordinating contract can be actually implemented, which happens due to acceptability rules.

6.2. Acceptability Rules

The notion of acceptability rules implied in supply chain contracts was described by Hezarkhani and Kubiak, 2010. According to their research, two approaches towards formulating the acceptability conditions exist in literature. The first approach supposes that, in order to be acceptable, the contract should lead to the each member's utility being above a certain acceptable level. This level can take the form of reservation profit, opportunity costs, outside options or status quo utilities, i.e. an agent should not be in worse situation with a new contract than it was with the existing one.

This approach was followed by Gan et al., 2004, who defined coordinating contract as a contract which the agents of a supply chain agree upon, while the optimizing decisions of the agents under the contract should satisfy each agent's reservation payoff (minimum acceptable utilities) constraint and lead to Pareto-optimal decisions and Pareto-optimal sharing rule. This definition formulates the acceptability condition according to the first approach stated earlier, as satisfaction of minimum acceptable utilities. Nevertheless, it has a sufficient drawback as it does not indicate how one contract should be preferred over the others in case of multiple contracts with Pareto-optimal sharing rules which satisfy the agent's minimum acceptable utilities.

The second approach implies that the contract should not only guarantee some minimum acceptable level of utility to all the members, but also allocate extra benefits from the contract to its members in some fair manner. The notion of fairness here provides that the profit is allocated among members proportionally to their investments, i.e. share of costs (Hezarkhani and Kubiak, 2010).

This approach was adopted by Cachon, 2003, who stated that there are three conditions that a supply chain contract should meet in order to be coordinating:

1. With a coordinating contract, the set of supply chain optimum decisions should be a pure Nash equilibrium;

2. Coordinating contract should divide the supply chain profits arbitrarily among agents;
3. Coordinating contract should be worth adopting.

The first condition implies that no member should have an incentive to deviate from the set of optimal actions. Ideally, the described equilibrium should be unique, with cooperation being the most profitable alternative for all members. Gan et al., 2004. Finally, the third condition articulates that coordinating contracts with highest efficiency may not be the best option for the supply chain, as sometimes non-coordinating contracts with high efficiency ratio can be preferred by the supply chain members.

Despite the different interpretations of the acceptability condition of a coordinating contract by Cachon, 2003 and Gan et al., 2004, the fundamental notion in both definitions is similar. That is, with the coordinating contract, agents' optimum decisions must be the same as the supply chain's optimum decisions, and the contract should divide the resultant payoffs among the supply chain members so that all agents are satisfied and, as the result, they would accept the contract (Hezarkhani and Kubiak, 2010).

Therefore, two variations of the concept of coordinating contract were formulated by Hezarkhani and Kubiak, 2010:

* *Weak Coordination*: If a contract could achieve the equivalence of agents' optimal decisions (pure Nash equilibrium) and the supply chain's optimal solution, and at the same time it satisfies the minimum acceptable utilities for all agents, then the contract is weakly coordinating.
* *Strong Coordination*: If a contract could achieve the equivalence of agents' optimal individual decisions (pure Nash equilibrium) and the supply chain's optimal solution, and at the same time it could divide the total supply chain payoff in any manner among the agents, then the contract is strongly coordinating.

The relationship between the two definitions is that if a weakly coordinating contract is also flexible, then it is strongly coordinating as well.

7. Coordination in Supply Network

A typical model that is used for analyzing supply chain coordination with contracts is a newsvendor model - a standard one-period one-product one-echelon (i.e. consisting of two firms, a supplier and a buyer) setting for modeling order quantity decisions under stochastic demand, presented in Figure 4 below.

In this framework the supplier (manufacturer) produces one type of product at a constant cost c and sells it to the buyer (dealer) at a wholesale price $w(Q)$ per unit. In turn, the buyer resells this product to the market at a retail price r. In the newsvendor model, the action to coordinate the supply chain is the buyer's order quantity Q, as, while facing stochastic demand, the buyer must determine an order quantity Q before the start of the selling season. Cachon, 2003 emphasizes that a contract is said to coordinate the supply chain if the set of supply chain optimal actions is Nash equilibrium, i.e. no firm has a profitable unilateral deviation from the set of supply chain optimal actions.

This model is a building block for a large stream of the research modeling and scientific literature on supply chain contracts. According to Khouja, 1999, the traditional newsvendor setting lies in the basis of the majority of other more complex

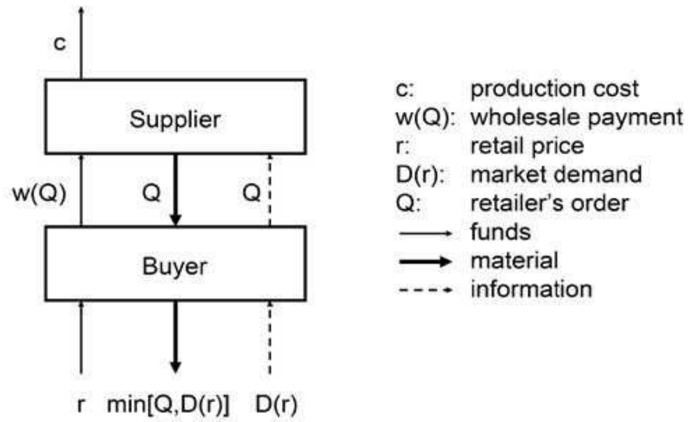

Fig. 4: Basic one-period one-product supply chain model (Hohn, 2010)

models developed for more complicated configurations of parameters. Thus, the described newsvendor model will be used for current research purposes as a basis for supply network model, presented in Figure 5 and discussed further.

This is a one-period one-product topology with one upstream firm that supplies two downstream firms. In this framework manufacturer produces one type of product at a constant cost c and sells it to the dealers at wholesale prices $w1$ and $w2$ per unit of good. In turn, dealers resell this product to the open market at a retail prices $p1$ and $p2$ accordingly. Product is homogeneous and neither of the dealers has any technical advancements, i.e. they have the same marginal costs. Thus, they compete with each other in the open market with demand function defined as $D(p1, p2)$. Full summary of parameters used in the model is described below.

Table 1: Parameters for Supply Network Model

C:	production cost
$w_1(Q_1)$:	wholesale payment of the 1st dealer
$w_2(Q_2)$:	wholesale payment of the 2d dealer
Q_1:	1st dealer's order
Q_2:	2d dealer's order
p_1:	1st dealer's retail price
p_2:	2d dealer's retail price
$D(p_1, p_2)$:	Market demand
q_1:	1st dealer's sales
q_2:	2d dealer's sales
$\rightarrow$:	Financial flows
$\longrightarrow$:	Material flows
$- \rightarrow$:	Information flows

For the purposes of the current research, it is assumed that the dealers compete under the rules of Bertrand competition model, which examines interdependencies between rivals' decisions in terms of pricing. According to this model, there are two firms, selling homogeneous goods with the same marginal costs, which have to take

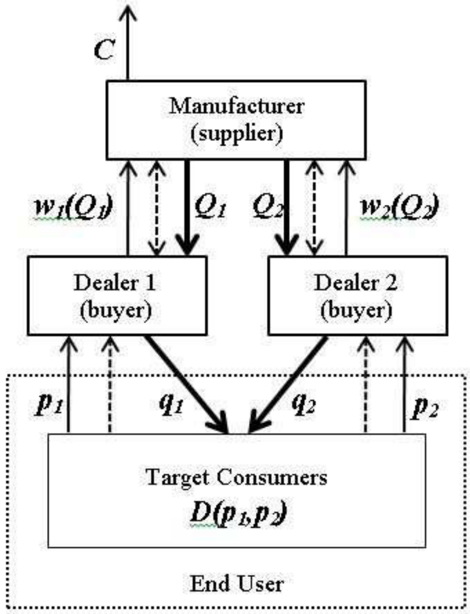

Fig. 5: Supply network model

simultaneous decisions on setting a retail price based on their assumptions on the expected price of their rival. Then the market determines the quantities bought from each firm dependent on the prices they have previously set.

Thus, assuming that dealers compete on prices, the quantity Q_i ordered by a dealer i from manufacturer can be described by the demand function $q_i(p_1, p_2)$ (1).

$$Q_i(p_1, p_2) = \theta k_i - \delta_i p_i + \gamma(p_j - p_i), \qquad i = 1, 2, i \neq j \tag{1}$$

Q_i stands for the order quantity of a dealer i at a given period of time under the conditions of price competition with dealer j, with p_i being retail price of dealer i. θ represents the potential size of the market, where k_i is market share of the dealer i, provided that $k_1 + k_2 = 1$. δ_i and γ are parameters of the demand function.

In the stated model (1) it is assumed that there are two types of customers forming the market: switching customers and marginal customers. Switching customers will always buy the good at a cheapest possible price. Marginal customers will only buy the good if its price is lower than a certain minimum price. Therefore, parameter γ describes the behavior of switching customers and stands for demand leakage, while parameter δ_i characterizes marginal customers, who can be attracted by lowering the price. Total demand can be defined as follows (2).

$$D(p_1, p_2) = \sum_{i=1}^{2} q_i = \theta - \sum_{i=1}^{2} \delta_i p_i \tag{2}$$

Following the assumptions of Bertrand competition model, both dealers have the same marginal costs c, nevertheless manufacturer's operating expenses to fulfill their orders are different and equal to s_1 and s_2 per unit of good accordingly.

Relationships between the members of a supply network can be formalized by a two-tier hierarchical game leader – competing followers, where manufacturer sets contract parameters, while dealers compete on prices in the next round after the choice is made. Taking into consideration assumptions and specifications of the developed supply network model, this would be a non-zero sum game under conditions of perfect information.

In order to formalize the discussion, let us introduce a game:

$$\Gamma =< N, \{X_i\}_{i\in N}, \{\pi_i\}_{i\in N} > \tag{3}$$

where $N = \{S, B_1, B_2\}$ is a set of players, with S being a supplier (manufacturer) and B_i being buyer i (dealer), X_i is a set of strategies available for a player i, π_i is a payoff function of a player i defined by the profit function of a given company. To make it clearer, let us define the manufacture's payoff as π_m.

Each of the discussed contract types is formalized into a separate game, where the goal of the first-tier player is to choose a dependent contract parameter according to the definition of the coordinating contract and, therefore, define the transfer payment. Meanwhile, the chosen parameter is a function of the dealers' retail prices, e.g. a function of followers' strategies.

The set of manufacturer's strategies (player S) in a game number k, where k is correspondent to a specific contract type, will look as follows:

$$X_1^k = \{T^k = (T_1^k(p_1, p_2), T_2^k(p_1, p_2))\}, k = 1, 2, 3, 4 \tag{4}$$

where $T_i^k(p_1, p_2) \in C^2(p_1, p_2)$ is a function of dealer's i transfer payment (player B_i) in a contract k, which is a double continuously differentiable function on p_1,p_2. Each dealer has his own transfer payment function. Notably, manufacturer determines the formulas for these functions, while dealers, in their turn, use given functions to solve the competition problem on the second stage of the game. Therefore, manufacturer's strategy is a choice of vector comprised of transfer payment functions for a distinct contract type. These functions are chosen according to the rule determined by a definition of a coordinating contract. The vector T^k, in turn, stands for the chosen contract.

The dealer i strategy (for each contract type k, e.g. in any given game) is a choice of the retail price p_i under the rules of Bertrand competition model. The order quantities $Q_i(p_1, p_2)$ are uniquely defined by the demand function (5):

$$X_2 = \{p_1\}_{p_1\geq 0}, X_3 = \{p_2\}_{p_2\geq 0}. \tag{5}$$

Let us consider the set of deales' payoffs, which are equivalent to their profit functions. $\pi_i(p_1, p_2)$ is a function of dealer i profit and equals to (6, 7):

$$\pi_1(p_1, p_2, T_1^k) = Q_1(p_1, p_2)(p_1 - c) - T_1^k(p_1, p_2), \tag{6}$$

$$\pi_2(p_1, p_2, T_2^k) = Q_2(p_1, p_2)(p_2 - c) - T_2^k(p_1, p_2) \tag{7}$$

for $Q_2(p_1, p_2)$ being defined by (1), Q_ip_i being dealer i profit, cQ_i being total dealer's expenses on purchase, storage and sales of Q_i units of good and T_i being a transfer payment from dealer i to manufacturer according to the terms of contract $T^k(p_1, p_2)$. The manufacturer's profit function is a sum of two local profit functions in simple supply chains manufacturer - dealer(8):

$$\pi_m(p_1, p_2, T_1^k, T_2^k) = \pi_{m1} + \pi_{m2}. \tag{8}$$

The local profit functions equal to (9) and (10) accordingly, where $s_i Q_i(p1, p2)$ are operational costs to fulfill the order of a dealer i.

$$\pi_{m1}(p_1, p_2, T_1^k) = T_1^k(p_1, p_2) - s_1 Q_1(p_1, p_2), \tag{9}$$

$$\pi_{m2}(p_1, p_2, T_2^k) = T_2^k(p_1, p_2) - s_2 Q_2(p_1, p_2). \tag{10}$$

The total supply network profit can be concurrently divided into two streams: $P(p1, p2) = P_1(p1, p2) + P_2(p1, p2)$, where P_1 and P_2 are local supply chain profit functions resulting from interaction between manufacturer and an associated dealer.

$$P(p_1, p_2) = Q_1(p_1, p_2)(p_1 - c - s_1) + Q_2(p_1, p_2)(p_2 - c - s_2), \tag{11}$$

$$P_1(p_1, p_2) = \pi_1(p_1, p_2) + \pi_{m1}(p_1, p_2) = Q_1(p_1, p_2)(p_1 - c - s_1), \tag{12}$$

$$P_2(p_1, p_2) = \pi_2(p_1, p_2) + \pi_{m2}(p_1, p_2) = Q_2(p_1, p_2)(p_2 - c - s_2). \tag{13}$$

Let us assume the direct manufacturer's payoff function equals to the total supply network profit function P. Then, the main goal of the manufacturer is coordination of supply network as a whole or coordination of the two affiliated supply chains separately, if supply network coordination is impossible. Therefore, the main concern of the manufacturer is choice of such contract parameters that the maximum supply network profit is achieved. For the research purposes, that sort of contract is called coordinating.

Thus, contract $T^k(p_1, p_2)$ is called *strongly coordinating* if it meets the conditions (14):

$$\begin{cases} \arg\max\limits_{p_1} \; P(p_1, p_2) = \arg\max\limits_{p_1} \; \pi_1(p_1, p_2), \\ \arg\max\limits_{p_2} \; P(p_1, p_2) = \arg\max\limits_{p_2} \; \pi_2(p_1, p_2). \end{cases} \tag{14}$$

Following the conditions for coordinating contract, an optimal solution of the dealer's market competition problem optimal prices p_1^*, p_2^*, should also be an optimal solution for the supply network coordination problem, as in this point the maximum of the supply network profit function (11) should be attained. This is achieved through manufacturer's choice of the dependent parameter of the contract and, respectively, the transfer payment function, compliant with (14). Therefore, we have defined the rule for manufacturer's strategy choice by introducing the optimality principle, which is supply network profit maximization.

Notably, in some situations with specific contract types the optimal solution to supply network coordination problem cannot be found analytically. For this cases the optimization criteria is lowered, allowing maximization of the profit function separately for each supply chain constituting the network. Therefore, contract $T^k(p_1, p_2)$ is assumed to be weakly coordinating if it meets the conditions (15).

$$\begin{cases} \arg\max\limits_{p_1} \; P_1(p_1, p_2) = \arg\max\limits_{p_1} \; \pi_1(p_1, p_2), \\ \arg\max\limits_{p_2} \; P_2(p_1, p_2) = \arg\max\limits_{p_2} \; \pi_2(p_1, p_2). \end{cases} \tag{15}$$

By introducing conditions (14) and (15), we introduce the rule for manufacturer's optimal strategy choice, which states that manufacturer will always choose such contract parameters that guarantee supply network profit maximization. In other words, while dealers pursue to maximize their individual profits, manufacturer integrates the supply network in order to maximize the profit of the whole system. Thus, supply network is fully coordinated if it meets the conditions (16) and the problem of the game is to find corresponding strategies of all the players, so that these conditions are met.

$$\begin{cases} \arg\max\limits_{p_1} \; P(p_1, p_2) = \arg\max\limits_{p_1} \; \pi_1(p_1, p_2), \\ \arg\max\limits_{p_2} \; P(p_1, p_2) = \arg\max\limits_{p_2} \; \pi_2(p_1, p_2), \\ \max\limits_{p_1} \; \pi_1(p_1, p_2, T_1^*), \\ \max\limits_{p_2} \; \pi_2(p_1, p_2, T_2^*). \end{cases} \tag{16}$$

Consequently, supply network is weakly coordinated if it only meets lowered optimization criteria (17).

$$\begin{cases} \arg\max\limits_{p_1} \; P_1(p_1, p_2) = \arg\max\limits_{p_1} \; \pi_1(p_1, p_2), \\ \arg\max\limits_{p_2} \; P_2(p_1, p_2) = \arg\max\limits_{p_2} \; \pi_2(p_1, p_2), \\ \max\limits_{p_1} \; \pi_1(p_1, p_2, T_1^*), \\ \max\limits_{p_2} \; \pi_2(p_1, p_2, T_2^*). \end{cases} \tag{17}$$

8. Contract Decision-Making

Based on the choice of the contract parameters, there are several types of coordinating contracts recognized in literature that can be applied in a newsvendor setting. These are revenue-sharing, buy-back, price-discount, quantity-flexibility, sales-rebate, two-part tariff and quantity discount contracts. Studies of Cachon, 2003, Hohn, 2010, and Arishinder, 2011 synthesize the main findings and give summarizing reviews on the existing supply chain contract topologies. Behzad et al. (2010), in turn, provides a detailed overview of coordinating contract in literature and presents the state of art research in this field. According to his study, two broad classes of coordination contracts can be identified in literature: quantity dependent contracts and price dependent contracts.

As the supply network model, presented in the previous section, is based on the model of Bertrand price competition, the scope of this paper is restricted to the price dependent contracts, including wholesale, buy-back, price-discount, revenue-sharing, sales-rebate, quantity-discount and two-part tariff contracts.

Namely, four contracts chosen for the modeling and analysis are wholesale, revenue-sharing, quantity-discount and two-part tariff contracts, which are described later in this chapter. Each contract has dependent and independent variables, which determine how the profit is distributed between manufacturer and dealers. The decision on the independent variables is taken as a result of negotiations between the agents (manufacturer and dealer), strictly after the retail prices were set by the dealer and the dependent contract variables were chosen by the manufacturer. In a general case, dealer i has to pay a transfer payment T^k (18) to the manufacturer, where k refers to the specific contract type.

$$T_i^k(p), i = \{1, 2\} \tag{18}$$

where $p = (p_1, p_2)$ is a price vector.

8.1. Wholesale Contract

Under a wholesale contract a dealer buys goods in quantity Q_i from the manufacturer at a wholesale price per unit w_i and then resells them at a retail price p. Therefore, the transfer payment looks as follows (19):

$$T_i^1(p) = w_i(p)q_i(p), \qquad i = \{1, 2\}. \tag{19}$$

This type of contract only has one dependent variable, chosen by manufacturer, and no independent variables to be negotiated later. Wholesale contract is the least flexible type of contract among all chosen for the analysis, as the supply chain profit is distributed uniquely between manufacturer and dealer. Consequently, manufacturer's (20) and dealers' (21) profit functions can be formalized as follows:

$$\pi_{mi}(p, T_i^1) = Q_i(p)(w_i(p) - s_i), \tag{20}$$

$$\pi_i(p, T_i^1) = Q_i(p)(p_i - c - w_i(p)). \tag{21}$$

This type of contract is the most commonly observed in practice, as it is the simplest to set out and to administer, so it is usually assumed as a basic model for supply chain contract studies with all other types of contracts being derived from it.

8.2. Revenue-Sharing Contract

Under a revenue-sharing contract a dealer buys goods in quantity Q_i from the manufacturer at a wholesale price per unit w_i plus pays a percentage of his revenue. Notably, the supply chain revenue is assumed to include salvage revenue as well. In the end of the selling season dealer receives ϕ share of the revenue, while manufacturer receives the remaining part $(1 - \phi)$. Both parameters are specified before the order quantity Q_i is decided by the dealer. The transfer payment with this type of contract is equal to (22).

$$T_i^2(p) = (1 - \phi_i)Q_i(p)p_i + w_i(p)Q_i(p). \tag{22}$$

Therefore, manufacturer's (23) and dealers' (24) profit functions look as follows:

$$\pi_{mi}(p, T_i^2) = Q_i(p)((1 - \phi_i)p_i + w_i(p) - s_i), \tag{23}$$

$$\pi_i(p, T_i^2) = Q_i(p)(\phi_i p_i - c - w_i(p)). \tag{24}$$

Notably, profits of the separate supply chains and supply network as a whole will be the same for all the studied contracts (11), (12), (13), as the finite function is not dependent on the specific contract parameters due to the fact that transfer payment T is shortcut in the process of mathematical computations.

Revenue-sharing contract is one of the most widely applied in practice, as it has clear interpretation and explicit formulas for ϕ and w, which enforce the coordination in one-echelon supply chains under the rules of Cournot competition model. Moreover, this type of contract allows flexible allocation of profit between manufacturer and dealer, which is highly valued.

8.3. Quantity-Discount Contract

Under a quantity-discount contract a dealer buys goods in quantity Q_i from the manufacturer paying the wholesale price per unit $w_i(Q)$, which decreases with the increase of Q_i. In other words, this means that the discount is dependent on the quantity ordered. In general case transfer payment for this type of contract can be presented as $T(Q) = w(Q)Q$. Nevertheless, in case of two competing dealers quantity-discount contract becomes more complex, where transfer payment may be expressed by the following correlations (25).

$$T_i^3(p) = \begin{cases} w_i(p)Q_i(p) - \frac{1}{2}v_iQ_i^2(p), & \text{if } Q_i(p) \leq \overline{Q_i}(p) = \dfrac{w_i(p) - s_i}{v_i}, \\ T(\overline{Q_i}(p)) + s_i(Q_i(p) - \overline{Q_i}(p)), & \text{otherwise,} \end{cases} \tag{25}$$

where s_i are manufacturer's operating costs to produce and deliver a unit of good for dealer i, $w_i(p)$ is a wholesale price per unit for dealer i, v_i represents a discount (independent parameter) obtained by dealer i, compliant with the following criteria (Cachon and Kok, 2010): $v_i \in [0, \bar{v})$ $\bar{v} = min(\frac{2\delta_1}{\delta_0}, \frac{2\delta_2}{\delta_0})$, where

$$\delta_0 = \delta_1\delta_2 + \gamma(\delta_1 + \delta_2). \tag{26}$$

Notably, that this contract allows flexible allocation of profit between manufacturer and dealer and is included in the multitude (19) with $?_i = 0$. Profit functions of manufacturer (27) and dealers (28) in each case are as follows:

$$\begin{cases} \pi_i(p, T_i^3) = Q_i(p)(p_i - c) - w_i(p)Q_i(p) + \frac{1}{2}v_iQ_i^2(p), & \text{if } Q_i(p) \leq \overline{Q_i}(p), \\ \pi_{mi}(p, T_i^3) = w_i(p)Q_i(p) - \frac{1}{2}v_iQ_i^2(p) - s_iQ_i(p), \end{cases} \tag{27}$$

$$\begin{cases} \pi_i(p, T_i^3) = Q_i(p)(p_i - c) - T(\overline{Q_i}(p)) - s_i(Q_i(p) - \overline{Q_i}(p)), & \text{if } Q_i(p) > \overline{Q_i}(p), \\ \pi_{si}(p, T_i^3) = T(\overline{Q_i}(p)) - s_i\overline{Q_i}(p). \end{cases} \tag{28}$$

8.4. Two-Part Tariff Contract

Two-part tariff is actually a particular case of the wholesale price contract. Manufacturer sells the produced goods in quantity Q_i to the dealer at a wholesale price w_i and charges and additional fee equal to F_i. While the wholesale price is chosen by manufacturer, the parameter F is independent (negotiated) and should be paid at the end of the selling season disregarding the actual dealer's profit. Thus, transfer payment may be formalized as follows:

$$T_i^4(p) = F_i + w_i(p)Q_i(p). \tag{29}$$

Therefore, profit functions of manufacturer (30) and dealers (31) can be presented as follows:

$$\pi_{mi}(T_i^4) = Q_i(p)(w_i(p) - s_i) + F_i, \tag{30}$$

$$\pi_i(p, T_i^4) = Q_i(p)(p_i - c - w_i(p)) - F_i. \tag{31}$$

9. Optimization Results for Quantitative Modeling

Let us formulate the general rule for solving the game in terms of finding the appropriate players' strategies. As manufacturer is a leading player, he would have to analyze the current situation, taking into consideration competition between dealers and their next move, and based on this knowledge take a decision on dependent contract parameters to choose. The resulting parameters should ensure strong or weak coordination of the supply network, provided retail prices and order quantities chosen by the dealers.

9.1. Optimal parameters for the wholesale contract

After the manufacturer's first move, on the second stage of the game each dealer would maximize his profit function, using the first-order conditions, provided that the profit function is strictly concave:

$$\frac{\partial \pi_i}{\partial p_i} = 0.$$

Therefore, manufacturer has to choose contract parameters in such a way, that condition (32) is fulfilled (if weaker criterion of optimality is chosen):

$$\frac{\partial P_i}{\partial p_i} = \frac{\partial \pi_i}{\partial p_i}. \tag{32}$$

If this condition is fulfilled in the point $p_i = p_i^*$ (e.g. optimal dealer's price under competition) the supply chain profit would also hit its maximum, as manufacturer's and dealer's profit functions would match due to specific choice of contract parameters. If both functions are strictly concave on the price of the dealer i, there is no need to check the second-order conditions to demonstrate that point of extremum is a maximum.

Therefore, let us illustrate that both profit functions are strictly concave on the price p_i:

$$\pi_i(p) = Q_i(p)(p_i - c - w_i(p)) = (\theta k_i - \delta_i p_i + \gamma(p_j - p_i))(p_i - c - w_i(p)),$$

$$\frac{\partial \pi_i}{\partial p_i} = (\theta k_i - \delta_i p_i + \gamma(p_j - p_i))(1 - \frac{\partial w_i}{\partial p_i}) - (\delta_i + \gamma)(p_i - c - w_i),$$

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = -2(\delta_i + \gamma)(1 - \frac{\partial w_i}{\partial p_i}) - \frac{\partial^2 w_i}{\partial p_i^2}(\theta k_i - \delta_i p_i + \gamma(p_j - p_i)) < 0.$$

If these conditions are met on the function $w_i(p)$, the dealers' profit functions are strictly concave on p_i. Correspondingly:

$$P_i = Q_i(p)(p_i - c - s_i),$$

$$\frac{\partial^2 P_i}{\partial p_i^2} = -2(\delta_i + \gamma) < 0.$$

The conditions for expressing contract parameter w_i from (32) for each of the supply chains can be formalized as follows:

$$\theta k_i - \delta_i p_i + \gamma(p_j - p_i) - (\delta_i + \gamma)(p_i - c - w_i) = \theta k_i - \delta_i p_i + \gamma(p_j - p_i) - (\delta_i + \gamma)(p_i - c - s_i),$$

$$w_i = s_i.$$

This leads to a conclusion, that the wholesale contract does not coordinate a supply chain due to the fact that the first-order condition is fulfilled only when the wholesale price is equal to the manufacturer's operational costs, meaning that manufacturer would get a zero profit. Taking into consideration, that this is the simplest type of coordinating contracts, the first criterion of optimality was used. In other words, the chosen contract parameters for each dealer should maximize the total supply network profit function P independently on p_1 and p_2:

$$\frac{\partial \pi_i}{\partial p_i} = \frac{\partial P}{\partial p_i}, \tag{33}$$

$$\theta k_i - \delta_i p_i + \gamma(p_j - p_i) - (\delta_i + \gamma)(p_i - c - w_i) =$$

$$= \theta k_i - \delta_i p_i + \gamma(p_j - p_i) - (\delta_i + \gamma)(p_i - c - s_i) + \gamma(p_j - c - s_j),$$

$$(\delta_i + \gamma)(w_i - s_i) = \gamma(p_j - c - s_j),$$

$$w_i(p) = s_i + \frac{\gamma}{\delta_i + \gamma}(p_j - c - s_j). \tag{34}$$

Now the optimal wholesale price is higher than manufacturer's operational costs, providing the positive profit for the manufacturer and ensuring coordination in a supply network. Moreover, in this case w_i does not depend on the price p_i, which means that

$$\frac{\partial w_i}{\partial p_i} = 0, \frac{\partial^2 w_i}{\partial p_i^2} = 0,$$

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = -2(\delta_i + \gamma) < 0.$$

Notably, total supply network profit function P is also strictly concave on p_i:

$$\frac{\partial^2 P}{\partial p_i^2} = -2(\delta_i + \gamma) < 0.$$

In order to solve the problem of competition, knowing the manufacturer's choice on $w_i(p)$, let us insert $w_i(p)$ into the dealer's i profit function (21):

$$\pi_i = (\theta k_i - \delta_i p_i + \gamma(p_j - p_i))(p_i - c - s_i - \frac{\gamma}{\delta_i + \gamma}(p_j - c - s_j)).$$

While choosing the optimal retail price p_i^*, the dealer i would maximize his profit function, thus, applying the first-order condition:

$$\frac{\partial \pi_i}{\partial p_i} = 0 \tag{35}$$

$$\Rightarrow \frac{\partial \pi}{\partial p_i} = \theta k_i + (\delta_i + \gamma)(c + s_i - 2pi) + (2p_j - c - s_j) = 0.$$

As there are two dealers in the model, their reaction functions (describing the reaction of a dealer on the price set by his competitor), can be derived from the first-order conditions (35) and formalized as follows:

$$\begin{cases} p_1 = \frac{1}{2}(\frac{\theta k_1}{\delta_1 + \gamma} + c + s_1 + \frac{\gamma}{\delta_1 + \gamma}(2p_2 - c - s_2)), \\ p_2 = \frac{1}{2}(\frac{\theta k_2}{\delta_2 + \gamma} + c + s_2 + \frac{\gamma}{\delta_2 + \gamma}(2p_1 - c - s_1)). \end{cases} \tag{36}$$

Then, the optimal competitive prices are derived by expressing p_1 in terms of p_2:

$$p_1 = \frac{1}{2}(\frac{\theta k_1}{\delta_1 + \gamma} + c + s_1 +$$

$$+ \frac{\gamma}{\delta_1 + \gamma}((\frac{\theta k_2}{\delta_2 + \gamma} + c + s_2 + \frac{\gamma}{\delta_2 + \gamma}(2p_1 - c - s_1) - c - s_2)),$$

$$p_1 = \frac{1}{2}(\frac{\theta k_1}{\delta_1 + \gamma} + c + s_1 + \frac{\gamma}{(\delta_1 + \gamma)(\delta_2 + \gamma)}(\theta k_2 + \gamma(2p_1 - c - s_1)).$$

If we denote δ_0 as $\delta_1\delta_2 + \gamma(\delta_1 + \delta_2)$, then:

$$p_1 \left(1 - \frac{\gamma^2}{(\delta_1 + \gamma)(\delta_2 + \gamma)}\right) =$$

$$\frac{1}{2}\left(\frac{\theta k_1}{\delta_1 + \gamma} + c + s_1 + \frac{\gamma}{(\delta_1 + \gamma)(\delta_2 + \gamma)}(\theta k_2 - \gamma(c + s_1))\right),$$

$$p_1 = \frac{(\delta_1 + \gamma)(\delta_2 + \gamma)}{2\delta_0}) \left(\frac{\theta k_1}{\delta_1 + \gamma} + c + s_1 + \frac{\gamma}{(\delta_1 + \gamma)(\delta_2 + \gamma)}(\theta k_2 - \gamma(c + s_1))\right),$$

$$\begin{cases} p_1^* = \frac{\theta(\gamma + k_1\delta_2)}{2\delta_0} + \frac{1}{2}(c + s_1), \\ p_2^* = \frac{\theta(\gamma + k_2\delta_1)}{2\delta_0} + \frac{1}{2}(c + s_2). \end{cases} \tag{37}$$

After expressing optimal retail prices (37) in context of market competition, we can consequently determine the optimal order quantities $Q_i^* = Q_i(p_1^*, p_2^*)$.

After all the players choose their strategies, we can evaluate the expected values of manufacturer's, dealers', supply chains and total supply network profit functions based on (8), (11), (20), (21).

9.2. Optimal parameters for the revenue-sharing contract

This contract has a more complicated structure, than the one discussed above. Thus, if the first-order conditions (33) are used to find optimal parameters (e.g. strong criterion of optimality), there will be no explicit solution to the game. Therefore, weak criterion of optimality is applied. Parameter ϕ defines specific shares in which revenue is divided between manufacturer and dealer in the supply chain i (Cachon and Lariviere, 2005):

$$\pi_i = \phi P_i.$$

In order to find contract parameters, let us assume that the correlation between the parameters is valid for the model with two dealers, meaning that each dealer receives a share of total supply network profit correspondent to his profit generated in a supply chain:

$$\pi_i = \phi_i P_i. \tag{38}$$

Consequently:

$$\frac{\partial \pi_i}{\partial p_i} = \phi_i \frac{\partial P_i}{\partial p_i}.$$

Possibility of coordination is predetermined by the contract type and the first-order condition results from (38). Hence, while choosing optimal retail price under competition, a dealer, as well, maximizes his local supply chain profit. Let us illustrate that under the revenue-sharing contract dealers' profit functions are strictly concave on the price pi:

$$\pi_i(p) = Q_i(p)(\phi_i p_i - c - w_i(p)) = (\theta k_i - \delta_i p_i + \gamma(p_j - p_i))(\phi_i p_i - c - w_i),$$

$$\frac{\partial \pi_i}{\partial p_i} = (\theta k_i - \delta_i p_i + \gamma(p_j - p_i))(\phi_i - \frac{\partial w_i}{\partial p_i}) - (\delta_i + \gamma)(\phi_i p_i - c - w_i),$$

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = -2(\delta_i + \gamma)(\phi_i - \frac{\partial w_i}{\partial p_i}) - \frac{\partial^2 w_i}{\partial p_i^2}(\theta k_i - \delta_i p_i + \gamma(p_j - p_i)) < 0.$$

Let us express $w_i(\phi_i)$ out of (38), when the coordination in supply chains is attained (Cachon and Lariviere, 2005):

$$Q_i(p)(\phi_i p_i - c - w_i) = \phi_i Q_i(p)(p_i - c - s_i),$$

$$Q_i(p)(c + w_i) = \phi_i Q_i(p)(c + s_i),$$

$$w_i = \phi_i(s_i + c) - c. \tag{39}$$

This formula allows us to find relevant w_i, which maximizes profit in the local supply network and, therefore, ensures coordination, dependent on the ϕ_i, resulting from negotiations between parties. Nevertheless, w_i is not dependent on prices. Thus, dealers' profit functions are strictly concave:

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = -2\phi_i(\delta_i + \gamma) < 0.$$

In order to solve the problem of competition, knowing contract conditions, let us insert w_i into the dealer's i profit function (24):

$$\pi_i = Q_i(p)(\phi_i p_i - c - w_i) = (\theta k_i - \delta_i p_i + \gamma(p_j - p_i))(\phi_i p_i - c - \phi_i(s_i + c) + c).$$

Consequently, first-order conditions are expressed as follows:

$$\frac{\partial \pi_i}{\partial p_i} = -(\delta_i + \gamma)(\phi_i p_i - \phi_i(s_i + c)) + \phi_i(\theta k_i - \delta_i p_i + \gamma(p_j - p_i)) = 0.$$

Then, the reaction functions can be formalized as:

$$-2\phi_i p_i(\delta_i + \gamma) + \phi_i((\delta_i + \gamma)(s_i + c) + \theta k_i + \gamma p_j) = 0,$$

$$\begin{cases} p_1 = \frac{1}{2}(s_1 + c + \frac{\theta k_1 + \gamma p_2}{\delta_1 + \gamma}), \\ p_2 = \frac{1}{2}(s_2 + c + \frac{\theta k_2 + \gamma p_1}{\delta_2 + \gamma}). \end{cases} \tag{40}$$

Finally, let us express optimal retail prices:

$$p_1 = \frac{1}{2}(s_1 + c + \frac{\theta k_1 + \frac{\gamma}{2}(s_2 + c + \frac{\theta k_2 + \gamma p_1}{\delta_2 + \gamma})}{\delta_1 + \gamma}),$$

$$(2p_1 - s_1 - c)(\delta_1 + \gamma) = \theta k_1 + \frac{\gamma}{2}(s_2 + c) + \frac{\gamma(\theta k_2 + \gamma p_1)}{2(\delta_2 + \gamma)},$$

$$p_1(4\delta_0 + 3\gamma^2) = 2(s_1 + c)(\delta_0 + \gamma^2) + 2\theta k_1(\delta_2 + \gamma) + \gamma(s_2 + c)(\delta_2 + \gamma) + \gamma\theta k_2,$$

$$\begin{cases} p_1^* = \frac{2(s_1 + c)(\delta_0 + \gamma^2) + \gamma\theta + \theta k_1(2\delta_2 + \gamma) + \gamma(s_2 + c)(\delta_2 + \gamma)}{4\delta_0 + 3\gamma^2}, \\ p_2^* = \frac{2(s_2 + c)(\delta_0 + \gamma^2) + \gamma\theta + \theta k_2(2\delta_1 + \gamma) + \gamma(s_1 + c)(\delta_1 + \gamma)}{4\delta_0 + 3\gamma^2}. \end{cases} \tag{41}$$

With the expression of optimal retail prices (41), we can find out optimal order quantities and the expected values of all the profit functions.

9.3. Optimal parameters for the quantity-discount contract

In this case formula for transfer payment calculation is divided into two parts, dependent on the order quantity, which should be reflected in the analysis. For the research purposes, weak criterion of optimality is used.

Let us consider the first situation:

$$T_i^3(p) = w_i(p)Q_i(p) - \frac{1}{2}v_i Q_i^2(p), \quad Q_i(p) \leq \overline{Q_i}(p) = \frac{w_i(p) - s_i}{v_i}. \tag{42}$$

Assume that $Q_i(p) \leq \overline{Q_i}(p)$. In this case, let us define dealer's profit function and show that it is strictly concave on p_i, then evaluate the dependent parameter w_i, wherein the coordination in local supply chains is attained.

$$\pi_i(p) = Q_i(p_i - c - w_i(p) + \frac{1}{2}v_i Q_i),$$
$$\frac{\partial \pi_i}{\partial p_i} = (\delta_i + \gamma)(c + w_i(p) - p_i) + Q_i(p)(1 - \frac{\partial w_i}{\partial p_i} - v_i(\delta_i + \gamma)),$$
$$\frac{\partial^2 \pi_i}{\partial p_i^2} = (\delta_i + \gamma)(2\frac{\partial w_i}{\partial p_i} - 2 - v_i(\delta_i + \gamma)) - Q_i \frac{\partial^2 w_i}{\partial p_i^2} < 0.$$

If $w_i(p)$ is chosen according to the conditions above, dealers' profit functions are strictly concave.

Let us state the first-order conditions:

$$\frac{\partial \pi_i}{\partial p_i} = \frac{\partial P_i}{\partial p_i},$$
$$(\delta_i + \gamma)(c + w_i - p_i) + Q_i(1 - v_i(\delta_i + \gamma)) = -(\delta_i + \gamma)(p_i - c - s_i) + Q_i,$$
$$w_i(p) = v_i Q_i(p) + s_i, \tag{43}$$

which is equivalent to $Q_i = \dfrac{w_i - s_i}{v_i}$, meaning that in this case coordination is achieved only on the threshold value of the interval for Q_i. Let us show the fulfillment of conditions for $w_i(p)$, which ensure that dealers' profit functions are strictly concave:

$$\frac{\partial w_i}{\partial p_i} = -v_i(\delta_i + \gamma),$$
$$\frac{\partial^2 w_i}{\partial p_i^2} = 0,$$
$$\frac{\partial^2 \pi_i}{\partial p_i^2} = -(\delta_i + \gamma)(2 + v_i(\delta_i + \gamma)) < 0.$$

In order to solve the problem of competition and find optimal retail prices, under condition that transfer payment equals to (42), let us instead of $w_i(p)$ insert into the dealer's i profit function (27) its value according to (43):

$$\pi_i = Q_i(p_i - c - v_i Q_i - s_i + \frac{1}{2}v_i Q_i) = Q_i(p_i - c - s_i - \frac{1}{2}v_i Q_i),$$
$$\frac{\partial \pi_i}{\partial p_i} = -(\delta_i + \gamma)(p_i - c - s_i - \frac{1}{2}v_i Q_i) + Q_i(1 + \frac{1}{2}v_i(\delta_i + \gamma)) = 0.$$

Then, the reaction functions can be formalized as:

$$\begin{cases} p_1 = \dfrac{(c + s_1)(\delta_1 + \gamma) + (\theta k_1 + \gamma p_2)(1 + v_1(\delta_1 + \gamma))}{(2 + v_1(\delta_1 + \gamma))(\delta_1 + \gamma)}, \\ p_2 = \dfrac{(c + s_2)(\delta_2 + \gamma) + (\theta k_2 + \gamma p_1)(1 + v_2(\delta_2 + \gamma))}{(2 + v_2(\delta_2 + \gamma))(\delta_2 + \gamma)}. \end{cases} \tag{44}$$

Finally, let us express optimal retail prices:

$$p_1 = \frac{(c+s_1)(\delta_1+\gamma)}{(2+v_1(\delta_1+\gamma))(\delta_1+\gamma)} +$$
$$+\frac{(\theta k_1 + \gamma(\frac{(c+s_2)(\delta_2+\gamma)+(\theta k_2+\gamma p_1)(1+v_2(\delta_2+\gamma))}{(2+v_2(\delta_2+\gamma))(\delta_2+\gamma)}))(1+v_1(\delta_1+\gamma))}{(2+v_1(\delta_1+\gamma))(\delta_1+\gamma)},$$

$$p_1(\delta_0+\gamma^2)(2+v_1(\delta_1+\gamma))(2+v_2(\delta_2+\gamma)) = (c+s_1)(\delta_0+\gamma^2)(2+v_2(\delta_2+\gamma))+$$
$$+(1+v_1(\delta_1+\gamma))(2+v_2(\delta_2+\gamma))(\delta_2+\gamma)\theta k_1 + \gamma(c+s_2)(\delta_2+\gamma)(1+v_1(\delta_1+\gamma))+$$
$$+\gamma(1+v_1(\delta_1+\gamma))(1+v_2(\delta_2+\gamma))(\theta k_2+\gamma p_1),$$

$$p_1((\delta_0+\gamma^2)(2+v_1(\delta_1+\gamma))(2+v_2(\delta_2+\gamma)) - \gamma^2(1+v_1(\delta_1+\gamma))(1+v_2(\delta_2+\gamma))) =$$
$$= (c+s_1)(\delta_0+\gamma^2)(2+v_2(\delta_2+\gamma)) + (1+v_1(\delta_1+\gamma))(2+v_2(\delta_2+\gamma))(\delta_2+\gamma)\theta k_1+$$
$$+\gamma(1+v_1(\delta_1+\gamma))(1+v_2(\delta_2+\gamma))\theta k_2 + \gamma(c+s_2)(\delta_2+\gamma)(1+v_1(\delta_1+\gamma)).$$

Let us denote $\delta_i + \gamma$ as α_i , then optimal prices equal to:

$$\begin{cases} p_1^* = \frac{\gamma(1+v_1\alpha_1)\{\alpha_2(c+s_2)+(1+v_2\alpha_2)\theta k_2\}+\alpha_2(2+v_2\alpha_2)\{\alpha_1(c+s_1)+\theta k_1(1+v_1\alpha_1)\}}{\delta_0(4+2v_2\alpha_2+2v_1\alpha_1+v_1v_2\alpha_1\alpha_2)+\gamma^2(3+v_2\alpha_2+v_1\alpha_1)}, \\ \\ p_2^* = \frac{\gamma(1+v_2\alpha_2)\{\alpha_1(c+s_1)+(1+v_1\alpha_1)\theta k_1\}+\alpha_1(2+v_1\alpha_1)\{\alpha_2(c+s_2)+\theta k_2(1+v_2\alpha_2)\}}{\delta_0(4+2v_2\alpha_2+2v_1\alpha_1+v_1v_2\alpha_1\alpha_2)+\gamma^2(3+v_2\alpha_2+v_1\alpha_1)}. \end{cases} \tag{45}$$

It can be clearly seen that in this case optimal retail prices are dependent on contract parameter ?i, which stands for a discount defined during the negotiation period. All other parameters, such as optimal prices, wholesale price, order quantities and profits, are determined according to the chosen discount.

Now let us consider the second situation when $Q_i(p) > \overline{Q_i}(p)$. In this case dealers' profit functions can be formalized according to (28):

$$\pi_i = Q_i(p_i - c - s_i) - \frac{w_i - s_i}{v_i}\left(\frac{s_i}{2} + (w_i - s_i)\left(\frac{w_i}{v_i} - \frac{1}{2}\right)\right) =$$
$$= P_i - \frac{w_i - s_i}{v_i}\left(\frac{s_i}{2} + (w_i - s_i)\left(\frac{w_i}{v_i} - \frac{1}{2}\right)\right)$$
$$\Rightarrow \frac{\partial \pi_i}{\partial p_i} = \frac{\partial P_i}{\partial p_i}.$$

Therefore, coordination is achieved no matter what contract parameters are chosen. In this sense manufacturer focuses on those parameters, which yield higher supply chain profit, and makes his choice based on the analysis of these two cases.

$$\frac{\partial \pi_i}{\partial p_i} = -(\delta_i+\gamma)(p_i - c - s_i) + \theta k_i - \delta_i p_i + \gamma(p_j - p_i) = 0.$$

The reaction functions:

$$\begin{cases} p_1 = \dfrac{c+s_1}{2} + \dfrac{\theta k_1 + \gamma p_2}{2(\delta_1+\gamma)}, \\ p_2 = \dfrac{c+s_2}{2} + \dfrac{\theta k_2 + \gamma p_1}{2(\delta_2+\gamma)}. \end{cases} \tag{46}$$

Now let us express optimal prices:

$$p_1 = \frac{c+s_1}{2} + \frac{\theta k_1 + \gamma \left(\dfrac{c+s_2}{2} + \dfrac{\theta k_2 + \gamma p_1}{2(\delta_2+\gamma)} \right)}{2(\delta_1+\gamma)},$$

$$4p_1(\delta_0+\gamma^2) = 2(c+s_1)(\delta_0+\gamma^2) + 2\theta k_1(\delta_2+\gamma) + \gamma(\delta_2+\gamma)(c+s_2) + \gamma\theta k_2 + \gamma^2 p_1,$$

$$\begin{cases} p_1^* = \dfrac{2(c+s_1)(\delta_0+\gamma^2) + (\delta_2+\gamma)(2\theta k_1 + \gamma(c+s_2)) + \gamma\theta k_2}{4\delta_0 + 3\gamma^2}, \\ \\ p_2^* = \dfrac{2(c+s_2)(\delta_0+\gamma^2) + (\delta_1+\gamma)(2\theta k_2 + \gamma(c+s_1)) + \gamma\theta k_1}{4\delta_0 + 3\gamma^2}. \end{cases} \tag{47}$$

The comparison of formulas (47) and (45) leads to a conclusion that in the first case the optimal retail price would be always higher, which means that, according to the law of demand, in the second case lower prices enforce higher order quantity. This proves that in the second case dealers' order quantities would meet the requirement $Q_i > \overline{Q_i}$.

In order to determine optimal strategy for the manufacturer, it is necessary to compare supply chain profit $Q_i(p_i - s_i - c)$ under both (47) and (45) for each separate case. It is also possible to insert in profit function $Q_i(p_i - s_i - c)$ equations dependent on v_i (47) and, under first-order conditions, find through market values and players' costs expressions for optimal discounts v_1, v_2 for the first situation.

9.4. Optimal parameters for the two-part tariff contract

In this case strong criterion of optimality gives the same result as for the wholesale contract:

$$\pi_i(p) = Q_i(p)(p_i - c - w_i) - F_i,$$

$$\frac{\partial \pi_i}{\partial p_i} = \theta k_i - \delta_i p_i + \gamma(p_j - p_i) - (\delta_i + \gamma)(p_i - c - w_i),$$

$$\frac{\partial P}{\partial p_i} = \frac{\partial \pi_i}{\partial p_i},$$

$$\theta k_i - \delta_i p_i + \gamma(p_j - p_i) - (\delta_i+\gamma)(p_i - c - w_i) =$$
$$= \theta k_i - \delta_i p_i + \gamma(p_j - p_i) - (\delta_i+\gamma)(p_i - c - s_i) + \gamma(p_j - c - s_j),$$

$$w_i = s_i + \frac{\gamma}{\delta_i+\gamma}(p_j - c - s_j). \tag{48}$$

Expression for the contract parameter is similar to the case of the wholesale contract, with the only difference in controlling parameter F_i, which is an independent contract parameter allowing to redistribute supply chain profit between manufacturer and dealer, while under the wholesale contract maximum profit can be distributed in an exclusive and predefined way.

Therefore, due to similarity in formulas for these two contracts, optimal retail prices are equivalent to (37).

10. Bargaining Power in Contract Decision-Making

According to Kannan (2011), the final choice on the type of contract to be implemented is based on supply network profit allocation between the participating members. This leads to the notion of the bargaining power and the ways it can be distributed among the supply network members, as, following Choi and Triantis (2012), when two parties enter into a contract, their relative bargaining power affects the terms of their deal.

Although bargaining power is often cited as a critical determinant of contractual terms, neither the meaning of power nor the path of its influence is very clear (Choi and Triantis, 2012). The slipperiness of the term is due, at least partly, to the fact that bargaining power frequently boils down to a tautology: one party had bargaining power when the resulting agreement is more favorable to that party than its counterpart.

To understand what a bargaining power is, consider price is a function of the manufacturer's and dealer's respective perceptions of the two reservation prices (each party's own and that of her counterpart). The perceived bounds for the bargaining range, and the price ultimately chosen within this range, are determined by a mix of factors that might be exogenous or endogenous to the negotiations. Choi and Triantis (2012) divide these factors into five distinct categories:

- Demand and supply conditions
- Market concentration
- Private information
- Patience and risk aversion
- Negotiating skills and strategy

The first category of exogenous factors consists of the demand and supply conditions in the relevant market. When there is a significant increase in the demand for the product or reduction in the supply, the market price will tend to increase and manufacturer is often said to have increased bargaining power.

Second category of exogenous factors is market concentration. A monopolist's market power is often referred to as its bargaining power. A dealer's no-agreement alternative is limited by the fact that there are no other manufacturers available in the market and his reservation price is correspondingly higher than if he could purchase the same good from a competitor. Typically, market concentration on the seller side increases price and concentration on the buyer side decreases it.

A third category of exogenous factors contains informational advantages that one party may enjoy by knowing more about the other party, the market or by concealing information about itself. A party with private information can be thought of as having a type of monopoly originating from having private access to valuable information.

Fourth category is containing company's characteristics, such as patience and risk aversion, that may determine where the agreed price will fall within a given bargaining range. Bold parties, for example, may do better than timid players, and the patient negotiator typically enjoys higher returns than the impatient opponent. Patience may be, in turn, a function of other factors, such as the solvency and liquidity constraints, or the ability to diversify risks of an unfavorable bargaining outcome.

In the fifth category, there are various negotiating tactics that can change the actual or perceived reservation price of either party, so as to induce a favorable shift in the bargaining range. For example, a party might take steps to worsen (or appear to worsen) the opponent's outside opportunities, through credible threats or otherwise. Strategic negotiators also exploit the cognitive biases and errors of their opponents, particularly the tendency of some individuals to escalate commitment and be overconfident in their abilities.

In any given contract transaction, one or more of these factors might be in action. Which ones are present usually determines the exact path by which unequal bargaining power affects given contract design. This means that a factor or a combination of certain factors gives one party the opportunity to influence the contract terms in his own favor. In other words, having more bargaining power refers to *the ability of one party to influence the choice of contract parameters in such a way, that this party receives more benefits from the contract.*

For the purposes of the current research, it is assumed that the party, which enjoys more bargaining power in negotiations, uses it to receive additional benefits from the contract in terms of winning a bigger share of total supply network profit. This means, that during the negotiation period, contract parameters will be chosen in favor of the most powerful party, nevertheless, being accepted as an optimal solution by all the supply chain members.

11. Contract selection modelling in Supply networks

Based on the results of the theoretical studies, presented in Chapter 9, special software for the improved contract selection methodology was developed. It computes optimal parameters for all four types of contracts, studied in current paper, so that these contracts coordinate a given supply network and return the highest possible profits, according to their type. Moreover, this software tool also allows graphical representation of supply network profit function dynamics, while changing certain parameters for manufacturer, dealers and the market.

Software tool was developed in Visual Studio 2012 using C# programming language. Graph construction was carried out in ZedGraph frame. Its functionality is presented in the Figure 6 below.

It is necessary to give some comments on revenue-sharing contract parameters approximation, as well as optimal discounts computation. Analytically, for the revenue-sharing contract, the problem was solved in terms of weak coordination, as if the criterion (49) for strong coordination is followed, then equation for w_i would take the following form (50):

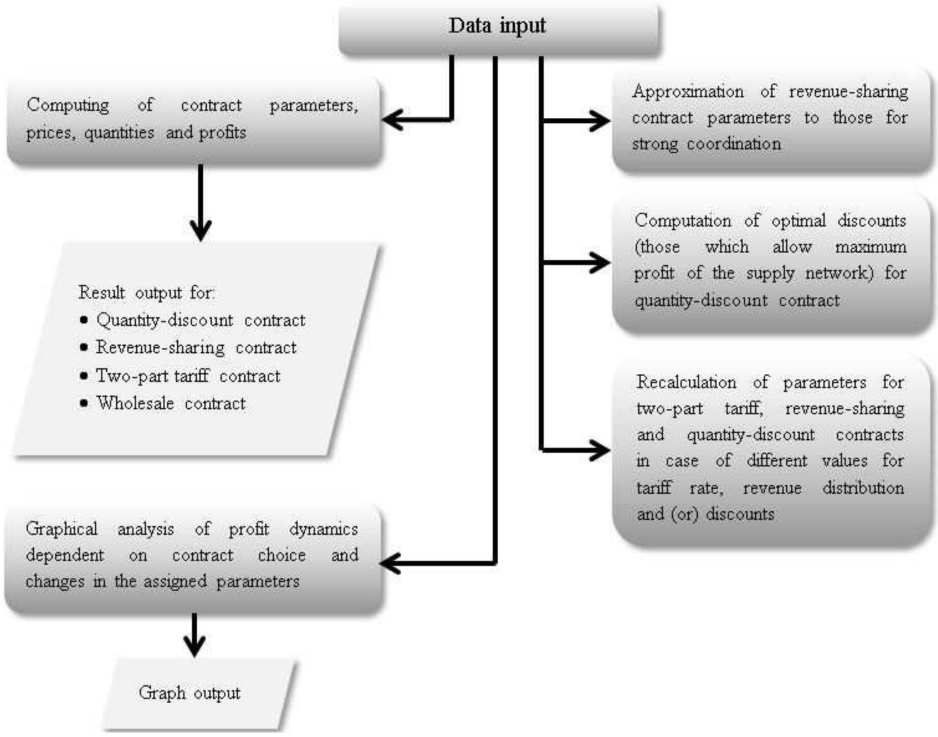

Fig. 6: Software functionality

$$\frac{\partial \pi_i}{\partial p_i} = \phi_i \frac{\partial P}{\partial p_i}, \tag{49}$$

$$w_i = \phi_i(s_i + c) - c + \frac{(\delta_1 + \gamma)(s_1 - w_1)}{2p_1(\delta_1 + \gamma) - \gamma p_2}. \tag{50}$$

Equation (50), in turn, led to problem insolvability due to the last additive component. Therefore, the proposed algorithm of approximation chase is based on the method of drawing near this last component, initially assuming that it equals to 0 and then gradually increasing its value in different combinations. Since profit function is concaved, the chase goes on until supply network profit keeps growing. As soon as the next iteration gives value for a profit function, which is smaller than the one given at a previous step, search cycle is stopped. Hence, approximation for w_i in terms of strong coordination would look as follows (51).

$$\begin{cases} w_1 = \phi_1(s_1 + c) - c + e_1, \\ w_2 = \phi_2(s_2 + c) - c + e_2, \end{cases} \tag{51}$$

where e_1, e_2 are the algorithmically found approximations. Testing showed that these approximations return higher values for supply network profit function than previously used weak coordination parameters.

As for optimal discounts computation, the problem solution has resulted in two different options of pricing and quantity decisions, namely, when dealer i orders quantity $Q_i \leq \overline{Q_i}$ and when dealer i orders quantity $Q_i > \overline{Q_i}$. These two sets of decision options result in four separate cases. Software makes computations of optimal discounts for each case and then returns the one, which maximizes supply network total profit.

Initially, the program requires to input certain parameters, including market parameters: θ potential market size, δ_1, δ_2 and γ demand function parameters, k_1, k_2 first and second dealer's market shares accordingly; and cost parameters: c dealers' marginal costs, s_1, s_2 manufacturer's operating expenses to fulfill the orders of an associated dealer. For certain contracts it is also necessary to insert additional initial parameters, such as shares of revenue for revenue-sharing contract and discounts for quantity-discount contract.

Developed software will be first applied for modelling numerical examples to show the mechanics and draw some conclusions, which then will be tested on real-life cases. Both modeling examples and cases were selected to cover the notion of different bargaining power distribution between the supply chain members, which was discussed in previous Chapter. Consequently, first example assumes the situation of strong manufacturer and is later illustrated with Audi Russia (Volkswagen group) case study, second example assumes the situation of strong dealers and is supported by ProtechDry Portugal (Impetus group) case study and the last example assumes negotiation between equally powerful parties, which is illustrated with Heineken (Local wholesaler) case study.

11.1. Coordinating Contract with Strong Manufacturer

Let's consider the situation of initially strong manufacturer, who can insist on contract parameters in his own favor. In this case manufacturer tries to gain a relatively bigger share of profit from the supply network, while dealers would accept these unfavorable conditions, as they have limited bargaining options.

Table 2 below summarizes market conditions and contract parameters that would correspond to a described situation.

Table 2: Initial data set for the case of strong manufacturer

θ	γ	δ_1	δ_2	k_1	k_2	c	s_1	s_2	ϕ_1	ϕ_2	v_1	v_2	F_1	F_2
200	0.5	1	1	0.45	0.55	10	35	40	0.3	0.3	0.1	0.15	160	170

The potential market size equals to 200 conditional units, which is more or less equally divided between dealers, as first dealer has 45% market share, while second covers the remaining 55%. Given γ, ranging from 0.1 to 0.9, reflects market elasticity, while $\gamma + \delta_i$ shows price demand elasticity of a given dealer. Therefore, it is assumed that initially market is characterized by medium elasticity.

From mathematical point of view, manufacturer's power would directly affect given contract parameters, such as ϕ_1 and ϕ_2, which are relatively low in order to reflect lower dealer's profits. Similarly, dealer's discounts v_1 and v_2, which can range from 0 to 1, will be quite small as well.

Given the initial data set, Table 3 below gives an overview for the resulting prices and order quantities, while Table 3 aims to summarize the modeling results in terms of listing the profits achieved by all the participants of a supply network under different contract rules.

Table 3: Optimal prices and quantities for the case of strong manufacturer

	p_1^*	q_1^*	w_1^*	p_2^*	q_2^*	w_2^*
Wholesale contract	70	24	44.17	77.5	29	48.33
Revenue-Sharing	65,2	29	4.7	72,7	34	5.4
Quantity-discount	66,4	28	37,8	75	31	44,65
Two-part tariff	70	24	44.17	77.5	29	48.33

Table 4: Profit function values summary for the case of strong manufacturer

	π_1^*	π_2^*	π_{s1}^*	π_{s2}^*	π_s^*	P_1^*	P_2^*	P^*
Wholesale contract	380	555.8	220	241.6	461,7	600	797.5	1397,5
Revenue-Sharing	141,1	218,3	445,3	554,7	1000	586,5	773	1359,5
Quantity-discount	560	703,9	39,3	72	111,3	599,2	776	1375,2
Two-part tariff	220	385,8	380	411,7	791,7	600	797.5	1397,5

As for the Table 4 and further in this Chapter, π_1^*, π_2^* - are optimal profits for the first and the second dealers accordingly; π_{m1}^*, π_{m2}^* - are optimal profits of the manufacturer in the distinct supply chains with each dealer; π_m is total optimal manufacturer profit; P_1^*, P_2^* - are separate supply chains optimal profits; P^* - is optimal total supply network profit under a specific contract type.

It can be clearly derived from the obtained results that quantity-discount and wholesale contracts would not be chosen in a situation, when manufacturer is a strong party, as both these contacts provide him with less than average profits. Revenue-sharing and two-part tariff contracts are more suitable for manufacturer, knowing that he can claim a bigger part of the total profit.

To understand how these contracts will be able to coordinate supply network relationships under changing market conditions, developed software is enabled with Graphical analytical toolkit. Notable, that graphs show only three types of contracts wholesale, revenue-sharing and quantity-discount, as two-part tariff contract behaves perfectly identical to wholesale contract with Y-shift equal to the value F. The resulting graphs showing the behavior of the profit function for the case of strong manufacturer are presented in the Appendix 1, while main findings are summarized below.

If marginal costs c increase from 7 to 15 monetary units (see Appendix 1, Fig. 31 - 33), supply network's total profit, as well as manufacturer's and dealers' profits, tend to decrease. Notably, the safest contracts for manufacturer in this case are wholesale and quantity-discount, as manufacturer's profit function is less sensitive to negative effects under their conditions.

At the same time, with the increase in manufacturer's operating expenses for the order fulfillment from 30 to 40 monetary units (see Appendix 1, Fig. 34 - 36), total profit of supply network is decreasing. Profit of the second dealer is growing proportionally to decrease in first dealer's profits. As for manufacturer, all the studied contracts share little sensitivity to negative effects. Therefore, as quantity-discount and wholesale contracts return the smallest profits, revenue-sharing and two-part tariff are most suitable in this case.

The influence of market parameters is defined by γ, ranging from 0.1 to 0.9, parameter that is connected to switching customers' behavior and reflects market elasticity, and δ_i, ranging from 0.3 to 2, parameter that is connected to marginal customers' behavior and reflects price sensitivity. Thus, $\gamma + \delta_i$ shows price demand elasticity for the dealer i.

With γ increase (see Appendix 1, Fig. 37 - 39), elasticity of the market increases accordingly, which leads to further weakening of both dealers. In such market conditions, wholesale or two-part tariff contracts would be the most suitable option for the manufacturer, as under these contracts profit function stays within a specified frame, having a corridor with maximum and minimum borders, instead of constantly falling down, like it happens under all other contracts. Moreover, wholesale contract in a situation of increasing market elasticity considerably drives up manufacturer's own profits. Thus, in case of strong manufacturer and increasing market elasticity γ, two-part tariff contract is the most suitable option.

On the other hand, while price sensitivity for first dealer's products δ_1 is increasing (see Appendix 1, Fig. 40 - 42), total profit of supply network is decreasing, as well as profit of the first dealer itself. A steep increase in price sensitivity results in a heavy decrease in the manufacturer's profit under both wholesale and quantity-discount contracts. In this sense, revenue-sharing contract gives the best safety to manufacturer, as it has low sensitivity to changes in both δ_i and in γ.

Other parameters from the set of external environment features that might influence the profit function are dealers' market shares k_1 and k_2. In other words, a way the market is divided between the two players. In order to track changes in the profit function values, market share of the first dealer is increased from 0.3 to 0.8, while market share of the second dealer decrease accordingly from 0.7 to 0.2 (see Appendix 1, Fig. 43 – 45). This parameter reflects market concentration in a way it is possible to do so for an oligopolistic market.

As it was expected, with the increase of a market share the profit of an associated dealer is increasing as well, while its competitor is losing his profit. Another obvious conclusion is that the more severe is competition the smaller is total supply network profit, as both dealers have strong incentives to lower prices following the rules of Bertrand competition. In other words, it can be stated that supply network profit increases proportionally with the increase in market concentration.

As for the manufacturer, in a situation of low market concentration and, therefore, equal and relatively weak dealers, most optimal decision would be to operate under two-part tariff contract, as it returns the highest profit. At the same time, in a situation of high market concentration, with one dealer being sufficiently more powerful then another, but still less powerful then manufacturer, revenue-sharing would be more favorable.

From the conducted research it can be concluded that for the case of a strong manufacturer, when he has sufficient bargaining power to pursue contract decisions in its own favor, two-part tariff contract is the most optimal contract choice, as it behaves identically to the wholesale contract, nevertheless, allowing profit reallocation in favor of a powerful manufacturer. Revenue-sharing contract has fewer advantages, but generates much stable revenue streams under volatile market conditions and, therefore, can be considered as an optimal choice for some specific markets.

11.2. Coordinating Contract with Equal Power Participants

In turn, let's consider the situation when manufacturer and dealers initially have almost equal bargaining power and, consequently, none of the supply network participants can claim a bigger share of profit. Therefore, given contract parameters, such as revenue shares ϕ_1 and ϕ_2, as well as dealers' discounts v_1 and v_2 and tariff rates F_1 and F_2 would be considerably fairer.

Table 5 below summarizes market conditions and contract parameters that would correspond to a given situation. It is assumed that potential market conditions stay similar to the ones described in a previous example.

Table 5: Initial data set for the case of equal power distribution

θ	γ	δ_1	δ_2	k_1	k_2	c	s_1	s_2	ϕ_1	ϕ_2	v_1	v_2	F_1	F_2
200	0.5	1	1	0.45	0.55	15	35	40	0.5	0.5	0.4	0.45	100	110

Given the initial data set, Table 6 gives an overview for the resulting prices and order quantities, while Table 7 summarizes the results in terms of listing the profits achieved by all the participants of a supply network under different contracts applied.

Table 6: Optimal prices and quantities for the case of equal power distribution

	p_1^*	q_1^*	w_1^*	p_2^*	q_2^*	w_2^*
Wholesale contract	72,5	21	43,3	80	26	47,5
Revenue-Sharing	68,2	26	11,2	75,7	31	12,9
Quantity-discount	72,9	21	43,4	81,7	24	50,8
Two-part tariff	72,5	21	43,3	80	26	47,5

Table 7: Profit function values summary for the case of equal power distribution

	π_1^*	π_2^*	π_{s1}^*	π_{s2}^*	π_s^*	Π_1^*	Π_2^*	Π^*
Wholesale contract	297,5	455	175	195	370	472,5	650	1122,5
Revenue-Sharing	236,9	321,4	236,9	321,4	601,9	473,8	642,9	1116,7
Quantity-discount	392,9	511,2	88,2	129,6	217,8	481,2	640,7	1121,9
Two-part tariff	197,5	345	275	305	580	472,5	650	1122,5

It can be derived from the obtained results that quantity-discount contract unevenly distributes total supply network profit between the participants in favor of dealers. Wholesale contract seems to have this drawback as well, although it is considerably less overbalanced. Such uneven profit allocation might be compensated with the wise application of two-part tariff contract by choosing appropriate tariff rates. Revenue-sharing contract, in turn, divides profit in a perfectly balanced way, according to predefined negotiated shares. Examination on how these contracts will coordinate supply network relationships under changes in different parameters can be found in the Appendix 2.

Considering changes in marginal costs c from 10 to 20 (see Appendix 2, Fig. 46 - 48) and in operating expenses s_i from 30 to 40 (see Appendix 2, Fig. 49 - 51), the results are similar to the case of strong manufacturer. In a situation of equal power participants, from the dealer's perspective, revenue-sharing contract is the one least sensitive for cost increase, while manufacturer might favor wholesale and quantity-discount as being safer.

Notably, with changes in γ from 0.1 to 0.9 (see Appendix 2, Fig. 52 - 54) some mixed results are observed. On the one hand, supply network profit is the most stable towards changes in the market conditions under quantity-discount and wholesale contracts, while under revenue-sharing contract there a slight decrease in profit is evidenced. On the other hand, under the wholesale contract profit dynamics for manufacturer and dealers tend to be completely the opposite manufacturer's profit is drastically increasing with increase in market elasticity, while dealers' profits suffer significant decrease at the very same moment. This situation seems to be completely inacceptable in a case of equally distributed bargaining power. Therefore, revenue-sharing contract is more reasonable here, as the behavior of manufacturer's and dealers' profit functions follows the same patterns.

All studied types of contracts reacted similarly to changes in δ_1 from 0.3 to 2 (see Appendix 2, Fig. 55 - 57). When price sensitivity is increasing, total profit of supply network is decreasing, as well as first dealer's and manufacturer's profit, which is identical to the results obtained for the case of strong manufacturer earlier. In this situation, revenue-sharing contract gives the best safety to manufacturer, while second dealer would give credit to the wholesale contract.

Expectedly, the observed dynamics for changes in dealers' market shares k_i, with market share of the first dealer growing from 0.3 to 0.8, while market share of the second dealer is decreasing from 0.7 to 0.2 (see Appendix 2, Fig. 58 - 60), are similar to the case of a strong manufacturer. The more severe is competition, the smaller is total supply network profit, as both dealers have strong incentives to lower their retail prices (following the rules of Bertrand competition), disregarding what type of contract is applied. In other words, supply network profit increases proportionally with the increase in market concentration. For the dealers, revenue-sharing contract is the most stable in terms of profit allocation, while for manufacturer, in a situation of equal and relatively weak dealers, most optimal decision would be to operate under two-part tariff contract, and in a situation of one dealer being sufficiently more powerful then another, revenue-sharing would be more favorable.

From the studied example, it can be concluded that for the case of equally distributed bargaining power, optimal contract choice would be revenue-sharing, as it allocates the supply network profit exactly according to the negotiated shares. In addition to that, revenue-sharing contract is less sensitive to changes in market conditions and preserves the same tendencies for both dealers' and manufacturer's profit functions, which is important. Two-part tariff might also be used, if tariff rate is tuned to the supply network needs, but it suffers more sensitivity to costs escalation and unfavorable market environment.

11.3. Coordinating Contract with Strong Dealers

The last numerical example considers the situation, when dealers initially have more bargaining power then the manufacturer and, therefore, impose their decisions on the supply network in terms of business arrangements. Dealers' bargaining power would again influence given set of contract parameters, such as revenue shares ϕ_1

and ϕ_2, dealers' discounts v_1 and v_2 and tariff rates F_1 and F_2. Table 3.5 below summarizes market conditions and contract parameters that would correspond to the situation. It is assumed that potential market conditions stay similar to the previous examples.

Table 8: Initial data set for the case of strong dealers

θ	γ	δ_1	δ_2	k_1	k_2	c	s_1	s_2	ϕ_1	ϕ_2	v_1	v_2	F_1	F_2
200	0.5	1	1	0.45	0.55	17	35	40	0.8	0.9	0.9	0.85	30	40

Given the initial data set, Table 9 below gives an overview for the resulting prices and order quantities, while Table 10 summarizes the modeling results in terms of listing the profits achieved by all the participants of a supply network under different contract types applied.

Table 9: Optimal prices and quantities for the case of strong dealers

	p_1^*	q_1^*	w_1^*	p_2^*	q_2^*	w_2^*
Wholesale contract	73,5	20	43	81	25	47,2
Revenue-Sharing	69.5	25	25.7	77,5	29	35.7
Quantity-discount	72.9	21	43.4	81,7	24	50.8
Two-part tariff	73,5	20	43	81	25	47,2

Table 10: Profit function values summary for the case of strong dealers

	π_1^*	π_2^*	π_{s1}^*	π_{s2}^*	π_s^*	Π_1^*	Π_2^*	Π^*
Wholesale contract	270	420,8	160	179,2	339,2	430	600	1030
Revenue-Sharing	349,1	533,6	87,3	59,3	146,6	436,4	592,8	1029,2
Quantity-discount	304,1	415,8	50,4	72,7	123	419,3	569,3	988,5
Two-part tariff	240	380,8	190	409,2	580	430	600	1030

Behavior of the profit functions of a supply network, manufacturer and dealers is summarized in the Appendix 3. In a situation of changing marginal costs c from 12 to 25 (see Appendix 3, Fig. 61 - 63) and manufacturer's operating expenses s_i from 30 to 40 (see Appendix 3, Fig. 64 - 66), the behavior of profit functions is similar to the cases discussed before. In general, it can be concluded that all types of contracts have little sensitivity for costs escalation, with revenue-sharing contract being the most stable option in terms of revenue streams.

As for the influence of market parameters, profit function behavior under the changes in price sensitivity δ_i, ranging from 0.3 to 2, (see Appendix 3, Fig. 70 – 72) also has insignificant differences from the cases discussed earlier. Nevertheless, changes in market elasticity, ranging from 0.1 to 0.9, (see Appendix 3, Fig. 67 - 69) bring some new interesting insights.

With increase in market elasticity γ, both revenue-sharing and quantity-discount contracts react less intensively than wholesale and two-part tariff contracts, which would probably be an attractive option for powerful dealers. In addition to that, revenue-sharing contract can even provide some growth in total supply network profit due to a slight increase in manufacturer's profit. Nevertheless, quantity-discount is less sensitive to changes in market conditions. Therefore, if dealer runs a risk of losing a part of his market share, optimal choice would be apply a quantity-discount contract, while if he is expecting some growth, application of a revenue-sharing contract would enforce a steeper profit growth.

As it was expected, the observed dynamics for changes in the dealers' market shares k_i, from 0.3 to 0.8 for the first dealer and from 0.7 to 0.2 for the second dealer, (see Appendix 3, Fig. 73 - 75), are similar to the previous cases. The more severe is competition, the smaller is total supply network profit, as both dealers have strong incentives to lower their prices (following the rules of Bertrand competition), no matter what type of contract is chosen. In other words, supply network profit increases proportionally with the increase in market concentration. For the dealers, revenue-sharing contract is the most stable option in terms of profit allocation, while for the manufacturer, in a situation of equal and relatively weak dealers, most optimal decision would be to operate under two-part tariff contract, and in a situation of one dealer being sufficiently more powerful then another, revenue-sharing contract would be preferred.

From the current example, it can be concluded that in any case, quantity-discount contract tends to allocate profit in favor of dealers, no matter what costs and discounts are chosen. Moreover, this type of contract has some characteristics, which might be of use in a situation of strong dealers. Thus, this contract would be an optimal choice in this case. Wholesale contract, in turn, allocates too much profit to the manufacturer, which is very doubtful to be accepted by the dealers enjoying higher bargaining power. At the same time, revenue-sharing contract allows dealers to receive an exact share of total supply network revenue according to the negotiations. However, it this contract type is very sensitive to changes in external market conditions and, therefore, is applicable only for some specific situations.

11.4. Audi Group Case Study

This is a case study based on the data of the year 2010, which was obtained from the interview with a CEO of one of the Audi dealership centers in Saint-Petersburg, Sergey P. Ticholiz, on 16.04.2012. In addition to that, public company reports, as well as open-source data were used in order to obtain some data for the modeling purposes. Detailed information can be found in the research paper "Supply Chain Coordination with revenue-sharing contract: Audi dealers case" (M. Koroleva, 2012).

This is a case of a strong international manufacturer selling its goods through small, compared to manufacturer size, local dealers, who have to compete for the same local market with each other. In 2010, Audi's importing department, in terms of their own branded dealership network, included 46 points of sales in 35 cities across Russia, of which 9 were in Moscow and 3 in Saint-Petersburg.

Volkswagen Group Rus usually encloses 5-year long-term revenue-sharing contracts with its associated dealers. According to the contract terms, an official dealer of Audi, based on his own demand estimations, buys a specific amount of branded Audi cars Q_i from the importing company at a price w_i per car and then resells these cars to the market at a price p_i per unit. The difference between w and p is called dealer's percentage and, therefore $p_i = \phi_i w_i$. Moreover, if a dealer sells more than a certain amount of cars, he receives a bonus from the manufacturing company, which might be interpreted as having a quantity-based discount.

To evaluate the potential market size, Audi sales statistics of the year 2010 was used. Due to lacking information, it is impossible to estimate, how many cars exactly were ordered by dealers in 2010. Therefore, it is assumed that the number of sold cars equals to the number of ordered cars and salvage value therefore equals to zero. Thus, combined dealers' order quantity Q for the region of Saint-Petersburg in 2010 was equal to 1430 car units.

Concerning the actual retail prices, there is a lot of volatility in the car market due to a number of possible car grades, which can range from simple to luxurious. To overcome this problem, retail price p was assumed to be equal to the mathematical average between the lowest and the highest prices of a specific model.

Based on these data, potential market size θ for Audi cars in 2010 in Saint-Petersburg can be estimated as equal to 2 834 885 795 Rub. For the purposes of the current study, it is assumed that all dealers have equal market shares, as, according to the interview (Sergey Ticholiz, 2012), their competition is quite intense. An additional analysis, presenting different possible levels of market concentration will be presented later in this section.

Concerning market elasticity γ, as car buyers are very likely to switch between dealers in case of lower prices, γ is assumed to be equal to 0,7 in order to reflect the situation. At the same time, price sensitivity δ tends to be medium and equals to 1, as Audi cars fall into a category of luxury goods with less price-sensitive customer audience.

Following Sergey Ticholiz (2012), dealer's marginal costs c can be approximately estimated as being equal to 70 000 000 Rub, while operating expenses of Audi Russia (s_i) in 2010 were 474 000 000 euros (from Audi Group Annual Financial Statement). Euro exchange rate for 31.12.10 was equal to 40.3 Rub / Euro. Therefore, costs of Audi Russia to fulfill all the associated dealers' orders in 2010 were equal to 19 102 200 000 Rub, while the costs s_i to fulfil the order of one dealer in Saint-Petersburg can be estimated as 181 925 700 Rub.

As it was mentioned earlier, in terms of bargaining power, this is the case of having a strong manufacturer at one side and a number of small, competing dealers on the other side, which is reflected in contract parameters. According to Sergey Ticholiz (2012), in 2010 the distribution of profit between manufacturer and dealer was 90% manufacturer's share and 10% dealer's share (ϕ). At the same time, quantity based discount available (v) was 5% at maximum. As for F_1, F_2, after modelling the wholesale contract, their initial values were set equal to 150 000 Rub, as to redistribute the profit according to the situation of extremely strong manufacturer.

Table 11 below summarizes market conditions and contract parameters that would correspond to the described situation, with M standing for million Rubles.

Table 11: Initial data set for Audi case

θ	γ	δ_1	δ_2	k_1	k_2	c	s_1	s_2	ϕ_1	ϕ_2	v_1	v_2	F_1	F_2
2834M	0.7	1	1	0.5	0.5	70M	181M	181M	0.1	0.1	0.05	0.05	0.15M	0.15M

Main results for the profit allocation are summarized in Table 12. The results are presented in million Rubles.

Table 12: Profit function values summary for Audi case

	π_1^*	π_2^*	π_{s1}^*	π_{s2}^*	π_s^*	Π_1^*	Π_2^*	Π^*
Wholesale contract	199420	199420	139594	139594	279188	339015	339015	678030
Revenue-Sharing	31692	31692	285232	285232	570349	316924	316924	633722
Quantity-discount	310062	310062	12678	12678	25276	322700	322700	645401
Two-part tariff	494200	49420	289594	289594	579188	339015	339015	678030

As it can be clearly seen from the Table 12, Audi Russia has chosen revenue-sharing contract to be the one coordinating their supply network, as initially it returns the company, as a manufacturer, the highest profit in absolute terms. Nevertheless, wholesale and two-part tariff contract results show that there still exists room for supply network optimization in terms of increasing system-wide total profit. Moreover, with the application of a two-part-tariff contract, this profit can be reallocated according to the power distribution with the usage of corresponding tariff rate, which would result in higher total supply network profits, as well as higher profits for both dealers and Audi Group.

Let's now see how these contracts will be able to coordinate Audi's supply network relationships under changing market conditions. In case of changes in dealer's marginal costs c, in a range from 40M to 100M Rub., profit functions would look as follows (Fig. 7 – 9):

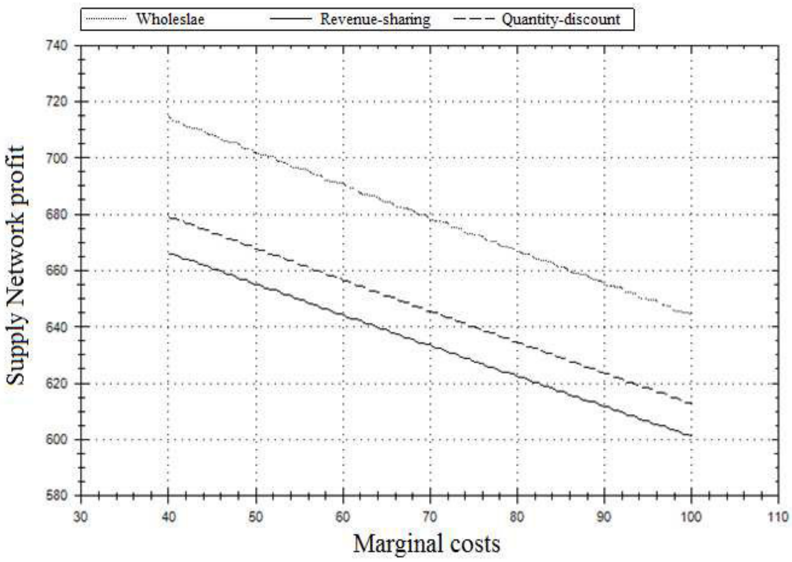

Fig. 7: Audi supply network profit function under volatility of marginal costs

The behavior of profit functions for Audi's supply network is similar to that of a numerical example for strong manufacturer and brings the same conclusions: the

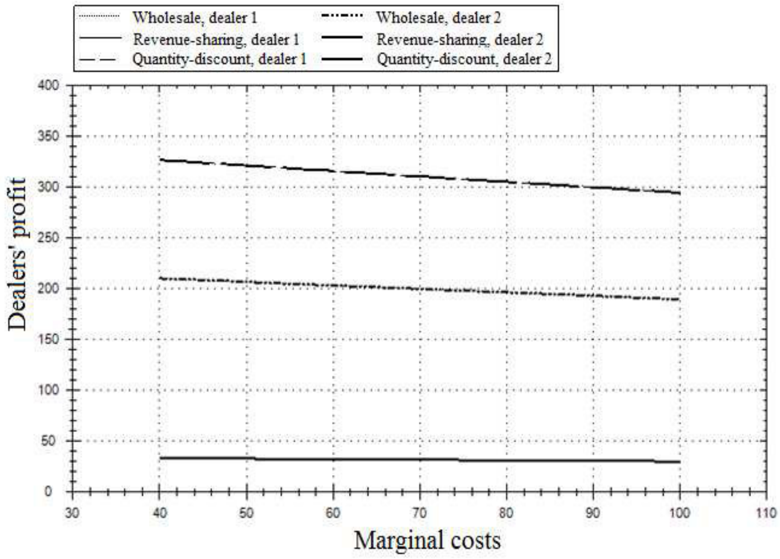

Fig. 8: Audi dealers' profit function under volatility of marginal costs

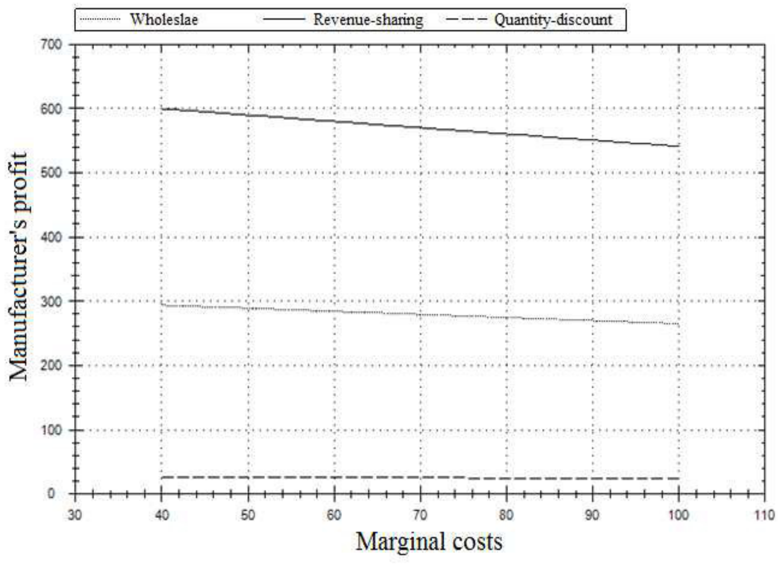

Fig. 9: Audi profit function under volatility of marginal costs

safest contracts for Audi would be wholesale and quantity-discount, as company's profit function is less sensitive to the dealer's costs escalation.

Consequently, the behavior of profit function in case of changes in Audi's operating expenses to fulfill the orders of their dealers, from 160M to 200M Rub, would be similar to that of a studied example as well (see Appendix 4, Fig. 77 - 79). With the increase in Audi Russia operating expenses for order fulfillment, total profit of the whole supply network is decreasing. Moreover, profit of the second dealer is growing proportionally to the decrease in first dealer's profits. All of the studied contracts share little sensitivity to negative effects of the changes in cost structure, thus, revenue-sharing and two-part tariff are the most suitable contracts for Audi Group in this case.

As for the influence of different market parameters, including market elasticity γ, ranging from 0.1 to 0.9, and price sensitivity δ_i, ranging from 0.3 to 2, the behavior of Audi supply network profit functions can be found in the Appendix 4 (Fig. 80 - 84). In general, main results are similar to those, attained for the numerical example.

Notably, γ was initially quite high in the Audi case, reflecting the situation of equally small dealers weakened by their intense competition. In a situation of growing market elasticity γ, wholesale or two-part tariff contracts would be the most suitable options for Audi Russia, as they save company's profits from constantly falling down, keeping the profit function within the corridor, as opposed to other contract types. Nevertheless, increase in price sensitivity δ leads to a heavy decrease of Audi's profit under wholesale and quantity-discount contracts. In this sense, revenue-sharing contract gives the best safety in a volatile market situation, as it has low sensitivity to changes in both δ_i and in γ.

Now the assumption of initially equal market shares k_1 and k_2 is to be tested. Market share of the first dealer would be increased from 0.3 to 0.8, while market share of the second would decrease from 0.7 to 0.2 accordingly. Consequently, the behavior of profit function would look as follows (Fig. 10 - 12):

As it was expected, with the increase of his market share, profit of an associated dealer is increasing as well, while its competitor's profit is falling proportionally. Another evident conclusion is that the more severe is the competition, the smaller is total Audi supply network profit, as under the rules of Bertrand competition both dealers have strong incentives to lower their prices in order to attract consumers. For Audi, in a situation of facing equally weak dealers, the most optimal decision would be to operate under two-part tariff contract, as it returns the highest profit. If concentration on the market would be eventually increasing, revenue-sharing contract becomes more favorable for the Audi Group.

All in all, this case study goes in line with the results attained in a numerical example earlier in this Chapter. According to the data available, for Audi Russia Group, the most optimal contract choice is two-part tariff contract, as it provides enough safety towards volatile market conditions, while optimizing the supply network economic performance in terms of returning the highest possible total profit, in addition allowing profit reallocation in favor of manufacturer. Revenue-sharing contract, which is currently used by company, is suitable for specific market conditions, such as growing price sensitivity of customers, which might be the case during the economic crisis or due to political environment.

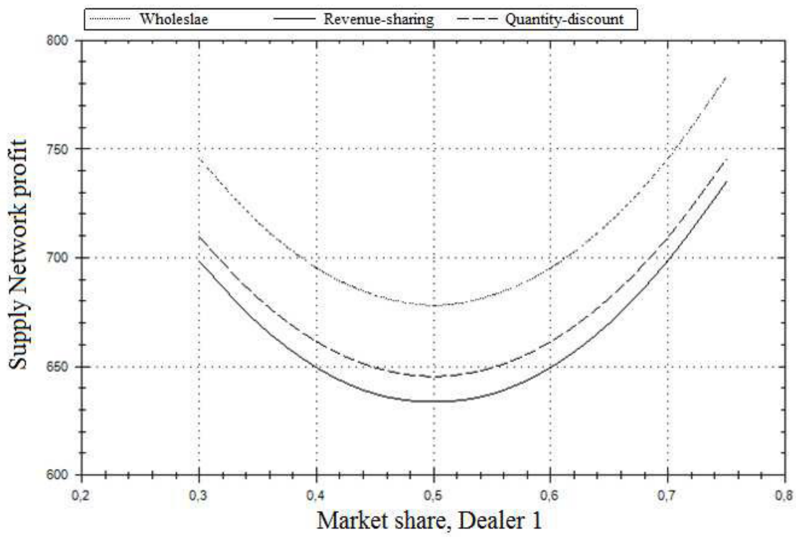

Fig. 10: Audi supply network profit function under changes in market concentration

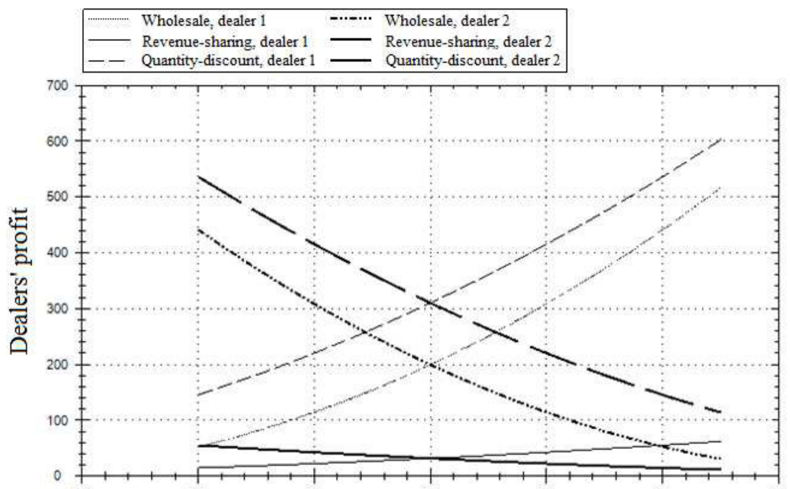

Fig. 11: Audi dealers' profit function under changes in market concentration

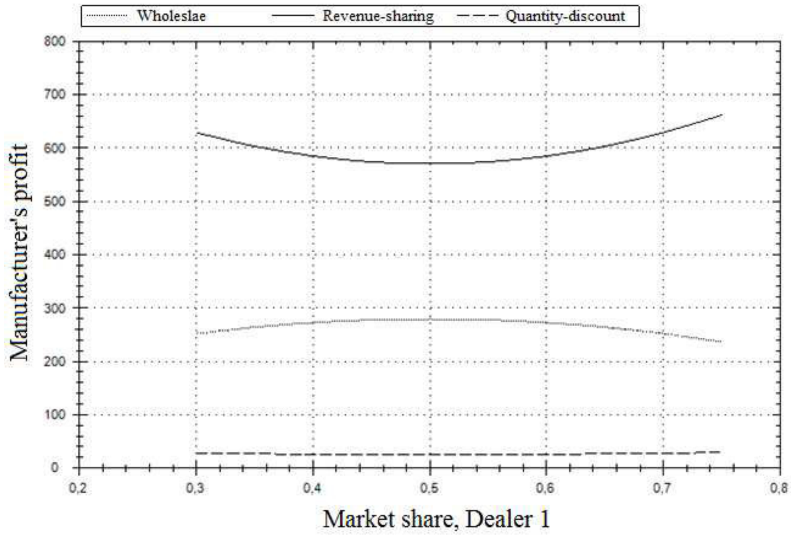

Fig. 12: Audi profit function under changes in market concentration

11.5. Heineken Case Study

Heineken case study is based on the data of the year 2015, which was obtained from the confidential interview with a middle manager responsible for procurement and logistics of the medium chain pub in Saint-Petersburg on 21.08.2015. In addition to that, company's contract offers and warehouse documentation were used in order to attain the necessary data.

This is a case of equal power parties, with a beer wholesaler selling products to separate pubs in the center of Saint-Petersburg. Concerning the dealers' side, restaurant and foodservice market in Saint-Petersburg is extremely competitive, with huge chain players dominating the market at one side and medium-to-small local companies altogether comprising the majority of the market (more than 50% market share) at another. According to Rosstat, the number of cafes, restaurants, and other food outlets in Russia currently stands at about 88,000 and almost 88 percent of outlets are independent non-chain cafes and restaurants.

As for the supplier side, big alcohol manufacturers, such as Heineken Group, have specific distribution requirements, which are more or less similar worldwide. In order to have direct relationships with Heineken Group any buying company should purchase and realize certain volumes of their product on a monthly or a weekly basis. If company cannot satisfy a minimum qualification level, it has to purchase Heineken products through wholesalers, who accumulate orders from numerous smaller companies. Being able to cumulate the required purchase volume, these wholesalers make purchases directly from the Heineken Group and redistribute down the supply system.

Therefore, in terms of bargaining power distribution, this is a case of a medium wholesale company reselling branded beer to two medium pubs, which reflects the situation of equal power participants.

Consequently, the two pubs chosen for analysis are situated next to each other in one street in the Saint-Petersburg city center. Both are buying certain amount of beer Q_i from the wholesaler at a price w_i per liter of product and then resell it in their point of sales at a price p_i per liter. It is assumed that they are serving the same sort of beer "Heineken" in exactly the same way, so that it is completely indistinguishable to consumers. Moreover, it is assumed that consumers make their buying decision based on the retail price, disregarding pubs' location (as they are situated next to each other), reputation, interior design, etc. Nevertheless, the consumers are characterized by a certain degree of loyalty to one of the pubs, as this is an important notion for the foodservice industry.

From the interview with a company manager (2015), a small-to-medium pub in the city center is able to sell up to 1000 liters of one specific well-known brand of beer per week. Therefore, as beer is an FMCG product, it is assumed that all beer ordered from the wholesaler is realized during the same week. Thus, total potential demand for Heineken sort of beer in that specific place of the city is equal to 2000 liters per week. Placing more efforts in promotion, first pub enjoys a slightly bigger market share $k_1 = 0.6$, compared to the rival's $k_2 = 0.4$. Therefore, order quantities are $Q_1 = 1200$, $Q_2 = 800$ liters of Heineken per week.

The retail price p of a Heineken beer is 180 Rub per 0,33 liters, which makes it 540 per liter in retail. This subsumes the potential market value θ to be equal to 1 080 000 Rub per week. At the same time bar's marginal costs c equal approximately 150 000 Rub. per week, while wholesaler costs to fulfill the order are 190 Rub. per liter of Heineken. Then, $s_1 = 228\,000$ Rub. and $s_2 = 152\,000$ Rub. per week.

Concerning market elasticity γ, consumers are not very likely to switch between small pubs in case of price decrease, as there is a significant percentage of loyal customers in the target audience. Therefore, γ is assumed to be equal to 0.3, reflecting considerably low market elasticity. On the other hand, price sensitivity δ tends to be medium, as the target audience seems to be not very price-sensitive to out-of-home FMCG products, and therefore equals to 1.

In terms of bargaining power, equal power distribution is reflected in contract parameters in a following manner. Thus, the distribution of profit between the wholesaler and the pubs is assumed to be 50% share of the wholesaler and 50% pub's share (ϕ). Similarly, quantity based discount (v) is 50% for the first pub and 40% for the second one. As for F_1, F_2, after modelling the wholesale contract, their initial values were set as 900 and 1000 Rub. accordingly, as to reallocate profit more evenly.

Table 13 below summarizes market conditions and contract parameters that would correspond to the described situation, with T standing for thousand rubles.

Table 13: Initial data set for Heineken case

θ	γ	δ_1	δ_2	k_1	k_2	c	s_1	s_2	ϕ_1	ϕ_2	v_1	v_2	F_1	F_2
1080T	0.3	1	1	0.6	0.5	150T	228T	152T	0.5	0.5	0.5	0.4	900	1000

Main results are summarized in Table 14, in thousand rubles.

As it can be clearly seen from the Table 14, supply network in terms of total profit will be optimized under the wholesale or two-part tariff contract. While the

Table 14: Profit function values summary for Heineken case

	π_1^*	π_2^*	π_{s1}^*	π_{s2}^*	π_s^*	Π_1^*	Π_2^*	Π^*
Wholesale contract	11789	4466	2439	2012	4452	14229	6497	20708
Revenue-Sharing	6956	3270	6956	3270	10227	13913	6541	20454
Quantity-discount	11180	5620	2412	1155	3567	13833	6776	20609
Two-part tariff	10889	3466	3339	3012	6352	14229	6497	20708

wholesale contract better suits the interests of the stronger pub, wholesaler would favor revenue-sharing contract. At the same time, relatively weaker pub would choose quantity-discount contracts. Most probably, this indicates that final decision on the contract type will be made based on specific market conditions or negotiation power, as the cases of completely even bargaining power distribution are extremely rare.

Nevertheless, from the results of modeling on a numerical example, it was conducted that quantity-discount and wholesale contracts unevenly distribute total supply network profit. Namely, these contracts allocate a bigger share of profit to the dealers. Therefore, the most balanced contract is two-part tariff, in case the tariff rates are chosen appropriately. In turn, revenue-sharing contract as well divides total supply network profit in a balanced way, according to predefined negotiated shares.

Let's examine how these contracts will be able to coordinate supply network relationships in foodservice industry under changing market conditions. The behavior of profit function in case of changes in pubs' marginal costs c from 100 to 200 thousand Rub. would look as follows (Fig. 13 - 15):

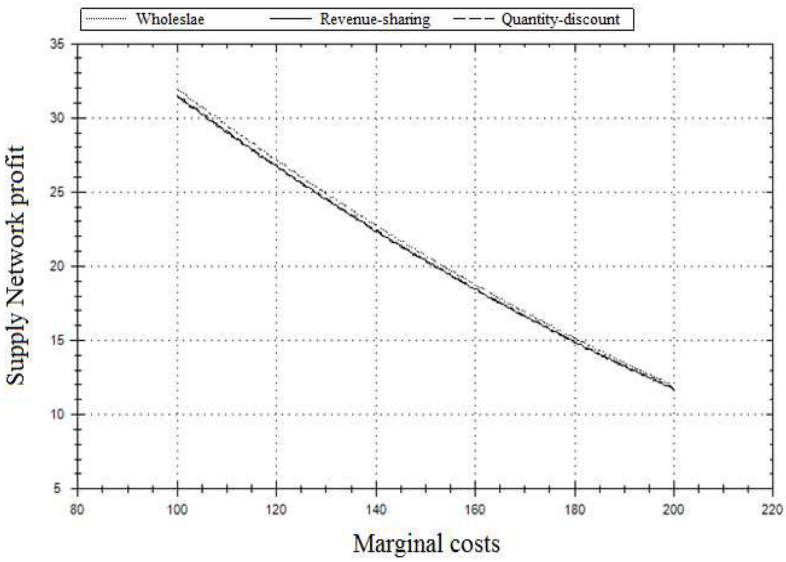

Fig. 13: Heineken supply network profit function under volatility of marginal costs

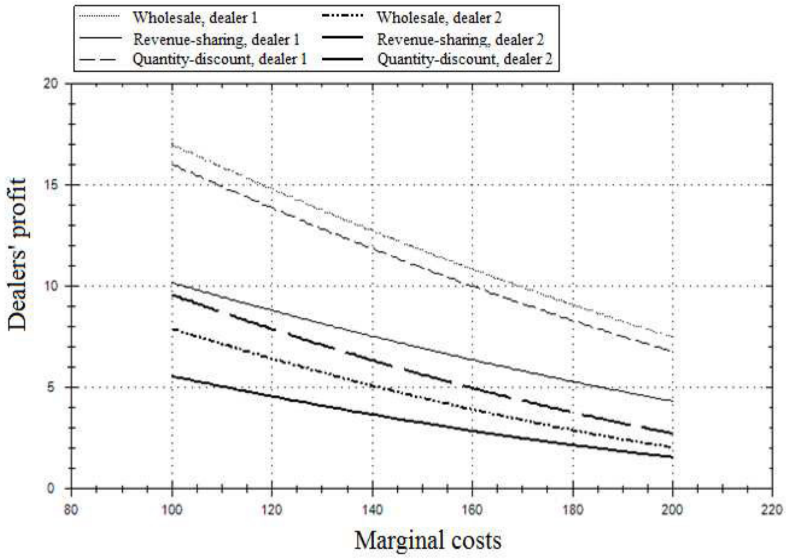

Fig. 14: Bar profit function under volatility of marginal costs

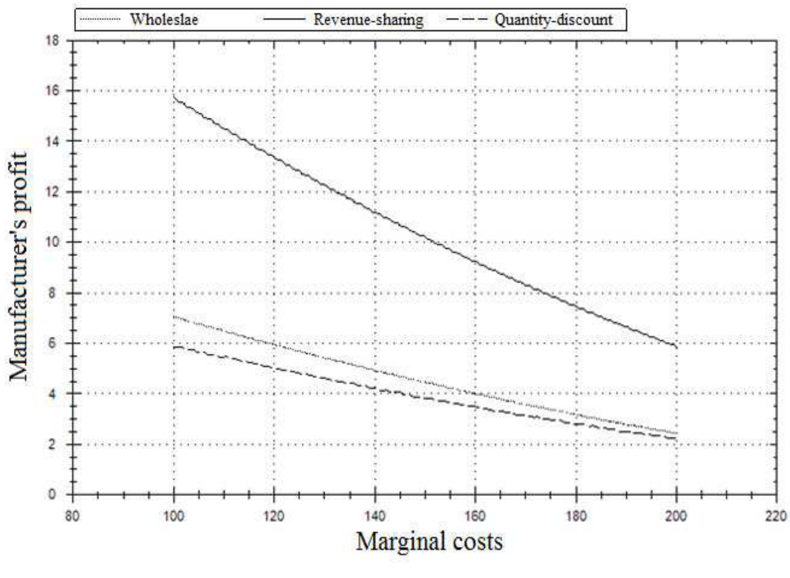

Fig. 15: Beer wholesaler profit function under volatility of marginal costs

The behavior of profit function in case of changes in the wholesaler's operating expenses to fulfill the orders of the pub with a higher market share, growing from 200 to 250 thousand Rubles, would be as follows (Fig. 85 - 87):

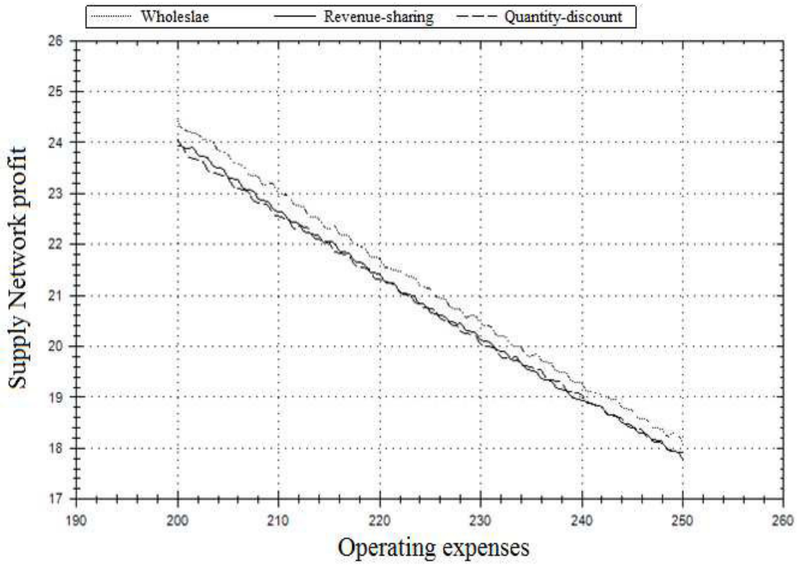

Fig. 16: Heineken supply network profit function under volatility of operating expenses

In line with the results obtained from a numerical example, from the pub's perspective, revenue-sharing contract shows the smallest sensitivity for costs escalation, while beer wholesaler would favor wholesale and quantity-discount contracts as providing more safety.

Graphs, showing the reaction of the profit function on changes in market elasticity and consumer price sensitivity δ can be found in the Appendix 5 (Fig. 88 – 90). Supply network profit proved to be the most stable in terms of profit towards changes in the market conditions under quantity-discount and wholesale contracts. Nevertheless, under the wholesale and two-part tariff contracts, profit dynamics for the beer wholesaler and pubs proved to be completely the opposite. Such a situation seems to be inacceptable in case of equally distributed bargaining power. Therefore, revenue-sharing contract is more reasonable, as the wholesaler's and pubs' profit functions follow the same tendencies.

The last set parameters that might influence the profit function is market concentration, reflected by pubs' market shares k_1 and k_2. Assume, that the market share of the first pub is increasing from 0.3 to 0.8, while market share of the second dealer is decreasing accordingly from 0.7 to 0.2. Thus, the behavior of profit function in case of changes in the market shares would be as follows (Fig. 19 - 21):

The observed dynamics again prove that the more severe is competition, the lower is total supply network profit, no matter what type of contract is applied. In other words, supply network profit increases proportionally with the increase in

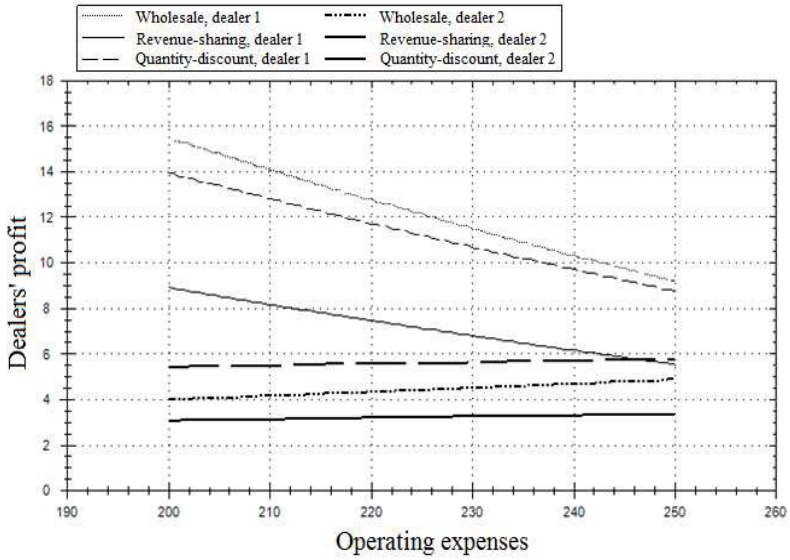

Fig. 17: Bar profit function under volatility of operating expenses

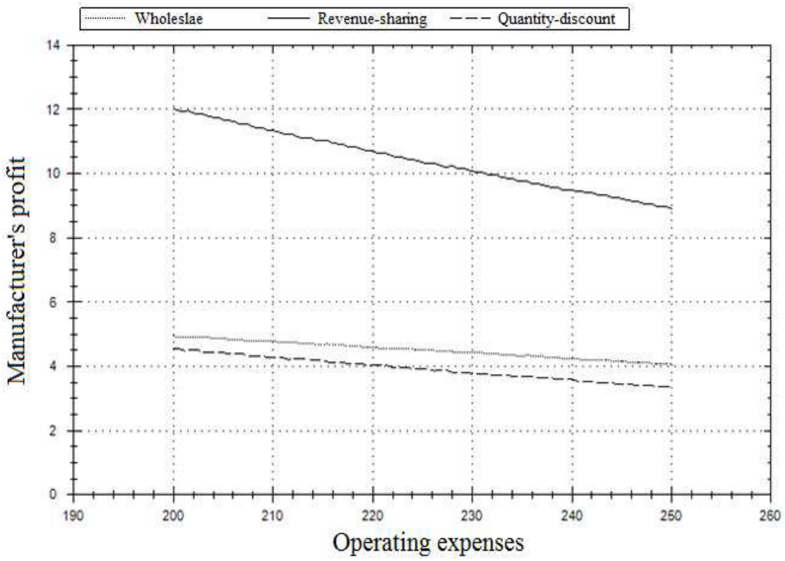

Fig. 18: Beer wholesaler profit function under volatility of operating expenses

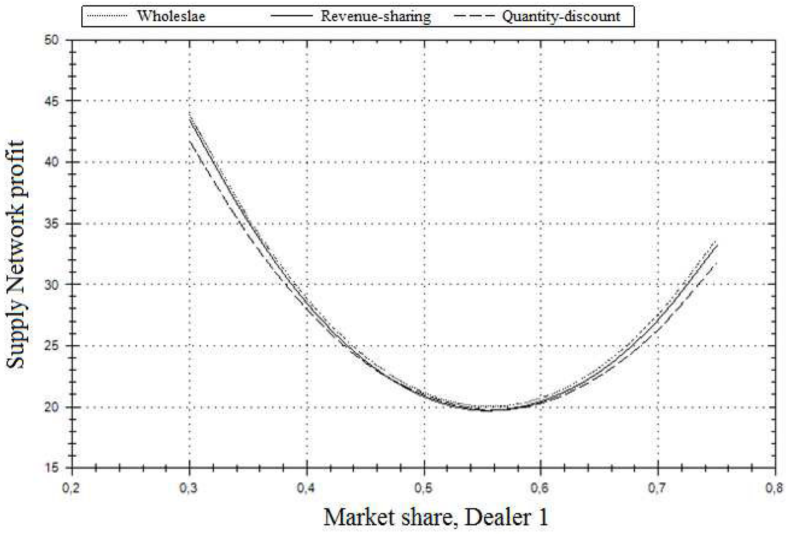

Fig. 19: Heineken supply network profit function under changes in market concentration

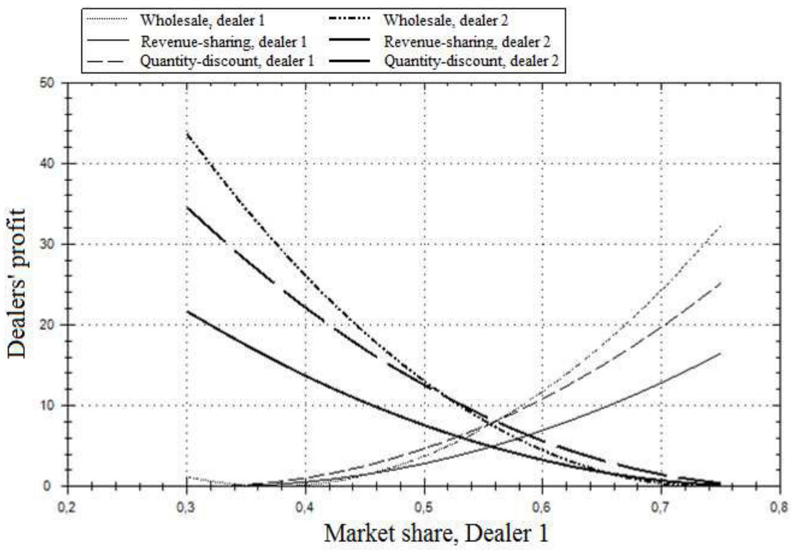

Fig. 20: Bar profit function under changes in market concentration

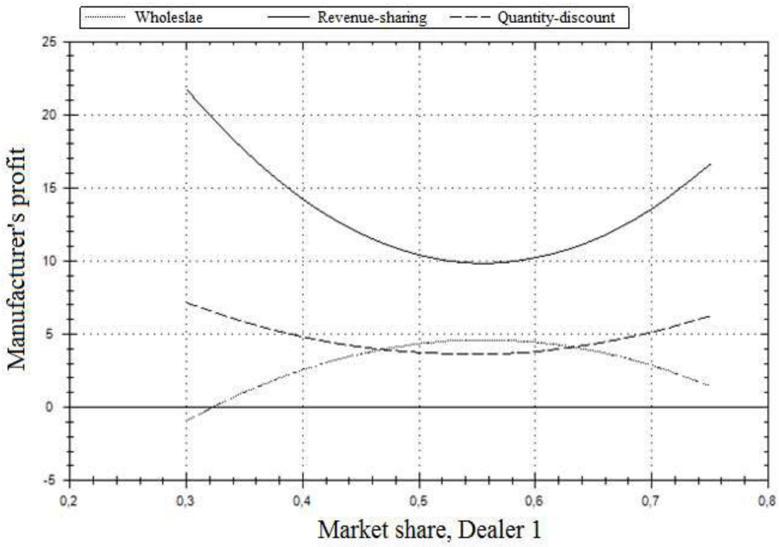

Fig. 21: Beer wholesaler profit function under changes in market concentration

market concentration. For the bars, revenue-sharing contract is the most stable in terms of profit allocation and revenue streams, while for the beer wholesaler optimal choice depends on the market concentration. Thus, in a situation of equally small and relatively weak bars, the optimal decision for the beer wholesaler would be to operate under a two-part tariff contract, however, in a situation when one bar has a sufficiently bigger market share, for the wholesaler revenue-sharing contract is more favorable.

As for the case of a Heineken beer wholesaler reselling products to different pubs of Saint-Petersburg, optimal choice is a revenue-sharing contract, as it allocates the supply network profit exactly according to the negotiated shares. In addition to that, revenue-sharing contract is less sensitive to volatile market conditions and preserves the same tendencies for both bars and the wholesaler, which should be taken into account assuming that parties have equal bargaining power. Bars should also favor revenue-sharing contract, as it provides them with the most stable revenue streams, ensuring a considerate protection in case of costs escalation.

11.6. ProtechDry Case Study

This is a case study based on the data of the year 2015, which was obtained from the interview with a senior manager of ProtechDry company on 11.03.2015 for Integrated Marketing Communications course in Nova SBE, Portugal. As an additional source of information, ProtechDry and Impetus Group reports and financial statements for the years 2014-2015 were studied. Detailed information can be found in the research paper "ProtechDry Integrated Marketing Communications" (M. Koloreva, N. Kowalczyk, T. S. Baena, F. M. de Mello, M. B. Moura Costa, 2015).

ProtechDry is a Portuguese brand that belongs to Impetus Group, specializing in the production of Cut and Sew and seamless products. ProtechDry is an innovative solution, developed in 2010 by the Impetus R and D department and launched in the Portuguese market as a separate entity. ProtechDry is ultra-absorbent, washable and anti-odor underwear that was specially designed for people with light incontinence and is supposed to replace the need of using pads.

As ProtechDry is legally separated from their parent company, they had to develop their own distribution network, not connected to the one used by Impetus Group. As a strategic decision, for the past few years, ProtechDry was sold alongside other incontinence products through grocery retail channels. Grocery retail in Portugal is heavily dominated by domestic players, with few international companies operating in the market. The two largest chain retailers are Sonae Modelo Continente and Jeronimo Martins, which together captured a substantial 36% share of the overall value sales in grocery retail in Portugal during 2014.

This is one of the perfect examples of unequal bargaining power distribution, when a small unknown brand faces huge retail chains, which completely dominate the market and therefore are able to set their own rules.

According to the ProtechDry manager (2015), big retail chains buy small quantities Q_i of ProtechDry underwear for placing it on shelves at a price w_i per package. Then retailers resell the product in the stores at a price p_i per package. As the brand is new to the market, retailers do not buy any sufficient quantities for storage, therefore, it is assumed that they sell everything they buy and salvage value equals to zero.

According to the company report, in 2014 ProtechDry has sold 12 000 packages of underwear, which is taken as combined retailers order quantity Q. Concerning the actual retail prices, there is an even price of 24.99 euros per package, which is set by the company and does not vary over different stores and retail chains. 23% of the price is due to Value Added Tax, the retailer margin is about 7 euros and the distribution costs account for 3.5 euros per unit ($c = 3.5 * Q$). The costs of materials and production compose 4 euros per unit of product ($s = 4 * Q$). The contribution margin of is around 5.9 euros.

For the purposes of the current study, potential market size θ equals in units to order quantity Q and, therefore, is estimated to be around 299 880 euros. Concerning market elasticity γ, consumers in Portugal are extremely likely to switch between retailers in case of lower prices offered by any competitor. Thus, γ is assumed to be equal to 0,8 to reflect this situation. At the same time, price sensitivity δ tends to be medium, as compared to its competitors ProtechDry is in a category of luxury goods, characterized by less price-sensitive consumer audience. Moreover, as this is a niche product serving very specific need of people with light urinary incontinence, which ensures that target customers are even less price sensitive, because the number of available solutions is very limited. Therefore, price sensitivity is set to be $\delta = 0{,}5$.

This is the case of supply network relationships between strong retail chains at one side and a small local brand on the other side, which is reflected in contract parameters. According to the data, received from the interview with the company manager (2015), the profit distribution between ProtechDry and retail chains in 2014 was as follows: 25% share of profit was allocated to ProtechDry and the remaining 75% was retailers' share (ϕ). At the same time, quantity based discount (v) is assumed to be 90%, as all the retail chains in Portugal apply heavy discount policies. As for tariff rates F_1, F_2, after modeling the wholesale contract, their initial values were set as 100 euros, although it seems to be very unlikely that a strong retail chain would pay any tariff to a small company.

As it was already stated above, the competition in grocery retail is very intense, which is reinforced by the fact that market is more or less equally shared between five to six retail giants. In this sense, it is assumed that dealers have equal market shares k_1 and k_2, while an additional analysis with different levels of market concentration will be presented later in this paragraph.

Table 14 below summarizes market conditions, and contract parameters that would correspond to the described situation, with t standing for thousand euros.

Table 15: Initial data set for ProtechDry case

θ	γ	δ_1	δ_2	k_1	k_2	c	s_1	s_2	ϕ_1	ϕ_2	v_1	v_2	F_1	F_2
300t	0.8	0.5	0.5	0.5	0.5	21t	24t	24t	0.75	0.75	0.9	0.9	100	100

Main results of possible profit allocations are summarized in Table 16, in thousand euros.

Table 16: Profit function values summary for ProtechDry case

	π_1^*	π_2^*	π_{s1}^*	π_{s2}^*	π_s^*	Π_1^*	Π_2^*	Π^*
Wholesale contract	3138	3138	5021	5021	10043	8160	8160	16320
Revenue-Sharing	4967	4967	1655	1655	3311	6622	6622	13245
Quantity-discount	5861	5861	2142	2142	4284	8004	8004	16008
Two-part tariff	3038	3038	5121	5121	10243	8160	8160	16320

As it can be derived from the Table 16, ProtechDry's supply network would be optimized under the wholesale or two-part tariff contracts, in terms of total profit. Nevertheless, it is quite obvious that those types of contracts allocate far too big share of profit in favor of a manufacturer, which is a small weak brand in this case. Retail giants, such as Continente and Jeronimo Martins, being able to dictate their own terms, would never accept such conditions. Therefore, most probably, a quantity-discount contract will be chosen for supply network optimization, as it shows much better results than revenue-sharing contract.

The behavior of the profit functions of the supply network, ProtechDry and the retail chains in case of changing retailers' marginal costs c and manufacturer's operating expenses s_i is presented in the Appendix 6 (Fig. 91 - 96). All types of studied contracts show little sensitivity for costs escalation, while an interesting observation is that due to the low consumers' price-sensitivity δ and high market elasticity γ, showing that many customers can be attracted by price reduction, profit decreases less intensively in a situation of increasing costs.

As for the influence of changes in price sensitivity δ_i, within the range from 0.3 to 2, the behavior of profit function would look as follows (Fig. 22–24):

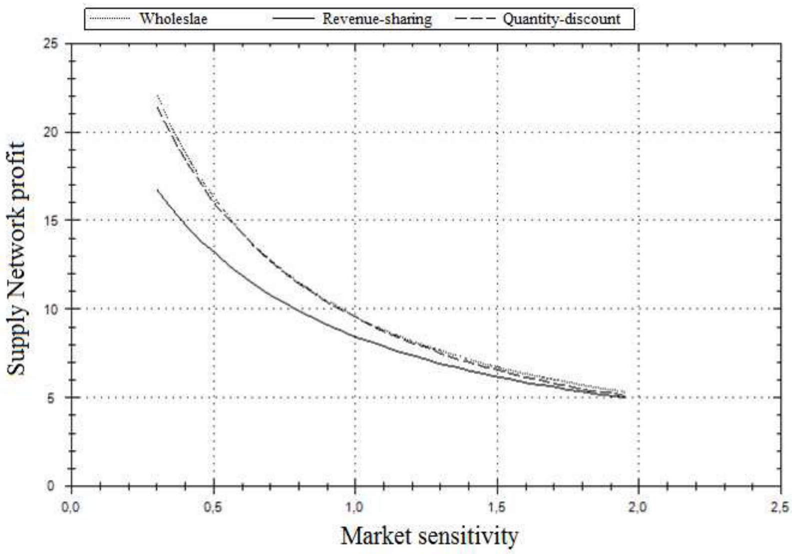

Fig. 22: ProtechDry supply network profit function under changes in price sensitivity

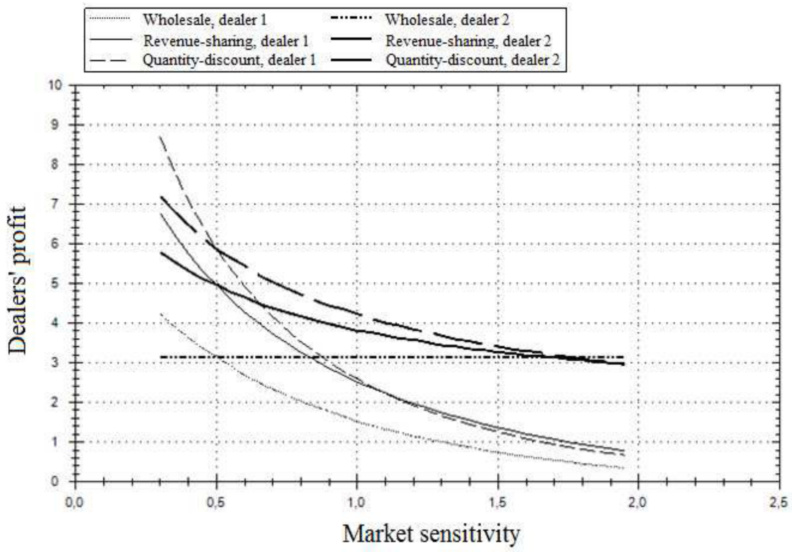

Fig. 23: ProtechDry dealers' profit function under changes in price sensitivity

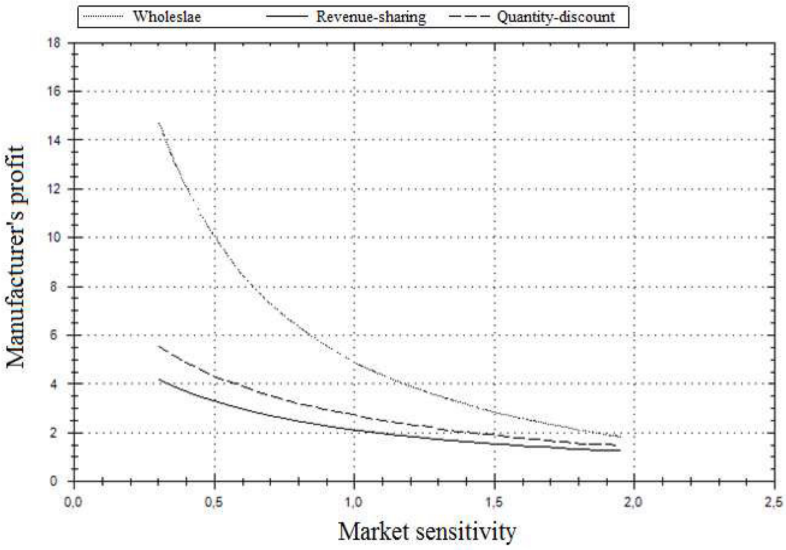

Fig. 24: ProtechDry profit function under changes in price sensitivity

ProtechDry, retail chains' and supply network aggregated profit functions in a situation of changing market elasticity γ, ranging from 0.1 to 0.9, would look as follows (Fig. 25–27):

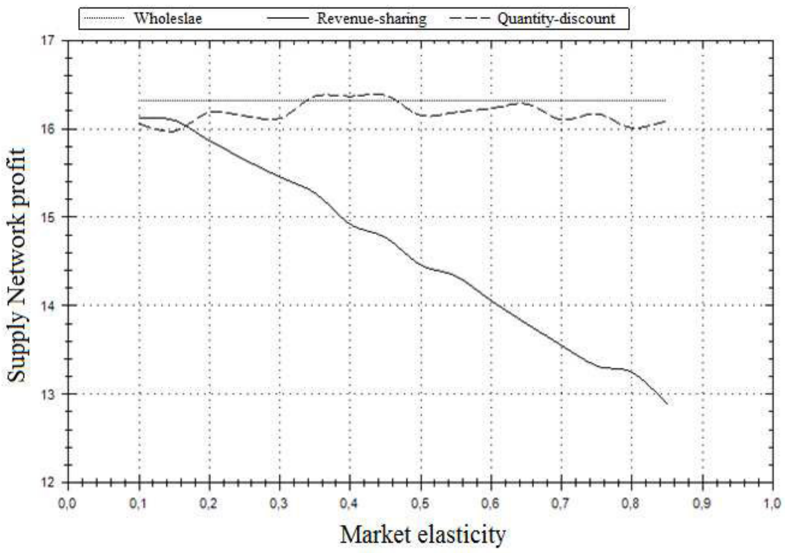

Fig. 25: ProtechDry supply network profit function under changes in market elasticity

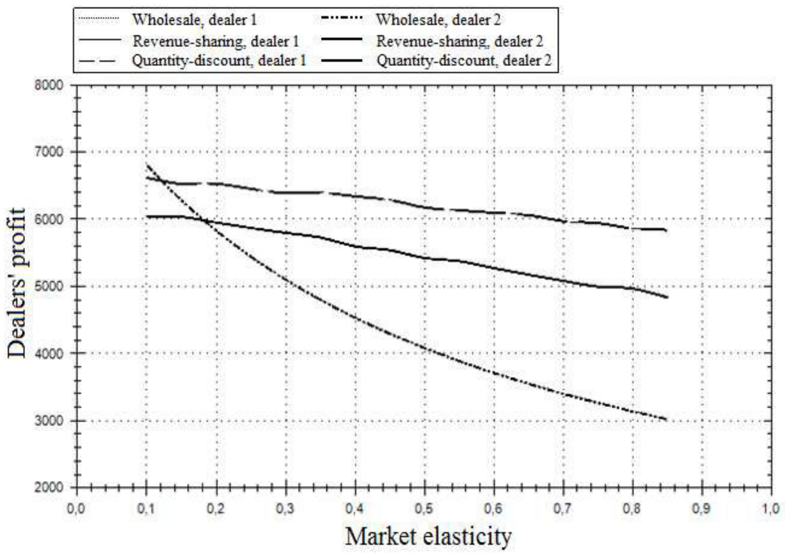

Fig. 26: ProtechDry dealers' profit function under changes in market elasticity

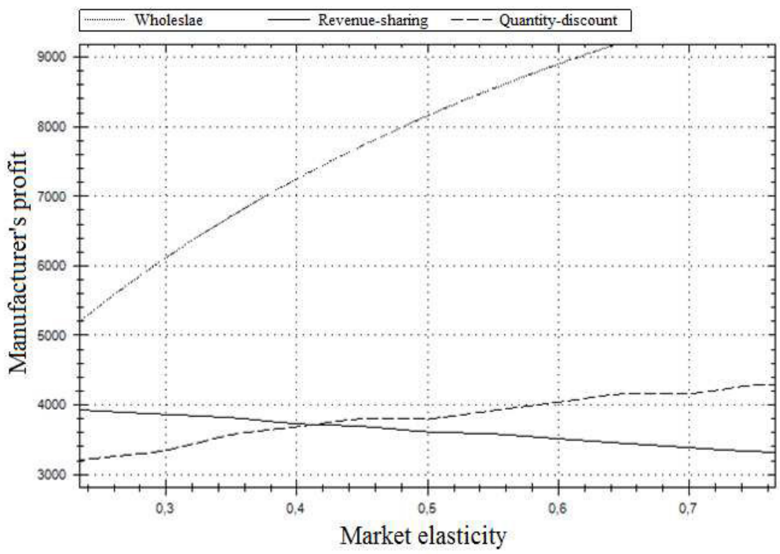

Fig. 27: ProtechDry profit function under changes in market elasticity

Conclusions that can be derived from the Figures 25 - 27, follow the ones made earlier in this paragraph. With increase in market elasticity γ, revenue-sharing and quantity-discount contracts react less intensively than wholesale and two-part tariff contracts, which would attract cautious retailers. At the same time, quantity-discount contract provides some growth in total supply network profit due to slight increase in ProtechDry's profit. Thus, if a retailer runs a risk of losing a part of its market share, optimal choice would be application of a quantity-discount contract, while if a retailer is expecting some growth, revenue-sharing contract would enforce a steeper profit growth. For ProtechDry quantity-discount contract would serve better in highly elastic market.

Considering the possible changes in retailers' market shares k_1 and k_2, from 0.3 to 0.8 for the first retail chain and from 0.7 to 0.2 for the second accordingly, the behavior of profit function would look as follows (Fig. 28 - 30):

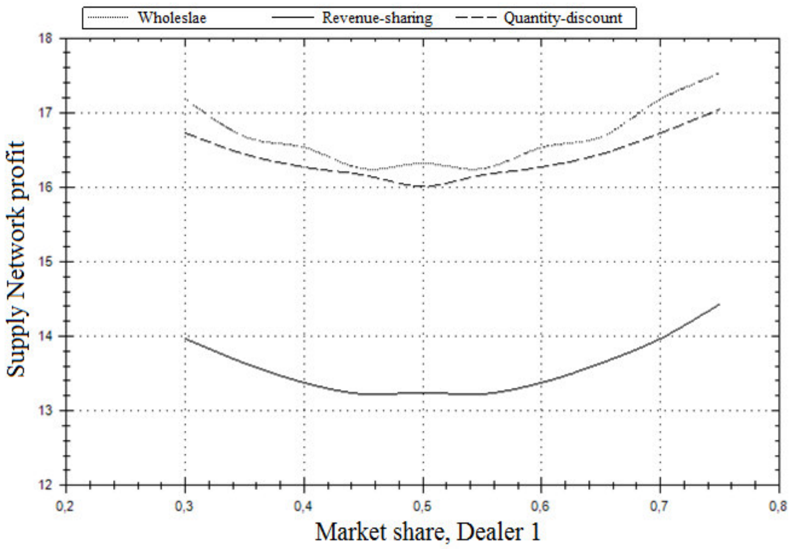

Fig. 28: ProtechDry supply network profit function under changes in market concentration

Following all the other studied cases and examples, with an increase in a market share, profit of the associated retailer is increasing as well, while its competitor's profit is decreasing. Moreover, the more severe is the competition the smaller is total ProtechDry supply network profit. In any case, quantity-discount contract tends to allocate supply network profit in favor of dealers, no matter what costs and discounts were chosen during the negotiation process. This type of contract has some characteristics, which might be useful in a situation of powerful retail chains, especially when they are operating in a highly competitive market, like Portuguese.

Thus, quantity-discount contract would be an optimal choice for powerful retail chains in case of ProtechDry. Wholesale contract allocates too much profit to manufacturer, which is very doubtful to be accepted. At the same time, as opposed to the numerical example, revenue-sharing contract has shown the worst performance in a situation of strong retailers in terms of both total supply network profit and stability towards changing market conditions.

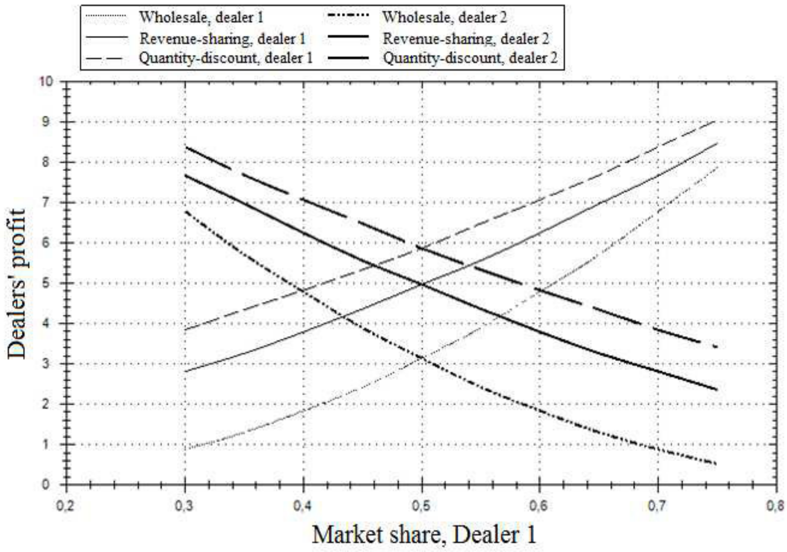

Fig. 29: ProtechDry dealers' profit function under changes in market concentration

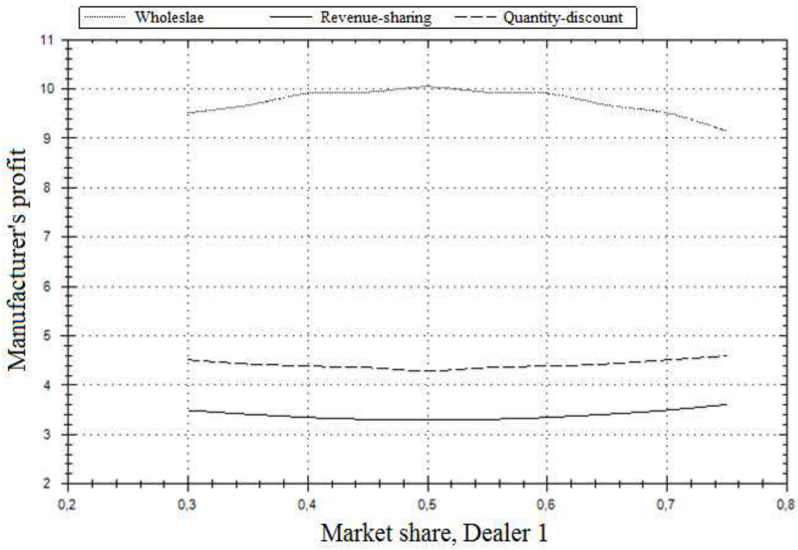

Fig. 30: ProtechDry profit function under changes in market concentration

Most probably, ProtechDry would not have a possibility to affect the choice of the contract type due to extremely low bargaining power. However, despite the fact of returning the lowest profits for the company in absolute terms, quantity-discount contract shows good performance in highly elastic market, providing ProtechDry with opportunities for faster growth.

12. Conclusion

This is the final section, which aims to give an overview of the conducted research and formulate the main conclusions. Hence, discussion concerns in turn main results of the study, limitations to those results and managerial implications.

The objective of the article was methodology improvement of contract selection in cooperative supply networks for achieving higher supply network economic performance, where economic performance stands for total supply network profit. Supply network management is a new line of research within a broader field of supply chain management. Therefore, as a starting point of contract decision-making methodology improvement, the conceptual understanding of supply network phenomena was extended. In general, supply network can be described as a system comprised of individual supply chains, united by an integrated flow of products, services, finances and (or) information, provided that at least two of its members are direct competitors. As a result of defining supply network conceptual framework, the standard newsvendor setting was improved and adapted in order to reflect the situation of competing retailers (dealers).

Nevertheless, in light of adding a new factor of competition, the problem of supply network optimization through coordination could be solved only partially, as the existing methodologies only allow achieving coordination of distinct supply chain pairs (manufacturer-dealer) separately. This suggests that there exists some space for methodology improvement.

The stated objective was successfully achieved by the application of a new supply network setting to the supply chain cooperative game, which was solved regarding the new initial input in the form of competing retailers. Thus, the methodology of contract selection in a supply chain or a set of supply chains was widened by devising a mechanism that allows not only coordination of distinct simple supply chains but also coordination at a system-wide level in the context of competing retailers (dealers). In addition to that, the improved methodology embraces the notion of bargaining power and enables building different scenarios based on the estimation of the negotiation power disposed by the supply network members.

Based on the game-theoretical and mathematical modeling, resulting in the improved methodology, a quantitative software tool was developed aimed at facilitation of methodology application. With the help of this tool, the improved methodology was tested on the real-life cases, matching three main alternatives of bargaining power allocation: strong manufacturer, strong retailers (dealers) and equal power participants. All three cases showed potential for supply network economic performance improvement, in terms of increasing total profit of the system itself, as well as individual profits of each supply network member, which can be achieved through methodology application as a means for coordinating contract selection.

The main results of the study can be summarized as follows:

1. The standard newsvendor model was improved and adapted in order to reflect the situation of competing retailers (dealers), referring to the supply network concept;
2. The methodology of coordination contracts decision-making was improved by devising a mechanism for contract selection for the case of multi-echelon supply network with two competing dealers enabling coordination at a system-wide level;
3. Economic performance improvement potential of developed contract decision-making methodology was empirically proved by testing it on the real-life cases of Audi Russia, Heineken Saint-Petersburg and ProtechDry Portugal;
4. For each case a set of recommendations on contract selection for optimizing system-wide performance was formulated, giving attention to the bargaining power and, therefore, decision priority of all members.

Nevertheless, these findings have some important limitations that are not to be neglected, as they are primarily related to the applicability of the developed methodology in different circumstances.

First of all comes the group of the limitations originating from the newsvendor setting, a supply chain model widely used for studying coordinating contracts. Supply network model developed in the present master thesis was designed as an improved and widened newsvendor model, assuming that retailers compete with each other. Therefore, application of the studied model is limited to one product and one period. This means that if a company sells a range of different products down the supply chain, the improved methodology of contact selection would be able to find optimal solutions for each product separately as if those were separate supply networks with no possibility of interconnections, combinations, cross discounts, mutual contracts, etc. The same is true for the time horizon.

In addition to that, another limitation originating from the newsvendor setting is assumption of perfect information throughout the supply network. Hence, it is not clear, whether the model can be successfully applied in case of incomplete information or in case of the opportunistic behavior, when participants are trying to use their access to private information as a way to receive an advantage.

Moreover, the developed model does not cover the situation of products from competing companies (manufacturers) being distributed through the same retailers. Influence of these products should be studied more thoroughly in order to derive any conclusions on the possible effects concerning the methodology of contract decision-making.

Second set of limitations is related to the rules of market competition applied in the model. For the purposes of the current study it was assumed that retailers set their prices following the rules of Bertrand competition, which limits model application to the markets to a greater extent satisfying these conditions. As a direction for future research, studied methodology can be improved further by application of Cournot competition rules. In addition to that, directly linked to Bertrand competition rules used in the model, come limitations of specific contract types. As due to these rules, the methodology considers only coordinating contracts belonging to a group of price-dependent contracts. Therefore, it would be interesting to study also those contracts related to quantity-dependent group.

However, present research paper derives important theoretical and practical implications.

From the theoretical perspective, this paper develops the research related to the supply network conceptual framework, originated from Bryant (1980) and then developed by Deneckere, Marvel and Peck (1997). Most directly related to the current research are papers of Birge et al. (1998) Carr et al. (1999) and van Mieghem and Dada (1999), who consider the special case of the supply network model with two competing retailers.

Research originality of the paper is granted by an applied procedure that fills in the research gap in papers devoted to development of specific contract selection mechanisms, which are applicable in real life situations. The particular novelty of the research lies in in the improved methodology of contract selection, which is able to achieve a system-wide coordination under the conditions of competing retailers (dealers). Thus, the paper widens the field of supply chain coordination, however, upscaling it to coordination of supply systems, as a broader scope of relationships between companies.

Theoretical implications of the research therefore include extended concept of supply network phenomena and an improved methodology of contract decision-making for a specific case of competing retailers, which was tested and proved to be applicable to the real-life situations. The studied methodology opens a broad area for future research, as it might be improved further in a range of different courses, such as including additional coordinating contracts, applying different competition rules, extending the time horizon or product range, etc.

In the array of managerial implications the most important result is an improved methodology of contract selection and a quantitative software tool, that enable companies to choose a specific contract type in order to maximize supply network economic performance as well as to distribute total profit in a specific desired way. The improved methodology by the means of a software tool was tested on real-life cases and proved to give corresponding results, as well as demonstrated a significant economic performance improvement potential.

Resulting from the case study analysis, which was encompassed by bargaining power distribution between the supply network members, it was noted that the more power is concentrated in the hands of one supply network member and the more he is able, in terms of negotiating abilities, financial resources and personal involvement, to integrate the entire system in pursuing his own goals, the more efficient this supply network becomes from the perspective of total profit. This observation underlines the idea of importance of coordination mechanisms application as a means to improve supply chain efficiency and sustain company's competitiveness in the modern market economy.

Acknowlegments The authors are grateful to N. A. Zenkevich for research guidance and useful discussions on the subjects.

Appendix

1. First Appendix

The behavior of profit function in case of changes in marginal costs c from 7 to 15 would look as follows (Figures 31–33):

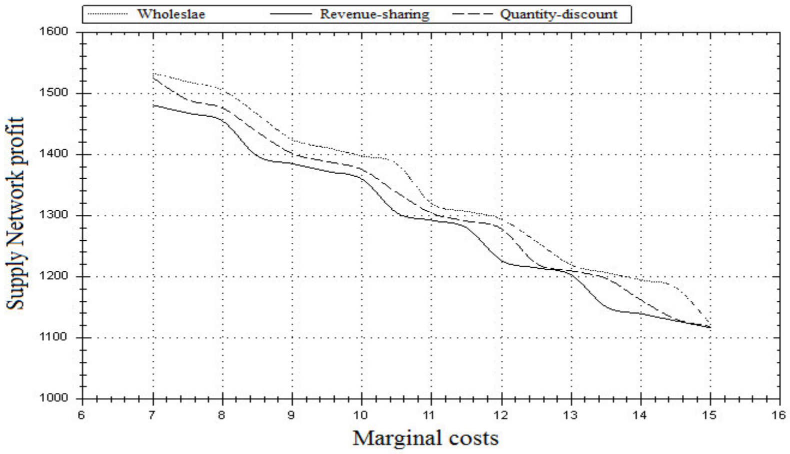

Fig. 31: Supply network profit function under volatility of marginal costs

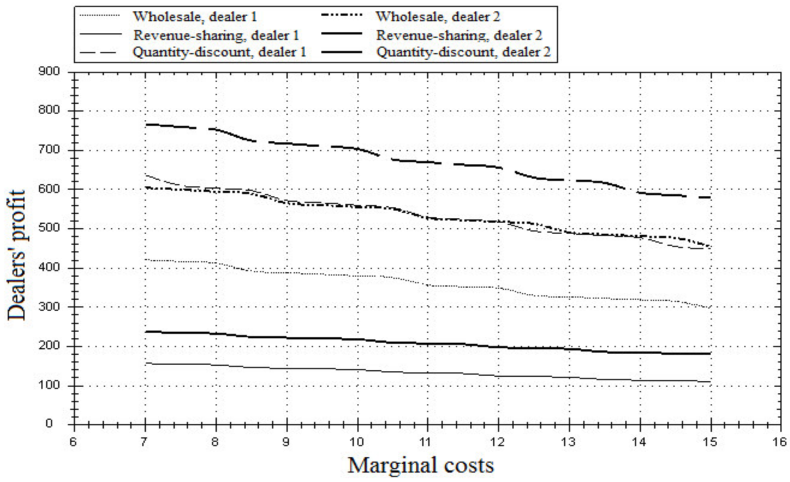

Fig. 32: Dealers' profit function under volatility of marginal costs

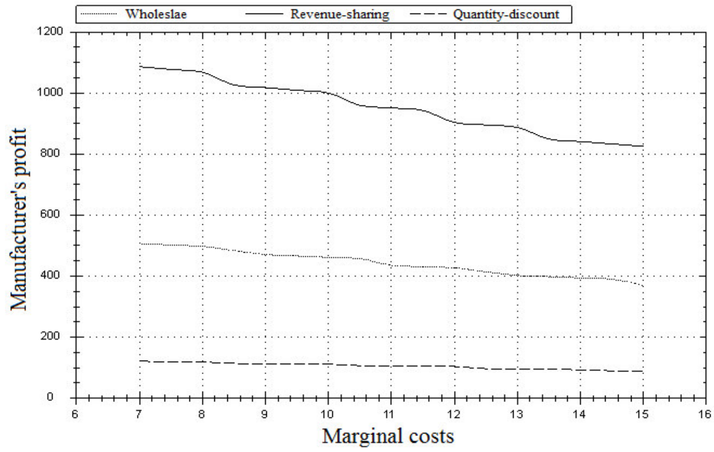

Fig. 33: Manufacturer profit function under volatility of marginal costs

The behavior of profit function in case of changes in manufacturer's operating expenses to fulfill the orders of, say, dealer 1, s_1 from 30 to 40 per unit of good would be as follows (Figures 34–36):

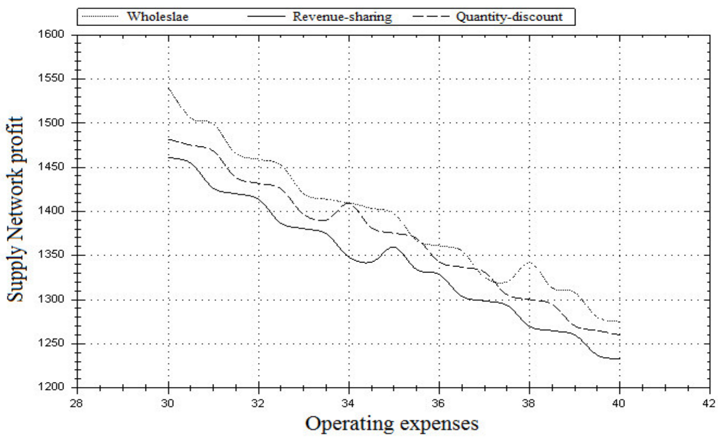

Fig. 34: Supply network profit function under volatility of operating expenses

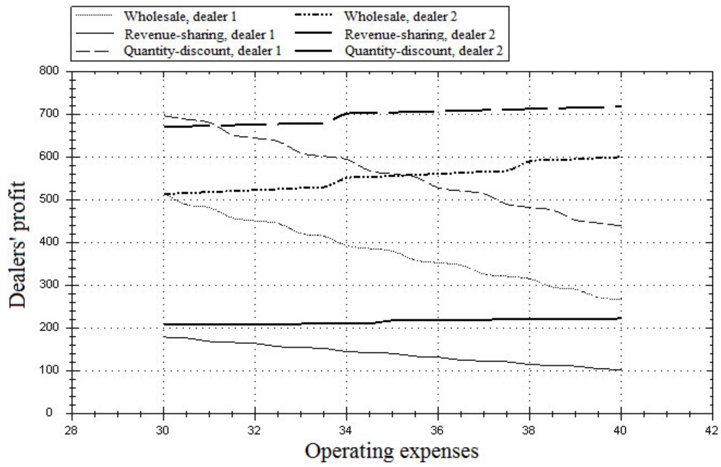

Fig. 35: Dealers' profit function under volatility of operating expenses

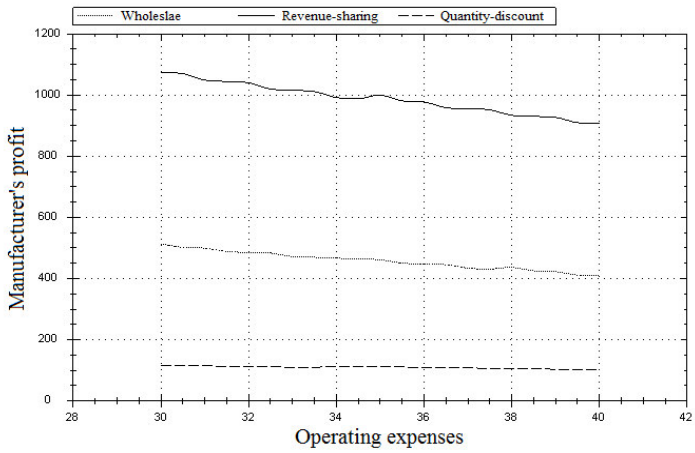

Fig. 36: Manufacturer profit function under volatility of operating expenses

The behavior of profit function in case of changes in market elasticity γ from 0.1 to 0.9 would look as follows (Figures 37–39):

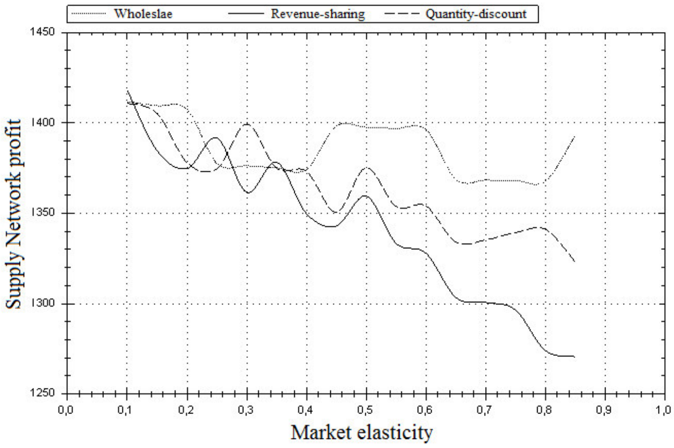

Fig. 37: Supply network profit function under changes in market elasticity

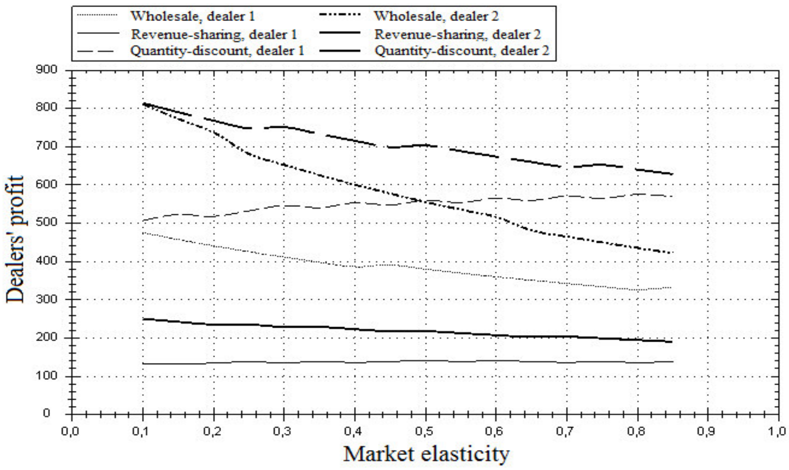

Fig. 38: Dealers' profit function under changes in market elasticity

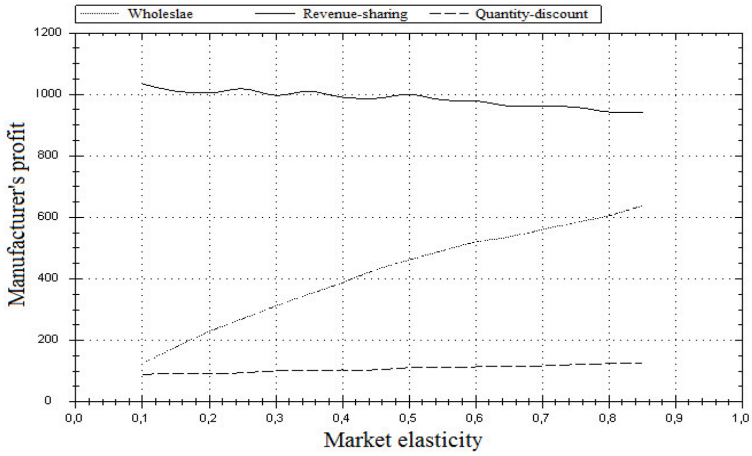

Fig. 39: Manufacturer profit function under changes in market elasticity

The behavior of profit function in case of changes in price sensitivity for dealer 1 products k_1 from 0.3 to 2 would look as follows (Figures 40–42):

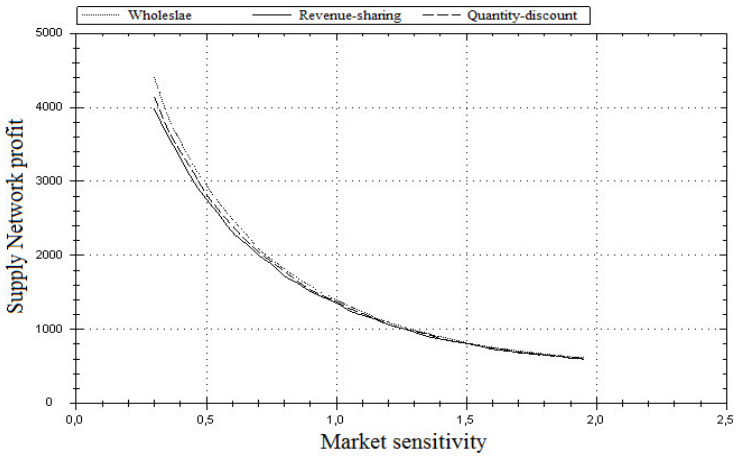

Fig. 40: Supply network profit function under changes in price sensitivity

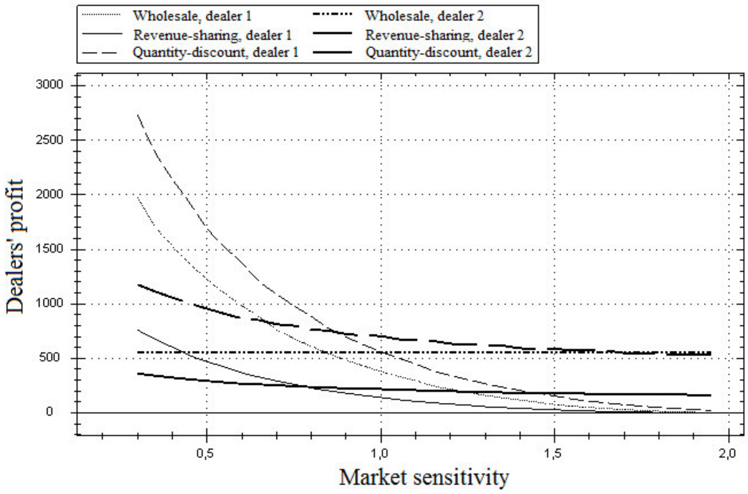

Fig. 41: Dealers' profit function under changes in price sensitivity

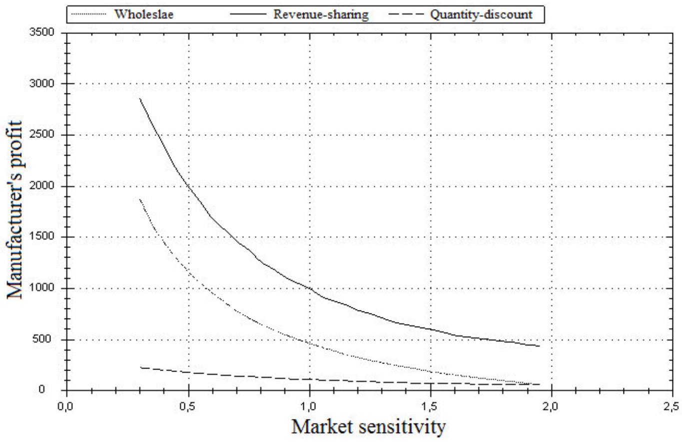

Fig. 42: Manufacturer profit function under changes in price sensitivity

The behavior of profit function in case of changes in dealers' market shares would be as follows (Figures 43–45):

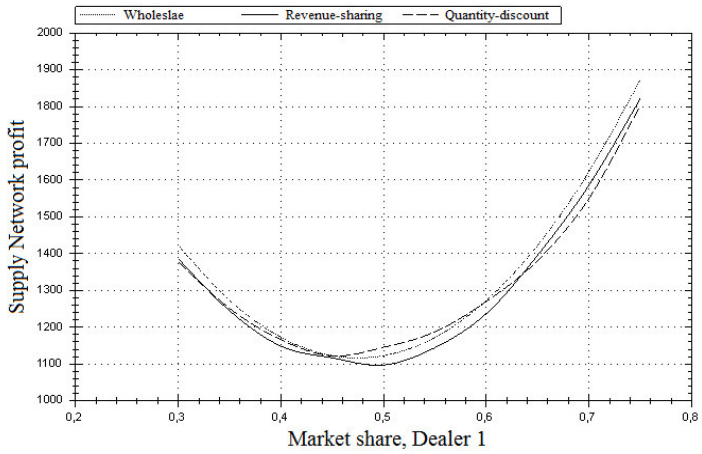

Fig. 43: Supply network profit function under changes in market concentration

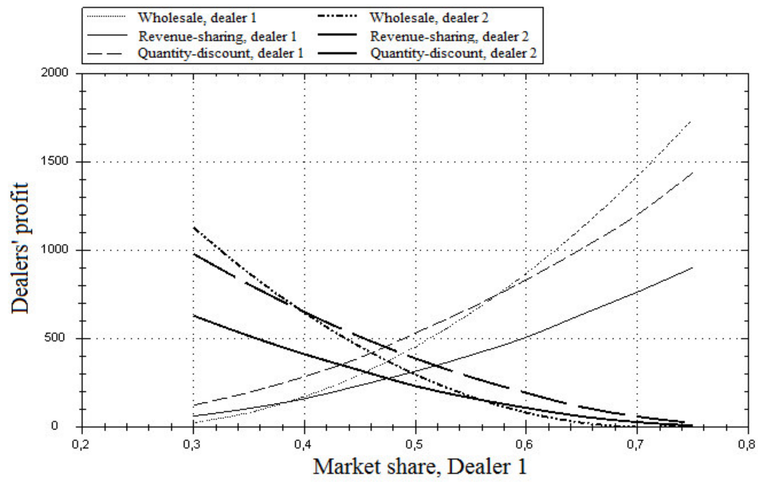

Fig. 44: Dealers' profit function under changes in market concentration

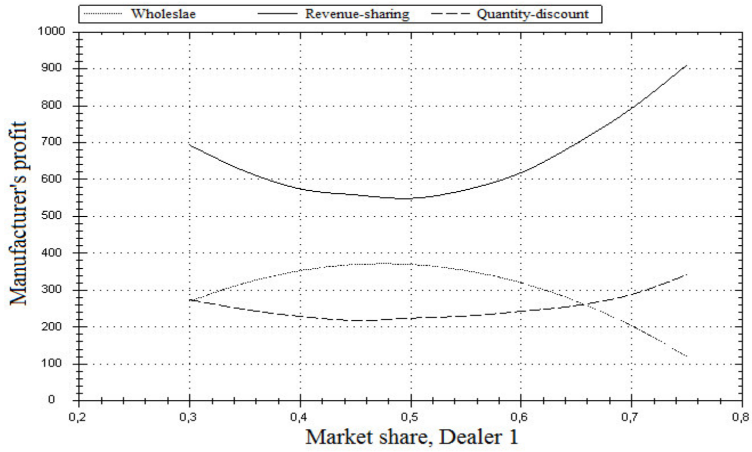

Fig. 45: Manufacturer profit function under changes in market concentration

2. Second Appendix

The behavior of profit function in case of changes in marginal costs c from 10 to 20 would look as follows (Figures 46–48):

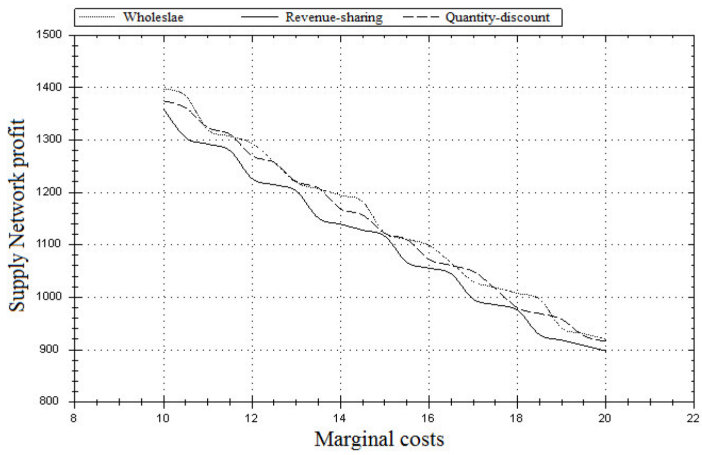

Fig. 46: Supply network profit function under volatility of marginal costs

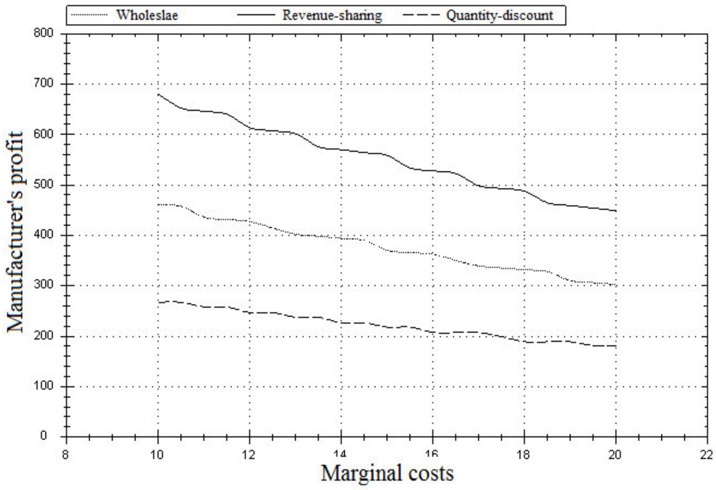

Fig. 47: Dealers' profit function under volatility of marginal costs

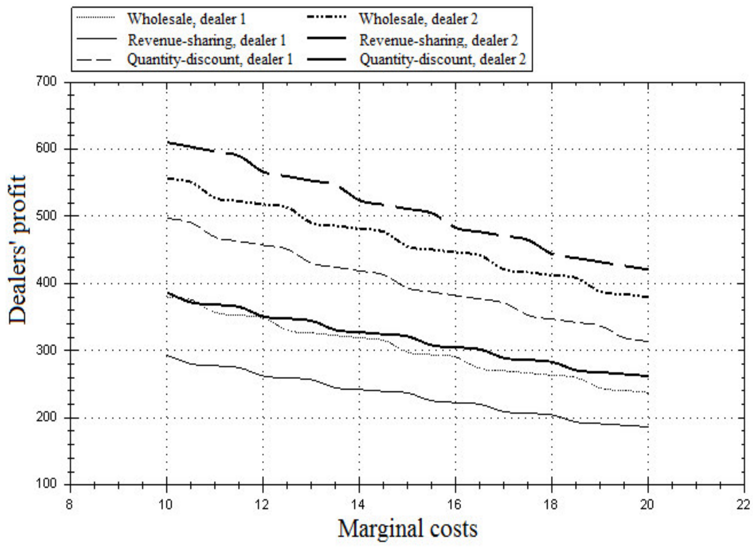

Fig. 48: Manufacturer profit function under volatility of marginal costs

The behavior of profit function in case of changes in operating expenses to fulfill the orders of dealer 1 (s_1) from 30 to 40 per unit of good would be as follows (Figures 49–51):

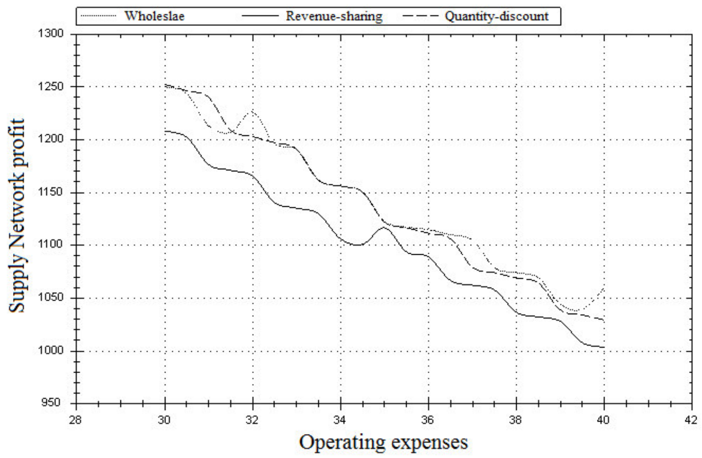

Fig. 49: Supply network profit function under volatility of operating expenses

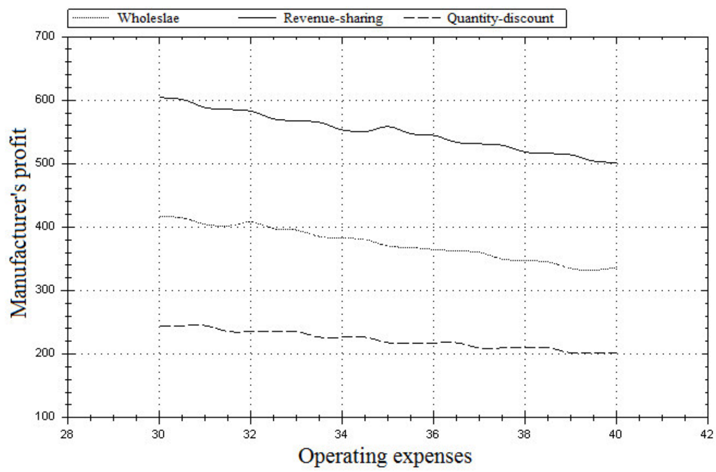

Fig. 50: Dealers' profit function under volatility of operating expenses

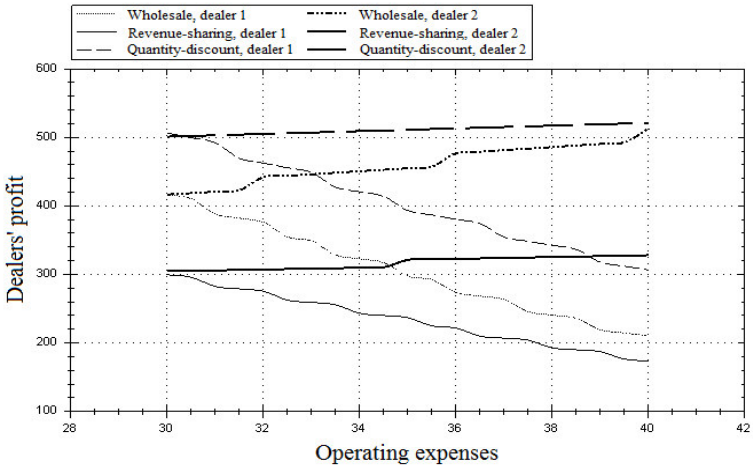

Fig. 51: Manufacturer profit function under volatility of operating expenses

The behavior of profit function in case of changes in market elasticity γ from 0.1 to 0.9 would look as follows (Figures 52–54):

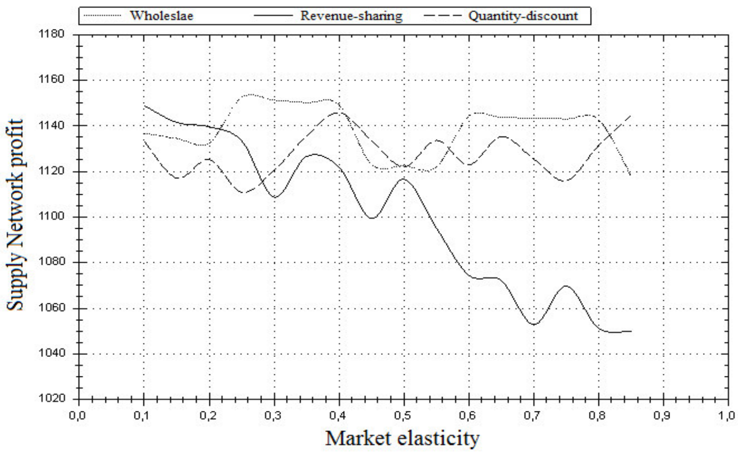

Fig. 52: Supply network profit function under changes in market elasticity

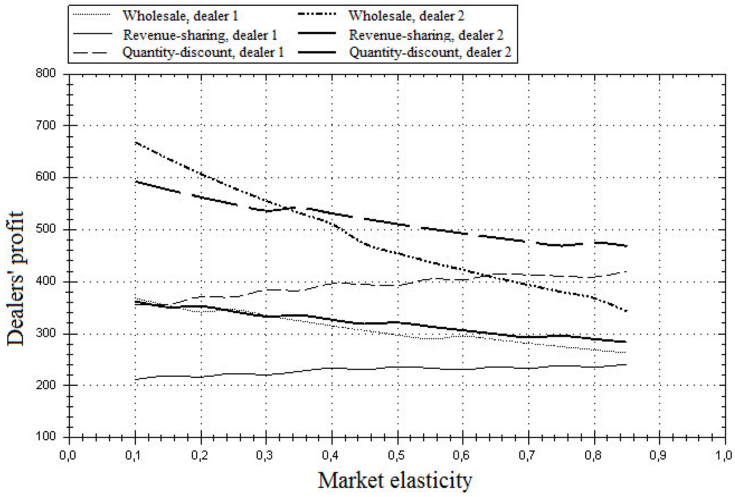

Fig. 53: Dealers' profit function under changes in market elasticity

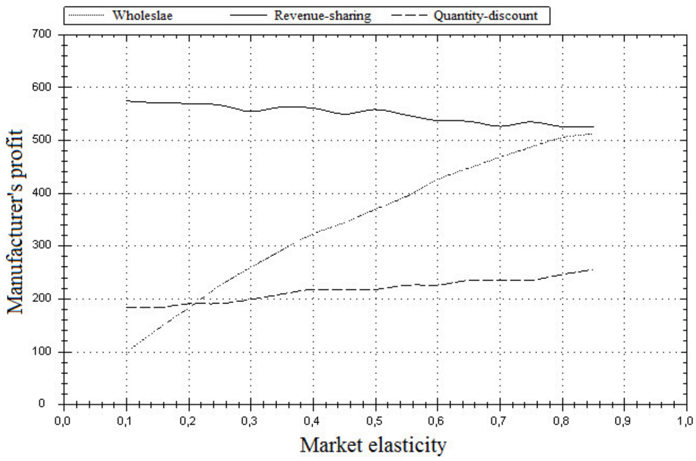

Fig. 54: Manufacturer profit function under changes in market elasticity

The behavior of profit function in case of changes in price sensitivity for dealer 1 δ_1 from 0.3 to 2 would look as follows (Figures 55–57):

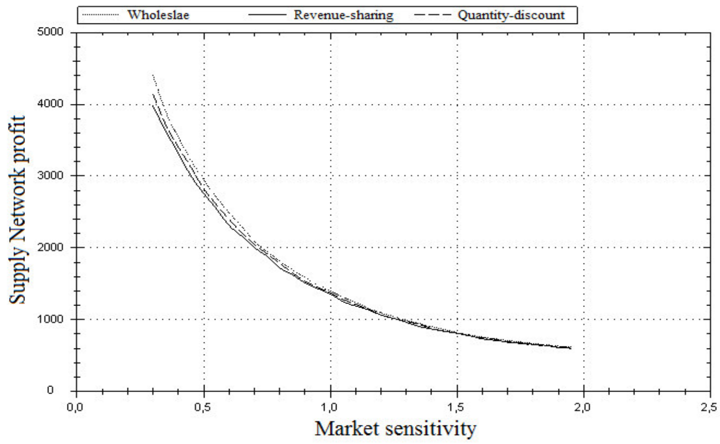

Fig. 55: Supply network profit function under changes in price sensitivity

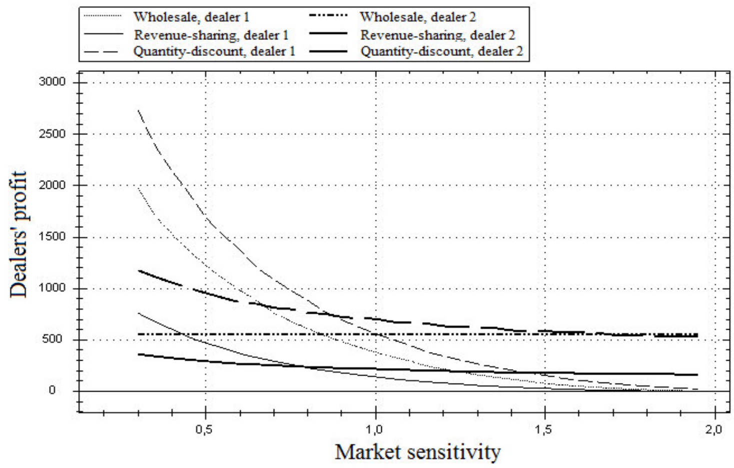

Fig. 56: Dealers' profit function under changes in market elasticity

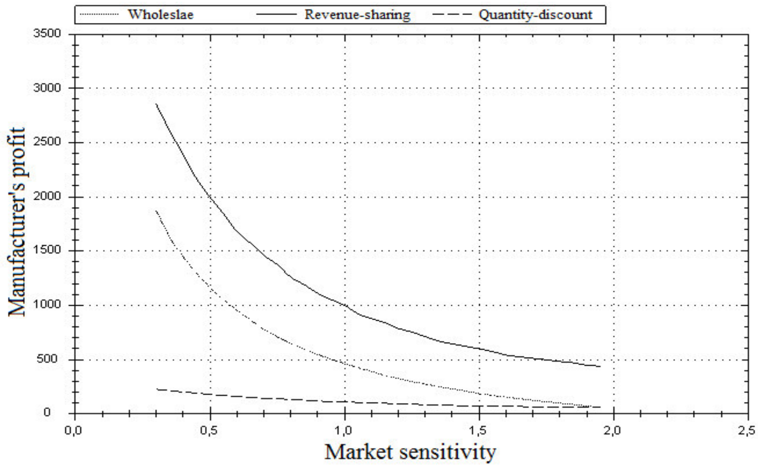

Fig. 57: Manufacturer profit function under changes in market elasticity

The behavior of profit function in case of changes in dealers' market shares would be as follows (Figures 58–60):

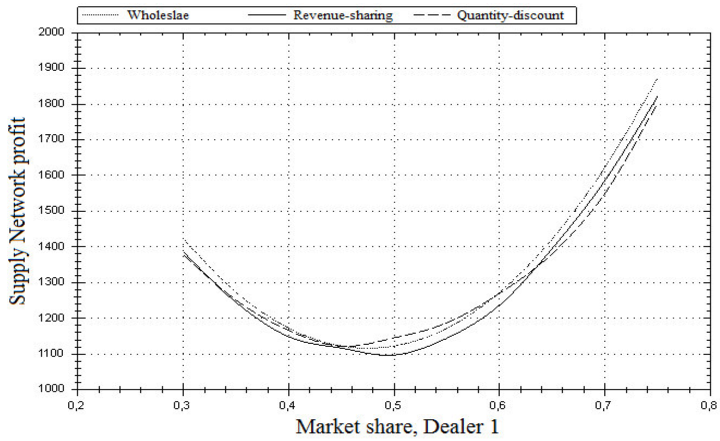

Fig. 58: Supply network profit function under changes in market concentration

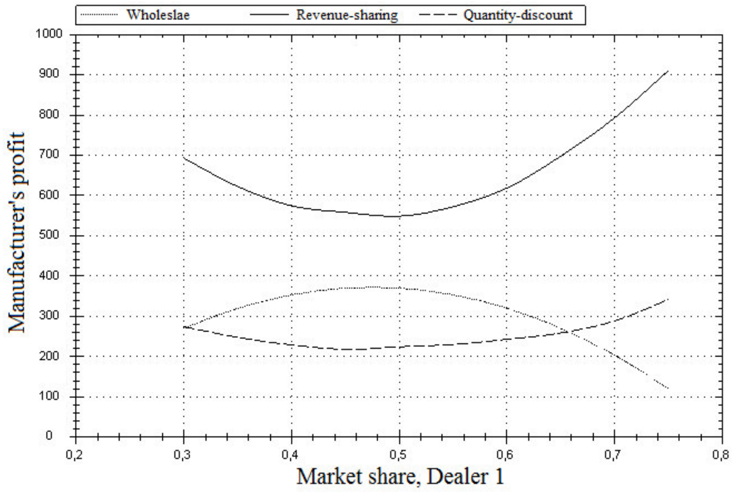

Fig. 59: Dealers' profit function under changes in market concentration

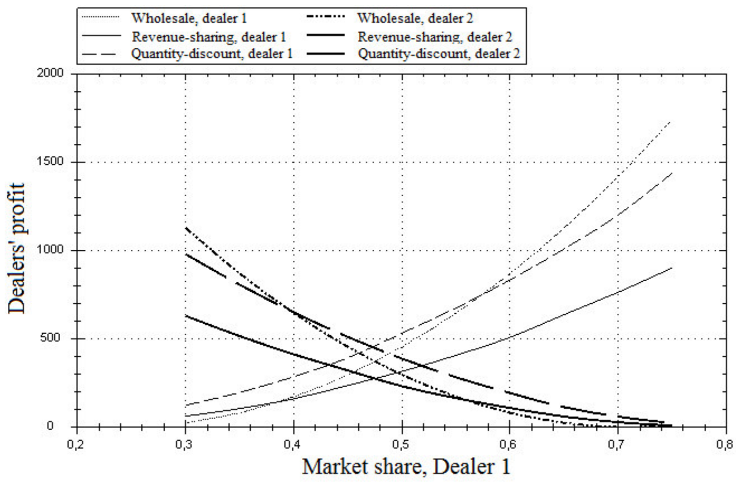

Fig. 60: Manufacturer profit function under changes in market concentration

3. Third Appendix

The behavior of profit function in case of changes in marginal costs c from 12 to 25 would look as follows (Figures 61–63):

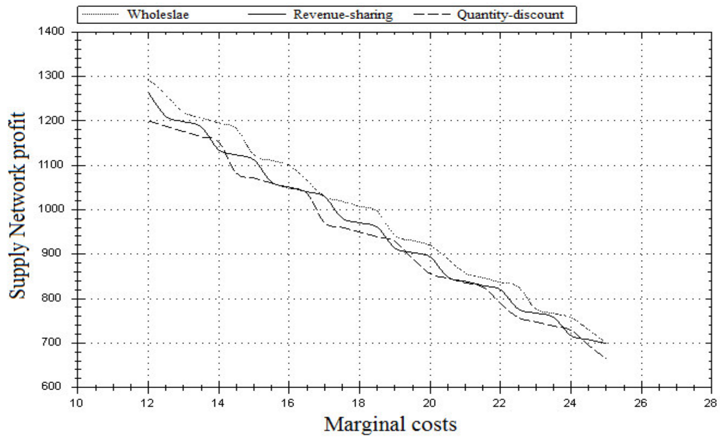

Fig. 61: Supply network profit function under volatility of marginal costs

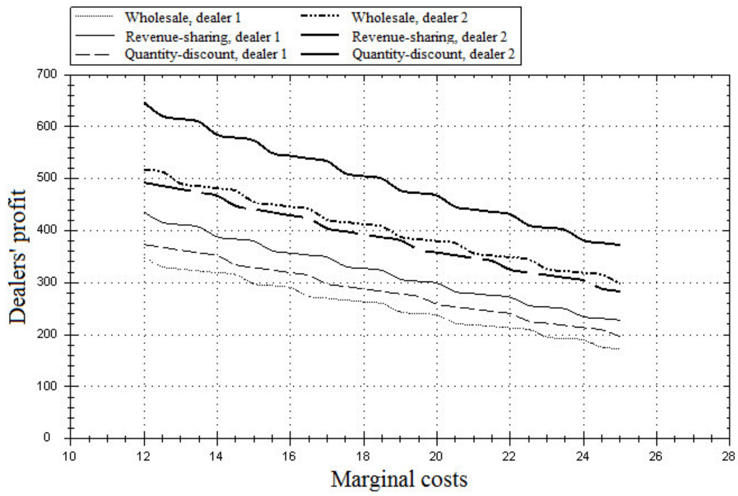

Fig. 62: Dealers' profit function under volatility of marginal costs

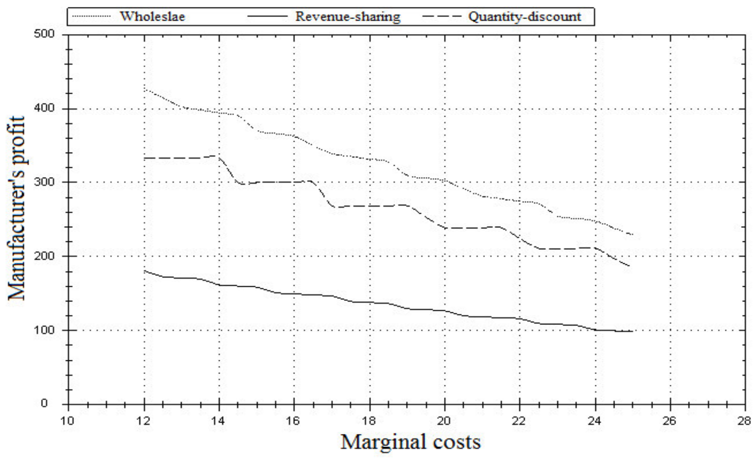

Fig. 63: Manufacturer profit function under volatility of marginal costs

The behavior of profit function in case of changes in manufacturer's operating expenses to fulfill the orders of, say, dealer 1, s_1 from 30 to 40 per unit of good would be as follows (Figures 64–66):

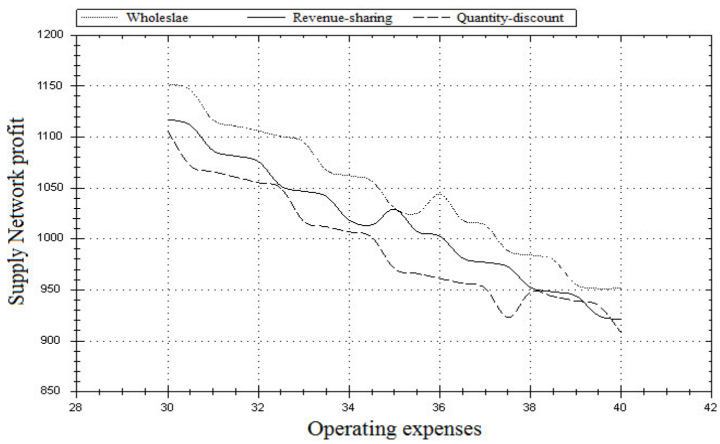

Fig. 64: Supply network profit function under volatility of operating expenses

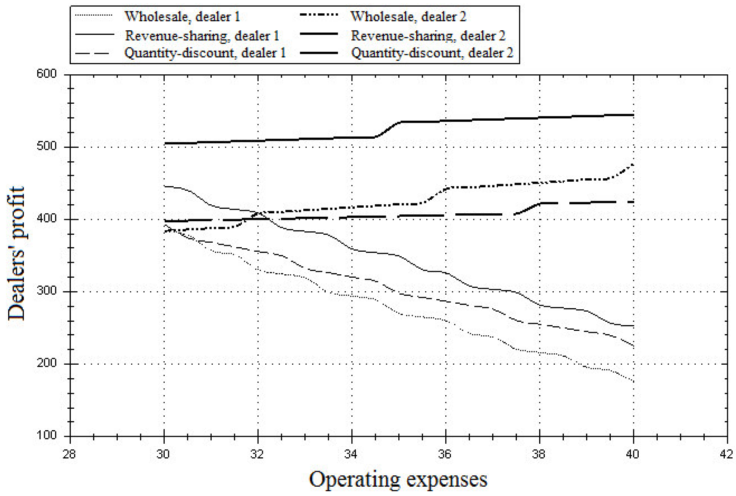

Fig. 65: Dealers' profit function under volatility of operating expenses

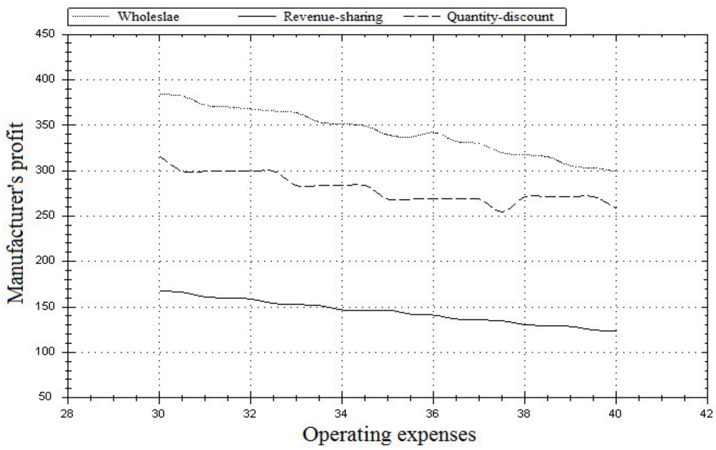

Fig. 66: Manufacturer profit function under volatility of operating expenses

The behavior of profit function in case of changes in market elasticity γ from 0.1 to 0.9 would look as follows (Figures 67–69):

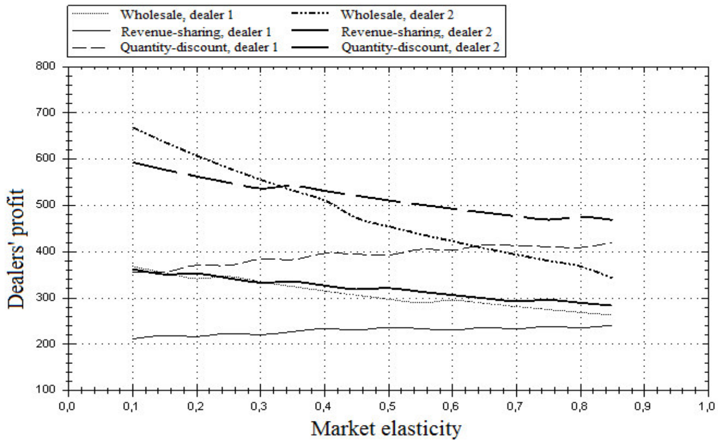

Fig. 67: Supply network profit function under changes in market elasticity

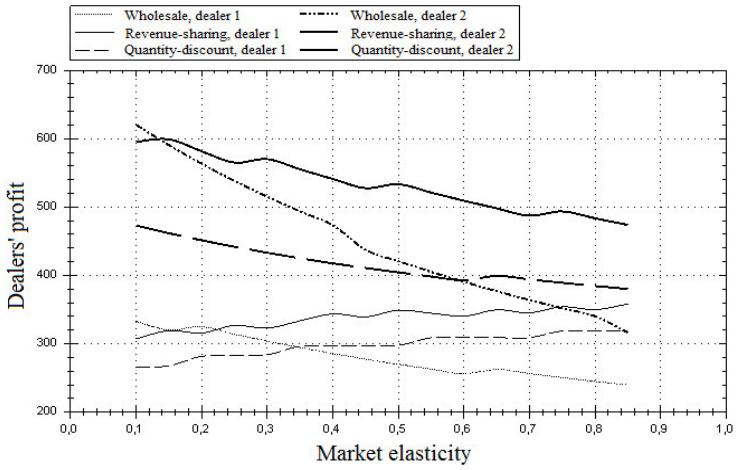

Fig. 68: Dealers' profit function under changes in market elasticity

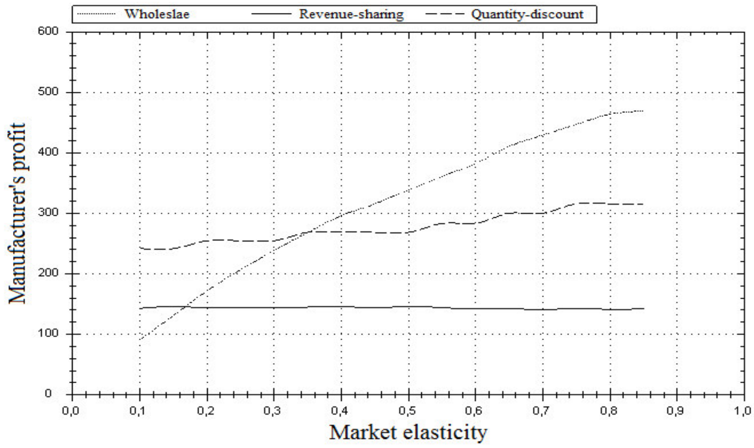

Fig. 69: Manufacturer profit function under changes in market elasticity

The behavior of profit function in case of changes in price sensitivity for dealer 1 products δ_1 from 0.3 to 2 would look as follows (Figures 70–72):

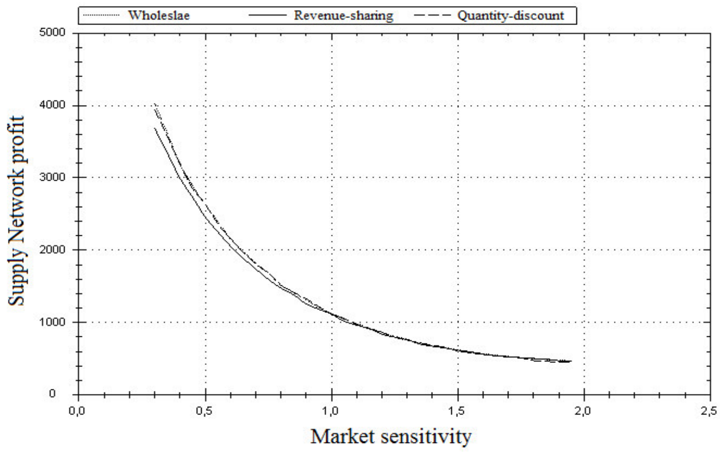

Fig. 70: Supply network profit function under changes in price sensitivity

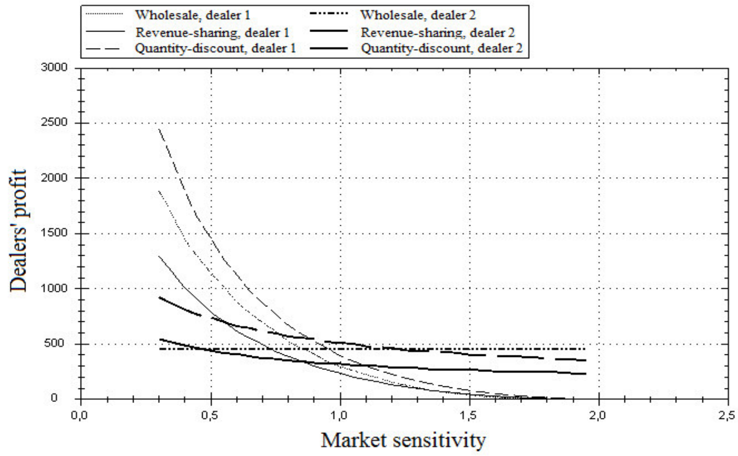

Fig. 71: Dealers' profit function under changes in price sensitivity

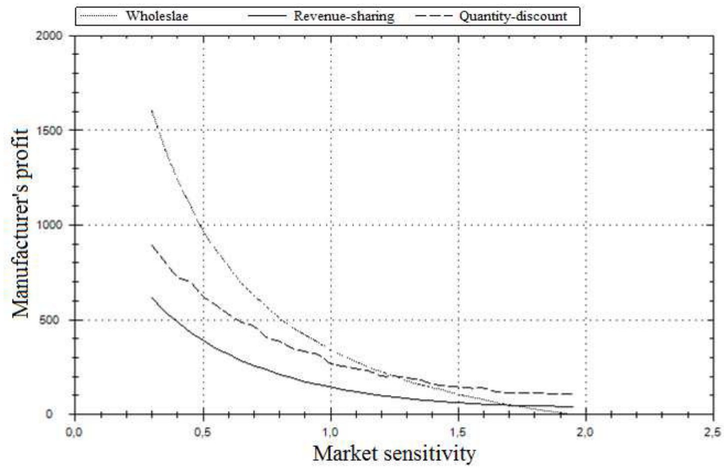

Fig. 72: Manufacturer profit function under changes in price sensitivity

Considering changes in dealers' market shares k_1 and k_2 from 0.3 to 0.8 and from 0.7 to 0.2 accordingly, the behavior of profit function would be as follows (Figures 73–75):

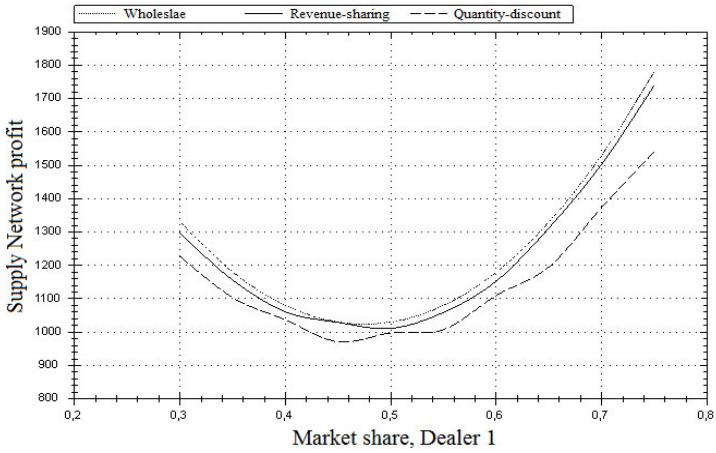

Fig. 73: Supply network profit function under changes in market concentration

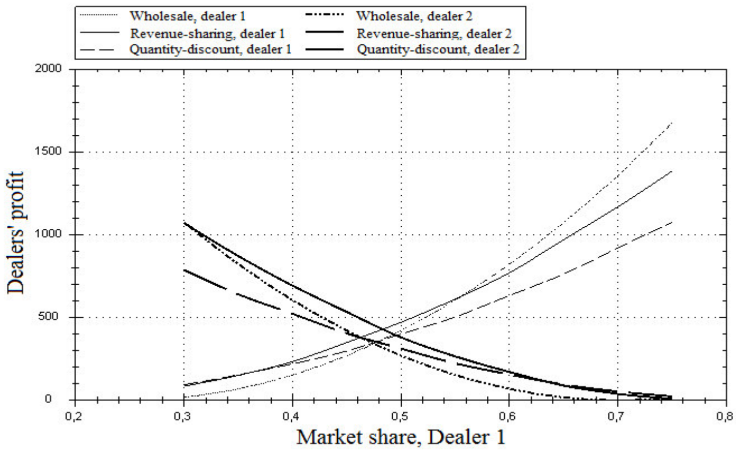

Fig. 74: Dealers' profit function under changes in market concentration

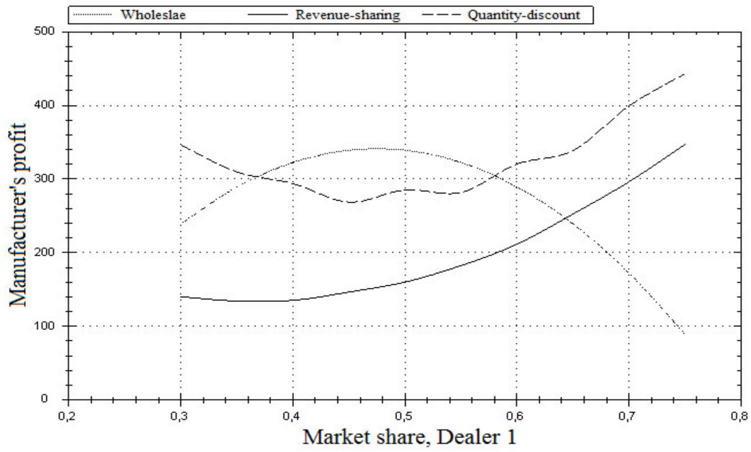

Fig. 75: Manufacturer profit function under changes in market concentration

4. Fourth Appendix

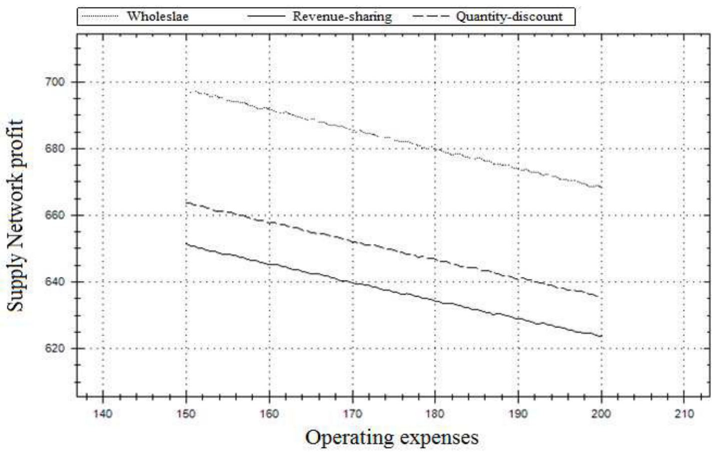

Fig. 76: Audi supply network profit function under volatility of operating expenses

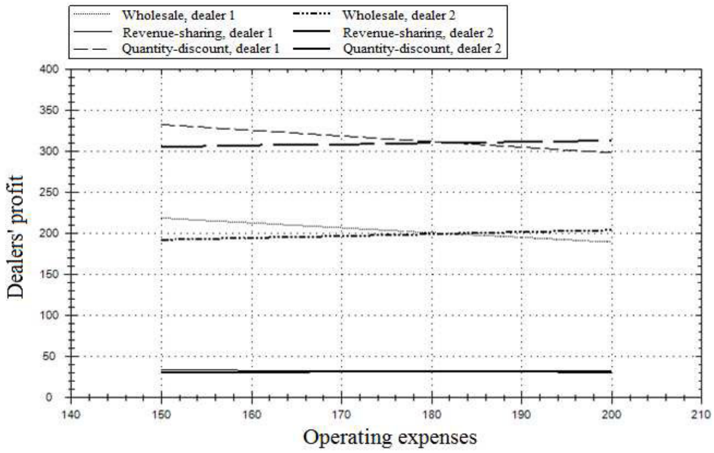

Fig. 77: Audi dealers' profit function under volatility of operating expenses

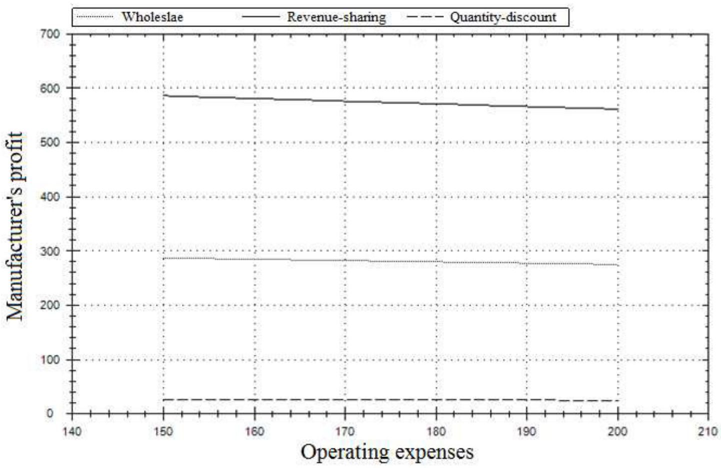

Fig. 78: Audi profit function under volatility of operating expenses

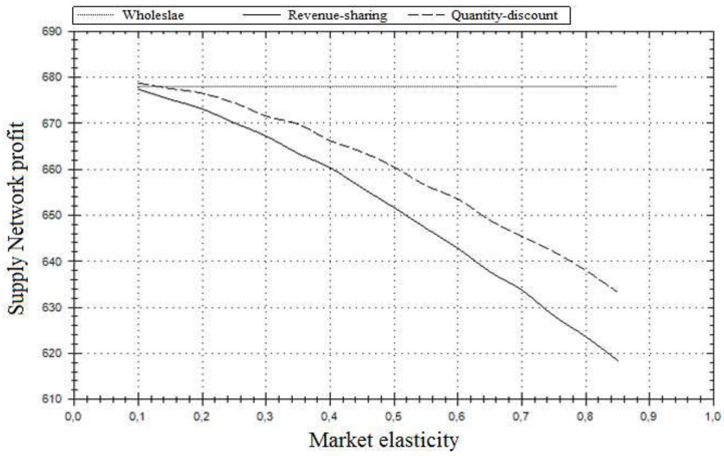

Fig. 79: Audi supply network profit function under changes in market elasticity

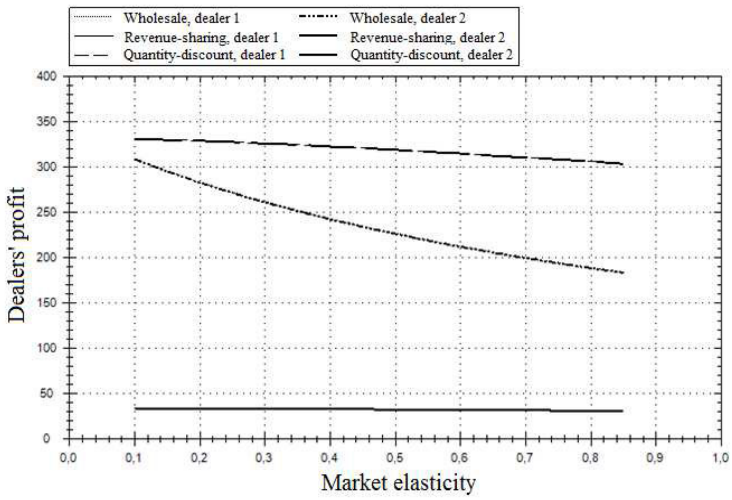

Fig. 80: Audi dealers' profit function under changes in market elasticity

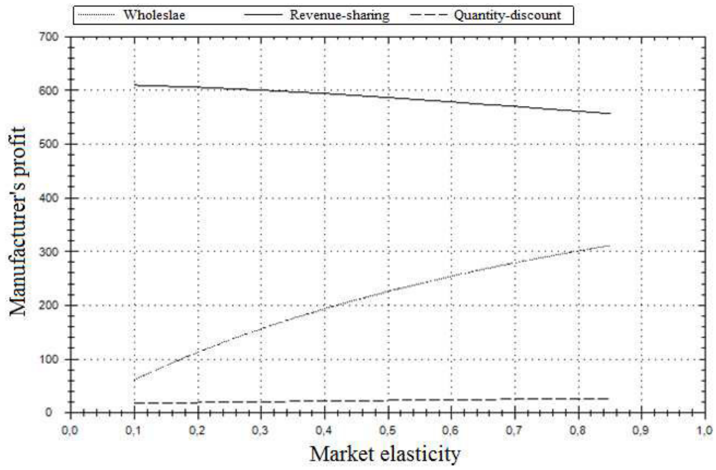

Fig. 81: Audi profit function under changes in market elasticity

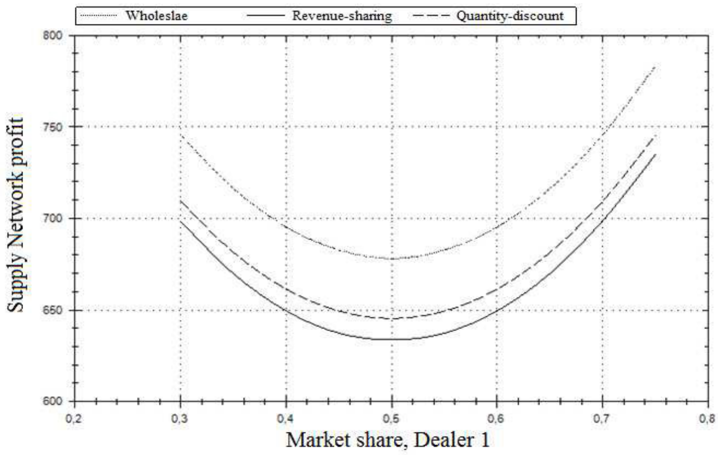

Fig. 82: Audi supply network profit function under changes in price sensitivity

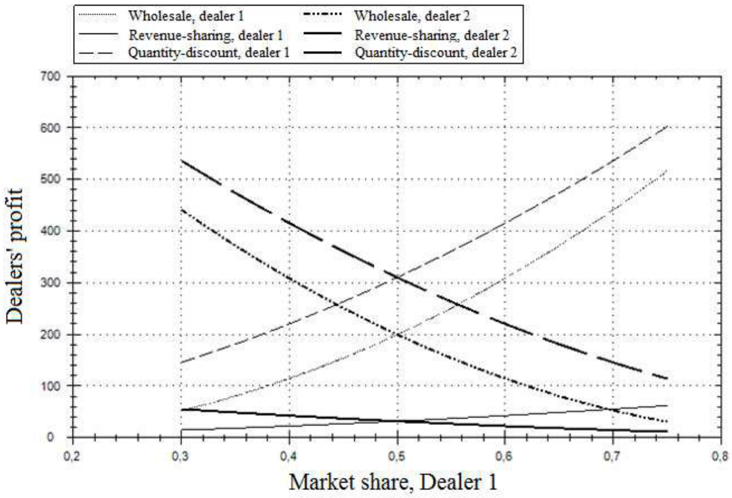

Fig. 83: Audi dealers' profit function under changes in price sensitivity

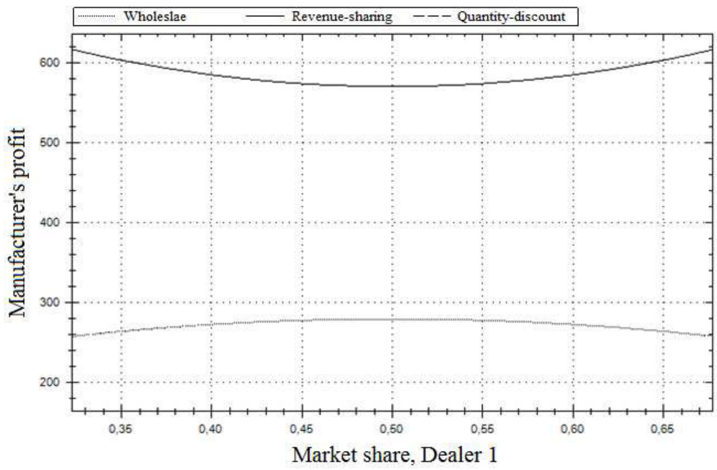

Fig. 84: Audi profit function under changes in price sensitivity

5. Fifth Appendix

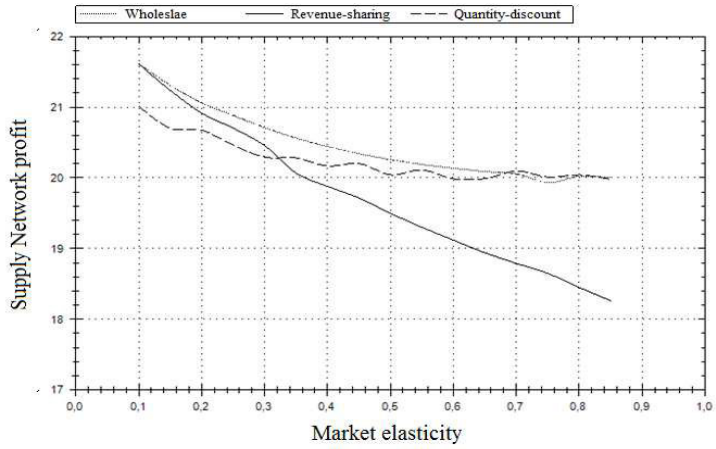

Fig. 85: Heineken supply network profit function under changes in market elasticity

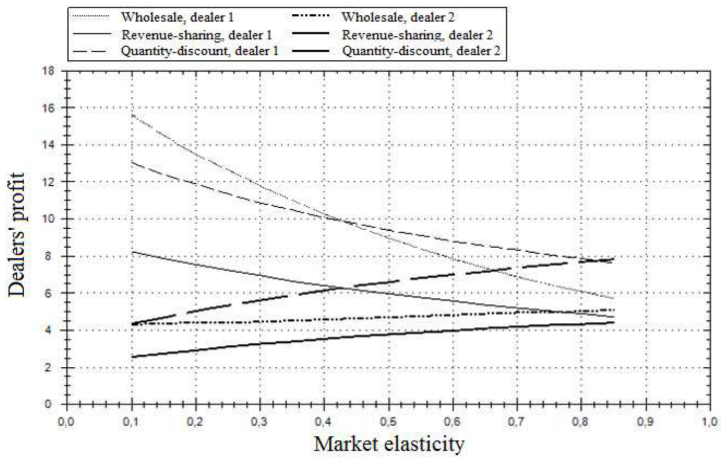

Fig. 86: Bar profit function under changes in market elasticity

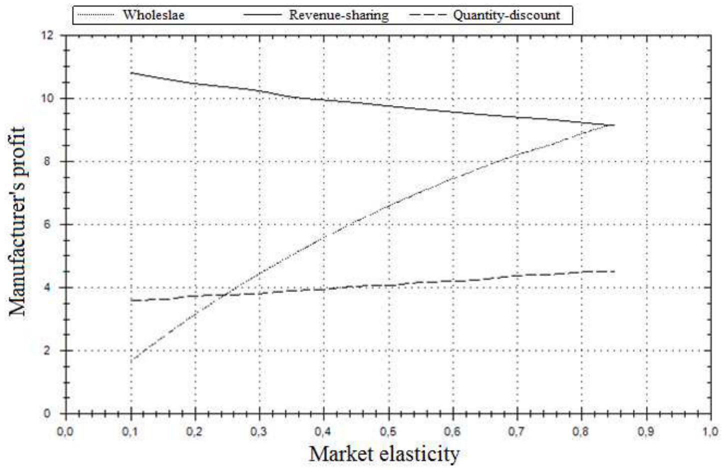

Fig. 87: Beer wholesaler profit function under changes in market elasticity

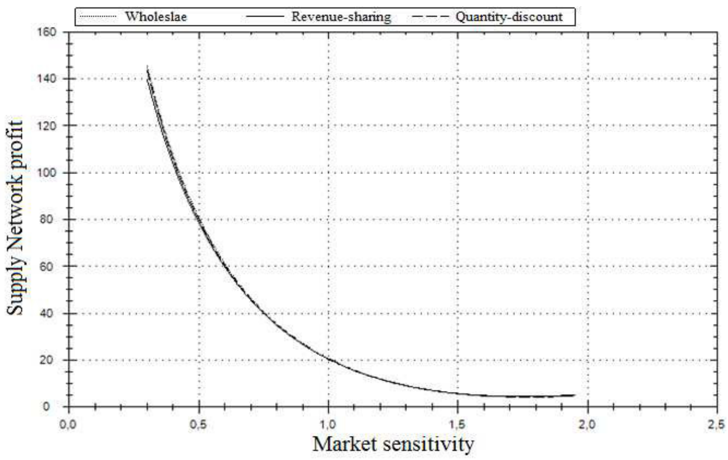

Fig. 88: Heineken supply network profit function under changes in price sensitivity

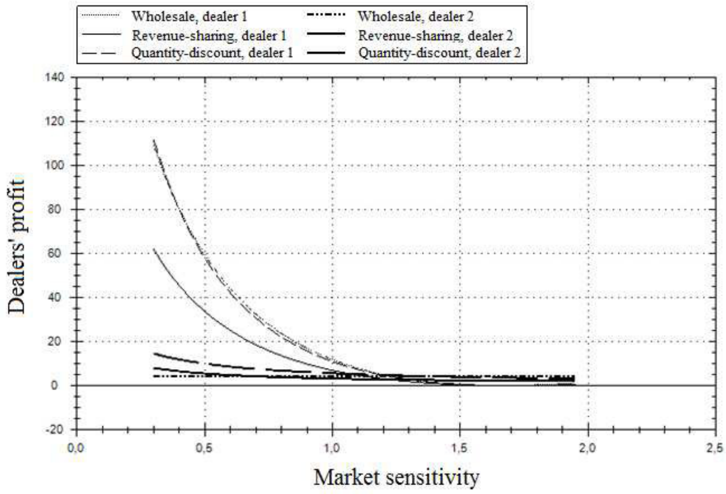

Fig. 89: Bar profit function under changes in price sensitivity

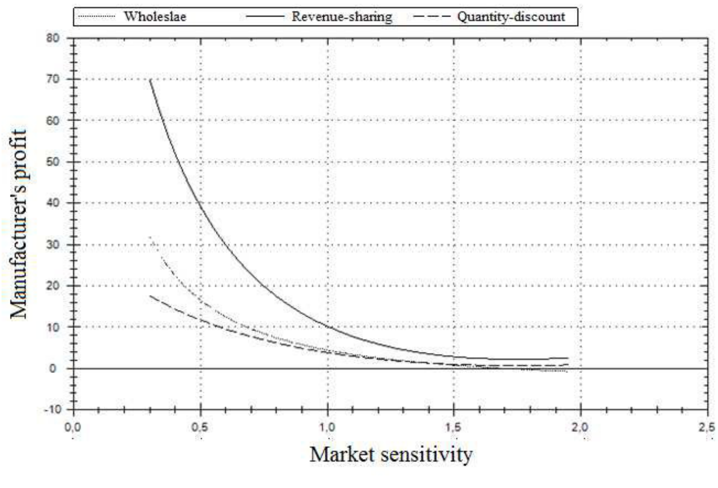

Fig. 90: Beer wholesaler profit function under changes in price sensitivity

6. Sixth Appendix

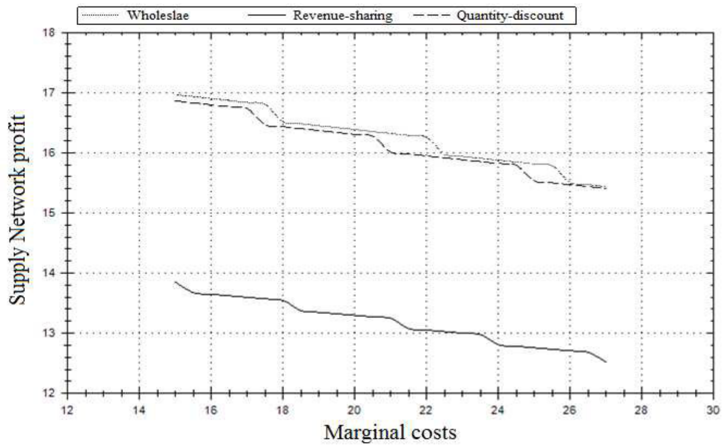

Fig. 91: ProtechDry supply network profit function under volatility of marginal costs

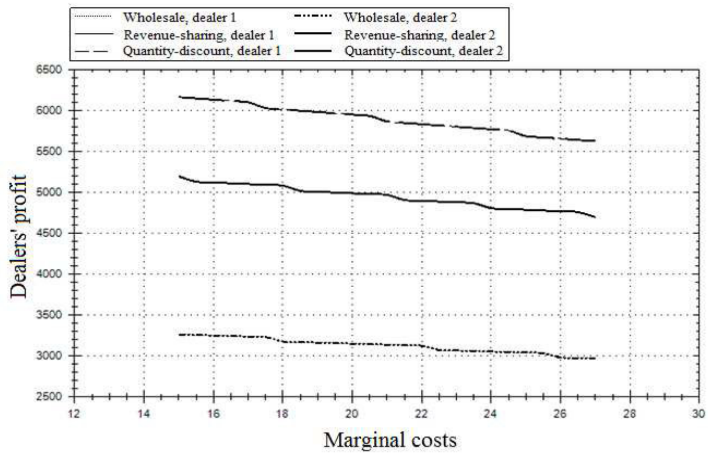

Fig. 92: ProtechDry dealers' profit function under volatility of marginal costs

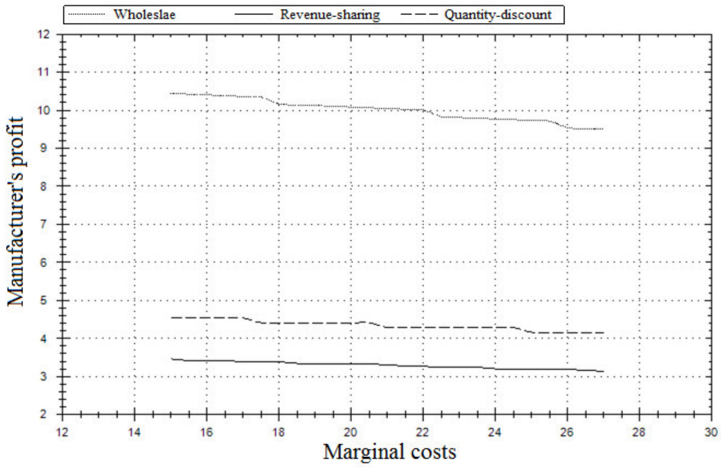

Fig. 93: ProtechDry profit function under volatility of marginal costs

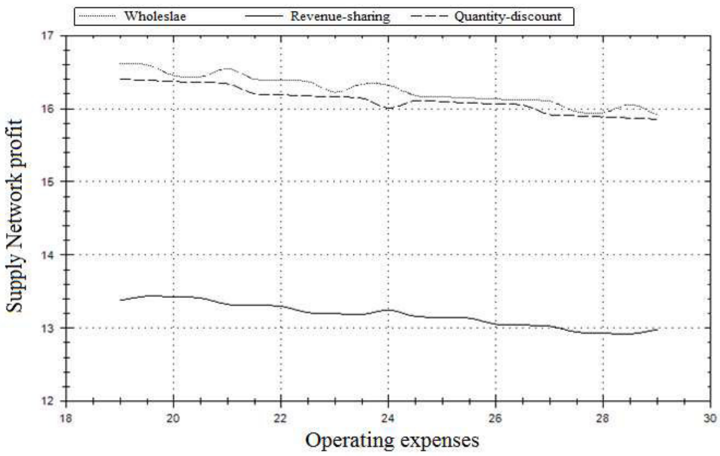

Fig. 94: ProtechDry supply network profit function under volatility of operating expenses

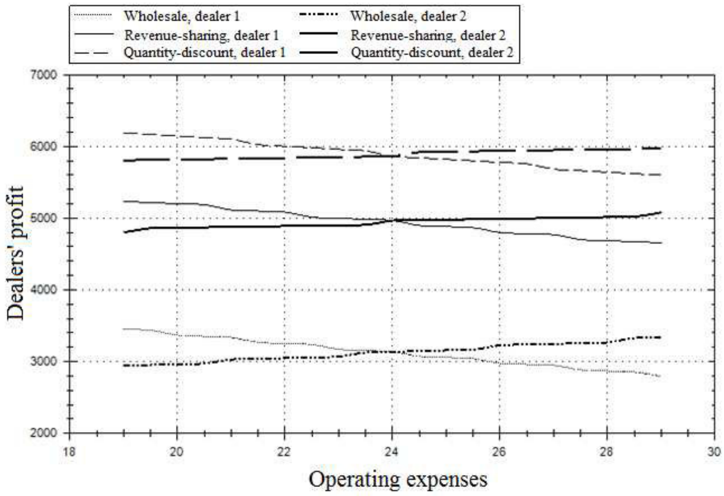

Fig. 95: ProtechDry dealers' profit function under volatility of operating expenses

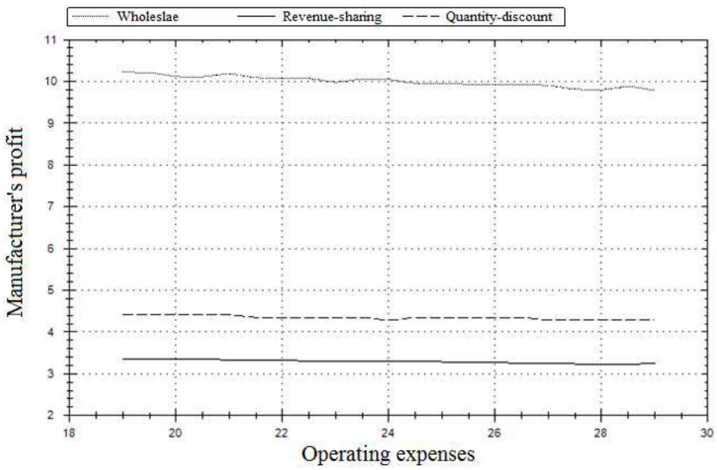

Fig. 96: ProtechDry profit function under volatility of operating expenses

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Phenomenon of a “Snag“ in Financial Markets and its Analysis via the Cooperative Game Theory

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Abstract The paper describes the development of financial markets and changes in the nature of economic growth using the theory of cooperative games. These issues have developed since the early 1950s under the influence of theoretical problems based on the game theory itself and interacting with real problems outside of the game theory (mostly from economics). It turned out that various applications and contexts correspond to numerous possible solutions of standard tasks, e.g. Nash (S, d) bargaining problem. Some of the significant solutions are responded to questions arising in the context of social welfare economic theory, respectively issues are related to the redistribution of wealth between different groups in population and the rationale of such reallocation. We show that under conditions of sufficiently effective financial markets the question of the relationship between efficiency and equality, which is typical of the theory of social welfare, may be replaced by the question of making full utilization of investment opportunities associated with the acquisition, preservation and application of human capital. We define “sufficient efficiency of financial markets” as ability to fully utilize investment opportunities related – to put it simply – to human development, regardless of its initial assets or income position. This is related to the fact that instead of different ways of reasoning for solution (S, d) of the problem we can take advantage of technical solution (based on the equality of marginal returns of investment opportunities, or rather based on sum payments maximization), e.g. the solution used in problem of optimal allocation of water (water allocation problem) (Brink, et al., 2011). The question of compensation payments in relation to solutions based on technical optimum has important interpretation. Sufficiently efficient functioning of financial markets (in the above mentioned sense) assumes also good functioning of such financial market instruments, e.g. human capital contracts associated with the use of transferred prices and mediated utilization of transferred prices. In case of full utilization of those tools, compensation would not be necessary. Above mentioned topics are part of wider research focused on changes in the nature of economic growth. This research is based on the hypothesis that the existing possibilities of economic growth have become exhausted and that it is necessary to transition towards the economy based on the dominant role of productive services, i.e. services which have immediate effect on the acquisition, preservation and utilization of human capital (e.g. education, health care etc.)(Friedman, 1957). The development of financial markets in the above direction is prerequisite to economic growth.

Keywords: Nash bargaining problem, investment opportunities, human capital; financial markets; cooperative games; investment opportunities

1. Introduction

In our contribution, we are pointing out an interesting and from the practical point of view important area of interpretation of solving cooperative games (especially the Nash (S, d) problem) (Nash, 1950), namely from the perspective of relation between the level of reality (practical application), a definition of assumptions (in the language of micro-economy), a drawing up of a concept (by defining assumptions based on the cooperative game theory) and a setting of a corresponding axiomatic system. The application of the cooperative game theory apparatus for financial markets, specifically for the analysis of supply and demand of investment funding and investment opportunities is original and innovative. As part of the defined objective, we identify a general cause of a certain type of problems that arise in financial markets (we call it a "snag" in financial markets) and we point out a practical purpose of this phenomenon. The achieved results are well applicable at searching an answer to the question of what causes some of the phenomena that we are currently encountering in financial markets. At present, they are being applied in the financial markets research which is being carried out by the University of Finance and Administration based in Prague.

We will take the following steps:

1. We will point out the microeconomic and practical dimension of the problem and apply a numerical model to it.
2. We will identify the "snag" phenomenon.
3. We will describe it by means of the cooperative game theory and point out some interesting and from the practical point of view important characteristics of this phenomenon.
4. We will discuss our results with respect to some methodological issues of the cooperative game theory (relation between a theoretical solution and its practical purpose) and with respect to possible interpretations of various solutions of the Nash (S, d) problem and with respect to what we encounter in financial markets in the Czech Republic.
5. Lastly, we will outline a possibility of substantial expansion of the area of application of the processes which we deal with in our contribution. Since the single steps that we make require their permanent relation to a practical context, or more precisely identification of practical relevance of the rolling results, we will put emphasis on comprehensibility and a clear picture for a wide circle of experts, as it results from, inter alia, our experience of cooperation with specialists in financial markets.

2. Introduction to the problem – Microeconomic view of financial markets

We consider a simple model of the financial market which includes two entities, each of them having investment opportunities and investment funding. Combination of a particular amount of investment funding and a particular investment opportunity results in a particular yield. We consider the current income that the business entities have at disposal to be investment funding. The future income that they will receive by combining investment opportunities and investment funding is considered to be their yield. We assume that both business entities will maximise their future yield and therefore they will utilise investment opportunities in the order of their rate of return, i.e. the function of the marginal rate of return on investment opportunities is a function non-increasing in its whole domain. Functions of the marginal rate of return on investment opportunities of both entities are continuous, when the minimum of one of the functions is smaller than the maximum of the second function and the maximum of the first function is greater than the minimum of the second function, see Fig. 1:

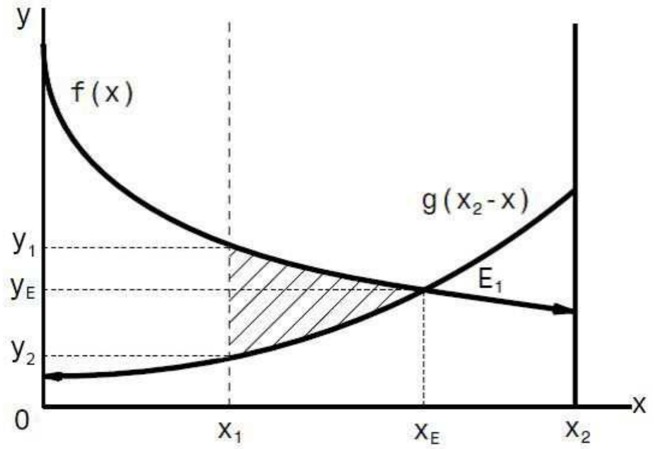

Fig. 1: Supply and demand of investment funds and investment opportunities

Here: $x_1,\ x_2 - x_1$ are the quantities of investment funding that business entities 1 and 2 have available, y is the future yield in marginal quantities, $f(x)$, $g(x)$, or $g(x_2 - x)$ non-deceasing continuous functions of the marginal yield on investment opportunities; $g(x)$ is modified for a better graphical illustration of the situation in question.

$E_1(x_E, y_E)$ is the point in which $f(x) = g(x) = f(x_2 - x) = g(x_2 - x)$; in this point all the investment opportunities of both entities are used depending on the rate of return. The hatched area shows a size of the maximum possible Pareto improvement as a result of the financial market effect if one of the entities gives up his less profitable investment opportunities and provides funding to the second entity.

The total yield of the first (analogically the second) business entity is as follows:

$$\int_0^{x_1} f\,(x)\,d\,(x) = \int_{x_1}^{x_2} f\,(x_2 - x)\,d(x),$$

resp.

$$\int_{0}^{x_1} g(x)\, d(x) = \int_{x_1}^{x_2} g(x_2 - x)\, d(x).$$

In the event that the cost of investment funding is determined by equality of marginal yields, i.e. by the fact that $f(x) = g(x)$, investment opportunities of both entities will be utilised depending on the rate of return on them. The compensation of the entity that provided his investment funding to implement an investment opportunity of the other entity will equal to $y_E(x_E - x_1)$.

Such process is satisfactory for the microeconomic approach. It has reached the Pareto equilibrium; both entities improved their positions compared to the previous ones; the origin and volume of interest have been clarified (as a compensation for use of investment funding of the other entity to implement their own investment opportunities). This solution seems to be problem-free.

$y(1) = \int_{0}^{x_1} f(x)\, d(x)$ is the function of the payoff of the entity 1,

$y(2) = \int_{1}^{x_2} g(x_2 - x)\, d(x)$ is the function of the payoff of the entity 2.

Distribution of the yields in Figure 1 can be mathematically described as follows:

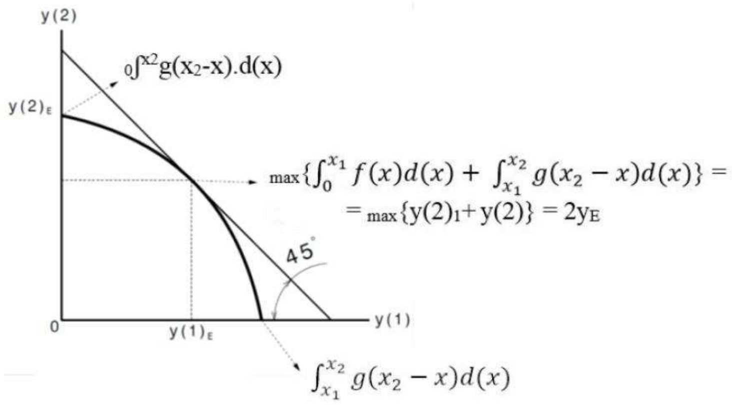

Fig. 2: Limit of achievable payoffs of both entities

where:

$\int_{0}^{x_1} f(x)\, d(x)$, $\int_{x_1}^{x_2} g(x_2 - x)\, d(x)$ are maximum payoffs of business entities,

$\int_{0}^{x_1} f(x)\, d(x) + \int_{x_1}^{x_2} g(x_2 - x)\, d(x)$ is a function delimiting a set of permissible payoffs,

$\max\{\int_{0}^{x_1} f(x)\, d(x) + \int_{x_1}^{x_2} g(x_2 - x)\, d(x)\}$ is the maximum total of business entities' payoffs.

The points inside the area of delimitation by the curve of the sum of the payoffs can be interpreted as points corresponding to business entities' payoffs in the situation when they did not use all of their investment funding.

As long as the financial market is in operation (investment funding of one business entity can be used for implementing investment opportunities of the other business entity), both entities can increase their payoffs if the cost of investment funding – marked with the letter y – is in the interval between $f(x_1) = y_1$ and $g(x_1) = y_2$, i.e. if it is true that $g(x_1) < y_i < f(x_1)$. At the cost of investment opportunities equal to y_E, all the investment opportunities prioritised by their return on them will be used.

2.1. Microeconomic model testing

Before we applied the apparatus of the cooperative game theory to the microeconomic issues of relation between supply and demand of investment funding and investment opportunities, we had created a simple numerical model to test various situations. To understand the achieved results better and to illustrate how financial markets work from the viewpoint considered by us, we will point out some findings from the numerical model testing.

$$y' = 2(5-x),\ y = 10x - x_2,$$

$$x_1 = 2,\ {y'}_1 = 6,\ y_1 = 16,$$

$$x_2 = 5,\ {y'}_2 = 0,\ y_1 = 25,$$

$$y_1 = 2.6 + 2.4.$$

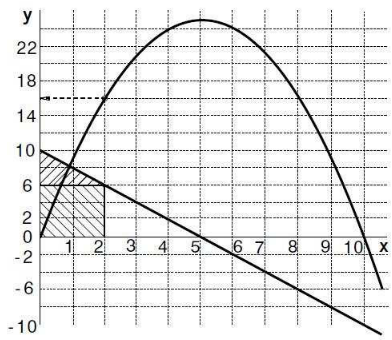

Fig. 3: Quadratic production function simulating a decrease of the marginal yield on investment opportunities

First part of the Figure 4 on the left side shows supply and demand of investment funding and investment opportunities of two entities in case of a quadratic production function of both entities, when one of its possible situations is presented in Figure 3. In first part of the right side of Figure 4. you can see the Pareto improvement in the graph, where axes ${}^1y$ and ${}^2y$ show yields of individual entities. Notice the typical "heart" shape which appears in most models and implies that the maximum yield of individual entities is usually not in the point of full utilization of investment opportunities. Because of significance of the above-mentioned, we present one more Figure (5,6) which shows the area of Pareto improvements in detail:

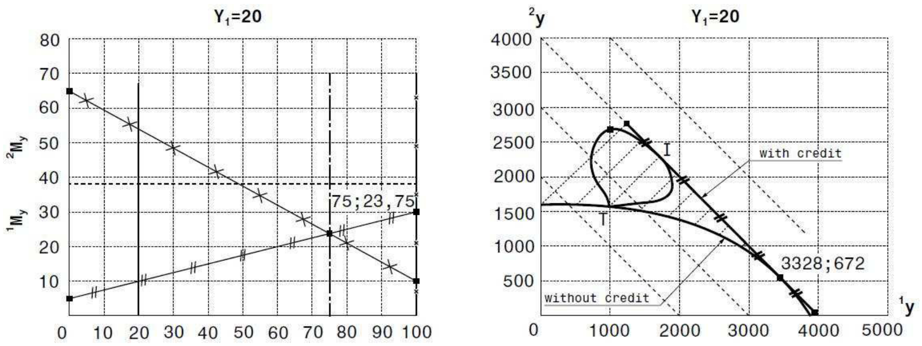

Fig. 4: Supply and demand of investment funding and investment opportunities of two entities in case of a quadratic production function

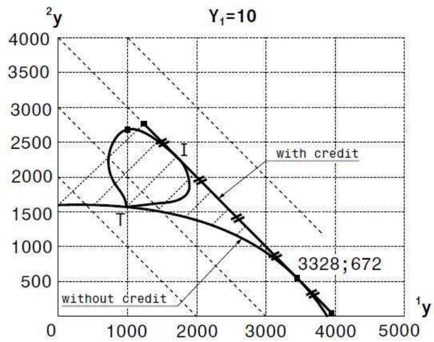

Fig. 5: Detail of Pareto improvement areas of financial market stakeholders

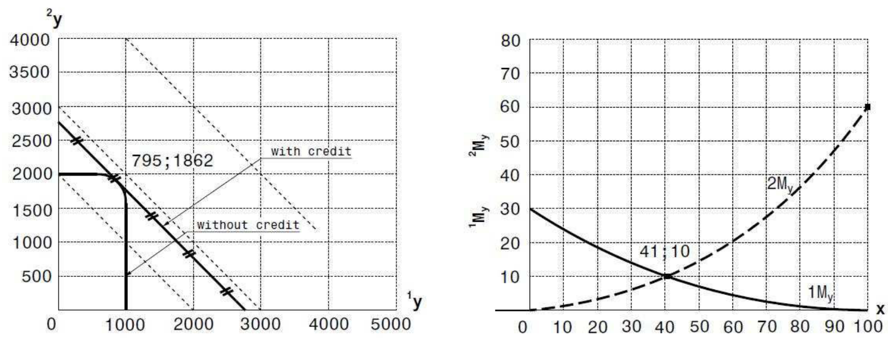

Fig. 6: Example of another marginal yield functions (quadratic function of decreasing marginal yields)

The concept we have used to illustrate the relation of supply and demand of investment funding and investment opportunities can have other interpretations as well. One of them is the water problem, for details see e.g. (Beal, 2013), (Brink, 2011), (Houba, 2013). From the practical point of view, we have two very different tasks, however, in terms of description by an abstract concept, the problems are almost identical. We will comment on this topic at the end of our contribution. To monitor multiple possible interpretations during the presentation (and moreover without knowledge of what the presentation will bring) would make comprehensibility of our steps significantly harder.

3. Problem of the yield distribution in the language of cooperative games, Nash (S, d) problem

As we approach the area of game theory, we will call business entities players. If in the point $(y(1)_E, y(2)_E)$ both players achieved the maximum payoff at the costs of investment funding being in the bounded interval $\langle y_1, y_2 \rangle$ Fig. 1. If in that point both players achieved the maximum payoff at the costs of investment funding being in the bounded interval, assumption of the individual rationality would suffice to regard the point $(y(1)_E, y(2)_E)$ as an intuitively acceptable solution of the respective cooperative task. However, this need not to be true, see Figure 7:

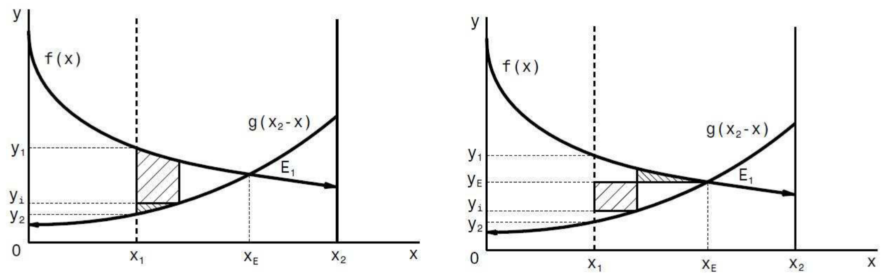

Fig. 7: Players' payoffs at changing costs of investment funding

The figure on the left side shows an increase of the first player's and second player's payoff at the cost of investment funding y_i. The next figure shows the change which would occur if the cost of investment funding changed from y_i to y_E. As seen in Figure 7, at the cost of investment funding y_E (i.e. at the cost which we consider equilibrium from the microeconomic point of view), the payoff of the first player will decrease as a result of the decline in the cost of investment funding (interest, compensation) more (see the hatched area 45° in the figure on the left side) than it increases as a result of the fact that more investment opportunities will be utilised (see the thickly hatched area 135° in the figure on the left side).

This is a very significant moment. It turns out that the individual rationality assumption need not suffice to find an unequivocal solution. Therefore, it is useful to convert the problem encountered by us to the form of the Nash (S, d) bargaining problem.

Now, let us look at the following Figure 8.

It corresponds to Figure 3-7; however, it shows the compensation area. We regard compensation as the part of the yield which the person who owns utilisable investment opportunities will transfer to the person who lent investment funding (i.e. in our case interest paid by the debtor to the creditor as well as the cost of

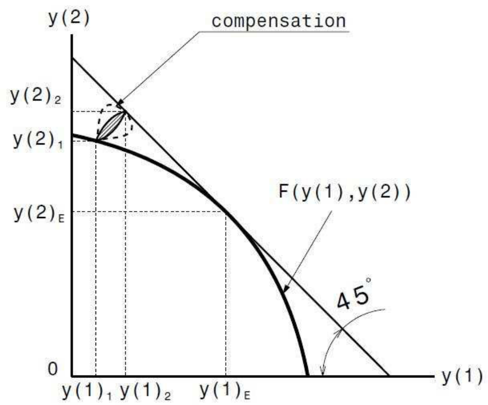

Fig. 8: Compensation area

investment funding). The size of compensation naturally depends on the interest rate, i.e. on the cost of investment funding.

The particular task can be also viewed as the Nash (S, d) bargaining problem. In the following figure we will illustrate the problem which occurs at compensations. For this purpose, we will look at the compensation area in more detail. Figure 9 is an enlarged section of Figure 8.

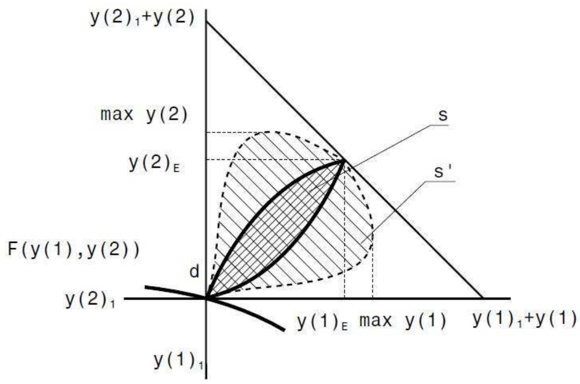

Fig. 9: Compensation area as the Nash (S, d) problem

S is a set of payoff distribution possibilities in the situation when $\max y(1)$ and $\max y(2)$ are smaller than $y(1)_E$ and $y(1)_E$. S' is a set of payoff distribution possibilities in the situation when $\max y(1)$ and $\max y(2)$ are greater than $y(1)_E$ and $y(1)_E$. Pareto improvement functions are in Graph 5. The solid-line curve represents the situation when $\max y(1)$ and $\max y(2)$ are smaller than $y(1)_E$ and $y(1)_E$; the broken-line curve the situation when $\max y(1)$ and $\max y(2)$ are greater than $y(1)_E$ and $y(1)_E$. In the first situation (provided that the cost of investment funding is constant and use is made of all the investment opportunities the yield of which is greater than this cost), the solution of the cooperative task in question is univocally determined by the individual rationality assumption.

In the second example, the individual rationality assumption does not suffice. A "**snag**" (term introduced by us) occurs at **utilisation of investment opportunities.** It is possible to apply a number of potential approaches to a solution of the cooperative game in question. Let us view the particular problem (the cooperative task in question) in more detail.

Each of the players may claim his maximum payoff: $\max y(1)$, or as the case may be $\max y(2)$. The bargaining will then take place in the bounded interval between $y1_{max}$ and $y2_{max}$ (which is the cost of investment funding at which one or the other of the players will achieve the maximum payoff), see Figure 10.

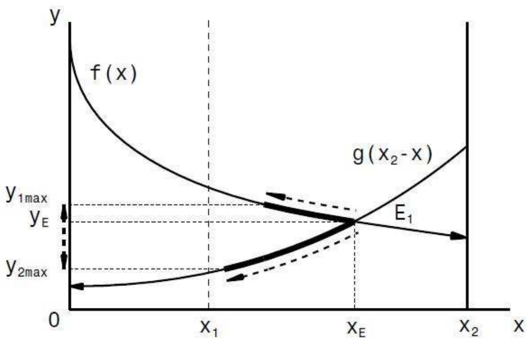

Fig. 10: Possiblity for bargaining

The same can be illustrated in Figure 11.

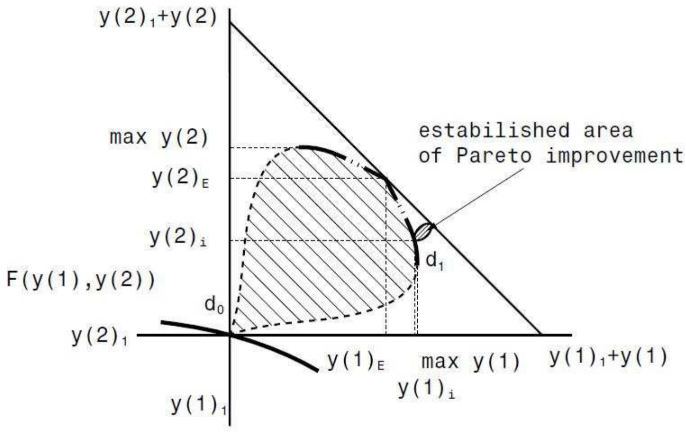

Fig. 11: Area of Pareto improvement

Only the points on thick solid lines in both pictures correspond to the requirements (axioms) of achievability and collective rationality.

Each cost of investment funding which the players opt for or more precisely agree on corresponds to a particular distribution of yields between them. However, only if the cost equals to y_E, use will be made of all the investment opportunities prioritised by their rate of return.

It means that if the players opt for any solution of the cooperative game which results in a different cost of investment funding than y_E, they have a chance to act in such a way that they will improve their position compared to this solution of the cooperative game, see Figure 10.

If they opt for the solution that results in the cost y_i, then the hatched 45° area of Pareto improvements is offered. We can show this hatched 45° area in the Figure 11 as well.

If we want to meet the requirement of collective rationality, i.e. to exploit all the Pareto improvements, to use the language of microeconomy, then in the situation when the (S, d) solution of the Nash bargaining problem does not result in the cost when the sum of the payoffs is maximised, another sequential improvement is always possible at a different cost of investment funding. In the limit situation, sequential solutions constructed this way and derived from any type of the cooperative solution of the Nash bargaining problem will reach the line of the maximum sum. All the investment opportunities, regardless to which of the entities (players) they belong, are utilised depending on their rate of return. Nevertheless, the distribution of players' payoffs need not (and usually will not) correspond to the point of the maximum sum in the primary task, see Figure 12.

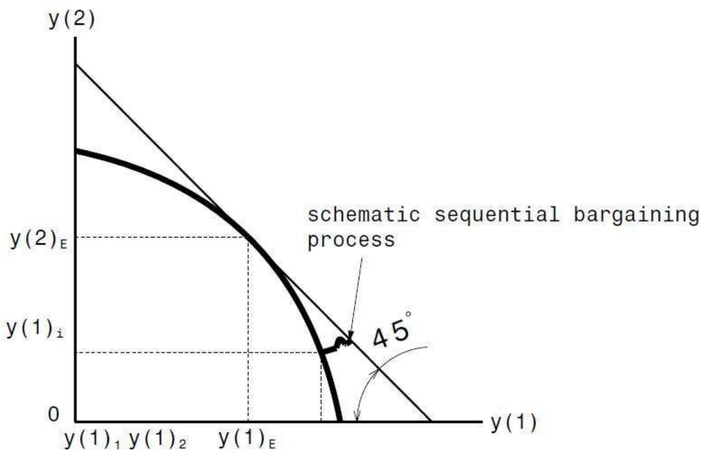

Fig. 12: Sequential bargaining process

4. Results and Discussion

The task that we are dealing with has a number of interpretations. With regard to what we know especially two of them come into focus:

- Financial markets, i.e. supply and demand of investment funding and investment opportunities.

- Water problem – supply of water and a possibility of its use in order to realise a yield in a various way.

Generally, this task can be understood as an application of a production factor in different ways if we have two owners who have different possibilities of applying the respective production factor in order to achieve some yield.

We assume that each of the owners of the resource is in question and he uses his own opportunities to use this resource according to the rate of return on the opportunities that he owns. The question is whether or under what conditions opportunities prioritised by their rate of return are utilised regardless of who is the owner of such opportunities for application of the production factor (and who is the owner of the production factor). From this point of view, let us repeat our initial assumption – business entities (players) use their investment funding to implement investment opportunities prioritised by their rate of return. Let us proceed from this assumption to an analysis of assumptions under which investment opportunities prioritised by their rate of return are implemented regardless which of the business entities (players) owns them. And it turns out that it is a very difficult topic.

4.1. Recapitulation of the findings and a discussion about the theoretical and methodological aspect of the topic in question

If the cost of the production factor may change in the sense that it is paid (as a compensation) by the person who uses the resource for his own opportunities, at costs at various stages of bargaining and rolling agreements (realised acts of exchange), then all the investment opportunities prioritised by their rate of return are used (regardless to whom they belong); the solution has a sequential character.

The requirement of a single cost for all resource units and simultaneous utilisation of all the opportunities of resource use prioritised by the rate of return (i.e. if we want both requirements to be applied simultaneously) is met by the only solution at which the sum of yields is maximised right in the first step. Any other solution conflicts with a simultaneous meeting of both requirements. If we do not insist on the requirement of a single cost, there exist an endless number of different sequential solutions at which the sum of yields is maximised by gradual compensations.

On the basis of a given task, a great number of currently known solutions can be interpreted – dictator's, egalitarian, Kalai-Smorodinski's, Raiffa's solution, etc. At a certain shape of the function delimiting yields in the point of the maximum sum (unless the function is smooth), some of the solutions merge into one (those of Nash (1950), Kalai-Smorodinski (1977, 1975), Raiffa (1953)).

As the first step for evaluating the presented result theoretically, it is possible to compare the currently known solutions of the Nash (S, d) bargaining problem with the possibility of interpreting them from the perspective of their relevance (information values) for the problem of the "snag" in financial markets:

- Which of the known solutions have a real interpretation?

- For which do we lack an interpretation?

- Are there any of the solutions of the Nash (S, d) bargaining problem which, for principal reasons, cannot have any interpretation in a given area and why?

One of the extremely interesting applications is a survey of the financial market development which can be mentioned based on the above-mentioned facts. It is as follows:

– Imperfections of the financial market can be understood as impossibility of using all the resources (investment funding) according to their rate of return.

– One reason may be that problems can not be solved on the basis of maximization the amount and at the same time, only acceptable solutions are those which set a single cost.

– These solutions can be described empirically, notionally, conceptually and presumably axiomatically as well. It means that each imperfection agrees with a certain type of a cooperative game solution which can be expressed at a various level of abstraction.

The cooperative game theory then gives a highly effective instrument for describing financial markets and their development:

– Identification of standard situations.

– Identification of standard transitions between different stages and ways of the financial market development.

– Definition of ways of improving financial markets (or, on the contrary, their degeneration), etc.

It would be interesting to see whether there are other tasks which would similarly induces a wide spectrum of problems associated with a search and comparison of various cooperative games in terms of relation among reality, concept, model and different levels of abstraction, or transition up to its axiomatic expression. At this moment, we consider the area that we have touched upon particularly interesting from the above-mentioned point of view.

From theoretical, methodological and philosophical perspectives, these are interesting issues of relation of our real world on one hand and possible worlds on the other hand that are partly a result of our abstractions and partly corresponding to possible changes of our real world.

4.2. Outline of one of the possible applications of achieved results to examine current problems in financial markets in the Czech Republic

One of the conclusions arising from the attained findings can be briefly formulated as follows: If entities in the market of supply and demand of investment funding and investment opportunities are in an asymmetric position, when the stronger position belongs to the entity offering investment funding, this stronger entity will offer investment funding for differentiated costs in such a way to increase his yield as much as possible to the detriment of the other entity. After he uses investment opportunities of the other entity at the cost which is the most advantageous for the entity offering investment opportunities, (only then) he will offer his additional investment opportunities at a lower cost. This process can repeat.

Now, let us look at one of the topical or rather acute problems of the Czech financial market (to discuss the question of whether it shows the financial market "snag" problem by us revealed and analysed, or not). We will show that the concept of analysing the snag in financial markets can be used at analysing real situations which occur at present. Real situations always comprise multiple, various influences. A good model facilitates their precise identification. Let us try to do so in the context of the following situations that occurred in the Czech financial market.

Continuation in interventions of the Czech National Bank as the most visible part of its monetary policy under the current macroeconomic conditions results in accomplishing new "records" in some key monetary indicators. In recent days, the volume of ready money in circulation has exceeded CZK 500,000 million and we may discuss or rather speculate about reasons for its permanent growth without regard to the real economic growth.

On 13 November 2015, the total value of all banknotes and coins which are currently in circulation ("money in circulation") exceeded the amount of CZK 500,000 million for the very first time. This amount represents more than 2,000 million banknotes and coins in circulation. In the long term, the most numerous representations among Czech money have been had by the one-thousand-koruna banknote and the one-koruna coin, as shown in the statistics of the Czech National Bank.

A new record was also shown in CNB deposits of domestic commercial banks which recently for the very first time exceeded CZK one trillion, i.e. one million million. The fact that this amount is twice as big as the total value of all the banknotes and coins which are currently in circulation is related to the effect of the money multiplier, or more precisely to multiplication of deposits.

In the past, a long-term massive surplus of liquidity in domestic banks was invested in Czech state bonds that account for a majority of the securities in their portfolio structure. A high demand for Czech state bonds resulted in a slump of their yield, across maturities. In shorter maturities (up to two years), yields of Czech state bonds gradually moved to red figures and recent state bond auctions show that a negative yield appears in longer maturities (three to five years) as well. However, lower yields of Czech state bonds, also when compared with German bonds that are generally considered the lowest-risk state bonds, do not mean that the Czech Republic is a more solvent debtor than Germany, but they reflect the fact that almost all foreign investors expect the Czech koruna appreciation after the CNB's interventions end.

The fact that the time of foreign currency interventions is quickly drawing to an end and at the same time some members of the Bank Board admit aloud that interventions may continue even after the originally announced half of the year 2016 gives the impression of the Bank Board of the Czech National Bank not having any exit strategy from the intervention regime drawn up, let alone possible exit scenarios. At the high quality of work of Czech National Bank' analysts, it is strange that the possibility of a jump appreciation of the Czech koruna after the end of interventions is considered highly unlikely, opposed to a majority of economic experts. The development of the Czech economy in relation to its main business partners has been showing a convergent character in the long term and an artificial prevention of fluency of this convergence (through foreign currency interventions) only increases the future pressure on koruna appreciation. The situation when at comparable work productivity and costs of living the labour force receives a wage of third or fourth the amount (when compared to developed European countries) is not sustainable in the long run. And jumps and shocks of all kind rank among the factors that are not beneficial to any normally functioning economy. Without comparing the Czech Republic and the Swiss Confederation, the jump appreciation of the Swiss franc after the end of interventions that we witnessed is at least food for thought (not only for our central bankers). A permanent inflow of euros from the EU funds which will be higher than the outflow still for few years results in additional demand for Czech koruna and a pressure on its appreciation. Moreover, the sale of Czech state bonds with a negative yield at primary auctions (organised by the Czech National

Bank) indicates that all the "big players" in financial markets already expect the appreciation of koruna.

The current structure of the CNB' liabilities in		[million Kč]	
1.	Bank notes and coins in circulation	495 873	-829
2.	Liabilities to the IMF	49 655	918
3.	Liabilities abroad	21 792	-1 641
3.1	Loans from foreign banks	6 335	-3 000
3.2	Other liabilities abroad	15 457	1 359
4.	Liabilities to domestic banks	955 876	1 625
4.1	Loans received	395 300	81 100
4.2	Minimum reserve requirement	81 959	3 117
4.3	Other liabilities to banks	478 617	-82 592
5.	Liabilities to the state and other public institutions	1 913	107
6.	Other liabilities	12 174	147
7.	Reserves	282	0
8.	Registered capital and reserve funds	15 561	0
9.	Revaluation differences	11 670	0
10.	Profit or loss for the previous period	0	0
11.	Profit or loss for the accounting period	5 475	10 875
	LIABILITIES IN TOTAL	1 570 271	11 202

The current structure of the CNB' liabilities is illustrated in the table.

Now, let us view the topic in question from the perspective of two players of a cooperative game, when one of the players are commercial banks and the other player the other financial market stakeholders, including the Czech National Bank. It is rather simplified, especially with regard to the fact that:

– Commercial banks compete with each other.

– The other stakeholders in the financial market represent a substantially heterogeneous mix (starting from the already-mentioned Czech National Bank through the state bond market up to companies who are granted loans by commercial banks and last but not least households as the financial market stakeholders at whom consumer loans and mortgage loans in particular target).

With awareness of the reservations we have mentioned, it is however evident that the following behaviour prevails in commercial banks (that, as it turns out, act in a certain agreement):

– They first select such entities whom they can grant loans at a very high interest rate.

– They differentiate among them depending on how big the loans are, how high the related risk is, what the transaction costs are and how transparent the information about these stakeholders is.

– Subsequently, they invest a huge amount through deposits in the central bank at very low interest; (however, in the context of this particular investing as a cost of sacrificed opportunity in relation to the other investment possibilities) they strengthen their asymmetric or privileged position against the other owners of investment opportunities.

To what extent this phenomenon is related to our analysed phenomenon of the "snag" in financial markets is a subject of another analysis. It is necessary to take into consideration various aspects that we mentioned in connection with a reduction of the task to two players of the cooperative game. A certain connection undoubtedly exists here and the instruments of the financial market "snag" analysis discussed by us (as well as those that we will acquire by further examination directing the discussion towards theoretical and methodological aspects of the respective problem) may produce immensely interesting findings.

5. Conclusions

Since the early 1950s the cooperative game theory has been developing under the influence of theoretical problems arising both in the field of the game theory itself and in interaction with real problems outside the game theory (mostly in the economic field). It turned out that different applications or contexts correspond to different potential solutions of standard tasks, e.g. Nash (S, d) bargaining problem. Also, a number of different axiomatic systems describing the (S, d) problem have been established. Some of the solutions responded to the issues arising in the then important economic theory of social welfare, or the issue related to redistribution of wealth among various groups of population and justification of such redistribution. To do so, some of the arbitrary solutions of the Nash (S, d) bargaining problem were mostly applied.

In our contribution, we have presented an interesting and significant area of interpretation of different solutions of the Nash (S, d) bargaining problem in connection with some other issues of the cooperative game theory and the instruments employed by this theory (e.g. in relation to the water problem solution) at analysing supply and demand of investment funding and investment opportunities. And this both at the general level and in specific conditions of the Czech Republic, where we interpreted this task as a cooperative game of two players in which one of the players is commercial banks and the other player is the other financial market stakeholders, including the Czech National Bank. The interpretation and the subsequent discussion about the theoretical and methodological aspect of the topic in question have produced many interesting findings concerning the relation among the level of reality (practical application), a definition of assumptions (in the language of microeconomy), a drawing up of a concept (by defining assumptions based on the cooperative game theory) and a setting of a corresponding axiomatic system. Equally, the application to the issues of the Czech financial market and the related discussion about achieved results shows that it is a useful and promising topic.

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Cross-Border Collaboration in European-Russian Supply Chains: Integrative Approach of Provision on Design, Performance and Impediments

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Abstract The primary goal of this research paper is to provide new insights in the research area of supply chain collaboration. The research aims to deliver novel evidence if supply chain collaboration has a positive effect on improvement of firm performance and what mediates and moderates such effects in the case of the contextual cross-border inter-firm (EU15-Russia) research design based on a survey questionnaire (quantitative). The empirical results showed that in a cross-border inter-firm context, as in the case of EU15-Russia, supply chain collaboration has a direct positive impact on improvement of operational performance and achieving collaborative advantages. Furthermore, these collaborative advantages have a bifurcated effect on the relationship between supply chain collaboration and improvement of firm performance. Conclusively, in market situations and environments with impediments in the form of collaborative barriers and cross-border business barriers the positive effects of supply chain collaboration are amplified and strengthened due to unfavorable environmental dissimilarities in the market which makes it more difficult and harder to obtain them.

Keywords: Supply chain collaboration, collaboration barriers, firm performance, operational performance, collaborative advantage, cross-border business barriers, mediation, interaction moderation.

1. Introduction

International trade and efficient supply chain management and operations are a valid and essential method towards firm growth, increase of sales and firm performance, and to reach higher levels of internationalization for many companies in today's globalized world. Disruptions of the status quo, changes in technology, and globalization of products and services have resulted in an increase of dynamic markets and uncertain environments. Nowadays, customers are better informed, have greater access to a wider choice of products, and have access to new products emerging at a faster pace. These developments and alterations in the current situation have significant consequences and implications on the network and design of supply chains and business operations in general, and its influence on the value chain of supplier-buyer dyads (Wuyts and Geyskens, 2005). The everlasting fierce competition in global markets, the introduction of products with shorter product life cycles (PLCs), and the heightened demands of customers have forced firms to invest in resources and to pay more attention to stronger mutually beneficial relationships and supply chains. (Deloitte, 2012). Facing and dealing with uncertain developments and dynamic environments, firms are striving to achieve greater collaboration in supply chains to leverage the resources and knowledge of their suppliers and buyers (Fawcett and Magnan, 2004; Lejeune and Yakova, 2005). Therefore, in today's global market, firms no longer compete as independent entities but rather as integrative parts of collaborative networks, grouping entities for allowing them to seize opportunities and possibilities to which a single entity could not achieve alone (Msanjila and Afsarmanesh, 2010).

To meet the requisites and the demands of the current markets and customers, firms try to develop value-added processes that deliver innovative, high-quality, low-cost products on time, with short cycle times and greater responsiveness than ever before. In order to do so, firms are transitioning from transactional supplier relationships to more transparent and collaborative relationships designed and constructed to achieve mutually beneficial outcomes (Deloitte, 2012).

Hence, firms began to perceive that it is not enough to improve efficiencies within their organization alone. Firms are looking outside their internal organizations for opportunities to collaborate with supply chain members to ensure that the supply chain is efficient and responsive to dynamic market's needs. The future belongs to integration and collaboration of supply chains. As business increasingly relies on other firms, especially in industrial and consumer products industries, the need to effectively manage external relationships is of considerable importance. The ability to achieve effective collaboration becomes a strategic imperative in the era of information and globalization.

The debates in contemporary supply chain management (SCM) literature centers around supply chain collaboration (SCC) (Siew et al., 2012; Cao and Zhang, 2011; Christopher, 2011; de Leeuw and Fransoo, 2009) and its impact on firm performance (Stank, Keller and Daugherty, 2001). Despite the success stories (Hofman and Aronow, 2012), SCC inconsistencies have also been reported by researchers and scholars (Bragg et al., 2011). Hence, the key question of whether SCC has a specific positive impact on firm performance is still a subject of debate (Gunasekaran and Ngai, 2012).

These inconsistencies lead to the purpose and objective of this research paper to contribute to the SCC domain by testing SCC peculiarities of theoretical concepts in the existing latest academic literature by means of SCC design and its relationship and effects on operational performance and firm performance. The emphasis of this study is to research how these theories work in practice in a specific cross-border inter-firm supply chain context and how impediments in the form of collaboration barriers and cross-border business barriers moderate and mediate the effects of SCC on operational and firm performance.

2. Inception and Ascent of Supply Chain Collaboration

In today's business world, which is characterized by globalization, increased customer responsiveness, customer expectation, channel integration and advances in information and communication technologies under increasing uncertainty (Schoenherr, 2009), firms have no other option than participating in a supply chain. Thereupon, collaboration between firms plays a significant role for improving firm performance and to capitalize on sustained competitive advantage (Grant, 2012; Gunasekaran and Ngai, 2012; Hassini et al., 2012; Cao and Zhang, 2011), which then can help and improve economic and financial development (Mefford, 2011).

When talking about collaboration, a substantial amount of previous scholars and researchers cite mutuality of benefits, rewards and risk sharing together with the exchange of information as the foundation of collaboration (Stank et al., 1999; Barratt and Oliveira, 2001). Considered as an establishment for governing organizations in firms (Stein, 1982), it is believed that collaboration in supply chains could realize exceptional benefits (Mena et al., 2009).

In conclusion, it can be said that collaboration in supply chain means that different firms involve themselves in the flow of products and information from raw materials to end consumer in order to fulfill customer needs. The areas and functions in which firms can collaborate are, for instance, supply chain design (procurement, transportation and distribution); manufacturing (planning, inventory management, product design and development) and order fulfillment (including order processing, sales, customer service and demand management) (Anderson and Lee, 2001; Ellaram, 1995; Horvath, 2001).

Collaboration in business can be found in both inter-firms and intra-firms and ranges from simple partnership to complex multinational corporation. The importance of inter-firm coordination and integration are examined by most scholars and researchers as key tasks for SCM. Coordination and integration is achieved by collaboration among actors in the supply chain. In fact, Horvath (2001) argues that collaboration is a prerequisite to achieve SCM; without collaboration, there can be no SCM. There are several ways to categorize SCC (Simatupang and Sridharan, 2005). Holweg et al. (2005) classified SCC into four types based on inventory and planning coordination (Mena et al., 2009).

On the other hand, Barratt (2004) distinguishes between internal and external collaboration and whether it is vertical or horizontal. Vertical collaboration performs both internally or along the supply chain. In case of external collaboration, along the supply chain, it means working more closely with trading partners to improve each other's efficiency for collective advantages and benefits. The focus is on giving and gaining visibility into each other's processes so that each of the supply chain members can do a better job. The study in this research paper only deals with external collaboration in a vertical direction. Thus, external inter-firm vertical collaboration.

3. Supply Chain Collaboration Synopsis

As the purpose of collaboration is to optimize profitability, supply chain members need to plan, execute, and control key decisions related to defining and delivering products to the end customers. By practicing and executing SCC dimensions, firms have the opportunity to obtain and achieve collaborative advantages over its competitors. In addition, the dimensions of SCC and the achieved collaborative advantages are expected to have positive effects, consequences and outcomes on both operational performance and firm performance. The direct relationships between these constructs may be mediated through these collaborative advantages and moderated by the constructs collaboration barriers and cross-border business barriers, which are expected to have a negative direct main effect on its respective dependent performance variables.

3.1. Supply Chain Collaboration Dimensions

In SCM there are different dimensions of collaborative approaches such as information sharing, incentive alignment and decision synchronization (Holweg and Pil, 2008; Akintoye et al., 2000; Spekman et al., 1998). SCM and collaborative performance system (CPS) requires information sharing, decision synchronization, and incentive alignment to monitor and improve actual firm performance. Information sharing reports data about performance status. Decision synchronization allows supply chain members to optimize performance metrics through effective joint decision making. Incentive alignment employs performance metrics to construct benefit and cost sharing agreements. Integrated supply chain processes provide feedback about the actual benefits of collaboration based on the status of physical supply chain events.

Information sharing refers to the access to private data in supply chain members' information systems enabling monitoring of the progress of products as they pass through each process in the supply chain (Simatupang and Sridharan, 2002). This activity covers data acquisition, processing, representation, storage, and dissemination of demand conditions, inventory status and locations, order status, transparency of costs, and performance status. Visibility of key performance metrics and process of data enables the participating supply chain members to obtain the bigger picture of the situation which includes important factors for making effective decisions. Effective decisions enable supply chain members to address product flow issues more quickly, and thereby allowing more agile demand planning to take place. Several criteria, such as relevancy, accuracy, timeliness, and reliability, can be used to judge the contribution of information sharing to the integration of supply chains.

The supply chain members are interested in the utility of information sharing rather than information for its own sake. What makes information sharing valuable to supply chain members is eventually the ability to make better decisions and to take actions on the basis of greater visibility (Davenport et al, 2001). Core guidelines are that visibility should inform action, and that action becomes visible if supply chain members understand better the underlying principles that link integrated information and performance drivers. Thus, information sharing generally facilitates decision synchronization through providing relevant, timely, and accurate information required to take effective decisions about supply chain planning and execution. It enables participating supply chain members to make use of integrated information to help fulfill demand more quickly with shorter order cycle times. For example, demand and inventory visibility can be used to eliminate stock-outs by accurately replenishing fast-movers (Fisher, 1997).

In connection with supply chain performance, information sharing provides data about the progress of collaboration and performance status to supply chain performance. Supply chain managers and professional can use this data to evaluate and construct new targets and performance metrics that are relevant to new and volatile market dynamics and situations. In conjunction with incentive alignment, information sharing provides visibility about the status of incentive scores of supply chain members. It also reveals the actual link between performance measures and incentives. Finally, integration of supply chain processes provides primary useful field data about product, process, and performance status.

Decision synchronization can be defined as the extent to which supply chain members are able to orchestrate critical decisions at planning and execution levels for optimizing supply chain profitability (Simatupang et al., 2002). The activity covers constructing joint decision making processes, including reallocating decision rights in order to synchronize supply chain planning and execution that seeks to match demand with supply. The method to assess effective decision synchronization hinge on the effects of accurate response towards fulfilling customer demand and supply chain profitability (Corbett et al., 1999). Face-to-face meetings and virtual discussion to take certain decisions are examples of possibilities to implement decision synchronization.

The significance of decision synchronization is embedded in the fact that supply chain members have different decision rights and expertise about supply chain operations. For example, a retailer may have the decision right to determine order quantity but not order delivery. In most of the situations supply chain members have conflicting criteria in making decisions resulting in solutions that are less optimal for the overall and whole supply chain (Lee et al., 1997). Supply chain members need to coordinate critical decisions that affect the way they achieve better performance. The use of joint decisions depends on the incremental sales that can be realized and the significant amounts of inventory costs that can be reduced from this joint decision making. Joint decisions may include sales and order forecasts, inventory, replenishment, order placement, order delivery, customer service level, and pricing. For example, vendor managed inventory (VMI) provides the supplier with decision rights to determine the frequency and quantity of orders that need to be delivered. This scheme enables the supplier to match supply with demand from the supply chain wide perspective and thereby improves profits for both supply chain members.

Decision synchronization provides feedback to supply chain performance concerning how performance metrics guide the supply chain members to make effective decisions. In relation to information sharing, decision synchronization aids and enhances information sharing to identify what kind of relevant data should be collected and transferred to the decision makers. In supporting incentive alignment, decision synchronization provides justification for incentive alignment to construct appropriate incentive schemes, because different supply chain members are responsible for different levels of decision making. Finally, decision synchronization helps supply chain members to carry out productive actions associated with integrated supply chain processes such as replenishment, transportation, and customer service.

Incentive alignment refers to the process of sharing costs, risks, and benefits among the participating supply chain members (Simatupang and Sridharan, 2002). This scheme motivates supply chain members to act in a consistent manner with their mutual strategic objectives, which includes making decisions that are optimal for the overall supply chain and revealing truthful private information. It covers calculating costs, risks, and benefits as well as formulating incentive schemes such as pay-for-performance and pay-for-effort. The contribution of incentive alignment can be judged based on compensation fairness and self-enforcement. Compensation fairness ensures that aligned incentives motivate supply chain members to share properly and equally the loads and benefits that result from collaborative efforts. An effective incentive scheme means that supply chain members are self-enforcing for aligning their individual decisions with the mutual objective of improving total profits. Expert systems, activity-based costing, and web-based technology can be used to trace, calculate, and display incentive scores (Kaplan and Narayanan, 2001; Simatupang and Sridharan, 2002).

The theory behind incentive alignment assumes that an individual supply chain member tends to act in a certain way based on the expectation that the act will result in mutual benefit and on the attractiveness of that benefit to individual supply chain members (Simatupang et al., 2002). An appropriate incentive scheme can be formed in a number of ways (Simatupang and Sridharan, 2002). Pay-for-effort is a scheme that links payment and effort. This assumes that rewarding effort would motivate the individual supply chain member to expand a given amount of effort which relates to a certain level of performance. Pay-for-performance is a scheme that links payment and performance. This scheme assumes that rewarding performance will motivate the individual supply chain member to achieve a particular level of performance. The supply chain members accept the importance of the potential rewards that can be obtained from collaboration although costs need to be shared. The interaction of incentive alignment with other SCC dimensions and indicators is very profound as it motivates supply chain members to align their actions to the mutual purpose of collaboration that would also enhance their individual profitability. Incentive alignment links performance scoreboards from supply chain performance to incentives. The more transparent the linkages between performance and incentives, the more effectively the given incentives are able to motivate the desired and required behavior. Information sharing is required to signal supply chain members that incentives are available, timely and proper. In conjunction with decision synchronization, incentive alignment provides incentives to motivate supply chain members to make effective decisions that reinforce the desired level of performance.

Recent research have focused on the development of SCC models that reflect the latest understanding of collaboration which includes four new dimensions in addition to the aforementioned three (Cao and Zhang, 2011; Nyaga et al., 2010; Ramanathan et al., 2011). Latest research topics on SCC dimensions have shown that the most dominant SCC dimensions consists of information sharing (Manthou et al., 2004), goal congruence (Angeles and Nath, 2001), decision synchronization (Stank et al., 2001), resource sharing (Sheu et al., 2006), and incentive alignment (Simatupang and Sridharan, 2005) among independent supply chain members. However, the study in this research paper defines SCC dimensions as the following seven intertwined components and indicators: information sharing, goal congruence, decision synchronization, incentive alignment, resources sharing, collaborative communication, and joint knowledge creation.

These seven dimensions of SCC are expected and ought to be interwoven with each other. Each of all the dimensions and components of SCC add value by reducing response time, leveraging resources and improving innovation. Besides the aforementioned initial three core SCC dimensions, contemporary research has identified the following SCC dimensions and components, namely goal congruence, resource sharing, collaborative communication and joint knowledge creation.

Goal congruence between supply chain members is the extent to which supply chain members notice their own goals and objectives are achieved and satisfied by accomplishing supply chain objectives. It can be said that it is the degree and level of goal agreement among supply chain members (Angeles and Nath, 2001). Supply chain members either feel that their objectives fully coincide with those of the supply chain, or if there is discrepancy, that their goals and objectives can be realized as a direct result of working toward the goals and objectives of the supply chain as a whole (Lejeune and Yakova, 2005).

Resource sharing indicates the process of leveraging capabilities and assets and investing in it with supply chain members. From the viewpoint of resource based view (RBV) theory, resources include physical resources, such as manufacturing equipment, facility, and technology (Harland et al., 2004). Practices of SCC models such as VMI are used to allow suppliers to assess stock level data and take the required replenishment actions (Lamming, 1996).

Collaborative communication encompasses the contact and message transmission process among supply chain members through frequency, direction, mode, and influence strategy. Tight and close inter-firm relationships are generally open, frequent, balanced, two-way and mutual, and multilevel communications (Goffin et al., 2006). On the contrary, Mohr and Nevin (1990) highlighted patterns of communication from a mechanistic perspective. Both provide evidence that collaborative communication has higher frequency, more bidirectional flows, better informal modes, and increased indirect influence. Frequency relates to the amount of contact between supply chain members. Direction concerns to the movement of communication up and down the supply chain (Mohr and Nevin, 1990; Prahinski and Benton, 2004). Mode of communication refers to method that is used to transmit information. Informal communication covers the degree and level to which communication among supply chain members is formed through a spontaneous and non-regulated way. While direct influence aims to change behavior by requesting specific actions from its supply chain members using recommendations, promises, and tends towards legal obligations, indirect influence focuses to change the supply chain members' belief and attitudes about their desires of intended behavior without explicit commanding or threats (Mohr and Nevin, 1990).

Joint knowledge creation is the extent to which supply chain members develop a better understanding of the market and exogenous factors that influence it by collaboration (Malhotra et al., 2005). According to Harland et al (2004), there are two types of knowledge creation activities: knowledge creation such as search and acquire new and relevant knowledge, and knowledge exploitation such as comprehend and apply knowledge. The capture, exchange, and absorption of knowledge among supply chain members allow innovation and enhance it to realize long-term competiveness (Harland et al., 2004). Supply chain members should not only be involved in creating and building a knowledge framework, but also be engaged in interpreting knowledge, which allows firms to create added value through developing new products, building brand image, and satisfying customers' needs (Kaufman et al., 2000). Recent research has shown that the value of SCC is not only limited towards efficiency improvements, moreover it has strategic benefits, which aids the value chain respond to competition and increasingly satisfy the needs of the customers (Sobrero and Roberts, 2001).

SCC accelerates a firm's ability and capability to capitalize swiftly on market opportunities (Uzzi, 1997). As an example, problem solving increases the velocity that products are introduced to the market by resolving and overcoming thresholds at a faster pace. Collaboration between supply chain members can eventually lead to unique sources that enhance new product ideas (Kalwani and Narayandas, 1995). Shared resources between supply chain members could result in reduction of sub-additive cost, or complementary resources, which increases super-additive value (Tanriverdi, 2006). SCC for resource sharing and replenishment will result in significant cost reduction in supply chain processes and activities. Such sources of business synergy can lead to competitive advantage outcomes.

Furthermore, information sharing among supply chain members guarantees on-time replenishment (Cachon and Fisher, 2000). It also supports supply chain members to be involved and engaged in inventory pooling and joint replenishment activities (Ramanathan, 2012). Overall, adequate information technology is reliant on the associated benefits of supply chains such as cost reduction or sales incentives (Toktay et al., 2000). Therefore, complementary to collaborative planning activities and collaborative decision making process, the activities and execution of the embedded dimensions and components of SCC help to improve and optimize supply chain processes.

3.2. Collaborative Advantages

SCC relates to the desired synergy outcomes of SCC activities that could not have been realized by a firm individually (Vangen and Huxham, 2003). Collaboration between supply chain members has the prospect to increase the size of joint benefits and to give each of the supply chain members a share of greater gain that could not be generated by a firm alone. The value that is created by collaboration could be in the form of either cost savings and/or cost avoidance, enhanced capacity and flexibility for collective actions, better decisions making and a surge in revenue by resource synergy and innovation through the combination and interpretation of ideas. Therefore, collaborative advantage comprises the following five dimensions: process efficiency, offering flexibility, business synergy, quality and innovation.

Bagchi and Skjoett-Larsen (2005) have shown that a firm's collaboration process with other SCC partners is cost competitive among its competitors. This form of process efficiency could be information sharing, joint logistics process, joint product development process or joint decision making process. Hence, process efficiency is a rate of success and a determinant influencing factor on a firm's profit (inventory turnover and operating cost). The benefits of collaboration include cost reductions and revenue enlargements (Lee et al., 1997).

Offering flexibility points out to how a firm's supply chain correlates and adapts to changes in product or service offerings, volume, speed, features, and specifications, in reaction to environmental and business changes. Generally, it is also called customer responsiveness based on the existing literature (Kiefer and Novack, 1999; Holweg et al., 2005). Offering flexibility encompasses the ability of the collaborating firm to swiftly change process structures or to accustom information sharing process for altering the features of a product (Gosain et al., 2004). Nowadays, firms pay more and more attention to customers and an increasing amount of firms use customer input at the design stage resulting in better product acceptance (Bagchi and Skjoett-Larsen, 2005).

Furthermore, supply chain members combine complementary and related resources to obtain considerable benefits in the form of business synergy. Ansoff (1988) found that synergy can lead to a combined return of resources that is greater than the sum of individual parts. This collaborative effect results from the process of making more efficiently use of resources in the total supply chain, including physical assets such as manufacturing facilities and intangible assets such as customer knowledge, technological competence, and organizational culture (Itami and Roehl, 1987).

It is expected that firms that are able to respond and adapt quick and agile to customer demand with high quality products, innovative design and perfect after sales service supposedly build customer loyalty, increase market share and finally receive higher profits. On the other hand, Garvin (1988) mentions eight dimensions of quality, namely: performance, features, reliability, conformance, durability, serviceability, aesthetics, and perceived quality, which are comprehensive but measures for each are difficult to establish and to create. By reason of shorter PLCs, firms have to innovate frequently and in small increments (Handfield and Pannesi, 1995; Kessler and Chakrabarti, 1996). In order to innovate, firms work jointly with their supply chain members in introducing new processes, products or services. Firms improve their ability to engage in process and production innovation by carefully managing their relationships with suppliers and customers (Kaufman et al., 2000). Firms have the opportunity and possibility to improve absorptive capacity which could lead to fast and frequent introduction of new products by systematically joint creativity capacities, joint organization learning, knowledge sharing and joint problem solving between supply chain members.

The imperative condition for SCC is that supply chain members are capable to increase the total gain due to synergy (Simatupang and Sridharan, 2005). Supply chain members will obtain financial benefits by enhancing responsiveness (Fisher 1997). Another advantage of SCC is to achieve cost savings and reduce and terminate non-value added activities and/or duplication of efforts (Lambert et al., 2004). The cooperation between supply chain members can foster greater knowledge and result in synergetic benefits. Initially, firms will obtain operational improvements through SCC such as higher order fulfillment, lower total logistics costs and less stock-out. In the long-term horizon, SCC will be beneficial through more competitive products and shorter time-to-market that will transform into possible competitive advantages and increased profits (Stuart and McCutcheon, 1996). Hence, collaborative advantages will eventually lead to improved operational performance and firm performance which comprises on how a firm fulfills its operational and financial goals compared with other firms (Yamin et al., 1999; Barua et al,. 2004; Li et al., 2006).

Besides, Frohlich and Westbrook (2001) observed that firms who have implemented an effective SCC enjoy the largest rates of performance improvement across multiple marketplaces (market share and profitability), productivity (cost and lead-time) and non-productivity (customer service, quality and delivery) measures.

3.3. Operational and Firm Performance

SCC has been linked to enhance and boost firm performance (Simatupang and Sridharan, 2004; Squire Cousins, Lawson and Brown, 2009; McLaren et al., 2002). By working with other supply chain members, firms are expected to multiply the outcomes of the effort from working alone (Wilding, 2006). Such outcomes and results consist out of a better level of responsiveness and service level improvements from their SCC programs (Holweg et al., 2005). One more expected benefit is the reduction of supply chain costs like those that reflect with inter-firm transactions, inventory and production (McLaren et al., 2002).

Many studies have concluded that a higher degree of SCC can improve firm performance (Nyaga et al., 2010; Robson et al., 2008) especially on their logistics activities (Ha et al., 2011). Further, success of collaboration could also lead to more collaborative actions in the future (Ramanathan and Gunasekaran, 2012). Moreover, higher levels of collaboration could lead to elimination of bullwhip effect, inventory reduction, better transport capacity utilization, and risk mitigation (Holweg et al., 2005).

Financial performance and operational performance can be measured in terms of firm performance (Chen and Paulraj, 2004). In addition, firm performance can also be viewed through the lens of service effectiveness and cost effectiveness (Richey et al., 2010). Performance can be also measured by cost, quality, delivery and flexibility (Krause et al., 2007). In general, supply chain performance measures include: order lead-time, inventory levels, time-to-market, quality, customer service, and flexibility (Bhatnagar and Sohal, 2005; Gunasekaran et al., 2001). Bagchi et al. (2005) measured performance in eight dimensions, namely: order fulfillment lead-time, order fill rate, production flexibility, total logistics costs, return processing costs, inventory days of supply/inventory turnover rate, on-time delivery, and rate of returns. Generally, logistics performance is a key determinant for maintaining relationship with the other supply chain members (Glenn Richey et al., 2010; Ellinger et al., 2006, 2000; Beamon, 1999). Hence, supply chain members gravitate to be more satisfied when their performance of logistics is improved (Gunasekaran et al., 2001).

Furthermore, according to Porter (1980), the two generic competitive strategies are cost advantage and differentiation. Cost advantage can be realized through reducing costs, while on the other hand differentiation increases profitability by providing increased levels of customization and service. By means of efficient and effective order capture, product availability, on-time delivery, information transparency and improved responsiveness, a firm can increase its level of service. Further, SCC creates elements of differentiation by means of customer value which is formed by superior service (Christopher and Peck, 2003). In addition, there is a positive relationship between high service levels and growth of sales volume and customer retention (Parasuraman et al., 1991; Mentzer et al., 1999; Ray et al., 2004). Therefore, this stipulates that SCC should be the silver bullet for reducing costs without negatively impacting customer service and improving service without exponentially increasing costs. Increase of operational performance and efficiency by means of cost savings, inventory reduction, planning accuracy and improved responsiveness could eventually lead to increase of sales and reduction of costs. Supplementary, successful SCC and firm performance can be determined in terms of market share and satisfaction of SCC (Mishra and Shah, 2009).

3.4. Collaboration Barriers

Although, according to the existing literature, SCC amongst independent firms often result in improvements and larger benefits from effectively satisfying end customers' needs, lack of awareness about the existence of barriers of collaboration burdens to grasp the benefits of it. Based on recent literature, there are several identified SCC barriers that have a negative direct effect on realizing collaborative advantages by SCC. The final list of collaborations barriers was partially adapted from the study of Ramesh, Banwet and Shankar (2009).

In most of the supplier-buyer dyads, trust is acknowledged as an essential element to bind independent firms (Agarwal and Shankar, 2003). Trust can only exist when firms believe that its supply chain member is reliable and benevolent (Heikkila, 2002). On the other hand, Chung et al. (2008) mentions that human relations like trust or long-term orientation are a tremendous important aspect in relationships.

Thus, according to Cetindamar et al. (2005), a lack of trust is the core argument behind the difficulties in establishing collaboration between firms.

In order to achieve success, when firms decide to collaborate with each other, it is important to educate and train employees about awareness of SCC and its core principles to exploit collaboration and to improve business processes to realize the advantages. A severe lack of understanding and awareness of SCC among the employees could result in significant diminishing of positive spillover effects of collaboration advantages (Ramesh, Banwet and Shankar, 2009).

Besides trust, commitment to collaboration and to relationship is also considered to be an important aspect as an enduring ambition and longing to maintain a fruitful relationship (Moorman et al., 1992). Morgan and Hunt (1994) concluded that commitment was the core component to successful long-term relationships.

The inability of vision and understanding of the supply chain is also a barrier to effective collaboration. As mentioned in the Introduction chapter, supply chains are getting more sophisticated which results in limited view and understanding of the entire and complete supply chain. Individual firms focus on their own functional areas and fail to recognize how collaboration with others, both inside and outside the firm, will improve overall performance (Mentzer et al., 2000). The inter-firm comfort levels of collaboration could increase as managers begin to understand the importance of integrated business processes and commit to working for the betterment of the whole supply chain.

The depth of collaboration also dependents on the supply chain members' technological capabilities. According to Kwan (1999), in cases that supply chain members are incapable to exchange information electronically due to low IT resources it hinders and forms barriers and thresholds to implement and optimize collaboration.

Information sharing is determined as a core requisite for collaboration. Multiple studies (Bowersox et al., 2003; Cannon and Perreault, 1999) pointed out that successful supplier-buyer relationships are connected with high level of information sharing. Low level and inadequate information sharing could lead to low level of trust and commitment that harms the efforts of collaboration.

For a long-term relationship focus between supplier-buyer, risk and reward sharing is an important factor. According to Spekman et al. (1998), firms collaborate to share risks and benefits in order to create competitive advantage. In addition, Kaufman et al. (2000) and Kotabe et al. (2003) emphasized that it is essential that channel participants in a supply chain share risks and rewards.

The lack and inconsistency of appropriate performance metrics and measurement systems results in the barrier for supply chain alignment between supply chain members (Fawcett and Magnan, 2001). This could lead to conflicts, because firms are focusing on improving their key performance indicators (KPI) and metrics rather than the performance metrics of the whole supply chain performance.

In conclusion, lack of awareness about the existence of barriers of collaboration hinders to realize the benefits of collaboration. Therefore, it is important to know and identify the barriers of collaboration so that the collaborative decision makers can focus on how to overcome and manage these collaboration barriers in order to obtain higher benefits out of SCC.

3.5. Cross-Border Business Barriers

Cross-border barriers can be defined as the attitudinal, structural, operational and other constraints that hinder a firm's ability to initiate, develop or sustain international operations (Koksal and Kettaneh 2011). Therefore, in the context of this paper, which covers cross-border SCC, it is important to achieve a better understanding of such barriers since these barriers waste resources of firms and threaten the efficiency, effectiveness and profitability of a firm's operations.

The cross-border business barriers that were the most suitable and appropriate for the study in this research paper were used and included, which are partially derived adapted from current literature, such as Leonidou (2011). This careful selection of cross-border business barriers, based on their relevance in context of the study in this research paper, are depicted in the table below.

Table 1: Frequent indicated cross-border business barriers

Cross-border business barrier	Authors
Strong international competition	Leonidou (2000); Da Silva et al. (2001); Ortega (2003); Ahmed et al. (2004); Altintas et al. (2007); Koksal et al. (2011); Mpinganjira (2011)
High business risk	Leonidou (2000); Kneller et al. (2011)
Different customer culture	Leonidou (2000); Altintas et al. (2007)
Unfamiliar foreign business practice	Leonidou (2000); Altintas et al. (2007)
High tariff and non-tariff barriers	Leonidou (2000); Ahmed et al. (2004); Altintas et al. (2007); Koksal et al. (2011)
Unfavorable foreign exchange rates	Leonidou (2000); Da Silva et al. (2001); Kneller et al. (2011)
Lack of government assistance	Leonidou (2000); Ahmed et al. (2004); Altintas et al. (2011)
Restrictive rules and regulation	Leonidou (2000); Mpinganjira (2011)
Transportation difficulties	Leonidou (2000); Mpinganjira (2011); Kneller et al. (2011); Koksal et al. (2011)
Bureaucratic requirements	Leonidou (2000); Altintas et al. (2007); Mpinganjira (2011)
Limited information about foreign markets	Leonidou (2000); Mpinganjira (2011); Koksal et al. (2011)

Source: Partially adapted from Leonidou (2011)

4. Hypothesis Development

From the theoretical background and the literature review, it has become apparent that there are several prominent dimensions in SCC that are pivotal in the integrative and integral process. By practicing and executing SCC dimensions, firms have the opportunity to obtain and achieve collaborative advantages over its competitors. In addition, the dimensions of SCC, and the achieved collaborative advantages, are expected to have positive consequences and outcomes on both operational performance and firm performance. The direct relationships between these constructs may be mediated through these collaborative advantages and moderated by the constructs collaboration barriers and cross-border business barriers, which are expected to have a negative direct main effect on its respective dependent performance variables.

The developed conceptual SCC framework suggests that supply chain members need to embrace SCC dimensions and to conduct and perform the dimensions of SCC properly. If a firm accomplishes to do so, the properly executed SCC dimensions will lead to efficient and effective collaborative planning activities, collaborative decision making processes and collaborative advantages, which in its turn will have a positive direct and indirect impact on firm performance and operational performance.

Based on the results of the literature review, several relevant latent constructs were identified and defined, namely: supply chain collaboration dimensions (SCCD), collaborative advantage (CA), operational performance (OP), firm performance (FP), cross-border business barriers (CBBB) and collaboration barriers (CB). Furthermore, each of the latent construct consists out of several pivotal and key variables and items. The latent construct SCCD has 7 variables, CA has 5 variables, OP has 5 variables, FP has 4 variables, and CBBB and CB have 9 variables. The identification of the latent constructs were converged to the following formulated hypotheses per latent construct.

Supply chain collaboration dimensions (SCCD):

H1a: Supply chain collaboration dimensions have a significant positive direct effect on operational performance

H1b: Supply chain collaboration dimensions positively impacts collaborative advantage at a significant level.

H1c: Supply chain collaboration dimensions have a positive significant direct impact on firm performance.

Collaborative advantage (CA):

H2a: Collaborative advantage has a positive direct significant impact on operational performance.

H2b: Collaborative advantage has a direct positive significant influence on firm performance.

H2c: Collaborative advantage positively mediates the positive relationship between supply chain collaboration dimensions and operational performance.

H2d: Collaborative advantage positively mediates the positive relationship between supply chain collaboration dimensions and firm performance.

Operational performance (OP):

H3: Operational performance has a direct positive significant impact on firm performance.

Collaboration barriers (CB):

H4a: Collaboration barriers positively moderate the positive effect and relationship between supply chain collaboration dimensions and operational performance.

H4b: Collaboration barriers positively moderate the positive effect and relationship between supply chain collaboration dimensions and collaborative advantage.

H4c: Collaboration barriers positively moderate the positive effect and relationship between supply chain collaboration dimensions and firm performance.

Cross-border business barriers (CBBB):

H5a: Cross-border business barriers positively moderate the positive effect and relationship between collaborative advantage and operational performance.

H5b: Cross-border business barriers positively moderates the positive effect and relationship between collaborative advantage and firm performance.

The identified theoretical latent constructs were conceptualized to study the effects and relationships. The conceptual SCC hypotheses framework that was conducted for the study in this research paper, including the relationships between the different constructs, mediation and interaction moderations, is visualized in Figure 1.

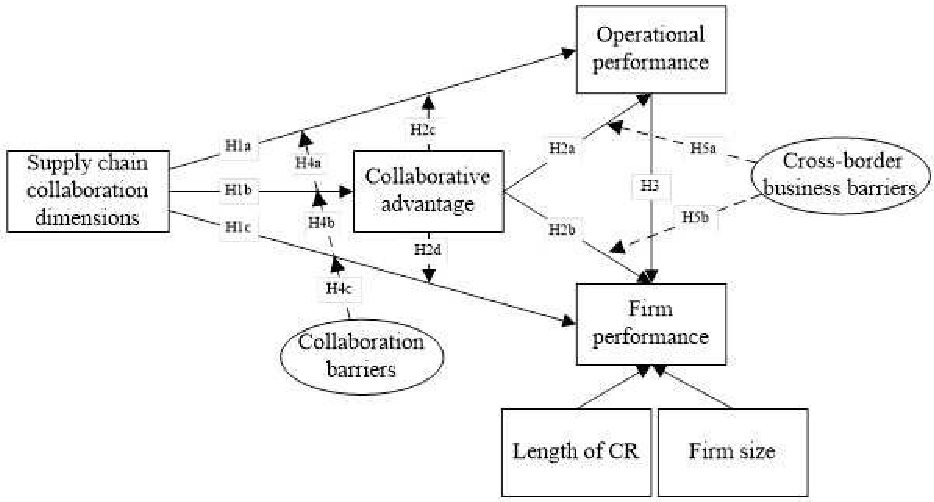


Fig. 1: Conceptual supply chain collaboration hypotheses framework
Source: partially adapted from Ramanathan and Gunasekaran (2012); Zhang and Cao (2011)

5. Research Approach and Design

The study in this research paper is considered to be explanatory and deductive, concerning the latent constructs in the conceptual SCC framework. According to Hussey et al. (1997), deductive research is a study in which a conceptual framework and theoretical structure is developed and then tested by empirical observation. For this reason, the deductive method is referred to as moving from the general theoretical concepts and theories of SCC in the existing literature to the particular usefulness in practice for supply chain managers, professionals and practitioners.

In this study the conceptual SCC framework was examined by means of a web-based survey questionnaire. Analyses were performed after collecting and compiling all the data. The literature on SCC inception, design, advantages, operational performance and firm performance, as well as on impediments and barriers, was reviewed first in order to formulate the hypotheses for this study.

5.1. Scope and Delimitations

The geographical scope was constrained to the initial 15 member states, which are: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, and the United Kingdom. The core argument and main reason to constraint the EU to its initial 15 member states, which nowadays comprises 28 member states, is that the integration of the initial EU15 member states is more mature and profound, and the economy and financial institutions are more interwoven and intertwined with each other than in comparison with the New Member States (NMS) of the Eastern enlargement. Furthermore, a study of European Association of Comparative Economic Studies (EACES 2013) enumerated that the largest bilateral trade flows still take place between neighboring sovereign countries and geographical areas: between Japan and China, the EU15 and Russia, the US and Mexico. Hence, for the EU15, Russia is a more important market than, for instance, China.

Strictly speaking, this development and situation is strengthened by the same study of EACES (2013) in figures by stating that the EU15 exports recorded their biggest increase in medium-high technology (MHT) sectors to Russia (machinery, motor vehicles and chemicals) and in high-technology (HT) goods to Russia (electronics and pharmaceuticals). In addition, there is also an increase observed in exports of capital goods to Russia, which received in 2008 almost one-third of the EU15 exports, and has thus overtaken China as its most important market. Russia was also the largest market for consumption goods, absorbing half the EU15 exports in 2008.

From a practical point of view, whereas the European Union (EU) facilitated economic and trade integration that makes the flow of goods and collaboration across national borders simple and smooth, the importance of cross-border context is still high in many parts of the world. Such as the eye-catching case of the EU and Russia, where the national and cultural differences in many aspects and elements are quite substantial. While Russia seems to offer low-cost production opportunities and possibilities for international manufacturers, the study of Hilmola et al. (2008) showed how difficult it is to achieve cost efficiency in Russian operations in comparison to similar operations in more mature markets. Furthermore, western markets are starting to get saturated and European firms are starting to look for new growth opportunities in emerging economies. Countries that used to be targeted for low cost sourcing are now changing into attractive end markets due to their increase in GDP and disposable income. Russia is one of those markets. However, besides the fact that the market is booming, its business environment shows differences in comparison with the European ones and poses specific challenges. This complicates the supply chain because it not only needs to be globally managed but also adapted to local conditions. Therefore, export and supply chain operations of semi-finished and final goods from the EU to Russia are likely to maintain and increase, accentuating the importance of cross-border context in terms of SCM. Conclusively, European firms perceive difficulties in the ease of doing business in Russia in areas of customs regulation, bureaucracy, uncertainty and logistics and transport (Finnish-Russian Chamber of Commerce, 2004).

In conclusion, Russia is a key factor for the EU15 performance in the aforementioned markets, which can be explained by the geographic proximity and the nature of the Russian import demand (capital goods, MHT and HT products). The trends in the EU15 export intensities to the large emerging economies show that Europe has by far outperformed the other suppliers in the Russian market. In the other large emerging markets, the positions of the EU15 tend to converge with the world average.

5.2. Data Collection and Research Methodology

In contemporary SCM research, little attention has been paid to the comprehensive, integrative and integral approach of SCC and its impact through the construct collaborative advantage on operational performance and firm performance. The construct SCC dimensions forms the cornerstone and backbone for the operational business processes of collaborative planning activities and collaborative decision making processes, and its impact on the mediation construct collaborative advantage, which impacts the dependent constructs operational performance and firm performance. Therefore, as mentioned throughout this research paper the high-level and abstract objective is to analyze and to discover the impact of SCC dimensions on firm performance and operational performance.

Historically, in most of the prior studies in the field of SCM, survey questionnaires have been the most popular research method (Mentzer and Kahn, 1995; Kotzab et al., 2005; Sachan and Datta, 2005; Burgess et al., 2006; Giunipero et al., 2008; Chicksand et al., 2012). Furthermore, the logic of researchin this study is deductive. Deductive research pursues a conscious direction from a general law to a specific case (Alvesson and Skoldberg, 1994; Andreewsky and Bourcier, 2000; Danermark, 2001; Kirkeby, 1990; Taylor et al., 2002). Thus, deductive research scans theory, derives logical conclusions from this theory and presents them in the form of hypotheses. These hypotheses are tested in an empirical setting and presents general conclusions based on the corroboration or falsification of the self-generated hypotheses (Arlbjorn and Halldorsson, 2002; Kirkeby, 1990; Wigblad, 2003).

The subject of study in this research paper is SCC in the contextual cross-border inter-firm design in a supplier (EU15) - buyer (Russian firm) dyad, whereas the EU15 supplier is the focal firm. Therefore, the object of study is the focal European firms of the EU15 member states. The unit of analysis is a sample of EU15 firms which was primarily extracted from the Amadeus database.

The tool applied for collecting primary data was a web-based survey questionnaire. The sample respondents were expected to have experience in doing business in Russia. For each construct and their indicators and items, a Likert (1932) type method of summated five-point scale was used to assess and review its perceived level and degree of perception on several propositions. This Likert scale was apt, because it provides an interval. This is the most powerful scale for statistical analysis (Hair et al., 2010). The potential sample respondents were collected by means of database Amadeus of Bureau van Dijk. To improve response rate, four waves of emails were sent once a week. Out of the 72 respondents, 66 were considered as useable for data analysis.

6. Data Analysis of Depth and Scope of Collaboration

The scope of collaboration encompasses the number of business processes and activities that are collaborating, and the depth of collaboration measures the integration of processes that are collaborating. Therefore, the web-based survey questionnaire included multiple different business processes and departmental variables measuring the scope and depth of collaboration. Furthermore, the web-based survey questionnaire also included multiple indicators of the independent constructs operational performance and firm performance to measure the perceived improvements as a result of collaboration. The respondents were asked to estimate the level of collaboration and involvement of their Russian buyer in several organizational areas of SCC.

Pearson correlation coefficients of the collaboration areas and firm performance and operational performance were calculated to make some preliminary conclusions about the effects and relationship between these dependent variables (firm performance and operational performance) and independent variables (collaboration areas). The results of the Pearson correlation coefficients are shown in table 2.

Table 2: Pearson correlation of collaboration areas, operational and firm performance

Dependent / independent	**Production**	**Inventory management**	**Distribution**	**R&D**	**Procurement**	**Supply chain design**	**Product development**
Sales growth	.082	.177	.316**	.018	-.005	.123	.070
Market share	.183	.209	.149	.138	.074	.035	.157
ROI	.134	.289*	.218	.129	.091	.137	.085
On-time delivery	-.037	.276*	.507**	.211	.024	.296*	.268*
Order fulfillment lead time	.047	.312*	.471**	.329**	.022	.291*	.276*
Total logistics cost	.024	.293*	.193	.222	.109	.315*	.137
Inventory turns	.089	.346**	.133	.211	.109	.129	.254*
Stock-outs	.110	.451**	.157	.178	.117	.204	.239

**. Correlation is significant at the 0.01 level (2-tailed).
*. Correlation is significant at the 0.05 level (2-tailed).

As illustrated in table 2, statically significant correlations were measured in several collaboration areas, namely: inventory management, distribution, R&D, supply chain design and product development. All of the statistically significant correlations were positive. Collaboration in distribution led to strong significant correlation with strong change in operational performance on-time delivery (.507**) and moderate significant correlation with order fulfillment lead-time (.471**) and sales growth (.316**). Further, collaboration in inventory management resulted in weak significant correlation with ROI (.219*), on-time delivery (.276*), and total logistics costs (.293*), moderate significant correlation with order fulfillment lead-time (.312*) and inventory turns (.346**) and stock-outs (.451**). Collaboration in the area of inventory management showed the most relationships and significant effects with dependent variables. Also, collaboration in R&D showed a moderate significant correlation with order fulfillment (.329**). In addition, collaboration in supply chain design led to moderate significant correlation with total logistics cost (.315*) and weak significant correlation with on-time delivery (.296*) and order fulfillment lead-time (.291*). Last but not least, collaboration in the area of product development showed weak significant correlation with on-time delivery (.268*), order fulfillment rate (.276*) and inventory turns (.254*). Absolutely no significant correlations were found in the collaboration areas production and procurement. Interestingly, also no significant correlation was found in the dependent firm performance variable market share growth.

By computing the composite variables through summing the collaboration area variables and firm performance and operational performance variables, correlation was analyzed between these two composited variables. Interestingly, the sum of collaboration areas had a significant moderate correlation with the sum of firm performance and operational performance (.345**). Hence, it can be concluded that there is indeed a moderate correlation between the scope and depth of collaboration with firm performance and operational performance.

However, to get a better detailed understanding of the effect of the parameters and elements of SCC to operational performance, and operational performance to firm performance, multiple regression analyses were performed. Based on prior conducted researches and studies, the cut-off and threshold value for the adjusted R square was set on .10 (Bagchi et al., 2005). In addition, there were a numerous number of significant strong correlations among the independent variables of collaboration areas. This could lead to multicollinearity, which is an undesirable situation where the correlations among the independent variables are strong. Therefore, the indicators of collaboration areas, operational performance and firm performance were tested for multicollinearity by means of variation inflation factor (VIF). For this study, VIF between 5 and 10 indicates high correlation that may be problematic. And if the VIF is above 10, the regression parameter estimates and coefficients are poorly estimated due to multicollinearity. The VIF values of the collaboration areas were in the range of 1.204 to 1.981, therefore, the variables were not subject to multicollinearity. In the case of operational performance indicators, the VIF values were in the range of 1.500 to 5.776. Only some variables of operational performance had a VIF value higher than 5 which might have caused some minor multicollinearity, but none of them were above 10, so these were negligible, therefore, no adjustments were made. The results of multiple regressions of collaboration areas as independent variables and operational performance indicators as dependent variables are presented in table 3.

Table 3: Multiple regressions of collaboration areas and operational performance

Operational performance variables	**Collaboration area variables**	**Regression parameter estimate (Beta)**	**Adjusted R square**
On-time delivery	Distribution**	.395	.304
Order fulfillment lead time	Distribution**	.346	.295
	Procurement*	-.316	
Total logistics cost	N.A.	N.A.	.088
Inventory turns	N.A.	N.A.	.070
Stock-outs	Inventory management**	.530	.158

***. P < 0.001, **. P < 0.01, *. P < 0.05 and N.A.=Not Applicable

Concerning the results of the multiple regression analysis of collaboration areas on operational performance, the operational performance variable on-time delivery was significant correlated with collaboration area distribution. The same observation was made with regards to the operational performance variable order fulfillment lead-time. Looking back on the Pearson correlation results, the results were partially expected in these collaboration areas. Collaboration in distribution activities and processes, such as order deliveries in a cross-border context, improves supply chain performance and efficiency in on-time delivery and order fulfillment lead-time. The multiple regression analysis also illustrated that collaboration in collaboration area inventory management had a significant positive correlation with stock-outs. Again, this correlation was logic and coherent with its inherent nature of activities and processes. Collaboration by means of inventory management leads to better understanding and synchronization of inventories in the pipeline between supplier and buyer. All these efforts of collaboration in inventory management therefore results in minor and/or less frequent stock-outs. An interesting and kind of unexpected result was the significant negative correlation between collaboration area procurement and order lead-time fulfillment. The explanation of the negative sign of the correlation could be that collaboration in procurement improves similar and alike processes and activities which will lead to higher demands from the purchasing and procurement department to their planning and LSP to meet service-levels of order fulfillment lead-times. Another explanation could be that due to improvement and optimization of activities within the department, the information exchange and sharing with other departments and external parties changed from the current situation which could lead to initial misunderstandings and errors.

Again, by computing the composite variables through summing the collaboration area variables and operational performance variables, a linear regression analysis was conducted between these two composited variables. The same as with the Pearson correlation analysis, the sum of collaboration areas variables had a significant parameter estimate with the sum of operational performance variables (.379***). Furthermore, the adjusted R square is higher than the cut-off value of .10, namely: .130. Therefore, it can be stated that there is indeed a positive effect between the scope and depth of collaboration with operational performance.

The results of multiple regression of firm performance as dependent variables are presented in table 4 below.

Table 4: Multiple regressions of collaboration areas and firm performance

Firm performance variables	**Collaboration area variables**	**Regression parameter estimate (Beta)**	**Adjusted R square**
Sales growth	N.A	N.A.	.035
Market share growth	N.A	N.A.	-.033
ROI	N.A	N.A.	.007

***. P < 0.001, **. P < 0.01, *. P < 0.05 and N.A.=Not Applicable

The second result of multiple regression analysis of collaboration areas on firm performance did not show any significant regressions between the variables of collaboration areas as independent variables and firm performance indicators as dependent variables. For all the firm performance variables the adjusted R square was lower than the cut-off value of .10. However, there were no collaboration area variables that had a p-value that was lower than .05 to any of the firm performance variables. Hence, none of the independent variables of collaboration areas had a significant regression with the dependent variables of firm performance.

However, by computing the composite variables through summing the collaboration areas variables and firm performance variables, a linear regression analysis was conducted between these two composited variables. The composite variable collaboration area had a positive parameter estimate with the composite variable of firm performance (.220), but it was not significant. Furthermore, the adjusted R square was lower than the cut-off value of .10, namely: .033. Therefore, it can be stated that there was no significant positive effect between collaboration areas on firm performance.

Last but not least, the results of multiple regressions of operational performance as independent variable and firm performance indicators as dependent variable are presented in table 5.

Table 5: Multiple regressions of operational performance and firm performance

Firm performance variables	**Operational performance variables**	**Regression parameter estimate (Beta)**	**Adjusted R square**
Sales growth	N.A.	N.A.	.355
Market share growth	N.A.	N.A.	.247
ROI	N.A.	N.A.	.374

***. $P < 0.001$, **. $P < 0.01$, *. $P < 0.05$ and N.A.=Not Applicable

The results of multiple regression analysis of operational performance on firm performance did not show any significant regressions between the variables of operational performance as independent and firm performance indicators as dependent. However, the adjusted R square was moderate above the cut-off value of .10, while in comparison with the adjusted R square of the collaboration areas indicators on firm performance the indicators the adjusted R square was around zero.

To provide an integrative and all-embracing analysis of the scope and depth of collaboration in the context of EU15-Russian supply chains, a path diagram of the multiple regressions was conducted. The independent variables of all the different collaboration areas were combined to one latent construct, which is named: collaboration areas, whereas the latent constructs operational performance and firm performance were determined as dependent variables. The results of the multiple regression analysis and its extension to the visualized path diagram are presented in Appendix 1. Table 6 shows the results of only significant relationships between variables.

The table above and the path diagram in Appendix 1 highlight and accentuate that there was a positive significant relationship and effect between latent construct collaboration areas and latent construct operational performance (.351*). However, there was no significant positive relationship between latent construct collaboration areas and latent construct firm performance. Unexpectedly, there was a negative effect and relationship between latent construct collaboration and latent construct firm performance (-.020), but not significant. On the other hand, operational performance had a strong positive significant effect and relationship with latent construct firm performance (.576***). Furthermore, the control variables firm size (.130) and length of customer relationship (.128) had a weak positive effect on firm performance, but not significant. Moreover, the control variables contributed and explained a higher degree of variance of the latent construct firm performance,

Table 6: Regression parameter estimates of depth and scope of collaboration

Relationship	**Regression parameter estimate (Beta)**	**P-value**
Collaboration → Operational performance	**.351***	**.016**
Collaboration → Firm performance	-.020	.879
Operational performance → Firm Performance	**.576*****	*******
Firm size → Firm performance	.130	.242
Length of CR → Firm performance	.128	.246
Collaboration → Inventory management	.520***	***
Collaboration → Distribution	.338*	.033
Collaboration → R&D	.785***	***
Collaboration → Procurement	.560***	***
Collaboration → Supply chain design	.361*	.014
Collaboration → Product development	.803***	***
Firm performance → Sales growth	.901***	***
Firm performance → Market share growth	.922***	***
Firm performance → ROI	.825***	***
Operational performance → On-time delivery	.918***	***
Operational performance → Order fulfillment LT	.970***	***
Operational performance → Total logistics cost	.606***	***
Operational performance → Inventory turns	.567***	***
Operational performance → Stock-outs	.597***	***

***. $P < 0.001$, **. $P < 0.01$, *. $P < 0.05$ and N.A.=Not Applicable

thereby, increasing the reliability of other predictors on the dependent variable firm performance.

In addition, another model was constructed by compositing all variables and indicators of collaboration areas, operational performance and firm performance, the relationship and effect between collaboration and operational performance was positive and significant (.379***). Furthermore, the relationship and effect between operational performance and firm performance was also positive and significant (.654***). In this model the control variables firm size (.228) and length of customer relationship (.104) were also positive, but again not significant. The visualized structural path model of the composite observed variables is included in Appendix 2.

6.1. Concluding Remarks

Overall, the tendency shows that the scope of collaboration, by the number of business process and activities that are collaborating, was quite moderate, while the depth of collaboration, by the level and degree of integration of the process in collaboration, can be determined between low and moderate in the challenging cross-border contextual design. Hence, the depth and scope of collaboration in the

EU15-Russian supply chain setting can be defined and concluded as moderate and modest.

As reported by the Pearson correlation and regression tables, it seems that collaboration in the areas of distribution and inventory management had the most positive significant effect on primarily operational performance indicators such as on-time delivery, order fulfillment lead-time and stock-outs. Nevertheless, it must be underlined that in many collaboration areas no severe and substantial results were reported from collaboration. Conclusively, it is clear that distribution and inventory management are the collaboration areas where EU15 suppliers should collaborate with their Russian buyer. In the train of thought, supply chain design can also be considered as a viable and feasible option for collaboration to enhance and complement mainly operational performance parameters and metrics.

In addition, by compositing all variables of collaboration areas, operational performance and firm performance, the relationship and effect between collaboration and operational performance was positive and significant (.379***). Furthermore, the relationship and effect between operational performance and firm performance was also positive and significant (.654***). In this model the control variables firm size and length (.228) of customer relationship (.104) were both positive but not significant.

Additionally, an integrative structural model was conducted to measure the path coefficient and relationship between the unobserved latent constructs collaboration areas and operational performance (.351*), and collaboration areas and firm performance (-.020). Furthermore, the effect and relationship between operational performance and firm performance (.576***) was analyzed.

Hence, it can be concluded that if the latent construct collaboration goes up by one standard deviation, the latent construct operational performance goes up by a standard deviation of .351 at the 5 percent level of significance. Thus, more depth in collaboration, especially in distribution and inventory management, leads to a significant positive effect on operational performance. Following-up, if the latent construct operational performance goes up by one standard deviation, the latent construct firm performance goes up by a standard deviation of .576 at the 0.1 percent level of significance.

The control variables firm size and length of customer relationship were positive, respectively .130 and .128, but not significant. However, the control variables do explain more of the latent construct firm performance' variance and adjusted the effect of the latent construct operational performance and collaboration areas on firm performance.

7. Structural Equation Model of Supply Chain Collaboration

In consideration to test the depicted conceptual SCC hypotheses framework that is visually presented in figure 1, the two-step approach was used for assessing the structural model (Anderson and Gerbing, 1988).

The two-step approach advocates that in order to test a structural regression model, the measurement part of the model was firstly identified and consequently the structural part of the model. Hence, the suitability of the formulated conceptual model in this research paper was tested before the eventual structural path relationships in the conceptual SCC hypotheses framework were examined to test the hypotheses. Hence, first of all, a confirmatory factor analysis (CFA) was conducted for the measurement part of the model of the indicators of the latent constructs: SCCD, CA, OP and FP, including the interaction moderation latent constructs CB and CBBB.

CFA is a multivariate statistical procedure that is used to test how well the measured predefined variables represent the above mentioned latent constructs. For the study in this research paper it was felt that the two-step approach would be the best, because the conceptual SCC hypotheses model is partially adapted from the studies of Ramanathan and Gunasekaran (2012) and Zhang and Cao (2011). The CFA evaluates a priori hypotheses and relies heavily on existing theory of previous researchers and scholars. Therefore, the number of latent constructs and indicators are partially determined in advance (Thompson, 2004).

7.1. Reliability and Validity Tests

First of all, it is preferable to determine if a measurement instrument is able to produce consistent results every time it is conducted under similar circumstances. Statistically, reliability is defined as the percentage of the inconsistency in the responses to the survey questionnaire which is the result of differences in the respondents. This implies that responses to a reliable survey questionnaire will vary, because respondents have different opinions, not because the survey questionnaire questions are confusing or ambiguous. Therefore, the predefined indicators of each of the latent constructs were tested to remove confusing indicators in order to improve reliability. In this study, one of the methods that were used to test reliability was Cronbach's alpha for each latent construct and its indicators. Furthermore, a Cronbach's alpha of all the indicators of all the latent constructs was also calculated. Generally, Cronbach's alpha of >0.7 is the cut-off and threshold value (Cooper and Schindler 2006; Malhotra and Birks 2006). The results of the reliability analyses are illustrated in table 7.

Table 7: Cronbach's alpha reliability test

Latent construct	**Number of indicators**	**Cronbach's alpha**
SCCD	7	.895
CA	5	.736
FP	4	.926
OP	5	.897
CB	9	.928
CBBB	11	.844
All indicators	49	.901

The results of the Cronbach's alpha test indicated that all the latent constructs had a Cronbach's alpha above the cut-off and threshold value of 0.7. Hence, based on the preliminary reliability test of Cronbach's alpha, all the latent constructs and its indicators were included in the CFA for further reliability and validity analysis.

However, due to the large number of indicators in the latent construct CBBB and the distinction in the nature and dimension of the barriers, a principal component with varimax rotation factor analysis was conducted. The results of the principal component analysis are included in Appendix 3. The KMO is .818 and Bartlett's Test of Sphericity was significant, therefore, the set of variables were suitable for factor analysis. As shown in Appendix 3, there are some indicators that had a poor and low loading, and loaded in multiple factors. Therefore, the indicators with the lowest loading and that loaded in multiple factors were deleted to rerun the factor analysis. Furthermore, due to the fact that only two indicators loaded in the last factor, the number of factors was constrained to two. After deleting the indicators 'unfavorable foreign exchange rates' (.341) and 'strong international competition' (.542), the KMO increased to .836.

The results of the revised factor analysis showed that the two factors can be categorized and interpreted in indicators with an environmental dimension, and indicators with a market dimension. Hence, the revised CFA, in which the latent construct CBBB was separated in market dimension (MD) indicators and environmental dimension (ED) indicators, is included in Appendix 4.

The outcomes of the CFA analysis functioned as input to conduct composite reliability, and convergent and discriminant validity tests. If the latent constructs do not show adequate validity and reliability, the structural model will be of less good fit, thereby, unreliable. Hence, more thoroughly validity and reliability tests were conducted, such as composite reliability (CR), average variance extracted (AVE), maximum shared variance (MSV), and average shared variance (ASV) (Hair et al., 2010). The same authors established the following cut-off and threshold values that were used to measure and determine reliability and validity, which are presented in the table below.

Table 8: Reliability and validity threshold values

Reliability and validity tests	Cut-off value
Composite reliability	> 0.70
Convergent validity	CR $>$ AVE
	AVE > 0.50
Discriminant validity	MSV $<$ AVE
	ASV $<$ AVE

Source: Hair et al. (2010)

In order to calculate the above mentioned reliability and validity tests, the correlation table and standard regression weight table of the CFA, including all the latent constructs, were used as input values. By means of an Excel macro (Gaskin, 2014), the outcomes of the CFA were used as input to calculate the reliability and validity tests. The total results of the test are illustrated in the table below.

Table 9: Reliability and validity test results of CFA

LC	CR	AVE	MSV	ASV	ED	SCCD	CA	FP	OP	CB	MD
ED	0.890	0.623	0.215	0.096	0.790						
SCCD	0.888	0.534	0.320	0.131	0.276	0.731					
CA	0.741	0.370	0.320	0.127	-0.138	0.566	0.609				
FP	0.928	0.763	0.429	0.172	-0.239	0.366	0.516	0.873			
OP	0.879	0.600	0.429	0.142	-0.057	0.487	0.390	0.655	0.775		
CB	0.929	0.596	0.335	0.102	0.452	0.133	-0.032	-0.234	-0.022	0.772	
MD	0.798	0.508	0.335	0.112	0.464	0.037	-0.016	-0.303	-0.173	0.579	0.713

The results in the table above demarcates that there was one element of convergent validity that did not meet the cut-off value, which is shown by the red font color. The low AVE number of CA can be explained by the low factor loadings of some of the observed variables and indicators, such as business synergy (.47), quality (.53) and innovation (.57), as shown in Appendix 4. In addition, it is noteworthy to mention that due to the relatively low sample size (n=66), the factor loadings of each indicator on the latent construct should be approximately around .60 - .65 in order to meet the requirements of the reliability and validity test (Hair et al., 2010). Nevertheless, in the spirit of the study and the low impact on model fit of only one criterion that did not meet the reliability and validity requirements, all the indicators of the latent construct CA were included despite the aforementioned low loadings of some of these indicators.

As shown in Appendix 4, besides the observed indicators and variables of the aforementioned latent construct CA, the remaining observed indicators had a loading of approximately .60 and higher to their respective latent constructs. The measurement model met almost all the cut-off and threshold values of composite reliability, convergent validity and discriminate validity. Hereinafter, the measurement model of the CFA was used to test the common method bias by means of common latent factor (CLF).

The common method bias test indicates bias in the dataset due to something external to the measures. Something external to the question may have influenced the given answers. For example, collecting data using a single (common) method, such as a web-based survey questionnaire, may introduce systematic response bias that will either inflate or deflate responses. Significant common method bias is one in which a majority of the variance can be explained by one single factor. The CLF was used to capture the common variance among all observed variables and indicators in the measurement model. The CLF was implemented in the measurement model and the standardized regression weights of the measurement model with and without the CLF were compared. The measurement model with the CLF is included in Appendix 5. According to Podsakoff et al. (2003), if the difference between the adjusted common bias standardized regression weights with CLF and the standardized regression weights without CLF is greater than 0.2 then the standardized regression weight results with the CLF should be used. The results of comparison are included in Appendix 6. As shown in Appendix 6, the difference between the standardized regression weights of CLF and without CLF was not greater than the threshold value of 0.2; therefore the measurement model without CLF was used for the next step of the structural SCC hypotheses model.

7.2. Structural Equation Model

After the first step was conducted and the amended measurement model without the CLF was approved, the next step, structural model, was done in order to test the conceptual SCC hypotheses framework by means of SEM.

SEM is a robust statistical analysis technique that is used for multivariate analysis. SEM is a set of linear equations for testing the hypothesis about the relationship between observed indicators and latent constructs (Hair et al., 2010). SEM is widely known for the following advantages. First, SEM makes assumptions, unobserved latent construct, and hypothesized relationships. Second, SEM enhances a degree of precision, since it contains clear definition of latent constructs and the functional relationship between them. Third, SEM offers a formal framework for constructing and testing both theories and measures, and selection of sample size through the use of estimation methods. Comprehensively, the main goal of SEM is to find the extent to which a hypothesized model fits or at least adequately describes sample data.

Model fit of the measurement and structural model was tested by using a number of goodness-of-fit (GOF) indices. These GOF indices aim to measure the distance or difference between sample covariance and fitted covariance. Hair et al. (2010) recommend that in order to establish a robust and vigorous analysis more than one fit index is mandatory. Hence, the table 10 shows the GOF indices and their cut-off values that were used in this study to measure model fit. Furthermore, as a side mark, it is important to point that GOF is inversely related to sample size and the number of variables in the structural model.

Table 10: GOF indices for structural model

GOF measure	**Cut-off value**
Chi-square/degree of freedom	< 3 good; < 5 sometimes permissible
CFI	> .95 great; > .90 moderate; > .80 sometimes permissible
NFI	> .90
AGFI	> .80
RMSEA	< .05 good; .05 - .10 moderate; > .10 bad

Source: Hair et al. 2010

By using the measurement model of the CFA without the adjusted CLF, a hybrid structural model was constructed, which is included in Appendix 7. The hybrid structural model showed that the latent construct SCCD had a significant positive influence and effect on latent construct CA (.588***) and on the latent construct OP (.442*). The latent construct CA had significantly positive effect on the latent construct FP (.300*). Furthermore, the latent construct OP had a significant relationship and impact on FP (.524***). Considering the impediments, the moderator CB had negative direct effects and impacts on CA (-.104) and FP (-.111), and a positive direct effect on OP (.078). In case of the cross-border business barriers, ED had a negative direct effect and impact on OP (-.105) and FP (-.090), but both of them were not significant. As for MD, the direct effect and impact on OP (-.180) and FP (-.095) were all negative and not significant.

Consequently, the structural path model was conducted by comprising the unobserved latent constructs into observed variables, which then does not account for measurement error as in the hybrid model, because it is just a structural path model between the newly created imputed composite observed latent constructs. Furthermore, the control variables firm size and length of customer relationship were also included. The structural path model between the observed constructs is included in Appendix 8. The standardized regression weight results of both the hybrid structural model and the structural model are included in table 11.

The structural model, which includes the imputed composited observed variables, did not include measurement errors, as in the case with the hybrid structural model. The results of the structural path model showed that there were significant positive effects and relationships on SCCD to OP (.472***), SCCD to CA (.651***), CA to FP (.429***) and OP to FP (.579***). One surprising observation was the

Table 11: Standardized estimates between hybrid model and structural path model

Relationship	**Hybrid model**	**Structural path model**
SCCD → OP	.442*	.472***
SCCD → CA	.588***	.651***
SCCD → FP	-.022 (ns)	-.180*
CA → OP	.115 (ns)	.122 (ns)
CA → FP	.300*	.429***
OP → FP	.524***	.579***
CB → CA	-.104 (ns)	-.129 (ns)
CB → OP	.078 (ns)	.100 (ns)
CB → FP	-.111 (ns)	-.121 (ns)
ED → OP	-.105 (ns)	-.126 (ns)
ED → FP	-.090 (ns)	-.010 (ns)
MD → OP	-.180 (ns)	-.201 (ns)
MD → FP	-.095 (ns)	-.070 (ns)
Firm size → FP	N.A	.122 (ns)
Length of CR → FP	N.A	.070 (ns)

***. $P < 0.001$, **. $P < 0.01$, *. $P < 0.05$ and ns=not significant

significant negative sign of the standardized estimate of the relationship between SCCD to FP (-.180*). The most likely explanation for this significant negative sign and magnitude of the effect of SCCD on FP is that SCC dimensions and the inherent business activities and processes require resources that have to be implemented and put into place. The direct main effect and impact of implementing, establishing and executing SCC business activities is negative on firm performance, because in the structural model with the included mediation variable CA, it implies that no advantages were obtained through SCC. Thus, if a firm implements SCC dimensions and business practices, but it does not manage to obtain advantages through collaboration these resources are wasted and have a negative direct main effect on firm performance, because the wasted resources increases the total costs of operations and does not lead to an increase in profits. Therefore, increased total costs of operations increases the marginal costs of the firm' product which dampens the profits and decreases the profitability and competiveness of the firm.

As an intermezzo, the table below presents the GOF and model fit of the aforementioned conducted measurement model and structural path model to give an overview about the development of model fit from the CFA to the structural path model.

Based on the results in the above table, it can be concluded that the reliability and validity test improved model fit significantly from the CFA to the structural path model, including the imputed composite observed constructs. The CFI increased from 0.784 to 0.846, NFI from 0.531 to 0.806 and AGFI from 0.557 to 0.627, while RMSEA increased from 0.099 to 0.180 in the structural model and decreased to 0.099 in the CFA and hybrid model. On a side note and remark, it has to be mentioned that the added interaction moderation constructs CB, ED and MD had a negative impact on model fit. The reason is that these moderation constructs are exogenous variables that have a direct main effect on one or several dependent variables and does not explain for all the variance. Hence, hypothetically, the same

Table 12: Model fitting indices of SEM two-step approach

GOF measure	**Cut-off value**	**CFA**	**Hybrid model**	**Structural path model**
Chi-square/df.	< 3 G; < 5 P	1.635	1.640	3.099
CFI	$> .95$ G; $> .90$ M; $> .80$ P	0.784	0.782	0.846
NFI	$> .90$	0.531	0.592	0.806
AGFI	$> .80$	0.557	0.530	0.627
RMSEA	$< .05$ G; .05 - .10 M; $> .10$ B	0.099	0.099	0.180

G=Good, M=Moderate, P=Permissible and B=Bad

structural model was run without the moderation constructs to determine model fit. Almost all of the GOF measures met the cut-off and threshold values. The results were as follows: 0.985 for CFI, 0.956 for NFI, 0.854 for AGFI and RMSEA was 0.082. However, in light of the study and the formulated hypotheses, which are included in the conceptual SCC framework, no adjustments and revisions were made despite the moderate and modest model fit of the structural model, including the impediments moderation variables.

7.3. Mediation Effect of Collaborative Advantages

Due to the mediation latent construct CA in the conceptual SCC hypotheses framework, which was used to measure the chains of causation, a mediation analysis was conducted. Mediation in SEM is generally used to provide a more accurate explanation of the causal effect of the independent variable on the dependent variable. The mediator variable is in most of the cases bridging the gap in a causal chain. For instance, the latent construct SCCD had a positive main effect on OP and a negative direct effect on FP, but not in all contextual situations, as not all SCCD activities always lead to either OP or FP. Hence, some mediation variable, such as the latent construct CA explains this effect and relationship. Thus, it can be expected that collaborative advantages positively mediates the relationship between SCCD and FP and/or OP. This means that the relationship between SCCD with OP and FP is better explained through the mediation variable CA. The same structural path model was used to analyze the mediation of the latent construct CA. The results of the mediation analyses are shown in the table below.

Table 13: Mediation effect of CA on SCCD to OP and FP

Path	**Direct without CA mediator**	**Direct with CA mediator**	**Indirect effect**	**Conclusion**
SCCD $\rightarrow$ CA $\rightarrow$ OP	.559***	.472***	.079 (ns)	No mediation
SCCD $\rightarrow$ CA $\rightarrow$ FP	.125 (ns)	-.180*	.599**	Bifurcated

***. $P < 0.001$, **. $P < 0.01$, *. $P < 0.05$ and ns=not significant

According to the table above, the latent mediation construct CA had no mediation effect on the path from latent construct SCCD to OP. However, on the other hand, the latent construct CA had a strong bifurcated mediation effect on the latent construct FP. The results can be explained by the fact that the dimensions of SCC have a direct main effect and impact on the relationship with OP indicators. For instance, information sharing, decision synchronization and/or collaborative communicative have a direct positive significant effect on operational performance indicators such as order-fulfillment lead-time, total logistics cost and/or on-time delivery.

On the other hand, the dimensions of SCC are fully bifurcated by the mediation variable CA. This means that the direct main effect of the dimensions of SCC without the mediation variable CA was not significant (.125) on FP. However, when the mediation variable CA was included between the direct path of SCCD and FP, the direct main effect and impact of SCCD on FP was significant and negative (-.180*). On the contrary, the indirect effect of SCCD through the mediation variable CA on FP was positive and significant (.599**). Thus, it can be strongly implied that, for instance, by conducting and practicing the dimensions of SCC, collaborative advantages can be accomplished and realized. These collaborative advantages by means of indicators such as offering flexibility, process efficiency, quality and innovation can result into sustainable competitive advantages which on its turn will lead to a stronger competitive position in the marketplace in comparison with its competitors. Therefore, the firm might be able to outperform and outcompete its competitors to increase its firm performance metrics, such as market share growth and sales growth.

Conclusively, the direct main effect of SCCD on OP (.559***) without the mediation construct CA was significant and positive. However, when the mediation construct CA was added the direct main effect of SCCD on OP (.472**) was slightly dampened, because the mediator CA accounted for some of this effect and impact (.079), but not significant. Therefore, it can be concluded that the mediator CA had no mediation effect on the relation between SCCD and OP.

Furthermore, the direct main effect of SCCD on FP (.125) is positive but not significant. When the mediator CA was added the direct effect of SCCD to FP (-.180*) was negative and significant. However, through the mediator CA the indirect main effect of SCCD through CA on FP (.599**) was strongly positive and significant. This type of mediation is called bifurcation. Hence, if a firm practices SCCD and realizes CA it has a strong positive effect on FP, because it might obtain and realize sustainable competitive advantages over its competitors that increases the metrics of firm performance such as sales growth and market share growth.

7.4. Interaction Moderation Effects of Impediments

There are several identified collaboration barriers that are supposed to have a negative direct main effect on CA and a positive moderating effect on the realized CA by means of the independent variable SCCD. Therefore, an interaction moderation analysis was conducted to determine the moderating effect on the positive relationship of SCCD on CA and OP, and on the negative relationship of SCCD on FP. The interaction moderation of the impediments was conducted in several steps and consisted out of a two-way and three-way interaction moderation analyses. The two-way interaction moderation analysis was done with the independent variable SCCD and the moderating variable CB to CA, OP and FP, and the three-way moderation analysis was done with the independent variables CA and the moderating variables MD and ED to OP and FP. The interaction moderation is further explained by plotted slopes following the procedures outlined by Dawson and Richter (2006).

Factoid, interaction moderation effects are in theory actually joint effects of two predictor variables in addition to the individual direct main effects (Hair et al., 2010). Interactions enable more precise explanation of causal effects by providing a method for explaining not only how SCCD directly affects CA, OP and FP, but also under what circumstances the effect of SCCD changes depending on the interaction moderating variable CB. Basically, the interaction regression equation specifies that the slope of the line relating SCCD to CA, OP and FP changes at different continuous interaction moderation levels of CB, or equivalently, that the slope of the line relating SCCD to CA, OP and FP changes at different levels of CB.

First of all, the two-way interaction moderation was conducted with the independent variable SCCD and the moderation variable CB to the dependent variables CA, OP and FP. The main effect variables were standardized before forming the interaction terms (Cohen et al., 2003). The table below shows the results of the first and second step that were conducted to perform the moderation analysis.

Table 14: Two-step interaction moderation of collaboration barriers

Relationship	**Model 1**	**Model 2**
Step 1: Main effects		
CB → CA	-.129 (ns)	-.175 (ns)
CB → OP	.100 (ns)	.090 (ns)
CB → FP	-.121*	-.143 (ns)
SCCD → CA	.651***	.635***
SCCD → OP	.472***	.475**
SCCD → FP	-.180*	-.170 (ns)
Step 2: Two-way interaction effects		
CB x SCCD → CA		.251**
CB x SCCD → OP		.025 (ns)
CB x SCCD → FP		.054 (ns)
Firm size → FP	.123 (ns)	.124 (ns)
Length of CR → FP	.086 (ns)	.196 (ns)

***. $P < 0.001$, **. $P < 0.01$, *. $P < 0.05$ and ns=not significant

The results of two-way interaction moderation of CB in the table above shows that in the first model the main effect of CB on FP was negative and significant (-.121*). The main and direct effect of CB on CA (-.129) was also negative, while the main effect on OP (.099) was positive, but both of them were not significant. The second model included the interaction effects of the multiplied standardized predictors CB and SCCD which showed that the interaction effect of CB and SCCD had a positive significant impact on CA (.251**). The interaction effect on OP (.025) and FP (.054) were also positive but not significant.

In addition, the results of the second model, including the interaction effects, were used to conduct plots to help to interpret and to understand the interaction moderating effect of CB on the relationship between SCCD and CA, OP and FP better. Gaskin (2014) created an Excel worksheet to visualize and interpret two-way interaction moderation effects which was used to conduct plots.

The interaction moderation effect of CB on the relationship between SCCD and CA can be explained as follows. Although, at first glance, the results in figure 2 might seem to go against the grain, intuition and rationale, CB strengthens the positive relationship between SCCD and CA. Furthermore, it demonstrated that the relationship between SCCD and CA is always positive. The slope of high CB is steeper and stronger than the slope of low CB. This means that if CB increases, in other words, the effect of collaboration barriers are getting stronger, the effect between SCCD and CA is also getting stronger. In other words, in a scenario where there are high collaboration barriers the effects of low SCCD to high SCCD are the most severe and significant on the dependent variable CA. Exactly, the same effect occurred between SCCD and OP, but the magnitude and impact was less strong in comparison to the effect between SCCD and CA.

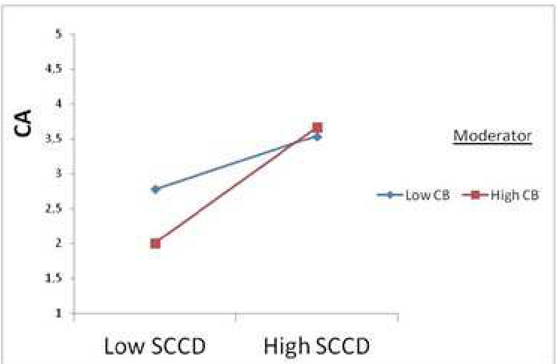


Fig. 2: CB moderation On SCCD and CA

Exactly, the same effect occurred between SCCD and OP, but the magnitude and impact was less strong in comparison to the effect between SCCD and CA.

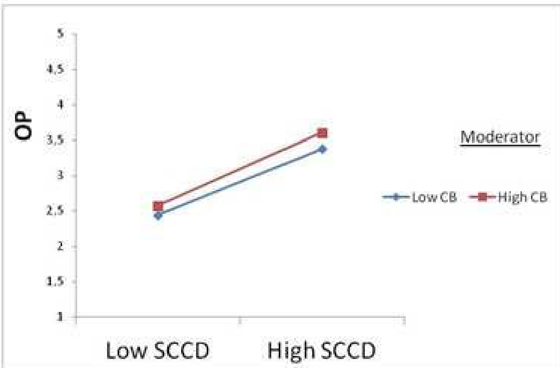


Fig. 3: CB moderation On SCCD and OP

On the other hand, CB dampens the negative effect and relationship between SCCD and FP. Hence, high CB has less impact and effect on the negative relationship between SCCD and FP. In other words, this can be explained by the fact that if a firm decides to practice SCC dimensions such as goal congruence, decision synchronization, incentive alignment and resource sharing, the elements of SCC could have a negative direct main effect and impact on a firm if it does not attain any collaborative advantages. In the scenario of high collaboration barriers, these aforementioned dimensions of SCC are hindered and affected by the collaboration barriers, which will work contradictory and lead to disincentives for a firm to conduct SCC practices. Therefore, a firm will synchronize, align and adapt less towards what is best for the overall and whole supply chain, and will focus more on its own individual goals and objectives which dampens the negative firm performance effects of the individual firm.

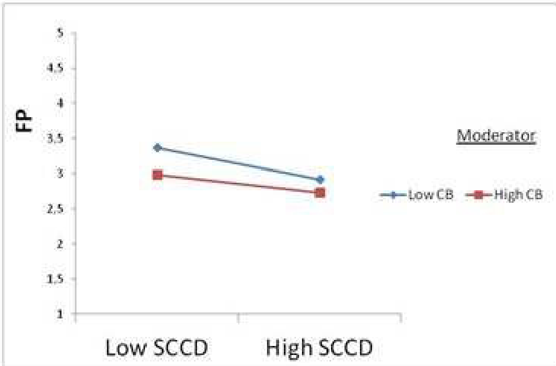


Fig. 4: CB moderation On SCCD and FP

Secondly, the three-way interaction moderation was conducted with the independent variable CA, and the moderation variables ED and MD to the dependent variables OP and FP. The same approach was used as with the two-way interaction moderation. The main effect variables were standardized before forming the interaction terms. The results of the first, second and third step are depicted in the table below.

According to the results that are depicted in the table above, the first model consisted only out of independent variable CA and moderating variables ED and MD which measured the main and direct effects on the dependent variables OP and FP. The effect and relationship between CA and FP was positive and significant (.429***). Furthermore, the effect and relationship between CA and OP was also positive but not significant (.122). The interaction moderation variables ED and MD showed weak negative direct main effects within the bandwidth of -.010 to -.201 to both OP and FP but also not significant.

For the second model the standardized multiplied two-way interaction effects were included. In comparison with the first model that only included the main effects and not the interaction effects, the positive effect in the relationship between CA and FP decreased (.395***). On the contrary, the positive effect in the relationship between OP and FP amplified (.599***). The main direct effects of ED and MD

Table 15: Three-step interaction moderation of cross-border business barriers

Relationship	Model 1	Model 2	Model 3
Step 1: Main effects			
CA → OP	.122 (ns)	.084 (ns)	.001 (ns)
CA → FP	.429***	.395***	.403***
ED → OP	-.126 (ns)	-.175 (ns)	-.220 (ns)
ED → FP	-.010 (ns)	-.037 (ns)	-.039 (ns)
MD → OP	-.201 (ns)	-.184 (ns)	-.147 (ns)
MD → FP	-.070 (ns)	.014 (ns)	.016 (ns)
OP → FP	.579***	.599***	.600***
Step 2: Two-way interaction effects			
CA x ED → OP		.254*	.314*
CA x ED → FP		-.121 (ns)	-.123*
CA x MD → OP		-.112 (ns)	-.186 (ns)
CA x MD → FP		.215*	.218*
ED x MD → OP		-.132 (ns)	-.104 (ns)
ED x MD → FP		-.025 (ns)	-.025 (ns)
Step 3: Three-way interaction effects			
CA x ED x MD → OP			.182 (ns)
CA x ED x MD → FP			.003 (ns)
Firm size → FP	.122 (ns)	.119 (ns)	.122 (ns)
Length of CR → FP	.086 (ns)	.080 (ns)	.082 (ns)

***. $P < 0.001$, **. $P < 0.01$, *. $P < 0.05$ and ns=not significant

were also negative in the second model, but not significant, except for MD on FP (.014). Intriguingly, the interaction effect of CA and ED showed a significant positive effect and impact on OP (.254*). Additionally, the interaction effect of CA and MD had a significant positive effect and impact on FP (.215*).

In the last model, the three-way interaction constructs were included to analyze and to determine the final consequences and effects of the impediments ED and MD on both OP and FP. Equally interesting, the interaction moderations of the impediments MD and ED on the relationships CA to OP and FP were also included. As shown in table 15 in the last column on the previous page, the main and direct effects of both MD and ED were negative on OP and FP, except for MD on FP (.016) which was positive, but all the these main effects and direct paths were not significant. The main and direct effect of CA on FP from model one (.429***) to model three (.403***) dampened by including the two-way and three-way interaction effect constructs. On the contrary, the effect of OP on FP amplified from .579** in the first model to .600*** in the third model. As for the three-way interactions, the interaction effects of MD, ED and CA on both OP (.182) and FP (.003) were positive, but as expected not significant. It has to be pointed out that, in general, interaction moderation variables are rarely significant. Moreover, it is equally interesting to see the moderations of OP and FP at different levels of SCCD, ED and MD.

To conclude, the two-step interaction moderation analyses of CB on the relationships between SCCD and CA, OP and FP indicated that the direct main effects of the interaction moderation construct CB had a minor negative direct effect on CA (-.175). Interaction effects of the moderator CB on the relationship between SCCD and CA (.251**), OP (.025) and FP (.054) were all positive, but only significant for the interaction between the relationship SCCD and CA. Thus, it can be stated that the impediment construct variable CB had a negative direct main effect on the dependent variables CA and FP and a positive direct main effect on the dependent variable OP. Furthermore, only the interaction moderation of the product of the standardized variables SCCD and CB on CA was significant (.251**). Therefore, it can be concluded that the interaction moderation variable CB had a significant positive moderation impact on the relationship SCCD and CA. In other words, in the scenario of high CB, the impact and effect of low SCCD on CA is the lowest primarily due to high CB. However, in the scenario of high CB, if a firm manages to accomplish a high level of SCCD, the effect of SCCD on CA will be strengthened and a firm will obtain even higher collaborative advantages than in the case of low CB. So to speak, firms retrieve and attain more valued, unique and inimitable collaborative advantages in the scenario of high CB if a firm realizes to practice the indicators of SCCD on a high level and scale.

The three-step interaction moderation analyses of the interaction moderation variables ED and MD on the structural paths and relationships of CA to OP and CA to FP determined that the direct and main effects of ED were negative and low on OP (-.220) and on FP (-.039), but not significant. The same findings were stemmed from the interaction moderating analysis for the moderation variable MD on OP (-.147). However, the direct main effect of MD on FP (.016) was positive but also not significant. Interestingly, the three-way interaction moderators ED and MD had a positive effect and impact on the structural path and relationship from CA to OP (.182) and FP (.003). The most reasonable explanation of this finding can be given by the fact that in the case and scenario of high international and cross-border barriers, firms can realize an even higher positive effect and result on OP and FP if they manage to attain and realize high levels of CA. Nevertheless, the three-way interaction results between ED, MD and CA were statistically not significant. Thus, these three attributes combined interactively do not predict OP and FP. Even though the three-way interaction constructs were statistically not significant, it is still interesting to determine and interpret the interaction effects between ED, MD and CA on OP and FP.

The relationships and effects between the independent variable CA and the interaction moderation variables ED and MD on the dependent variables OP and FP can be better understood by visualization and plotting the results. Hence, due to these sophisticated and complicated relationships, the separate plots of the interaction moderation of ED and MD on OP and FP are included below to clarify the implications and results of the three-way interactions.

In the figure 5 the horizontal axis shows the independent variable CA and the vertical axis indicates the dependent variable OP. The figure comprises the relationship between CA and OP moderated by both the impediments ED and MD. The above plot and graph encrypts and demystifies the interaction effects of the moderation variables ED and MD on OP.

As shown in figure 5, there is almost no change of effect in the relationship between CA on OP when both ED and MD are low. This stagnated and horizontal slope can be clarified and explained by the fact that if there are low cross-border barriers by means of ED and MD the effect and relationship between CA and OP (.001) tend to be small and not significant, as shown in table 15 in the last column.

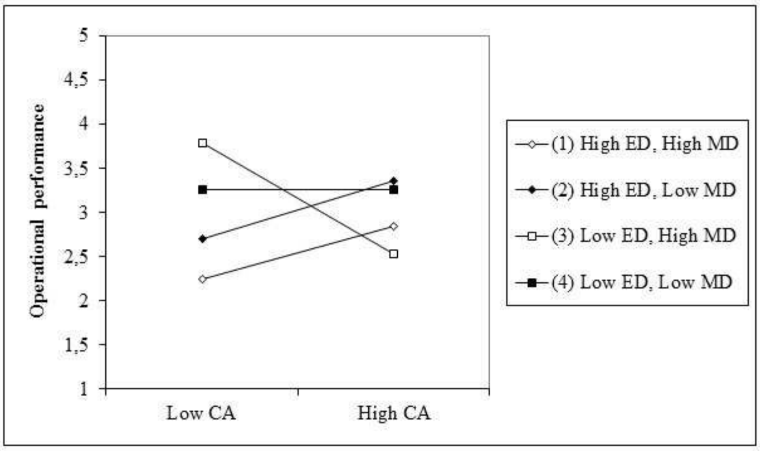


Fig. 5: Plot of regression slopes for three-way interaction on OP
Source: Author's own

The effect of CA on OP is larger in the scenario and situation when ED was high. The findings indicated that under the conditions of high ED the effect of achieved and attained CA were stronger on OP. Rationally, if a firm operates in a business environment that is affected and categorized by negative market conditions by means of high cross-border environmental barriers and impediments, such as bureaucratic requirements, transportation difficulties and restrictive rules and regulations, a firm perceives and experiences difficulties executing and practicing its business operations and activities. These environmental barriers and impediments form burdens and hurdles for the firm which leads to deoptimization and to non-value added business activities in order to get the job done. Hence, if a firm manages to realize and establish CA such as process efficiency, offering flexibility and quality by means of highly reliable and highly quality products, and innovation under the conditions of high ED and low MD barriers, these positive CA effects will be stronger on OP parameters and metrics such as on-time delivery, order fulfillment lead-time and total logistics costs.

The effect of CA on OP showed almost the same tendency and steep of slope for the market conditions when both MD and ED were high. However, there was a shift of the slope downward meaning that the overall effect of CA on OP was lower. Thus, in market conditions of both high ED and MD, the effect of CA on OP showed almost the same steep of slope, but the slope shifted downward. In general, the effects of CA on OP are lower due to high MD barriers. The most likely explanation that MD barriers decreased the effects of CA on OP is that MD indicators such as high business risk, different customer culture, unfamiliar foreign business practices and limited information about markets have a negative direct main effect on OP. If a firm experiences a business environment that is characterized by high MD indicators such as high business risk, different customer culture and limited information about markets, the firm will perceive difficulties to accurately plan demand and to determine which product types are the most suitable and demanded in the market. Therefore, the firm might be experiencing a higher probability of risk in that it will experience unexpected higher and frequent stock-outs, lower inventory turns, fluctuations and undesired on-time deliveries and order fulfillment lead-time which will also lead to higher total logistics costs. Nonetheless, the relationship of CA on OP was positively moderated by the also high ED indicators.

In addition, the effect of CA on OP decreased when MD was high and ED low. This means that in the case of high CA the effect was lower than it was in the case of low CA. Therefore, the effect of high CA worked contradictory on OP. This finding can be explained by the fact that due to high MD indicators, such as high business risk, different customer culture and limited information about markets, a low level of CA by means of offering flexibility, process efficiency, innovation and quality, had a higher effect on OP indicators such as total logistics costs, on-time delivery, order fulfillment lead-time and inventory turns. For instance, a high level of business risk and limited information about markets, and a high level and degree of innovation by means of rapid product development and low time-to-market will lead to more transactions and operational activities in comparison with low CA. Therefore, solemnly looking at the operational performance indicators and metrics, the context in which MD is high and ED is low, the effect of CA has a negative effect and impact on OP.

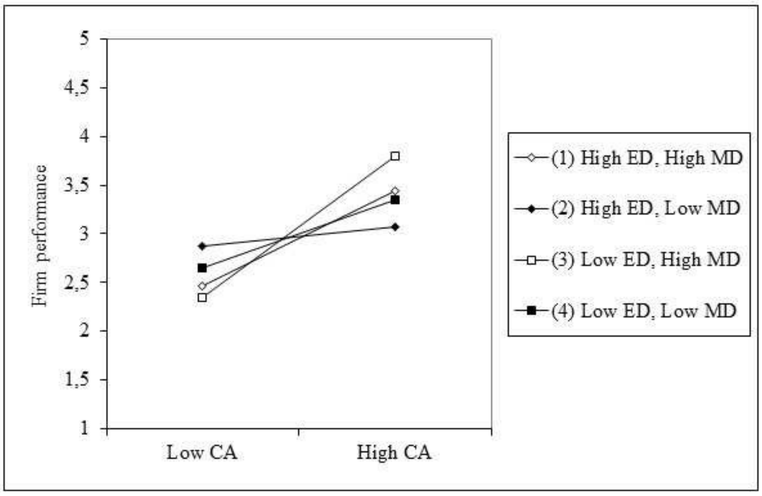


Fig. 6: Plot of regressions slopes for three-way interaction on FP
Source: Author's own

As in the case of the plot of regressions slopes for three-way interaction on FP, the horizontal axis shows the independent variable CA and the vertical axis indicates the dependent variable FP. The figure comprises the relationship between CA and FP moderated by both the impediments ED and MD.

Figure 6 illustrates and presents that the effect of CA on FP is the largest in the situation of low ED and high MD market circumstances. This can be explained through the logic and rationale of that high MD indicators, such as limited information, high business risk and different customer culture are strengthening the effect of CA such as offering flexibility, process efficiency, and especially, quality and innovation on FP. Specifically, if a firm perceives high CA through SCC the effects on sales growth, market share growth and ROI will be bigger, because the perceived CA such as quality and innovation leads to rapid product development, low time-to-market and frequent innovations which had an even higher positive impact on FP in a business environment and market conditions with high MD barriers and obstacles. The findings tend to show that it might be more difficult and rarer to attain such collaborative advantages in these types of market circumstances and therefore the positive effects of CA on FP are even more rewarding than in other market situations.

The second largest effect of CA on FP was in the situation when both MD and ED were high. Hence, for both market situations in which MD was high the effect of CA on FP had the largest effects. This finding seems to be plausible due to the fact that CA had a bifurcated mediation effect on the path SCCD-¿CA-¿FP. Thus, the positive effects of SCCD on FP were going through the mediation effect CA. Therefore, CA had a strong positive effect on FP. Furthermore, due to high ED barriers, the effect of CA on FP was dampened and the slope was less steep than in the situation of low ED barriers.

In case of low MD and ED, the effect of CA on FP was even less strong. Despite the fact of low ED and MD barriers and impediments, the effect of CA on FP was still positive. However, since MD was low the effect is less strong, therefore, firms do not have the additional and incremental possibility and opportunity to overcome these business environment and market conditions by collaborative advantages and to achieve an even more rare, unique and inimitable sustainable competitive advantages as in a market situation with high MD impediments. Thus, probably in this market situation, CA is easier to be attained and achieved by firms; therefore, it does not substantially lead to a tremendous positive effect of CA on FP.

The effect of CA on FP was the least and marginal in a market situation with high ED and low MD. Nevertheless, the effect of CA on FP was still positive. The most likely explanation that the effect of CA on FP was the least in a market landscape in which there are relatively high ED and low MD impediments was that ED indicators such as bureaucratic requirements, restrictive rules and regulations and transportation difficulties dampens the positive effect of CA. However, in the situation of low CA, low MD and high ED, the relatively high FP can be partially explained by low MD and this same low MD dampened and limited the effect of high CA on improved FP.

To put the whole sequence and process of structural equation model of supply chain collaboration in a nutshell, the final measurement model was transformed and computed in the final structural path model. This final structural path model was used for mediation analyses of the mediation construct CA, and for the interaction moderation analyses of the interaction moderators CB, ED and MD to test the formulated hypotheses in the conceptual SCC hypotheses framework. The full SEM model results of the standardized regression weights of the structural direct paths and the mediation and interaction moderations are shown in figure 7.

The results of the structural path model showed that there were significant positive effects and relationships on SCCD to OP (.472***), SCCD to CA (.651***), CA to FP (.429***) and OP to FP (.579***). One surprising observation was the significant negative sign of the standardized estimate of the relationship between SCCD to FP (-.180*). The most likely explanation for this significant negative sign and magnitude of the effect of SCCD on FP is that SCC dimensions and the inherent

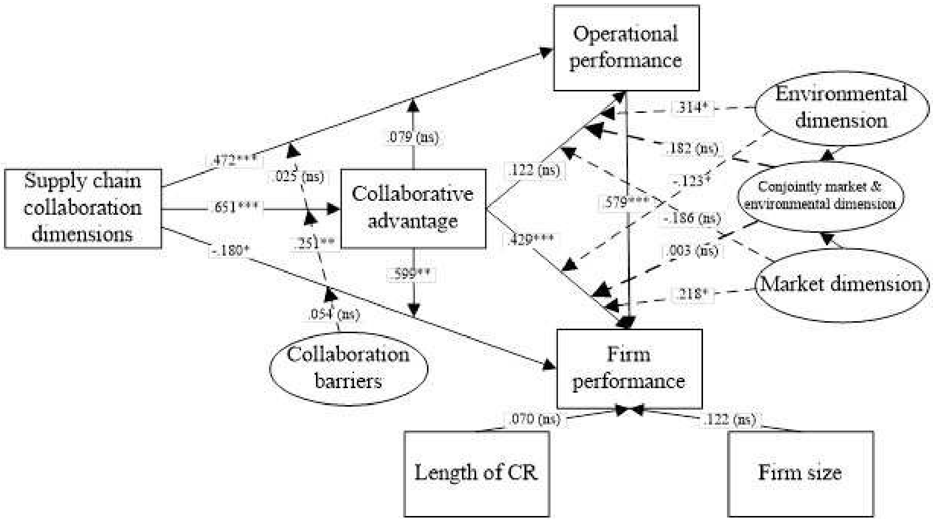


Fig. 7: SEM full model results of conceptual SCC hypotheses framework
Source: Author's own

business activities and processes require resources that have to be implemented and put into place. The direct main effect and impact of implementing, establishing and executing SCC business activities is negative on firm performance, because it implies that no advantages were obtained through SCC. The results of the SEM model test the hypotheses of structural direct paths and the mediation variable CB on the relationships SCCD to OP and SCCD to FP. Furthermore, the two-way interaction variable of the moderation variable CB, and the two-way and three-way interaction effects of the moderation variable MD and ED are also included. The results of the hypotheses testing are shown in the table 16.

Conclusively, in the contextual cross-border inter-firm collaboration in the case of EU15-Russian supply chains, the integrative and integral SEM full model results show that the different dimensions of SCC have a significant positive impact and effect on realizing and achieving collaborative advantages and improving operational performance directly. These collaborative advantages are a form and type of sustainable competitive advantage in which a firm is able to distinguish itself from its competitors to improve its uniqueness and inimitableness, thereby, increasing its competiveness. Increased competiveness of a firm leads to significant direct positive effects on improvement of firm performance, and a positive not significant effect on improvement of operational performance. Improved operational performance primarily and significantly through the different dimensions of SCC and marginally and not significantly through collaborative advantages, have a positive significant effect and impact on improvement of firm performance. The mediation variable CA had a marginal positive insignificant effect and impact on the path SCCD to OP, and a bifurcated mediation on the path SCCD to FP. All the interaction moderation impediment variables had a not significant negative direct impact on their respected dependent variables except for CB on OP. The interaction effect of CB was only significant on the path SCCD to CA. Last but not least, the three-way

Table 16: Hypotheses testing and results

	Chi-square/df.	CFI	NFI	AGFI	RMSEA
Structural model	3.099	0.846	0.806	0.627	0.180
Hypotheses	**Structural path**			**Std. est.**	**Result**
H1a	SCCD ⇒ OP			.472***	Supported
H1b	SCCD ⇒ CA			.651***	Supported
H1c	SCCD ⇒ FP			-.180*	Rejected
H2a	CA ⇒ OP			.122 (ns)	Rejected
H2b	CA⇒ FP			.429***	Supported
H2c	SCCD⇒ CA ⇒ OP			.079 (ns)	Supported
H2d	SCCD ⇒ CA⇒ FP			.599**	Supported
H3	OP ⇒ FP			.579***	Supported
H4a	SCCD ⇒ CB ⇒ OP			.025 (ns)	Supported
H4b	SCCD ⇒ CB⇒ CA			.251**	Supported
H4c	SCCD ⇒ CB ⇒ FP			.054 (ns)	Supported
H5a	CA ⇒ ED & MD ⇒ OP			.182 (ns)	Supported
H5b	CA⇒ MD ⇒ OP			-.186 (ns)	Rejected
	CA ⇒ ED ⇒ OP			.314*	Supported
H5b	CA⇒ ED & MD ⇒ FP			003 (ns)	Supported
	CA ⇒ MD⇒ FP			.218*	Supported
	CA ⇒ ED⇒ FP			-.123*	Rejected

Source: Author's own

interaction effects of MD and ED were both positive, but not significant on either OP or FP, as expected.

7.5. Concluding Remarks and Findings

The SEM full model results of structural direct paths of the developed conceptual SCC hypotheses model, which was partially adapted from Ramanathan and Gunasekaran (2012) and Zhang and Cao (2011), supported and replicated the results of prior studies of prominent and well-known scholars and researchers in the research area SCC of SCM literature that in general SCC achieves improvement and positive changes in firm performance.

The structural paths of the full SEM model showed that the dimensions of SCC by means of information sharing, decision synchronization, incentive alignment, resource sharing, collaborative communication, joint knowledge creation and goal congruence achieved positive direct changes on operational performance. The dimensions information sharing and collaborative communication were considered as pivotal and imperative pillars for SCC in the cross-border and inter-firm context.

Information sharing between supplier and buyer increases visibility of key performance metrics and process data which enables them to obtain the bigger picture of the as-is situation that takes into account important factors in making effective decisions. These effective decisions by both supplier and buyer enable them to address product flow issues and hiccups more quickly, thereby allowing more agile demand planning to take place. Complementary on information sharing, collaborative communication enhances the tight and close inter-firm relationship. Hence, as Mohr and Nevin (1990) highlighted, the patterns of collaborative communication increases the intensity and frequency, has more bidirectional flows, better informal modes, and increased indirect influence. Therefore, information sharing and collaborative communication in conjunction lead to more frequent contact points and moments and increases the accuracy and relevancy of the content. Another spillover effect of the SCC dimensions information sharing and collaborative communication is that it increases trust and commitment by means of social exchange processes. Especially, within a cross-border and international business context this snowball effect of events accumulates and will have a positive direct main effect on primarily operational performance.

Conclusively, information sharing through optimized, smooth and lean collaborative communication increases the ability to make better decisions and to take actions on the basis of greater visibility (Davenport et al. 2001). The core cornerstone and backbone of information sharing and collaborative communication provides the opportunity to link integrated information and performance drivers. Hence, information sharing and collaborative communication provide a platform to stimulate joint knowledge creation and decision synchronization by means of relevant, timely and accurate information. As decisions are incrementally more synchronized between supplier and buyer, incentive alignments come into place which employs performance metrics to construct benefit and cost sharing agreements. This new form of business environment of integrated information between supplier and buyer helps to fulfill demand more quickly with shorter order cycle times.

The results of the study in this research paper showed that the dimensions information sharing and collaborative communication are paramount for SCC, especially in a cross-border and inter-firm context. These dimensions incentivize the more deeply involved dimensions of SCC such as decision synchronization, joint knowledge creation, incentive alignment and goal congruence.

To summarize, the full SEM model results showed that SCC by means of the aforementioned seven dimensions has a positive direct effect and impact on both operational performance and collaborative advantages. The improvement of operational performance and the established and realized collaborative advantages by SCC dimensions have a positive effect and impact on firm performance which increases the profitability and competitiveness of the firm.

8. Conclusions and Implications

The research of this paper provided comprehensive understanding of the relationships and effects of SCC and its effects on both operational performance and firm performance by testing the latest theoretical concepts of SCC. The novelty and topicality of this study is the integrative and integral empirical study. Prior studies only included the direct main effects of SCC activities and dimensions on mainly firm performance KPIs. This study included collaborative advantages as a mediation variable to determine the mediation effects of SCC on operational performance and firm performance.

Due to the international business dimension of the cross-border inter-firm contextual design of the study, the impediments, collaboration barriers and cross-border business barriers, in the form of market dimension and environmental dimension, were included as interaction moderation variables. Again, these interaction moderation variables offer novel insights which have not been analyzed before. These interaction moderation variables measured the change of effect of different hypothesized direct main effects in a variety of different market environments and situations by means of low and high impediment factors. Furthermore, the direct and main effects of these interaction moderation variables were also included to conclude their impact on collaborative advantages, operational performance and firm performance.

The final results of the novel introduced mediation analysis showed and implied that the actual direct main effect of SCC has a negative impact and effect on firm performance. The most plausible and reasonable explanation for this direct main effect is that SCC requires tangible and intangible assets of the firm. If the firm does not achieve any form of collaborative advantages and/or improvements in operational performance, SCC does not add any value. Hence, these opportunity costs could have been used for different purposes. However, the relationship SCC through the mediator collaborative advantages shows positive changes and improvements on firm performance. Thus, collaborative advantages bifurcates the effect of SCC on firm performance. On the other hand, there is no mediation effect between the relationship SCC and operational performance. Hence, it can be concluded that collaborative advantage as a mediator explains the relationships and effects better than prior conducted studies on both operational performance and firm performance.

Another novel element of the study in this study was the moderator collaboration barriers and cross-border business barriers by means of market dimension and environmental dimension. The results of the full SEM model of the moderator collaboration barriers showed that collaboration barriers moderate positively the positive effect of SCC on CA. Thus, SCC has an even greater effect and impact on realizing collaborative advantages. Although, the direct main effect of collaboration barriers on collaboration advantages is negative. Therefore, it can be concluded that under the presence of collaboration barriers, SCC will experience difficulties to be conducted properly and firms are disincentified and discouraged to conduct SCC. However, if a firm manages to realize SCC, the effects on collaborative advantages will be even more profound and stronger. These stronger and more profound collaborative advantages can be interpreted as more unique and rare collaborative advantages under challenging collaboration business environments which leads to even greater improvements in firm performance.

The results of the moderators market dimension and environmental dimension showed that the positive effect of collaborative advantages on operational performance are even more profound and stronger in market situations which are categorized by environmental impediments and barriers, while on the other hand, the positive effect of collaborative advantages on firm performance dampens. As for market dimensional impediments and barriers, the positive effect of collaborative advantages on firm performance amplifies in market situations that are characterized by market impediments and barriers. In conjunction, both market dimensions and environmental dimension strengthens the positive relationship between collaborative advantage and operational performance and firm performance. The strengthening effects tend to be slightly stronger on operational performance than on firm performance due to the stronger negative direct effects of both market dimensions and environmental dimensions on operational performance.

Based on the results of the interaction moderation analyses of market dimension and environmental dimension impediments, the figure below was constructed.

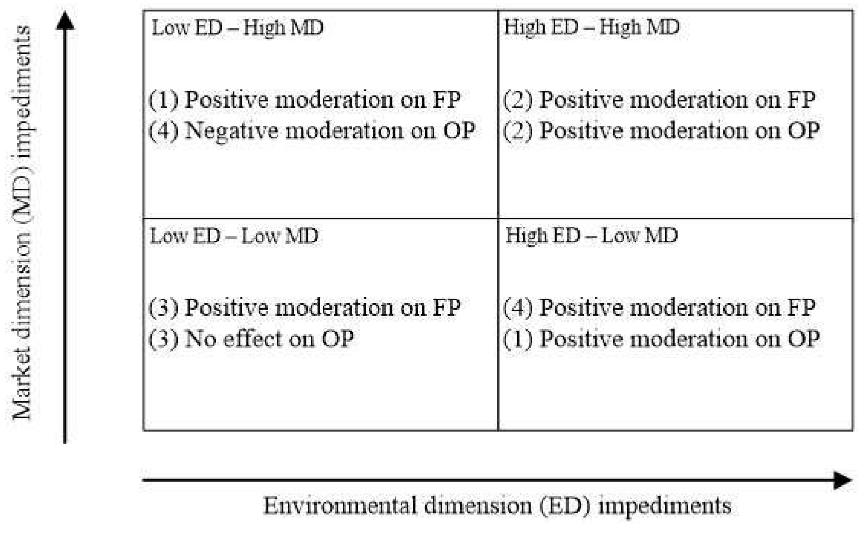


Fig. 8: Moderation changes in different market situations on OP and FP
Source: Author's own

This figure of four quadrants interact the low and high moderation effects and changes of market dimension and environmental dimension on the effects and relationship between collaborative advantages and operational performance and firm performance. The numbers within the parentheses show the rank and magnitude of the interaction moderation effect on the relationship between collaborative advantages and operational performance and firm performance. In other words, the higher the rank (1), the higher the moderation effect of collaborative advantages on operational performance or firm performance.

The figure 8 shows that in a market situation with high environmental and market impediments and barriers, the effect of collaborative advantages is positively moderated on both operational performance and firm performance.

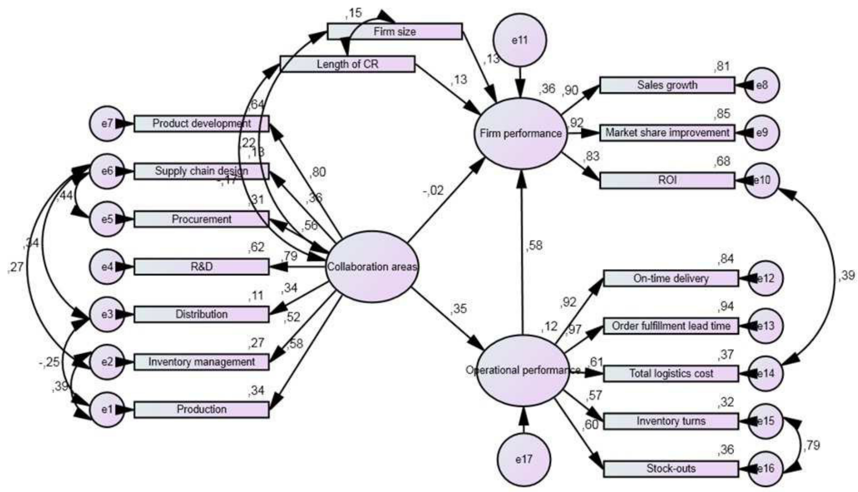
The final results showed and implied that the actual direct main effect of SCC has a negative effect on firm performance. The most plausible and reasonable explanation for this direct main effect is that SCC requires tangible and intangible assets of the firm. If the firm does not achieve any form of collaborative advantages and/or improvements in operational performance, SCC does not add any value. Hence, these opportunity costs could have been used for different purposes. However, the relationship SCC through the mediator collaborative advantages shows positive changes and improvements on firm performance. Thus, collaborative advantages bifurcates the effect of SCC on firm performance. On the other hand, there is no mediation effect between the relationship SCC and operational performance. Hence, it can be concluded that collaborative advantage as a mediator explains the relationships and effects better than prior conducted studies on both operational performance and firm performance.

In conclusion, after finalizing all the empirical and statistical analyses and formulating the conclusions and implications, the strict and precise contribution and proposition of this study on the depicted and formulated problem statement, which stated that it is unclear and ungrounded and that there is no definite and conclusive answer if SCC has a positive direct impact and effect on operational performance and on firm performance, and which mechanisms mediate and moderate such impacts and effects in the case of the contextual cross-border inter-firm (EU15-Russia) design, is as follows.

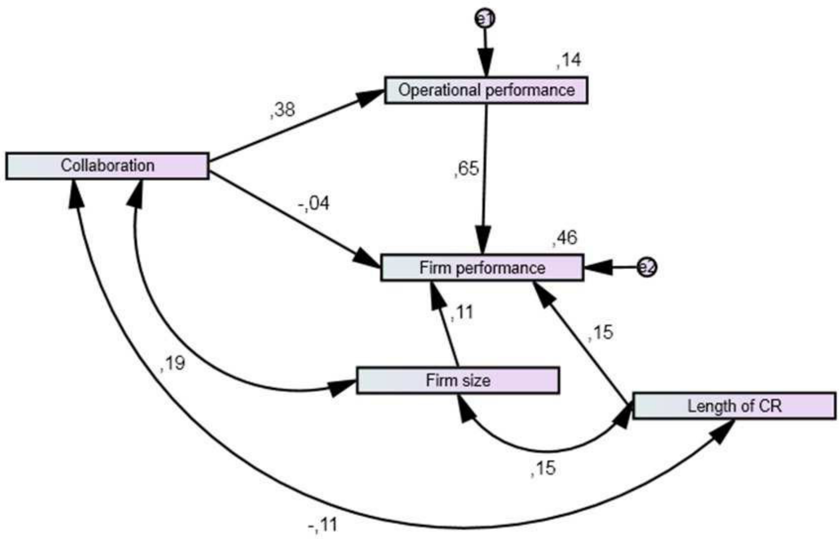
In the contextual cross-border inter-firm (EU15-Russia) design, SCC has a positive direct impact and effect on operational performance and indirectly through collaborative advantages on firm performance. The effects of SCC are bifurcated by collaborative advantages on firm performance. Furthermore, collaboration barriers and cross-border business barriers have negative direct main effects, but strengthen and amplify the effect of collaborative advantages on operational performance and firm performance.

Appendix

1. Path Diagram of Scope and Depth of Collaboration



2. Path Diagram of Scope and depth of Collaboration (Composite Observed)



3. Factor Analysis Latent Construct CBBB

Table 17: Rotated Component Matrix[a]

	Component		
	1	2	3
Restrictive rules and regulation	,890		
Bureaucratic requirements	,856		
Lack of government assistance	,840		
Transportation difficulties	,783		
High tariff and non-tariff barriers	,587	,327	
Unfamiliar foreign business practice		,871	
Different customer culture		,861	
High business risk		,621	
Limited information about markets	,402	,576	
Strong international competition			,808
Unfavorable foreign exchange rates	,348		,795

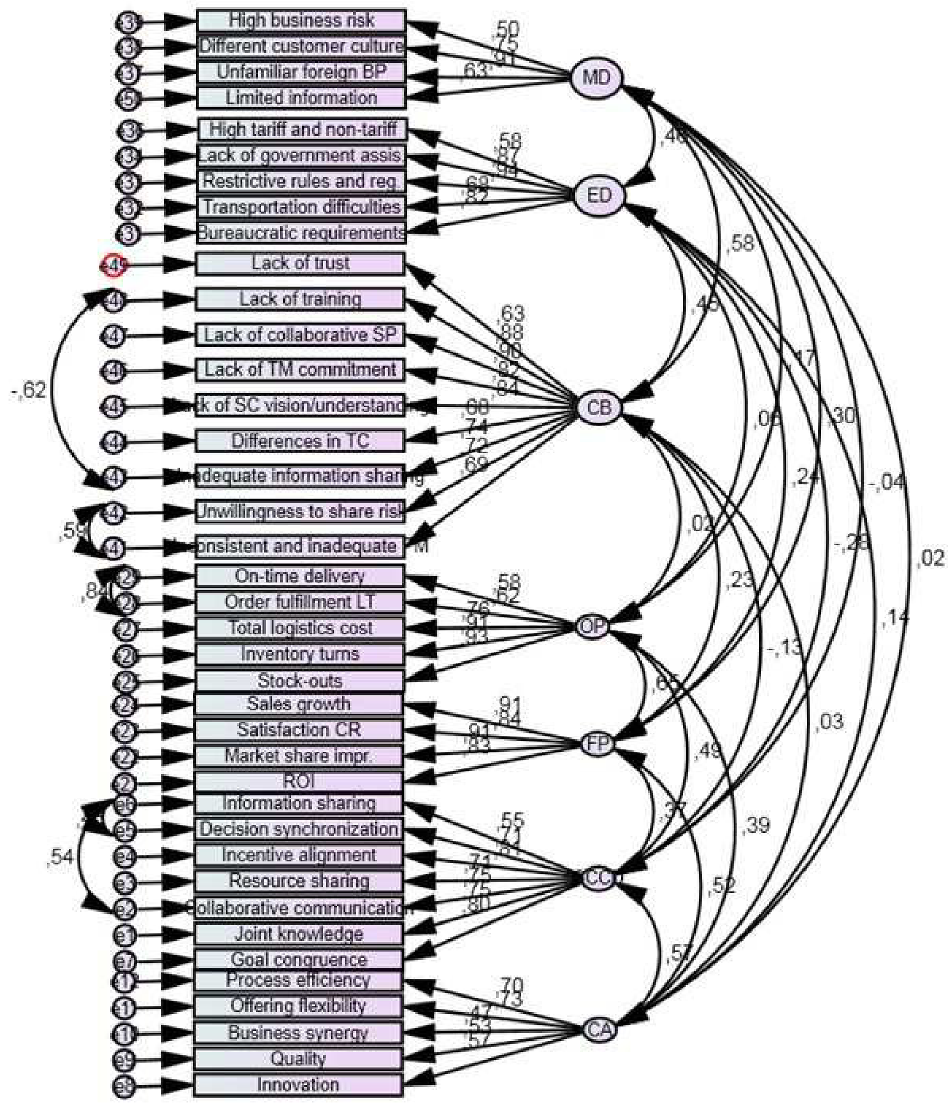
Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.
a. Rotation converged in 5 iterations.

Table 18: Rotated Component Matrix[a]

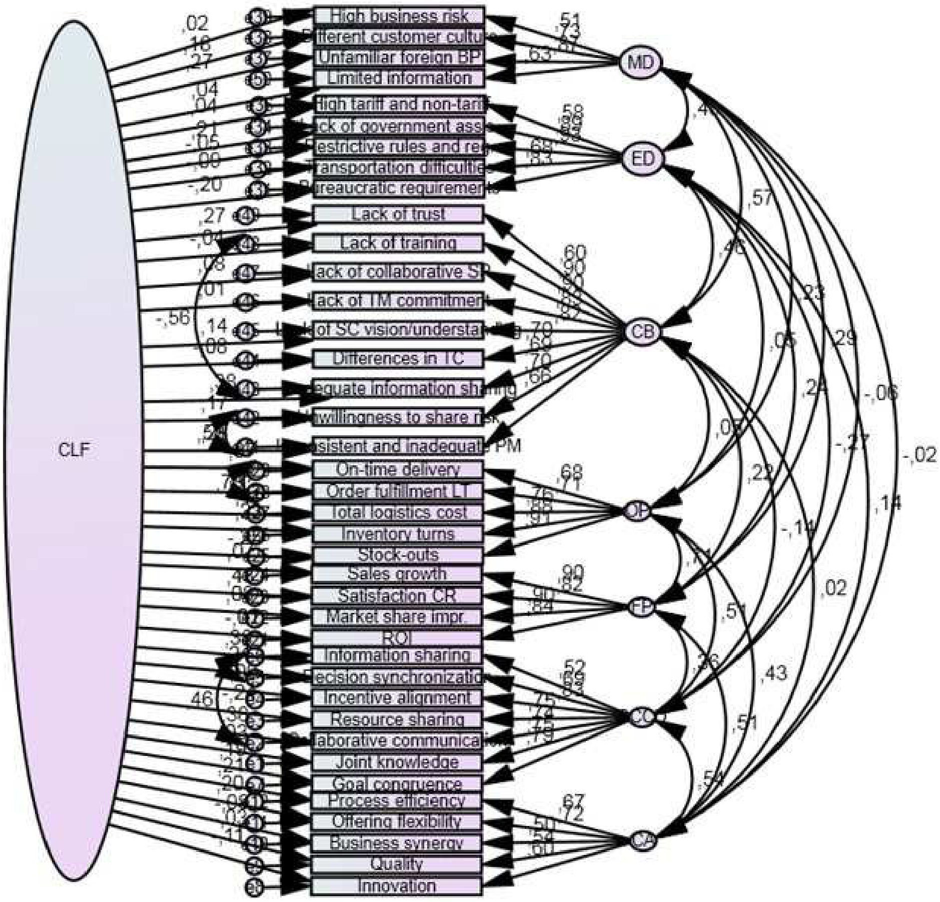
	Component	
	1	2
Restrictive rules and regulation	,904	
Bureaucratic requirements	,869	
Lack of government assistance	,845	
Transportation difficulties	,767	
High tariff and non-tariff barriers	,594	
Different customer culture		,878
Unfamiliar foreign business practice		,868
High business risk		,626
Limited information about markets		,593

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.
a. Rotation converged in 3 iterations.

4. Confirmatory Factor Analysis



5. Common Method Bias - CFA with CLF

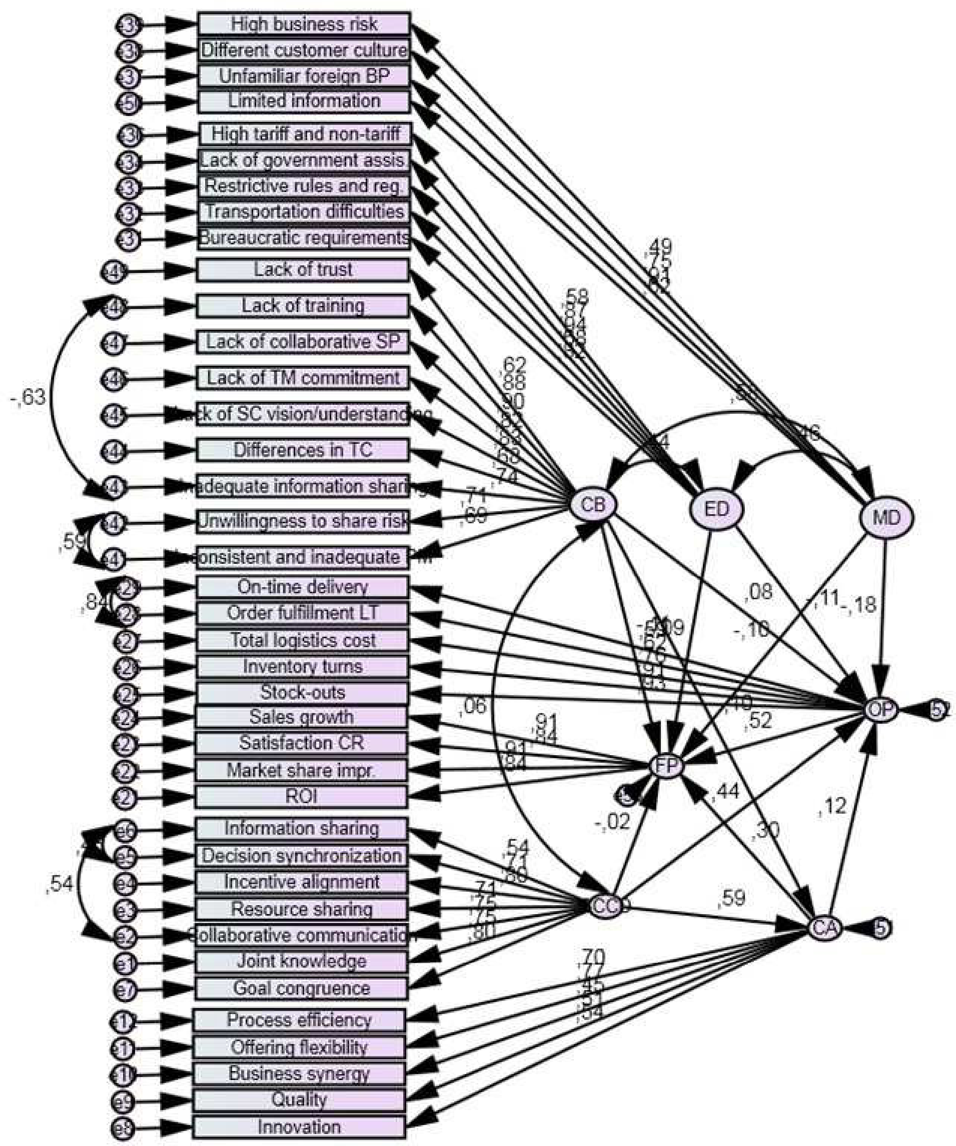


6. Comparison between CLF with and without CLF

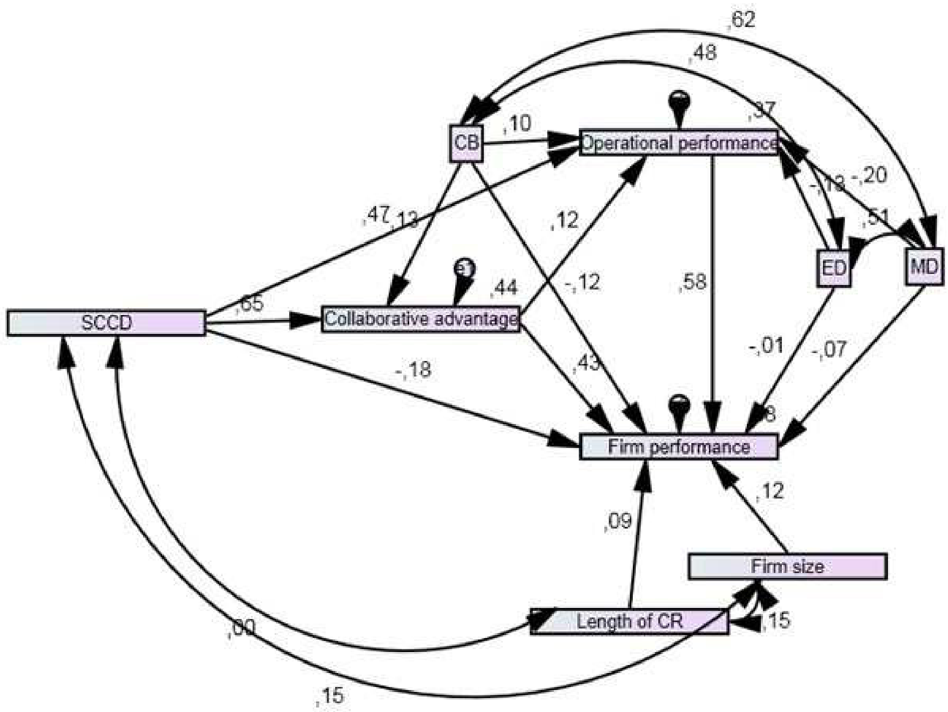
Standardized Regression Weights **Standardized Regression Weights:**

With CLF			**Without CLF**		
	Estimate			Estimate	
var46 <— SCCD	0,747		var46 <— SCCD	0,755	0,008
var45 <— SCCD	0,736		var45 <— SCCD	0,754	0,018
var44 <— SCCD	0,748		var44 <— SCCD	0,707	-0,041
var43 <— SCCD	0,828		var43 <— SCCD	0,81	-0,018
var42 <— SCCD	0,688		var42 <— SCCD	0,709	0,021
var41 <— SCCD	0,521		var41 <— SCCD	0,55	0,029
var47 <— SCCD	0,795		var47 <— SCCD	0,799	0,004
var53 <— CA	0,603		var53 <— CA	0,568	-0,035
var52 <— CA	0,539		var52 <— CA	0,534	-0,005
var51 <— CA	0,498		var51 <— CA	0,467	-0,031
var50 <— CA	0,715		var50 <— CA	0,733	0,018
var49 <— CA	0,672		var49 <— CA	0,699	0,027
var81 <— FP	0,84		var81 <— FP	0,832	-0,008
var80 <— FP	0,899		var80 <— FP	0,91	0,011
var79 <— FP	0,818		var79 <— FP	0,839	0,021
var78 <— FP	0,904		var78 <— FP	0,909	0,005
var88 <— OP	0,909		var88 <— OP	0,933	0,024
var87 <— OP	0,879		var87 <— OP	0,913	0,034
var86 <— OP	0,755		var86 <— OP	0,757	0,002
var85 <— OP	0,713		var85 <— OP	0,619	-0,094
var84 <— OP	0,68		var84 <— OP	0,584	-0,096
var111 <— CB	0,663		var111 <— CB	0,693	0,03
var109 <— CB	0,693		var109 <— CB	0,736	0,043
var108 <— CB	0,7		var108 <— CB	0,682	-0,018
var107 <— CB	0,824		var107 <— CB	0,836	0,012
var106 <— CB	0,827		var106 <— CB	0,819	-0,008
var105 <— CB	0,903		var105 <— CB	0,905	0,002
var104 <— CB	0,896		var104 <— CB	0,884	-0,012
var103 <— CB	0,596		var103 <— CB	0,626	0,03
var110 <— CB	0,7		var110 <— CB	0,716	0,016
var98 <— ED	0,928		var98 <— ED	0,936	0,008
var100 <— ED	0,828		var100 <— ED	0,822	-0,006
var97 <— ED	0,892		var97 <— ED	0,872	-0,02
var99 <— ED	0,68		var99 <— ED	0,681	0,001
var95 <— ED	0,585		var95 <— ED	0,584	-0,001
var94 <— MD	0,872		var94 <— MD	0,91	0,038
var93 <— MD	0,732		var93 <— MD	0,753	0,021
var92 <— MD	0,508		var92 <— MD	0,497	-0,011
var101 <— MD	0,634		var101 <— MD	0,626	-0,008

7. Hybrid Structural Model



8. Structural Strandardized Regression Model



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A Game-Theoretic Model of Pollution Control with Asymmetric Time Horizons*

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Abstract In the contribution a problem of pollution control is studied within the game-theoretic framework (Kostyunin et al., 2013; Gromova and Plekhanova, 2015; Shevkoplyas and Kostyunin, 2011). Each player is assumed to have certain equipment whose functioning is related to pollution control. The i-th player's equipment may undergo an abrupt failure at time T_i. The game lasts until any of the players' equipment breaks down. Thus, the game duration is defined as $T = \min(T_1, \dots, T_n)$, where T_i is the time instant at which the i-th player stops the game.
We assume that the time instant of the i-th equipment failure is described by the Weibull distribution. According to Weibull distribution form parameter, we consider different scenarios of equipment exploitation, where each of player can be in "an infant", "an adult" or "an aged" stage. The cooperative 2-player game with different scenarios is studied.

Keywords: differential game, cooperative game, pollution control, random duration, Weibull distribution.

1. Introduction

When considering game-theoretic problems of pollution control it is important to take into account the fact that the game may end abruptly. The reason for this can be an equipment failure, an economical break-down or a natural disaster among many others. In this paper we consider one particular case when the game duration is determined by the life duration of the equipment. Typically, when describing the life circle of a technical system one considers three different stages: the "infant" stage, the "adult" or regular stage, and the "aged" or weared-out stage. It is well known that the life-time for all these stages can be well described by the Weibull distribution (Weibull, 1951).

In this paper, we consider a pollution control problem for n players. We assume that the equipment of each player at the beginning of the game can be in any of three states ("infant", "adult" or "aged"). Thus the life-time of the equipment differs for each player. The game ends with the occurence of the first failure.

The proposed approach is illustrated by an example of pollution control problem with two players.

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2. The Problem Statement

Consider a game-theoretic model of pollution control based on the models (Breton et al., 2005; Shevkoplyas and Kostyunin, 2011). There are n players (countries) involved in the game. Each player i manages his emission $e_i \in [0, b_i]$, $b_i > 0$, $i = \overline{1,n}$. Each country is assumed to have certain equipment whose functioning is related to pollution control.

The game starts at the time instant t_0. The i-th player's equipment may undergo an abrupt failure at time T_i. The game lasts until any of the players' equipment breaks down. Thus, the game duration is defined as $T = \min(T_1, \ldots, T_n)$, where T_i is the time instant at which the i-th player stops the game.

We assume that the time instant of the i-th equipment failure is the random variable T_i with known probability distribution function $F_i(t)$, $i = \overline{1,n}$ (Petrosjan and Murzov, 1966). Assume also that $\{T_i\}_{i=1}^n$ – independent random variables. It is obvious, that $T = min\{T_1, T_2, \ldots, T_n\}$ is a random variable too. Using the cumulative distribution functions of the random variables $\{T_i\}_{i=1}^n$, we can write the expression for $F(t)$.

Proposition 1. *Let $\{T_i\}_{i=1}^n$ – independent random variables, with probability distribution functions $\{F_i(t)\}_{i=1}^n$. Then probability distribution function $F(t)$ of the random variable $T = min\{T_1, T_2, \ldots, T_n\}$ has the following form:*

$$F(t) = 1 - \prod_{i=1}^{n} (1 - F_i(t)) \,. \tag{1}$$

Proof. According to the distribution function definition:

$$F(t) = P\{T < t\} \,.$$

Here

$$P\{T < t\} = 1 - P\{T \geq t\} \,.$$

The random variable T is defined as $T = min\{T_1, T_2, \ldots, T_n\}$, so:

$$P\{T \geq t\} = P\{min\{T_1, T_2, \ldots, T_n\} \geq t\} \,.$$

$\{T_i\}_{i=1}^n$ – independent random variables, then

$$P\{min\{T_1, T_2, \ldots, T_n\} \geq t\} = P\{T_1 \geq t\} P\{T_2 \geq t\} \ldots P\{T_n \geq t\} \,.$$

Using again the distribution function definition, we have

$$P\{T_1 \geq t\} P\{T_2 \geq t\} \ldots P\{T_n \geq t\} = (1 - F_1(t))(1 - F_2(t)) \ldots (1 - F_n(t)) \,,$$

i.e.:

$$F(t) = 1 - \prod_{i=1}^{n} (1 - F_i(t)) \,.$$

□

The net revenue of player i at time instant t is given by quadratic functional form:

$$R_i\left(e_i\right) = e_i(t)\left(b_i - \frac{1}{2}e_i(t)\right),\ t \in [t_0, T], \tag{2}$$

$$b_i > 0,\ i = \overline{1,n}. \tag{3}$$

Denote the stock of accumulated net emissions by P(t). The dynamics of the stock is given by the following equation with initial condition:

$$\dot{P}(t) = \sum_{i=1}^{n} e_i(t),\ t \in [t_0, T], \tag{4}$$

$$P(t_0) = P_0. \tag{5}$$

The expected integral payoff of the player i can be represented as the following mathematical expectation:

$$K_i\left(P_0, t_0, e_1, e_2, \ldots, e_n\right) = E\bigg(\int_{t_0}^{T} \big(R_i(e_i) - d_i P(s)\big) ds\bigg),$$

where $d_iP(t)$ – is a cost of player i for decreasing of his emission at the moment t.

Then we have the following integral payoff for player i:

$$K_i\left(P_0, t_0, e_1, e_2, \ldots, e_n\right) = \int_{t_0}^{\infty}\int_{t_0}^{t} \big(R_i(e_i) - d_i P(s)\big) ds dF(t). \tag{6}$$

After simplification of the integral payoff (Kostyunin and Shevkoplyas, 2011), we get

$$K_i\left(P_0, t_0, e_1, e_2, \ldots, e_n\right) = \int_{t_0}^{\infty} \big(R_i(e_i) - d_i P(s)\big)\big(1 - F(s)\big) ds. \tag{7}$$

Denote the described game starting at the time instant t_0 from the situation P_0 by $\Gamma(t_0, P_0)$. Let the game $\Gamma(t_0, P_0)$ develops along the trajectory $P(t)$. Then at the each time instant $\theta \in [t_0; T]$ players enter new game (subgame) $\Gamma(\theta, P(\theta)))$ with initial state $P(\theta)$ and duration $(T - \theta)$. The expected payoff of the player i under the condition that the game is not finished before the moment θ can be calculated by following formula:

$$K_i\left(P(\theta), \theta, e_1, e_2, \ldots, e_n\right) = \frac{1}{1 - F(\theta)} \int_{\theta}^{\infty} \big(R_i(e_i) - d_i P(s)\big)\big(1 - F(s)\big) ds. \tag{8}$$

Further we assume an existence of a density function:

$$f(t) = F^{'}(t), \tag{9}$$

and using the Hazard function $\lambda(t)$ which is given by the following definition:

$$\lambda(t) = \frac{f(t)}{1 - F(t)}, \tag{10}$$

we have

$$1 - F(s) = e^{-\int\limits_0^s \lambda(\tau) d\tau}. \tag{11}$$

We can prove the following proposition.

Proposition 2. *Let* $\{T_i\}_{i=1}^n$ *– independent random variables with probability distribution function* $F_i(t)$, $\ i=\overline{1,n}$ *and the hazard functions* $\{\lambda_i(t)\}_{i=1}^n$. *Then for the random variable* $T = min\{T_1, T_2, \dots, T_n\}$ *the hazard function* $\lambda(t)$ *can be calculated by the following formula:*

$$\lambda(t) = \sum_{i=1}^{n} \lambda_i(t). \tag{12}$$

Proof. As we say in (1):

$$F(t) = 1 - \prod_{i=1}^{n} (1 - F_i(t)), \tag{13}$$

using (11), we have:

$$\prod_{i=1}^{n} (1 - F_i(t)) = e^{-\int\limits_0^t \lambda(\tau) d\tau}. \tag{14}$$

Taking the log of both sides in (14), we obtain:

$$\ln\big(\prod_{i=1}^{n} (1 - F_i(t))\big) = -\int\limits_0^t \lambda(\tau)\, d\tau. \tag{15}$$

Then we have:

$$\sum_{i=1}^{n} \ln\big(1 - F_i(t)\big) = -\int\limits_0^t \lambda(\tau)\, d\tau. \tag{16}$$

Similarly, we see that

$$1 - F_i(t) = e^{-\int\limits_0^t \lambda_i(\tau) d\tau}, \tag{17}$$

and

$$\ln\big(1 - F_i(t)\big) = -\int\limits_0^t \lambda_i(\tau)\, d\tau. \tag{18}$$

Substituting (18) in (16), we obtain:

$$\sum_{i=1}^{n} \int\limits_0^t \lambda_i(\tau)\, d\tau = \int\limits_0^t \lambda(\tau)\, d\tau. \tag{19}$$

So we can conclude that

$$\lambda(t) = \sum_{i=1}^{n} \lambda_i(t).$$

□

One of probability distributions that can be used for description of random variables T_i is Weibull Law. The Weibull failure rate function is given by:

$$\lambda(t) = \lambda\delta t^{\delta-1}, \quad t > 0;\ \lambda > 0;\ \delta > 0, \tag{20}$$

where δ is the shape parameter and λ is the scale parameter of the distribution. Using Weibull distribution allows to consider three "scenarios" of the game in the sense of behaviour of the random variables T_i:

1. $\delta < 1$ corresponds to "burn-in" period, when the equipment failure is mostly caused by deficiencies in design (new equipment);
2. $\delta = 1$ corresponds to "adult" period, when failures are due to random events;
3. $\delta > 1$ corresponds to "wear-out" period (worn-out equipment).

For Weibull distribution we have:

$$1 - F_i(s) = e^{-\int\limits_0^s \lambda_i \delta_i \tau^{\delta_i - 1} d\tau} = e^{-\lambda_i s^{\delta_i}}. \tag{21}$$

Then the payoff of player i in subgame $\Gamma(\theta, P(\theta))$ can be represented as:

$$K_i\left(P(\theta), \theta, e_1, e_2, \ldots, e_n\right) = e^{\sum\limits_{i=1}^n \lambda_i \theta^{\delta_i}} \int_\theta^\infty \big(R_i(e_i) - d_i P(s)\big) e^{-\sum\limits_{i=1}^n \lambda_i s^{\delta_i}} ds. \tag{22}$$

Suppose that players are agree to cooperate and maximize the joint payoff:

$$K_1 + K_2 + \cdots + K_n = \frac{1}{1 - F(\Theta)} \int_\theta^\infty \big(R_1(e_1) + R_2(e_2) + \cdots + R_n(e_n) - \\ - (d_1 + d_2 + \cdots + d_n) P(s)\big)\big(1 - F(s)\big) ds. \tag{23}$$

According to the Proposition 1:

$$F(s) = 1 - \prod_{i=1}^n \left(1 - F_i(s)\right),$$

then:

$$1 - F(s) = e^{-(\lambda_1 s^{\delta_1} + \lambda_2 s^{\delta_2} + \cdots + \lambda_n s^{\delta_n})}. \tag{24}$$

As a result, the expected total payoff of players in the subgame $\Gamma(\theta, P(\theta))$ with initial state $P(\theta)$ and duration $(T - \theta)$ is given by the following equation:

$$\sum_{i=1}^n K_i = e^{\lambda_1 \theta^{\delta_1} + \lambda_2 \theta^{\delta_2} + \cdots + \lambda_n \theta^{\delta_n}} \int_\theta^\infty \big(R_1(e_1) + R_2(e_2) + \cdots + R_n(e_n) - \\ - (d_1 + d_2 + \cdots + d_n) P(s)\big) e^{-(\lambda_1 s^{\delta_1} + \lambda_2 s^{\delta_2} + \cdots + \lambda_n s^{\delta_n})} ds. \tag{25}$$

3. 2-player game

Consider 2-player cooperative game-theoretic model of pollution control.

Let T_1 – the time instant of the equipment failure for the player 1 with probability distribution function $F_1(t)$ and failure rate function $\lambda_1(t)$. T_2 – the time instant of the equipment failure for the player 2 with probability distribution function $F_2(t)$ and failure rate function $\lambda_2(t)$.

The game duration is defined as $T = min\{T_1, T_2\}$.

The game is considered over time $t \in [0, T]$, where T is a random variable with known probability distribution function $F(t) = 1 - (1 - F_1(t))(1 - F_2(t))$ and failure rate function $\lambda(t) = \lambda_1(t) + \lambda_2(t)$.

We use Weibull distribution as a distribution of random variables T_1 and T_2. The players are assumed to have the identical scale parameter $\lambda = \lambda_1 = \lambda_2$.

Using (25), we get the equation for the joint payoff of players:

$$K_1 + K_2 = \int_0^{\infty} \big(R_1(e_1) + R_2(e_2) - (d_1 + d_2)P(s)\big)e^{-\lambda(s^{\delta_1}+s^{\delta_2})}ds. \quad (26)$$

To find the optimal emissions $\overline{e}_1,\ \overline{e}_2$ for players 1, 2, we apply Pontrygins maximum principle.:

$$\max_{\substack{e_1\in[0,b_1],\\ e_2\in[0,b_2]}} (K_1 + K_2) = \int_0^{\infty} \big(R_1(\overline{e}_1) + R_2(\overline{e}_2) - (d_1 + d_2)P(s)\big)e^{-\lambda(s^{\delta_1}+s^{\delta_2})}ds, \quad (27)$$

where

$$\begin{aligned} K_i =&\ K_i\left(P_0, e_1, e_2\right), \quad i = 1,2,\\ \dot{P}\left(t\right) =&\ e_1\left(t\right) + e_2\left(t\right),\\ P\left(0\right) =&\ P_0. \end{aligned}$$

The Hamiltonian for this problem is as follows:

$$\begin{aligned} &H(P, e_1, e_2, \Lambda) =\\ &= \left(e_1(t)(b_1 - \frac{1}{2}e_1(t)) + e_2(t)(b_2 - \frac{1}{2}e_2(t)) - (d_1 + d_2)P(t)\right) e^{-\lambda\left(t^{\delta_1}+t^{\delta_2}\right)} +\\ &\qquad\qquad + \Lambda\left(t\right)\left(e_1(t) + e_2(t)\right). \quad (28) \end{aligned}$$

From the first-order optimality condition

$$\frac{\partial H}{\partial e_i} = \left(b_i - e_i\right)\, e^{-\lambda\left(t^{\delta_1}+t^{\delta_2}\right)} + \Lambda(t) = 0,\ i = 1,2, \quad (29)$$

we get the following formulas for optimal emissions:

$$\overline{e}_i(t) = b_i + \Lambda(t)e^{\lambda\left(t^{\delta_1}+t^{\delta_2}\right)},\ i = 1,2. \quad (30)$$

Adjoint variable $\Lambda(t)$ can be found from the from the differential equation:

$$\dot{\Lambda} = -\frac{\partial H}{\partial P}. \quad (31)$$

Then

$$\Lambda(t) = (d_1 + d_2)\int_0^t e^{-\lambda\left(s^{\delta_1}+s^{\delta_2}\right)}ds + c. \quad (32)$$

We consider the problem with time $t \in [0, \infty)$ and the condition for $\Lambda(t)$ has a form:

$$\lim_{t\to\infty} \Lambda(t) = 0. \quad (33)$$

3.1. Optimal emissions

Different scenarios with possible conditions of players' equipment are considered in this section.

The normal operating mode of the equipment Let's consider the case, when the equipments of both players are used in the normal operating mode. It means that $\delta_1 = \delta_2 = 1$ ("adult" scenario).

Using (30), we have the following form for optimal emissions

$$\overline{e}_i(t) = b_i + \Lambda(t)e^{2\lambda t}. \tag{34}$$

Adjoint variable in this case has a form:

$$\Lambda(t) = (d_1 + d_2)\int\limits_0^t e^{-2\lambda s}ds + c = -\frac{(d_1 + d_2)}{2\lambda}e^{-2\lambda t} + \frac{(d_1 + d_2)}{2\lambda} + c, \tag{35}$$

where c can be found from (33):

$$c = -\frac{(d_1 + d_2)}{2\lambda}. \tag{36}$$

So

$$\Lambda(t) = -\frac{(d_1 + d_2)}{2\lambda}e^{-2\lambda t}. \tag{37}$$

Then substituting (37) in (34) we obtain the two optimal strategies:

$$\overline{e}_i(t) = \begin{cases} 0, & b_i \leq \dfrac{d_1 + d_2}{2\lambda}; \\ b_i - \dfrac{d_1 + d_2}{2\lambda}, & b_i > \dfrac{d_1 + d_2}{2\lambda}, i = 1, 2. \end{cases} \tag{38}$$

The mode of normal operation of the equipment and worn-out equipment Assume now that the equipment of the first country is in the normal operating mode ($\delta_1 = 1$) and the second one uses the worn-out equipment ($\delta_2 > 1$). Without loss of generality we assume $\delta_2 = 2$ (the Rayleigh distribution).

Using (30), we have the following form for optimal emissions:

$$\overline{e}_i(t) = b_i + \Lambda(t)e^{\lambda(t+t^2)}. \tag{39}$$

Adjoint variable in this case has a form:

$$\begin{aligned}\Lambda(t) = (d_1 + d_2)\int\limits_0^t e^{-\lambda(s+s^2)}ds + c = \\ = \frac{(d_1 + d_2)\sqrt{\pi}e^{\frac{1}{4}\lambda}}{2\sqrt{\lambda}}\left(\operatorname{erf}(\sqrt{\lambda}t + \frac{1}{2}\sqrt{\lambda}) - \operatorname{erf}(\frac{1}{2}\sqrt{\lambda})\right) + c,\end{aligned} \tag{40}$$

where c can be found from (33). Then we get

$$\Lambda(t) = \frac{(d_1 + d_2)\sqrt{\pi}e^{\frac{1}{4}\lambda}}{2\sqrt{\lambda}}\left(\operatorname{erf}(\sqrt{\lambda}t + \frac{1}{2}\sqrt{\lambda}) - 1\right). \tag{41}$$

Denote by

$$\widehat{e}_i(t) = b_i + \frac{(d_1 + d_2)\sqrt{\pi}e^{\frac{1}{4}\lambda}e^{\lambda(t+t^2)}}{2\sqrt{\lambda}}\left(\operatorname{erf}(\sqrt{\lambda}t + \frac{1}{2}\sqrt{\lambda}) - 1\right),$$

and

$$A_{\widehat{e}_i} = \{t \mid \ \widehat{e}_i(t) < 0\}.$$

Then we get:

$$\overline{e}_i(t) = \begin{cases} 0, & \text{if } t \in A_{\widehat{e}_i}; \\ \widehat{e}_i(t), & \text{otherwise.} \end{cases} \tag{42}$$

The new equipment for both countries Assume that both of countries use the new equipment, the shape parameters in this case are $\delta_1 = \delta_2 = \frac{1}{2}$.

Using (30), (32),(33), we have:

$$\overline{e}_i(t) = b_i + \Lambda(t)e^{2\lambda\sqrt{t}}, \tag{43}$$

$$\Lambda(t) = -\frac{(d_1 + d_2)(2\lambda\sqrt{t}e^{-2\lambda\sqrt{t}} + e^{-2\lambda\sqrt{t}} - 1)}{2\lambda^2} + c, \tag{44}$$

where

$$c = -\frac{(d_1 + d_2)}{2\lambda^2}. \tag{45}$$

Then we get

$$\Lambda(t) = -\frac{(d_1 + d_2)(2\lambda\sqrt{t}e^{-2\lambda\sqrt{t}} + e^{-2\lambda\sqrt{t}})}{2\lambda^2}. \tag{46}$$

Let's find optimal emissions in this case. If $b_i \leq \frac{d_1+d_2}{2\lambda^2}$, then

$$\overline{e}_i(t) = 0. \tag{47}$$

If $b_i > \frac{d_1+d_2}{2\lambda^2}$, then

$$\overline{e}_i(t) = \begin{cases} 0, & t \geq \left(\dfrac{2\lambda^2 b_i - d_1 - d_2}{2\lambda(d_1 + d_2)}\right)^2; \\ b_i - \dfrac{(d_1 + d_2)(2\lambda\sqrt{t} + 1)}{2\lambda^2}, & 0 \leq t < \left(\dfrac{2\lambda^2 b_i - d_1 - d_2}{2\lambda(d_1 + d_2)}\right)^2. \end{cases} \tag{48}$$

The worn-out equipment for both countries Consider the problem in the case when equipment of both countries is worn-out ($\delta_1 > 1$, $\delta_2 > 1$). Fix the following values of the shape parameters: $\delta_1 = \delta_2 = 2$.

Using (30), (32),(33), we have:

$$\overline{e}_i(t) = b_i + \Lambda(t)e^{2\lambda t^2}, \tag{49}$$

$$\Lambda(t) = \frac{(d_1 + d_2)\sqrt{2\pi}\operatorname{erf}(\sqrt{2\lambda}t)}{4\sqrt{\lambda}} + c, \tag{50}$$

where c can be found from (33).

Then we get the following form for adjoint variable:

$$\Lambda(t) = \frac{(d_1 + d_2)\sqrt{2\pi}(\operatorname{erf}(\sqrt{2\lambda}t) - 1)}{4\sqrt{\lambda}}. \tag{51}$$

Denote by

$$\widetilde{e}_i(t) = b_i + \frac{(d_1+d_2)\sqrt{2\pi}e^{2\lambda t^2}(\mathrm{erf}(\sqrt{2\lambda}t)-1)}{4\sqrt{\lambda}},$$

and

$$A_{\widetilde{e}_i} = \{t \mid \ \widetilde{e}_i(t) < 0\}.$$

Then we get:

$$\overline{e}_i(t) = \begin{cases} 0, & \text{if } \ t \in A_{\widetilde{e}_i}; \\ \widetilde{e}_i(t), & \text{otherwise.} \end{cases} \tag{52}$$

The graphic representation of the optimal emissions for four scenarios of the game we can see at the Fig. 1.

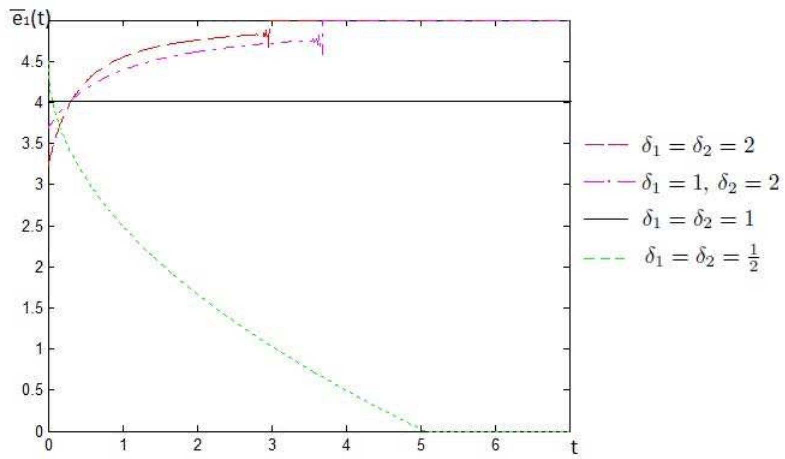

Fig. 1: Optimal emissions of the player 1 for four scenarios of the game.

4. Conclusion

In the paper a problem of pollution control was studied within the game-theoretic framework. Each player was assumed to have certain equipment whose functioning is related to pollution control. The i-th player's equipment may undergo an abrupt failure at random time T_i which is described by the Weibull distribution with different parameters corresponding to different modes of operation of the equipment. The game lasts until any of the players' equipment breaks down. Thus, the game duration is defined as $T = \min(T_1, \ldots, T_n)$, where T_i is the time instant at which the i-th player stops the game.

A cooperative 2-player game with different scenarios was studied in detail.

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Supply Chain Cooperation Modeling: Trends and Gaps*

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Abstract The aim of this work is firstly to provide a comprehensive overview of the current trends in supply chain cooperation modeling and secondly to highlight the fruitful research avenues in this field based on a systematic literature review. As a result, it was found that in the previous years the research work on supply chain management has primarily focused on the study of materials and information flows and very little work has been done on the study of upstream and downstream flows of money. It is shown, that the evolution of the research in the field of supply chain cooperation modeling has evolved from centralized cooperative models through decentralized coordination models to collaborative models. Moreover, the unit of modeling has become significantly more complex from unconnected supply chains to multi-echelone systems. From the authors point of view, the further step ahead is development of models of collaborative supply chain networks, especially in the field of financial supply chain management.

Keywords: supply chain management, supply chain cooperation, supply chain modeling, thematic trend, methodological trend.

1. Introduction

1.1. Justification of the Research

The field of supply chain management (SCM) has developed as an academic discipline in the last 30 years, as can be observed by the growing number of academic journals and articles that focus on it. This research explores theoretical developments in this discipline by analyzing the existing stream of literature, what allows the authors to spot trends and gaps in the literature, and to identify fruitful areas for future research.

In order to inform future SCM development, it is helpful to reflect on where the gaps are in current theoretical perspectives. The following discussion is not meant to be an exhaustive list; rather, it is more a consideration of potential avenues of thought that may have saliency for SCM in general and supply chain collaboration (SCC) in particular.

SCM revolves around coordination and cooperation among several business partners that are linked through flows of material, money and information. These partners include suppliers of basic raw materials and component parts, manufacturers, wholesalers, distributors, transporters, retailers, banks and financial institutions. In general, the materials, component parts and finished goods flow downstream

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although the returned merchandise flows upstream. The money in contrast flows upstream in a supply chain whereas the information flows in both directions. For an effective supply chain system, the management of upstream flow of money is as important as the management of downstream flow of goods (Gupta and Dutta, 2011). Nevertheless, the research work on supply chain management has primarily focused on the study of materials flow and very little work has been done on the study of upstream flow of money.

The reminder of the paper is organized as follows: the next section describes the literature search procedures. The following section presents theoretical background in terms of SCC meaning, outlines the difficulties faced by SC members in adoption of SCC and possible causes of lack of coordination in SC. The next section deals with different mechanisms of SCC. In the next section SCC models are summarized. The last section concludes the paper and suggests an agenda for future research.

1.2. Research Questions and Objectives

The goal of the paper is to provide a comprehensive overview of the current trends in supply chain cooperation modeling and highlight the fruitful research avenues in this field based on a systematic literature review. To achieve the above formulated goal the following objectives are to be fulfilled:

1. To analyze the evolution of the key concepts in the field of supply chain management: supply chain, supply chain management, supply chain cooperation, supply chain coordination, supply chain collaboration, supply chain performance on the grounds of theoretical and methodological identification and systematization.
2. To analyze the metrics of supply chain cooperation performance, financial supply chain cooperation performance on the grounds of theoretical systematization.
3. To analyze the existing supply chain cooperation models, financial supply chain cooperation models and identify their strengths and limitations on the grounds of theoretical systematization.

1.3. Methodology of the Research

As the goal of the paper is to provide a snapshot of the diversity of the research being conducted in the field of supply chain management and especially financial supply chain management in order to outline further research paths on the basis of theoretical and methodological gap identification, only the journals ranked 4* or 4 (top journals in the field) in the Chartered Association of Business Schools Academic Journal Guide 2015 research were used for the initial search, namely: Journal of Operations Management, International Journal of Operations and Production Management, Production and Operations Management (in the field of Operations and Technology Management). It has been suggested that top-ranked journals should communicate, diffuse and archive scholarly knowledge more effectively than other journals.

The period of search was set from 2010 till 2015 year. An initial keyword search for articles containing any of the terms of the phrase financial supply chain management (limited to citations and abstracts of periodicals) was then subsequently limited to the exact phrase, financial supply chain management.

The papers in response to the above-mentioned objectives were gathered and systematically analyzed.

1.4. Limitations of the Research

The limitations of the following research are generally related to the method that we used to obtain the literature sample. Despite the fact that the aforementioned journals belong to the top-ranked specialist journals in the field, it however limits the external validity of our study and the possibility of extending the conclusions.

2. Trends and Gaps in Supply Chain Collaboration

2.1. Concept of Supply Chain Management

Globalization, technology boom, organizational consolidation as well as quickly altering government policy and regulation made it very important for companies to be familiar with the concept of supply chains (SC) that function inside and around the company. That is the reason why in recent years the area of supply chain management (SCM) has become very popular. This is evidenced by marked increase in practitioner and academic publications, conferences, professional development programs and university courses in the area. While interest in SCM is immense, it is clear that much of the knowledge about SCM resides in a narrow fields such as purchasing, logistics, IT and marketing. At least partly as a result of this, there appears to be little consensus on the conceptual and research methodological bases of SCM. This has contributed to the existence of a number of gaps in the knowledge base of the field. Thus, from a conceptualization perspective, the definition of the term is unclear.

According to Beamon (1998), a simple supply chain (SC) may be defined as an integrated process wherein a number of various business entities (i.e., suppliers, manufacturers, distributors, and retailers) work together in an effort to: 1. acquire raw materials, 2. convert these raw materials into specified final products, and 3. deliver these final products to retailers. This chain is traditionally characterized by a forward flow of materials and a backward flow of information (Beamon, 1998).

At its highest level, a SC can be decomposed to two basic, integrated processes: 1. the Production Planning and Inventory Control Process, and 2. the Distribution and Logistics Process. These processes, illustrated in Fig. 1 provide the basic framework for the conversion and movement of raw materials into final products.

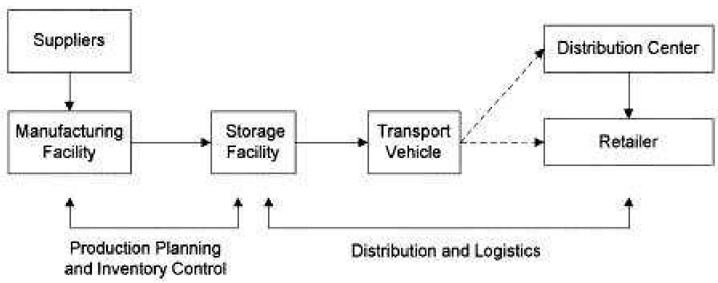


Fig. 1: Simple supply chain processes (adopted from Beamon, 1998)

The Production Planning and Inventory Control Process comprises of the manufacturing and storage sub-processes, and their interfaces. More specifically, production planning describes the design and management of the entire manufacturing process (including raw material scheduling and acquisition, manufacturing process design and scheduling, and material handling design and control). Inventory control describes the design and management of the storage policies and procedures for raw materials, work-in-process inventories, and usually, final products (Beamon, 1998).

The Distribution and Logistics Process determines how products are retrieved and transported from the warehouse to retailers. These products may be transported to retailers directly, or may first be moved to distribution facilities, which, in turn, transport products to the retailers. This process includes the management of inventory retrieval, transportation, and final product delivery (Beamon, 1998).

These processes interact with one another to produce an integrated SC. The design and management of these processes determine the extent to which it works as a unit to meet the required performance objectives.

Definition of an integrated SC was affirmed by Akkermans (2003). He stated that SC is a network that consists of suppliers, manufacturers, distributors, retailers, and customers. This network is supported by three types of flows (material, information and financial) and requires more careful planning and closer coordination.

The evolution of the concept of SC took 30 years. Internal supply chain integration transitioned to external supply chain integration as there was a limited amount of performance improvement that could be achieved without involving suppliers and customers. External supply chain integration transitioned to goal directed network supply chains as firms understood that supply chains were non-linear networks and that there would be benefit for non-strategic (or non-integrated) suppliers to have visibility of demand. It is generally supposed, that by now we are facing the process of undergoing a transition to devolved, collaborative supply chain clusters. It is suggested that this transition is occurring due to the increased complexity, risk and costs that are being borne by focal firms who are attempting to manage large networks. By effectively outsourcing elements of this management to lead suppliers, there is devolvement of the collaboration into clusters.

The evolution of SC concept displayed in the previous paragraph can be used further and implemented to the concept of supply chain management, namely the evolution of SCM shown in Fig. 2.

Today one of the most wide-spread definitions of SCM is one produced the Council of by Supply Chain Management Professionals (CSCMP): SCM encompasses the planning and management of all activities involved in sourcing and procurement, conversion, and all logistics management activities. Importantly, it also includes coordination and collaboration with channel partners, which can be suppliers, intermediaries, third party service providers, and customers.

Mentzer et al. (2001) define SCM as the systemic, strategic coordination of the traditional business functions and the tactics across these business functions within a particular company and across businesses within the supply chain, for the purposes of improving the long term performance of the individual companies and the supply chain as a whole.

Given that the aim of this paper is not to review the numerous definitions of SCM in extant literature, it simply adopts one that of Mentzer et al. (2001) since it contains all the key elements (strategic coordination, collaboration across the whole supply chain and long-term performance), while dealing not only with material and information flows, but also with financial ones.

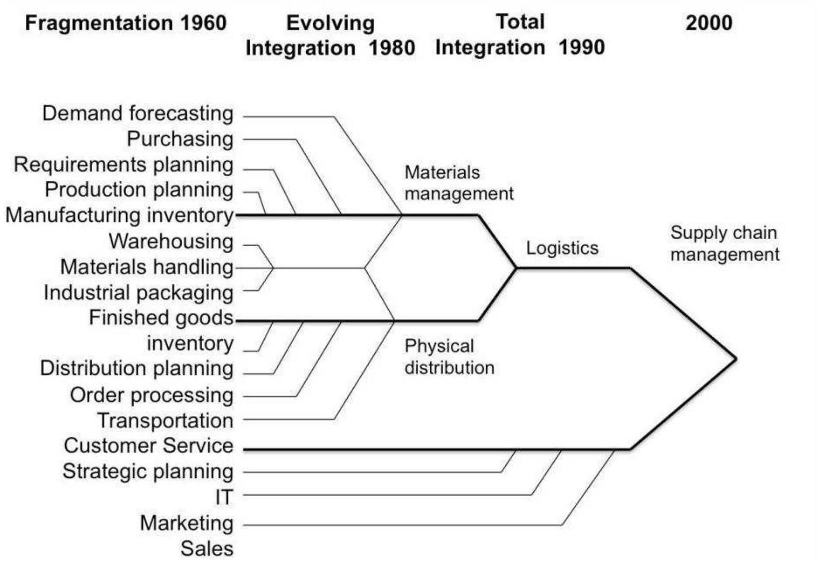


Fig. 2: Evolution of SCM concept (adopted from Coyle, et al., 2013)

2.2. Cooperation, Coordination and Collaboration in Supply Chains

A good indication of the maturity level of a field is the attitude of researchers to the definition of key concepts. In a mature field, most researchers would use existing standard definitions. In our case, there is no clear convergence among the authors on a single definition (although most were based on themes associated with operations research). Though, there are efforts in literature regarding collaboration of different functions of the SC, the study of coordinating functions in isolation may not help to coordinate the whole SC. It appears that the study of SC collaboration (SCC) is still in its infancy. Though, the need for collaboration is realized, a little effort has been reported in the literature to develop a holistic view of coordination.

Supply chains are generally complex and are characterized by numerous activities spread over multiple functions and organizations, which pose interesting challenges for effective SC collaboration. To meet these challenges, SC members must work towards a unified system and cooperate with each other. Collaboration is an amorphous meta-concept that has been interpreted in many different ways by both organizations and individuals. SC collaboration has proven difficult to implement although still has the potential to offer significantly improved performance. It is suggested that many of the problems related to SC collaboration are due to a lack of understanding of what collaboration actually implies. This poor understanding is further increased due to the association of collaboration with the hype surrounding e-business whereby technology has been promoted as the key to enabling wide scale inter-organizational collaboration.The evolution of the collaboration concept from simple generic integration concept can be tracked through the evolution of SCM strategies, tools, and techniques in time.

It is argued that SCM developed from a baseline of functional (independent) silos and the first level of integration was across functions (akin to process integration). This then moved to full internal integration involving a seamless flow through the internal supply chain, and finally to external integration embracing suppliers and customers. The primary benefits were identified as improved customer service and reduced inventory and operating costs. What has changed since the introduction of the concept of SCM is the context within which supply chains operate, and the enablers of change and performance improvement. As a result the relevance of narrow, linear-based supply chain models has been challenged as firms have looked more and more toward networked and collaborative supply chain strategies to deliver superior performance.

SCM as a discipline has evolved rapidly. The early focus of SCM began when organizations began to improve their inventory management and production planning and control. The aim of these practices was to improve production efficiencies and ensure that the capacity of capital assets and machinery was utilized efficiently. This extended upstream to include the management of transport of raw materials at a time when firms were relatively vertically integrated.

The early definition of integration is provided by Frohlich and Westbrook: At the tactical level, there are two interrelated forms of integration that manufacturers regularly employ. The first type of integration involves coordinating and integrating the forward physical flow of deliveries between suppliers, manufacturers, and customers. The other prevalent type of integration involves the backward coordination of information technologies and the flow of data from customers to suppliers (Frohlich and Westbrook, 2001)

The next phase in the evolution of SCM was the systematization of materials, production, and transport management. This began with materials requirement planning (MRP) focusing on inventory control. MRP expanded to become MRPII by incorporating the planning and scheduling of resources involved in manufacturing. Both MRP and MRPII were conceived in the 1960s but did not gain prominence until the 1980s. MRP and MRPII evolved to become ERP, in an attempt to gain greater visibility over the entire enterprise (Stevens and Johnson, 2016).

The mid to late 1980s brought intense retrospection from western firms concerning the threat of Japanese firms that were perceived to be more competitive due to higher productivity. This period led to the implementation of Japanese practices such as total quality management (TQM) and lean by firms. These practices focused on reducing inventory through improving quality and flow and involving suppliers in product and process design. At this point, one can say, that cooperation is a substantial prerequisite for further coordination and collaboration.

The next phase in the evolution of SCM included the introduction of other process improvement practices (e.g. six sigma) that sought to provide a more concrete improvement method compared to TQM or lean. As process improvement, and the standardization of products and processes that facilitated it, took place, there was increasing awareness that end customers were requiring ever increasing levels of choice and differentiation. This led firms to consider that they had become too lean and rigid and should be focusing on creating agile supply chains to adapt to changing demand. The agile approach was blended with lean as demand could be decoupled into push and pull to create greater choice for the customer while still retaining some control (Stevens and Johnson, 2016).

The most commonly accepted definition of coordination in the literature is the act of managing dependencies between entities and the joint effort of entities working together towards mutually defined goals (Malone and Crowston, 1994).

The 1990s also saw a focus upon core competences within firms. This led to a rise in increased outsourcing of non-core activities to lower cost economies. Political factors such as unilateral liberalization measures and the removal of formal free trade barriers have contributed to the growth of developing countries exporting to high wage economies, encouraging firms to source from lower cost economies. This, in turn, fuels both demand for products from developed economies and the competition to supply. This changed the topology of the supply chain as well as the magnitude, profile and direction of material, and information flows. Significant changes have also taken place around the understanding of how a firm secures a competitive position. Traditionally, superior competitive advantage was seen to be a function of how a firm organized its resources to differentiate itself from the competition and its ability to operate at a lower cost. The prevailing tendency was to control as much of its upstream and downstream activities as possible, often leading to high levels of vertical integration (i.e. within a firm rather than with suppliers). Thus, firms focused more on managing, in-house, core competences, i.e. those competencies or capabilities that deliver value (as perceived by the customer) and outsourcing non-core activities to specialist often lower cost third parties. This resulted in the advent of 3PL providers and supply chain integrators.

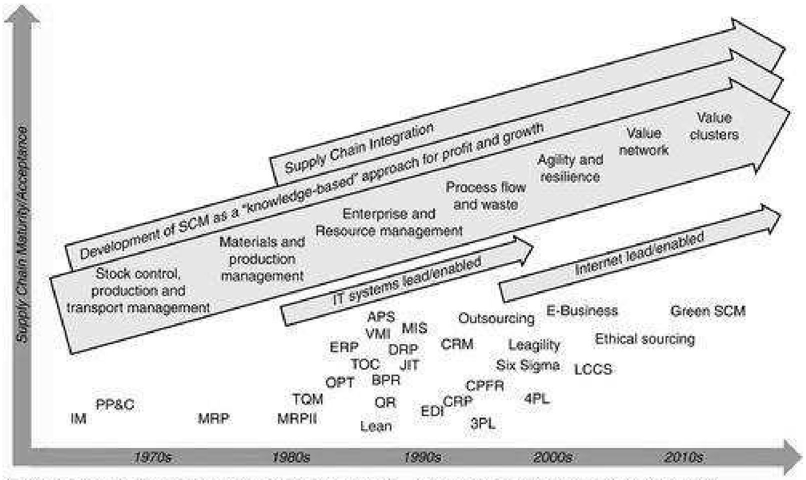
Supply chains are inherently unstable in terms of inevitable challenges of forecasting and data integrity. Technology has been used to good effect to improve information flows. However, the increased remoteness of a global market and supply base, together with the need to manage an increasingly complex network has exacerbated the challenge. In addition to the issues caused by information distortion and a global supply base, the twenty-first century is a time when organizations are facing pressure from consumers and other stakeholders to have green and ethical supply chains. This requires organizations to become more transparent in terms of disclosing their sources of supply, which increases costs and may place pressure on moving away from the lowest cost economies where labor rights can be poor. At this period of time the concept of collaboration evolved.

Collaboration is a very broad term and when it is put in the context of the supply chain it needs yet further clarification. When talking about collaboration many authors mention mutuality of benefit, rewards and risk sharing on the basis of the exchange of information. There seems to be no unique definition of SCC, although different perspectives have been presented in literature for coordinating SC:

– Collaborative working for joint planning, joint product development, mutual exchange information and integrated information systems, cross coordination on several levels in the companies on the network, long-term cooperation and fair sharing of risks and benefits.
– A collaborative SC simply means that two or more independent companies work jointly to plan to execute SC operations with greater success than when acting in isolation.
– A win/win arrangement that is likely to provide improved business success for both parties.
– A strategic response to the challenges that arise from the dependencies SC members.

M. Simatupang and R. Sridharan introduced one of the most cited definitions of SC collaboration in 2002. According to authors: A collaborative supply chain simply means that two or more independent companies work jointly to plan and execute supply chain operations with greater success than when acting in isolation (Simatupang and Sridharan, 2002). But this definition is limited by the boundaries of the inter-organizational processes. To overcome this problem B. Flynn reflected more spread definition of Supply Chain Collaboration (SCC): as the degree to which a manufacturer strategically collaborates with its supply chain partners and collaboratively manages intra- and inter-organization processes. The goal is to achieve effective and efficient flows of products and services, information, money and decisions, to provide maximum value to the customer at low cost and high speed (Flynn, Hou and Zhao, 2010). This definition more precisely outlined that collaboration in supply chain can happen not only between several companies but also at the level of one company.

Summing up, there seems to be no standard definition of SCC. Various perspectives on SCC as reported in the literature are testimony to this, but basically they fall into two groups of conceptualization: process focus and relationship focus. Some of these perspectives present the inherent capability or intangibles required to coordinate like responsibility, mutuality, cooperation and trust. The other perspectives can be visualized, based on the coordination effort required in achieving common goals in various activities of SC. Since the activities are different, the coordination requirements also vary with the complexity of the activity. The most challenging coordination perspective is to extend the concept of coordination from within an organization to coordination between organizations.



Notes: 3PL, third party logistics provider; 4PL, fourth party logistics provider; APS, advanced planning systems; BPR, business process re-engineering; CPFR, continuous planning, forecasting, and replenishment; CRM, customer relationship management; DRP, distribution resource planning; EDI, electronic data interchange; ERP, enterprise resource planning; IM, inventory management; LCCS, low-cost country sourcing; MIS, management information systems; MRP, material requirements planning; MRPII, manufacturing resource planning; OPT, optimized production technology; PP&C, production planning and control; QR, quick response; TOC, theory of constraints; TQM, total quality management; VMI, vendor managed inventory

Fig. 3: A timeline of SCM strategies, tools, and techniques (adopted from Stevens and Johnson, 2016)

This all points toward an explosion in SCM thinking over the last 25 years. Fig. 3 presents a timeline of SCM strategies, tools, and techniques. The dates in the figure are based upon when these practices were popularized, not introduced (Stevens and Johnson, 2016). Fig. 4 outlines the transition of collaboration.

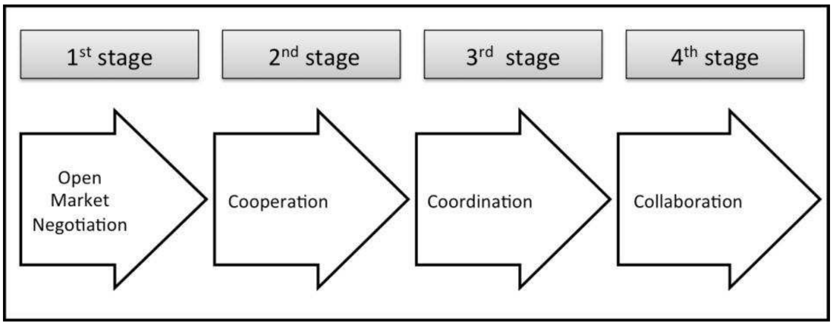


Fig. 4: Transition of collaboration (adopted from Mentzer at al., 2001)

If the collaboration is to be sustainable then there are a number of strategic elements, which must be present. Synthesizing the literature, supply chain collaboration consists of seven interconnecting components: 1. information sharing, 2. goal congruence, 3. decision synchronization, 4. incentive alignment, 5. resources sharing, 6. collaborative communication, and 7. joint knowledge creation. These dimensions are expected to be inter-correlated with each other, although there might be causal relationships among them (Barratt, 2004).

Information sharing refers to the extent to which a firm shares a variety of relevant, accurate, complete, and confidential information in a timely manner with its supply chain partners. Information sharing is described as the heart, lifeblood, nerve center, essential ingredient, key requirement, and foundation of supply chain collaboration. Information sharing can be defined as the willingness to make strategic and tactical data such as inventory levels, forecasts, sales promotion, strategies, and marketing strategies available to firms forming supply chain nodes.

Goal congruence between supply chain partners is the extent to which supply chain partners perceive their own objectives are satisfied by accomplishing the supply chain objectives. It is the degree of goal agreement among supply chain partners. In the case of true goal congruence, supply chain partners either feel that their objectives fully coincide with those of the supply chain, or, in case of disparity, believe that their goals can be achieved as a direct result of working toward the objectives of the supply chain.

Decision synchronization refers to the process by which supply chain partners orchestrate decisions in supply chain planning and operations that optimize the supply chain benefits (Simatupang and Sridharan, 2005). Planning decisions are required to determine the most efficient and effective way to use the firm's resources to achieve a specific set of objectives. There are seven key supply chain management planning decision categories: operations strategy planning, demand management, production planning and scheduling, procurement, promise delivery, balancing change, and distribution management (Barratt, 2004). Joint planning is used to align collaborative partner and to make operating decisions including inventory replenishment, order placement, and order delivery.

Incentive alignment refers to the process of sharing costs, risks, and benefits among supply chain partners (Simatupang and Sridharan, 2005). It includes determining costs, risks, and benefits as well as formulating incentive schemes. Successful supply chain partnerships require that each participant share gains and losses equitably and the outcomes of the collaboration are quantifiably beneficial to all. Incentive alignment requires a careful definition of mechanisms that share gains equitably, which means gains are commensurate with investment and risk (Barratt, 2004).

Resource sharing refers to the process of leveraging capabilities and assets and investing in capabilities and assets with supply chain partners. Resources include physical resources, such as manufacturing equipment, facility, and technology.

Collaborative communication is the contact and message transmission process among supply chain partners in terms of frequency, direction, mode, and influence strategy. Open, frequent, balanced, two-way, multilevel communication is generally an indication of close inter-organizational relationships (Barratt, 2004).

Joint knowledge creation refers to the extent to which supply chain partners develop a better understanding of and response to the market and competitive environment by working together. There are two kinds of knowledge creation activities: knowledge exploration (i.e., search and acquire new and relevant knowledge) and knowledge exploitation (i.e., assimilate and apply relevant knowledge).

There are multiple benefits accruing from effective SCC. Some of these include: elimination of excess inventory, reduction of lead times, increased sales, improved customer service, efficient product developments efforts, low manufacturing costs, increased flexibility to cope with high demand uncertainty, increased customer retention, and revenue enhancements.

These expected benefits of SCC motivated the researchers and practitioners to develop and test the concept of elements of collaboration, but further research is required to develop a deeper understanding of the relationships between these elements of collaboration.

Despite the popularity and potential benefits of SCC, many attempts fall short of the participants expectations. It was previously observed by Sabath and Fontanella (2002) that collaboration arguably has the most disappointing track record of the various supply chain management strategies introduced to date (Cao and Zhang, 2011). The difficulties faced in SCC activities may be visualized in the following way:

- There exist differences in the interest of SC members as the members work out of habit as an individual firm based on local perspective. Such an opportunistic behavior results in mismatch of supply and demand (Arshinder et al., 2008).
- The following types of conflicts may exist: conflicting goals and objectives (goal conflict), disagreements over domain of decisions and actions (domain conflict) and differences in perceptions of reality used in joint decision making (perceptual conflict) between SC members.
- The traditional performance measures based on the individual performance may be irrelevant to the maximization of SC profit in a collaborative manner.
- The traditional policies, particularly rules and procedures, may not be relevant to the new conditions of inter-organizational relationship. Moreover, there has been over-reliance on technology in trying to implement Information Technology (IT).

The consequences of lack of coordination may result in poor performance of SC as a whole, particularly in inaccurate forecasts, low capacity utilization, excessive inventory, inadequate customer service, inventory turns, inventory costs, time to market, order fulfillment response, quality, customer focus and customer satisfaction (Arshinder et al., 2008). These problems are solved by implementing some mechanisms in SC activities, which may result in the improvement of some performance measures. These mechanisms include: joint decision making, information sharing, resource sharing, implementing IT, joint promotional activities, etc. The other motivation seems to be the ability of SC members to share the risks and subsequently share the benefits. Further these mechanisms are discussed in detail.

SC Contracts. SC members coordinate by using contracts for better management of supplierbuyer relationship and risk management. The objectives of SC contracts are:

- to increase the total SC profit,
- to reduce overstock/understock costs, and
- to share the risks among the SC partners.

In buyback contract, the buyer is allowed to return the unsold inventory to some fixed amount at agreed upon prices. The manufacturers accept the returns from the retailers when the production costs are sufficiently low and demand uncertainty is not too great (Cachon and Lariviere, 2005).

In the revenue-sharing contracts, the supplier offers the buyer a low wholesale price when the retailer shares fraction of his revenue with supplier, which helps partners in selecting order quantities that are optimal for the whole SC (Cachon and Lariviere, 2005).

In the quantity flexibility contracts, the supplier and the buyer accepts some of the inventory and stock out cost burden. The supplier allows the buyer to change the quantity ordered after observing actual demand. The buyer commits to a minimum purchase and the supplier guarantees a maximum coverage (Tsay, 1999). The coordination achieved by the contracts provides incentives to all SC members and improves the service level.

There are a number of extensions to buyback contracts are presented in the literature like two period supply contract model for decentralized assembly system (Zou et al., 2008) and flexible returns policies in three-level SC (Ding and Chen, 2008) to fully coordinate SC members.

Information technology. IT is used to improve inter-organizational coordination and in turn, inter-organizational coordination has been shown to have a positive impact on select firm performance measures, such as customer service, lead time and production costs. IT helps to link the point of production seamlessly with the point of delivery or purchase. It allows planning, tracking and estimating the lead times based on the real-time data. Advances in IT (e.g. internet, EDI (electronic data interchange), ERP (enterprise resource planning), e-business and many more) enable firms to rapidly exchange products, information, and funds and utilize collaborative methods to optimize SC operations. The various coordination problems handled by information systems are:

- little value to the supplier because of competitive bidding,
- forced implementation of IT,
- incompatible information system at different levels of SC,

– greater lead times,
– inefficient purchase order, and
– misaligned e-business strategies and coordination mechanisms (Arshinder et al., 2008).

Information sharing. The SC members coordinate by sharing information regarding demand, orders, inventory, POS data, etc. Timely demand information or advanced commitments from downstream customers helps in reducing the inventory costs by offering price discounts and this information can be a substitute for lead time and inventory (Reddy and Rajendran, 2005). The value of information sharing increases as the service level at the supplier, supplier-holding costs, demand variability and offset time increase, and as the length of the order cycle decrease.

Joint decision making. Joint decision making consists of several key procedures:

– replenishment,
– inventory holding costs with dynamic demand,
– collaborative planning,
– costs of different processes,
– frequency of orders,
– batch size,
– product development to improve the performance of SC.

A coherent decision making helps in resolving conflicts among SC members and in exceptions handling in case of any future uncertainty.

There are many factors involved in achieving coordination like human, technology, strategies, relationship, rewards, sharing of knowledge, sharing benefits, aligning goals, scheduling of frequent meetings of stakeholders for conflict resolution, understanding of nature of intermediates and knowledge of SC concepts, status or power difference and resistance in following the instructions of other organizations.

Even though SCC improves the performance of the SC, it may not always be beneficial to coordinate all the SC members. The high adoption costs of joining inter-organizational information systems and information sharing under different operational conditions of organizations may hurt some SC members. Therefore, it is essential to investigate the conditions under which SCC is beneficial, so that it should not result in higher SC costs and imprecise information.

Cooperation Forms and Dimensions. Based on this definition, SCM can be broken into two parts: internal (which entails cross-functional coordination and collaboration within the company) and external. External SCM can further be broken into two parts: upstream, which has to do with coordination and collaboration with suppliers, and downstream, which has to do with coordination and collaboration with customers. In the SCM literature, these three parts can be referred to as internal integration, supplier integration and customer integration (Flynn et al., 2010; Wong et al., 2013; Yu et al., 2013) or supplier relationship management, internal SCM and customer relationship management (Dey and Cheffi, 2013).

Whilst many organizations have integrated various internal interfaces, e.g. marketing and logistics, purchasing and manufacturing, there are still few organizations that have achieved complete internal integration, i.e. purchasing-manufacturing-logistics-marketing (Fawcett and Magnan, 2002). Mentzer et al. (2001) classify these early forms of integration as predominantly based on interaction, in the sense that functional departments hold meetings and attempt to share more information. What are missing from such initiatives are the joint goals, shared resources, and common vision that is espoused by the collaborative approach. A potential danger of internal collaboration is that organizations could achieve internal integration, and have simply created a larger albeit organizational silo (Barratt, 2004).

External collaboration presents a number of potential opportunities for vertical supply chain collaboration on the downstream side of the supply chain (customer relationship management (CRM); collaborative demand planning (which includes collaborative forecasting, CPFR, etc.); demand replenishment; and shared distribution) as well as on the upstream side of the supply chain (supplier relationship management (also referred to as supplier development, e.g. VMI, CRP); supplier planning and production scheduling; collaborative design (which could include new product introduction); and collaborative transportation).

Supply Chain Cooperation Performance. There is a growing recognition among company executives that today's business competition is no longer between individual firms, but between SCs. If a SC is properly managed, its whole value can be greater than the sum of its parts. Not surprisingly, there is an increasing demand for both scholars and business practitioners to make SCM more financially accountable. Optimizing financial performance along the SCs should be the ultimate goal of any SCM strategy. The existing literature has shown SCM's great potential to enhance a firm's key financial outcomes. To demonstrate the financial accountability of SCM activities a number of SCM drivers for firm-level financial performance are identified (Shi and Yu, 2013).

On the basis of collaborative management of relationships between the organizations that constitute the value chain and integrated coordination of processes from the ultimate supplier to the ultimate customer, SCM aims to create more value for customers, as well as for the supply chain partners, thus improving performance not only within each organization, but also across the whole chain (Shi and Yu, 2013). A SCM system entails the implementation of a set of practices that can be defined as activities deployed in an organization in order to enhance the effective management of its supply chain. Despite the constantly growing attention to SCM, contributions to the link between supply chain management practices (SCMPs) and performance are very diverse in scope and nature, and most often remain dispersed and incomplete.

The existing studies on the financial impacts of SCM have enabled the researchers to formulate some empirical patterns, with which we identify a number of performance drivers attributing to firm financial performance, in particular: sourcing strategy, information technology (IT), system integration, and external relationship.

Sourcing strategy. When a firm develops its sourcing strategy in the SCM context, it constantly weighs the total costs associated with the make-or-buy decisions. A well-developed SC sourcing strategy allows SC partners to focus on their key competitive advantages, thus resulting in a win-win situation for all involving parties. According to TCE, successful SC sourcing strategy should be able to reduce production costs and increase process flexibility since firms no longer need to commit to asset specificity (Williamson, 1981).

According to Shi and Yu (2013), the performance implications of SC sourcing strategy are widely debated in the literature. On one hand, several empirical studies have shown its positive contributions to firms' financial performance. It was discussed, how purchasing and supply management affect financial performance such as business growth, profitability, cash flow, and asset utilization. On the other hand, not all the studies are able to establish positive relationship between sourcing strategy and financial performance. It was previously found, that firms performing more aggressive outsourcing practices do not experience significant and direct performance improvements. In addition, firm strategy and environmental dynamism are found to moderate the relationships between outsourcing intensity and financial performance. (Shi and Wei, 2013). Overall, SC sourcing strategy generate positive contributions to financial performance. However, an optimal level of outsourceability may exist to maximize the benefits.

Information technology. According to transaction cost economics (TCE), the main purpose of IT in SCM is to enhance SC collaboration and reduce coordination costs along SC by increasing SC visibility and transparency. Meanwhile, there is a debate on whether the IT capability can really serve as a catalyst in improving firms' performance. The skeptics' major argument is that particular SC technology can be easily duplicated by competitors, making it difficult for the investing firms to gain competitive advantages over their competitors. According to resource based view (RBV), therefore, the increasing investments in IT capability do not guarantee performance improvements. Blankley (2008) provides a comprehensive literature review relevant to the impacts of IT on the financial performance. He proposes a conceptual model to demonstrate how an effect chain is extended from SCM technology to a firm's financial performance. Therefore, the following empirical finding regarding the financial impacts of IT can be derived: Information technology in SCM makes positive contributions to financial performance, but IT alignments and implementations could affect financial outcomes.

System integration. An integrated SCM system enhances a firm's capability to coordinate all business processes within and beyond the firm's boundary. Enterprise resource planning (ERP) system, which integrates internal and external information flows and management functions within and across involving SC participants, is a typical example.

By collecting survey results from Korean and Japanese firms, Kim (2009) uses SEM approach to examine the causal relationship among SC activities, competitive strategy, SC integration, and firm performance. For both Korean and Japanese samples, there exists a significant relationship between SCM activities and competition capability. However, the mechanism of how SC integration impacts firm performance is different in Korean and Japanese samples due to firm sizes and levels of SC integration. In Korean firms, the interrelationship between SCM practices and competition capability enhances SC integration, which in turn has a direct effect on firm performance. On the other hand, some studies are not able to establish positive relationship between SCM integration and firms' performance. Hendricks et al. (2007) report mixed results concerning the impacts of ERP, SCM, and customer relationship management (CRM) on firms' long-term financial performance. Specifically, they find some improvements in firms' financial metrics (ROA and ROS) for the ERP and SCM adopters, but not for the CRM adopters. To partly explain this performance puzzle, some studies suggest that the SCM systems be integrated with other IT infrastructures to achieve the best performance. An integrated SCM system represents a firm's general capability to coordinate all business processes within and beyond the firm's boundary and improve overall financial performance.

Summing up, system integration in SCM achieves optimal financial performance when it is implemented together and aligned with IT infrastructures and overall business strategies.

External relationships. As a firm's unique resource and valuable asset, external relationships in SCM, including supplier and customer management, is expected to be highly associated with financial performance. As a matter of fact, it can be argued that the quality of external relationships with upstream and downstream partners is one of the most important drivers of financial performance. The association between external and internal contextual SCM factors and various performance measures in the information industry was earlier investigated in Taiwan. Several studies focus on the specific components of external relationships in SCM. For example, Flynn et al. (2010) especially investigate the impact of supplier-customer-internal (SCI) relationship on firms' performance in China. Empirical analysis shows that the SCI relationship is positively associated with both operational and financial performance.

SC collaboration and mutual trust are especially important to manage external relationships with suppliers and customers. Cao and Zhang (2011) investigate SC collaboration and its impact on firm performance. The empirical results indicate that SC collaboration considerably improve collaborative advantage, which in turn, has significant positive effect on firms' financial performance. In particular, the mediator role of collaborative advantage is stronger for small firms than medium and large firms. Therefore, we have following empirical finding: as a firm's unique resource and valuable assets, SC external relationships are highly associated with financial performance.

Over the past few decades, more and more executives have realized the strategic importance of SCM and recognized the distinctive competitive advantages that a well-managed SC can bring to the company. SCM has therefore attracted substantial investments across various industries recently and company executives not only need to know whether SCM is able to make positive contributions to firm-level financial performance, but also want to know how to direct their SC investments to enhance competitive advantages and optimize financial outcomes. SCM managers, therefore, are obliged to demonstrate SCM's positive financial contributions and justify relevant expenses.

As we constrain this study on the financial impacts of SCM practices, only accounting- and market-based financial measures are discussed in this section.

The accounting-based financial measures are direct indicators of a firm's financial conditions from different perspectives. For example, return on assets (ROA), return on equity (ROE), and return on investment (ROI) are usually used to examine a firm's asset and capital utilization, while profit margin, cost of goods sold (COGS), and economic value added (EVA) are common measures of a firm's capability to make profits. Some accrual measures, such as ROA, ROI, and profit margin, are particularly popular in the SCM literature. However, it is worth noting that the accrual measures are not always appropriate in performance measurement due to their own limitations. First, most accrual measures are not able to catch intangible or non-cash benefits associated with SCM practices, such as market share, market reputation, and company goodwill. Second, they are used to measure the past performance but are not forward-looking indicators. Third, they are relatively easy to be manipulated by accounting frauds and illegal practices. A few studies, therefore, propose financial measures based on cash flow to directly evaluate a firm's profits and liquidity. To better catch the company-wide effects of SCM practices, several studies develop comprehensive financial measures by combining multiple corporate income and balance sheet values together.

As an essential complement to accounting-based financial measures, market-based measures focus on shareholder value. Shi and Yu (2013) state, that in one of the early studies investigating the impacts of SC strategy on shareholder value, Christoper and Ryals (1999) define the shareholder value as the financial value created for shareholders by the companies in which they invest. Since SCM activities are strongly associated with revenue growth, operating cost reduction, fixed and working capital efficiency, they are expected to impose significant effects on shareholder values. It is consistent with studies in other disciplinaries. Swink et al. (2010) employ Sharpe ratio to characterize how well the excess return of SCM excellence compensates the stockholder for the risk taken. As the most popular market-based measure, abnormal stock return documents the difference between the expected stock return and the actual stock return, which is often triggered by special SCM events (see event study in research method section for details). In a widely-cited study, Hendricks and Singhal (2003) propose a framework to link SC performance to shareholder value through operational metrics and intangible assets. In an efficient financial market, the improved SC performance eventually will be reflected on shareholder values. Johnson and Templar (2011) develop a unified performance proxy composing of different elements in profitability, liquidity, and productivity. Since a significant proportion of firm value today lies in intangible assets, market-based measures provide a more objective approach than the accounting-based measures. In the absence of deep understanding of SCM's contributions to shareholder value, SCM professionals have great impediments to assess the true value of SCM activities and justify the continuous SCM investments.

Fig. 5 summarises all the paths that link learning and growth perspective and internal process perspective (SCMPs and some operational non-financial performance measures) to the customer and financial perspectives (customer satisfaction, product quality and financial performance), which constitute a firms strategic objectives.

Theoretical Gaps in Supply Chain Cooperation. Despite research confirming the positive benefits of supply chain integration, and its importance to a firms success (Flynn et al., 2010), ambiguity remains as to what constitutes supply chain collaboration (Fabbe-Costes et al. 2014).

Currently there exists a gap in the SCM literature to link theoretical background and empirical evidences. A few authors have attempted to lay theoretical foundations for SCM by employing a variety of organizational theories, such as TCE, RBV, agency theory, institutional theory, network theory, game theory, and strategic choice theory (Chatha and Butt, 2015). With the exception of TCE and RBV, most theories, however, did not receive sufficient empirical supports in the literature. Thus, the following points can become starting points for further research:

1. *More diverse theoretical foundations.* Most of current empirical studies formulate their hypothesis in the framework defined by either TCE or RBV. Several other organizational theories, such as principle-agent theory and network theory, are discussed in the SCM context. Apparently, more diverse theoretical

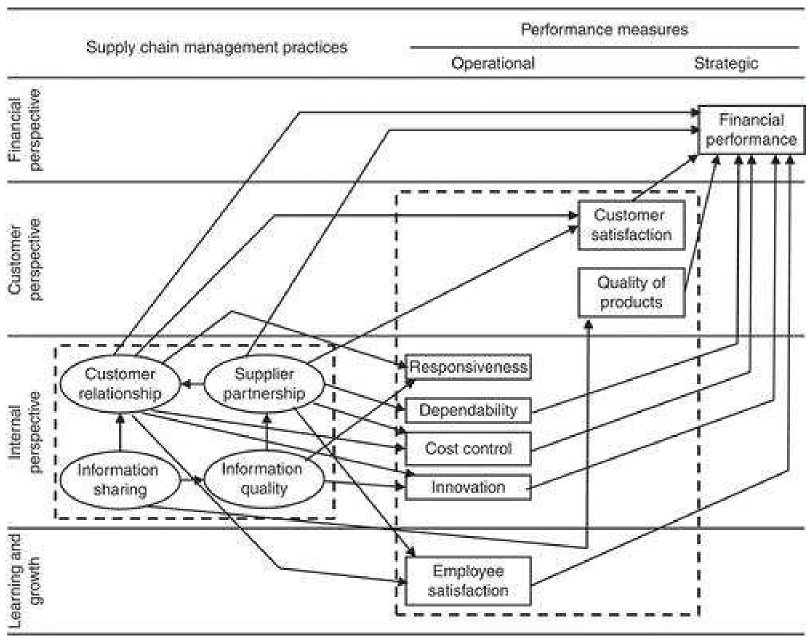


Fig. 5: Linkage of SCM practices on performance (adopted from Okongwu, Brulhart and Moncef, 2015)

foundations will enhance our understanding of SCM's financial impacts from different perspectives.

2. *Narrow focus.* Regarding the fundamental question of which SCM practices impact individually or collectively on which performance measures, most studies often focus on only one or few aspects (or parts) of the supply chain such as the upstream network (Eltantawy et al., 2015) or the internal relationships (Williams et al., 2013). In this field, two research streams can be distinguished: first, studies that aim to establish a link between two variables (a SCM practice and a performance measure) based on a unique construct of SCM and performance, Second, studies focusing on the impact of two or more SCM practices (considered separately or collectively) on one or several performance variables.
3. *Under-researched SC variables.* Besides the discussed variables, more SC variables should be empirically examined on its contribution to financial performance. For example, what quality characteristics are available to drive SCM improvement and what is their financial impact?
4. *Robustness of empirical results.* As stated in the previous section, this is an emerging research area and most studies reviewed in this paper are published recently. Therefore, the robustness of the empirical findings should be tested under different environmental settings. For example, what is the role of SCM under different macroeconomic climates? Are the financial contributions from effective SCM enhanced or weakened during economic recessions? What are the SC variables attributable to the performance change?
5. *Corporate bond market.* For empirical studies based on the secondary data, most of them employ the financial data from stock market. The influential corporate bond market is largely ignored in the literature. The inclusion of corporate bond

market is necessary to extend our understanding beyond the stock market. For example, how SCM activities affect a firm's cash flows and its ability to raise capitals from the corporate bond market?

2.3. Financial Supply Chain Cooperation

What emerges from the definitions and associated discussions on supply chain and supply chain cooperation is a broad concept that focuses on the flow of physical goods and services supported by business processes that run along the full extent of the supply chain from the end user to the raw materials suppliers and includes every organization involved in the design, manufacture, distribution and retail of the product or service. To contribute to the development of research into financial supply chains and to set out the broad scope of the case study, a formal definition of financial supply chains is proposed.

A financial supply chain (FSC) is the network of organizations and banks that coordinate the flow of money and financial transactions via financial processes and shared information systems in order to support and enable the flow of goods and services between trading partners in a product supply chain (Blackman, Holland, and Westcott, 2013).

Lately the importance of understanding the relationship between physical and financial supply chains has arised among supply chain finance practitioners such as finance providers, corporate, commercial and small and medium-sized (SME) clients, market investors, regulators or legal practitioners as well as it and infrastructure providers (Standard Definitions for Techniques of Supply Chain Finance, 2016). According to this document as one of the first attempts to establish this link, the Financial Supply Chain (FSC) is the chain of financial processes, events and activities that provide financial support to physical supply chain participants. Financial Supply Chain Management (FSCM) refers to the range of corporate management practices and transactions that facilitate the purchase of, sale and payment for goods and services, such as the conclusion of contractual frameworks, the sending of purchase orders and invoices, the matching of goods sent and received to these, the control and monitoring of activities including cash collections, the deployment of supporting technology, the management of liquidity and working capital, the use of risk mitigation such as insurance and guarantees, and the management of payments and cash-flow. FSC management involves the orchestration of a range of contributors to meeting FSC needs such as internal corporate functions, trading parties, and service providers in the area of supply chain automation and in the whole range of financial services.

In order to reduce vagueness in the term, it is needed to introduce master definition of a supply chain finance (SCF) provided in Standard Definitions for Techniques of Supply Chain Finance, (2016): the Supply Chain Finance is defined as the use of financing and risk mitigation practices and techniques to optimise the management of the working capital and liquidity invested in supply chain processes and transactions. The following aspects of this definition are highlighted by the authors:

– *Portfolio.* SCF is a portfolio of financing and risk mitigation techniques and practices that support the trade and financial flows along end-to-end business supply and distribution chains, domestically as well as internationally. This is emphatically a holistic concept that includes a broad range of established and evolving techniques for the provision of finance and the management of risk.

- *Parties.* Parties to SCF transactions consist of buyers and sellers, which are trading and collaborating with each other along the supply chain. As required, these parties work with finance providers to raise finance using various SCF techniques and other forms of finance. The parties, and especially anchor parties on account of their commercial and financial strength, often have objectives to improve supply chain stability, liquidity, financial performance, risk management, and balance sheet efficiency.
- *Event driven.* Finance providers offer their services in the context of the financial requirements triggered by purchase orders, invoices, receivables, other claims, and related pre-shipment and post-shipment processes along the supply chain. Consequently, SCF is largely event-driven. Each intervention (finance, risk mitigation or payment) in the financial supply chain is driven by an event or trigger in the physical supply chain. The development of advanced technologies and procedures to track and control events in the physical supply chain creates opportunities to automate the initiation of SCF interventions in the related financial supply chain.
- *Evolving and flexible.* SCF is not a static concept but is an evolving set of practices using or combining a variety of techniques; some of these are mature and others are new or leading edge techniques or variants of established techniques, and may also include the use of traditional trade finance. The techniques are often used in combination with each other and with other financial and physical supply chain services.

There is clearly a close and reciprocal relationship between physical and financial processes within a supply chain. The crucial importance of business processes in manufacturing supply chain management and that business processes run throughout the supply chain and connect separately owned companies was identified. The financial business process is defined as the set of activities involved in the coordination of financial transactions within and between separate companies that comprise a manufacturing supply chain and their banking partners. This could include, for example, invoices, domestic and international payments, foreign exchange transactions and remittance advice. In general, financial business processes operate in tandem with manufacturing and logistics processes because typically money flows mirror product flows in a supply chain.

Given that financial supply chains operate in parallel with product supply chains it is reasonable to adapt the framework proposed by Mentzer et al. (2001) and use it as the basis for our further research as these authors identified three interdependent supply chain dimensions: business processes, management components and network structure.

Financial Cooperation Forms and Dimensions. The goal of FSC structure is to increase the transparency and the level of automation of business processes along the financial value chain. The purpose is to save processing costs and reduce the working capital of the company. This definition does not consider where the financial supply chain actually begins and ends, because there are also analytical processes that are not directly related to a business process but which belong nonetheless to the financial supply chain.

According to Weiss (2011), the financial supply chain is different from the physical supply chain because it deals with the flow of cash instead of goods. Just as in the physical supply chain, though, every day that is lost in the cash-to-cash cycle equals lost revenue. Besides a number of rather operational problems, there are several concrete key performance indicators and metrics that can be used to analyze financial supply chain. The financial supply chain stretches across different business processes, which are, in a broader sense, the two processes: order-to-cash and purchase-to-pay. The *order-to-cash* process includes, from the perspective of a supplier (or creditor), the following business process steps: 1. creditworthiness check, 2. invoice creation, 3. cash forecast, 4. financing of working capital, 5. processing of dispute cases, 6. cash collection, 7. settlement and payment, 8. account reconciliation.

From the perspective of a customer (or debtor), the *purchase-to-pay* process consists of the following business processes: 1. procurement, 2. cash forecast, 3. financing of working capital, 4. receipt of invoices, 5. resolution of discrepancies or exceptions, 6. invoice approval, 7. settlement and payment, 8. account reconciliation.

There are a number of operational factors within the order-to-cash and purchase-to-pay processes that can serve as indicators of a suboptimal financial supply chain:

- The number of paper-based business processes is very high and there are several changes in medium (for example, the creation of invoices).
- The straight-through processing rate is low, which means that there are multiple manual interventions and process steps.
- Companies struggle with a large number of dispute cases during the creation of invoices, and it takes them a lot of time to process these.
- There is a large amount of uncollectable receivables on the balance sheet, and many employees in receivables or collections management are involved in the resolution process.
- Enterprises have not implemented a consistent credit management policy, which results in a number of bad debt losses.
- Management has difficulties in predicting cash flows.
- There is no centralized cash management to control payment streams, and the company maintains too many bank connections.

The business process construct maps directly onto financial supply chains. Management components is concerned with the integration and coordination of business processes. In a financial supply chain, financial business processes are managed through information technology based systems and the sharing of information within and between organizations. To reflect the critical role of information technology combined with management systems, the term financial and banking information systems is used in place of management components. Network structure has been identified as a key feature in the supply chain literature and this concept applies equally to the network structure of organizations and banks involved in the financial supply chain.

If to look closer to the operations that are the essential parts of the companies that are using FSCM approach, they could be generally defined into several categories:

Receivables Purchase. Receivables discounting is a form of Receivables Purchase, flexibly applied, in which sellers of goods and services sell individual or multiple receivables (represented by outstanding invoices) to a finance provider at a discount.

Forfaiting is a form of Receivables Purchase, consisting of the without recourse purchase of future payment obligations represented by financial instruments or payment obligations (normally in negotiable or transferable form), at a discount or at face value in return for a financing charge.

Factoring is another form of Receivables Purchase, in which sellers of goods and services sell their receivables (represented by outstanding invoices) at a discount to a finance provider (commonly known as the factor). A key differentiator of factoring is that typically the finance provider becomes responsible for managing the debtor portfolio and collecting the payment of the underlying receivables.

Payables finance is provided through a buyer-led programme within which sellers in the buyers supply chain are able to access finance by means of Receivables Purchase. The technique provides a seller of goods or services with the option of receiving the discounted value of receivables (represented by outstanding invoices) prior to their actual due date and typically at a financing cost aligned with the credit risk of the buyer. The payable continues to be due by the buyer until its due date.

Loan, or Advance-based. Loan or Advance against Receivables is financing made available to a party involved in a supply chain on the expectation of repayment from funds generated from current or future trade receivables and is usually made against the security of such receivables, but may be unsecured.

Distributor finance is financing for a distributor of a large manufacturer to cover the holding of goods for re-sale and to bridge the liquidity gap until the receipt of funds from receivables following the sale of goods to a retailer or end-customer.

Loan, or Advance against Inventory is financing provided to a buyer or seller involved in a supply chain for the holding or warehousing of goods (either pre-sold, un-sold, or hedged) and over which the finance provider usually takes a security interest or assignment of rights and exercises a measure of control.

Pre-shipment finance is a loan provided by a finance provider to a seller of goods and/or services for the sourcing, manufacture or conversion of raw materials or semi-finished goods into finished goods and/or services, which are then delivered to a buyer. A purchase order from an acceptable buyer, or a documentary or standby letter of credit or a Bank Payment Obligation, issued on behalf of the buyer, in favour of the seller is often a key ingredient in motivating the finance, in addition to the ability of the seller to perform under the contract with the buyer.

Financial Supply Chain Cooperation Performance. There is a diversity of approaches and different frameworks to measure the performance of supply chains, taking into account financial and nonfinancial measurements, operational performance, strategic performance and highlevel measures of overall firm performance such as profitability.

One of the adapted frameworks to measure financial supply chain performance is the framework proposed by Gunasekaran et al. (2004) to measure the performance of physical supply chains. The advantages of using this rather broad framework are that it allows the researchers scope to examine the performance characteristics over three main performance areas (operational/tactical, quality and strategic) without being overly prescriptive at this relatively early stage of theory development concerning financial supply chains. In the context of financial supply chains, the operational/tactical performance includes measurements such as reduction in international payments from offsetting, efficiency of the foreign exchange process, the lead-time for the payment cycle within the banking system and the reduction of variability of customersupplier settlement dates. Six sigma quality measurement concepts from manufacturing map directly onto financial processes, for example to measure the quality of payment and foreign exchange transactions. Examples of strategic outcomes from a financial supply chain are increased cohesion in the global financial supply chain and the development of a global payment factory.

The financial supply chain strategy is logically related to the manufacturing and logistics supply chain strategy and this is captured in the model by the interdependence between the financial supply chain strategy and the manufacturing and logistics supply chain strategy (Heuser and Brockwell, 2009). There is a two-way influence where the manufacturing activities place demands on the financial systems, and in turn the financial activities enable the functioning and operation of the manufacturing supply chain. This means that changes in the product supply chain such as new suppliers, increased globalization of operations and new commercial arrangements place demands on the financial supply chain. Similarly, new capabilities in the financial systems such as certainty of payment on a specific future date, guaranteed in a local currency and at a fixed exchange rate may enable better trading relationships.

The financial supply chain strategy is an adaptation of the supply chain model proposed by Lambert et al. (1998) and is defined by the set of inter-related theoretical constructs: financial business processes, financial and banking information systems, and financial network structure. The performance of the financial supply chain is defined in terms of the quality of operations measured by six sigma techniques, financial operational benefits such as reduced cash balances and better foreign exchange rates, and strategic outcomes such as the development of a global payments factory. The important aspect of performance is that it should be measured dynamically and related to the evolution of the financial supply chain strategy in order to develop a better understanding of how changes in the financial supply chain strategy are empirically related to performance. To understand how the model operates in practice it is applied in a global setting and the methodology and data collection are described in the next section.

In order to define the interdependency of the financial supply chain and the manufacturing supply chain research into Motorola has been made (Blackman, Holland and Westcott, 2013). The case data clearly shows that it is only possible to build a sophisticated global treasury management and payments system in tandem with a global production network, because the financial system uses core supply chain data to support its business processes. The interdependency between manufacturing and financial supply chains also makes strategic changes more complex. This partly explains the long time-scales involved in the implementation of standardized financial processes based on automated systems.

The empirical evidence that demonstrates the relationship between financial supply chain strategy and performance is mapping out of the evolutionary timeline of the financial supply chain strategy and relating key events and strategy changes to performance outcomes. Changes in the financial supply chain strategy can then be related to qualitative improvements in areas such as financial process innovations and better relationships with suppliers and banks, and also to quantitative, operational performance improvements, for example time-series payment volumes and six sigma levels.

Summing up, the overwhelming trend is towards a standard financial supply chain model to coordinate international banking and payments throughout the physical supply chain. The movement of products and services encapsulated by the manufacturing supply chain is now supported by parallel financial and banking systems. As close collaboration is required between the trading partners within the supply chain to meet customer needs, the movement of funds has evolved to track the movement of goods in a concomitant manner rather than as a distinct and separate management function.

Theoretical Gaps in Financial Supply Chain Cooperation. Similar to other rapidly developing subject areas, there is no consensus or agreed definition of the concept. Global supply chain management systems rely on financial processes in addition to manufacturing, logistics and marketing activities to coordinate the flow of goods, services and money between separate stages in the supply chain. Financial supply chains are therefore an integral component of supply chains and yet there is very little research that specifically addresses the strategy, implementation and performance of global financial supply chains. Financial processes such as invoices, payments, foreign exchange and banking transactions have received very little attention in the supply chain literature because previous research has tended to focus almost exclusively on the movement of products and services in the supply chain and largely ignores the movement of money and related financial activities.

The literature in this area is only just emerging and is fragmented across academic and business publications. For example, in the academic literature Fairchild (2005) examined the integration of data from financial and physical supply chains to explore how companies can increase the efficiency of financial processes by integrating data from physical processes involved in the movement of goods and services with financial processes. Gupta and Dutta (2011) modelled the dynamics of financial supply chains in terms of the flow of money between customers and suppliers. Hofmann (2011) has analyzed two specific aspects of financial supply chains, risk and supplier financing in the automotive industry.

From a consultancy perspective, Hartley-Urquhart (2006) argued that companies should manage financial supply chains as closely as they manage physical supply chains in order to deal with the inherent complexity and risk of global production systems: as companies operate in a global environment where outsourcing and sourcing arrangements lead to more complex supply chain arrangements and risk management strategies (Chopra and Sohdi, 2004), the financial processes concerned with payments, visibility of the financial process, foreign exchange and risk management need to be much more sophisticated and integrated with the product supply chain. Heuser and Brockwell (2009) addressed similar issues, though from a banking perspective. Their focus was on the treasury management aspects of financial supply chains. In terms of early research originating from industry and management practice, there are parallels with the early development of the supply chain management concept, which was influenced by consultancy practice and industry specific research.

The logic of considering financial supply chains as an integral component of supply chains is that the flow of money and related financial and banking services is coordinated by shared financial processes that connect each stage of the supply chain in much the same way that manufacturing and logistics processes manage the flow of products from raw material suppliers through manufacturing, distribution and retail. Financial processes are therefore inextricably linked to the supply chain activities defined by Mentzer et al. (2001).

There is very little research that directly addresses the subject of financial supply chains that takes an overview of the topic and attempts to define a conceptual framework and illustrate it with significant empirical evidence over a period of time. There is also little consensus regarding the formal definition of the financial supply chain concept. An important element of the research is to understand the strategic evolution of financial supply chains in the context of the manufacturing supply chain over a significant time period, synthesize the performance metrics of a financial supply chain and set out a research agenda for financial supply chains. To start to address the lack of research into financial supply chains, it is necessary to define a framework that captures the core elements of the financial supply chain concept and relates it to the broader literature on manufacturing and logistics. In the next section a review of the literature is presented that forms the basis for the development of a research framework.

The literature that specifically addresses financial supply chains is scant and typically focuses on one specific aspect of the financial supply chain. Finance papers have tended to focus on the technical aspects of financial supply chains (Gupta and Dutta, 2011) and failed to address the strategic and operations management issues. Other research is also very specialized in nature. For example, researchers have examined the integration of manufacturing and financial data (Fairchild, 2005), currency hedging (Hofmann, 2011), financing arrangements (Hofmann, 2005) and technical risk from electronic payments (Johnson, 2008). However, none of these authors provide a conceptual framework or definition of financial supply chains. A broader view has been offered by practicing managers see for example Heuser and Brockwell (2009) who proposed a model of treasury management in the supply chain from a banking perspective but did not provide evidence for its use in practice.

Based on the following gaps defined there is number of research opportunities existing. Research into financial supply chains is in its infancy when compared to research into manufacturing supply chains. An agenda for future research opportunities is therefore proposed. The research frameworks used earlier were effective at capturing the principles of financial supply chains but the model could be extended in terms of additional variables, for example the nature of the strategic change process and project management techniques, and also into the nature of the relationships between the variables in the research framework.

Another important area is the detailed modeling of the flow of payments, akin to the modeling of product flows based on shared information between manufacturers and suppliers (Blackman, Holland and Westcott, 2013; Gupta and Dutta, 2011). What are the benefits to suppliers of receiving advance notification of payments with a certainty that they will receive funding in their own currency on a specific and guaranteed date? How should benefits such as reduced borrowing and foreign exchange requirements be quantified? How will the frequency of payments change in the future as it becomes possible to manage financial exchanges between trading partners at the level of individual items on a purchase order because of lower transactional costs from advances in information technology and banking systems?

In terms of formulating strategy for financial supply chains, moving away from a standard adversarial stance to a cooperative partnership approach with suppliers and banks requires a significant shift in the mind-set of senior finance managers who are typically accustomed to maximizing financial benefits within the organizational boundary of their own firm rather than looking to the competitive nature of the supply chain as a whole. However, the strategic benefits such as managing finances on a global scale and better relationships with suppliers, coupled with evidence from previous research that shows supply chain management capabilities are correlated with firm performance (Johnson and Templar, 2011) should encourage finance specialists to work closely with manufacturing and logistics managers to realize the benefits of closer integration across functional areas within the company and along the supply chain. In an economic environment where the availability and cost of bank funding are becoming significant problems, particularly for smaller companies, supply chain financing based on closer financial ties between large organizations and their supply networks becomes an attractive and strategically important opportunity. Empirical research in other global financial supply chains is needed to tackle these types of questions convincingly.

Another very important issue is the inability of key performance indicators (KPIs) of the FSCM defining. There are various key performance indicators that are relevant for measurement in financial supply chain management. One key metric is the cash flow cycle, which defines the period from delivery by suppliers until the cash collection of receivables from customers . It is the time period required for the company to receive the invested funds back in the form of cash. The cash flow cycle can be divided into the operating cycle which is the time period between delivery by suppliers and the actual cash collection of receivables, and the cash flow cycle which is the time period between the cash payment for inventory and the cash collection of receivables. The longer the cash flow cycle, the greater is the working capital requirement of a company, which means that a reduction of the cash flow cycle will immediately free up liquidity. However, the motivation as well as KPIs for an effective financial supply chain is very unobvious to define (Weiss, 2012).

In summary, it can be said that, empirical supply chain research has a limited focus on FSCM and is thus lagging behind. Similarly, scholars focusing on trade finance rather investigate the topic from a corporate risk perspective than a supply chain perspective (e.g. Chauffour and Malouche, 2011) and thus often omit the interplay of financial and operational flows in supply chains (Protopappa-Sieke and Seifert, 2010). Since FSCM by definition has a broad scope, the purpose of this paper can only be an initial attempt at investigating FSCM.

3. Trends and Gaps in Supply Chain Cooperation Modeling

3.1. Typology of Supply Chain Cooperation Models

As supply chain members are often separate and independent economic entities, a key issue in SCM is to develop mechanisms that can align their objectives and coordinate their activities so as to optimize system performance. In our research we are going to implement the typology of SCC models introduced by Li and Wang in 2007.

According to it, ideally, a decision in a supply chain can be made by a centralized decision maker with access to all available information to optimize system performance. This is possible when the entire supply chain is under the control of a single decision maker, or the coordination benefits can be fairly distributed among the individual members by a central planner. When such a solution can be implemented, the system is referred to as a centralized system. However, in general, neither a supplier nor a buyer can control the entire supply chain. Each supply chain member has its own state of information and decisions that can be made use to optimize its own interest. When the supply chain members are separate and independent economic entities, they will act independently and opportunistically to optimize their individual benefits. In this case, an action plan has to be complemented with an incentive scheme that can allocate the benefits of coordination among the supply chain members so as to align their objectives of coordination. Such a system is regarded as a decentralized supply chain system.

In a supply chain, entities such as suppliers, manufacturers, distributors, and retailers, can belong to a single organization or independent organizations. However, the distinction between centralized and decentralized systems is more properly related to the incentive structures within the chain. At the most basic level, in a centralized supply chain, there is a central planner who makes decisions for the entire system, while each entity in a decentralized system functions as an autonomous unit. Decentralized control policies can be easily implemented and analyzed at the local level (function, department, firm, etc.), however coordinated planning of the individual entities in a way that optimizes the value of the overall supply chain (system) is a difficult undertaking. Research tools that are used for planning such systems include network flow models and Mixed Integer Programming (MIP) models.

1. Centralized supply chain systems. The objective is to develop a production/inventory policy to minimize system cost. It is typically assumed that demand occurs at a buyer/retailer continuously at a constant rate, and no backlogging, lost sales, or transshipment is permitted anywhere in the system. Early studies have focused on the existence and development of optimal policies. However, such policies are usually difficult to characterize and implement. Recent studies have focused on approximate policies that are nearly optimal and practically useful.

1.1. Deterministic systems. 1.1.1. No time coordination. The problem of optimizing a multi-echelon inventory system is a classical one. When the planning horizon is finite, an optimal lot-sizing policy exists. This optimal policy is typically non-stationary. Discrete-time lot-sizing problem was solved by developing various algorithms. The continuous-time version of the problem can be solved approximately by a discrete-time algorithm with a very small base planning period. When the planning period is infinite, however, an optimal policy is very difficult to characterize when there is more than one buyer.

1.1.2. Time coordination. The optimal replenishment policy of a multi-echelon inventory system, however, typically entails a very complex non-stationary structure and thus is difficult to obtain and of little practical use. As such, previous studies have considered heuristic policies by restricting the timing of orders for the supplier and buyers so as to meet the above necessary properties for an optimal solution. Specifically, early studies have focused on stationary-nested or single-cycle policies. A policy is called stationary if each facility orders at equally-spaced points in time and in equal amounts. A policy is nested if each facility orders every time any of its immediate suppliers does, and perhaps at other times as well (Li and Wang, 2007).

Stationary and nested policies are attractive because they are easy to implement. However, such policies may result in very bad results in some cases.

A special case of the above model is the classical joint replenishment problem (JRP). Consider an inventory system in which multiple items are ordered from a common source. A major ordering cost is incurred each time an order is placed to the common source, independently of the number of items that are included in the order, and a minor ordering cost is incurred for each item that is included in the order. Obviously, ordering cost savings can be obtained when several items are replenished jointly. The key issue is then how to group these items. Many studies adopted group replenishment at constant intervals of time.

1.2. Stochastic systems. In reality, a stochastic model that specifies demand as a stochastic process is often more accurate than its deterministic counterpart the economic order quantity (EOQ) model (Li and Wang, 2007). However, a barrier to the application of a stochastic model is that the optimal policy does not have a simple structure, and is not easy to implement even if it does exist. This implies that appropriate coordination mechanisms are especially necessary.

Following the developments of multi-level production/ inventory systems, two classes of inventory control policies have been used for supply chain inventory management: an operationally simple, but not optimizing system performance installation policy (control of inventory is decentralized in the sense that each member makes its inventory decision separately based entirely on the local inventory position) and echelon stock policy that replenishes inventory based on the echelon inventory position (the sum of the local inventory position and the inventory positions at all its downstream members). Echelon base-stock policies are optimal in a periodic-review finite-horizon setting when there are no economies of scale in placing orders at all the stages except the most upstream stage in a serial inventory system. This result was later generalized to an infinite-horizon setting and assembly systems. Nevertheless, optimal echelon stock policies are extremely difficult to characterize when there are economies of scale in placing orders at all stages. Because of this difficulty, most previous studies have considered heuristic policies for serial inventory systems.

Obviously, as the echelon stock policy incorporates downstream agents inventory information for inventory control, it is superior to an installation policy.

Unfortunately, neither the installation stock nor the echelon stock completely characterizes the inventory state of a supply chain. To optimize system performance, inventory should be replenished at the supplier based on the exact inventory positions at the buyers. Nonetheless, this requires that demand and stock information at each stocking point be shared on a real time basis between the supplier and buyers in the supply chain. With the recent advances in information technology such as electronic data interchange (EDI) and other related developments, this is now possible. In fact, these developments have had a substantial impact upon SCM. As the time and cost to process orders are substantially lowered, impressive improvements in supply chain performance have been obtained. It is now a general belief that capturing and sharing real-time demand and stock information is the key to improving supply chain performance.

1.2.1. Independent and exogenously determined demand process. In a recent research by Sazvar (Sazvar et al., 2014) a stochastic mathematical model id developed in order to propose a new replenishment policy in a centralized supply chain for deteriorating items. In this model, they consider inventory and transportation costs, as well as the environmental impacts under uncertain demand. The paper (Rezapour and Farahani, 2010) develops an equilibrium model to design a centralized supply chain network operating in markets under deterministic price-depended demands and with a rival chain present. The two chains provide competitive products, either identical or highly substitutable, for some participating retailer markets. They model the optimizing behavior of these two chains, derive the equilibrium conditions, and establish the finite-dimensional variational inequality formulation, and solve it using a modified projection method. Correlated demand process

2. Decentralized distribution system. Although more and more firms have realized that collaboration with their supply chain partners can significantly improve their profits, the centralization of inventory and production decisions for a decentralized supply chain is often unrealistic. The challenge, then, is to devise coordination mechanisms that are not only able to coordinate the activities but also able to align the objectives of independent supply chain members (Chen et al., 2000).

2.1. Deterministic systems. Previous research on the coordination of decentralized deterministic systems has focused on using quantity discounts to induce independent buyers to increase their order quantities.

Many studies have been done independently from the viewpoints of inventory and production management and marketing channel coordination. The studies in the two areas differ in their focuses and model assumptions. Specifically, previous studies in the inventory and production management literature have typically focused on improving channel efficiency in managing inventory and production activities under the assumption that annual demand is exogenously determined. In contrast, studies in the marketing literature have typically focused on sales profit maximization under the assumption that inventory and production costs are independent of the pricing decision. Various discount pricing policies have been developed.

In general, it is assumed that the external demand rate, which could be constant or price-sensitive, occurs at a retailer continuously over an infinite horizon, and the supplier has symmetrical information about the annual demand and relevant cost parameters of a buyer. The objective is to determine the inventory and quantity discount policies to minimize cost or maximize profit.

2.1.1. The case of a single retailer. Many existing studies have analyzed quantity discount policies in the setting of a supplier and a single buyer. Although a supplier normally faces many buyers in reality, this setting has been adopted for simplicity of analysis.

In addition to quantity discount policies, profit sharing mechanisms have also been proposed. Under this proposal, the system performance is first optimized and the resultant benefit is then shared between the supplier and the buyer. This solution can be considered as a cooperative solution. Its implementation, however, depends on the development of a profit sharing scheme that is acceptable to both parties.

The model proposed by Li, Wang and Cheng (2010) investigates the sourcing strategy of a retailer and the pricing strategies of two suppliers in a supply chain under an environment of supply disruption, characterizing the sourcing strategies of the retailer in a centralized and a decentralized system. As a result, they derive a sufficient condition for the existence of an equilibrium price in the decentralized system when the suppliers are competitive. Based on the assumption of a uniform demand distribution, the authors obtained an explicit form of the solutions when the suppliers are competitive.

2.1.2. The case of heterogeneous retailers. When there are many buyers, an important issue for the coordination of a decentralized supply chain is whether incentive schemes can be designed on an individual basis. However, such a coordination mechanism with a unified incentive scheme is difficult to develop. There are two reasons. First, as discussed previously, a suppliers optimal inventory replenishment policy when facing a group of heterogeneous buyers typically entails non-stationary replenishment intervals and, thus, does not admit an explicit formulation. Second, a unified discount policy must be designed according to buyers cost and demand structures, as well as their economic behaviors, so as to fully exploit the benefits of coordination. When individual incentive schemes are permissible, a straightforward solution to the problem that is able to optimize system performance is for the supplier to negotiate a separate discount policy with each buyer, fixing the lot size and annual volume at the quantities that optimize system profit and selecting a price that is agreeable to both parties.

Suppliers in reality usually offer a common pricing policy that contains multiple break points to all buyers. Other than legal considerations, a common pricing policy is desirable not only for fairness of trade but also for ease of implementation. Multiple break points are offered to accommodate different cost and demand structures of heterogeneous buyers. However, a general discrete quantity discount is difficult to develop. As such, early studies adopted continuous approximations.

The models above, however, suffer from a common weakness that a heuristic inventory policy or simply an approximation of the inventory related cost function is assumed for the supplier. Obviously, neither a lot-for-lot policy nor a heuristic replenishment policy is desirable for the supplier.

2.2. Stochastic systems. In view of the difficulties in managing centralized stochastic multi-echelon inventory systems, it is an understatement that it is a challenge to coordinate a decentralized supply chain with stochastic demand. It is then not surprising that the literature in this category is scattered. As most real supply chain inventory systems fall into this category, this of course represents challenges and opportunities for future research.

Xu et al. (2014) investigate the impact of establishing a dual-channel supply chain coordinating contract when the supply chain agents are risk aversion under a mean-variance model. They present an analytical framework for marking price decisions in a centralized and a decentralized dual-channel supply chain with risk-averse, and analyze the impact of risk tolerance on the manufacturer and retailer's pricing decisions. The results show that the price set by a risk-averse dual-channel supply chain is lower than the one set by a risk-neutral dual-channel supply chain. Furthermore, compared with a centralized system, the vertical and horizontal competition in a decentralized system tends to result in channel inefficiency. To achieve channel coordination, the two-way revenue sharing contract is proposed that demonstrates the coordination of the dual-channel supply chain with risk-averse, and then it is analyzed how the risk attitude changes the parameters of the coordinating contract.

3.2. Typology of Financial Supply Chain Cooperation Models

In the field of supply chain management cooperation and collaboration are linked through flows of goods, information and finance business partners (basic raw materials and components suppliers, manufacturers, distributors, transporters, banks and financial institutions, etc.) and are core concepts. Thus, in terms of paradigm shift from competition to cooperation supply chains are often viewed as a networks of integrated companies (Mentzer et al., 2001). For an effective supply chain the management of upstream flow of money is as important as the management of downstream flow of goods (Gupta and Dutta, 2010). The problem of flow of goods in supply chains has been studied widely. But mainly the research on supply chain systems has focused on inventory cost, transportation cost and cost related to goods procurement. However, there has been very little research work that focuses on the upstream and downstream flows of money (Kouvelis et al., 2006). Scholars only recently began to demonstrate in formal analytical models how planning, managing and controlling financial flows along supply chains positively affect supply chain profitability (Raghavan and Mishra, 2011). Even though the aforementioned studies provide an analytical framework to evaluate the financial impact of supply chain performance, they are based on implicit assumptions, such as joint decision-making, absenteeism of opportunism and perfect information sharing, which are rarely applicable. Considering the theoretical basis of the proposed research, there is a need of further step toward understanding the supply chain in terms of integration of financial, material and information flows (Mentzer et al., 2001, Wuttke et al., 2013).

According to Gupta and Dutta (2011) the research on flow of money in a supply chain has not yet attracted the attention of mainstream Operations Management scholars even though the problem is important and bears a great resemblance to flow of material. The money flow problem has primarily been studied as the problem of cash circulation, cash management and cash balance. Based on the available literature, the research work under the rubric of financial supply chains can be divided into the following three categories:

- Cash flow systems analogous to ERP systems.
- Models for cash management based on inventory concepts.
- Cross functions models integrating manufacturing and finance decisions.

Cash flow systems analogous to ERP systems. There is a plenty of literature on financial supply chains that has primarily focused on the use of technology in improving the cash flow process similar to that of ERP in a manufacturing environment. The main focus of these studies is on the improvement of actual business process interactions across multiple organizations in financial supply chain systems. Although, this approach of cash management may not be applicable in value-added-service operations where it is very difficult to pin point the exact return for each and every purchase and investment. In many cases such purchase and investment are made for strategic advantages, with no immediate clear-cut return. We believe that the flow of cash needs to be managed as an overall problem rather than trying to map which upstream flow results in which downstream flow and then make decisions. Such mapping approach may result in a non-optimal performance of the overall business in terms of cash situations of the company. The studies that deal with cash flow process or the C2C research do not develop a scheme for an optimal or near-optimal management of cash flow in financial supply chain system, as we have done in this paper. They do not optimize the payment schedule. These studies could be considered complimentary to the contribution of this paper because our paper specifies the optimal payment schedule whereas these studies focus on efficient processes.

Models for cash management based on inventory concepts. Another stream of research in cash management literature has borrowed concepts from inventory theory. In general, an organization maintains a portfolio of assets that include liquid cash, treasury bills, commercial papers, etc. The optimal cash policies for these organizations can be determined by minimizing costs of holding cash and various transaction costs to convert from one asset type to another. The mathematical models for cash balance primarily focus on balancing the cash in hand with the liquid asset like marketable securities based on firms needs for cash and predictability of such needs.

The cash balance problems addressed in this type of models are orthogonal to the problem of our research. The cash balance problems in these papers deal with internal cash management of an organization so that transaction cost is minimized or higher return can be found from these transactions. However, the problem we are studying focuses on management of external cash transactions such as cash received from downstream partners and cash payables to upstream partners.

Cross functions models integrating manufacturing and finance decisions. Some papers have emphasized that financial supply chain decisions should be integrated with advanced planning and scheduling decisions. These papers developed mixed integer linear programming based formulations for cash management in a chemical process industry. Cash management problem studied in these papers is based on maximizing the cash position by combining profit and the cost of making that profit. This approach may be applicable for manufacturing industries. However, in service industries such an approach may not be plausible. Our research bears some similarity to the approach presented in these papers. However, we address the problem of cash management to prioritize the payment schedule based on incoming revenue stream and pending invoices to be paid. The results of this study can be applied between any two levels of upstream and downstream partners, in both manufacturing and service industry in a supply chain.

The majority of the most recent research in financial supply chain management belongs to this stream of works. The outcome of the research by Blome, Paulraj and Schuetz (2014) is the analysis of the deviation from an optimal profile of supply chain collaboration and its detrimental effect on sustainability performance as well as market performance. The model obtained shows that the effects of alignment on performance measures are mediated by the firm's internal sustainable production. The research by Cao and Zhang (2011) inspects the nature of supply chain collaboration and explore its impact on firm performance based on a paradigm of collaborative advantage. As the result, valid model of these constructs was developed through empirical analysis which shows, that supply chain collaboration improves collaborative advantage and indeed has a bottom-line influence on firm performance, and collaborative advantage is an intermediate variable that enables supply chain partners to achieve synergies and create superior performance. A further analysis of the moderation effect of firm size reveals that collaborative advantage completely mediates the relationship between supply chain collaboration and firm performance for small firms while it partially mediates the relationship for medium and large firms. In their work Schoenherr and Swink (2012) cross-validate Frohlich and Westbrook's framework (Frohlich and Westbrook, 2001) utilizing multi-dimensional performance measures collected from supply chain managers. They also extend Frohlich and Westbrook's study by investigating the moderating role of internal integration on the relationships between arcs of integration and performance. In accordance with information processing theory, the results indicate that internal integration strengthens the positive impacts of external integration on both delivery and flexibility performance. The model obtained by Hadid and Afshin Mansouri (2014) lean constructs are identified and operationalized to establish their interrelation and impact on organizational performance. This paper synthesizes a comprehensive set of lean technical practices, lean supportive practices, inhibitors and expected outcome of lean service. Moreover, six influential contextual variables on the lean-performance relation are identified based on a review of the management accounting literature, organizational strategy literature and diversification literature to overcome limitations of previous studies.

3.3. Gaps in Financial Supply Chain Cooperation Modeling

Different models of SCC have been proposed considering isolated activities or different functions of SC, nevertheless these models appear to be fragmented efforts.

A great deal has been written on centralized supply chain systems. When demand and lead time are deterministic, exact optimal coordination policies for many buyers are still difficult to characterize. However, power-of-two and integer-ratio policies provide a highly effective and practically useful framework to coordinate supply chain inventory activities. Nevertheless, similar coordination mechanisms have not been developed for a supply chain system when demand and lead time are stochastic. Although power-of-two and integer-ratio policies can also be applied, their applicability and effectiveness have not yet been fully established in this case. A future research area is then to develop optimal, or nearly-optimal but practically useful, inventory policies for supply chain systems with uncertain demand and/or lead time. For example, a suppliers (optimal) inventory policy when facing multiple heterogeneous buyers with uncertain demand and/or lead time is still an open issue.

With the recent advances in information technology, real time data exchange has become feasible and affordable. As a result, an equally important issue for SCC is to incorporate information into a coordination policy. The issue, however, is no longer whether information is useful, as this has been demonstrated by many previous studies. Rather, future research should focus on what information to be shared among supply chain members and how to use such information. Previous studies adopted different coordination policies and, as a result, obtained very different assessments for the benefits of information sharing. Apparently, this shows that optimal supply chain inventory policies depend on the information structure. When demand and stock information can be shared among all members in real time, neither the installation policy nor the echelon stock policy is optimal. Future research must then identify the desirable information structures and coordination policies under various supply chain structures.

In comparison to centralized supply chains, the literature on decentralized supply chain systems is less extensive. The coordination of decentralized supply chain systems is more difficult: facing the same challenge to optimize system performance and also requiring a scheme to reallocate the benefits of coordination so as to maintain the interest and participation of all independent supply chain members. When demand is deterministic, many incentive schemes have been studied. Among these incentive schemes, quantity discounts stand out to be the most widely employed mechanism to entice the cooperation of independent supply chain members. However, quantity discounts are usually not able to optimize system performance when there are heterogeneous buyers and/or multiple products. There is a need to develop more effective and practically useful incentive schemes. Furthermore, as an action plan to coordinate supply chain decisions and activities often lead to unbalanced cost burdens to different supply chain members, the incentive scheme and the coordination policy must be developed together as a single mechanism.

Finally, real research opportunities exist for the coordination of decentralized supply chain systems with stochastic demand and/or lead time. As compared to the above categories, the amount of literature in this area is severely unbalanced. Although a few previous studies have developed non-cooperative (Nash equilibrium) solutions, the coordination issue represents a real challenge. In view of the previous studies, a coordination mechanism for a decentralized supply chain system should include at least three components: 1. an operational plan to coordinate the decisions and activities of the supply chain members, 2. a structure to share information among the members, and 3. an incentive scheme to allocate the benefits of coordination so as to entice the cooperation of all members.

4. Conclusion

This literature review offers implications for both researchers and practitioners. For SCM research, this study makes contributions to existing knowledge by providing a state-of-the-art picture on the relationship between SCM and firm-level financial performance. On the one hand, effective SCM enhances both accounting- and market-based performance measures through the improvements in revenue growth, operating costs reduction, and working capital efficiency. On the other hand, disruptive SCM causes substantial financial losses in both short-term and long-term periods. The slow recovery from SC disruptions makes the firms even more vulnerable in this time-sensitive business environment.

Although, there is an emergent stream of literature which has highlighted the need to improve that kind of integration (Fairchild, 2005, Gupta and Dutta, 2011), these attempts are rather scant and fragmented. The review addresses a distinct gap in the operations and supply chain management literature by proposing that the improvement of supply chain performance and the optimisation of working capital along the supply chain requires a holistic understanding of the flow of physical and financial resources across supply networks.

This study pays particular attention to the problem, that over the past two decades the operations and supply chain management literature has focused primarily on the flows of physical goods and information, rather than financial supply chains (Fairchild, 2005; Gupta and Dutta, 2011). The financial supply chain, which runs parallel to the flow of goods and information, is common to all economic supply networks, and its integration with the physical supply chain is therefore a critical and ubiquitous aspect. It is shown, that the evolution of the research in the field of supply chain cooperation modeling has evolved from centralized cooperative models through decentralized coordination models to collaborative models. Moreover, the unit of modeling has become significantly more complex from unconnected supply chains to multi-echelone systems. From the authors point of view, the further step ahead, which is expected to be a fruitful avenue of thought, is development of models of collaborative supply chain networks, especially in the field of financial supply chain management.

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Concept of Joint Venture's Stability: Case of Renault–Nissan Alliance

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Abstract In this paper the problem of alliance in the form of joint venture stability evaluation is considered. The longevity of joint ventures existence is determined by long term motivation of alliance partners to achieve stated goals, which in turn, is determined by alliance stability. Analysis of the existing papers on alliance stability topic showed that the research is very fragmented. Most well-known papers in the area are mainly focused on the investigation of different internal and external factors that can influence joint venture stability. At the same time, no joint venture stability definition and concept has been suggested. For this reason, authors developed original approach to the definition of strategic alliance stability and its conceptualization which allows evaluation of stability, taking into account influence of both, internal and external factors.The introduced concept is implemented to the analysis of the Renault-Nissan strategic alliance.

Keywords: strategic alliance, joint venture, joint venture stability, internal stability, external stability, Renault-Nissan alliance.

1. Introduction

In recent decades, globalization of the economy has been growing rapidly, boosting rapid changes in the market structure, business competition and information environment. At the same time, consumers are becoming more aware and raise demands for quality and consumer properties of products. In such circumstances, it is extremely difficult for a manufacturing company to meet its needs for resources, technologies, skills, competencies, knowledge and information to achieve competitive advantage. Therefore, strategic alliances in general and joint ventures (JV) in particular are an effective strategic tool of competition as they allow companies to merge their resources (Shuvalova, 2008). Joint ventures enable companies to partner and to expand their geographic presence rapidly, to enter new markets, to gain access to new knowledge, information, skills, technologies and the necessary competence. Nowadays more and more manufacturing companies see a joint venture as a source of their competitive advantage in terms of globalization of markets. However, the percentage of international strategic alliances fulfilling the tasks stated prior to the relationships development remains extremely low, while failure rate grows (Zafar et al., 2014).

The term "strategic alliance" in this article refers to a long-term cooperative agreement on cooperation between partner companies in order to achieve economic benefits, while maintaining their legal and economic independence; and the term "joint venture" refers to a form of a strategic alliance incorporating raising parts of the capital of two or more partner companies forming one that may have either equal or unequal stakes in the company. To be more precise, partner companies in equity strategic alliances establish an independent company (a joint venture) by combining some of its assets to start long-term relationships and the transmission of tacit knowledge.

During the study of dynamic development of joint ventures, stability of the alliance is considered as a necessary condition to achieve the best result of the joint venture. It is clear that in order to maintain the stability of a JV the motivation to cooperate should be maintained during the whole period of its existence, even though the motivation of the partners may be affected by various internal and external factors. Therefore, the term of JV stability is referred to the success of its activities during the period of implementation under the continuous motivation of each partner company to achieve maximum results.

Analysis of existing approaches to the assessment of the stability of alliances has shown that there is currently no relevant methodology for assessing the stability which takes into account both external and internal factors (most of them suggest univariate analysis). It should be noted that a model designed to predict econometric analysis cannot offer practical recommendations for overcoming the instability of alliances (stabilization of a joint venture), and, therefore, managerial application of the results presented in research papers remains low. Thus, the problem of one unified methodology for assessing the stability of joint ventures is still unsolved.

This paper suggests a JV stability concept for assessing the JV stability based on the external stability forecasting and game-theoretic modeling of the internal stability of the alliance. It appears that this approach to define stability can become widely used for joint ventures in various sectors of the economy.

The structure of the paper is organized as follows. The first section introduces an overview of the concept of strategic alliances and joint ventures. The second section is dedicated to a concept of strategic alliances stability. In the third section the proposed concept was used on a retrospective analysis of stability of Renault and Nissan JV. The main findings are presented in conclusion.

2. Strategic Alliances and Joint Ventures

2.1. The Concept of Strategic Alliance

In order to start with the assessment of stability of joint ventures, it is essential to clarify the terms and understand what forms of cooperation fall into this category. Joint venture is defined as a special case of a strategic alliance, which is a broader class of inter-firm cooperation. Historically, the term of strategic alliance appeared in the scientific literature a little less than a hundred years ago. In the economic literature it was first introduced by Hoxie (1923) for the different types of professional associations. After 30 years Estey (1955) also used the terminology of Hoxie to study the effectiveness of trade unions activities. Therefore, the initial interpretation of the term of strategic alliance meant a union between a "dependent" organization (in need of help to achieve its goals), and economically self-sufficient firms able to provide the dependent side with the necessary assistance by their high position in the industry.

Early explorers emphasized that the strategic alliance is not necessarily formed on the basis of a formal agreement and is often informal, verbal or even secret. Despite the fact that the term of strategic alliance started to be actively used only in the 1980s, researchers and business consultants of the 1930s became interested in forms of inter-firm cooperation. For example, in the famous Gault work (Gault, 1937) the author studied the problem of cooperation of business groups in the field of marketing and gave a prediction about growth of large-scale inter-firm cooperation in the coming decades based on the analysis of benefits from the joint actions in the form of strategic alliances.

In the middle of the 20th century, another area of research was the study of inter-firm cooperation in the form of joint ventures. With the development of communication and transport, domestic markets of other countries have become more readily available for different companies. New markets in Europe and Asia were developing especially actively getting open for US companies, which introduced a new way of entering foreign markets through an establishment of a joint venture, all assets of which belonged to two or more independent companies. Therefore, almost all the works of this period are devoted to the analysis of international joint ventures established for the purpose of expansion - entering foreign markets. Until the late 1970s joint ventures were considered by economists as an additional tool for the company's expanding into new geographical markets. At the same time, strategic aspects of cooperation between companies remained outside of the analysis.

The approaches to the definition of a strategic alliance differ and there is no uniform understanding. Currently, the term of strategic alliance has a broad meaning that embraces many forms of inter-organizational cooperation. The alliance is an organizational form that includes more than just a market transaction; it is a form of contractual relations, a merger or an acquisition along with forms of inter-organizational cooperation, such as joint ventures, licensing agreements, joint projects in product development, joint purchase or production (Inkpen, 2001).

As already mentioned above, there is currently no common understanding of the term of strategic alliance and the approach to the classification of strategic alliances. Table 1 shows only some definitions of strategic alliance widely used in studies in chronological order.

Analysis of the definitions shows that researchers emphasize different peculiarities of alliances. In all cases, it is stated that an alliance is a cooperative agreement between two partner companies. This emphasizes voluntary participation in the alliance (Gulati, 1998; Das and Teng, 2000); the fact that companies receive economic benefits or the benefits of cooperation (Rangan and Yoshino, 1996; Clarke-Hill et al., 1998b; Das and Teng, 2000; Ireland et al., 2002; Todeva and Knoke, 2005); and the necessity of participation of two or more organizations (Rangan and Yoshino, 1996; Osborn et al., 1998; Dussauge et al., 2000; Pyka and Windrum, 2003; Contractor and Lorange, 2002; Ireland et al., 2002; Clarke-Hill et al., 1998a; Todeva and Knoke, 2005). The attachment of other cooperative forms created outside of the alliance to the new alliances remains arguable.

The diversity of views on the strategic alliance in the scientific literature correlates with goals of a particular research. Summing up views on the concept of a strategic alliance of different researchers, it is possible to identify the characteristic properties to be satisfied by the term of strategic alliance; e.g. strategic alliance is a form of implementing the cooperative strategy of partner companies, which have common goals and can also have their own private purposes; it requires the participation of at least two partner companies; partner companies remain legally independent; partner companies are jointly implementing management control and distribute the benefits of cooperation.

One important characteristics of an alliance is a formal independence of partner companies (Inkpen, 2001), whereas cooperation makes partners interdependent within the alliance (Inkpen, 2001; Ireland et al., 2002). This fact indicates the difference between the alliance and mergers or acquisitions. The interdependence of partner companies gets complicated by uncertainty about the behavior of partners (Inkpen, 2001).

Table 1: Different interpretations of the term of strategic alliance.

Authors	Interpretation
Hill, Hwang, Kim, 1990, p. 218	Strategic alliances can be placed on a continuum where contractual agreements lie on one end of the continuum, representing low control and low resource commitment, whereas joint ventures lie on the other end of the continuum, representing high control and high resource commitment
Varadarajan, Cunningham, 1995, p. 282	Strategic alliances, a manifestation of inter-organizational cooperative strategies, entails the pooling of specific resources and skills by the cooperating organizations in order to achieve common goals, as well as goals specific to the individual partners
Rangan, Yoshino, 1996, p. 7	A strategic alliance is an arrangement that links specific facets of the business of two or more firms. The basis of the link is a trading partnership that enhances the effectiveness of the participating firms' competitive strategies by providing for the mutually beneficial exchange of technologies, products, skills or other types of resources
Clarke-Hill, Robinson, Bailey, 1998, p. 300	Strategic alliance is a coalition of two or more organizations to achieve strategically significant goals and objectives that are mutually beneficial
Gulati, 1998, p. 293	Define strategic alliances as voluntary arrangements between firms involving exchange, sharing, or co-development of products, technologies, or services
Das, Teng, 2000, p. 33	Strategic alliances are voluntary cooperative inter-firm agreements aimed at achieving competitive advantage for the partners
Dussauge, Garrette, Mitchell, 2000, p. 99	Strategic alliance is an arrangement between two or more independent companies that choose to carry out a project or operate in a specific business area by coordinating the necessary skills and resources jointly rather that either operating on their own or merging their operations
Elmuti, Kathawala, 2001, p. 205	A strategic alliance is an agreement between firms to do business together in ways that go beyond normal company dealings, but fall short of a merger or a full partnership
Contractor, Lorange, 2002, p. 486	An alliance is any inter-firm cooperation that falls between the extremes of discrete, short-term contracts and the complete merger of two or more organizations

Ireland, Hitt, Vaidyanath, 2002, p. 413	Strategic alliances are cooperative arrangements between two or more firms to improve their position and performance by sharing resources
Pyka, Windrum, 2003, p. 245	Cooperative agreement between two or more autonomous firms pursuing common objectives or working towards solving common problems through a period of sustained interaction
Todeva, Knoke, 2005, p. 125	A strategic alliance involves at least two partner firms that remain legally independent after the alliance is formed, share benefits and managerial control over the performance of assigned tasks and make continuing contributions in one or more strategic areas, such as technology or products
Hitt, Ireland, Hoskisson, 2007, p. 269	Strategic alliances are partnerships between firms whereby their resources, capabilities, and core competencies are combined to pursue mutual interests in designing, manufacturing, or distributing goods or services

For the purpose of further analysis of the strategic alliances stability, the paper is focused more on long-term agreements, long-term cooperation with partner companies (for at least 5 years) opposed to co-market transactions or short-term agreements. It should be noted that very often in the literature, the term of strategic alliance is used to refer to long-term cooperation, yet, in fact, it is a temporary form of organization (Das, 2006).

In this paper, the strategic alliance is understood as *a long-term cooperative agreement between the partner companies that are legally independent after the formation of the alliance, share the benefits of their co-operation and management control to implement the tasks and activities carried out continuously in one or more strategically important areas such as technologies or products.*

2.2. The Concept of Joint Ventures

Moving to the concept of a joint venture, it is essential to return to what was stated earlier. A JV in this paper is considered as a special case of a strategic alliance in the framework of their classification to attract capital of partner companies within the alliance. One should distinguish between alliances involving capital and without it.

The strategic alliances with capital raising (equity strategic alliance) partner companies create a legally independent company (JV) by combining some of their assets. Joint ventures are typically created to establish long-term relationships between partners as well as to transfer their tacit knowledge. They can have both equal and different stakes in the company.

Strategic alliances without raising capital (non-equity strategic alliances) include agreements for the supply, production or distribution of products and services without the formation of a joint venture and cooperative agreements in the fields of marketing activities or knowledge sharing.

Alliances of this type are less formal and carry fewer risks. However, such alliances are not always suitable for complex joint projects requiring effective ways of long-term use of shared resources and the exchange of competencies between the partners.

Because of the special historical role of the JV development in Russian economy, authors observed confusing of strategic alliances and joint ventures notions in a number of domestic research papers, which complicated the reader's understanding of differences in terminology. For example, Strovsky (1999) confused the definitions of the joint venture in the wide and narrow sense in his work, essentially repeating the definition of strategic alliances and joint ventures.

In this paper, the joint venture is understood *as a form of equity strategic alliance of two or more partner companies, which may have equal or different shares of participation in the venture*. In equity strategic alliances partner companies create an independent company, combining some of their assets for establishing long-term relations between partners and transferring tacit knowledge (Zenkevich et al., 2014, p. 34-35).

What is more, only strategic alliances with capital raising (JV) are considered in this paper, so the terms "strategic alliance" and "joint venture" can be seen as synonyms, unless otherwise stated.

A classification of joint ventures is also introduced there. The first category can be distinguished by the number of partner companies within a joint venture: there could be two (bilateral) and numerous partners (multilateral). The first type of a joint venture is the simplest and more common in business practice (Das and Teng, 2002). Multilateral alliances are less studied, because they are more complex in terms of management. Recently, in addition to the literature of multilateral alliances the term of strategic network can be found, referring to the organizational forms of multilateral organizations having entered into several cooperative agreements (Hitt et al., 2007). Joint ventures can be created by companies operating in the same industry (intra-alliance) and in different sectors (intersectional alliance).

During the analysis of the terms of strategic alliance and joint venture there is an opportunity to track certain evolution of the concept of strategic alliances and joint ventures. The first attempts to analyze the strategic aspects of cooperation of companies were made in the late 1970s. In 1976, Pfeffer published an article that explains the creation of joint ventures by the theory of resource dependence (Pfeffer and Nowak, 1976; Pfeffer and Salancik, 1978), exploring the relationships of resource and power companies that had access to the resources themselves or from other companies. According to the theory of resource dependence, the company has two reasons for establishing a joint venture (Das and Teng, 2000; Grant and Baden-Fuller, 2004):

1) to have an access to important resources of a partner;

2) to increase the control over partner-companies.

The next decade could be characterized by the prosperity of joint ventures in business practice in variety of forms and objectives of cooperation. Therefore, in the late 1980s - early 1990s the phenomenon of strategic alliances formation (joint ventures in particular) was explained in terms of several theories of resource concepts, theories of dynamic capabilities, the concept of market power and the theory of transaction costs.

According to the concept of resources, the main reason for the formation of a joint venture is the desire to maintain or achieve the desired possession of resources. In other words, companies have access to their partners' resources that allows them to purchase a set of resources within the joint venture that will have the greatest value for the alliance (Eisenhardt and Schoonhoven, 1996; Das and Teng, 2000; Harrison et al., 2001).

The distinction between the concept of resource and resource dependence theory is that the first one studies the internal resources, as companies need a set of rare and valuable resources for their competitiveness. Joint ventures can be used to accomplish this task. At the same time, the theory of the resource dependence focuses solely on the resources that can be obtained from the environment (Barringer and Harrison, 2000). This, in its turn, leads to the creation of mechanisms depending on the resources and power management.

Other researchers used the theory of market power to explain the reasons for the JV formation (Kogut, 1988; Hagedoorn, 1993; Barringer and Harrison, 2000). According to the theory of market forces, the company creates a joint venture in order to improve its competitive position in relation to its competitors (Kogut, 1988). This includes not only the improvement of the competitive position of the company, but also contributes to preventing attempts of competitors to improve their position. In addition, joint ventures are used by companies as means for increasing market power. Empirical studies have shown (Hagedoorn, 1993) that the companies use alliances to enter the market and as an instrument of influence in the market structure.

According to the theory of transaction costs, joint ventures and alliances are one of the ways to avoid inefficient transaction costs. The transaction may not be effective for two reasons: first, the market transaction makes the company dependent on other companies (Kogut, 1988); secondly, markets themselves may be ineffective, resulting in high transaction costs. Manufacturing companies also may not be effective, for example, due to lack of knowledge, competencies, technology and human resources. Therefore, in case production costs are high and markets are inefficient, mergers and acquisitions entail additional costs, such organizational forms as a joint venture may be the most promising strategic alternative.

The end of the last decade of the 20th century was characterized by the new direction of the development of the resource concept - the concept of dynamic capabilities (Teece et al., 1997; Sanchez, 2001). Within this concept, joint ventures are created with the purpose of gaining access to rare poorly reproducible resources and intangible assets of its partners (including knowledge). Being a part of a joint venture companies have access to the intangible assets of their alliance partners, and they are able to internalize (Dussauge et al., 2000). Some interesting research focuses on aspects of learning within the alliance. (See, e.g.: Hamel, 1991; Inkpen and Beamish, 1997; Dussauge et al., 2000).

At the turn of the 21st century, when the creation of strategic alliances has become an integral part of the company's strategy and the number and complexity of the alliances was constantly increasing (some companies have about a thousand current strategic partnerships with different companies), new social approach to the study was developed in two research areas: relational approach and the approach of network structures.

Relational approach is considering a joint venture in terms of the interaction of social systems, as in the real world a joint venture is established not only for economic, but also attitudinal characteristics, such as trust, reputation and communication. According to this approach (Seabright et al., 1992), the relationship between the partners is forming, evolving, deteriorating and eventually terminating as a result of repetitive attitudinal interactions that affect the success or failure of the alliance as a whole.

One of the most important attitudinal factors is the trust (Arino and de la Torre, 1998; Inkpen and Currall, 2004). Trust defines the company's expectations with regard to the behavior of their partners. In particular, the relationship between the partners can be honest and not be opportunistic (Gulati, 1995). The article (Koza and Lewin, 1998) based on the study shows that in successful joint ventures partners trust each other, while in unsuccessful ventures trust was lost or missing.

The network approach studies the interaction of the market elements, where a lot of companies (components), interact with each other one way or another (Gulati, 1998; Wilkinson and Young, 2002). These elements and their relationships form a social network, covering the investigated company and its alliances (including joint ventures) in which it is present (Gulati, 1998). The social network of the company, in which it is present, affects its behavior and activities. Gulati gives some examples of the impact of social networking on companies. Thus, social network allows an observer to see new possibilities for creating a joint venture for the company and determines how often and with whom the company forms joint ventures. In addition, if the two companies form an alliance, their position in the network is changed, and it determines new proximity of alliance management. Unlike previously described approaches, the networking approach is primarily a mechanism to identify links and relations of the company.

In the new millennium, the prevailing approaches to the analysis of strategic alliances and joint ventures were the concept of dynamic capabilities, network and relational approaches. The main focus in all the above mentioned theories was placed on identifying reasons of firms to establish joint ventures, as well as factors affecting the results of joint venture. Questions of prosperity of a joint venture in terms of a long temporal gap, the causes of success and failure, termination of the JV existence are still poorly understood. Currently, in the scientific literature there is no suitable theory that could predict the future development of alliances and joint ventures in dynamic.

One reason for the existing situation is that existing approaches use static cases, therefore, it is not always possible to explain the reality and effectiveness of the operation of alliances in the long run. What is more, this is also the reason why explanations of partner companies behavior in joint venture on the basis of these approaches is not convincing; joint ventures had to be analyzed over time, as each stage takes its substantial period of time, formation of a joint venture takes from few months to several years, it operates for years or even decades before ceases to exist. The life cycle of a joint venture includes three stages: the formation of the alliance, the implementation phase and the completion stage.

The first phase includes intra preparation, choice of a partner and negotiations on the composition of a joint venture and cooperative agreements within its framework. In the second phase (implementation) the joint venture starts to operate, and its management should be able to sustain and contain the pressure of adverse internal and external factors which can have impact both on the current results and prospects of the continuation of the joint venture activity. Finally, in the third phase (completion) the joint venture ends its existence that does not necessarily mean its inconsistency.

The dynamic perspective of the alliance is related directly to the interaction of the alliance partners over the lifetime of the alliance. The determining factors of the relationship between the partners in the joint venture are: trust, involvement, balance and coordination between partners. Although these factors are often the subject of research (Doz, 1996; Arino and de la Torre, 1998; Koza and Lewin, 1998; Das and Teng, 2002), the development of partners' relations in the long term still has not been paid enough attention to (Inkpen, 2001).

Determining the extent of the JV success, the researchers focus specifically on its results. However, this area of research is problematic, because so far there is no consensus about which set of factors determines the activity of the joint venture, nor an agreement on how to assess its results.

There are several ways to determine the activity of a joint venture in terms of success or failure. Indicators of the success of a joint venture may be indicators of its survival and duration of existence (Pangarkar, 2003). On the one hand, the long period of the joint venture existence may be indicative of its success and satisfaction of its partnership companies. Long period of existence enables partners to share knowledge and achieve good results within the alliance. On the other hand, the premature termination of the alliance does not mean its failure as well as the prolonged existence of a joint venture does not always imply its success. Therefore, the duration of the alliance can be an indicator of the success of the joint venture, but not of a particular one. The results of the joint venture can also be determined by the following approaches:

– Achievement of strategic goals (Parkhe, 1993). The results of a joint venture are considered in terms of achieving the objective pursued by the company during the formation of the alliance.
– Financial indicators of the results of a joint venture (Combs and Ketchen, 1999). Thus, the costs, sales growth, profits are often considered as financial indicators for assessing the performance of the joint venture.
– Knowledge sharing among the partners of the alliance (Kale et al., 2002). Quite often the company decides to create a joint venture in pursuit of subjective non-financial goals, such as getting a new technology or knowledge of a new market. Should one, several or all of the partners have not reached the goal, such an alliance can be considered unsuccessful for partners, despite the fact that they can be satisfied with the financial results of the joint venture. However, subjective measures to assess the performance of the joint venture have been criticized for being biased in the definition and evaluation due to their inaccuracy.

It should be noted that usage of these approaches can have positive results for the joint venture, but it still does not guarantee that the partnership will be successful in cooperation and constructive relations, and vice versa. These considerations lead to the need for the formulation of a different approach evaluating the success of the joint venture, including its analysis from the standpoint of a dynamic perspective.

3. Alliance Stability

3.1. The Concept of Stability of Alliances

As alternative indicators of the results of a joint venture indicators of stability of the alliance can be considered (Mohr, 2006). Stable joint ventures may serve longer than unstable ones, while within the first companies the likelihood of achieving desired goals and strong financial performance being members of the alliance increases. However, despite the increasing number of papers analyzing performance of different forms of alliances, joint ventures turn to be fails more often than activities of independent firms, established branches and subsidiaries, as well as the mergers and acquisitions (Alexandrovsky and Fomenkov, 2011). In this regard, there is a need for a more detailed study of the concept of stability of a joint venture.

Many researchers regard an assessment of the alliance's performance an indicator of the stability of the joint venture (Geringer and Hebert, 1991). However, according to the authors, it is more correct to consider the stability of a joint venture not as a way to assess its performance, but as a way to determine the achievement of the desired results. Stability is vital for development, evolution and survival of a joint venture, as it is both a prerequisite and a good estimate for the benefits of the JV success (Beamish and Inkpen, 1995; Dussauge and Garrette, 1995). Once the joint venture is created, its resistance becomes a "prerequisite for extracting competitive benefits of participation in strategic alliances" (Bidault and Salgado, 2001, p. 619).

Currently, in literature there is no single point of view on the concept of stability of a strategic alliance as a whole and joint ventures in particular. This largely explains the fact that the stability of alliances is one of the least studied aspects in modern business literature.

The term "stability of a joint venture" was first introduced in 1971 in the work by Franko (1971). Basing on the empirical study of 170 joint ventures operating in the United States, L. Franko analyzed the cases in which, in his opinion, there was no resistance. The main conclusion of the work is very obvious and has no practical implementation, it lies in the fact that the alliance is unstable when partner changes or sells its stake in the alliance or alliance is liquidated.

Existing approaches to assessing the stability of joint ventures are listed in Table 2. The conventional time division into two periods of work due to the fact that earlier studies rely primarily on the degree of the foreign partner business peculiarities in the home country of the alliance (Kogut, 1989; Beamish and Inkpen, 1995). Research projects related to the later period consider market power of each of the partners as a key factor in the stability of the joint venture (Inkpen and Beamish, 1997; Sim and Ali, 2000; Gill and Butler, 2003).

Cross-cultural differences between the two companies are an integral part of intercultural alliances, defined as partnerships between companies from different countries to meet mutual interests and sharing of resources and capabilities (Doz and Hamel, 1998; Yan and Luo, 2001).

The following example can be given: in autumn of 1991 the company Vitro, SA, a Mexican glass manufacturer, signed an agreement establishing a joint venture with the American company Corning Inc. Over the next few years the company existed in a conflict of interests. First of all, there were problems associated with the JV management- in Mexico top management made all the decisions, that slowed the pace of work significantly for those accustomed to the American style of doing business. Vitro's marketing strategy has been less aggressive than the one of the company Corning Inc. This was due to the fact that Mexican company has been conducting its activities in a closed economy for a long time. Although the choice of partners was strategically right (Corning Inc. specialized in melting glass and cookware and Vitro - in the manufacture of glassware) and the alliance was economically attractive for both sides, in 1994 partners decided to return money which was invested in the joint venture and joint activities were terminated due to irreconcilable contradictions (Doh, 2003).

Table 2: Overview of research on joint venture's stability.

Authors	Approach	Stability factors and hypothesis
Inkpen, Beamish, 1997	Conceptual model	Bargaining power of each of the partners is a key factor in the stability of the alliance
Yan, 1998	Conceptual model	Factors of instability: the unexpected circumstances, lack of economic benefits, low market power of one of the alliance partners, lack of training opportunities within the alliance
Yang, Zeng, 1999	Conceptual model	In order to understand the stability of the strategic alliance it is necessary to compare the results with results that could be achieved in the absence of co-operation
Das, Teng, 2000	Conceptual model	Cooperation/competition, rigid structure/flexibility, focus on short-term/long-term results as the main factors of stability
Sim, Ali, 2000	Empirical study	The psychological distance between the partners and their willingness to cooperate are the main factors of stability of the joint venture
Bidault, Salgado, 2001	Case method	The organizational complexity of the joint venture leads to a deviation from the basic goal of its creation and, consequently, to its instability
Gill, Butler, 2003	Case method	Key factors: the confidence of partners, conflicts and power of their dependence on each other
Ernst, Bamford, 2005	Practical study	Constant restructuring and revision of the organizational structure of the joint venture guarantees stability
Nakamura, 2005	Empirical study	Transfer of experience inside of a joint venture may eventually change the market force of partners, leading to restructuring and, therefore, to its instability

The market power of the partners in the alliance largely determines the stability of the joint cooperation. This approach was first introduced in 1997 by Inkpen and Beamish (1997) in their scientific paper, in which, basing on the analysis of various joint ventures, they have come to the conclusion that the fundamental element of stability is the bargaining power of each of the partners. In the article by Nakamura (2005) 231 companies in a strategic alliance since the end of World War II and until 1971 were also studied; M. Nakamura discovered that changes in the market power of partners can lead to instability of a strategic alliance.

To be more precise, there is a possibility to consider an example of a strategic alliance between the Norwegian company Statoil and British BP - one of the leading representatives of the oil and gas industry in the world. In 1991 companies merged to create a joint venture aimed at achieving long-term strategic goals for both partners. The BP, despite many years of international experience, was in a difficult financial situation which did not allow to continue a natural development. In its turn, Statoil had considerable material resources in absence of conducted international activities, which also gave it the opportunity to continue its development. The share of the company's participation in the joint venture was as follows: BP possessed 66.65%, and the Statoil held the remaining 33.35%. By the time Statoil has made significant progress in the international market by developing its own fields, it has increased its market power; the alliance has ceased operations in the distribution of activities they mentioned in the agreement. Changing forces of partners in this case had a significant impact on the stability of the joint venture.

Another direction of research of stability of joint ventures is associated with the study of external factors, such as changes in the external environment, unforeseen events, unfavorable economic situation, leading to deterioration in the financial results of the alliance (Yan, 1998). There was no comprehensive study of internal and external factors, despite the fact that in a situation of real business they must be taken into account.

In order to investigate the stability of existing joint ventures, static methods are used most commonly. The main approach is aimed at studying the results of the joint venture in its final stages. In the course of such a research factors that influenced the decision on cooperation of firms are analyzed. Special attention is given to the analysis of the literature on cases of JV instability and the factors affecting it. In this paper, the problem of the successful development of the joint venture is analyzed mainly in the long term starting with its implementation, which requires the study of the contractual relationship between companies (Jiang et al., 2008). Analysis of stability in the long term lets provide and reduce losses of each of the partner companies at the stage of the alliance formation.

Another obstacle in the field of stability in the JV research projects is the current lack of its rigorous conceptualization. For this reason, several problems arise:

– The stability of the joint ventures is understood differently in different works and may vary depending on the purpose of the research and the theoretical position of the scientist. This, in its turn, leads to contradictory assumptions, a wide variety of concepts, unconvincing arguments (De Rond and Bouchikhi, 2004);
– Scientists often do not distinguish between the concepts of stability and instability of a joint venture. For example, in the works devoted to the stability of the alliance scientists often go on to analyze the reverse phenomenon - the instability of the alliance - or a mix of these concepts;
– In many studies, stability of an alliance is regarded in its relation to a particular type of a joint venture, for example, international joint venture, which prevents generalization of the results to other forms of strategic alliances;
– Studying the stability (instability) is often not held by its evaluation, but only through the identification of significant factors of influence.

After the analysis of studies on the stability of joint ventures, it was possible to determine only one definition of stability and two definitions of the instability of the alliance (Table. 3).

As seen from the definitions above, all of them are quite abstract and difficult to operationalize in practice. Thus, in the definition of stability, proposed by Jiang, Li and Gao (2008) it is not clear what degree of effective relationships between partners this is and how it can be measured. According to Inkpen, Beamish (1997) and work by Das and Teng (2000) all joint ventures that do not meet the definitions of instability are stable. However, this argument is contentious.

In this paper the study of the dynamic stability of joint ventures of the alliance is seen as a prerequisite for achieving the best possible result of the joint venture. It seems that for the stability of a joint venture companies' motivation to cooperate during the whole period of existence must be maintained, while it should be noted

Table 3: Different interpretations of the term of strategic alliance.

Authors	Definition
Inkpen, Beamish, 1997, p. 182	Instability is defined as a major change in partner relationship status that is unplanned and premature from one or both partners' perspective
Das, Teng, 2000, p. 77	Alliance instabilities refer to major changes or dissolutions of alliances that are unplanned from the perspective of one or more partners
Jiang, Li, Gao, 2008, p. 178	Alliance stability is the degree to which an alliance can run and develop successfully based on an effective collaborative relationship shared by all partners

that motivation of the partners may be affected by various internal and external factors.

There are no specific differences in the phenomenon of stability of a joint venture and strategic alliance. In the view which was formed after the analysis, a stable strategic alliance is understood as the success of joint venture's activities during the implementation in a situation of continuous motivation of every partner company to achieve maximum results.

One of the fundamental theoretical approaches to the study of joint ventures is a resource concept (Oliver, 1990; Barringer and Harrison, 2000), aimed at assessing how the resources obtained during the signing of an agreement between the two companies help to reduce uncertainty and interdependence (Pfeffer and Salancik, 1978; Harrigan and Newman, 1990).

Methods of assessing the stability of joint ventures based on the concept of resource are widely represented in the scientific literature. The paper of 1980 by Provan, Beyer and Kruytbosch (1980) bears the idea that by forming a joint venture a company is increasing its market power, thereby reducing dependence on suppliers of resources through joint ventures. Later the correlation between the size of the company and gain from cooperation was proven – major partners get less from cooperative agreements, thus, the joint venture of the largest companies are less stable (Das et al., 1998).

At the beginning of the 21st century special attention was paid to international joint ventures in connection with the integration of resources in the world economy. International joint ventures were claimed to be stable when both partners were dependent on each other, but remained in strategic control of the company that has the greatest resources (Yan and Gray, 2001). The possibility of misappropriation of resources through the creation of inter-institutional relations and the formation of different security policies for small companies was analyzed in more recent studies (Katila et al., 2008).

The resource concept is often used in conjunction with the theory of networks (Gulati, 1995), as it applies the same approach to the assessment of partners' dependence, but pays more attention to the social context of relationships. In addition, it is used in conjunction with game theory and theories of organizational behavior to assess the strength of a partner (Saxton, 1997), with the theory of agency agreements to build the model of separation of control in the alliance (Kumar and Seth, 1998); theory of transaction to predict future action of each of the partners (Elg, 2000; Steensma et al., 2000). However, despite the wide applicability of the concept of a resource, there is a need for a more practical approach to evaluating the stability of joint ventures, which would allow a more accurate assessment, which could thereby adjust the strategic management of the company and reduce costs.

Other methods that are more theoretical in nature are based on an evaluation of external factors affecting the stability of joint ventures. Such factors include, for example, the structure of the alliance management, namely the division of administrative power (Dhanaraj and Beamish, 2004), and the credibility of each partner (Fryxell et al., 2002). Later works expand range of external factors affecting the stability by adding a degree of difficulty to achieve the objectives of the joint venture, the national peculiarities of partners, the experience of participation in the partner alliances, as well as a mismatch of expectations of partners (see, e.g., McCutchen et al., 2008). To test the hypothesis, an electronic survey of 490 employees of biopharmaceutical companies was conducted in 1998. The authors used two models of a binary logistic regression and polynomial regression; alliances were divided into two groups: those that have ceased to exist, and those that were underway or were completed successfully (the respondent had to estimate the values of the first three variables – 1 or 0, and the second set of variables – from 1 to 8). Due to the external factors mentioned, it is difficult to assess the perception of different employees because it differs for many reasons, often it is not related to the relationship between the two companies, therefore the results which are given by empirical study had no practical use and did not propose universal estimation procedure of stability of joint ventures.

Numerous studies show the influence of various internal or external factors on the stability of a joint venture, however, they do not offer a practical solution to overcome the instability. Among these factors there is the degree of information exchange between the partners (Stuart, 2000), trust of partners to each other (Deitz et al., 2010), the degree of involvement of partners in the activities of the joint venture (Meschi and Wassmer, 2013).

In the present paper while investigating dynamic development of joint ventures, stability of a JV alliance is regarded as a requirement for maximization of JV performance. It appears that in order to maintain stability of a JV, motivation to cooperate within the whole period of its existence should be preserved, while the motivation is affected by various internal and external factors.

3.2. External and Internal Stability of Joint Ventures

Analysis of existing approaches shows that in conditions of globalization and integration of resources there is an urgent requirement for creating methods to detect weaknesses of joint ventures and ways to address the evolving problems. In the long run, this allows companies to follow the revised strategic development plan and reduce operational costs. The model developed and introduced in the present paper includes assessment of external and internal stability of the JV, embracing analysis of all factors affecting it. Most significantly, the model is widely applicable and can be used in the evaluation of joint ventures from all industries. In our opinion, it is advisable not only to examine potential impact of specific factors on motivation of JV partners and, thereby, its stability, but also to introduce a comprehensive concept of stability which would let us take into consideration all basic factors that can affect the stability of the strategic cooperation between companies. Thus, stability is understood *as the success of the JV during its implementation period in terms of motivation of all partners to achieve the maximum results of the alliance activity.*

From the definition it follows that stability is a multiconceptional notion due to the fact that the nature of motivation of participants in various joint ventures is diverse and multicomponent. Therefore, the stability-related factors may have different character. The figure shows the structural configuration of the JV stability concept.

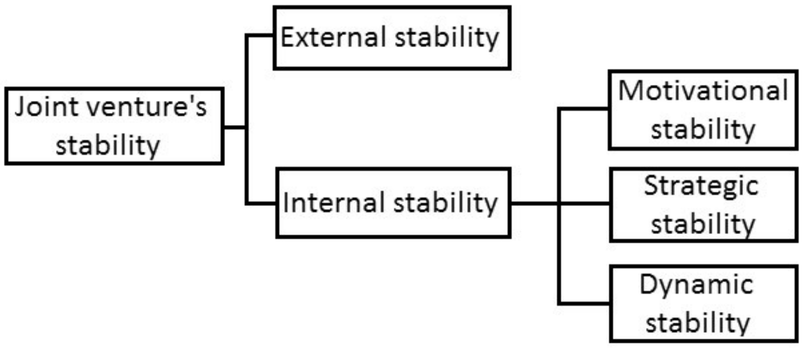

Fig. 1: Joint venture's stability concept

It should be noted that in the strategic management literature there is a variety of approaches to determining the success of alliances in general and joint ventures in particular. Some approaches allow for a qualitative assessment, others for a quantitative, based on economic performances of the JV.

Due to the fact that it is customary for companies to evaluate the achievements by means of economic indicators, let us compare the notion of the success of the joint venture and the concept of *external stability* of the joint venture thus wise: a joint venture has the external stability property, if its economic performances show an upward economic trend in the long term. Given a long term upward trend of a JV, the partners assume that their cooperation is successful and the alliance brings economic benefits. Thus, the partners keep their long-term motivation for cooperation being a tool for achieving the benefits. To illustrate the importance of the long-term economic trend influence let us provide a theoretical example. Supposing that some joint venture started operating in 2004, when its long-term trend of economic results was rising, however, along with the global financial crisis in 2008, the JV obtained negative economic outcomes that resulted in a short-term descending trend. Nonetheless, if all partners appraised the loss as temporary and in the long term saw an upward trend of economic benefits, they have then no motive to doubt rationality of their strategic partnership. If the situation caused by the crisis, did not allow to define the trend in economic outcomes as long-term positive, the advisability of cooperation may be questioned by the partners, leading to external instability of this joint venture. The concept of external stability provides an opportunity to assess stability of a JV as a separate economic entity. It helps to remember that the alliance was formed by the partners each being an economic entity and having its own economic interests.

Accordingly, the next step of assessing the stability of the JV should be consideration of the internal, or cooperative, stability. (Zenkevich et al., 2009b). It seems clear that internal stability is determined by numerous factors such as partner satisfaction, confidence in the choice of a partner, content with the distribution of benefits from the joint venture between the partners, etc. In the strategic management literature there is a number of studies (both theoretical and empirical) on the internal stability of a JV (e.g.: Dyer et al., 2008; Gill and Butler, 2003; Wong et al., 2005; Kumar, 2011). At the same time, in cooperative game theory strict definitions of the various types of internal stability are introduced; and the theoretical basis for their evaluation is developed in (Zenkevich et al., 2009b). Therefore it seems useful to combine the approaches of strategic management and the approach of game theory to the assessment of internal stability as an alternative path to the already existing directions in the theory of strategic management.

First, the stability of the motivation of participants to cooperate needs to be considered. So, one component of the internal stability of the joint venture will be its *motivational stability.* The success of cooperation between two or more institutions is influenced not only by internal and external economic and other factors, but also by interpersonal relationships of the alliance partners (Deitz et al., 2010). The motivation for the continuation of the joint activity is supported by corporate objectives as well as extent of partners' involvement in permanent cooperation. It was noted earlier that a number of factors, such as trust to each other and attention to intercultural differences between the partners, has an impact on the financial performance of the joint venture, namely its stability. In order to assess the internals stability of the JV, we must admit an assumption that the companies within the joint venture are rational: each of them tries to maximize its benefits. According to the definitions of a joint venture and a strategic alliance, JV is created in conditions where the results achieved by the partners are better than if they worked separately owing to the synergistic effect. Therefore, both partners should make each of the participants in the JV try to increase the total income of the cooperation by working in conditions of mutual understanding and trust. This approach is required to anticipate possible actions of partners, to maximize benefits from cooperation. However, it should be noted that there are not only positive benefits from collaboration, but also potential threats, such as loss of management control, control over technology and know-how. Consequently, the analysis of motivational stability is carried out to determine the motivation of participants of a joint venture to cooperate with regard to all possible circumstances. The authors also offer to consider other types of internal stability – strategic and dynamic, which are well conceptualized and examined in cooperative game theory (Zenkevich et al., 2009a). They allow considering important aspects of strategic cooperation within the alliance that affect its stability.

The importance of strategic stability can be illustrated by the following example. Given that several companies have created a joint venture, however, during the implementation phase one of the JV partners felt that the relationship with the selected partners does not bring the desired results, and that he could possibly achieve better results within a joint venture with other partners or as part of another type of interaction with companies. Nevertheless, the early exit from the joint venture inflicts a lot of costs. Even if one partner finds a better alternative to participation in this JV, they cannot always leave the alliance and realize it due to the fact that the benefits of other alternatives (taking into account the costs of early exit from the joint venture) may be less than the benefits that the partner derives from the cooperative agreement. *Strategic stability* suggests that none of the alliance partners finds it profitable to deviate from their cooperative agreement under conditions when all the partners stick to it. Starting the joint venture; each of the partners makes a strategic choice in favor of an alliance. During the implementation period of the JV partners may doubt the choice and, consequently, begin to consider the decision to withdraw from the JV.

To assess the stability associated with the distribution of gains from cooperation within a joint venture, it is necessary to consider the concept of dynamic stability, introduced in game theory. The term dynamic stability was first introduced in the academic literature by Leon Petrosjan (1977). According to this concept, when creating the JV partners imagine types and quantity of benefits they will receive at the time of concluding the cooperation agreement of sharing the benefits of the JV. *Dynamic stability* implies that when considering one partner in a moment of realization of a joint venture, the benefits he has already received in the framework of cooperation in conjunction with the benefits he still expects to receive prior to the expiration of the agreement shall not be less than the benefits he expected to obtain at the stage of signing the cooperation agreement. Thus, dynamic stability implies that the above-mentioned character of benefits will be fair for each of the partners in any review period of the JV implementation. This conceptualization of dynamic stability seems logical. Because if the partner finds that the benefits he will receive by the end of the existence of the joint venture will be less than expected, his motivation to participate in the JV may be severely reduced or even disappear. Conversely, if at any moment of the implementation of the JV a participant understands that they will reach the benefits initially planned by the end of the alliance existence, they will not have desire to withdraw from the agreement.

The proposed concept of stability of a strategic alliance enables assessment of cooperative agreements concerning presence of each component of the stability.

In the next chapter the stability of Renault–Nissan strategic alliance will be analyzed according to the introduced concept of stability.

4. Stability Analysis of the Renault–Nissan Joint Venture

We illustrate the above-described concept of the stability by an example of Renault and Nissan strategic alliance in a joint venture form.

4.1. Renault–Nissan Strategic Alliance

Strategic French–Japanese partnership in the field of motor industry between the French Renault and the Japanese Nissan came into force on the 27th of March, 1999. Implementation phase of the alliance began in 2003, when the partners moved on to joint activities and a JV was created. Thus, data between 2003 and 2012 is analyzed in the case; that allows us to assess the stability of this strategic alliance.

The beginning of the XXI century is characterized by fast pace of the motor industry development, which led to numerous agreements, amalgamations and takeovers. Thus, in 1999 Renault acquired a 36.8% stake in Nissan, and two years later increased the stake up to 44.4%. In its turn, the Japanese company bought a 15% stake in Renault in 2001. The established ownership structure was stable and survived to the present day. Renault–Nissan BV (RNBV) alliance located its head office in Amsterdam (the Netherlands), was established on a parity basis for the exchange of ideas and technologies, as well as for synergy effects maximization.

The alliance structure is presented in Figure 2.

Considering that there was an exchange of shares, it is clear that both companies are interested not only in the success of the jointly created enterprise, but also in the further development of a partner company.

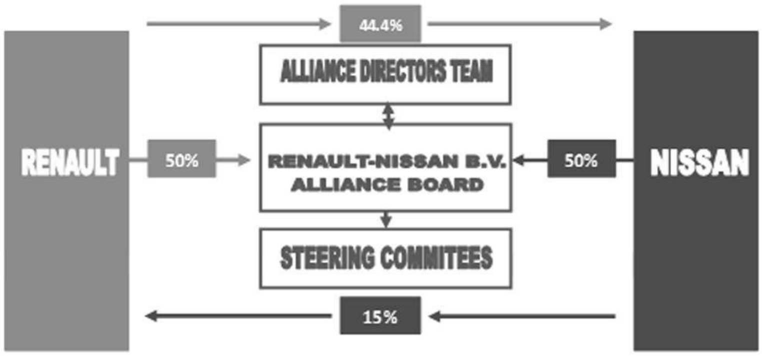

Fig. 2: The Renault–Nissan alliance structure

The aim of the alliance is to achieve a scale effect without sacrificing the uniqueness of each partner that is ensured by joint development of engines, batteries and other components. For example, the increase of Nissan market share in commercial vehicles segment resulted from adoption of elements of Renault vans such as Renault Kangoo/Nissan Kubistar, Renault Master/Nissan Interstar, Renault Trafic/Nissan Primastar. In addition, Renault is the engines constructor of almost all Nissan diesel engines in Europe. Cooperation between Renault and Nissan also focuses on capital-intensive research projects (transport with zero gas emission and the increase of sales in emerging markets such as Brazil, Russia and India). Moreover, partners reduce suppliers and logistics related expenses owing to the fact that procurements and supply chains are formed simultaneously for two companies. Product output of each partner in the region with no factory is carried out by the other partner's manufacturing division that also allows to maximize effect from economy of scale. In general, the alliance estimated cost reduction by means of synergy at 1.5 billion € in 2010.

Ten years after signing of the initial agreement, in 2009, under the strategy of Renault and Nissan partnership strengthening a separate group of experts from both companies was created. The role of the group was to promote closer cooperation in 16 different fields, including research and advanced engineering research, parts and accessories production as well as marketing. In 2012, sales of the alliance reached a record of 8.1 million units worldwide (a 1%-increase compared to the previous year), thereby continuing to grow for the fourth year running.

4.2. Prerequisites of a Strategic Alliance Establishment

By the mid-1990s Nissan has been experiencing significant financial difficulties, so its controlling stake was sold to Renault. Following the signing of the agreement, Carlos Ghosn assumed the position of President and CEO of Nissan. The set of reforms proposed by the new President and designed to save the stuck in debt company obtained the title "Nissan Revival Plan – NRP". It included job cuts, shutdowns of factories, reduction in the number platforms, investments in new technologies and creation of an efficient production system.

Traditionally, Nissan had stronger positions in the US and Japan, while Renault operation was more efficient in Russia, France and Brazil. Renault sales were mostly in Europe, in 1998 the company did not sell a single car in the USA and sold 2476 units in Japan. At the same time, both companies announced their intention to carry out joint development and production to increase their share in emerging markets such as China, India and Brazil. Thus, one of the goals of the JV was territorial expansion.

Due to the fact that the new technologies development was conducted jointly, both partners maximized synergy effects in the field of capital-intensive projects. For example, it was announced in 2013 that the company sold the 100.000th electric vehicle, and the Nissan Leaf has become the most popular electric car in the world. Additionally, Renault and Nissan agreed on joint control of the Russian "AVTOVAZ", which could be very difficult for each company on a stand-alone basis. Reduction of expenses and achieving maximum synergies in this area was also one of the goals of the alliance.

4.3. Stability Assessment of the Renault–Nissan Joint Venture

External Stability. According to the accepted definition, for an external stability assessment of the Renault–Nissan JV it is necessary to analyze whether the economic performance trend of the alliance is positive in the long run.

Figure 3 presents information about the economic performance of the alliance in the period from 2004 to 2011 in the form of a graph and an economic performance smooth curve with an interval of three years.

Let us consider trend lines presented in Figure 4, for the smooth curve of the strategic Alliance economic performance.

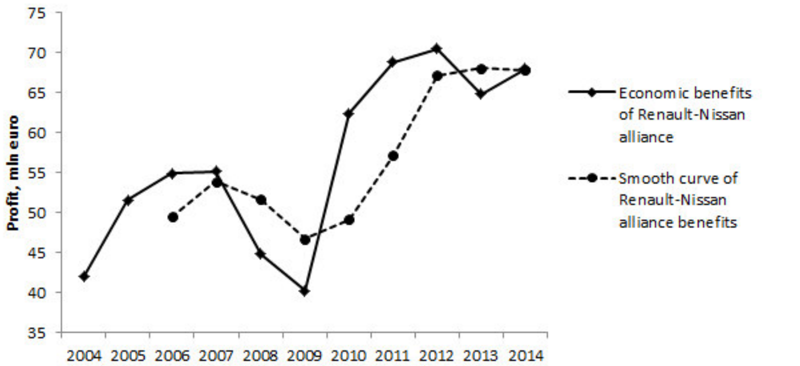

Fig. 3: Economic performance of the Renault–Nissan JV and its smooth curve

Figure 4 shows that trend lines 1 and 2 of the alliance economic performance are descending because of the economic crisis of 2008 and 2009. In 2006-2007 short-term instability was observed. However, this period did not last more than two years. As shown in Figure 3, in 2008 trend line 3 becomes upward. Renault–Nissan successfully overcame the crisis. Trend lines 4 and 5 are rising. It is evident that a long-term trend represented by line 6 is upward as well.

Despite a small fluctuation in the economic performance trend of the JV, the company hoped that the difficulties they had to face were temporary; nevertheless,

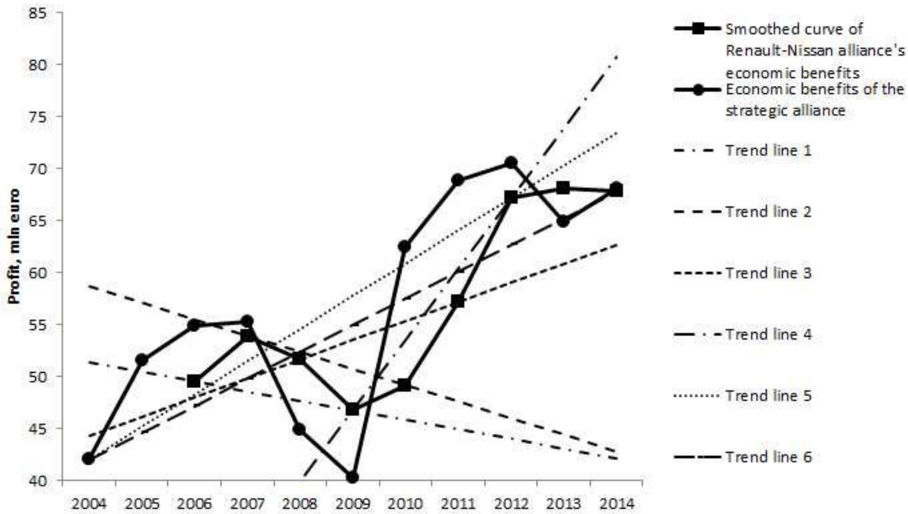

Fig. 4: Trend lines and economic performance smooth curve of the Renault–Nissan JV

the growth of economic results of the alliance is expected in long-term perspective. Thus, it can be argued that Renault–Nissan joint venture has the property of external stability.

Internal Stability. First, we should analyze motivational strength of the partners. Benefits and risks for the partners from participation in the alliance are presented in Table 4.

Table 4: Benefits and risks for Renault and Nissan in the alliance*.

	Benefits	Risks
Renault	Access to the markets of North America and Asia.	Loss of management control.
	Engineering solutions in the motor industry.	Cross-cultural conflicts.
	Nissan reputation in SUVs and pickup market.	Nissan debts.
Nissan	Access to the European market.	Loss of management control.
	Financial and managerial support.	Takeover threat.
	Skills in design and production, marketing and design. The expansion of the model range.	Cross-cultural conflicts.

Ranking of possible outcomes for Renault and Nissan according to their preference on the basis of benefits and risks comparison for both partners depending on the selected strategies combination was conducted by the expert consultant with work experience in M&A and Due Diligence for enterprises and an employee of the Product Management Department of Adam Opel AG.

The expert analysis results were processed using the instrument DSSS ASPID-3W (Table 5).

*Sources: authors' analysis based on information in Nissan Company Corporate Website, Renault Company Corporate Website.

Table 5: Outcome ranking in Renault and Nissan motivational game in the alliance.

Situation in a game	$(A;\ A)$	$(A;\ P)$	$(P;\ A)$	$(P;\ P)$
Renault outcome ranking	4	1.7	2.7	1.7
Nissan outcome ranking	4	2	1.7	2.3

Table 5 generates a motivational game set in Table 6. In this game the only strong equilibrium realized is in the situation $(A;\ A)$, which indicates a motivational stability of the Renault and Nissan companies JV.

Renault had to enter the markets of North America and Japan, which were the largest consumers of car manufacturers. Despite the fact that the French company owned a rich set of competencies including advanced knowledge in the field of marketing and design, for the further successful development of the company new engineering solutions and production skills were required. Apart from this, Nissan had stronger positions in a pickup and SUV market resulting from a wide representation of the products.

Table 6: Renault-Nissan alliance motivational game.

Nissan strategies / **Renault strategies**	Participation	Exit
Active position	$\boxed{(4;\ 4)}$	(1.7; 2)
Passive position	(2.7; 1.7)	(1.7; 2.3)

Each JV has a threat of loss of its management control, which appears in case of a passive strategy of behavior in the alliance. Although the stage of signing the agreement and preparation for its implementation lasted for nearly four years, for both partners there was a threat of cross-cultural conflicts associated with the special identity of the Japanese culture. In addition, Renault had to cope with debt obligations of Nissan, which entered its area of responsibility. This means that the failure of the "Nissan Revival Plan" could undermine its economic position.

Regardless of strong and stable Nissan positions in Japan and the US, the European market remained its weakness. As a result, the alliance with the French Renault pursued the goal of entering a new market where the partner had already had an influence. Along with the acquisition of numerous skills, for instance, in the field of development and production, marketing and design, Nissan received financial support and a new CEO who was able to save the company from a complete bankruptcy. Hence, there was a takeover threat related to the fact that Nissan was materially and morally bound by obligations with Renault. Model range expansion was gradual and did not give significant advantages to any of the companies specifically; however, it should be noted that partially the success of Nissan is stipulated by the use of body models of Renault vans.

Thus, both companies were encouraged to choose an active strategy and to participate in the work of the JV; that indicates motivational resistance, which was preserved throughout the time of the alliance implementation.

In 1998, when Renault and Nissan signed an agreement, both of them needed a strategic partner for its development. Since the stage of implementation of the

Alliance started 11 years ago, we may perform a retrospective analysis. Renault needed to enter the markets of North America and Asia in order to maintain its competitiveness in the long term. Alliances with American companies such as Ford, General Motors and Daimler-Chrysler seemed unlikely due to the fact that these companies were interested in mergers and takeovers. It was important to apply to companies widely represented in the Asian market that led to a partner search in Japan. However, Japanese companies Subaru and Isuzu were under the influence of General Motors, and Mazda - under Ford. The most suitable partner was Nissan, whose strategic plans in geographical expansion corresponded with those of Renault.

In its turn, competencies of the companies were of mutual necessity. Production optimization skills and engineering solutions of Nissan in combination with design solutions and marketing strategy of Renault provided a competitive advantage that the companies could not have obtained on their own. It should be noted that the companies made a share exchange (44,4% of the Japanese company was given to Renault, 15% of the French company passed to Nissan), which means closer cooperation between the partners. Total investments of Renault and Nissan to the alliance reached $2.1 billion. Besides, at present time Renault and Nissan have a common supply chain, and some models in different countries are manufactured in partner factories. Therefore, even if there is a more attractive alternative at the market, costs of exiting the alliance will be rather high. Let us proceed to the formalization of the game according to the above-described methodology of strategic stability assessing.

Similar to motivational stability, ranking of possible outcomes for Renault and Nissan according to their preference on the basis of benefits and risks comparison for both partners depending on the selected strategies combination was conducted by the expert consultant with work experience in M&A and Due Diligence for enterprises and an employee of the Product Management Department of Adam Opel AG. The rankings of the strategic outcomes of the conflict are given in Table 7.

Table 7: Renault and Nissan JV strategic conflict outcome rankings.

Situation in a game	$(P;\ P)$	$(P;\ E)$	$(E;\ P)$	$(E;\ E)$
Renault outcome ranking	4	1.7	2.7	1.7
Nissan outcome ranking	4	1.3	2.3	2.3

Table 7 gives rise to a strategic game represented in Table 8, in which the only strong equilibrium is realized in the situation $(P;\ P)$, which proves the stability of the Renault–Nissan strategic alliance.

Table 8: Renault-Nissan alliance strategic game.

Nissan strategies / **Renault strategies**	Participation	Exit
Participation	$\boxed{(4;\ 4)}$	(1.7; 1.3)
Exit	(2.7; 2.3)	(1.7; 2.3)

Renault–Nissan JV dynamic stability. Renault and Nissan not only established a joint venture in the framework of strategic cooperation, but also made mutual exchange of shares. Therefore, Renault and Nissan cooperation brings benefits in the form of JV profits and the joint cost of the companies themselves. As long as the partners are joint owners of each other, dividend payments must be considered as the benefits of cooperation as well.

Eventually, to estimate the total annual economic benefits of the Renault–Nissan alliance, we use the following formula:

$$Profit_{alliance} + ShEq_{R} + ShEq_{N} + Div_{R} + Div_{N},$$

where $Profit_{alliance}$ is a net profit of the Renault–Nissan alliance in the form of JV for a year; $ShEq_{R}$ – capitalization of Renault (assessed through equity share); $ShEq_{N}$ – capitalization of Nissan in the current year (assessed through equity share); Div_{R} – dividends paid by Renault in the current year; Div_{N} – dividends paid by Nissan in the current year. Total economic benefits of the Alliance in its implementation period from 2004 to 2014 are presented in Table 9.

Table 9: Renault and Nissan economic benefits, billion € *.

Year	JV net profit	Capitalization of Renault	Capitalization of Nissan	Renault dividends	Nissan Dividends	Total economic benefit
2004	5.26	15.86	18.40	1.80	0.70	42.03
2005	6.36	19.49	22.57	2.40	0.77	51.60
2006	5.33	21.07	24.55	3.10	0.90	54.95
2007	4.39	22.07	24.00	3.80	0.94	55.20
2008	1.25	19.42	23.35	0.00	0.83	44.85
2009	-4.07	16.47	27.81	0.00	0.00	40.21
2010	5.02	22.76	34.18	0.30	0.18	62.43
2011	4.10	24.57	38.47	1.16	0.57	68.85
2012	3.68	24.55	39.74	1.72	0.83	70.52
2013	2.82	23.21	36.22	1.72	0.87	64.84
2014	4.15	24.90	36.14	1.90	0.94	68.04

The benefits of participation in the alliance for each company are measured using the same components as for the benefits of the entire alliance; still, the share of mutual participation of partners in the capital stock of each other is also taken into account. Therefore, the economic benefits of Renault can be assessed by the following formula:

$$Profit_{renault} + 0,85ShEq_{R} + 0,444ShEq_{N} + 0,85Div_{R} + 0,444Div_{N},$$

where $Profit_{renault}$ is the Renault profit from participation in a JV with Nissan, $0,85ShEq_{R}$ – Renault benefits from the management of 85% of its own stake; $0,444ShEq_{N}$ – Renault benefits from possession of 44.4% stake in Nissan; $0,85Div_{R}$ – the dividends of Renault shareholders; $0,444Div_{N}$ – Renault dividends on 44.4% of Nissan's shares.

* Sources: Compiled information from Nissan Company Corporate Website, Renault Company Corporate Website and the annual reports of the companies from these websites.

Calculation of economic benefits from participation in the alliance for Nissan is performed similarly according to the formula

$$Profit_{nissan} + 0,15ShEq_{R} + 0,5596ShEq_{N} + 0,15Div_{R} + 0,5596Div_{N}.$$

Profit and economic benefits of Renault and Nissan in each year from 2004 to 2012 are shown in Table 10.

Table 10: Economic benefits of Renault–Nissan alliance, billion € *.

Year	Renault profit in JV	Nissan profit in JV	Renault economic benefit	Nissan economic benefit
2004	1.35	3.91	24.78	17.25
2005	1.18	5.19	30.07	21.53
2006	1.07	4.26	32.82	22.12
2007	1.45	2.95	34.41	20.78
2008	0.25	1.00	27.40	17.44
2009	-2.17	-1.91	24.08	16.13
2010	2.41	2.61	37.13	25.30
2011	0.81	3.29	39.86	28.99
2012	0.50	3.18	40.54	29.64
2013	-0.80	3.62	36.72	28.11
2014	0.44	3.71	39.55	28.48

Table 11 summarizes the results of the previous calculations of the distribution of benefits from cooperation (tables 9 and 10), and indicates the share of each company from the common benefits of the alliance for each year.

Table 11: Distribution of profits from participation in the alliance, billion €*.

Year	Renault economic benefits	Nissan economic benefits	Total economic benefits of the alliance	Renault share	Nissan share
2004	24.78	17.25	42.03	0.59	0.41
2005	30.06	21.53	51.60	0.58	0.42
2006	32.82	22.12	54.95	0.59	0.40
2007	34.41	20.78	55.20	0.62	0.38
2008	27.40	17.44	44.85	0.61	0.39
2009	24.08	16.13	40.21	0.60	0.40
2010	37.13	25.30	62.43	0.59	0.40
2011	39.86	28.99	68.85	0.58	0.42
2012	40.54	29.64	70.18	0.58	0.42
2013	36.72	28.11	64.83	0.56	0.43
2014	39.55	28.48	68.03	0.58	0.42

Distribution of shares presented in Table 11 is also illustrated in Figure 5.

* Sources: Compiled information from Nissan Company Corporate Website, Renault Company Corporate Website and the annual reports of the companies from these websites as well as authors' calculations.

Figure 5 shows that distribution of total benefits from cooperation within the framework of the strategic alliance was structurally stable: 58,96% for Renault and 41,04% for Nissan. There are small annual fluctuations, standard deviation from the specified distribution of which is $\sigma = 0.02\%$. Therefore, Renault–Nissan joint venture is dynamically stable, that reaffirms its general resistance as it possesses all types of stability – external and internal (motivational, strategic and dynamic).

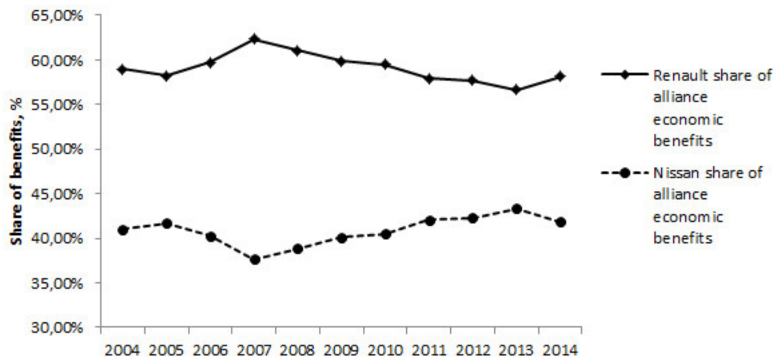

Fig. 5: Economic benefits of Renault and Nissan from participation in the alliance

5. Conclusion

In the first section of the article an overview of the literature and the evolution of the concept of strategic alliance and joint venture is provided; main tendencies of development of the given forms of cooperation of companies are revealed. We suggest author's definitions of the strategic alliance and joint venture, based on the analysis of the results of existing research, and meeting the requirements of the study; their classification is introduced and evolution of theories of strategic alliances and joint ventures is traced. The second section of the article describes the basic concepts of stability of strategic alliances and joint ventures, identifies their advantages and disadvantages. On the basis of the existing research we describe the factors affecting the stability of cooperative agreements. In addition, it is shown that modern stability theories cannot fully cover all possible factors and conditions peculiar to joint ventures, and address the JV stability problem from only one position that is of interest to researchers.

In addition, most existing approaches to the assessment of stability can hardly be called applicable. The reasons for this are various: from the inability to quantify the stability of the alliance to the problems with clear allocation of factors and criteria affecting the JV stability. In this regard, the authors propose their own approach to the definition of stability of a joint venture. The introduced concept of stability is a kind of synthesis of modern approaches to the alliance stability. Unlike all previous models, it does not consider the specific nature of motifs that influence creation or dissolution of an alliance. In the framework of this concept the existence of motifs for following or deviation from the cooperative path suffices. Thus, the author's concept of JV stability covers all previously listed theories and approaches to the definition of strategic alliance stability. Besides, it should be noted that this concept is a result of the integration of the strategic management theory and mechanisms of dynamic games theory. The advantages of this approach are obvious: it combines both the accuracy of game-theoretic analysis and its application to a variety of factors (determining the nature of motivation of the joint venture participants), the plenty of which can be found in the strategic management theory.

Finally, it was demonstrated that the stability concept can be applied to assessing the stability of existing ventures and predicting the stability of a potential JV, which helps to formulate recommendations and requirements of the agreement on formation of an alliance. The results of the research can serve as a theoretical base for future research in the field of stability analysis of strategic alliances.

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Strategic Support of the Shapley Value in Stochastic Games*

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Abstract We consider the cooperative behavior in stochastic games. We assume that players cooperate in the game and agree on realizing the Shapley value as an imputation of their total payoff. The problem of subgame (time) consistency of the Shapley value is examined. The imputation distribution procedure is constructed to make the Shapley value subgame consistent. We redefine the payoffs in stochastic game applying the imputation distribution procedure. The problem of strategic support of the Shapley value is examined. We prove that the cooperative strategy profile is the Nash equilibrium in the stochastic game with re-defined payoff functions when some conditions are satisfied. The theoretical results are demonstrated on the example of a data transmission game for a wireless network of a specific topology.

Keywords: cooperative stochastic game, time consistency, subgame consistency, imputation distribution procedure, strategic support

1. Introduction

We consider the class of stochastic games with discounted payoffs when players use stationary strategies. This class of games for two players was introduced by (Shapley, 1953a). Most papers devoted to stochastic games examine the non-cooperative behavior of the players, e. g. see (Herings and Peeters, 2004), (Jáskiewicz and Nowak, 2015), (Rosenberg et al., 2003). The cooperative model of a stochastic game was initially proposed by (Petrosjan, 2006). He investigated the problem of subgame consistency of cooperative solutions in a stochastic game played over a finite tree. The same problem was examined for discounted stochastic games when the set of states is finite and players use stationary strategies in (Petrosjan and Baranova, 2006). In this paper the method of finding a cooperative solution and verifying if the solution satisfies the principle of subgame (time) consistency. The problem of time consistency of the cooperative solution was proposed by Petrosyan in (Petrosyan, 1977). He proposed to modify the payment mechanism along cooperative trajectory of the initial game and introduced the IDP (imputation distribution procedure) to make the cooperative solution time-consistent (Petrosyan and Danilov, 1979). This idea was realised for the class of differential games but it is actual for stochastic games as well.

Two other principles of stable cooperation in dynamic games were formulated in (Petrosyan and Zenkevich, 2009) including the principle of strategic support of a cooperative solution. If the cooperative solution is strategically supported, then

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there exists the Nash equilibrium in trigger strategies with the players' payoffs equal to their payoffs in cooperation. The trigger strategy punishes the deviating player by allowing him to obtain the maxmin value in a subgame starting from the stage following the stage when the deviation has been observed (Petrosyan, 2008). The problem of strategic support is considered in (Parilina and Zaccour, 2015b) where the subgame perfect ε-equilibrium is constructed for the dynamic games played over event trees[1]. Another principle of stable cooperation is irrational-behaviour-proof (Yeung, 2006). It allows players to guarantee that their payoffs in cooperation will be not less than the payoffs when the cooperation breaks down at some stage and then players proceed playing the game as singletons. These three principles of stable cooperation were adopted to the discounted stochastic games (Parilina, 2015). Recently, the existence of cooperative solutions (Harsanyi, Shapley, Nash solutions) for discounted stochastic games was proved in (Kohlberg and Neyman, 2015).

In our paper we focus on two problems of cooperation in stochastic games: subgame consistency and strategic support. We prove the theorem which allows to construct the imputation distribution procedure to make the Shapley value subgame consistent. Then we define the behavior strategy profile in trigger strategies to support cooperation in case when some player deviates from the cooperative strategy profile. We need to mention here that initially we find the cooperative solution assuming that players use stationary strategies, but the construction of the trigger strategies requires considering the class of behavior strategies. Behavior strategies allow to observe the player's deviation and switch to a trigger mode of the trigger strategy.

As an example of a stochastic game we examine the problem of data transmission in the simple wireless network. The simple network of data transmission consisting of three nodes is taken as a basis of network topology. Two of the nodes generate data packages in each time slot with the corresponding probabilities. The third node is the destination one. The first two nodes are connected by a channel, the connection is one-way, i.e. the first node (first player) can transmit a package directly to node 3 or to node 2. For the transmission of a package to node 2 node 1 receives a nonnegative reward. The system of rewards and costs makes it possible to support cooperation between nodes 1 and 2 which are players 1 and 2 in the game, respectively. The described situation can be solved as a cooperative stochastic game.

Modeling data transmission as a stochastic game was introduced in (Altman et al., 2003, Parilina, 2010, Sagduyu and Ephremides, 2006). The game theory models of the behaviour in ad hoc wireless networks with emphasis on the development of cooperation mechanisms to stimulate package forwarding are considered in (Michiardi and Molva, 2003). Game theoretical models are useful for modeling the data transmission not only in ad hoc but also in CSMA networks (Benslama et al., 2013). The problem of constructing and analyzing the simple mechanism to stimulate the nodes for package forwarding is investigated in (Buttyan and Hubaux, 2003).

The rest of the paper is organized as follows: Section 2 describes the model of non-cooperative stochastic game, and Section 3 deals with the construction of the cooperative version of the stochastic game. Section 4 contains the description of the subgame consistency problem and the method to make the cooperative solu-

[1] The details of the specification of a game played over event trees may be found in (Haurie et al., 2012)

tion subgame-consistent. The idea of strategic support of a cooperative solution is investigated in Section 5. We provide an illustrative example of data transmission in wireless network in Section 6, and briefly conclude in Section 7.

2. Non-cooperative stochastic game

We consider a stochastic game with finite number of states when player's payoff is a discounted sum of the stage payoffs which players obtain along the realized trajectory (sequence of the realized action profiles). The game begins with the chance turn, i.e. with the choice of the initial state of the game which the game process begins with. The state of the stochastic game is determined as a normal form game of n players. One of the finite number of states is realized at each stage of the stochastic game. In the state some action profile is realized depending on the transition probabilities.

Definition 1. Let stochastic game G be determined by the set

$$\left(N, \left\{\Gamma^j\right\}_{j=1}^t, \delta, \pi^0, \left\{p(j,k;x^j)\right\}_{j=\overline{1,t}, k=\overline{1,t}, x^j \in \prod\limits_{i=1}^n X_i^j}\right), \tag{1}$$

where

- $N = \{1, \ldots, n\}$ is the set of players.
- $\Gamma^j = \langle N, X_1^j, \ldots, X_n^j, K_1^j, \ldots, K_n^j \rangle$ is a non-cooperative normal form game which defines the state j, $j = 1, \ldots, t$, X_i^j is the finite set of pure actions of player i in Γ^j, $K_i^j \left(x_1^j, \ldots, x_n^j\right) = K_i^j(x^j)$ is a payoff function of player i in state Γ^j, $j = 1, \ldots, t$.
- $p(j,k;x^j)$ is the probability that state Γ^k is realized if at the previous stage (in state Γ^j) action profile $x^j = (x_1^j, \ldots, x_n^j)$ has been realized, $p(j,k;x^j) \geqslant 0$ and $\sum_{k=1}^t p(j,k;x^j) = 1$ for each $x^j \in X^j = \prod\limits_{i \in N} X_i^j$ and for any $j, k = 1, \ldots, t$.
- $\delta \in (0,1)$ is the discount factor.
- $\pi^0 = (\pi_1^0, \ldots, \pi_t^0)$ is the vector of the initial distribution on states $\Gamma^1, \ldots, \Gamma^t$ where π_j^0 is the probability that state Γ^j is realized at the first stage of the game, $\sum_{j=1}^t \pi_j^0 = 1$.

For constructing the cooperative model of a stochastic game we need to define its subgame and the class of strategies which players use in the game.

Definition 2. Stochastic game (1) with vector $\pi^0 = (0, \ldots, 0, 1, 0, \ldots, 0)$ (the j-th component is equal to 1), i.e. game beginning from state Γ^j, is called a stochastic subgame G^j, $j = 1, \ldots, t$.

We assume that players realise stationary strategies in the game. Let $\Xi_i = \{\eta_i\}$ be the set of stationary strategies of player $i \in N$ in game G. Using stationary strategies a player chooses an action in each state from $\left\{\Gamma^1, \ldots, \Gamma^t\right\}$ depending only on which state is realized at this stage, i.e. $\eta_i : \Gamma^j \longmapsto x_i^j \in X_i^j, j = 1, \ldots, t$. Considering stochastic game in stationary strategies, and taking into account that the set of states is finite and the game has an infinite horizon, there are a finite number of subgames of game G. The number of subgames equals the number of the states.

Remark 1. Obviously, the stationary strategy of player i in game G is also a stationary strategy of the player in subgames $G^1, \dots, G^t$.

The payoff to the player in game G is a random variable. Consider the mathematical expectation of the player's payoff as his payoff in game G. Let $\bar{E}_i(\eta)$ be the expected payoff of player i in game G and $E_i^j(\eta)$ be the expected payoff of player i in subgame G^j when strategy profile η is realised in game G (subgame G^j). Let $E_i(\eta)$ be the vector $(E_i^1(\eta), \dots, E_i^t(\eta))'$.

The expected payoff to player i in subgame G^j satisfies the following recurrent equation:

$$E_i^j(\eta) = K_i^j(x^j) + \delta \sum_{k=1}^{t} p(j,k;x^j) E_i^k(\eta) \tag{2}$$

s. t. $\eta(\Gamma^j) = x^j$, i. e. $\eta(\cdot) = (\eta_1(\cdot), \dots, \eta_n(\cdot))$ where $\eta_i(\Gamma^j) = x_i^j \in X_i^j$, $x^j = (x_1^j, \dots, x_n^j)$ for any $j = 1, \dots, t$, $i \in N$. Hereinafter, let $\eta(\cdot) = (\eta_1(\cdot), \dots, \eta_n(\cdot))$ be the stationary strategy profile such as $\eta_i(\Gamma^j) = x_i^j \in X_i^j$ where $j = 1, \dots, t$, $i \in N$.

The transition matrix of stochastic game G when stationary strategy profile $\eta(\cdot)$ is realised is:

$$\Pi(\eta) = \begin{pmatrix} p(1,1;x^1) & \dots & p(1,t;x^1) \\ p(2,1;x^2) & \dots & p(2,t;x^2) \\ \dots & \dots & \dots \\ p(t,1;x^t) & \dots & p(t,t;x^t) \end{pmatrix}. \tag{3}$$

We can rewrite equation (2) in a matrix form using matrix (3) in the following way:

$$E_i(\eta) = K_i(x) + \delta \Pi(\eta) E_i(\eta), \tag{4}$$

where $K_i(x) = (K_i^1(x^1), \dots, K_i^t(x^t))$, and $K_i^j(x^j)$ is the payoff to player i in state Γ^j when action profile $x^j \in X^j$ is realized in this state.

Equation (4) is equivalent to the equation[2]:

$$E_i(\eta) = (\mathbb{I}_t - \delta \Pi(\eta))^{-1} K_i(x), \tag{5}$$

where $\mathbb{I}_t$ is an identity $t \times t$ matrix.

The expected payoff to player i in game G is calculated by formula:

$$\bar{E}_i(\eta) = \pi^0 E_i(\eta). \tag{6}$$

3. Cooperative stochastic game

Suppose that the players from the grand coalition N decide to cooperate and receive the maximal total payoff. Denote the strategy profile maximizing the sum of the expected players' payoffs in game G as $\bar{\eta}(\cdot) = (\bar{\eta}_1(\cdot), \dots, \bar{\eta}_n(\cdot))$:

$$\max_{\eta \in \prod_{i\in N} \Xi_i} \sum_{i \in N} \bar{E}_i(\eta) = \sum_{i \in N} \bar{E}_i(\bar{\eta}). \tag{7}$$

[2] Matrix $(\mathbb{I}_t - \delta \Pi(\eta))^{-1}$ always exists for $\delta \in (0,1)$. The proof follows. It is known that all the eigenvalues of stochastic matrix $\Pi(\eta)$ are in the interval $[-1, 1]$. For the existence of matrix $(\mathbb{I}_t - \delta \Pi(\eta))^{-1}$ it is necessary and sufficient that the determinant of matrix $(\Pi(\eta) - \frac{1}{\delta}\mathbb{I}_t)$ be not equal to zero. Thus matrix $(\Pi(\eta) - \frac{1}{\delta}\mathbb{I}_t)$ must not have the eigenvalue to be equal to $\frac{1}{\delta}$. The last condition takes place because $\frac{1}{\delta} > 1$, so this number cannot be the eigenvalue of stochastic matrix $\Pi(\eta)$.

Call the strategy profile $\bar{\eta}(\cdot)$ cooperative strategy profile.

The cooperative model of a non-cooperative game G is given by set (N, v), where N is the set of players and v is a real-valued function, called the characteristic function of the game, defined on the set 2^N (the set of all subsets of N), and satisfying the property: $v(\emptyset) = 0$. The value $v(S)$ is a real number which is assigned to coalition $S \subset N$, and may be interpreted as the worth or power of coalition S. The members of coalition S play together as a unit.

Define the characteristic function $\bar{v}(S)$ in stochastic game G using characteristic functions $v^j(S)$ of stochastic subgames G^j, $j = 1, \ldots, t$, as follows:

$$\bar{v}(S) = \pi^0 v(S) \tag{8}$$

for any coalition $S \subset N$ where $v(S) = (v^1(S), \ldots, v^t(S))'$. And $v^j(S)$ is the value of the characteristic function for subgame G^j calculated for coalition S. Now the problem is to define the characteristic function $v^j(S)$ for any coalition S. We use α-approach to define the characteristic function. According to this approach the value of characteristic function for coalition S is equal to the maximal total payoff of coalition S which this coalition can guarantee when the left-out players cooperate and minimize total payoff of coalition S.

First, consider coalition $S = N$. Bellman equation for $v(N)$ is:

$$v(N) = \max_{\eta \in \prod\limits_{i \in N} \Xi_i} \left[\sum_{i \in N} K_i(x) + \delta \Pi(\eta) v(N) \right] = \sum_{i \in N} K_i(\bar{x}) + \delta \Pi(\bar{\eta}) v(N),$$

where $\bar{\eta}(\cdot)$ is the cooperative strategy profile.

Therefore, the value $v(N)$ is:

$$v(N) = (\mathbb{I}_t - \delta \Pi(\bar{\eta}))^{-1} \sum_{i \in N} K_i(\bar{x}). \tag{9}$$

Second, consider coalition $S \subset N$, $S \neq \emptyset$. To define the value of characteristic function $v^j(S)$, $j = 1, \ldots, t$, for each subgame G^j, we consider a zero-sum stochastic game G^j_S with two players (coalitions S and $N \backslash S$) where coalition $S \subset N$ plays as a maximizing player and coalition $N \backslash S$ plays as a minimizing player. Define the value $v^j(S)$ for subgame G^j as a maxmin of the payoff of coalition S in stochastic game G^j_S (in fact, the lower value of matrix game):

$$v^j(S) = \max_{\eta_S} \min_{\eta_{N \backslash S}} \sum_{i \in S} E^j_i(\eta_S, \eta_{N \backslash S}), \tag{10}$$

where $(\eta_S(\cdot), \eta_{N \backslash S}(\cdot))$ is a stationary strategy profile such that $\eta_S(\cdot) = (\eta_{i_1}(\cdot), \ldots, \eta_{i_k}(\cdot))$ is a vector of stationary strategies of players $i_1, \ldots, i_k \in S$, $i_1 \cup \ldots \cup i_k = S$, $\eta_S(\cdot) \in \prod_{j=1}^k \Xi_{i_j}$, the set of stationary strategies of coalition $S \subset N$, and $\eta_{N \backslash S}(\cdot)$ is a vector of stationary strategies of players $i_{k+1}, \ldots, i_n \in N \backslash S$, $i_{k+1} \cup \ldots \cup i_n = N \backslash S$, $\prod_{j=k+1}^n \Xi_{i_j}$, the set of stationary strategies of coalition $N \backslash S$.

Third, consider $S = \emptyset$. Let the value of characteristic function be:

$$v^j(\emptyset) = 0. \tag{11}$$

Remark 2. Characteristic functions $\bar{v}(S)$ determined by (8) and $v^j(S)$ determined by (9)–(11) are superadditive.

Definition 3. Cooperative stochastic subgame G^j_{co} is a set (N, v^j), where N is the set of players and $v^j : 2^N \longrightarrow R$ is the characteristic function calculated by (9)–(11).

Definition 4. Cooperative stochastic game G_{co} is a set $(N, \bar{v})$, where N is the set of players and $\bar{v} : 2^N \longrightarrow R$ is the characteristic function calculated by (8).

Definition 5. Vector $\alpha^j = (\alpha^j_1, \dots, \alpha^j_n)$ satisfying the two following conditions:

1. $\sum_{i \in N} \alpha^j_i = v^j(N)$,
2. $\alpha^j_i \geqslant v^j(\{i\})$ for any $i \in N$,

is called an imputation in subgame G^j_{co} $(j = 1, \dots, t)$. Denote the imputation set in cooperative subgame G^j_{co} as A^j.

Definition 6. The vector $\bar{\alpha} = (\bar{\alpha}_1, \dots, \bar{\alpha}_n)$, where $\bar{\alpha}_i = \pi^0 \alpha_i$, $\alpha_i = (\alpha^1_i, \dots, \alpha^t_i)$, and $(\alpha^j_1, \dots, \alpha^j_n) = \alpha^j \in A^j$ is called an imputation in game G_{co}. Denote the imputation set in cooperative stochastic game G_{co} as $\bar{A}$.

Suppose that the imputation set in any subgame G^j_{co}, is nonempty, $j = 1, \dots, t$. Therefore, the imputation set in cooperative stochastic game G_{co} is also nonempty.

4. Subgame consistency of the Shapley value

Suppose that players decide to cooperate in stochastic game and for every subgame G^j_{co} they agree to choose an imputation $\alpha^j = (\alpha^j_1, \dots, \alpha^j_n) \in A^j$. The problem is to realize payments to the players at each stage of the stochastic game to guarantee the expected payoff α^j_i for player i in stochastic subgame G^j. If players receive stage payoffs according to their payoff functions they hardly ever obtain the components of the chosen imputation in mathematical expectation sense. To solve this problem we should suggest the method of redistribution of the total players' payoff in every state which may be realized during the game. Initially, the method was proposed by (Petrosyan and Danilov, 1979), for differential games.

There are two principles of constructing the payment scheme in a dynamic game which can be applied to the theory of stochastic games:

1. The sum of the payments to the players in every state is equal to the sum of the players' payoffs in action profile realized in this state according to the cooperative strategy profile $\bar{\eta}(\cdot)$.
2. The expected sum of the payments to player i in the game G is equal to the ith component $\bar{\alpha}_i$ of the imputation $\bar{\alpha}$.

Taking into account that in stochastic game (1) with stationary strategies the number of subgames is equal to the number of possible states, we need to define the vector $\beta_i = (\beta^1_i, \dots, \beta^t_i)$ for every $i \in N$, where β^j_i is a payment to player i in state Γ^j, $j = 1, \dots, t$. If these payments satisfy the two mentioned principles, they are called imputation distribution procedure (IDP) (see (Petrosyan and Danilov, 1979)). We are interested in constructing the subgame-consistent (time-consistent) IDP.

Definition 7. We call the IDP subgame-consistent in stochastic game G if for any subgame of game G the vector of the expected discounted sums of the payments to the players $1, \dots, n$ belong to the same cooperative solution[3].

[3] Let the cooperative solution be a singleton like the Shapley value. Then in any subgame we consider the Shapley value as a cooperative solution. The case where the cooperative solution is the set (e.g., the core) is considered in details by Parilina and Zaccour, 2015a for the games played over event trees.

In the paper we examine the Shapley value as a cooperative solution. Therefore, the subgame-consistent IDP guarantees any player to obtain the corresponding component of the Shapley value in any subgame.

Theorem 1. *Let the components of the IDP be calculated by equation:*

$$\beta_i = (\mathbb{I}_t - \delta \Pi(\bar{\eta}))\alpha_i, \tag{12}$$

where $\alpha_i = (\alpha_i^1, \ldots, \alpha_i^t)$, *and* $(\alpha_1^j, \ldots, \alpha_n^j) = \alpha^j$ *is the Shapley value in the cooperative game* G_{co}^j *with characteristic function* $v^j(S)$. *Then the IDP is subgame-consistent.*

Proof. First, prove that β_i, $i \in N$, calculated by equation (12) is the IDP. Taking into account equation (9) we obtain

$$\sum_{i \in N} \beta_i = (\mathbb{I}_t - \delta \Pi(\bar{\eta})) \sum_{i \in N} \alpha_i = (\mathbb{I}_t - \delta \Pi(\bar{\eta}))v(N) = \sum_{i \in N} K_i(\bar{x}).$$

Second, we calculate the expected sum of the payments to player i in the game G according to equation (12). Denote this sum for player i by $\bar{B}_i$ and it satisfies the equation:

$$\bar{B}_i = \pi^0 B_i = \pi^0 (B_i^1, \ldots, B_i^t)',$$

where B_i^j can be found from equation:

$$B_i^j = \beta_i^j + \delta \sum_{k=1}^{t} p(j, k; x^j) B_i^k,$$

or in a vector form:

$$B_i = \beta_i + \delta \Pi(\bar{\eta}) B_i. \tag{13}$$

Equation (13) is equivalent to the following one:

$$B_i = (\mathbb{I}_t - \delta \Pi(\eta))^{-1} \beta_i. \tag{14}$$

Taking into account equation (12), we prove that $B_i = \alpha_i$ and then $\bar{B}_i = \bar{\alpha}_i$. The equity $B_i = \alpha_i$ proves the subgame consistency of the IDP determined by equation (12).

Remark 3. Equation (12) is equivalent to the following one:

$$\alpha_i = \beta_i + \delta \Pi(\bar{\eta}) \alpha_i. \tag{15}$$

The second summand at the right-hand side of the equation (15) is the expected value of the component of the Shapley value in subgame starting from the next stage. Therefore, any player will receive his component of the Shapley value in any subgame if the payments to the players are the IDP satisfying equation (12).

Obviously, if players realise the cooperative strategy profile $\bar{\eta}(\cdot)$, the expected payoff of player i in stochastic game G with new payments in cooperative action profiles is equal to the expected value of the correspondent component of the Shapley value in cooperative stochastic game G_{co}.

Now for the imputation $\bar{\alpha} = (\bar{\alpha}_1, \ldots, \bar{\alpha}_n)$, where $\bar{\alpha}_i = \pi^0 \alpha_i$, $\alpha_i = (\alpha_i^1, \ldots, \alpha_i^t)$, $(\alpha_1^j, \ldots, \alpha_n^j) = \alpha^j \in A^j$, we determine the regularization of game G in the following way.

Definition 8. Noncooperative stochastic game G_α (subgame G^j_α, $j = 1, \dots, t$) is called α-regularization of stochastic game G (subgame G^j), if for any player $i \in N$ in state Γ^j payoff function $K_i^{\alpha,j}(x^j)$ is defined as follows:

$$K_i^{\alpha,j}(x^j) = \begin{cases} \beta_i^j, & \text{if } x^j = \bar{x}^j; \\ K_i^j(x^j), & \text{if } x^j \neq \bar{x}^j, \end{cases} \tag{16}$$

where IDP $\beta = (\beta_1, \dots, \beta_n)$ satisfies equation (12)[4].

We suggests a method of construction of the new payoff function in the game G (subgame G^j) in every state when the action profile is cooperative. Here we may ask a question: "Do the players agree to redefine the payoff function in the game?" Our answer is "Yes", if they want to make the payoff functions in the states subgame-consistent in the sense of Definition 7. Redistributing the payoffs using the IDP $\beta_i^1, \dots, \beta_i^t$ in the states $\Gamma^1, \dots, \Gamma^t$ respectively, player i receives the same sum (in terms of mathematical expectation) in game G_α (G^j_α) as he has planned to receive in the cooperative stochastic game G_{co} (G^j_{co}). Moreover, in any subgame his expected payoff will be the corresponding component of the Shapley value. In this case, we can state the *subgame consistency* (*time consistency*) of the chosen cooperative solution.

5. Strategic support of the Shapley value

In this section we need to consider the additional notations. Let $\Gamma(k) \in \{\Gamma^1, \dots, \Gamma^t\}$ be the state realized at stage k of game G_α. Let $x(k)$ be the action profile realized in state $\Gamma(k)$. Denote the subgame of game G_α starting from state $\Gamma(k)$ as $G_\alpha^{\Gamma(k)}$. Call the sequence $((\Gamma(1), x(1)), (\Gamma(2), x(2)), \dots, (\Gamma(k-1), x(k-1)))$ the history of stage k and denote it as $h(k)$. Let T be the set $\{(\Gamma^1, \bar{x}^1), (\Gamma^2, \bar{x}^2), \dots, (\Gamma^t, \bar{x}^t)\}$.

In this section we consider stochastic game G_α as the game with perfect information in the sense that at each stage k ($k = 1, 2, \dots$) all players know state $\Gamma(k)$ and the history of stage k. We would like to prove that the cooperative strategy profile in the game G_α is the Nash equilibrium in trigger strategies. To construct the Nash equilibrium we need to consider the sets of behavior strategies Φ_i, $i \in N$.

Definition 9. We call the behavior strategy profile $\varphi^* = (\varphi_1^*, \dots, \varphi_n^*)$ the Nash equilibrium in game G_α if for any player $i \in N$ the inequality

$$\bar{E}_i^\alpha(\varphi^*) \geqslant \bar{E}_i^\alpha(\varphi^* \parallel \varphi_i) \tag{17}$$

is true for any behavior strategy $\varphi_i \in \Phi_i$ of player i, and $\bar{E}_i^\alpha(\cdot)$ is the expected payoff of player i in α-regularization G_α.

The following theorem gives the condition when the cooperative strategy profile in the game G_α is the Nash equilibrium in the α-regularization G_α of game G.

Theorem 2. *If in the α-regularization G_α the following inequality is true for any coalition player $i \in N$:*

$$\beta_i \geqslant (\mathbb{I}_t - \delta \Pi(\bar{\eta})) F(\{i\}), \tag{18}$$

[4] The IDP for stochastic games was initially proposed in (Petrosjan, 2006) when the game process is realised on a graph and in (Petrosjan and Baranova, 2006) when the number of states in stochastic game is finite and players use stationary strategies.

where $F(\{i\}) = (F^1(\{i\}), \dots, F^t(\{i\}))$,
$F^j(\{i\}) = \max\limits_{\substack{x_i^j \in X_i^j \\ x_i^j \neq \bar{x}_i^j}} \left\{ K_i^j(\bar{x}^j \parallel x_i^j) + \delta \sum\limits_{l=1}^{t} p(j,l;\bar{x}^j \parallel x_i^j) v^l(\{i\}) \right\}$, *then in the game* G_α *there exists the Nash equilibrium with payoffs* $(\bar{\alpha}_1, \dots, \bar{\alpha}_n)$.

Proof. Consider the behavior strategy profile $\hat{\varphi}(\cdot) = (\hat{\varphi}_1(\cdot), \dots, \hat{\varphi}_n(\cdot))$ in game G_α:

$$\hat{\varphi}_i(h(k)) = \begin{cases} \bar{x}_i^j, & \text{if } \Gamma(k) = \Gamma^j, j = \overline{1,t}, h(k) \subset T; \\ \hat{x}_i^j(p), & \text{if } \Gamma(k) = \Gamma^j,\ j = \overline{1,t},\ \exists\ l \in [1, k-1] \\ & \text{and } p \in N,\ p \neq i\text{: } h(l) \subset T, \\ & \text{and } (\Gamma(l), x(l)) \notin T, \\ & \text{but } (\Gamma(l), (x(l) \parallel \bar{x}_p(l))) \in T, \\ \text{anyone} & \text{in other cases,} \end{cases} \tag{19}$$

where $\hat{x}_i^j(p)$ is an action of player i in state Γ^j which with actions x_k^j, $k \neq i$, $k \neq p$ forms the strategy of coalition $\{N \backslash p\}$ in zero-sum game against player p in subgame G^{Γ^j}.

The proof of the theorem is based on the proof of any folk theorem (for example, see (Dutta, 1995)) using the structure of the trigger strategy (19). We prove that $\hat{\varphi}(\cdot) = (\hat{\varphi}_1(\cdot), \dots, \hat{\varphi}_n(\cdot))$ determined in (19) is a Nash equilibrium in stochastic game G_α.

If the player p does not deviate from the cooperative strategy profile $\bar{\eta}$, then taking into account the definition of the strategy (19), the expected payoff of player p in the subgame G_α^j, $j = 1, \dots, t$, is

$$E_p^j(\hat{\varphi}(\cdot)) = E_p^j(\bar{\eta}(\cdot)).$$

Let $E_p(\hat{\varphi}(\cdot))$ be equal to the vector $(E_p^1(\hat{\varphi}(\cdot)), \dots, E_p^t(\hat{\varphi}(\cdot)))$, then for any player $p \in N$ the equation is true:

$$E_p(\hat{\varphi}) = (\mathbb{I}_t - \delta \Pi(\bar{\eta}))^{-1} \beta_p. \tag{20}$$

Consider the strategy profile $(\hat{\varphi}(\cdot) \parallel \varphi_p(\cdot))$, $p \in N$, when player $p \in N$ deviates from strategy $\hat{\varphi}_p(\cdot)$. Let stage k be such that there exists number $l \in [1, k-1]$ such that history $h(l) \subset T$ and state $(\Gamma(l), x(l)) \notin T$ but $(\Gamma(l), (x(l) \parallel \bar{x}_p(l))) \in T$. Without loss of generality, we suggest that $\Gamma(k) = \Gamma^j$. Calculate the payoff of player p in game G_α in strategy profile $(\hat{\varphi}(\cdot) \parallel \varphi_p(\cdot))$ as

$$\bar{E}_p^\alpha(\hat{\varphi} \parallel \varphi_p) = \pi^0 E_p^\alpha(\hat{\varphi} \parallel \varphi_p),$$

where

$$E_p^\alpha(\hat{\varphi} \parallel \varphi_p) = E_p^{\alpha,[1,k-1]}(\hat{\varphi} \parallel \varphi_p) + \delta^{k-1} \Pi^{k-1}(\hat{\varphi} \parallel \varphi_p) E_p^{\alpha,[k,\infty)}(\hat{\varphi} \parallel \varphi_p), \tag{21}$$

where $E_p^{\alpha,[1,k-1]}(\hat{\varphi} \parallel \varphi_p)$ is the expected payoff of player p at the first $k-1$ stages of game G_α, and $E_p^{\alpha,[k,\infty)}(\hat{\varphi} \parallel \varphi_p)$ is the expected payoff of player p in the subgame of game G_α starting from stage k. Since there were no deviations of any players from the cooperative strategy profile $\bar{\eta}(\cdot)$ up to stage $k-1$ inclusive as it was shown before, the following equalities holds for the elements of the right side of (21):

$$E_p^{\alpha,[1,k-1]}(\hat{\varphi} \parallel \varphi_p) = E_p^{\alpha,[1,k-1]}(\bar{\eta}),$$

$$\Pi^{k-1}(\hat{\varphi} \parallel \varphi_p) = \Pi^{k-1}(\bar{\eta}).$$

In the second term of the right side of (21) by $E_p^{\alpha,[k,\infty)}(\hat{\varphi} \parallel \varphi_p)$ we mean vector $(E_p^{\alpha,1}(\hat{\varphi} \parallel \varphi_p), \ldots, E_p^{\alpha,t}(\hat{\varphi} \parallel \varphi_p))$ where $E_p^{\alpha,j}(\hat{\varphi} \parallel \varphi_p)$ is the expected payoff of player p in regularized subgame G_α^j starting from the state Γ^j.

Now we calculate the expected payoff of player p in subgame G_α^j starting from stage k when the state $\Gamma(k)$ is Γ^j:

$$E_p^{\alpha,j}(\hat{\varphi} \parallel \varphi_p) = K_p^j(\bar{x}^j \parallel x_p^j) + \delta \sum_{l=1}^{t} p(j,l;\bar{x}^j \parallel x_p^j) v^l(\{p\}), \tag{22}$$

because players from coalition $N \setminus p$ will punish player p playing in zero-sum game against player p beginning from stage $k+1$ according to the definition of strategy profile $\hat{\varphi}(\cdot)$.

Since the expected payoffs of player p in strategy profiles $\hat{\varphi}(\cdot)$ and $(\hat{\varphi}(\cdot) \parallel \varphi_p(\cdot))$ equal until stage $k-1$, then as a result of the deviation, player p can guarantee the increase of his payoff only at the sacrifice of the part of game G_α beginning with stage k, i.e. at the sacrifice of the expected payoff in subgame G_α^j, $j = 1, \ldots, t$. Player p in strategy profile $(\hat{\varphi}(\cdot) \parallel \varphi_p(\cdot))$ can guarantee the following expected payoff from stage k:

$$\max_{\substack{x_p^j \in X_p^j \\ x_p^j \neq \bar{x}_p^j}} \left\{ K_p^j(\bar{x}^j \parallel x_p^j) + \delta \sum_{l=1}^{t} p(j,l;\bar{x}^j \parallel x_p^j) v^l(\{p\}) \right\}. \tag{23}$$

According to the definition of IDP, the expected payoff of player p in the subgame G_α^j in strategy profile $\hat{\varphi}(\cdot)$ can be calculated by the equation:

$$E_p^\alpha(\hat{\varphi}) = (\mathbb{I}_t - \delta \Pi(\bar{\eta}))^{-1} \beta_p, \tag{24}$$

where $E_p^\alpha(\hat{\varphi}) = (E_p^{\alpha,1}(\hat{\varphi}(\cdot)), \ldots, E_p^{\alpha,t}(\hat{\varphi}(\cdot))$. Taking into account the inequality (18) and (23), (24) and we prove the inequality

$$E_p^\alpha(\hat{\varphi}(\cdot)) \geqslant E_p^\alpha(\hat{\varphi}(\cdot) \parallel \varphi_p(\cdot)).$$

Therefore, the behavior strategy profile (19) is the Nash equilibrium in α-regularization of game G. The expected payoff of player p in game G_α in strategy profile $\hat{\varphi}(\cdot)$ is equal to $\bar{\alpha}_p$ where $\bar{\alpha}_p = \pi^0 \alpha_p$ and vector $\alpha_p = (\alpha_p^1, \ldots, \alpha_p^t)$ consists of p-th components of the Shapley value $\alpha^1, \ldots, \alpha^t$ calculated for the cooperative subgames $G^1, \ldots, G^t$ accordingly. This completes the proof.

6. Data transmission game

6.1. Model

In this section we introduce an example of a stochastic game application in telecommunication systems. We consider a slotted synchronous system in which nodes 1 and 2 independently generate packages in each time slot with probabilities a_1 and a_2, respectively, provided that their individual queues were empty at the end of the previous time slot. The graph of wireless network is depicted in Fig. 1. Some assumptions about this system are as follows:

1. Nodes 1 and 2 (players 1 and 2, respectively) are going to send their packages to a common destination (node 3).
2. The maximum buffer capacity of any node equals one. The destination node can accept only one transmitted package in one time slot. We do not assume multiple package transmissions or simultaneous transmissions and reception by any node in any time slot.
3. If players simultaneously transmit packages to the destination node, the last one rejects these packages and they return to their initial nodes, i.e. at the next time slot no new packages can be generated in nodes 1 and 2.
4. All transmitted packages have the same length, and it requires one time slot to transmit a package from one node to the other which has the direct channel with the first one.
5. Player 1 chooses between sending a package directly to node 3 or relying on node 2 to forward the package to the final destination (node 3).
6. If player 1 (node 1) transmits a package to player 2 (node 2) which has already had a package in its queue, player 2 rejects this package. Otherwise, player 2 decides on whether to accept or reject the package from player 1.

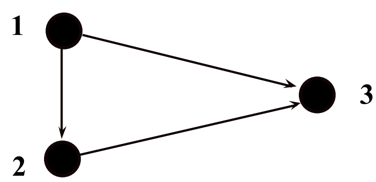

Fig. 1: Topology of a wireless network.

We suggest the following system of rewards and costs:

- $f \geqslant 0$ is a reward to player 1 or player 2 for each successful transmission to the destination node.
- Player 1 receives a reward $c \geqslant 0$ from player 2 for delivering a package to player 2 which can obtain the value f only after successful transmission of that particular package to the final destination in a subsequent time slot.
- Each time slot of package delay results in an additional cost $d \geqslant 0$ for the node that has that particular package in its queue (regardless of that package source).
- D_{ij} is an energy cost of one package transmission from node i to node j.

We suppose that the game ends in any time slot with the probability $0 < q < 1$. The probability $1-q$ can be interpreted as a discount rate. The transmission problem in a wireless network can be solved as a stochastic game. Denote the pair (Q_1, Q_2) as the state of the stochastic game where Q_i is a queue content of node i, $i = 1, 2$. The queue content Q_i can be equal to 0 or 1 if no or one package is present at the queue of node i, respectively.

The set of states in stochastic game is

$$Q = \{(0,0); (0,1); (1,0); (1,1)\}.$$

Consider the game in a cooperative setting meaning that the players' actions are coordinated by one center to improve the work of the network. The coordination of the device actions are useful to increase the speed of data transmission. For solving the cooperative version of the stochastic game, we assume players have information not only on their own queues but also on the other player's queue.

Now we need to describe the states, i. e., the games in normal form corresponding to the states:

1. State $(0,0)$: Player 1 has a unique action W (waiting), player 2 has the same action W (waiting). The payoffs to the players are $(0,0)$.
2. State $(0,1)$: Player 1 has a unique action W (waiting), player 2 also has a unique action $T3$ (transmission to node 3). The payoffs to the players are $(0, f - D_{23})$.
3. State $(1,0)$: Player 1 has two actions: i) $T3$ (transmission to node 3), ii) $T2$ (transmission to node 2). Player 2 has two actions: i) Ac (accepting a package from node 1), ii) Rej (rejecting a package from node 1).
   The payoffs to the players are represented in the matrix:
$$\begin{pmatrix} (f - D_{13}, 0) & (f - D_{13}, 0) \\ (c - D_{12}, -c) & (-d - D_{12}, 0) \end{pmatrix}$$
4. State $\Gamma(1,1)$: Player 1 has two actions: i) $T3$ (transmission to node 3), ii) W (waiting). Player 2 has two actions: i) $T3$ (transmission to node 3), ii) W (waiting). The payoffs to the players are as follows:
$$\begin{pmatrix} (-d - D_{13}, -d - D_{23}) & (f - D_{13}, -d) \\ (-d, f - D_{23}) & (-d, -d) \end{pmatrix}$$

6.2. Transition matrix

Assume the players use the stationary strategies. In the game defined in stationary strategies the players' choice of an action in the states depends neither on the history, nor on the time slot, in which the game is at present, but depends only on the state. In applications of stochastic games it is important to use a simple set of strategies for decreasing the number of calculations of players' expected payoffs.

Denote the set of mixed stationary strategies of player i as $\bar{\Xi}_i$, $i = 1, 2$. According to the game structure the player 1's mixed stationary strategy assigns him to choose action W with probability one in the states $(0,0)$, $(0,1)$, action $T3$ with probability p_{11} in the state (1, 0), and action $T3$ with probability p_{12} in the state (1, 1). The player 2's mixed stationary strategy assigns him to choose action W with probability one in the state (0, 0), action $T3$ in the state (0, 1), action Ac with probability p_{21} in the state (1, 0), and action $T3$ with probability p_{22} in the state (1, 1). Denote a player i's mixed stationary strategy as $\eta_i = (p_{i1}, p_{i2})$. A stationary strategy profile is $\eta = (\eta_1, \eta_2) = (p_{11}, p_{12}, p_{21}, p_{22})$.

The transition matrix when players realise stationary strategy profile η is

$$\Pi(\eta) = \{p(k, l; x^k)\}_{k=1,\ldots,t; l=1,\ldots,t}, \tag{25}$$

where
$p(1, 1; x^1) = (1 - a_1)(1 - a_2)$,

$p(1,2;x^1) = (1-a_1)a_2,$
$p(1,3;x^1) = a_1(1-a_2),$
$p(1,4;x^1) = a_1a_2,$
$p(2,1;x^2) = (1-a_1)(1-a_2),$
$p(2,2;x^2) = (1-a_1)a_2,$
$p(2,3;x^2) = a_1(1-a_2),$
$p(2,4;x^2) = a_1a_2,$
$p(3,1;x^3) = p_{11}(1-a_1)(1-a_2),$
$p(3,2;x^3) = p_{11}(1-a_1)a_2 + (1-p_{11})p_{21}(1-a_1),$
$p(3,3;x^3) = p_{11}a_1(1-a_2) + (1-p_{11})(1-p_{21})(1-a_2),$
$p(3,4;x^3) = p_{11}a_1a_2 + (1-p_{11})p_{21}a_1 + (1-p_{11})(1-p_{21})a_2,$
$p(4,1;x^4) = 0,$
$p(4,2;x^4) = p_{12}(1-p_{22})(1-a_1),$
$p(4,3;x^4) = (1-p_{12})p_{22}(1-a_2),$
$p(4,4;x^4) = p_{12}p_{22} + (1-p_{12})(1-p_{22}) + p_{12}(1-p_{22})a_1 + (1-p_{12})p_{22}a_2.$

6.3. Payoff functions

If the stationary strategy profile η is realized, the payoff to player 1 in the stochastic game is

$$K_1(x) = (K_1^1(x^1), K_1^2(x^2), K_1^3(x^3), K_1^4(x^4))',$$

where
$K_1^1(x^1) = K_1^2(x^2) = 0,$
$K_1^3(x^3) = p_{11}(f-D_{13}) + (1-p_{11})p_{21}(c-D_{12}) + (1-p_{11})(1-p_{21})(-d-D_{12}),$
$K_1^4(x^4) = p_{12}p_{22}(-d-D_{13}) + p_{12}(1-p_{22})(f-D_{13}) + (1-p_{12})(-d).$

If the stationary strategy profile η is realized, the payoff to player 2 in the stochastic game is

$$K_2(x) = (K_2^1(x^1), K_2^2(x^2), K_2^3(x^3), K_2^4(x^4))',$$

where
$K_2^1(x^1) = 0,$
$K_2^2(x^2) = f - D_{23},$
$K_2^3(x^3) = (1-p_{11})p_{21}(-c),$
$K_2^4(x^4) = p_{12}p_{22}(-d-D_{23}) + (1-p_{12})p_{22}(f-D_{23}) + (1-p_{22})(-d).$

We consider the set of pure stationary strategies which is denoted as Ξ_i, $i=1,2$. For example, player 1's pure stationary strategy $\eta_1 = (1,0)$ assigns player 1 to choose action $T3$ in the state $(1,0)$ and action W in the state $(1,1)$. Each player has 4 pure stationary strategies in the stochastic game, therefore, there are 16 pure stationary strategy profiles. For each pure stationary strategy profile $\eta = (\eta_1, \eta_2)$ the transition matrix $\Pi(\eta)$ is determined by (25).

For example, for the pure stationary strategy profile $\eta^1 = (1,1,1,1)$ the transition matrix is

$$\Pi(\eta^1) = \begin{pmatrix} (1-a_1)(1-a_2) & (1-a_1)a_2 & a_1(1-a_2) & a_1a_2 \\ (1-a_1)(1-a_2) & (1-a_1)a_2 & a_1(1-a_2) & a_1a_2 \\ (1-a_1)(1-a_2) & (1-a_1)a_2 & a_1(1-a_2) & a_1a_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

For each strategy profile $\eta \in \Xi = \prod_{i=1}^2 \Xi_i$ we can calculate the expected players' payoffs for subgames which are denoted as: $E_i(\eta) = (E_i^{(0,0)}(\eta), E_i^{(0,1)}(\eta), E_i^{(1,0)}(\eta),$

$E_i^{(1,1)}(\eta))'$, and

$$E_i(\eta) = (\mathbb{I}_t - (1-q)\Pi(\eta))^{-1} K_i(x), \tag{26}$$

where $K_i(x)$, $\Pi(\eta)$ are determined above.

The expected payoff to player i in the whole game including the chance move is

$$\bar{E}_i(\eta) = \pi^0 E_i(\eta), \tag{27}$$

where $\pi^0 = (\pi^0_{(0,0)}, \pi^0_{(0,1)}, \pi^0_{(1,0)}, \pi^0_{(1,1)})$ is a vector of the initial probabilities, and π^0_k is the probability that the first state in the stochastic game will be $k \in Q$. Vector π^0 is given.

6.4. Algorithm of solving cooperative stochastic game

In this section we describe the steps of solving cooperative stochastic game of data transmission in a wireless network of topology represented in Fig. 1.

1. For any state $k \in Q$ and any pure strategy profile $\eta = (\eta_1, \eta_2)$, $\eta_i \in \Xi_i$, $i = 1, 2$, calculate the expected players' payoffs $E_i^k(\eta)$ in subgame G^k by equation (5) and their expected payoffs in the whole game $\bar{E}_i(\eta)$ by equation (6).
2. Find the cooperative strategy profile $\bar{\eta}$ by equation (7).
3. Calculate the values of the characteristic functions $v^k(S)$ for any state $k \in Q$ and any coalition $S \subset N$ using equations (9), (10), (11). Then calculate the values of the characteristic function $\bar{v}(S)$ for any $S \subset N$ by (8).
4. Calculate the Shapley values $\alpha^k = (\alpha_1^k, \ldots, \alpha_n^k)$ for any subgame G^k starting from state $k \in Q$ using formula (Shapley, 1953b):

$$\alpha_i^k = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(|N| - |S| - 1)!}{|N|!} \left(v^k(S \cup \{i\}) - v^k(S)\right). \tag{28}$$

   Then calculate the Shapley value for the whole game $\bar{\alpha} = (\bar{\alpha}_1, \bar{\alpha}_2)$ using equation $\bar{\alpha}_i = \pi^0 \alpha_i$.
5. Calculate the components of the IDP β_i^k, $i = 1, 2$ and $k \in Q$ by equation (12).

6. To construct the subgame-consistent Shapley value we determine the α-regularization G_α re-defining the payoff functions by equation (16).
7. Verify if there exists the Nash equilibrium in behavior strategies with payoffs $(\bar{\alpha}_1, \bar{\alpha}_2)$ using inequality (18).

6.5. Numerical illustration

We introduce the numerical example of the data transmission game for wireless network. We identify the parameters of the simulation. The probability of package appearance at node 1 is higher than in node 2: $a_1 = 0.4$, $a_2 = 0.1$. The probability of a game end is $q = 0.01$ which is equivalent to the discount rate 0.99. The rewards and costs are $f = 1$, $d = 0.1$, $c = 0.3$, $D_{12} = 0.1$, $D_{13} = 0.6$, $D_{23} = 0.2$. We may notice that the cost of package transmission from node 1 to node 3 is three times more than the cost of package transmission from node 2 to node 3. Therefore, the cooperation of nodes 1 and 2 may be profitable. Let the game begin from any state with equal probability, i. e., $\pi^0 = (0.25, 0.25, 0.25, 0.25)$.

Table 1 represents the expected players' payoffs

$$E_i(\eta) = \left(E_i^{(0,0)}(\eta), E_i^{(0,1)}(\eta), E_i^{(1,0)}(\eta), E_i^{(1,1)}(\eta)\right)'$$

for any pure stationary strategy profile η for any player $i = 1, 2$, and the sum of the expected payoffs. The last column in Table 1 is $\bar{E}_1 + \bar{E}_2$ which is the total expected players' payoff in the whole game taking into account the vector of initial probabilities π^0.

The cooperative strategy profile maximizing the total players' payoff is

$$\bar{\eta} = \eta^{11} = (0, 0, 1, 1),$$

in which the player 1's strategy $\eta_1^{11} = (0, 0)$ assigns him "not to transmit to node 3, but transmit to node 2" in state (1,0) when there is a package at node 1 and there is no package at node 2. In this state the player 2's strategy $\eta_2^{11} = (1, 1)$ assigns her "to accept the package" from player 1 in state $(1, 0)$. When the game in state $(1, 1)$, player 1 "waits" and player 2 "transmits" to node 3.

The maximum of the total expected players' payoff in the whole game is

$$\max_{\eta \in \Xi} \sum_{i \in N} \bar{E}_i(\eta) = \sum_{i \in N} \bar{E}_i(\bar{\eta}) = 26.9472.$$

Table 1: Expected payoffs in the stochastic game.

η	$E_1(\eta)$	$E_2(\eta)$	$\bar{E}_1+\bar{E}_2$
$\eta^1=(1,1,1,1)$	-53.0129 -53.0129 -52.6129 -70.1000	-22.9935 -22.1935 -22.9935 -30.0000	-81.7048
$\eta^2=(1,1,1,0)$	15.8400 15.8400 16.2400 16.2400	6.76818 7.56818 6.76818 7.27732	23.1355
$\eta^3=(1,0,1,1)$	14.7353 14.7353 15.1353 14.8563	7.9200 8.7200 7.9200 8.7200	23.1855
$\eta^4=(1,0,1,0)$	-5.10968 -5.10968 -4.70968 -10.0000	-7.02581 -6.22581 -7.02581 -10.0000	-13.8016
$\eta^5=(1,1,0,1)$	-53.0129 -53.0129 -52.6129 -70.0000	-22.9935 -22.1935 -22.9935 -30.0000	-81.7048
$\eta^6=(1,1,0,0)$	15.8400 15.8400 16.2400 16.2400	6.76818 7.56818 6.76818 7.27732	23.1355
$\eta^7=(1,0,0,1)$	14.7353 14.7353 15.1353 14.8563	7.9200 8.7200 7.9200 8.7200	23.1855
$\eta^8=(1,0,0,0)$	-5.10968 -5.10968 -4.70968 -10.0000	-7.02581 -6.22581 -7.02581 -10.0000	-13.8016
$\eta^9=(0,1,1,1)$	-64.7464 -64.7464 -65.9794 -70.0000	-27.3398 -26.5398 -27.9446 -30.0000	-94.3241

η	$E_1(\eta)$	$E_2(\eta)$	$\bar{E}_1 + \bar{E}_2$
$\eta^{10} = (0,1,1,0)$	11.5347 11.5347 11.8060 12.0060	13.4228 14.2228 13.6218 13.8218	25.4926
$\eta^{11} = (0,0,1,1)$	4.90248 4.90248 5.04298 4.87602	21.4759 22.2759 21.8337 22.4792	26.9472
$\eta^{12} = (0,0,1,0)$	−8.93504 −8.93504 −9.06741 −10.0000	−8.73596 −7.93596 −8.97396 −10.0000	−18.1458
$\eta^{13} = (0,1,0,1)$	−64.2491 −64.2491 −65.4128 −70.0000	−26.728 −25.928 −27.248 −30.000	−93.4537
$\eta^{14} = (0,1,0,0)$	−8.4855 −8.4855 −8.8128 −7.6828	5.60847 6.40847 5.57380 6.13680	−2.43475
$\eta^{15} = (0,0,0,1)$	−18.532 −18.532 −19.010 −18.910	7.9200 8.7200 7.9200 8.7200	−10.426
$\eta^{16} = (0,0,0,0)$	−10.559 −10.559 −10.917 −10.000	−8.8313 −8.0313 −9.0826 −10.000	−19.49526

We calculate the values of the characteristic functions for subgames by equations (9), (10), (11):

$$v(\{1\}) = (-5.10968, -5.10968, -4.70968, -10.0)',$$
$$v(\{2\}) = (-8.735960, -7.93596, -8.97396, -10.0)',$$
$$v(\{1,2\}) = (26.3784, 27.1784, 26.8766, 27.3553)',$$

The characteristic function of the whole game is found by equation (8):

$$\bar{v}(\{1\}) = -6.23226,$$
$$\bar{v}(\{2\}) = -8.91147,$$
$$\bar{v}(\{1,2\}) = 26.9472.$$

Then we may calculate the Shapley Values for the subgames and the whole game using equation (28):

– for subgames:
  - $\alpha_1 = (15.0023, 15.0023, 15.5705, 13.6776)'$,
  - $\alpha_2 = (11.376, 12.176, 11.3062, 13.6776)'$,

– for the whole game:
  - $\bar{\alpha}_1 = 14.8132$,
  - $\bar{\alpha}_2 = 12.134$.

The cooperative payoff distribution procedure β_1 for player 1 and β_2 for player 2 are found by equations (12) using the Shapley values:

– $\beta_1 = (0, 0, 1.24274, -1.54974)'$,
– $\beta_2 = (0, 0.8, -1.34274, 2.24974)'$,

where β_i^k is a payment to player i in the state k. Remind that the payoffs of the players in the states defined in the matrix forms are as follows:

– $K_1 = (0, 0, 0.2, -0.1)'$,
– $K_2 = (0, 0.8, -0.3, 0.8)'$,

We may notice that in states $(0, 0)$ and $(0, 1)$ the components of IDP coincide with the payoffs according to the payoff functions K_1 and K_2. But in states $(1, 0)$ and $(1, 1)$ there is a redistribution of the total payoffs among the players. In state $(1, 0)$ players obtain -0.1 together and according to the IDP player 1 receives 1.24274 instead of 0.2, and player 2 receives -1.34274 instead of -0.3. Therefore, player 2 gives 1.04274 to player 1 to make IDP subgame-consistent. In state $(1, 1)$ players obtain 0.7 together and according to the IDP player 1 receives -1.54974 instead of -0.1, and player 2 receives 2.24974 instead of 0.8. Therefore, player 1 gives 1.44974 to player 2 to make IDP subgame-consistent.

Thus, the Shapley Value $\bar{\alpha} = (14.8132, 12.134)'$ is subgame consistent if the payoffs to the players in the states are made according to the IDP $\beta_1 = (0, 0, 1.24274, -1.54974)'$, and $\beta_2 = (0, 0.8, -1.34274, 2.24974)'$.

Now we need to examine the problem of strategic support of the cooperative strategy profile. First, calculate values $F^k(\{i\})$ for $i = 1, 2$ and $k \in Q$ determined in Theorem 2:

1. $F(\{1\}) = (-5.10968, -5.10968, -4.70968, -5.28632)'$,
2. $F(\{2\}) = (-8.73596, -7.93596, -8.97396, -8.1858)'$.

Second, verify if the inequalities (18) are true. For player 1 the inequality (18) takes the form:

$$\begin{pmatrix} 0 \\ 0 \\ 1.24274 \\ -1.54974 \end{pmatrix} \geqslant \begin{pmatrix} -0.186662 \\ -0.186662 \\ 0.418853 \\ -0.566649 \end{pmatrix}$$

We notice that the inequality is not true. In state $(1, 1)$ there is an intense for deviation of player 1 as his payoff according to IDP in this state -1.54974 is less than his payoff in case of deviation -0.566649. This means that the cooperation cannot be supported strategically by the behavior strategy profile (19).

For player 2 the inequality (18) takes the form:

$$\begin{pmatrix} 0 \\ 0.8 \\ -1.34274 \\ 2.24974 \end{pmatrix} \geqslant \begin{pmatrix} -0.0718427 \\ 0.728157 \\ -1.01842 \\ 0.620393 \end{pmatrix}$$

Again, we notice that the inequality is not true. In state $(1, 0)$ there is an intense for deviation of player 2 as her payoff according to IDP in this state -1.34274 is lower than her payoff in case of deviation -1.01842. We state that the behavior strategy profile determined by (19) cannot strategically support the cooperative payoffs.

7. Conclusion

We have examined the problem of cooperation in a dynamic game having a stochastic structure. First, we construct the subgame consistent cooperative solution of the game by redefining players' state payoff functions using the imputation distribution procedure. Second, we provide the conditions to verify if the cooperative solution can be supported strategically. All theoretical results are demonstrated by the example of a stochastic game modeling data transmission in an ad hoc wireless network with a simple topology. Numerical simulations show the actuality of an application of a game-theoretical model to telecommunication problems because it proposes the method of cost reduction.

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A Design of Strategies in Alternative Pursuit Games

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Abstract In this work we consider the games where P can terminate pursuit at will on any of two terminal manifolds. If the optimal feedback strategies for every variant of termination are known, an obvious pursuit strategy assigns the control that corresponds to the alternative with less value at every state. On the manifold with equal alternative values, this strategy may become discontinuous even when the value functions themselves are smooth. We describe smooth approximations for the minimum functions that allow to construct smooth alternative strategies and to deal with generalized solutions for differential equations with discontinuous right-hand sides. However, as shown by an example, the state may stay on a equivalued manifold and the game never terminates.

Keywords: approximations of minimum and maximum functions, alternative pursuit, generalized solutions for differential equations with discontinuous right-hand sides.

1. Introduction

Differential games advanced far beyond the initial findings of their founders. However, finding solutions for concrete games still involves more art than craft. In this paper, we study a method for generating pursuit strategies and evaluation of their guaranteed results when the goal functions represent the minimum of two value functions.

2. Smooth approximations for minimum functions and their derivatives

Certain values between v_1 and v_2 that described, e.g., as

$$F_\alpha(v_1, v_2) = \alpha v_1 + (1-\alpha)v_2,\ 0 < \alpha < 1$$

may be considered as "rough" approximations for $\min(v_1, v_2)$ from above or for $\max(v_1, v_2)$ from below. In "more accurate" approximations, α depends on v_1 and v_2, and $\alpha(v_1, v_2)$ takes the value close to 1 for $\min(v_1, v_2)$ and to 0 for $\max(v_1, v_2)$ if $v_i < v_{3-i}$, $i = 1, 2$. Thus,

$$\overline{F}^{\xi}_{\lambda}(v_1, v_2) = \frac{\lambda_1 v_1^{\xi} v_2 + \lambda_2 v_1 v_2^{\xi}}{\lambda_1 v_1^{\xi} + \lambda_2 v_2^{\xi}},\ v_1, v_2, \xi \in \mathbb{R}^+, \tag{1}$$

and

$$\underline{F}_{\lambda}^{\xi}(v_1, v_2) = \frac{\lambda_1 v_1^{\xi+1} + \lambda_2 v_2^{\xi+1}}{\lambda_1 v_1^{\xi} + \lambda_2 v_2^{\xi}},\ v_1, v_2, \xi \in \mathbb{R}^+, \tag{2}$$

which correspond to F_α with

$$\alpha(v_1, v_2) = \frac{\lambda_2 v_2^{\xi}}{\lambda_1 v_1^{\xi} + \lambda_2 v_2^{\xi}}$$

and

$$\alpha(v_1, v_2) = \frac{\lambda_1 v_1^{\xi}}{\lambda_1 v_1^{\xi} + \lambda_2 v_2^{\xi}}$$

approximate $\min(v_1, v_2)$ from above and $\max(v_1, v_2)$ from below, $0 < \lambda_i < 1, \sum_{i=1}^{2} \lambda_i = 1,\ i = 1, 2$, respectively; see, e.g., (Stipanović et al., 2009). Moreover, their partial derivatives approximate the corresponding partial derivatives of the minimum and maximum functions where they exist; see, e.g., (Shevchenko, 2009, 2012).

Since, e.g.,

$$\min(v_1, v_2, \ldots, v_n) = \min(v_1, \min(v_2, \ldots, v_n)),\ n > 2, \tag{3}$$

approximations for the arbitrary number of arguments minimum functions may be easily constructed with use of approximations for $\min(v_1, v_2)$ and $\max(v_1, v_2)$.

A more general approach is based on using monotonic functions; see, e.g., (Stipanović, 2012, 2014). Let g be a strictly decreasing non-negative differentiable function $\mathbb{R}^+ \to \mathbb{R}^+$ and $v_{i_0} = \min(v_1, \ldots, v_n)$. Then,

$$v_{i_0} \le v_i,$$

$$g(v_{i_0}) \ge g(v_i),$$

$$\lambda_i g(v_{i_0}) \ge \lambda_i g(v_i),\ 0 < \lambda_i < 1, \sum_{i=1}^{n} \lambda_i = 1,$$

$$\sum_{i=1}^{n} \lambda_i g(v_{i_0}) \ge \sum_{i=1}^{n} \lambda_i g(v_i),$$

$$g(v_{i_0}) \ge \sum_{i=1}^{n} \lambda_i g(v_i),$$

$$\sum_{i=1}^{n} \lambda_i g(v_{i_0}) \ge \sum_{i=1}^{n} \lambda_i g(v_i).$$

Since g is an invertible function,

$$v_{i_0} = g^{-1}(g(v_{i_0})) \le g^{-1}(\sum_{i=1}^{n} \lambda_i g(v_i)).$$

Let G_{λ}^{g} be the following symmetric function $(\mathbb{R}^+)^n \to \mathbb{R}^+$

$$G_{\lambda}^{g}(v_1, \ldots, v_n) = g^{-1}(\sum_{i=1}^{n} \lambda_i g(v_i)). \tag{4}$$

If $0 < \lambda_i < 1$ and $\sum_{i=1}^{n} \lambda_i = 1$, (4) approximates $\min(v_1, \ldots, v_n)$ from above since

$$\min(v_1, \ldots, v_n) < G_\lambda^g(v_1, \ldots, v_n), \text{ if } v_i \neq v_j,\ i \neq j, \tag{5}$$

In addition,

$$\min(v, \ldots, v) = G_\lambda^g(v, \ldots, v) = v. \tag{6}$$

Similarly,

$$g(v_{i_0}) \leq \sum_{i=1}^{n} \lambda_i g(v_i),\ \lambda_i \geq 1,$$

$$v_{i_0} = g^{-1}(g(v_{i_0})) \geq g^{-1}(\sum_{i=1}^{n} \lambda_i g(v_i)).$$

If $\lambda_i \geq 1$, (4) approximates $\min(v_1, \ldots, v_n)$ from below since

$$\min(v_1, \ldots, v_n) > G_\lambda^g(v_1, \ldots, v_n), \text{ if } v_1 \neq v_2, \tag{7}$$

$$\min(v, \ldots, v) > G_\lambda^g(v, \ldots, v) = g^{-1}(\sum_{i=1}^{n} \lambda_i g(v)). \tag{8}$$

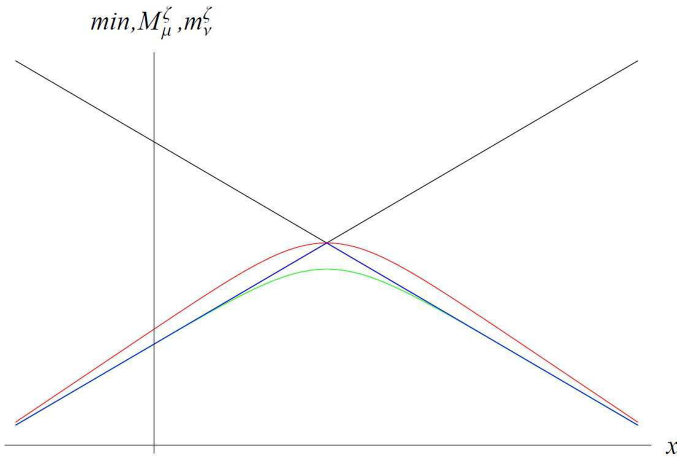


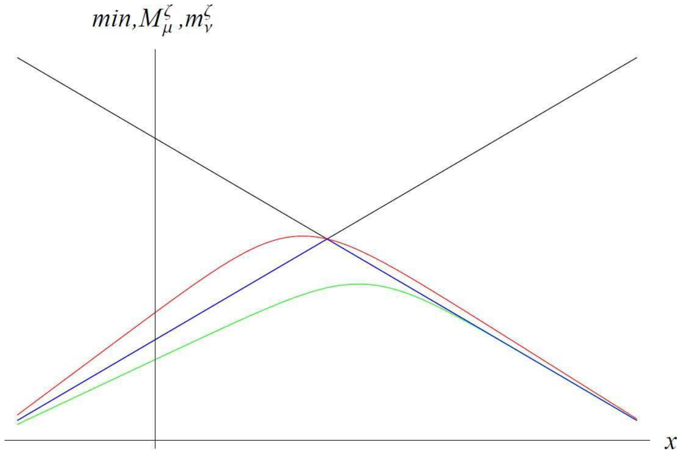
Fig. 1: Approximations for the minimum function; $\xi = 5$; $\mu_1 = 0.5, \mu_2 = 0.5$ (upper); $\nu_1 = 1, \nu_2 = 1$ (lower)

Certain upper (M_μ^ξ) and lower (m_ν^ξ) approximations for $\min(v_1, v_2)$ may be generated, e.g., with use of the family $\{v^{-\xi}\}_{\xi > 0}$,

$$M_\mu^\xi(v_1, v_2) = \left(\mu_1 v_1^{-\xi} + \mu_2 v_2^{-\xi}\right)^{-\frac{1}{\xi}},\ 0 < \mu_i < 1, \sum_{i=1}^{2} \mu_i = 1, \tag{9}$$

$$m_\nu^\xi(v_1, v_2) = \left(\nu_1 v_1^{-\xi} + \nu_2 v_2^{-\xi}\right)^{-\frac{1}{\xi}},\ \nu_i > 1, \tag{10}$$

see Figs. 1–2.



(a) (b)

Fig. 2: Approximations for the minimum function; $\xi = 5$; $\mu_1 = 0.3, \mu_2 = 0.7$ (upper); $\nu_1 = 3, \nu_2 = 1$ (lower)

The derivatives of G_λ^g are described as

$$\frac{\partial G_\lambda^g}{\partial v_j}(v_1, \ldots, v_n) = \lambda_j \frac{g'(v_j)}{g'\left(g^{-1}\left(\sum_{i=1}^n \lambda_i g(v_i)\right)\right)}, \tag{11}$$

and

$$\frac{\partial G_\lambda^g}{\partial v_j}(v, \ldots, v) = \lambda_j,$$

if $0 < \lambda_i < 1$ and $\sum_{i=1}^n \lambda_i = 1$.

For example, the derivatives of $\min(v_1, v_2)$ (where they exist) are approximated by the derivatives of M_μ^ξ since

$$\lim_{\xi \to +\infty} \frac{\partial M_\mu^\xi}{\partial v_j}(v_1, v_2) = \begin{cases} 1 \text{ if } v_j < v_{3-j}, \\ 0 \text{ if } v_j > v_{3-j} \\ \mu_j \text{ if } v_j = v_{3-j}. \end{cases}$$

Now, let g be a strictly increasing non-negative differentiable function $\mathbb{R}^+ \to \mathbb{R}^+$. Obviously, if $0 < \lambda_i < 1$ and $\sum_{i=1}^n \lambda_i = 1$, (4) approximates $\max(v_1, \ldots, v_n)$ from below and if $\lambda_i \geq 1$ from above. All other mentioned formerly results related to the use of (4) for approximations of the minimum function may be easily reproduced for approximations of the maximum function.

3. Alternative pursuit

Let $Z \subseteq \mathbb{R}^m$ be an open set, $\overline{Z}$ be a playing space and $\partial Z = \overline{Z} \backslash Z$ be its boundary. Let $z_P(t) \in \mathbb{R}^{n_P}$ and $z_E(t) \in \mathbb{R}^{n_E}$ meet

$$\dot{z}_P(t) = f_P(z_P(t), u_P(t)), \quad z_P(0) = z_P^0, \tag{12}$$
$$\dot{z}_E(t) = f_E(z_E(t), u_E(t)), \quad z_E(0) = z_E^0, \tag{13}$$

where $t \geq 0$, $u_P(t) \in \mathsf{U}_P \subset \mathbb{R}^{m_P}$, $u_E(t) \in \mathsf{U}_E \subset \mathbb{R}^{m_E}$, U_P and U_E are compact sets, $f_P : \mathbb{R}^{n_P} \times \mathsf{U}_P \to \mathbb{R}^{n_P}$ $f_E : R^{n_E} \times \mathsf{U}_E \to R^{n_E}$, $z_P^0 \in \mathbb{R}^{n_P}$ $z_E^0 \in R^{n_E}$ are initial states. Let $z(t) = (z_P(t), z_E(t)) \in Z \subseteq \mathbb{R}^n$, $n = n_P + n_E$,

$$\dot{z}(t) = f(z(t), u_P(t), u_E(t)),\ z(0) = z^0, \tag{14}$$

where $z(0) = z^0 = (z_P^0, z_E^0) \in Z$, $f(z, u_P, u_E) = (f_P(z_P, u_P), f_E(z_E, u_E))$. We assume that f is jointly continuous and locally Lipschitz with respect to z for all $u_P \in \mathsf{U}_P$ and $u_E \in \mathsf{U}_E$.

A strategy is a rule to determine the control depending on available information at any instant of the game. For a given strategy, the equation (14) is used to generate a pencil of all potential motions and evaluate the guaranteed payoff over all admissible countering actions.

For $z^0 \in Z$, $\Delta = \{t_0, t_1, \ldots, t_i, t_{i+1}, \ldots\}$ and a strategy S_P, let $Z_P(z^0, S_P, \Delta)$ be a pencil of piecewise constant solutions of the inclusion

$$\dot{z}(t) \in \mathrm{co}\{f(z(t_i), u_P(t_i), u_E) : u_E \in \mathsf{U}_E\}, \tag{15}$$

where $t \in [t_i, t_{i+1})$, $i \in \mathbb{N}$, $t_0 = 0$, $t_i \to_{i\to\infty} \infty$, $z : R^+ \to Z$ is a continuous function that has an absolutely continuous restriction to $[0, \theta]$ for any $\theta > 0$ and meets (15) for almost all $t \in [0, \theta]$.

A pursuit game is called alternative if

- from any internal state $z \in Z$, it can be terminated by P *at will* on any of two given terminal manifolds $M^a \subset \partial Z$ or $M^b \subset \partial Z$,
- for every alternative termination, the payoffs of Boltza type differ only in their terminal parts,
- for every alternative termination, the optimal feedback pursuit $(S_P^a(\cdot), S_P^b(\cdot))$ and evasion $(S_E^a(\cdot), S_E^b(\cdot))$ strategies and the value functions $(V^a(\cdot), V^b(\cdot))$ are known.

Among the games that may be considered as alternative are the obstacle tag (Isbell, 1967) and successive pursuit (Breakwell and Hagedorn, 1979) games.

For a given alternative terminal manifold M_l, let the payoff functional be defined as

$$\mathcal{P}_l^\varepsilon(z(\cdot)) = \begin{cases} \tau_l^\varepsilon + K_l(z(\tau_l^\varepsilon)), \text{ if } \tau_l^\varepsilon = \tau_l^\varepsilon(z(\cdot)) < \infty, \\ \infty \text{ otherwise}, \end{cases} \tag{16}$$

where

$$\tau_l^\varepsilon(z(\cdot)) = \begin{cases} \min\{t_i \in \Delta : z(t_i) \in M_l^\varepsilon\}, \text{ if } \exists t_i \in \Delta : z(t_i) \in M_l^\varepsilon, \\ \infty \text{ otherwise}, \end{cases} \tag{17}$$

M_l^ε is the ε neighbourhood of M_l, $M_l^\varepsilon = \{z : z \in Z, \min_{z' \in M_l} ||z - z'|| \leq \varepsilon\}$, $K_l : Z \to \mathbb{R}^+$, $l \in L = \{a, b\}$. Then the guaranteed result may be evaluated as

$$\check{\mathcal{P}}_l(z^0) = \lim_{\varepsilon \to 0+} \mathcal{P}_l^\varepsilon(z^0), \tag{18}$$

where $\check{\mathcal{P}}_l^\varepsilon(z^0) = \inf_{S_P} \check{\mathcal{P}}_l^\varepsilon(z^0, S_P)$,

$$\check{\mathcal{P}}_l^\varepsilon(z^0, S_P) = \lim_{|\Delta| \to +0} \check{\mathcal{P}}_l^\varepsilon(z^0, S_P, \Delta),\ |\Delta| = \sup_{i \in \mathbb{N}}(t_{i+1} - t_i),$$

$$\check{\mathcal{P}}_l^\varepsilon(z^0, S_P, \Delta) = \sup_{z(\cdot) \in Z_P(z^0, S_P, \Delta)} \mathcal{P}_l^\varepsilon(z(\cdot)).$$

Let $\hat{\mathcal{P}}_l : Z \to \mathbb{R}$ be a similar index for E and $V^l(z^0) = \check{\mathcal{P}}_l(z^0) = \hat{\mathcal{P}}_l(z^0), \forall z^0 \in Z$. Then the value function $V^l : Z \to \mathbb{R}^+$ represents a joint guaranteed result for both players.

If $V^a(\cdot)$ and $V^b(\cdot)$ are continuous in $\overline{Z}$, satisfy the terminal conditions

$$V^l(z) = K_l(z),\, z \in M^l, \tag{19}$$

and are continuously differentiable in Z, then the Isaacs' main equation

$$H^l(z, DV^l(z)) + 1 = 0 \tag{20}$$

is satisfied, where

$$H^l(z, DV^l(z)) = \min_{u_P \in U_P} \max_{u_E \in U_E} \lambda^l f(z, u_P, u_E) = \max_{u_E \in U_E} \min_{u_P \in U_P} \lambda^l f(z, u_P, u_E)\, l \in L.$$

If there are bounded $u_P^* : Z \times \Lambda \to U_P$ and $u_E^* : Z \times \Lambda \to U_E$ such that

$$u_P^*(z^l, \lambda^l) \in Arg \min_{u_P \in U_P} \left(\max_{u_E \in U_E} \lambda^l f(z, u_P, u_E) \right), \tag{21}$$

$$u_E^*(z^l, \lambda^l) \in Arg \max_{u_E \in U_E} \left(\min_{u_P \in U_P} \lambda^l f(z, u_P, u_E) \right), \tag{22}$$

the optimal feedback strategies are designed as follows

$$S_P^l(z) = u_P^*(z, DV^l(z)), \quad S_E^l(z) = u_E^*(z, DV^l(z)). \tag{23}$$

When solving such kind of games, a standard problem is to combine S_P^a and S_P^b into a pursuit strategy that guarantees a result less or equal to $\min(V^a(z), V^b(z))$ for every state $z \in Z$ (Shevchenko, 2009).[1] An obvious candidate strategy for alternative pursuit is

$$S_P^{a|b}(z) = \begin{cases} S_P^a(z) \text{ if } V^a(z) < V^b(z), \\ S_P^b(z) \text{ if } V^b(z) < V^a(z), \\ u_P \in [S_P^a(z), S_P^b(z)] \text{ or} \\ u_P \in \{S_P^a(z), S_P^b(z)\} \text{ if } V^b(z) = V^a(z), \end{cases} \tag{24}$$

or

$$S_P^l(z) = \begin{cases} u_P^*(z, D\min(V^a(z), V^b(z))) \text{ if } V^b(z) \neq V^a(z) \\ u_P \in [u_P^*(z, DV^a(z)), u_P^*(z, DV^a(z))] \text{ or} \\ u_P \in \{u_P^*(z, DV^a(z)), u_P^*(z, DV^a(z))\} \text{ if } V^b(z) = V^a(z), \end{cases} \tag{25}$$

where $[v_1, v_2] = \{v : \kappa v_1 + (1-\kappa)v_2, \kappa \in [0, 1]\}$. To evaluate the guaranteed payoff for $S_P^{a|b}$, one needs to determine pencils of solutions for differential equations with discontinuous right-hand sides. The generalized solutions (Krasovskii and Subbotin, 1988) include all possible absolutely continuous motions for all values of the control at a discontinuity point (as, e.g., the Filippov's solutions (Filippov, 1988)).

[1] A similar problem arises when optimal feedbacks are constructed with the use of a synthesis procedure based on the main equation and its smooth characteristics within the Isaacs' approach (Isaacs, 1967).

The constructive motions (Krasovskii and Subbotin, 1988) are absolutely continuous limits of motions along the Euler broken lines. The constructive motions are usually included into the generalized solutions and provide better guaranteed results. However, they are less stable (Krasovskii and Subbotin, 1988).

One way to avoid discontinuous controls at the regular states is to use smooth upper approximations of $\min(V^a(z), V^b(z))$ in (23) as, e.g., (9). Then with

$$S_P^l(z) = u_P^*(z, DM_\mu^\xi(z)),\ S_E^l(z) = u_E^*(z, DM_\mu^\xi(z)) \tag{26}$$

some subsets of the generalized motions are obtained.

Practically in all games solved with use of the Isaacs' approach, the value functions are not smooth globally (Isaacs, 1967). Call a state $z \in Z$ regular with respect to known value functions V^a and V^b if S_P^l and S_E^l meet (20)–(23) with $V^l, H^l \in C^2, u_P^* \in C,\ l \in L$, in some neighbourhood of z. A set of such states is also called regular. Let $\mathcal{E}^{a|b} = \{z \in Z : V^a(z) = V^b(z)\}$ be a two-sided regular smooth hypersurface separating two regular sets. Moving along any direction η from $z \in \mathcal{E}^{a|b}$ for a small enough time, the state shifts to a regular state z' on $\mathcal{E}^{a|b}$ $(V^a(z') = V^b(z'))$ or in Z $(V^a(z') < V^b(z'))$ or in Z $(V^a(z') > V^b(z'))$.

If

$$\frac{\partial}{\partial z}(V^a(z) - V^b(z)) f(z, S_P^a(z), S_E^a(z)) > 0, \tag{27}$$

$$\frac{\partial}{\partial z}(V^b(z) - V^a(z)) f(z, S_P^b(z), S_E^b(z)) > 0, \tag{28}$$

on $\mathcal{E}^{a|b}$ and P uses (25) or (26) in its close neighbourhood, the state may stay there for some time (Shevchenko, 2014).

4. Constructing strategies in a simple alternative pursuit game

Three points P, E_1, and E_2, $E = (E_1, E_2)$, with bounded velocities move on the plane as

$$\dot{z} = (u_P, u_e),\ z = z^0,$$
$$z_P, z_1, z_2 \in \mathbb{R}^2,\ z_e = (z_1, z_2) \in \mathbb{R}^4,\ z = (z_P, z_e) \in \mathbb{R}^6,$$
$$z^0 = (z_P^0, z_e^0),\ z_e^0 = (z_1^0, z_2^0),$$
$$u_P \in U_P,\ u_e = (u_1, u_2) \in U_e,$$
$$U_P = \{u_P : ||u_P|| \le 1\},\ U_e = \{u_e : ||u_1|| \le \beta_1 < 1, ||u_2|| \le \beta_2 < 1\}$$

It's required to determine the minimal guaranteed time $\tau^{1|2}$ for P to approach one of E's by a given distance $r > 0$ and the corresponding strategy.

The value of the game at the initial state $z^0 \in Z$ may be evaluated as

$$V(z^0) = \min(V^1(z^0), V^2(z^0)), \tag{29}$$

where

$$V^i(z^0) = \frac{||z_i^0 - z_P^0|| - r}{1 - \beta_i}, \tag{30}$$

the optimal feedback pursuit strategy is

$$S_P^{i(z^0)}(z) = -\frac{\partial V^{i(z^0)}}{\partial z_{i(z^0)}}(z) / || \frac{\partial V^{i(z^0)}}{\partial z_{i(z^0)}}(z) ||,\ z, z^0 \in Z, \tag{31}$$

and $i(z^0)(= 1 \vee 2)$ satisfies the condition

$$V^{i(z^0)}(z^0) = \min(V^1(z^0), V^2(z^0)). \tag{32}$$

It was noted in (Krasovskii and Subbotin, 1988) that in this game with $\beta_1 = \beta_2$ and $r = 0$ there is a dispersal line where both pursuers are equidistant and it may become a singular line with the payoff equal to $+\infty$ due to the measurement errors.

Look at an alternative version of the game with the pursuit strategy

$$S_P^{1|2}(z) = -\frac{\partial V^{i^*(z)}}{\partial z_{i^*(z)}}(z)/||\frac{\partial V^{i^*(z)}}{\partial z_{i^*(z)}}(z)||,\ z \in Z, \tag{33}$$

where $i^*(z)(= 1 \vee 2)$ meets the condition

$$V^{i^*(z)}(z) = V^{1|2}(z) = \min(V^1(z), V^2(z)). \tag{34}$$

This strategy is discontinuous in z on

$$\mathcal{E}^{1|2} = \{z : V^1(z) = V^2(z), z \in Z\}.$$

To construct smooth in z approximations for $S_P^{1|2}$, M_μ^ξ may be used as follows

$$S_P^\xi(z) = -\frac{\partial M_\mu^\xi}{\partial z_P}(V^1(z), V^2(z))/||\frac{\partial M_\mu^\xi}{\partial z_P}(V^1(z), V^2(z))||,\ z \in Z, \tag{35}$$

where

$$\frac{\partial M_\mu^\xi}{\partial z_P}(V^1(z), V^2(z)) = \sum_{i=1,2} \frac{\partial M_\mu^\xi}{\partial v_i}(V^1(z), V^2(z))\frac{\partial V^i}{\partial z_P}(z),$$

$$\frac{\partial V^i}{\partial z_P}(z) = -\frac{1}{1-\beta_i}\frac{z_i - z_P}{||z_i - z_P||},\ z \in Z, i = 1, 2.$$

Note that the time derivatives of $V^{1|2}$ and M_μ^ξ along a trajectory that corresponds to the strategies S_P, S_1, S_2 are described by the following expressions (where exist)

$$\frac{dV^{1|2}}{dt} = \sum_{i=1,2} \frac{\partial \min}{\partial v_i}(V^1(z), V^2(z))\frac{\partial V^i}{\partial z}(z)\frac{dz}{dt},\ \frac{\partial \min}{\partial v_i} \in \{0, 1\},$$

$$\frac{dM_\mu^\xi}{dt} = \sum_{i=1,2} \frac{\partial M_\mu^\xi}{\partial v_i}(V^1(z), V^2(z))\frac{\partial V^i}{\partial z}(z)\frac{dz}{dt},\ \frac{\partial M_\mu^\xi}{\partial v_i} \in [0, 1]$$

where

$$\frac{dz}{dt} = (S_P, (S_1, S_2)),\ z^0 = (z_P^0, (z_1^0, z_2^0)).$$

A locally optimizing strategy corresponds to the case when instant controls minimize/maximize the time derivative of a goal function for any $z \in Z$. Obviously, $S_P^{1|2}$ (see (33)) is locally optimizing for $V^{1|2}$ as well as

$$S_i^{\max} = \beta_i \frac{z_i - z_P}{||z_i - z_P||},\ z \in Z, i = 1, 2, \tag{36}$$

for V^i. S_P^ξ, $\xi > 0$, (see (35)) may be considered as an approximation for $S_P^{1|2}$.

Let E_i use $S_i^{\max}(z) = \beta_i e(\psi_i^{\max})$ and P use $S_P^{\mathcal{E}} = e(\varphi^{\mathcal{E}})$. The state moves along $\mathcal{E}^{1|2}$ if $\varphi^{\mathcal{E}}$ meets the condition

$$\frac{\beta_1}{1-\beta_1} - \frac{\beta_2}{1-\beta_2} - \left(\frac{e(\psi_1^{\max})}{1-\beta_1} - \frac{e(\psi_2^{\max})}{1-\beta_2}\right) e(\varphi^{\mathcal{E}}) = 0. \tag{37}$$

If E_i uses $S_i^{\max}$, $i = 1, 2$, and $\beta_1 = \beta_2 = \beta$, then $S_P^\xi = e(\varphi^\xi)$ keeps the state on $\mathcal{E}^{1|2}$ with the corresponding payoff

$$\tau^{1|2}(z, S_P^\xi, (S_1^{\max}, S_2^{\max})) = +\infty,\ z \in \mathcal{E}^{1|2}, \xi > 0,$$

see Fig. 3.

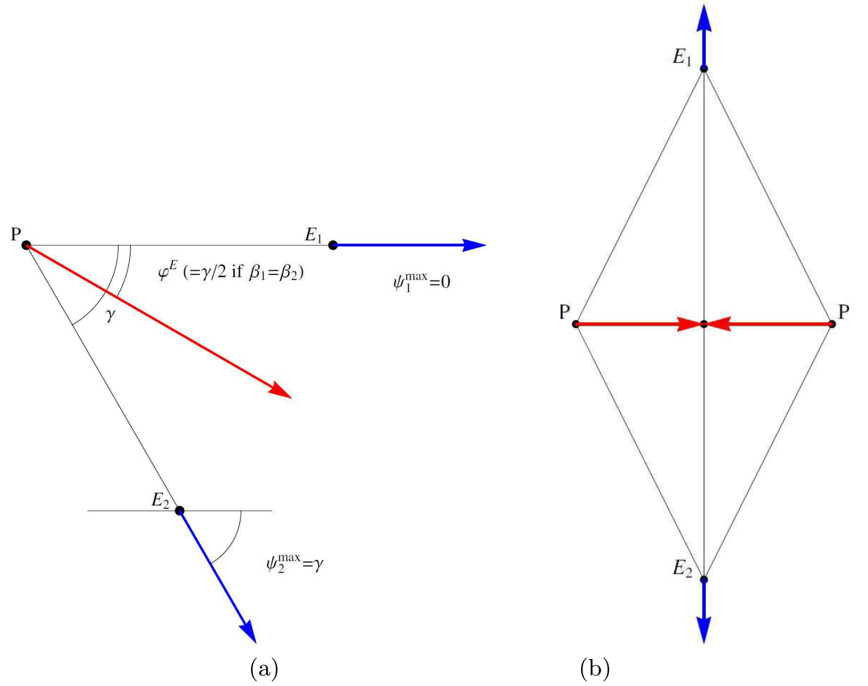


Fig. 3: Instant velocities on $\mathcal{E}^{1|2}$ (a) and trajectories in vicinity of a point of attraction (b)

The inequality

$$\frac{dM_\mu^\xi}{dt} < 0, \tag{38}$$

holds on $\mathcal{E}^{1|2}$ only if $0 \le \gamma < 2\arccos\beta$. Whereas, for $S_P^{i(z^0)}$, $S_1^{\max}$ and $S_2^{\max}$,

$$\frac{dV^{i(z^0)}}{dt} = -1,\ z \in Z. \tag{39}$$

5. Conclusions

In this paper, we provide an analysis on smooth approximations of the minimum function thus allowing construction of the corresponding strategies for a pursuer in an alternative pursuit game with two evaders. As a drawback of the procedure we show through an example how the state may never leave a manifold where two value functions are equal and thus the game never ends if the pursuit strategy is applied.

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Minimax Estimation of Value-at-Risk under Hedging of an American Contingent Claim in a Discrete Financial Market*

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Abstract The game problems between seller and buyer of an American contingent claim relate to large scale problems because a number of buyer's strategies grows overexponentially. Therefore, decomposition of such games turns out to be a fundamental problem. In this paper we prove the existence of a minimax monotonous (in time) strategy of the seller in a loss minimization problem considering value-at-risk measure of loss. The given result allows to substantially decrease a number of constraints in the original problem and lets us turn to an equivalent mixed integer problem with admissible dimension.

Keywords: decision making under uncertainty, value-at-risk, scenario tree, stopping time, hedging.

1. Introduction

A seminal series of papers (Merton, 1973; Black and Scholes, 1973; Shiryaev, 1999) initiated an extensive number of studies on financial asset pricing and minimization of risk associated with failure of contingent claim hedging (building a portfolio of assets to exceed the claim value). The authors assumed that trades occur continuously in time. Consideration of discrete models of a financial market for solving investment problems allowed to apply new methods, particularly ones of mathematical programming and game theory. This is due to the fact that the number of market scenarios is finite.

The first discrete models of contingent claims valuation were examined in (Harrison and Kreps, 1979). This paper proposed a new concept of a discrete market applying stochastic programming approach. The novel idea was to describe a financial market with a scenario tree. They formulated the notions of arbitrage (market condition which permits investment strategies with a guaranteed profit), a self-financing strategy, hedging (implementing the contingent claim) basing on scenario tree framework. The fundamental theorem of asset pricing was presented as well. The problem of maximizing the expected value of terminal portfolio was formulated in (Pliska, 1997). The author derived analytically the amount of initial capital needed for perfect hedging of various contingent claims. In paper (King, 2002) the existence of arbitrage opportunities was analyzed using the duality theory. He stated the linear and nonlinear programming problems to determine optimal buyer

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and seller's strategies. In addition, the author proved a criterion for the existence of an optimal solution in the utility maximization problem.

SPAN system (Standard Portfolio Analysis of Risk) is a good example which illustrates practical use of the discrete models of a financial market. It was introduced at the Chicago Mercantile Exchange in 1988 (Chicago Mercantile Exchange, 1999). This is the portfolio risk assessment methodology which determines the minimum margin requirements to cover losses for one trading period. 16 market scenarios are simulated in the system representing possible ranges of percentage changes in price and volatility of the underlying asset.

The main feature of an American contingent claim is an uncertain moment of exercise (using the right to oblige a seller to make a transaction). So, American claims may be exercised by its buyer at any time $t = \{0, ..., T\}$ up to expiration date. Exercise time is usually considered as an uncertain factor in investment problems. As a result a zero-sum game between the seller and the buyer arises in this scope. Perfect hedging (with probability one) of an American contingent claim generally requires considerable initial endowment from the seller.

There are several common ways to assess the risk of imperfect hedging. The authors of (Föllmer and Leukert, 1999) suggested to use strategies of two types. The first one is quantile hedging. It allows to hedge the contingent claim with the highest probability. This approach does not take the investor's attitude towards the risk into account in contrast to the second type of strategies that minimize a linear function of losses associated with imperfect hedging. The authors proved existence of the optimal solutions for a continuous model of the market using Neyman-Pearson lemma. Perez-Hernandez formulated optimization problems of the described two types of imperfect hedging for financial markets with discrete time and an infinite number of states (Perez-Hernandez, 2007). In this paper he also stated new problems and proved the existence of their optimal solutions under minimizing the initial portfolio endowment and the fixed losses. The paper (Novikov, 1999) considers the analogous problem of minimizing the initial endowment. However, the constraints are more complicated to deal with. The probability of full hedging is bounded from below. There are two tradable assets: risky and riskless ones. It was assumed that the contingent claim can not be exercised until the specific time moment which is optimal for the buyer. Then, the optimal hedging strategies were found in (Lindberg, 2012) for a slightly more general model of the market but a set of exercise times was restricted in a foregoing way. The problem of imperfect hedging from the buyer's perspective was proposed in (Pinar, 2011). An alternative description of the decision making process connected with exercising the claim allowed to formulate the mixed-integer problem which is equivalent to the original one. The paper (Camci and Pinar, 2009) stated a theorem which leads to even more reduction and equivalently turns to finding the optimal solution to the relaxed problem. Pinar (2011) also provides numerical results using real data.

In a present paper we propose value-at-risk (VaR) as a risk measure to estimate the losses from imperfect hedging. It is equal to the minimum value such that the expected losses do not exceed it with a specified probability. In other words, VaR corresponds to the amount of uninsured risk which the seller can take, see first (Rockafellar and Uryasev, 2000) for the details. Nowadays VaR method meets the standards of banking regulation approved by the Basel Committee on Banking

Supervision. This measure is recommended primarily for monitoring market risks and effectiveness of hedging strategies.

VaR approach of risk estimation was widely studied in (Rockafellar and Uryasev, 2000). The authors proved that minimization of VaR, CVaR (conditional value-at-risk that is roughly interpreted as expected losses which exceed VaR value) and Markowitz problem have the same optimum under some conditions. The analytic formula of a CVaR value was obtained in (Rockafellar and Uryasev, 2002) for a discrete model of a financial market. The distribution of future losses was assumed to be known. The paper (Sarykalin et al., 2008) provided detailed comparison of VaR and CVaR. In short, advantages of VaR measure include the fact that it is not subject to errors in the measurement of the biggest losses, assessment of which is rather difficult. The disadvantage of VaR is its non-convexity (in contrast to CVaR) which complicates problem solving in practice.

The rest of the paper is organized as follows. We describe the discrete model of securities market and define the basic notions of subject area in Sect. 2. Sect. 3 formally defines a zero-sum game (strategies of players and a loss function) and introduces a problem of VaR minimization consisted in finding of minimax for the game. We state and prove the main result in Sect. 4. Then, we apply it showing how to substantially reduce a number of constraints in the original problem.

2. The Model of a Financial Market

The market consists of $d+1$ tradable securities, whose prices are denoted at each state n by a non-negative vector $S_n = (S_n^0, ..., S_n^d)$. We assume the security indexed by 0 to be riskless (a bank deposit or a bond), it has strictly positive prices at each state. We choose this asset to be the numeraire and introduce the discounts $1/S_n^0$. Let a vector $X_n = S_n/S_n^0$ denote the discounted security prices relative to the numeraire. Its zero entry X_n^0 equals 1 in any state n.

The set of states $\mathcal{N}$ of the market has a tree structure; see examples of it in Fig. 1 and in (Harrison and Kreps, 1979, p. 393; Pliska, 1997, p. 79). It is divided into pairwise disjoint subsets of states $\mathcal{N}_t$ which may occur at specific time moments $t = 0, ..., T$. The set $\mathcal{N}_0$ contains the only element – a root of the tree denoted by 0. Every node $n \in \mathcal{N}_t$, where $t = 1, ..., T$, has a unique parent $a(n) \in \mathcal{N}_{t-1}$. We put $a(0) = 0$, $a^0(n) = n$, $a^{s+1}(n) = a^s(a(n))$, $s = 1, ..., t$, for all $n \in \mathcal{N}_t$, $t = 1, ..., T$. Next, each node $n \in \mathcal{N}_t$, where $t = 0, ..., T-1$, has a set of child nodes $\mathcal{C}(n) \subset \mathcal{N}_{t+1}$. Let $\mathcal{D}(n)$ be a set of all the nodes which may occur after n, i.e. child nodes, their children and so on ($\mathcal{D}(0) = \mathcal{N} \backslash \{0\}$, $\mathcal{D}(n) = \mathcal{C}(n)$ for all $n \in \mathcal{N}_{T-1}$).

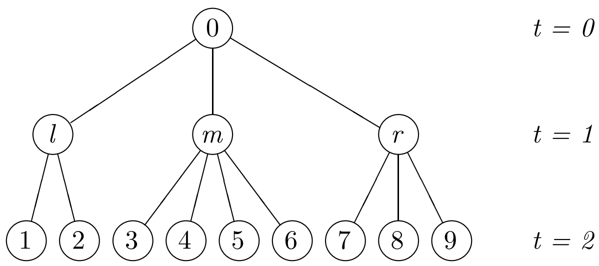

Fig. 1. A scenario tree ($T = 2$, $\mathcal{N}_1 = \mathcal{C}(0) = \{l, m, r\}$, $\mathcal{C}(l) = \{1, 2\} \subset \mathcal{N}_T = \{1, 2, ..., 9\}$).

A unique path $\omega = (n_0, n_1, ..., n_T)$ leads from the root to a leaf node $n \in \mathcal{N}_T$, where $n_0 = 0$, $n_{t-1} = a(n_t) \in \mathcal{N}_{t-1}$ for all $t = 1, ..., T$, $n_T = n$. These paths are interpreted as scenarios of market movement. They form atoms of probability space Ω. The set $\mathcal{N}_t$ partitions Ω into subsets (events). Each of them is defined by a node $n \in \mathcal{N}_t$ and consists of all the paths containing n. The partition generates an algebra $\mathcal{F}_t$ (algebra of events observed up to time moment t). Here, $\mathcal{F}_0 = \{\emptyset, \Omega\} \subset \mathcal{F}_1 \subset ... \subset \mathcal{F}_T$. The family of sets $\{\mathcal{F}_t\}$ is a filtration. Throughout the paper we will consider $\{\mathcal{F}_t\}$-adapted stochastic processes $b = \{b(t)\}$, where a random variable $b(t)$ takes values b_n, $n \in \mathcal{N}_t$, and, thus, is $\mathcal{F}_t$-measurable.

The probability measure $p = (p_n,\ n \in \mathcal{N})$ defined on Ω attaches values $p_n > 0$, $\sum_{n \in \mathcal{N}_T} p_n = 1$ to all the terminal states. The probabilities of other states can be defined consecutively: $p_n = \sum_{m \in \mathcal{C}(n)} p_m$ for all $n \in \mathcal{N}_t$, $t = T-1, ..., 0$. Note that $p_0 = 1$. Suppose that measure p defines true (statistical) probabilities of events. It can be uniquely determined by a probability distribution $p_T = (p_n,\ n \in \mathcal{N}_T)$. To define values of p it is convenient to set conditional measures at first:

$$p(\cdot|n) = (p(m|n) = p_m/p_n,\ m \in \mathcal{C}(n)).$$

They indicate the probabilities of turning from states $n \in \mathcal{N}_t$, $t = 0, ..., T-1$, to the next states $m \in \mathcal{C}(n)$. Then, values of p_n can be derived using the following formula:

$$p_n = \prod_{s=0}^{t-1} p(a^s(n)|\, a^{s+1}(n)).$$

3. Game Description

Let us consider a zero-sum game with two players: a seller of the contingent claim and its buyer. The seller is an investor in wide sense, he builds a trading strategy to hedge the American contingent claim. The buyer exercises the claim in some moment of time (i.e. obliges the seller to pay the claim value using his right specified in a contract). Next, we define strategies of players.

Seller's Strategy

We denote amount of security j held by the investor in state $n \in \mathcal{N}$ by θ_n^j. We will consider a portfolio process $\theta = \{\theta(t)\}$, where the portfolios $\theta_n = (\theta_n^0, ..., \theta_n^d)$, $n \in \mathcal{N}_t$, formed at stage t are the values of a random variable $\theta(t)$. So, the investor has an initial portfolio θ_0 at stage $t = 0$, then he forms a portfolio θ_n in state $n \in \mathcal{N}_1$ (buying some securities and selling others) and so on.

Portfolio process θ is called an investor strategy if a self-financing condition is satisfied:

$$X_n \cdot \theta_n = X_n \cdot \theta_{a(n)},\ \forall\, n \in \mathcal{N}_t,\ t = 1, ..., T.$$

Self-financing means that an investor does not spend money and does not get any revenue from outside. Let

$$Y_n = X_n - X_{a(n)},\ \forall\, n \in \mathcal{N}_t,\ t = 1, ..., T,$$

be the vector of increments of securities prices. Then, $Y_n \cdot \theta_{a(n)}$ means a discounted profit of investor from portfolio $\theta_{a(n)}$ in state n.

Portfolio value process $V = \{V(t)\}$ corresponds to a trading strategy θ. A random variable $V(t)$ takes values V_n equal to scalar products of price and portfolio vectors:

$$V_n = X_n \cdot \theta_n = \sum_{j=0}^{d} X_n^j \theta_n^j, \ \forall\, n \in \mathcal{N}.$$

It is easy to see (Föllmer and Schied, 2011, Prop. 5.7; Pliska, 1997, Prop. 3.2) that $V(t)$ can be represented in the following form for any strategy θ

$$V(t) = V_0 + \sum_{s=1}^{t} Y(s) \cdot \theta(s-1), \ \forall\, t = 1, ..., T.$$

Since each child node of the scenario tree has a unique parent, we may specify all the preceding nodes for each $n \in \mathcal{N}_t$, $t = 1, ..., T$. They are $a^t(n)$, $a^{t-1}(n)$, ..., $a(n)$. Hence, the portfolio values V_n, $n \in \mathcal{N}_t$, are equal to

$$V_n = V_0 + \sum_{s=1}^{t} Y_{a^{s-1}(n)} \cdot \theta_{a^s(n)}, \ \forall\, n \in \mathcal{N}_t, \ t = 1, ..., T.$$

It also follows from (Föllmer and Schied, 2011, Prop. 5.7). We will use these equations later to describe the relationship between trading strategy and portfolio values. To make the dependance more convenient we denote the amount of portfolio value increment up to state n by

$$(Y\theta)_n = \sum_{s=1}^{t} Y_{a^{s-1}(n)} \cdot \theta_{a^s(n)}, \ \forall\, n \in \mathcal{N}_t, \ t = 1, ..., T,$$

and let $(Y\theta)_0 = 0$.

It is said that the market has an arbitrage opportunity if there is a trading strategy θ such that $V_0 \leq 0$ and $V_n \geq 0$ for each $n \in \mathcal{N}\backslash\{0\}$ and at least one of these inequalities meets strictly. Following trading strategy θ, the investor loses nothing and yields a positive profit with a positive probability. Suppose further that there are no arbitrage opportunities in the market.

Strategy θ is called admissible if $V_n \geq 0$ for all $n \in \mathcal{N}$. We will consider only admissible trading strategies because they prevent the investor from ruin.

Remark 1. It is easy to show (King, 2002, p. 546) that strategy θ is admissible for arbitrage-free markets if $V_n \geq 0$ for all terminal states $n \in \mathcal{N}_T$. Indeed, otherwise suppose that portfolio value is negative in some state $m \in \mathcal{N}\backslash\mathcal{N}_T$ and $V_n \geq 0$ for all terminal states which follow m ($n \in \mathcal{N}_T \cap \mathcal{D}(m)$). Then, the investor may guarantee a positive profit for all future market scenarios.

Buyer's Strategy

Buyer's strategy is a moment of time when the contingent claim is exercised – a stopping time. Let us describe it with a random variable

$$\tau : \Omega \to \{0, ..., T\}$$

for which $\{\tau = t\} \in \mathcal{F}_t$. We use $\mathcal{T}$ to denote a finite set of all buyer's strategies.

Exercise time τ produces the only state $n_{\tau(\omega)} \in \mathcal{N}$, where stopping occurs for each simple event $\omega = (n_0, ..., n_T) \in \Omega$. Let us denote the set of such states as $\mathcal{N}_\tau$. It can be seen that a set $\tilde{\mathcal{N}} \subset \mathcal{N}$ conforms to some exercise time τ (in the sense that $\tilde{\mathcal{N}} = \mathcal{N}_\tau$) if there exists exactly one element of this set in each sequence of consecutive states $(n_0, ..., n_T)$.

A set of buyer's strategies grows very quickly while a number of trading periods T increases. A number of exercise times can be determined recursively. Let K_n denote it for the subtree with node n and other nodes $\mathcal{D}(n)$. Then

$$K_n = 1,\ \forall\, n \in \mathcal{N}_T,\ \ K_n = 1 + \prod_{m \in \mathcal{C}(n)} K_m,\ \forall\, n \in \mathcal{N}_t,\ t = T-1, ..., 0.$$

There is an exact formula for the value of $\mathcal{T}$ for specific cases when a number of child nodes $\mathcal{C}(n)$ is constant for all $n \in \mathcal{N} \backslash \mathcal{N}_T$ and equals 2 or 3. It is the following:

$$|\,\mathcal{T}| = \left[k^{c^T}\right], \tag{1}$$

where $[x]$ is an integral part of x, $k \approx 1{,}5028$ when $c = |\,\mathcal{C}(n)| = 2$ (Aho and Sloane, 1973), and $k \approx 1{,}2766$ when $c = |\,\mathcal{C}(n)| = 3$ (McGarvey, 2007). A number of buyer's strategies (exercise times) is shown in Table 1 for different values of T.

Table 1: A number of exercise times under different numbers of trading periods T.

T	0	1	2	3	4	5
$\lvert\mathcal{T}\rvert$, where $c = 2$	1	2	5	26	677	458330
$\lvert\mathcal{T}\rvert$, where $c = 3$	1	2	9	730	389017001	$\approx 5{,}9 \times 10^{25}$

American contingent claim

We describe an American contingent claim with a non-negative stochastic process $F = \{F(t)\}$, where a random variable $F(t)$ takes discounted values F_n with probability p_n, $n \in \mathcal{N}_t$, $t = 0, ..., T$. The simple example of a contingent claim is an option payment. Portfolio strategy θ hedges an American contingent claim F exercised in time τ if the corresponding portfolio value process V satisfies $V_n \geq F_n$ for all $n \in \mathcal{N}_\tau$.

Suppose that the seller does not have a necessary sum for perfect hedging and decides to manage with less initial endowment taking the risk of future losses. So, if the claim is exercised in state $n \in \mathcal{N}$ of the market, then seller's losses are equal to $(F_n - V_n)^+ = \max\{F_n - V_n;\ 0\}$. Let us evaluate seller's losses in exercise time τ using the value-at-risk function:

$$\mathrm{VaR}_\alpha\big((F(\tau) - V(\tau))^+\big) = \min\{B \in \mathbb{R}\,|\,\mathbb{P}\big((F(\tau) - V(\tau))^+ \leq B\big) \geq \alpha\},$$

where α is a preset level of significance which is usually not less than 95%.

Therefore, we defined a zero-sum game between a seller of the claim and its buyer. Let us state the optimization problem from the seller's side to find an optimal investment strategy (V, θ) which imperfectly hedges contingent claim F and minimizes a loss function VaR_α under uncertain exercise time τ. The given problem consists in finding a minimax value of the game and can be formulated in the following way:

$$\begin{aligned}&\min_{(V,\,\theta)}\max_{\tau\in\mathcal{T}}\mathrm{VaR}_\alpha((F(\tau)-V(\tau))^+)\\&V_n=v+(Y\theta)_n\geq 0,\ \forall\, n\in\mathcal{N}.\end{aligned}\tag{2}$$

4. Conversion of an Original Problem

Let us introduce an auxiliary variable u to bound the maximum of (2) from above. Then, we may rewrite the problem (2)

$$\min_{(V,\,\theta,\,u)} u$$
$$\begin{cases}u\geq \mathrm{VaR}_\alpha((F(\tau)-V(\tau))^+)\,, & \forall\,\tau\in\mathcal{T}\\ V_n=v+(Y\theta)_n\geq 0, & \forall\, n\in\mathcal{N}.\end{cases}$$

Next, we use the definition of VaR and introduce variables B_τ for all $\tau\in\mathcal{T}$. Hence:

$$\min_{(V,\,\theta,\,u)} u$$
$$\begin{cases}u\geq \min\limits_{B_\tau\in X(V,\tau)} B_\tau, & \forall\,\tau\in\mathcal{T}\\ V_n=v+(Y\theta)_n\geq 0, & \forall\, n\in\mathcal{N},\end{cases}\tag{3}$$

where $X(V,\tau)=\{B_\tau\in\mathbb{R}\,|\,\mathbb{P}((F(\tau)-V(\tau))^+\leq B_\tau)\geq\alpha\}$ for any fixed V and τ. Now we show that problem (3) can be equivalently reduced to the following one:

$$\min_{(V,\,\theta,\,u)} u$$
$$\begin{cases}\mathbb{P}((F(\tau)-V(\tau))^+\leq u)\geq\alpha, & \forall\,\tau\in\mathcal{T}\\ V_n=v+(Y\theta)_n\geq 0, & \forall\, n\in\mathcal{N}.\end{cases}\tag{4}$$

Indeed, for any $B_\tau\in X(V,\tau)$ and $u\geq B_\tau$

$$\alpha\leq\mathbb{P}\big((F(\tau)-V(\tau))^+\leq B_\tau\big)\leq\mathbb{P}\big((F(\tau)-V(\tau))^+\leq u\big)\,.$$

Conversely, for optimal solution (V^*,θ^*,u^*) of (4) we may put

$$B^*_\tau=\mathrm{Arg}\min_{B_\tau\in X(V^*,\tau)}B_\tau=\max_{n\in\mathcal{N}_\tau}(F_n-V^*_n)^+\leq u^*.$$

The first group of constraints in (4) shows that losses do not exceed u with probability not less than α for all the exercise times. So, when the seller determines his investment strategy, he separates all the states of the market into two groups whether planned losses exceed u or not. Let us incorporate binary variables $x_n\in\{0,1\}$ for all $n\in\mathcal{N}$ which represent the seller's choice of states. Then, the problem (4) has the following reformulation:

$$\min_{(x,\,V,\,\theta,\,u)} u$$
$$\begin{cases}\sum\limits_{n\in\mathcal{N}_\tau} p_n x_n\geq\alpha, & \forall\,\tau\in\mathcal{T}\\ V_n\geq x_nF_n-u, & \forall\, n\in\mathcal{N}\\ V_n=v+(Y\theta)_n\geq 0, & \forall\, n\in\mathcal{N}\\ u\geq 0, & \\ x_n\in\{0,1\}\,, & \forall\, n\in\mathcal{N}.\end{cases}\tag{5}$$

Indeed, $u \geq (F_n - V_n)^+$ if $x_n = 1$. The constraint $V_n \geq x_n F_n - u$ becomes redundant if $x_n = 0$.

Direct solving of (5) is complicated by a huge number of coupling constraints which correspond to all possible exercise times $\tau \in \mathcal{T}$. A lot of binary variables remains an issue to deal with as well. Next theorem proves the main outcome of this study – the existence of optimal solution for problem (5) such that x^* has a monotonic nature over time. Namely, we will show that

$$x_n^* \geq x_m^*,\ \forall\, m \in \mathcal{C}(n),\ n \in \mathcal{N} \backslash \mathcal{N}_T. \tag{6}$$

It can be interpreted in the following way. For each scenario $\omega = (n_0, ..., n_T)$, i.e. for each sequence of consecutive nodes of the scenario tree, leading from the root to a leaf node, the following is true: if $x_{n_t}^* = 0$, then $x_{n_s}^* = 0$ for each $s = t+1, ..., T$, hence we only want the portfolio value to be non-negative from a state n_t up to the terminal moment of time.

Theorem 1. *There will always be an optimal solution* $(x^*, V^*, \theta^*, u^*)$ *of* (5) *such that* x^* *satisfies the monotone condition* (6).

Proof. Let us fix an optimal solution $(x^*, V^*, \theta^*, u^*)$ and define a process $\tilde{x} = \{\tilde{x}(t)\}$ recursively for $t = T, ..., 0$:

$$\tilde{x}_n = x_n^*,\ \forall\, n \in \mathcal{N}_T,\ \tilde{x}_n = \min\left\{ x_n^*,\ \sum_{m \in \mathcal{C}(n)} p(m|n)\tilde{x}_m \right\},\ \forall\, n \in \mathcal{N}_t,\ t = T-1, ..., 0.$$

A stochastic process $\tilde{x}$ is analogous to Snell envelope (Föllmer and Schied, 2011, p. 285) and turns out to be a submartingale, i.e.

$$\tilde{x}_n \leq \sum_{m \in \mathcal{C}(n)} p(m|n)\tilde{x}_m,\ \forall\, n \in \mathcal{N}_t,\ t = 0, ..., T-1.$$

Besides, we conclude the following from the definition of process $\tilde{x}$:

$$\min_{\tau \in \mathcal{T}} \sum_{n \in \mathcal{N}_\tau} p_n x_n^* = \min_{\tau \in \mathcal{T}} \sum_{n \in \mathcal{N}_\tau} p_n \tilde{x}_n = \tilde{x}_0.$$

Next, we define the stopping rule $\tilde{\tau} \in \mathcal{T}$ for each scenario $\omega = (n_0, ..., n_T) \in \Omega$, where $n_t \in \mathcal{N}_t$:

$$\tilde{\tau}(\omega) = \begin{cases} T, \text{ if } x_{n_t}^* \geq \displaystyle\sum_{m \in \mathcal{C}(n_t)} p(m|n_t)\tilde{x}_m,\ \forall\, n_t \in \omega,\ t = 0, ..., T-1, \\ \min\left\{ t \,\middle|\, \tilde{x}_{n_t} = x_{n_t}^* < \displaystyle\sum_{m \in \mathcal{C}(n_t)} p(m|n_t)\tilde{x}_m \right\},\ \text{otherwise.} \end{cases} \tag{7}$$

According to definition of $\tilde{\tau}$ the buyer stops and exercises the contingent claim until T if $\tilde{x}_{n_t} = x_{n_t}^* = 0$. Therefore, (7) is equivalent to

$$\tilde{\tau}(\omega) = \begin{cases} T, & \text{if } x_{n_t}^* = 1,\ \forall\, n_t \in \omega,\ t = 0, ..., T-1, \\ \min\left\{ t |\, x_{n_t}^* = 0 \right\}, & \text{otherwise.} \end{cases}$$

In other words, we stop in the first possible state where $x_n^* = 0$ or in the final state if there was no stopping before that. Let us note that

$$\tilde{\tau} \in \operatorname*{Argmin}_{\tau \in \mathcal{T}} \sum_{n \in \mathcal{N}_\tau} p_n x_n^*.$$

For all states $m \in \mathcal{D}(n)$ which occur after $n \in \mathcal{N}_{\tilde{\tau}}$ we may put $x_m^* = 0$. Indeed, $V_m^* \geq F_m x_m^* - u^*$ and a value $\min_{\tau \in \mathcal{T}} \sum_{n \in \mathcal{N}_\tau} p_n x_n^*$ does not change. Minimum value of the objective function u remains unchanged too. Therefore, the optimal process x^* satisfies the monotone condition (6). $\square$

We use this theorem to reduce the problem (5) excluding the exercise times:

$$\min_{(x,\, V,\, \theta,\, u)} u$$
$$\begin{cases} \displaystyle\sum_{n \in \mathcal{N}_T} p_n x_n \geq \alpha, & \\ x_n \geq x_m, & \forall\, m \in \mathcal{C}(n),\, n \in \mathcal{N} \backslash \mathcal{N}_T \\ V_n \geq x_n F_n - u, & \forall\, n \in \mathcal{N} \\ V_n = v + (Y\theta)_n \geq 0, & \forall\, n \in \mathcal{N} \\ u \geq 0, & \\ x_n \in \{0, 1\}, & \forall\, n \in \mathcal{N}_T. \end{cases} \tag{8}$$

Corollary 1. *Any optimal solution of* (8) *is optimal for* (5) *too.*

Proof. A set of feasible solutions of (5) can be reduced by Theorem 1 incorporating the monotone condition. First, we show that a minimum value of the objective function does not change if we remove the constraint $x_n \in \{0, 1\}$ for all $n \in \mathcal{N} \backslash \mathcal{N}_T$. Non-negativity $x_n \geq 0$ for all $n \in \mathcal{N}$ follows from this system of inequalities

$$x_{a^T(n)} \geq \ldots \geq x_{a(n)} \geq x_n \geq 0,\ \forall\, n \in \mathcal{N}_T.$$

If there is a state $\tilde{n} \in \mathcal{N} \backslash \mathcal{N}_T$ such that $x_{\tilde{n}} > \max_{m \in \mathcal{C}(\tilde{n})} x_m$, then we may put $x_{\tilde{n}} = \max_{m \in \mathcal{C}(\tilde{n})} x_m$ decreasing $x_{\tilde{n}}$. Indeed,

$$V_{\tilde{n}} \geq F_{\tilde{n}} x_{\tilde{n}} - u > F_{\tilde{n}} \max_{m \in \mathcal{C}(\tilde{n})} x_m - u,\ \ x_{a(\tilde{n})} \geq x_{\tilde{n}} > \max_{m \in \mathcal{C}(\tilde{n})} x_m.$$

So, $x_n = \max_{m \in \mathcal{C}(n)} x_m$ for each $n \in \mathcal{N} \backslash \mathcal{N}_T$, that is why $x_n \in \{0, 1\}$ for all $n \in \mathcal{N}$. Decreasing of x does not change the values of x_T. Using monotone condition (6), we conclude that

$$\sum_{n \in \mathcal{N}_\tau} p_n x_n \geq \sum_{n \in \mathcal{N}_T} p_n x_n$$

for all exercise times $\tau \in \mathcal{T}$. Thus,

$$\sum_{n \in \mathcal{N}_T} p_n x_n = \min_{\tau \in \mathcal{T}} \sum_{n \in \mathcal{N}_\tau} p_n x_n \geq \alpha.$$

Therefore, turning to (8) is equivalent in the sense that the objectives achieve the same minimum values and any optimal solution of (8) remains optimal for (5). $\square$

Equivalent reduction to problem (8) allows to change $|\mathcal{T}|$ linear coupling constraints into $|\mathcal{N}|$ monotone conditions and one constraint which couples terminal values x_T. We remark here that a number of nodes $|\mathcal{N}|$ in the scenario tree grows exponentially with increasing number of trading periods T. For a constant number of child nodes ($|\mathcal{C}(n)| = c$ for all $n \in \mathcal{N}\backslash\{0\}$) it equals a sum of geometric series:

$$|\mathcal{N}| = \frac{c^{T+1} - 1}{c - 1}. \tag{9}$$

To estimate a number of monotone conditions if a number of child nodes is non-constant and does not exceed c we may bound $|\mathcal{N}|$ above with the fraction of (9). One should compare this formula with (1) to see clearly the effect of Theorem 1.

5. Conclusion

In this study we suppose that security trading in financial market occurs in deterministic time moments and a market has a finite number of scenarios. If there are a lot of trading periods and market scenarios, we deal with large-scale problems. There are no transaction costs during the trading and the market has no arbitrage opportunities.

Here for the first time we state the problem of finding the investment strategy which produces the minimal losses associated with imperfect hedging of American contingent claim using VaR measure. The main problem has a game form and consists in finding a minimax value of a specific zero-sum game. The main result of this study states that the seller always has a minimax strategy which is monotonous over time. It allows us to not only reduce the dimension of the original optimization problem, but also actually exclude the uncertainty associated with the time of exercising the contingent claim. The outcome can be used to create the software systems for financial institutions which deal with valuation and hedging of contingent claims, building trading strategies and so on.

In future research we are planning to investigate the formulations of the main game problem in which the seller may use mixed strategies and the buyer may use behavioral ones.

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CEO Incentive Plans Improvement in the U.S. Public Companies on the Base of Game Theoretical Modeling*

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Abstract The paper is aimed at improving the mechanism of forming the variable part of CEO compensation. The novelty of the given research paper is improving the methodology of evaluation the value of variable part of CEO compensation with the chosen model, so it can be applied on practice. The model is game theoretical interpretation of the principal-agent phenomenon whose objective is to model the variable part of CEO compensation to stimulate strategy implementation In detail, 14 company cases of the U.S. public companies in retail and technology industries were presented, the applicability of the model was proven and suggestions for methodology improvement were made.
Keywords: corporate governance, agency problem, CEO compensation, game theory, theoretical modeling, U.S. public companies.

1. Introduction

The research deals with the problem of CEO compensation value modeling which is one of the core issues of corporate governance. In theory, contracts should be designed by boards of directors to maximize company value. Contacts should attract and retain talented CEOs, incentivize them to exert high level of efforts to implement the company's strategy and ensure its competitive advantage.

To begin with, CEO compensation structure usually consists of base salary and variable part. Base salary of CEO is less dependent on performance compared to variable part of compensation and is usually determined by the reputation of a manager, his experience at managing companies, size of a considered company, certain industry specifics and the level of CEO base salary across the chosen industry. Contrary, variable part of CEO compensation is directly dependent on performance of a company. According to Frydman and Saks (2010), a variable part of top management compensation in form of option grants and cash bonuses has been prevalent since 1950s in the U.S. public companies.

Traditionally, a variable part of executive compensation is considered as a tool for solving the agency problem, that is caused by the conflict of interests between an agent (CEO) and a principal (company owners). Principal owns capital and delegates responsibility to manage it in his/her interest to the agent, however, because of the conflict of interests in separation of profits gained by the company between two parties, temptation of ex post opportunistic behavior occurs for the agent. That is why the mechanism of determining the value of variable part of CEO compensation, which eliminates motivation for opportunistic behavior, should be worked out.

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There are quite a lot of scientific studies on the topic of executive compensation studies. Prevailing optimal contract theory (pioneered by Holmstrom (1979), Grossman and Hart (1983)) claims that a compensation program can be constructed so that interests of shareholders and CEOs are aligned and the most talented CEOs at a competitive market are attracted and retained due to fair remuneration of their talents and efforts. On the other hand, managerial power theory (Bebchuk and Fried, 2004) argues that high value of executive compensation is the result of CEOs' rent-seeking behavior.

Despite vast research conducted on the matter of executive compensation, none of the existing theories offers a fully coherent explanation for historical evolution of executive compensation during the 1970s, some of the cross-sectional and time-series patterns in the data, and provides a convincing mechanism designing consistent compensation programs. Thus, the goal of the research paper was to improve the mechanism of forming the incentive plan of CEO compensation based on the existing theoretical models and approaches, and test the applicability of this mechanism for the U.S. public companies.

Mandatory disclosure of executives' compensation in the U.S. public companies added transparency to the issue in question in 1992 and mirrored disconnect between pay of executives and average employees. Additionally scandalous cases such as Enron, Tyco and WorldCom of the early 2000s and the Great Recession concluding the late 2000s attracted more attention to corporate governance issues, raising a question of necessary cuts in compensations and more rigorous monitoring of the CEO activities. In the 2000s compensations still remained generous but decreased in value in comparison to the peaking year 2000. One example of outrageous compensation is a case of the former J.C. Penney CEO compensation that in 2012 amounted to 53.5 million USD and exceeded an average worker wage by 1795 times

The U.S. publicly traded companies (without a controlling shareholder) were in focus of our analysis. Confirmed by research and scandalous media examples, when ownership and management are separated (like in public companies), CEOs might abuse substantial power to enjoy individual benefits without putting additional efforts into the company management.

Even though ownership structure in U.S. and Russian public companies is different, (scattered ownership in the U.S. vs. concentrated ownership in Russia) conclusions derived from the analysis of the U.S. compensation programs could be applied to some extent in the Russian environment.

2. Executive compensation problem

The subject of the research is executive compensation (interchangeably: compensation program or compensation package) for CEOs that incentivizes top managers to align their efforts with owners' expectations. Therefore, we refer to corporate governance, a system by which companies are controlled, directed and made accountable to shareholders and other stakeholders (Demirag, 1998). Since the emergence of formalization of the problem, academic literature on the subject has been enormous, spanning around accounting, finance, economics, law, strategy, organizational behavior and other disciplines.

As was mentioned earlier, the modern history of executive compensation research was evolving in parallel with theory on the principal-agent problem that was starting to be generally accepted in the early 1980s. To apply principal-agent approach to our research, we stated that under the agent we understand the CEO (interchangeably: chief executive or top manager) and under the principal we understand the shareholders and their proxy – the board of directors.

Both parties have utility functions. The utility of the principal depends on behavior of the agent: he wants the agent to behave in a way that maximizes his own, the principal's, utility. Since the agent maximizes his own utility and ownership stake in the company is rather small, his actions may contradict the interests of the principal who owns the company and would like its value to be maximized. Information asymmetry prevents the principal from obtaining direct information on the agent's efforts and actions. The agent's utility is assumed as his compensation less opportunity costs, a.k.a. efforts put into value creation, whereas the principal's utility function is the return on investment or value of the company.

Separation of ownership and control within the company is a cornerstone of corporate governance; it has been a central concern since the early 20th century (Berle and Means, 1932); therefore, the main driver of public companies analysis. Different problems arise due to the fact that interests of owners and managers vary whereas corporate governance tends to resolve these conflicts between different stakeholders in a public company (Kenneth, Nofsinger, 2004).

Another important aspect of executive compensation research is sensitivity to the company performance (pay-performance sensitivity). The earlier group of studies tried to find dependencies between changes in executive compensation and stock prices and was criticized to concentrate only on current remuneration but not on executive cumulative wealth (Murphy, 1985; Bernston, 1985). Jensen and Murphy (1990) integrated various factors and assessed relationship between the company performance and CEO wealth for large U.S. public companies for the time period of 1974-1986 (dollar change in wealth for a dollar change in the company value). Hall and Liebman (1998) continue research proposing to assess dollar-percentage change (equity-at-stake as measure of CEO incentives). Thus different measures in assessment of pay-performance sensitivity lead to different magnitude of incentives. Baker and Hall (2004) demonstrate that the measure of incentives is dependent upon CEO operations-company value relationship. Since 1990s the strength of pay-performance hypothesis has been questioned by various researchers, remaining one of the major issues of the executive compensation theory.

The principal's payoff – shareholders value – is understood differently in different models. Older models generally tend to consider company profit as the value to be maximized whereas contemporary models usually follow ideas of value-based management, so shareholders seek for company value maximization. Therefore, modern models of executive compensation use market capitalization instead of profit as the principal's value.

Within the principal-agent problem, traditionally, CEO compensation is either an instrument to solve the principal-agent problem (optimal contracting approach) or is itself a part of the principal-agent problem (managerial power approach).

3. Executive compensation model

Theoretical model

Current research studies on executives' compensation investigate dependencies compensation and other variables, including performance. The limitation of these research papers is that these models are used as purely theoretical, intended to get qualitative findings. As a result, there is lack of convincing explanations of compensation evolution starting from 1970s and explicit recommendations for construction of compensation packages, incentive plans in particular.

Under the requirements mentioned in theoretical background of the paper, a special theoretical model, developed by Casamatta and Guembel in 2007, was used in order to obtain quantitative results and practical recommendations for CEOs' incentive plan in 10 case studies. In their article Managerial Legacies, Entrenchment and Strategic Inertia Casamatta and Guembel consider two models. The first model implies one strategy for both periods but allows the principal to change the agent after the first period if he is not satisfied with his performance. The second model assumes that after the first period the principal can change the strategy and/or the agent. We have chosen a modified model since it appears more realistic. Usually after the first phase of strategy implementation if performance goals are not attained, the board of directors can question the effectiveness of the strategy and implementation efforts of the CEO.

The model is a game theoretical interpretation of the principal-agent phenomenon whose objective is to model the incentive plan of CEO compensation (performance-based pay component) to stimulate strategy implementation. The principal (owner, shareholder, investor) hires the agent (CEO, manager) to choose a company strategy to implement in the subsequent time, followed by the principal's decision to terminate or not the contract with the current CEO. The underlying assumption for the model is that the company strategy can be amended in both periods. In order to design the model the following assumptions were considered:

1. There are two players in the game – principal (owner / investor / shareholder; Board of directors can be a proxy for the owner) and agent (CEO / manager); interaction is happening within the company scope.
2. Interaction between shareholder and top manager happens during 2 periods, $t \in \{1, 2\}$.
3. At the beginning of the 1st period the principal hires the agent and signs a contract regarding his/her compensation, $w(R)$, where w is incentive plan of the agent's compensation and R is the Company performance during one period.
4. The agent can be of two types: H – high type and L – low type. The high type manager always chooses a successful strategy $S_0 = G$ whereas the low type manager chooses a poor, non-successful strategy $S_0 = B$. The probability that CEO is of high type H (before strategy implementation in the Company) is denoted as $q_0 \geq 0.5$ and called CEO's reputation. The type of CEO is not known to the principal or the agent him/herself. Reputation of the agent after the 2nd and the 1st period are denoted as follows: $q^{i,j} = prob\,(M = H \mid R_1 = R_i \text{ and } R_2 = R_j)$ and $q^i = prob\,(M = H \mid R_1 = R_i)$, $i, j \in \{l, h\}$ respectively.
5. In order to execute the chosen strategy the agent has to choose whether to exert high or low efforts $e_1 \in \left\{\underline{e_1}, \overline{e_1}\right\}$; efforts are non-observable for the principal (which reflect the essence of the principal-agent problem). High level of efforts $\overline{e_1}$ means individual costs c for the manager. The difference between high and low levels of efforts is expressed by the following formula:

$$\triangle e_1 = \overline{e_1} - \underline{e_1}.$$

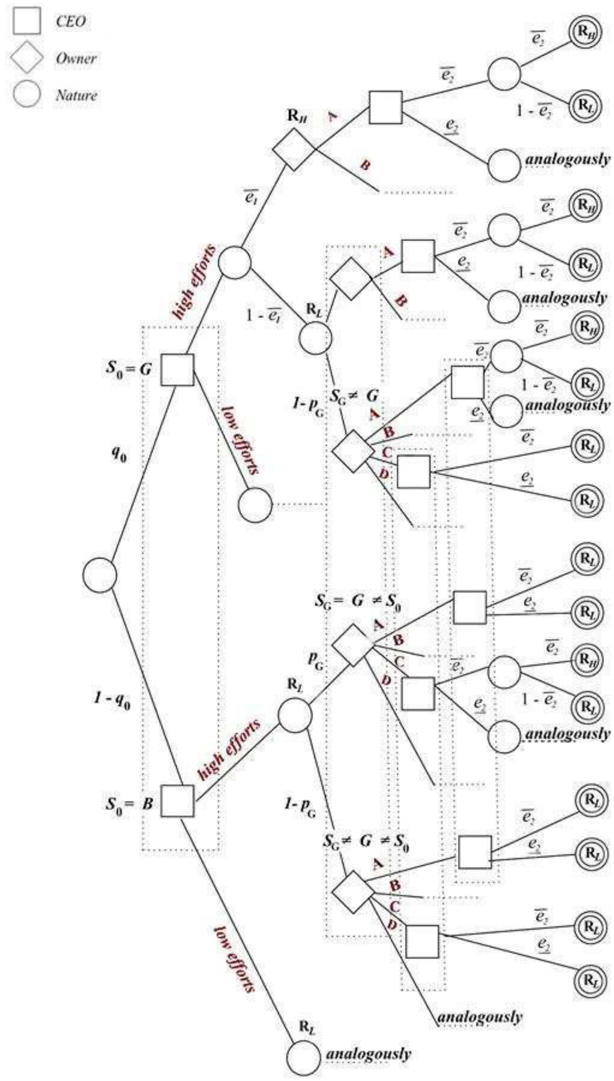

Fig. 1: Game tree

6. The nature also participates in the game. If CEO chooses the successful strategy $S_0 = G$, then the Company performance is high R_h with probability e_1 and low $R_l = 0$ with probability $(1 - e_1)$. If the chosen strategy is unsuccessful, $S_0 = B$, the Company performance is low $R_l = 0$ with probability equal to 1.
7. At the end of the 1st period the principal receives an information signal s_G regarding the needed strategy. We assumed that $p_G = Prob(s_G = G)$ is probability that the signal identifies the successful strategy.
8. The principal makes a decision related to the strategy choice for the 2nd period. If the Company performance after the 1st period is high R_h, there is no value in

changing the strategy, thus $S_1 = S_0 = G$. However if the Company performance is low $R_l = 0$, the principal considers the signal s_G: s/he observes whether the signal confirms the choice of the strategy. If $s_G = S_0$, the strategy is not to be amended; otherwise $S_1 \in \{s_G, S_0\}$.

9. Afterwards the owner decides whether to leave the CEO or terminate the contract with him and hire a new CEO.
10. In the 2nd period the CEO (old or new) decides whether to exert high or low efforts $e_2 \in \left\{\underline{e_2}, \overline{e_2}\right\}$; analogously efforts are non-observable for the owner. Again high efforts of the manager correspond to individual costs c for the manager. The difference between high and low levels of efforts is expressed by the analogous following formula:
$$\triangle e_2 = \overline{e_2} - \underline{e_2}.$$
11. If the applied strategy is successful $S_1 = G$, the Company performance is high R_h with probability e_2 and low R_l with probability $(1 - e_2)$. In case of the unsuccessful strategy $S_1 = B$ the Company performance is low R_l with probability equal to 1.

As it has been already mentioned, the chief executive cares not only for his monetary contract but also for his reputation after the strategy implementation or contract termination. Let us denote the CEO's reputation after period i as q_i, the definition of reputation is probability that the manager is of high type H provided the Company performs well or poorly (R_h or R_l respectively) and whether the Company strategy is amended or not in the 2nd period.

Let us denote the CEO value as $f(q)$ provided s/he has a reputation q; the formula representation is provided below:

$$f(q) = \alpha q, \tag{1}$$

where $\alpha > 0$.

The reputation of the agent keeps updating even if the contract with him/her was terminated after the 1st period. Only reputation of the first, old, CEO who made a strategic decision to implement is considered in the model. A new CEO has no reputational risks as he is not the one who chooses the strategy.

Let us find the value of reputation q with Bayes' formula:

1. If $R_1 = R_h$, also $S_1 = S_0$ and $R_2 = R_h$, then $q = q^h = 1$.
2. If $R_1 = R_l$, $S_1 = S_0$ and $R_2 = R_l$, then
$$q = q_0^{l,l} = \frac{q_0(1-e_1)(1-p_G)(1-e_2)}{q_0\,(1-e_1)\,(1-p_G)\,(1-e_2) + 1 - q_0}. \tag{2}$$
3. If $R_1 = R_l$, $S_1 \neq S_0$ and $R_2 = R_l$, then
$$q = q_1^{l,l} = \frac{q_0(1-e_1)(1-p_G)}{q_0\,(1-e_1)\,(1-p_G) + (1-q_0)(p_G\,(1-e_2) + (1-p_G))}. \tag{3}$$
4. If $R_1 = R_l$, $S_1 \neq S_0$ and $R_2 = R_h$, then $q = q_1^{l,h} = 0$.

Interaction between the owner and CEO is represented in the form of a decision tree in Fig. 1. Dotted lines incorporate the same information sets, in other words the player with the move cannot differentiate between nodes within the information set. Several branches are not depicted in detail due to the fact that the outcome will never occur. Branches where CEO exerts low efforts are analogous to branches where s/he exerts high efforts; the only difference is in probabilities. Also. There are 4 alternatives for the owner: A – not change the strategy nor the CEO; B – not change the strategy, hire a new CEO; C – change the strategy and hire a new CEO; D – change the strategy, leave the old CEO.

Payoffs of each player are described as follows:

1. If the contract with the agent is not terminated, then he receives a sum of payoffs for two periods. If he gets fired, he receives compensation only for the 1st period while the new manager receives compensation for the 2nd period. Let us denote the following:
   w^i is CEO's compensation for the 1st period provided $R_1 = R_i$, where $i \in \{h, l\}$;
   $w^{i,j}$ is CEO's compensation for the 2nd period provided $R_1 = R_i$, $R_2 = R_j$ where $i, j \in \{h, l\}$;
   $w^{i,j}_{new}$ is a new CEO's compensation for the 2nd period provided that a new manager is hired and $R_1 = R_i$, $R_2 = R_j$ where $i, j \in \{h, l\}$.
2. The principal's payoff is equal to a sum of Company performance figures for two periods less compensation of the agent(s).

Solution of the model. Compensation contract is accounted for the solution of the model. Equilibrium strategies for the principal and the agent constitute the overall Nash equilibrium; the model is solved by backward induction.

Let us consider the last move of the game where the top manager makes a decision about the level of efforts. In each sub-game the manager has 2 alternatives: exert high level of efforts $\overline{e_2}$ or shirk and exert low level of efforts $\underline{e_2}$. High efforts mean higher payoff for the principal.

Let us denote conditional probability that executed strategy of the 2nd period is successful (accounted for the Company performance in the 1st period and the fact whether the strategy has been changed or not) as p:

$$p = \begin{bmatrix} 1 \text{ if } R_1 = R_h \text{ or } s_G = S_0, \\ p^0 \text{ if } R_1 = R_l,\ s_G \neq S_0 \text{ and } S_1 = S_0, \\ p^1 \text{ if } R_1 = R_l,\ s_G \neq S_0 \text{ and } S_1 = s_G, \end{bmatrix} \tag{4}$$

where

$$P^0 = \frac{q_0(1 - e_1)(1 - p_G)}{q_0\,(1 - e_1)\,(1 - p_G) + 1 - q_0}, \tag{5}$$

$$P^1 = \frac{p_G(1 - q_0)}{q_0\,(1 - e_1)\,(1 - p_G) + 1 - q_0}. \tag{6}$$

In order to find compensation value we are required to solve linear programming problem: the principal maximizes his/her expected payoff for the 2nd period by minimizing the agent's expected compensation. The objective function looks as follows:

$$\min\left[p\left(\overline{e_2}\ w^{i,h} + (1 - \overline{e_2}\ )\, w^{i,l}\right) + (1 - p)w^{i,l}\right].$$

Subject to:

$$w^{i,h} - \ w^{i,l} \geq \frac{c}{p\triangle e_2} - \triangle f,$$

$$p(\overline{e_2}w^{i,h} - \ (1 - \overline{e_2})\, w^{i,l} + (1-p)w^{i,l} \geq c,$$

$$w^{i,h} \geq 0,\ w^{i,l} \geq 0.$$

There are four possible outcomes:

1. $R_1 = R_h$. It is not feasible to change the strategy and therefore results are equivalent to the Base game in App. 1:

$$w^{h,h} = \frac{c}{\triangle e_2}, \tag{7}$$

$$w^{h,l} = 0. \tag{8}$$

Compensation is the same for the old and new CEOs.

2. $R_1 = R_l$, $s_G = S_0$, then $p = 1$. Compensation for the old CEO is the following:

$$w^{l,h}_{S_1=s_G=S_0} = \max\left[\frac{c}{p\triangle e_2} - \triangle f; \frac{c}{\overline{e_2}}\right], \tag{9}$$

$$w^{l,l}_{S_1=s_G=S_0} = 0. \tag{10}$$

3. $R_1 = R_l$, $s_G \neq S_0$ but $S_1 = S_0$, then $p = p^0$, compensation for the old CEO is:

$$w^{l,h}_{S_1=s_G=S_0} = \max\left[\frac{c}{p^0\triangle e_2} - \triangle f; \frac{c}{p^0\overline{e_2}}\right], \tag{11}$$

$$w^{l,l}_{S_1=S_0} = 0. \tag{12}$$

4. $R_1 = R_l$ and the strategy was changed ($S_1 \neq S_0$).

The contract with old CEO is not terminated:

$$w^{l,h}_{S_1\neq S_0} = \frac{c}{p^1\triangle e_2} - \triangle f, \tag{13}$$

where

$$\triangle f = f\left(q^{i,h}\right) - f\left(q^{i,l}\right), \tag{14}$$

$$w^{l,l}_{S_1\neq S_0} = 0. \tag{15}$$

The contract with new CEO is the following:

$$w^{l,h}_{S_1\neq S_0,new} = \frac{c}{p^1\triangle e_2}, \tag{16}$$

$$w^{l,l}_{S_1\neq S_0,new} = 0. \tag{17}$$

Under these compensation values for the 2nd period the CEO will always exert high level of efforts, as his expected payoff accounted for high efforts is higher than in the case of low efforts.

Now let us consider the principal's move.

1. If after the 1st period the Company performance is high R_h or the performance is low $R_l = 0$ but the signal identifies that the initial strategy should be maintained $s_G = S_0$, the owner has two alternatives: pursue the initial strategy with the old or the new CEO. Base game solution presented in App. 1 demonstrates that hiring a new manager under the initial strategy is not optimal; therefore we assume that in such a case the owner always prefers to leave the old top manager in the Company.
2. If or the performance is low R_l and the signal confirms that the initial strategy will fail $s_G \neq S_0$, the owner has four alternatives:

A – not change the strategy nor the CEO
B – not change the strategy, hire a new CEO (non-optimal)
C – change the strategy and hire a new CEO
D – change the strategy, leave the old CEO (non-optimal)

Base game solution presented in App. 1 demonstrates that option B is not optimal. Let us consider alternatives C and D provided that the strategy is changed, $S_1 \neq S_0$. In this case compensation for the old and new CEOs should be compared (formulas (1.15) and (16) respectively, taking into account $\triangle f < 0$ in formula (14)). Compensation of the old CEO is higher than for the new CEO; that is why when a new strategy is adopted, the owner prefers hiring a new chief executive. Alternative D is therefore non-optimal, so the owner chooses between options A and C.

Under the condition that expected payoff of the owner in case of the initial strategy execution is higher than in case of a new strategy implementation in the 2nd period, he decides to follow the initial strategy (and leave the old CEO).

Let us consider the first move of the manager. He has 2 options in 2 sub-games: exert high or low level of efforts. In order to find optimal compensation incentivizing to exert high efforts, the following linear programming problem should be solved:

$$\min\left[q_0\left(\overline{e_1}\ w^h + (1-\overline{e_1}\ )\ w^l\right) + (1-q_0)w^l\right].$$

Subject to:

$$w^h - w^l \geq \frac{c}{q_0 \triangle e_1} - \overline{e_2}\left(w^{h,h} - w^{l,h}_{S_1=S_0}\right) - (1-\overline{e_2})\triangle f,$$

$$w^h \geq 0,$$

$$w^l \geq 0.$$

The problem solution is the following:

$$w^h = \max\left[0;\ \frac{c}{q_0 \triangle e_1} - \overline{e_2}\left(w^{h,h} - \mathrm{w}^{l,h}_{S_1=S_0}\right) - (1-\overline{e_2})\triangle f\right], \tag{18}$$

$$w^l = 0. \tag{19}$$

Considering these results it is transparent that the manager will exert high efforts in every sub-game in the 1st period in order to maximize his expected compensation. Therefore Nash equilibrium strategies for both players are as follows:

1. For the manager: in both periods he should exert high efforts $\overline{e_1}$ and $\overline{e_2}$.

2. For the owner: accounted for

$$P^0 \geq P^1 - \frac{P^1 \mathrm{w}^{l,h}_{S_1 \neq S_{0,new}} - P^0 \mathrm{w}^{l,h}_{S_1=S_0}}{R_2}. \quad (20)$$

He should not change the strategy or the manager. Otherwise, he should change the strategy and hire a new manager.

Let us calculate expected payoff for the owner for both periods:

1. If $S_1 = S_0$:

$$q_0 \left(\overline{e_1} \left(R - w^h + \overline{e_2}(R - w^{h,h})\right) + \right. \\ \left. +(1-\overline{e_1}) \left(p_G \overline{e_2}(R - w^{l,h}_{S_1=s_G=S_0} + (1-p_G)\overline{e_2}(R - w^{l,h}_{S_1=S_0})\right)\right). \quad (21)$$

2. If $S_1 \neq S_0$:

$$q_0 \left(\overline{e_1} \left(R - w^h + \overline{e_2}\left(R - w^{h,h}\right) + (1-\overline{e_1})\, p_G \overline{e_2} \left(R - \mathrm{w}^{l,h}_{S_1=s_G=S_0}\right)\right)\right) + \\ + (1-q_0) p_G \overline{e_2}(R - \mathrm{w}^{l,h}_{S_1 \neq S_0,new}). \quad (22)$$

The game solution is demonstrated in Fig.2.

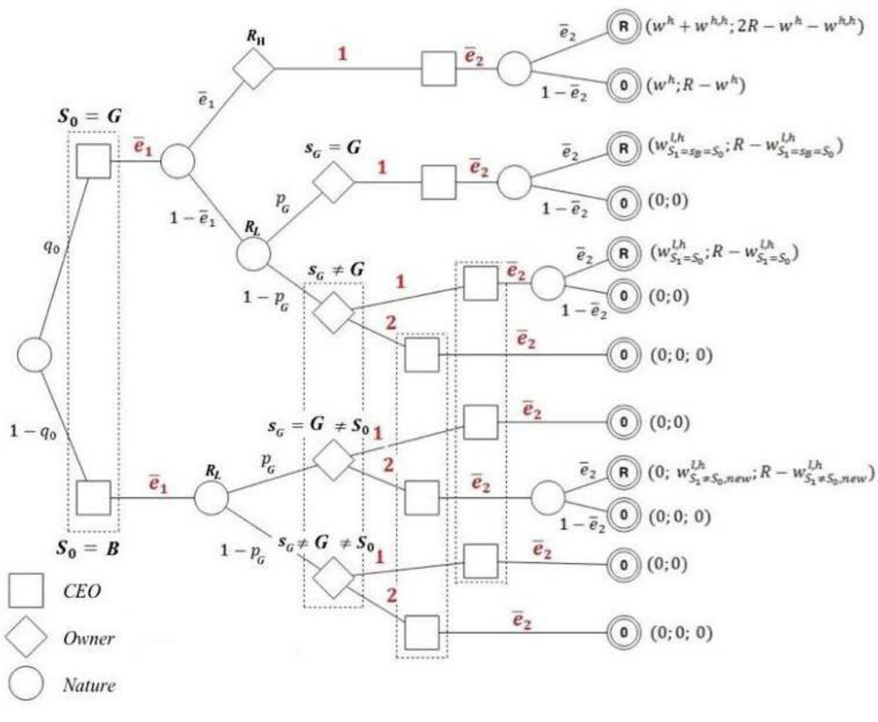

Fig. 2: Game solution

4. Specification of parameters for U.S. public companies

In order to make corresponding computations using the model, we needed to obtain data for corresponding variables or develop methods to approximate some of the variables.

Principal role. A very important issue is who to consider the principal while illustrating the model. In theoretical model we assumed that the principal can intervene and make a decision in regard to a strategy and CEO choice. In reality shareholders certainly have rights to monitor and oversee the CEO activities but with big limitations. Once there is a majority shareholder[1] in the company, i.e. investor that owns more than 50% of the company's outstanding shares, we can assess the probability of his/her intervention, dependent on individual behavior patterns (e.g. prior active participation in the company strategic decisions). Due to high control and voting interests in the company the majority shareholder is rather influential in business operations and strategic directions. However, as it has been mentioned U.S. public companies usually have scattered ownership and are, therefore, scarce for majority shareholders.

Due to the above mentioned reasons operational monitoring is delegated to the board of directors, so we approximate the role of the principal by the board of directors that is believed to execute actions in the shareholders' interest. We also can observe whether Chairman is independent director and how long he has been a part of the board, testing the assumption that independent directors are objective in pursuing shareholders' interest and are not captured by the CEO[2]. The underlying assumption based on literature review is that the longer chairman stays in his position, the more entrenched and the more dependent on CEO he becomes.

In either case we will consider ownership structure of the company under analysis.

Agent role. We have also underlined that the agent is a party who is delegated management of the principal's assets in order to maximize the principal's utility, i.e. maximize shareholders' value[3]. Therefore, it is natural that the CEO is assumed to be the agent in the model.

Strategy. Another essential aspect is definition of strategy in general, as well as strategy types. Strategy is strategy is the means by which individuals or organizations achieve their objectives (Grant, 2010). The strategy is focused on achieving certain goals (under resource constraint) that can be attained by pursuing critical actions that are consistent are cohesive with the decisions.

For our model it is essential to differentiate between successful and non-successful strategies. A successful strategy aims at achieving maximum economic results. However, high economic results are dependent not only on strategy choice but also on external factors (macro-environment, industry specifics) and different internal factors (including but not limited by efforts during the implementation). In reality, we understand that consistent long-term (over 3-5 years) above-industry average performance is results of a successful strategy implementation. Once again we will

[1] The majority shareholder is often the founder of the company, or in the case of long-established businesses, the founder's descendants.

[2] Gutierrez-Urtiaga M. (2000) Managers and Directors: a Model of Strategic Information Transmission. Working Papers from Centro de Estudios Monetarios Y Financieros; Cyert, Kang, Kumar (2002); Core, Holthausen, Larcker (1999)

[3] And/or if needed for utility maximization, optimize other Company parameters

stress that the working model focuses on incentivizing the manager at the phase of strategy execution.

There are different ways to categorize strategies described in strategic management academic resources. In a public company strategies can be divided into four layers (corresponding responsible managers are specified in the parentheses): corporate (CEO), division/ business (division president or executive vice president (VP)), functional (finance, marketing, manufacturing, R&D, HR, etc. manager) and operational (department, plant, etc. manager). Certainly, lower-level strategies should be in line with upper-level strategies. As follows from the paper name, we are focusing on corporate strategies in public companies.

The corporate strategy considers the following main elements: vertical scope (value chain), geographical scope, and product scope (Grant, 2010). A more thorough typology of strategies includes the following types: intensive in terms of product scope (market penetration, market development, product development), integration in terms of vertical and geographical scope (forward, backward, horizontal), diversification (concentric, conglomerate, horizontal), divestiture, retrenchment, liquidation, and a combination strategy.

According to Michael Porter (1980), there are two generic business strategy types – cost leadership and differentiation, which can lead to a competitive advantage defending against market forces of the industry. Whereas cost leadership means offering of standardized products, commodity, at low average unit cost, usually targeted at price-sensitive audience, product differentiation implies unique product offering desired by relatively price-insensitive customers. Cost leadership is aimed at wide range of customers while the product is distributed at the lowest price at the market. It usually highly correlates with high barriers of entry as the mentioned strategy requires economy of scale and, therefore, (prohibitively) high capital investment. Differentiation can incorporate several of the below mentioned dimensions: different design, brand image, number of features or different production technology. Additionally, the company can focus on a niche market achieving either a low cost advantage or differentiation in a narrow market segment.

Strategies can also be classified according to degree of activity: aggressive, defending and regressive.

Successful strategy is the result of simple, consistent, long-term goals; solid understanding of the competitive environment; objective appraisal of resources and effective execution efforts (Grant, 2010). The chosen model helps to incentivize the CEO to implement the strategy effectively.

Financial performance. In a general case while assessing the company performance shareholders usually care for the following aspects:

1. Their earnings (current and future)
2. Risk of their investment

In order to measure these parameters, we can assess the company performance – either financial or non-financial performance. However, we assume that non-financial metrics of company performance can be approximated by the financial ones[4]; therefore, let us consider types of financial performance metrics. Financial performance

[4]Even though company objectives can be expressed in non-monetary metrics too

indicators can be grouped into four categories. This classification is based on conventional financial analysis and corporate finance methods.[5]

Targets can be set for any of these metrics, hereby at the end of the periods in the model performance will be measured against these targets. Usually operational profitability performance metrics are set as targets for non-incentive equity plan (e.g. Operational profit, sales). As for performance-based stock awards, market ratios are usually taken into account while setting performance targets for this component (e.g. EPS).

In a specific situation, however, performance indicators are identified on the base of the strategy. Realized target values are the outcome of successfully implemented strategy.

Since financial targets chosen for specific cases usually combine several metrics, in case analysis we calculate multiples based on weights and values of metrics chosen by the Company to evaluate financial performance. Then we normalize performance indicator against the target figure.[6]

Table 1: Financial performance measures. Source: own rendering

Financial performance	
Group	*Most common variables*
Profitability incl. Investor ratios Market ratios	EBIT / Operating Profit, NI, revenue, costs ROI, ROE, ROIC EPS, P/E, P/B
Shareholder value	Intrinsic value, market cap, cash flows
Operations management	solvency, liquidity, business activity (efficiency) ratios
Gearing ratios	D/E, financial leverage

International aspect. The U.S. public companies chosen for the case study analysis should be involved in international commercial transactions that occur between two or more regions in order to be qualified as international (transnational or global) companies, i.e. sales, investments, logistics, etc. Since our research covers the largest U.S. public companies most of them have international operations, international suppliers or other logistics partners or hold international investments in their portfolio. We specify the international aspect of each company in the Company profile.

Compensation. In order to denote an unknown variable that corresponds to compensation package in the model, there are two approaches to measure compensation:

[5] Choi F., Frost C., Meek G. (2002) International Accounting. 4th Int.ed. Prentice Hall/Pearson Education Int.; Brigham E., Ehrhardt M. (2010) Financial Management: Theory & Practice. 13th ed. Thomson-South Western; Ross S, Westerfield R., Jordan B. (2008) Essentials of Corporate Finance. 6-th ed. McGraw Hill

[6] Therefore, as financial result figures we have 0 or R calculated for the specific Company

1. *Non-equity incentive plan* that is considered due to two reasons: it is a performance-based compensation component (can be short- and long-term); targets are usually rigorously described in the annual proxy statements.

2. *Performance-based stock units and Non-equity incentive plan* can be considered an integral incentive package. Targets for stocks awards component are also described in the annual proxy statements.

Other performance-based compensation components (stock options and time-based restricted stock units) are not considered in the scope of current research due to the following reason: these instruments are usually offered by the Company to retain the CEO in the Company for a particular time. Granting common shares (so that shares are realized[7], i.e they can be sold or be subject to any other transactions) usually has a downside risk since the owner of shares can also experience losses if the Company stocks are plummeting.

We use formulas (7) – (13), (14), (18) – (19) to calculate compensation values for the old CEO at the end of the 1st or 2nd period (accounted for the Company results and information signal regarding the applied strategy). Formulas (16) – (17) are used to calculate compensation of the new CEO if the decision was made to replace the old CEO after the 1st period.

Other variables used in the analysis. A full list of variables used in the model can be found in Tab. 3

Table 2: Additional model variables. Source: own rendering

Variable	**Brief description**	**Calculation method**
q	CEO reputation	See supplementary computation method of initial reputation q^0; Formulas (2) and (3)
f	CEO value	Formula (1)
$\triangle f$	Change in CEO value	Formula (14)
c	Cost of exerting high efforts	Planned bonus for the period; if no bonuses were paid out, mean bonus for the industry
e	Efforts exerted by CEO	See supplementary computation method below
p	Conditional probability of implementation of successful strategy in the 2nd period	Formulas (4) - (6)
p_G	Probability of identification of the successful strategy via signaling	See supplementary computation method below
Condition for changing the strategy		Formula (20)

Further clarifications should be made regarding evaluation of probabilities in the model.

Reputation of CEO. There are 3 methods of the reputation variable construction.

[7]Refer to Center on Executive Compensation (realized vs. realizable pay): http://www.execcomp.org/Issues/Issue/pay-for-performance/realized-pay

1. We assess *the whole prior history* of the CEO. Additional variables that need to be calculated are the following:
   - total number of years when the person in question was performing successfully as a CEO in *all previous companies;*
   - total number of years when the person in question was serving as CEO in *all previous companies.*

   Quotient of these two variables is the required probability.
2. We assess only *last CEO tenure* prior to the current position. We additionally calculate analogous variables:
   - number of years when the person in question was performing successfully as a CEO in *the previous company;*
   - number of years when the person in question was serving as CEO in *the previous company.*

   Quotient of these two variables is the required probability.
3. Rating of the CEO in the press, assessed by the industry experts (CEO rating divided by the maximum possible rating).

However, there are possible limitations to these calculation methods.

In case study analysis we considered for this research paper for most CEOs some of the prior positions did not include CEO position but executive position. Then we adjust calculations and calculate probabilities based on experience at other executive positions (trying to correspond executive's positions to appropriate performance metrics to assess his/her success).

There are also cases when prior work places were private companies or subsidiaries with non-disclosed performance figures. Then we adjust our calculations and use Method 2.

In case of prior history within the Company (we assess implementation of a new strategy but the CEO was serving in the Company as chief executive) we assess the period prior to evaluation as it was a case of a separate company in regard to the number of successful and total years.

Efforts of CEO. There are 2 methods to evaluate efforts level in the model.

1. Similar to the CEO reputation, this variable is based on historic behavioral patterns of the CEO. We assume that in order for the company to perform above industry average extra efforts from the CEO's side should be applied. We, therefore, find information on the following variables:
   - number of years when the company was performing above the industry average during the CEO tenure, by company;
   - number of years when the person was serving as CEO in the company, by company.

   We calculate corresponding quotients by company and choose the highest probability of high efforts and the lowest probability of low efforts.
2. Due to the fact that high efforts cause additional costs for CEO and we proxy these costs as Bonus assuming that additional efforts are reimbursed to the CEO in the amount of bonus, we can assess bonus history of the CEO in all prior companies. Quotient of number of years with bonuses over number of years s/he was performing as CEO but didn't receive any additional rewards for efforts, by company, will correspond to the required probabilities. Again, we choose the highest probability to represent high efforts and the lowest probability for low efforts.

There are also certain limitations to calculations. For some case study we can obtain information only on the prior work place. Then we consider different performance metrics and compare them against the industry average. Then analogous to the above mentioned method, we calculate corresponding quotients by performance metric; the highest probability represents high efforts and the lowest probability reflects low efforts.

Analogously, In case of prior history within the Company (we assess implementation of a new strategy but the CEO was serving in the Company as chief executive) we assess the period prior to evaluation as it was a case of a separate company in regard to the number of successful and total years.

Probability of successful strategy identification by the principal. This variable is computed based on analysis of the board of directors. The share of independent directors in the board should be used as approximation of successful strategy determination. Current academic studies such as Gutierrez-Urtiaga (2000), Cyert, Kang, Kumar (2002) and Core, Holthausen, Larcker (1999) suggest that independence of directors increases the quality of their responsibilities fulfillment. Since their duties include strategic monitoring and efficient CEO compensation programs, we assume that this quotient reflects probability of successful strategy identification.

5. Industries

In order to analyze the applicability of the considered theoretical approach it was necessary to narrow the research area to concentrate on several industries. Industry should have been representative that means companies should differentiate by size. That is, outcomes for the considered industries can be probably extrapolated on other industries. Realistically the industry incorporates not only public but also private companies, which compete along. However, lack of data regarding private companies' performance measures and compensation packages are not available for the public, so we considered only public companies. Moreover, conflicts in corporate governance in private companies are not as acute since the ownership is more concentrated. Another requirement for the examined industries is low volatility in examined year, so we chose the period between 2011 and 2013.

All public companies in the U.S. could be divided into 14 different key industries. For the purpose of our research, retail and IT-industry were chosen. The choice of sectors is interesting due to the following reason: retail is a relatively mature sector whereas information is rapidly growing sector. Therefore, such elements as demand, competition and product itself would differ; therefore, key success factors and strategies adopted in these industries would also be different. IT-industry is particularly interesting because key success factors here are brand development, fast product development and realization on the market, innovations, but mature industries can benefit from cost and scale efficiency, and low input costs. However, we considered top performing U.S. sectors, therefore, large players in mature industries also try to innovate and disrupt the course of conventional business operations

Overall, in the first research there were 16 companies from retail and technology industry, in the second research there were 80 companies from retail and 82 companies from IT-industry in our research. The data on such parameters as base salary, cash bonuses, stock awards, stock options, Non-equity incentive plan, other compensation, total compensation, market capitalization, CEO age and working experience in years was gathered. It was done in order to access industry average parameters included in the research, find companies for case studies and show in descriptive statistics that variable part of compensation package of CEO is very significant for those industries. So, for retail industry a variable part is 74,8% of total compensation of CEO in 2011-2013, and for IT-industry – 88,2%.

6. CEO incentive plan case studies analysis

Penney Company incentive plan practice

Company profile. J. C. Penney Company Inc. (JCP), incorporated on January 22, 2002, whose main operating subsidiary is J. C. Penney Corporation, Inc.[8] JCP encompasses selling merchandise and rendering services to consumers through department stores and online channel (jcp.com). The Company operates in the USA and internationally (1,104 department stores throughout in the USA and Puerto Rico).[9] Product offering includes: family clothes and footwear, accessories, jewelry, beauty products (Sephora) and home furnishings. Service offering consists of the following: styling, optical, photography and other services.

Ownership structure. Ownership structure can be found in the annual report and proxy statements. The majority of shares are owned by institutional stockholders (75.91% in monetary value) who are usually more long-term oriented than individual investors. 23.63% of total equity belongs to mutual funds and only 0.46% to insiders. According to Morningstar, the 20 largest owners (institutions and mutual funds) possess 58% of total shares.[10] Even though the figure is rather high, concentration of ownership is still rather low. Due to scattered ownership in the U.S. public companies we will use the Board of directors and its characteristics and guidelines for the model as a proxy of the principal.

Shareholders meetings (meetings of all stockholders) that happen annually mostly deals with matters regarding election of directors, approval of compensation plans, regulations and adopted-to-be documents that improve Company policies for tax benefits. Theoretically speaking, the Meeting can consider any other business properly brought before the meeting. However, it is certainly rather complex to be actively engaged in strategic planning of the Company. At each meeting of stockholders, the holders of a majority should constitute a quorum for the transaction of business. In the absence of a quorum the meeting may be adjourned until a quorum is present.[11]

The JCP Corporate Guidelines require stock ownership quota for the CEO: the goal in 2013 is 5x-6x of annual base salary within 5 years after being appointed (beforehand the goal was 10x of annual base salary).

Board of directors. Issues regarding corporate governance in the company are reflected in Corporate Governance guidelines where objectives and responsibilities of the stakeholders.

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[8] Refer to Reuters: http://www.reuters.com/finance/stocks/companyProfile?symbol=JCP
[9] As of February 2, 2013
[10] Refer to Morningstar:http://investors.morningstar.com/ownership/shareholders-overview.html?t=XNYS:JCP&region=usa&culture=en-US
[11] Refer to JCP Bylaws

One of the most important elements of corporate governance and interaction with CEO in particular is the board of directors. The size of the board should not be less than 3 directors; size of 10-15 directors is considered appropriate in the current Guidelines. The Board meets at least 6 times per annum unless called upon more frequently by the Chair.[12]

The Chairman of the Board may also serve as the JCP CEO, which underlines the source of possible asymmetry of information. However, the Board is comprised of a majority of independent directors (according to NYSE criteria for independence)

According to the Guidelines, business matters are managed under the supervision of the Board, which represents and is accountable to JCP stockholders. Among the Board's responsibilities, among others, are overseeing and regular evaluation of *strategy of JCP*, *the management effectiveness of strategy implementation* and *the selection, evaluation and setting of appropriate compensation for JCP CEO*.

There are five corresponding committees that treat corresponding issues and therefore execute delegated responsibilities: Audit, Corporate Governance, Finance and Planning, Human Resources and Compensation, and the Committee of the Whole.[13]

The independent directors committee, so-called Committee of the Whole, meets annually to assess the CEO's performance based on goals and objectives previously set out by the Committee of the Whole. The evaluation is usually conducted on the base on objective criteria (e.g. performance of the business, accomplishment of long-term strategic objectives, etc.) and used by the Committee of the Whole to construct CEO's compensation package (along with data and information regarding CEO compensation matters and a non-binding recommendation received from the Human Resources and Compensation Committee).[14]

In 2012 the Board consisted of 12 directors, thereof 11 were independent. Thomas J. Engibous was a non-employee, independent director. These figures are used for probability of successful strategy identification ($p_G = 0.92$). Hereby we believe that probability that the highly independent Board with independent Chair can determine the best possible decisions for the Company.

Problem description. Mr. Johnson was hired to lead rebranding of JCP to shake up the store's stodgy image and attract new customers by introducing upper class product portfolio of higher pricing and rejected a former policy of discounts on markup prices. While his rebranding effort was ambitious, he was said to have "had no idea about allocating and conserving resources and core customers. He didn't do test the concept on a sample market and his strategy failed.

New strategy in 2012: changes in pricing strategy, corporate branding, marketing, store layout and merchandise assortments, namely substantial changes in merchandise, edition and introduction of more global brands into the merchandise assortment, re-organization of department stores into curated unique specialty stores.[15]

[12] Refer to Corporate Governance Guidelines:
http://ir.jcpenney.com/phoenix.zhtml?c=70528&p=irol-govguidelines

[13] Refer to Investors relations web page:
http://ir.jcpenney.com/phoenix.zhtml?c=70528&p=irol-govcommcomp

[14] Refer to the Committee of the Whole Charter

[15] Refer to 10-K annual report

CEO profile: Ronald Ron B. Johnson, 54 yrs[16] (tenure: Nov. 2011 – 2013). Mr. Johnson has over 20 years of experience in retail and merchandising and impressive growth achievements in billion-dollar companies such as Apple and Target.

Career timeline:[17]

Nov, 2011 – Apr, 2013 – CEO at J.C. Penney Company, Inc.

2000 – 2011 - Senior Vice President, Retail for Apple, Inc. (Apple's retail strategy)

1985 – 2000 – Senior Vice President of Merchandising of Target Corporation and other senior management positions (initiatives for branding, marketing and merchandising)

Mr. Johnson's history of performance in the companies is provided in App. 7. Based on App. 7 reputation and probabilities of exerting high and low efforts are constructed for the model testing.

Current incentive plans. Due to prior long history of unsuccessful results and current transformation strategy CEO compensation structure in JCP is designed to tie compensation and performance. The target compensation mix of 2013 reflects the desired pay composition, including 29.8% of total pay in cash incentive awards and 26.6% in performance-based restricted stock units (PBRSUs), resulting in 56.4% of total pay in performance-based awards (against targets) and 78.4% in performance-based compensation (including stock options). History of actual compensation in Tab. 3 demonstrates that after rich initial payment in the form of stock awards to the new CEO in 2011, all incentive payments were equal to 0 (short-term and long-term incentives) due to outrageous bad performance of JCP and failure of implementation of the diversification strategy.

Table 3: CEO compensation at JCP. Source: rendering from DEF 14A proxy statements

Compensation, ths USD	**2011**	**2012**	**2013**
Salary	1 864,583	1 500	810,606
Bonus	0	0	0
Stock awards	64 056,935	0	0
Option awards	3 600	0	0
Non-equity incentive plan	2 111,302	0	0
All other compensation	16 210	388,587	1 582,024
Total compensation	87 842,827	1 888,587	2 392,630

Annual cash incentive awards. Cash incentives are paid out in accordance to annual Management Incentive Compensation Program. The incentive plan in 2012 was based only on Operating profit as an indicator of earnings and cost savings attainment whereas for the year 2013 performance metrics were broadened, then sales objectives were also included in the program for the CEO stimulation. In 2013 weights for performance metrics were 50% and 50% for operating profit and sales

[16] At the time when he started serving as CEO

[17] Refer to Bloomberg Businessweek: http://investing.businessweek.com/research/stocks/people/person.asp?personId=652443&ticker=AAPL

respectively. Once the target had been achieved, the CEO would have been paid out as a percentage from the base salary.[18]

Long-Term Incentive Awards. Long-term incentive awards are paid out corresponding to long-term incentive plan (3 years). In 2012 there were no PBRSUs offered to the CEO. Equity-based incentives in 2013 were offered to the CEO once he achieves Earnings/loss per share (EPS) target. The number of units granted was considered as a target award and this figure could be adjusted dependent on the actual EPS value.

Targets during the CEO tenure are also presented below in Tab. 4 along with actual figures. Target and actual figures are used to normalize JCP performance figure in order to obtain compatibility of numbers.

Table 4: Target and actual performance at JCP.Source: rendering from DEF 14A proxy statements and 10-K annual reports

For incentive plans	**2012**		Weights	**2013**		Weights
	T	**A**		**T**	**A**	
Operating profit target, bln USD	1,099	-1,016	100%	-0,106	-1,244	50%
Sales, bln USD	N/A	N/A		12,872	11,859	50%
EPS, USD per share	N/A	N/A		-1,22	-4,64	

Model illustration and reality check. The case is broken down into two periods: first period is year 2012 and the second is year 2013. Based on the methodology presented in section 4, we constructed variables in order to assess incentive compensation package as well as evaluate probability to change the company strategy and current CEO.

In order to calculate initial reputation of the CEO we used data from App. 7. Since Ron Johnson was previously working in Apple, we obtained data on operating profits and net sales of Apple during the years. For successful years we considered years of the company growth (8 successful years against 12 years overall). Therefore, the initial reputation $q_0 = 0.67$ according to Method 2 of reputation calculation and 0.75 based on Method 3 (Businessweek and Forbes expert qualitative valuation was put into scale).

Efforts were analyzed against industry average results for growth rates and operating margins. High efforts probability is, therefore, $\overline{e_1} = 0.92$ (11 successful years against 12 total years) and $\underline{e_1} = 0.42$ (5 successful years against 12 total years).

For the second period history for Mike Ullman was analyzed due to his replacement of Ron Johnson and effort figures were applied for him ($\overline{e_2} = 0.75$ and $\underline{e_2} = 0.3$).

Due to the fact that bonuses were not paid out in the Company for a number of years, we took an average bonus value for the retail industry ($c = 150$).

Using corresponding formulas (1) – (6) and (14) for amended reputations, conditional probabilities and value of the CEO we construct additional variables that can be found in Tab. 5. Then using formulas (7) – (13), (14), (18) – (19) and (16) – (17) we calculate all possible compensation values for the case (our model in Excel

[18]We point out again that independent directors set out targets and incentive opportunities(a corresponding multiple that translates objective into incentive) for the CEO, according to JCP Corporate Governance Guidelines

is constructed for the general case, therefore, it calculates all values), applicable for this case formulas are (15) and (17).

Overall model results are presented in Tab. 5 According to the model JCP Board of directors should have let go the CEO and immediately changed the strategy to improve the company financial performance. Along with an amended strategy, the Board should have also hired a new CEO who will be executing a new recovery strategy. Incentive package for current CEO should be 0 (compensation to a new CEO should also be 0).

Actual life was escalating similarly to what the model has predicted: the contract with the current CEO Ron Johnson was terminated and his successor (and predecessor Mike Ullman) came back as a CEO to get the company back on the feet. However, already for the past 7 years the company was stagnating (Operating profit) and until now the profitability situation hasn't improved. So Mr. Ullman hasn't obtained any incentive compensation yet since he returned back to his position.

Table 5: Model results for JCP case. Source: own rendering

q_0	$\overline{e_1}$	$\underline{e_1}$	$\overline{e_2}$	$\underline{e_2}$	p_G	c	R
0,67	0,92	0,67	0,75	0,29	0,92	150	1000

$\triangle e_1$	$\triangle e_2$	p^0	p^1	$q_0^{l,l}$	$f(q_0^{l,l})$	$q_1^{l,l}$	$f(q_1^{l,l})$
0,25	0,46	0,014	0,904	0,00346	0,104	0,043	1,28

$w^{h,h}$	$w^{l,h}_{S_1=S_0}$	$w^{l,h}_{S_1\neq S_0,new}$	$w^{h}_{S_1=S_0}$	$w^{l,h}_{S_1=s_G=S_0}$	$\triangle f$	Change?
323,077	23554,7	357,343	18316,3	293,181	29,896	Yes

Applied procedure for the theoretical model was tested on 10 case studies: 5 for companies of retail industry and 5 companies of IT-industry.

<u>Target Corporation</u>

Company profile. Target Corporation (TGT), incorporated on February 11, 1902, is engaged with selling general merchandise and food in stores (CityTarget and SuperTarget). TGT operates in three business segments based on product and geographical scope: U.S. Retail, U.S. Credit Card and Canadian (costs incurred in the U.S. and Canada related to its Canadian retail market). Product offering includes: everyday essentials and fashionable, differentiated merchandise at discounted prices.[19]

Ownership structure. Currently 70.8% of total equity (in monetary value) belong to institutional investors, 29.1% to mutual funds and only 0.1% to insiders. The largest 20 institutional and mutual fund investors hold 62.48%.[20] Even though the value is rather high, concentration of ownership is still considered low. Due to scattered ownership, the Board of directors again is used as a proxy for the principal's role.

[19] Refer to Reuters: http://www.reuters.com/finance/stocks/companyProfile?symbol=TGT

[20] Refer to Morningstar:http://investors.morningstar.com/ownership/shareholders-overview.html?t=TGT&region=usa&culture=en-US

The Board of directors. The appropriate size for a Board of Directors from Target perspective is 5 to 21 members. The Board believes that a membership of 11 directors is appropriate (however, it can vary in accordance with regular review).[21]

According to Governance Guidelines, the Compensation Committee of the Board of Directors annually evaluates CEO performance and its relationship to reward and provides recommendations. After that the independent members of the Board annually review the recommendations of the Compensation Committee and approve the CEO performance review along with compensation value and composition. The Compensation Committee also produces a report for inclusion in the Corporation's proxy statement in accordance with SEC rules and regulations.

The Board in year 2011 encompassed 11 members, 10 of which were independent.[22] These numbers will be used for calculation of probability of successful strategy identification once the model is tested in this case. Moreover, the CEO was also the Chairman of the Board.

Problem description. Mr. Steinhafel had to adjust to a more modest after-crisis shopper in the wake of the recession, Target's offerings had become more commonplace — heavy on food and other consumer staples. Fewer new products, especially creative unique to Target, were introduced. The product portfolio deteriorated; Target had to add pressure due to tough situation. Risk taking behavior also changed: Target became more risk cautious to new items. Rather than bet on the newest, most unique products, Target increasingly relied on a placement system that awarded prime shelf space to the highest bidders.[23]

CEO profile: Gregg W. Steinhafel, 52 yrs[24] (tenure: 2008 – 2014; 6 years). Mr Steinhafel was a genuine internally made CEO: he went through various job roles before he achieved top executive positions.

Career timeline[25]

2008 – CEO at Target

1999 – 2008 – President at Target

1994 – 1999 - Executive Vice President Merchandising at Target

1979 – 1994 – merchandise trainee at Target; variety of merchandising and operational management positions

Current incentive plan practice. After years of stagnating performance Target adjusted its compensation structure to be tightly linked to performance. According to proxy statements, performance-based compensation that is calculated against target performance measures (including performance-based restricted stock units (PBRSUs), performance share units (PSUs) and short-term incentive plan (STIP)) accounted for 57% in 2011 whereas in 2013 it amounts to 87% of total compensation.[26] Interestingly whereas the Company was using options awards as

[21] Refer to Board Committee web page:
http://investors.target.com/phoenix.zhtml?c=65828&p=irol-govcommittees

[22] Refer to DEF 14A Proxy statement (2012) retrieved from the U.S. SEC EDGAR database

[23] Hajewski D. (2008). Journal Sentinel (Bloomberg reporter). Steinhafel To Take Over at Target. Retrieved from: http://www.jsonline.com/business/29548034.html

[24] At the time when he started serving as CEO

[25] Refer to Bloomberg Businessweek: http://investing.businessweek.com/research/stocks/people/person.asp?personId=174446&ticker=TGT; Refer to App. 7 to find data on parameters evaluation

[26] Refer to proxy statements retrieved from U.S. SEC EDGAR database

remuneration element, it completely abandoned this component in 2013. Summary of compensation values and composition is presented in Tab. 6.

Table 6: CEO compensation at TGT. Source: rendering from DEF 14A proxy statements

Compensation, ths USD	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**
Salary	1345,769	1350	1500	1500	1500	1500
Bonus	447,68	0	1200	1250	0	0
Stock awards	6750,041	4425,064	8017,549	4857,502	5285,245	10224,12
Option awards	4074,038	3503,393	3189,299	3696,982	5248,573	0
Non-equity Incentive Plan	0	3250	4101	2205	2880	0
All other compensation	1020,642	778,177	5982,035	6197,623	5733,646	1229,094
Total compensation	13638,17	13306,63	23989,88	19707,11	20647,46	12953,21

Short-Term Incentives. STIP allows the CEO cash awards based on the following financial metrics, Earnings Before Interest and Taxes (EBIT) and Economic Value Added (EVA). These performance measures reflect objectives for profitability and investment discipline. The weights for these financial metrics are 50% and 50% respectively.

Long-Term Incentives. Long-term incentive plan is comprised of PSUs and PBRSUs. PSUs have a three-year performance period; they are granted in stock based on change in market share (calculated through net sales), EPS growth and return on invested capital (ROIC[27]) in equal proportions. PBRSUs are linked to total shareholders' return (TSR) in comparison to peers. Once the total magnitude of long-term performance-based compensation is identified, 75% of this value is granted in the form of PSUs and 25% in PBRSUs.[28]

Due to the fact that performance metrics are measured in rankings, we will need corresponding scale to interpret ranking results and then we will have to normalize he scale against the target value. Therefore, we would have to adjust measures twice, which is too much of value distortion. So we decided to test the model for short-term incentive plan only.

Table 7: Target and actual performance at TGT.Source: rendering from DEF 14A proxy statements and 10-K annual reports

For incentive plans	**2011**		**2013**		Weights
	T	**A**	**T**	**A**	
Operating profit target, bln USD	5,416	5,421	5,459	5,186	50%
EVA, bln USD	0,949	0,936	0,712	0,676	50%

Model illustration and reality check. We divided the real case in two periods: the first period of 2008-2011 and the second of 2012-2013. Analogously to

[27]New metric introduces in 2013; calculated as three year average net operating profit after-tax (NOPAT) divided by average invested capital

[28]Before 2013 the mix was the following: 50% stock options, 25% PSUs, and 25% RSUs.

JCP case we constructed all variables and calculated payoffs of the players based on methodology 1.5 and App. 7 figures.

According to the model the Board should have paid 2.05 mln USD to Target CEO in the first period and 0 in the second period due to overperformance in the first period and underperformance in the second period. The modeled results state that the strategy and the CEO shouldn't be changed.

In the actual situation Mr. Steinhafel also stayed in the company, so there was no change in the company strategy and CEO. Moreover, TGT CEO received non-equity incentive compensation during the first period (2.2 mln USD) and also obtained performance-based RSUs for the next three years as a long-term incentive plan. As performance was plateauing during the 2nd period, he didn't receive any non-equity incentive compensation in 2013.

Table 8: TGT case – model results. Source: own rendering

q_0	$\overline{e_1}$	$\underline{e_1}$	$\overline{e_2}$	$\underline{e_2}$	p_G	c	R
0,83	0,8	0,4	0,76	0,45	0,91667	1.25	1331

$\triangle e_1$	$\triangle e_2$	p^0	p^1	$q_0^{l,l}$	$f(q_0^{l,l})$	$q_1^{l,l}$	$f(q_1^{l,l})$
0,4	0,31	0,07692	0,84615	0,01961	0,58824	0,21552	6,46552

$w^{h,h}$	$w^{l,h}_{S_1=S_0}$	$w^{l,h}_{S_1\neq S_0,new}$	$w^{h}_{S_1=S_0}$	$w^{l,h}_{S_1=s_G=S_0}$	$\triangle f$	Change?
2.032	3.238	2.765	2.05	2	29,4118	No

CEO incentive plan in EMC Corporation

Company profile. EMC Corporation (EMC), incorporated on August 23, 1979, develops, de livers and supports the information and virtual infrastructure technologies, solutions and services, including IT as a service (ITaaS). EMC operates three segments as federated businesses: EMC Information Infrastructure (provider of information storage, intelligence and security solutions), Pivotal (vendor of application and data infrastructure software) and VMware Virtual Infrastructure (provider of virtualization infrastructure solutions).[29]

Ownership structure. Currently 69.25% of equity is owned by institutional 29% by mutual funds and 0.46% by insiders (based on monetary value of equity). Due to the fact that ownership is so scattered: the largest 20 shareholders (institutional; and mutual funds) own only 33.12% of total shares[30], according to Morningstar, it is impossible to consider any of the shareholders as the principal in the model. Therefore, we approximate the principal's role by the Board of directors.

In order to align the CEO's interests with shareholders' expectations, the CEO is required to own 650,000 shares of the Company's common shares.

Board of directors. The main responsibility of the EMC Board of Directors according to Corporate Governance Guidelines is to foster the long-term success of the Company and to build long-term value for the Company's shareholders, consis-

[29] Refer to Reuters: http://www.reuters.com/finance/stocks/companyProfile?symbol=EMC

[30] Refer to Morningstar:http://investors.morningstar.com/ownership/shareholders-overview.html?t=EMC&region=usa&culture=en-US

tent with the Board's fiduciary duties[31]. Therefore, the Board is also responsible for evaluation of the corporate strategy, challenges, industry situation and the Company performance. The Board also identifies potential candidates, selects and monitors performance of the CEO. The Company's strategy is presented by the CEO to the Board and evaluated and discussed on the regular basis.

The Board consists of no fewer than 8 nor more than 11 directors (it annually reviews the size of the Board). A majority of the Board should qualify as independent directors under the NYSE listing standards.[32]

Currently there are five standing committees of the Board: Audit Committee; Leadership and Compensation Committee; Finance Committee; Mergers and Acquisition Committee; and Corporate Governance and Nomination Committee. However, if needed, new committee may be established or old committee may be disassembled.[33]

According to the Corporate Governance policies and Committee's charter, the Leadership and Compensation Committee annually reviews and approves (either as a committee or together with the other independent directors) composition and value of compensation for the CEO. Additionally it should communicate in the annual Board Compensation Committee Report to shareholders the required disclosures.[34]

CEO may or may not annually serve on the Board as Chairman; however, necessity of his presence at the Board should be annually reviewed by the Board.

CEO profile: Joseph Joe Tucci, yrs[35] (tenure in the contract: 2001 – present; 14 years). Mr. Tucci is an aggressive and outspoken leader who

Career timeline[36]

2001 – present – CEO at EMC Corporation (Chairman since 2006)

2000 – COO at EMC Corporation

1993 – 1999 – CEO and Chairman at Wang Global (former bankrupt Wang Laboratories)

1990 – 1993 - Executive vice president of operations at Wang Global

1986–1990 - President of U.S. Information Systems at Unisys Corporation

1970–1986 - systems programmer, followed by several other positions at RCA Corporation

Problem description. Joe Tucci has already been the Company CEO for 8 years. Starting from 2003 EMC started to acquire specialized companies in order to become the leader in software-defined storage. Soon enough the EMC was expending not only in storage but in virtual infrastructure (VMWare) provision;

[31] Refer to Corporate Governance guidelines: http://www.emc.com/collateral/corporation/corp-gov-guide.pdf

[32] Refer to Corporate Governance guidelines: http://www.emc.com/collateral/corporation/corp-gov-guide.pdf

[33] Refer to Corporate Governance web page: http://www.emc.com/corporate/investor-relations/governance/board-committee.htm

[34] Refer to the Leadership and Compensation Committee Charter: http://www.emc.com/collateral/corporation/charter-compensation-committee.pdf

[35] At the time when he started serving as CEO

[36] Refer to Reference for business: http://www.referenceforbusiness.com/biography/S-Z/Tucci-Joseph-M-1947.html; Refer to App. 7 to find data on parameters evaluation

the Company is on the way to provide enterprises with an integral IT-as-a-Service (ITaaS) solution. Complexity and virtualization of the products were increasing from virtualized IT-owned application in customer companies through Enterprise critical applications to complete virtualization of IT business. The second stage of this transformation started in year 2009. The milestone phase was identified for the next year 2010 and further development was to be checked further along in year 2013.[37]

Current incentive plans. Compensation contract at EMC puts larger emphasis at long-term incentives that comprised from 2009 till 2013 48.8% and 77.3% respectively, which reflects intention to link remuneration of the CEO to attainment individual and corporate longer-term strategic objectives and alignment of CEO interest with the shareholders' interest. CEO compensation consists of the following parts: base salary, non-equity incentives (short-term and long-term) and equity incentives (performance-based stocks, time-based stocks and stock options). The Tab. 9 provides an overview of compensation value and composition in years 2009-2013.

Table 9: CEO compensation at EMC.Source: rendering from DEF 14A proxy statements

Compensation, ths USD	**2009**	**2010**	**2011**	**2012**	**2013**
Salary	872,308	1000	1000	1000	1000
Bonus	0	0	0	0	0
Stock awards	5995,8	7355,9	8408,713	12697,669	9426,404
Option awards	962,085	1337,077	1557,752	1310,657	650,417
Non-equity incentive plan	1068,42	2592	2140,869	1467,36	1260,058
All other copmensation	149,15	151,184	131,523	116,545	309,079
Total compensation	9047,763	12436,161	13238,857	16592,231	12645,958
% of STIP	10%	13,40%	12,80%	7,50%	77,30%
% of LTIP	48,80%	46,40%	47%	69%	12,20%

Non-equity incentive plan (Cash bonus plan): Non-equity incentive plans are annually designed to motivate the CEO to achieve specified corporate, strategic, operational and other financial performance goals. They require attainment of a threshold level performance to obtain compensation. For CEO non-equity incentive plan consists of two parts: the Corporate Incentive Plan (CIP) corresponding to longer-term goal achievement and the Management by Objectives Plan (MOP) mirroring short-term metrics and functional goals attainment. Through attainment of MBO objectives that are set out by the Compensation Committee the CEO receives semi-annual cash payments whereas through CIP top chief executive is semi-annually evaluated based on several metrics and can receive up to 200% of target bonus opportunity set out for him by the Compensation Committee (subject to negative discretion if needed). The performance targets used are Earnings per Share (non-GAAP adjusted EPS), Revenue and Free Cash Flows (FCF). The corresponding weights are 50%, 30% and 20% respectively. Actual performance against target metrics is presented in Tab. 3.8.

[37]Refer to EMC Investor relations web page: http://www.emc.com/corporate/investor-relations/strategy.htm

Performance stock units and performance stock options: Performance stock units and performance stock options are usually provided for 3-year vesting and then they become granted upon attainment of the performance targets. Performance targets used in this evaluation are also EPS and Revenue since the Compensation Committee believes that growing revenue and EPS leads to long-term shareholder value. The weights are 60% and 40% respectively. Actual performance against target metrics is presented in Tab. 10

Table 10: Target and actual performance at EMC.Source: rendering from DEF 14A proxy statements and 10-K annual reports

Performance	**2009**		**2010**		**2011**		**2012**		**2013**		**Weights**	
Target / Actual	T	A	T	A	T	A	T	A	T	A	**for NEIP**	**for LTEP**
Revenue, bln	15	14	16	17	19,6	20	22	21,7	23,5	23,2	30%	40%
EPS, USD per share	0,87	0,9	1,12	1,26	1,46	1,51	1,7	1,7	1,85	1,8	50%	60%
FCF, bln USD[38]	0,87	1,27	1,42	3,44	4	4,43	4,9	5,02	5,53	5,51	20%	

We test the model using long-term non-equity incentive plans as well as performance equity granted in year 2010 based on achievement of target performance goals. Therefore, we evaluate attainment of financial results based on two weighing scales (for non-equity incentive plan and for performance stock units and options).

Model illustration and reality check. Overall model results are presented in Tab. 11. According to the model EMC should not change the strategy of the Company (and therefore, the CEO) after the first period due to successful results and consent of the Board with the realized strategy. The modeled compensation after the first period should amount to 2.489 mln USD whereas the actual non-equity incentive plan in this period was equal to 2.592 mln USD. The modeled remuneration for the second period should have been 0 whereas in real case it was 1.260 mln USD. The game is finite, which is why we can hypothesize compensation for the second period is 0 whereas in real life we keep incentivizing the CEO to exert efforts and execute the chosen strategy.

Table 11: Model results for EMC case. Source: own rendering

q_0	$\overline{e_1}$	$\underline{e_1}$	$\overline{e_2}$	$\underline{e_2}$	p_G	c	R
0,86	0,7	0,44	0,8	0,4	0,91	150	138

$\triangle e_1$	$\triangle e_2$	p^0	p^1	$q_0^{l,l}$	$f(q_0^{l,l})$	$q_1^{l,l}$	$f(q_1^{l,l})$
0,26	0,4	0,14	0,78	0,03	0,95	0,38	11,25

$w^{h,h}$	$w^{l,h}_{S_1=S_0}$	$w^{l,h}_{S_1\neq S_0,new}$	$w^{h}_{S_1=S_0}$	$w^{l,h}_{S_1=s_G=S_0}$	$\triangle f$	Change?
375	2637,62	480	2489,07	345,95	29,05	No

[38]FCF was calculated on per share basis in years 2009 and 2010.

Incentive plan practice in eBay

Company profile. eBay Inc. (eBay), incorporated on March 13, 1998, operates at the commerce market through three business segments: Marketplaces (online commerce), Payments (financial services), and GSI (logistics). EBay provides platforms, tools and services to facilitate online and mobile commerce and payments. The revenue streams stem from transactions fees and advertising services.[39]

Ownership structure. The majority of equity in monetary terms (66.91%) belongs to the institutional investors; mutual funds won 26.69% of total equity whereas 6.4% belongs to insiders. According to Morningstar, the largest 20 institutional investors and mutual funds possess only 33.15%, which confirms our assumption on low concentration of ownership in the U.S. public companies.[40] Therefore, we will consider the Board of Directors as a determining decision-making force in strategy and compensation setting.

Board of Directors. Corporate Governance guidelines establish rules for the Board of Directors, so they act in the best interests of the shareholders and eBay itself. The size of the board is determined by the corresponding resolutions that evaluate the needs of business on a regular basis. The Board consists of at least the majority of independent directors. It is also recommended that the CEO is on the Board and up to several former executives serve at the Board for the best interests of the shareholders. The Board is responsible for selection and appointment of the CEO.[41]

There are five existing committee now: the Audit Committee, the Compensation Committee, the Corporate Governance and Nominating Committee, the Non-Officer Option Committee, and the Strategic Investment, Acquisition, and Disposition Committee.

According to the Committee Charter, the Compensation Committee sets compensation levels for the CEO; it conducts evaluation with assistance of with the independent compensation consultant (CEO is not present during these meetings).[42]

Within Say on Pay practice the Board increases investors engagement in reviewing and providing feedback for the compensation program. Shareholders cast their advisory vote and the Board is intending to increase provision of direct feedback in regard to remuneration packages.[43]

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[39] Refer to Reuters: http://www.reuters.com/finance/stocks/companyProfile?symbol=EBAY.OCuriously, eBay also created an open source platform to develop software and solutions for commerce (more than 800,000 members)

[40] Refer to Morningstar:http://investors.morningstar.com/ownership/shareholders-overview.html?t=EBAY&region=usa&culture=en-US

[41] Refer to eBay Investor relations web page: http://investor.ebayinc.com/corporate-governance-document.cfm?DocumentID=727

[42] Refer to the Compensation Committee Charter: http://files.shareholder.com/downloads/ebay/0x0x646152/b556f694-7b2c-4860-bd26-bf63ad018f6f/eBay_COMPEXHIBITA-CompCommitteeCharter_FINAL.pdf

[43] Refer to DEF 14A Proxy statement (2014) retrieved from the U.S. SEC EDGAR database

The Board in year 2011 encompassed 11 members, 9 of which were independent.[44] These numbers will be used for calculation of probability of successful strategy identification once the model is tested in this case. Furthermore, the CEO is not the Chairman of the Board (Mr. Omidyar is the Chair), which can mean lower probability of the Board being captured by the CEO and dictated in regard to strategic and compensation decisions.

CEO profile: John Donahoe, 47 yrs[45] (tenure: 2008 – present). Mr. Donahoe is a seasoned and highly qualified top manager who was prepared to become eBay CEO during Ms. Meg Whitman's tenure (being President of eBay Marketplaces).

Career timeline[46]

2008 – President, CEO and Director at eBay;

2005 – 2008 – President of eBay Marketplaces, responsible for eBay's global e-commerce businesses;

1999 – 2005 – CEO and Worldwide Managing Director at Bain & Company;

1982 – 1999 – Managing Director at Bain & Company.

Problem description. After eBay spectacular growth with Ms. Meg Whitman, the company was starting to struggle as its marketplace business was starting to slow down whereas PayPal business unit was gradually picking up. In the face of the crisis and increasing competition in the marketplace space Mr. Donahoe was to strengthen eBay retail position (acquisition of GSI Commerce in 2011) and keep growing financial services division. Whilst balancing the retail and financial services business units, Mr. Donahoe was then pursuing a growth strategy at the mobile commerce and mobile payments market, trying to capture a share not only at the online commerce market but commerce in general. Since the conventional online auction business still amounted to 7.4 bln USD against 5.6 bln USD generated by PayPal (2012), the forecasted relationship by 2015 is 52% to 48% (11.5 bln USD against 10.5 bln USD).[47]

In 2010 eBay was turning around the internal structure of businesses and assessing possible strategic directions. Therefore, it is an important milestone in turnaround strategy implementation.

Current incentive plans. CEO compensation structure in eBay is skewed toward performance-based components since the Compensation Committee believes in rewarding executives' efforts that lead to successful strategy implementation. In 2010 non-equity incentive plan and performance-based stock units accounted for 15% and 21% respectively whereas in 2013 the same components amounted to 12% and 40% respectively. This evolution of compensation structure within the company

[44] Refer to DEF 14A Proxy statement (2012) retrieved from the U.S. SEC EDGAR database

[45] At the time when he started serving as CEO

[46] Refer to Forbes: http://www.forbes.com/profile/john-donahoe/; Refer to App. 7 to find data on parameters evaluation

[47] Veverka M. (2013) Unplugged: Ebay's impressive run under CEO John Donahoe. USA Today. Retrieved from: http://www.usatoday.com/story/tech/columnist/veverka/2013/04/01/ebay-amazon-att-meg-whitman-john-donahoe/1995211/

shows stronger belief and focus on equity-based remuneration for performance. Total performance-dependent part of compensation exceeds 75% (including time-based stock units and options). Tab. 12 provides an overview of executive compensation throughout the period of 2008-2013.

Non-equity incentive plan. For eBay this part of compensation program reflects achievement of short-terms objectives, in other words, it aligns CEO remuneration with annual operational goals (however, the Compensation Committee can review and change the length of the performance period). Foreign-exchange neutral revenue (calculated on a fixed foreign exchange basis; FX-neutral), net income, net promoter score improvement, employee engagement improvement and individual performance are metrics against which yearly performance is assessed. Net promoter score improvement is a proxy for customer satisfaction; in such a customer oriented business it is important that it is properly measured and improved on year-by-year basis. Weights for evaluation are the following: 65% for the financial metrics in total (equally divided for two parameters), 25% is for individual performance, 5% is devoted to customer satisfaction metric and 5% is devoted to employee engagement. Since evaluation in 2013 slightly changed and employee engagement seem not to be taken into account, moreover, proxy statement doesn't reflect individual performance metric for the CEO, we recalculated weights of the above mentioned metrics.[48] Moreover, all metrics have a minimum threshold; if performance is below this threshold the CEO is not paid anything; otherwise he is paid according to a scale of multiples (in regard to target incentive plan that is linked to the base salary).

Table 12: CEO compensation at eBay. Source: rendering from DEF 14A proxy statements

Compensation, ths USD	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**
Salary	879,808	934,615	920,673	945,577	970,353	993,269
Bonus	500	522,917	736,538	0	0	0
Stock awards	5167,156	4450,388	5586,045	8854,607	23729,96	8855,064
Option awards	6364,098	2483,682	3735	3799,993	2000	2199,263
Non-equity incentive plan	0	1568,752	1158,575	2688,984	2844,346	1620,27
All other compensation	279,108	172,394	245,655	167,367	160,42	165,508
Total compensation	13190,17	10132,75	12382,49	16456,53	29705,08	13833,37

Performance-based restricted stock units (PBRSUs). The Compensation Committee offers two-year performance-based restricted stock units for the CEO based on attainment of several performance metrics, namely FX-neutral revenue, non-GAAP operating margin and return on invested capital (ROIC). PBRSUs are granted one or two years prior and vested to the CEO based on the performance. Thereby for example at the end of year 2010 the CEO was allocated (provided the goals are achieved) PBRSUs granted in year 2009 (performance period 2009-2010) and at the beginning of year 2010 (performance period 2010-2011).[49] Revenue and

[48]New weights are: 46.43% for financial metrics and 7.14% for customer satisfaction.

[49]Yet the portion of performance period 2010-2011 PBRSUs is not to be granted until after the end the performance period even though one-year targets are achieved.

operating margin metrics are weighted equally, then a resulting normalized measure is modified by a coefficient related to ROIC.

Performance share units. Another component that is paid out at some performance periods based on Total Shareholder Return (TSR). However, in 2013 he target value was not reached and in 2010 performance shares were not paid out, therefore, we do not analyze this component in detail. However, we hypothesize the same methodology as with non-equity incentive plan or performance-stock units can be applied to this compensation element.

Tab 13 demonstrates performance target metrics against the actual performance in two periods for case study analysis.

Table 13: Target and actual performance at eBay.Source: rendering from DEF 14A proxy statements and 10-K annual reports

For ST[50] incentive plan	**2010**		**2013**		Weights	**For LT[51] incentive plan**	**2010**		**2013**	
	T	**A**	**T**	**A**			**T**	**A**	**T**	**A**
Revenue, bln USD	8,337	9,16	15,16	16,15	46,43%	Revenue, bln USD	8,96	9,16	29,5*	29,85*
Net Income, bln USD	2,25	2,299	3,61	3,56	46,43%	Operating income	2,76	2,7	7,72*	8,25*
Customer satisfaction, points	7	Achieved	N/A	Achieved	7,14%	ROIC, %	23,90%	25%	23,1%*	23,5%*
						TSR, %	N/A	N/A	72,90%	73,20%

* Figures for the long-term incentive plan are calculated on two-year basis; therefore, for all financial metrics should take into account results of the year 2012 and 2013.

Model illustration and reality check. The case is broken down into two periods: first period is years 2008-2010 and the second period falls into years 2011-2013. App. 7 provides details on CEO's history and evaluated variables. While assessing the modeled short-term incentive plan compensation, we calculate the resulting figures in accordance to their weights in eBay methodology. First period was rather successful for the company (the normalized result is 1000[52]) and the second period also exceeded expectations (the normalized result is 115). The model results are presented in Tab. 3.12. According to the model results the Board does not need to change the strategy. The incentive plan in the first period is equal to 4.981 mln USD whereas in the second period it amounts to 1.875 mln USD. In actual case the amount that was paid to the CEO in the first period was 1.159 mln USD and 1.62 mln USD for the second period.

50 ST = short-term

51 LT = long-term

52 Calculated multiple corresponding to overperformance against the target metrics (metrics weighted in accordance to short-term and long-term incentive plan weights)

After recalculation of results for the long-term incentive plan (using another weighting scale) the model evaluation for the compensation was still the same. The possible reason for that is similar composition of both plans – short-term and long-term plans. Due to the similar assessment of performance in the short and long term, the model is insensitive to changes in financial performance variable. Therefore, resulted model numbers can be assessed as integral value of incentive plan. The actual figures for short- and long-term incentive plan is 4.496 mln USD for year 2010 and 7.175 mln USD for year 2013.

Based on this test we can derive the following insights. Firstly, it is arguable whether performance metrics for short- and long-term incentives should be the same. Certainly strategic goals (e.g. growth in the next 3 years) correlate with operational objectives (e.g. revenue growth per annum); however, it creates instruments for additional rent extraction. Once a strategic goal is broken down into series of operational objectives, remuneration mechanism should take into account overlapping of two metrics and compensation components. Secondly, once there is such an overlap in performance metrics, the model can be used for evaluation of integral incentive plan (short- and long-term incentive plans).

Table 14: Model results for eBay case. Source: own rendering

q_0	$\overline{e_1}$	$\underline{e_1}$	$\overline{e_2}$	$\underline{e_2}$	p_G	c	R
0,91	0,75	0,4	0,83333	0,56667	0,81818	500	1000

$\triangle e_1$	$\triangle e_2$	p^0	p^1	$q_0^{l,l}$	$f(q_0^{l,l})$	$q_1^{l,l}$	$f(q_1^{l,l})$
0,35	0,26667	0,3125	0,5625	0,07042	2,11268	0,58824	17,6471

$w^{h,h}$	$w^{l,h}_{S_1=S_0}$	$w^{l,h}_{S_1\neq S_0,new}$	$w^{h}_{S_1=S_0}$	$w^{l,h}_{S_1=s_G=S_0}$	$\triangle f$	Change?
1875	5972,11	3333,33	4981,04	1847,11	27,8873	No

Other cases

After the applied procedure was tested on 4 cases of U.S. public companies fromm retail and IT-industry, additional 10 cases of U.S. public companies from those industries were considered. The following result were presented in Table 15.

As can be seen from the Table 15, our model showed good results for the sun of two periods for five of the considered companies (Fred's, Dollar Tree, Barnes & Noble, Lowe's Corporation, Blackbaud), but is has some deviations in certain periods and, on the whole, is working better for the retail industry.

More than that, it worth mentioning that the model is working better in case of changing both strategy and CEO after the first period. It could be explained by the fact that the model suppose new CEO has no reputational risks and historical effects almost do not influence the incentive plan.

Also there is a practice of a partial payout of incentive packages in IT-companies even in case of failure to achieve the target performance goal set by the board of directors, but the model itself supposes for this case there is no incentive payout possible for a manager.

Moreover, there is a common tendency across 8 of 10 examined companies to overpay their CEO based on the results of theoretical modeling. Of course, some

Table 15: Summary results

Company	q_0	Change of strategy		Compensation after 1st period, million $		Compensation after 2st period, million $		Sum of compensation for two periods, million $	
		Fact	Model	Fact	Model	Fact	Model	Fact	Model
Fred's, Inc.	0,75	No	No	1,345	1,300	0,000	0,000	1,345	1,300
Dollar Tree, Inc.	0,545	No	No	1,800	3,000	1,900	0,450	3,700	3,450
Kohl's Corporation	0,75	No	No	2,145	1,750	0,535	0,000	2,680	1,750
Barnes & Noble, Inc.	0,625	Yes	Yes	0,000	0,000	2,604	2,848	2,604	2,848
Lowe's Companies, Inc.	0,6	No	Yes	2,225	2,181	1,500	0,525	3,725	2,706
Yahoo, Inc.	0,67	Yes	Yes	1,500	0,000	1,120	1,250	2,620	1,250
Blackbaud, Inc.	0,72	Yes	Yes	0,437	0,000	0,870	1,370	1,307	1,370
Blucora, Inc.	0,5	No	No	0,540	0,000	0,450	0,216	0,990	0,216
Linkedin Corporation	0,875	No	No	0,570	0,000	0,636	0,450	1,143	0,450
CA Technologies, Inc.	0,8	Yes	Yes	1,500	0,000	1,764	1,790	3,264	1,790

companies can save money and fire their CEO, but what happens in real practice is that this step would hurt the reputation of the company on the labor market of top-management. Also, companies do not limit their operation by one strategy only as considered in the model, but their business is rather diversified, so the board of directors often enough set a compensation package based on broader range of factors than those considered in the paper.

Besides, the model considers a game for two periods that sets huge reputational risks for those periods. In real business practice strategies are implementing for longer periods and it is possibly worth considering more periods in theoretical modeling as well to get more precise results, probabilities of outcomes and more smooth risks for players.

So, for the model to be more precise in cases of low business results it was suggested to introduce new coefficients $\varepsilon \;\; E$. Those parameters set the percentage of the maximum incentive package in case of either failure to achieve a target performance goal or achieving better result than that expected. And it worth mentioning that those coefficients are subject for individual setting for each company and should be determined by each board of directors.

7. Conclusion

The research paper represents total amount of 10 case studies of modeling of incentive packages for CEO of U.S. public companies in retail and IT-industries. It was demonstrated in the paper that the theoretical instrument could be applied as an instrument of valuation of incentive compensation for the better motivation of high level of efforts from CEO for corporate strategies implementation.

Furthermore, the chosen mechanism introduced reputation as an important factor of influence on manager's efforts application. Therefore, the CEO cares not only for monetary reward but also considers reputational risks in case of low performance, which is in line with current executive compensation research and corresponding concepts of talent.

We have also addressed the issue of setting performance objectives and goals and concluded that compensation composition can have adverse impact on efforts application (in the form of opportunistic behavior) by the CEO if similar performance metrics are used for design of short- and long-term incentive plans.

The novelty of the given research paper is formulation of methodology how to evaluate parameters in the chosen model, so it can be applied on actual company cases.

We realize that incentive methods are sensitive to international corporate governance regulations and current practices. Since European corporate governance differs with the U.S. practices, results of the methodology cannot be applied directly but taking into account European specificity methodology can be adapted and tested in different environments.

Since development of Russian public companies was following the U.S. example, we can assume that managerial implication for the Russian public companies is more structural and relevant for management control. In order to allow the board monitoring the CEO's efforts (hereby strengthening corporate governance), boards of directors should be composed of the majority of independent directors who represent shareholders' interest, not having other agenda in mind. There is lower opportunity to capture or collude with the board once composition is skewed toward independent directors.

Proven its applicability on example of the U.S. public companies in retail and technology industries, the model still has limitations and can be further improved. First of all, the game implies rationality of players, which is not always the case in reality. Even though we introduce the concept of reputation that implies nonmonetary stimulation, rationality in the model still remains an issue.

The applied procedure with minor amendments can be used as a secondary instrument in the U.S. public companies to evaluate incentive plans of CEO. Except for companies, some other researches like us could be interested in that methodology. Consulting companies could enrich their portfolio of instruments by introducing the considered model.

Appendix 1. The base model solution

This model is a base game theoretical interpretation of the principal-agent phenomenon whose objective is to model the incentive plan of CEO compensation (performance-based pay component). The principal (owner, shareholder or investor) hires an agent (CEO) to implement a company strategy (strategic decision) in the subsequent time, followed by the principal's decision to replace or leave the agent. This model is a non-cooperative dynamic game; a modification of this base model is analyzed with scrutiny in Chapter 1. In this game the company CEO is incentivized not only materially but also non-materially (he cares for his/her reputation).

The underlying assumption of the model is that the company strategy *cannot* be amended in the 2nd period after it has been chosen in the 1st period.

All variables and assumptions are similar to the game described in the text.

Interaction between the principal and the agent is represented in the form of a decision tree in Fig. 3. Dotted lines incorporate the same information sets, in other words the player with the move cannot differentiate between nodes within the information set. Several branches are not depicted in detail due to the fact that the outcome will never occur. Branches where CEO exerts low efforts are analogous to branches where s/he exerts high efforts; the only difference is in probabilities.

The following changes should be considered.

Based on Bayes' formula, reputation after the 1st period is calculated differently:

$$q^i = \left[\begin{array}{l} 1 \text{ if } R_1 = R_h, \\ q^l \text{ if } R_1 \ = R_l, \end{array}\right. \tag{23}$$

where reputation of the manager after the 1st period with low performance $R_l = 0$ (probability that the manager is good) is:

$$q^l = \frac{q_0(1-e_1)}{q_0\,(1-e_1)+1-q_0}. \tag{24}$$

Reputation after the 2nd period is the following:

$$q^{i,j} = \left[\begin{array}{l} 1\ if\ R_1 = R_h\ and/or\ \ R_2 = R_h, \\ \ q^{l,l}\ if\ R_1 \ = R_l\ and\ R_2 \ = R_l, \end{array}\right. \tag{25}$$

where reputation of the manager after the 2nd period in the light of two periods with low performance $R_l = 0$ (probability that the manager is good) is:

$$q^{l,l} = \frac{q_0(1-e_1)(1-e_2)}{q_0\,(1-e_1)\,(1-e_2)+1-q_0}. \tag{26}$$

Payoffs of each player are described the same as in the modified game in section 3.

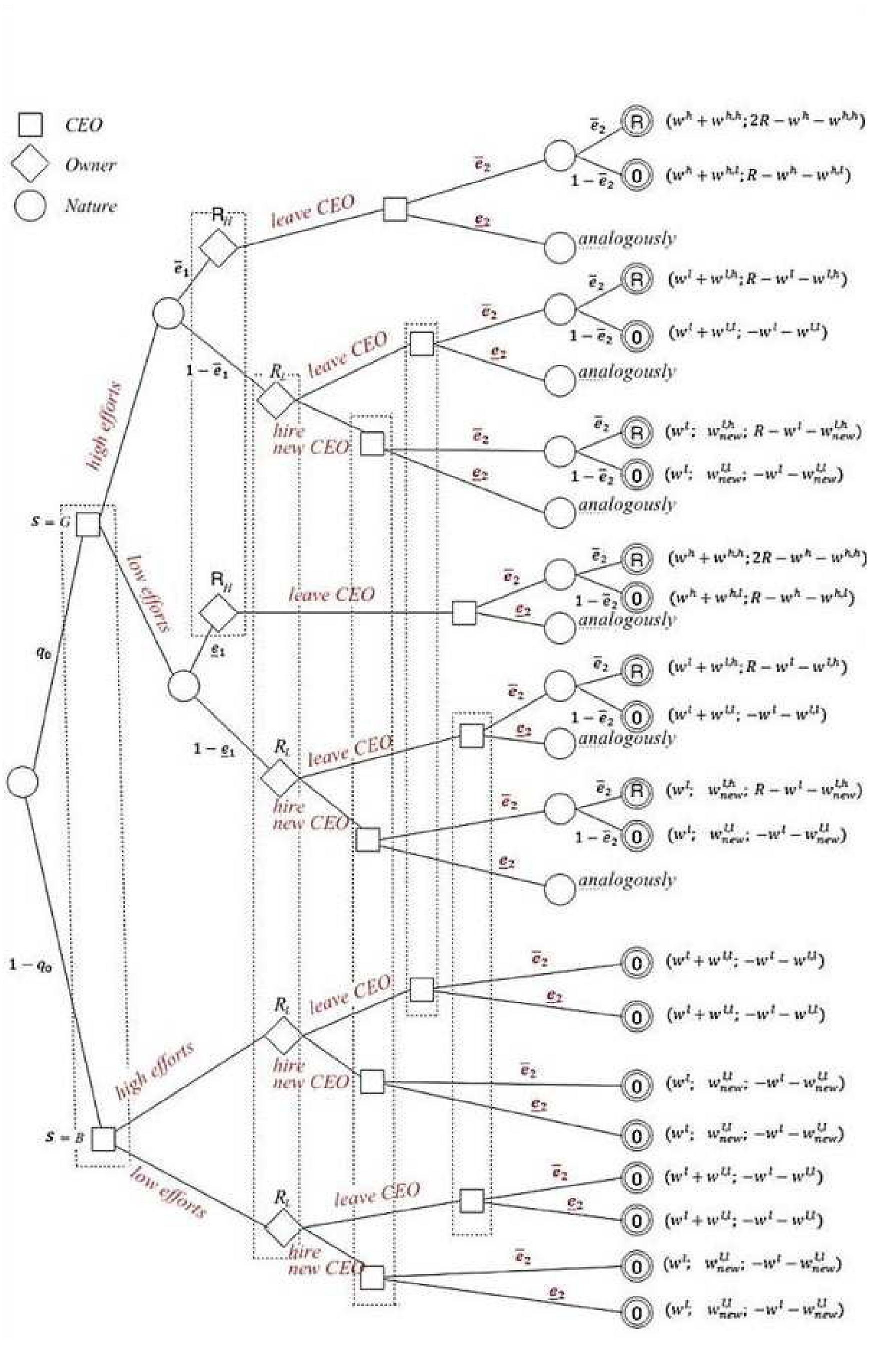

Fig. 3: Game tree

Solution of the model. Compensation contract is accounted for the solution of the model. Equilibrium strategies for the agent and the principal constitute the overall Nash equilibrium; the model is solved by backward induction.

Let us consider the last move of the game where the agent makes a decision about the level of efforts. In each sub-game the manager has 2 alternatives: exert high level of efforts $\overline{e_2}$ or shirk and exert low level of efforts $\underline{e_2}$. High efforts mean higher payoff for the principal and thus is more desirable.

Let us denote conditional probability that the chosen strategy is successful (accounted for the Company performance in the 1st period) as p^i:

$$p^i = \left[\begin{array}{l} 1 \text{ if } R_1 = R_h, \\ p^l \text{ if } R_1 = R_l, \end{array} \right. \tag{27}$$

where

$$P^l = \frac{q_0(1 - e_1)}{q_0\,(1 - e_1) + 1 - q_0}. \tag{28}$$

In order to find compensation value we are required to solve linear programming problem: the principal maximizes his expected payoff by minimizing the agent's expected compensation. The objective function looks as follows:

$$\min \left[p^i \left(\overline{e_2}\ w^{i,h} + (1 - \overline{e_2}\ )\, w^{i,l}\right) + (1 - p^i) w^{i,l}\right]$$

subject to the following constraints:

Constraint on incentives compatibility (the agent must exert high efforts):

$$w^{i,h} - \ w^{i,l} \geq \frac{c}{p^i \triangle e_2} - \triangle f. \tag{29}$$

In case of a new manager (no reputational risk):

$$w^{i,h} - \ w^{i,l} \geq \frac{c}{p^i \triangle e_2}. \tag{30}$$

Constraint (the agent's expected payoff should exceed costs under high efforts):

$$p^i (\overline{e_2} w^{i,h} - \ (1 - \overline{e_2})\, w^{i,l} + (1 - p^i) w^{i,l} \geq c. \tag{31}$$

Constraint on limited liability: $w^{i,h} \geq 0$, $w^{i,l} \geq 0$.

Let us consider possible outcomes:

1. $R_1 = R_h$, then $i = h$. Hereby $p^i = 1$, $\triangle f = f\left(q^{h,h}\right) - f\left(q^{h,l}\right) = 0$. Then the linear programming problem is the following: $\min \left[\overline{e_2}\ w^{h,h} + (1 - \overline{e_2}\ )\, w^{h,l}\right]$

subject to:

$$w^{h,h} - \ w^{h,l} \geq \frac{c}{\triangle e_2},$$

$$\overline{e_2} w^{h,h} - \ (1 - \overline{e_2})\, w^{h,l} \geq c,$$

$$w^{h,h} \geq 0, w^{h,l} \geq 0.$$

The first and fourth constraint are satisfied as equalities, therefore:

$$w^{h,h} = \frac{c}{\triangle e_2}, \tag{32}$$

$$w^{h,l} = 0. \tag{33}$$

Compensation is the same for the old and new CEOs.

1. $R_1 = R_l$, $i = l$, then $p^i = p^l$, so it can be calculated according to the formula (28). If the agent is not fired, then $\triangle f = f(1) - f\left(q^{l,l}\right)$. Condition (29) looks as follows:

$$w^{l,h} - w^{l,l} \geq \frac{c}{p^l \triangle e_2} - \triangle f. \tag{34}$$

Condition (31) looks as follows:

$$p^l (\overline{e_2} w^{l,h} - (1 - \overline{e_2})\, w^{l,l} + (1 - p^l) w^{l,l} \geq c. \tag{35}$$

We need to choose the lowest compensation that satisfies these conditions, then $w^{l,l} = 0$. If (34) becomes an equality, then

$$w^{l,h} = \max\left[0;\ \frac{c}{p^l \triangle e_2} - \triangle f\right].$$

These values satisfy the condition (35), i.e. $w^{l,h} \geq \frac{c}{p^l \overline{e_2}}$. If this condition is not satisfied, then (35) becomes an equality. Hereby optimal compensation is as follows:

$$w^{l,h} = \max\left[\frac{c}{p^l \triangle e_2} - \triangle f; \frac{c}{p^l \overline{e_2}}\right], \tag{36}$$

$$w^{l,l} = 0. \tag{37}$$

Under these compensation values for the 2nd period the CEO will always exert high level of efforts since his expected payoff accounted for high efforts is higher than in the case of low efforts.

Now let us consider the principal's move. If after the 1st period the Company performance is high R_h, then $p^h = 1$, i.e. the agent is good and the strategy is successful. Therefore, it is unreasonable to replace the agent after high performance in the 1st period, so in such a case the owner always prefers to leave the old top manager in the Company.

If or the performance is low R_l, the principal has two alternatives: leave or replace the agent. In order to find out the principal's strategy, we need to compare his/her payoffs in both cases. For the principal it is optimal to stimulate high efforts of the agent if and only if the Company financial performance is significantly high: $R \geq \frac{\overline{e_2} c}{p^i (\triangle e_2)^2}$.

Let us consider the first move of the agent. He has 2 options again: exert high or low level of efforts. In order to find optimal compensation incentivizing to exert high efforts, the following linear programming problem should be solved:

$$\min\left[q_0\left(e_1\ w^h+(1-e_1\ )\,w^l\right)+(1-q_0)w^l\right].$$

Subject to:

$$w^h-\ w^l\geq\frac{c}{q_0\triangle e_1}-\overline{e_2}\left(w^{h,h}-\mathrm{w}^{l,h}_{S_1=S_0}\right)-(1-\overline{e_2})\triangle f,$$

$$w^h\geq 0, w^l\geq 0.$$

The problem solution is the following when the first and the third inequalities become equalities:

$$w^h=\max\left[0;\ \frac{c}{q_0\triangle e_1}-\overline{e_2}\left(w^{h,h}-\mathrm{w}^{l,h}_{S_1=S_0}\right)-(1-\overline{e_2})\triangle f\right],\tag{38}$$

$$w^l=0.\tag{39}$$

Considering these results it is transparent that the manager will exert high efforts in every sub-game in the 1st period in order to maximize his expected compensation.

Therefore Nash equilibrium strategies for both players are as follows:

1. For the agent: in both periods he should exert high efforts $\overline{e_1}$ and $\overline{e_2}$.
2. For the owner: regardless of the Company result after the 1st period he should leave the agent.

Expected compensation of the agent for 2 periods is the following:

$$E\left(w\right)=q_0\left[\overline{e_1}\left(w^h+\overline{e_2}w^{h,h}\right)+(1-\overline{e_1})\overline{e_2}w^{l,h}\right].\tag{40}$$

The game solution is demonstrated in Fig. 4.

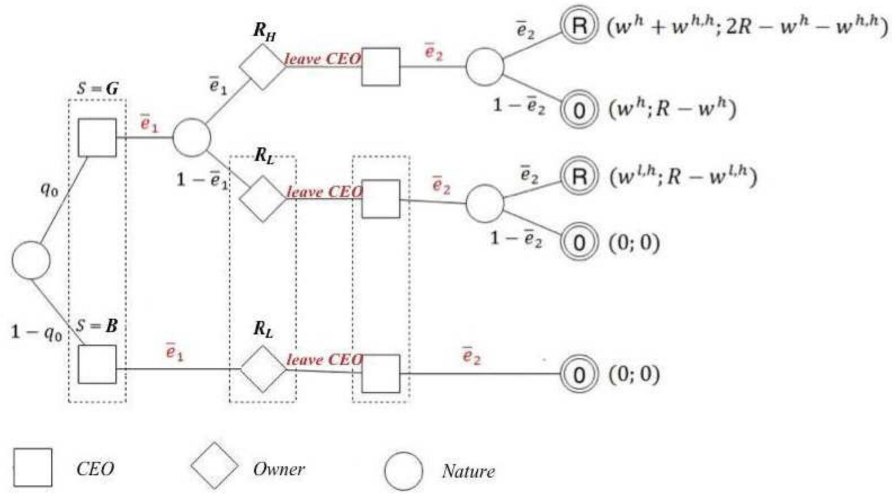

Fig. 4: Game solution

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Incentive Plans Improvement in Movie Value Chain: USA Motion Picture Industry *

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Abstract The purpose of the research is to investigate the motives of cooperation in the movie production and to improve the methodology of incentive income imputation formation on the basis of appropriate game-theoretical model construction. As a result a systematized methodology of the income imputations definition generated by the product of cooperation (movie), which can be used as a decision-making support tool in negotiations about shares of the income allocation among the participants of the cooperation of the movie creation, has been elaborated. It should be accentuated that it is not assumed that the implementation of the methodology will give the revenue imputations, which could be taken as per se, however, it can become a substantial help during negotiations of the parties involved about their participation in revenue. The applicability of the methodology has been tested on the cases from Hollywood practice.

Keywords: motion picture, members of motion picture industry, box-office, revenue-sharing contracts, cooperative game, imputation, nondominant revenue imputation, optimal imputation.

1. Introduction

Movie business is an extremely complex business, which involves a lot of people, a lot of interactions among various legal entities, high level of uncertainty about the outcomes of each single project, and thus leaves a lot of loopholes for unfair behavior and possibilities for manipulations. By analyzing the process of movie production, distribution and exhibition, we can trace a very important problem existent in the industry – the issue of optimal incentives for the participants of movie value chain. This issue can actually be broken down into two problems, which constitute two parts of the incentives alignment problem in the movie industry.

The first problem concerns the contract design among the participants of the movie value chain. The demand for the movie is very difficult to assess beforehand, and it can never be forecasted with 100% precision. At the same time the prevailing part of the investments is made at the initial stage, when the final result is completely unknown. The thing is that in the course of project realization some factors appear, which influence the revenue allocation, but which impede its division proportionally the financing of the budget. New groups of participants appear, such as leading actors and directors, which have the ability to substantially increase the

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box-office by their participation in the project. This gives them the advantage during negotiations concerning the level and the form of their compensation. Having a significant bargaining power, they can claim not only for the flat fee, but also for the percentage of revenue or profit (depending on the contract terms) of the project realization. At the same time actors and directors do not finance the budget, and rather on the contrary consume a part of it. According to Epstein (Epstein, 2011) sometimes, although the agreement on the shares is set, the parties may not be fully satisfied with it due to different reasons. For instance, in the case of the production of the movie “Terminator 3: Rise of the Machines” Arnold Schwarzenegger was given the unprecedentedly large share of revenue as a part of compensation without a reasonable basis from the perspective of producers, but they had to undertake this step due to the requirements of the investors. If the producers could underpin their decision with some tool, maybe the situation could have been different, and they would not have had to pay such a large compensation to Schwarzenegger. If a question of inclusion of certain agent of movie making into the allocation of the common revenue is left without attention, different internal problems may arise, such as conflicts between producers from one side and actors and directors – from another, which might activate the problem of lost opportunities and, consequently, smaller amounts of box-office. So here the central question is how to form the cooperative contract, i.e. which percentage of the revenues to assign to each participant.

The second problem concerns the contracts realization in the movie value chain. When the contracts are formed, and the shares of the chain participants are agreed on, they need to be implemented. The agreements among companies, which constitute the main links of the value chain, are in a form of participation contracts, meaning that everybody’s income is dependent on the final revenues, generated by the movie. However, currently the value chain has such a form, which allows different parties to behave opportunistically. This issue is thoroughly elaborated by Vogel (Vogel, 2015), who explains how parties tend to cut themselves a larger lump of the pie by artificially increasing their costs on the books, thus leaving a smaller amount for next participants to share. Wasko (Wasko,2003) and Eliashberg (Eliashberg, 2005) support this idea explaining that the weights of the chain participants are different, and some of them may deprive others from the part of the revenue they can obtain by manipulations with numbers or by exertion extra pressure on weaker players. In other words, the profit is allocated unfairly. For instance, producers can very often find themselves in an instable position, since the distributors, being the part of huge media conglomerates, have all the conditions to play with numbers on the books by artificially bloating costs, and at the same time the producers have to set agreements with the talent, who can also manipulate their remuneration by imposing special terms on their participation in the project. Therefore, the gap for the research appears: how to incentivize the participants to act fairly, and consequently to minimize the losses of the weakest players of the chain. Basically, the opportunistic behavior can be eliminated with the mechanisms of the contract implementation in the value chain. Special terms should be introduced, which would mitigate the risk of cheating, and motivate every participant in the chain not no overestimate the costs. Coordination contracts principles can help with the solution of this problem.

1.1. Movie making process

Movie production is a process which consists of a lengthy consequence of the unique creative decisions leading to creation of a product from the initial story till the release and the exhibition in the movie theaters, on television or in the Internet. Movie making process consists of several major stages: Development, Pre-production, Production, Post-production and Distribution.

Development. At this stage a producer selects a story, which may be based on a book, play, game, true story, be an original idea, which can vary from a general idea to a finished script (Vogel, 2010, Squire, 2004). In some cases a producer (or a studio) asks a scriptwriter to write a new (or to adapt an existing) script. However, usually the scriptwriter with the help of a literary agent gives the first draft to several independent or affiliated with the studio producers. If a producer is interested, both parties sign the option-type contract, which gives a producer a right to buy the finished script and a scriptwriter gets an up-front fee (a share of which is taken by the literary agent). From this point on substantial funding resources are required in order to start the project. The financing is not that problematic, if the producer is affiliated with the studio. When signing the contract with the studio, the producer usually gives up the studio a significant portion of rights, which are connected with the sequels, new episodes of the series, distribution. Funding is much more challenging, if a producer doesn't have an agreement with the studio, he needs to find the initial funds from other sources, which is a very difficult task, especially if there are no guarantees concerning the distribution (Eliashberg, 2005). The final version is submitted to investors, studios and other interested parties. For the assessment of the potential success of the movie even at the early stage a distributor can be attracted, who analyze the genre, the target audience, the success of the similar movies in the past, the actors and potential directors. All these factors imply some attractiveness for the spectators. Not every movie can be profitable only from theatrical exhibition, thus the production companies also take into consideration box-office in the world and the DVD sales. Producers and scriptwriters prepare the movie pitch, or treatment, and present it to potential investors. The parties involved negotiate the terms of the deal and sign the contract. As soon as the parties have met and the deal has been set, the movie may proceed to the pre-production stage.

Pre-production. Pre-production is the longest and definitely very important period. Producers hire the director, the actors and the crew, search for shooting locations, think of the design of the production set and the costumes, calculate the budget, based on such factors as script, expenditures for post-production (for instance, for special effects), starting salaries and funding potential. The production budget is compounded and the production expenditures are calculated. In case of massive projects, apart from anything else, the insurance is acquired for the protection of unforeseen circumstances. Then the producer hires the crew, which will be working on a movie production during several months. In many Hollywood blockbusters there are hundreds of people involved, while low-budget independent movies are sometimes created with only eight-nine people (Eliashberg, 2005).

Production. At this stage the crew is enlarged. The production period may last several months, but due to the high cost of this stage, producers try to minimize it by thorough planning and rational organization of the shooting process.

Post-production. At this stage the movie is assembled by the movie editor. The shot material and the sound are edited, and then all the sound elements are married to the picture and the work on the movie is finished (Weis & Belton, 1985). Usually post-production period lasts longer than the production stage, up to several months in duration. Editing process is often called the second director's production, because with its help it is possible to change the concept of the picture.

Distribution. At this stage the motion picture is released to the big screens. The massive marketing campaign starts. Upon the release distributors usually launch press-releases, interviews with press, preview screenings and film festival screenings. Movies are shown in predetermined cinemas and several months later they are released on DVD. The box-office is then allocated among the exhibitors, the distributors and the production company.

1.2. Producer-studio relationships

There exist different schemes of the relationships between the producers and the studios, which determine the income distribution among them. There are five basic financing-production and distribution options, described by Cones (Cones, 1997). Those are:

1. *In-house production/distribution.* Under this contract the studio (the distributor) finances all phases of the project. In this case a producer, who is responsible for a movie acts as an employee of the studio.
2. *PFD (production-financing-distribution) agreements.* In this case an independent producer comes to the studio/ distributor with a project, where all core elements are already defined, and the studio provides financing of production and distribution.
3. *Negative pickup agreements.* Under this agreement the distributor acquires the original negative with the distribution rights. In other words, it is responsible for distribution and pays the production costs.
4. *Acquisition deals.* The distributor is in charge of the distribution only, and the funds for production is given by other parties.
5. *Rent-a-distributor deals.* In this case practically all financing for the production and distribution has been provided by other parties, and the finished movie is ready for the distribution.

The difference in producer-studio relationship results in different value chains, and thus different problems, arousing on each of the links. This paper concentrates on movie production by an independent producer. First of all, the process is more complicated in this case, and the income distribution is not as obvious as in the case of studio-affiliated production, where a studio, being an extremely powerful player, imposes its own rules of the game, and no other party has enough power to command its own terms. Moreover, in case of studio-affiliated production, both the production and the distribution are under control of the studio, meaning that no contracts exist between the producer and the distributor, thus there is no problem of coordination of contracts. Basically, no contract disputes can occur in this situation. This situation is very advantageous for the studio, however, it is extremely disadvantageous for all other participants, who signed the agreement for sharing contract, because studios very often blow up the expenditures, and profitable movies in reality show no profit at all on the books. What is more, in the case when producer is affiliated with studio, he loses the rights for the movie, including the rights for the last word in the creative part of the production.

There are a lot of participants of this chain. Those are: the producer, the distributor, the exhibitors, the talent (actors, director, etc.), the scriptwriter, agents (of the scriptwriter, actors, etc.), the investors, the banks, the insurance company.

Rodnyansky (Rodnyansky, 2013) identifies following steps of the movie value chain from the idea creation till the delivery of the movie to the spectators:

1. The scriptwriter creates a script.
2. The scriptwriter finds an agent. If the agent considers the script promising, he starts offering it to the production companies.
3. The producer buys the script and develops it and simultaneously taking care of the search for actors and director.
4. When the script is ready, and the leading actors expressed their confirmation of the participation in the project, the work on the budget and calendar schedule starts.
5. The producer starts negotiations with American distributors.
6. The producer signs the contract with the distributor. If the potential of the movie is high, the contract may imply the payment of the minimum guarantee (MG). For the movie with a budget from \$15 mln. to \$60 mln. MG may constitute around \$5 mln., but such term is quite rare in the contracts. Usually, another guarantee is stipulated: the minimum number of copies, on which the movie will be released, and the minimum budget for the release (P&A).
7. With the mastered support of the American distributor, the producer starts negotiations with sales companies, which take care of the presales of the rights to the international distributors.
8. The location is defined.
9. The next very important step is the getting a completion bond, guaranteeing that even in the case of acts of God, the movie will be finished in accordance with the approved script, budget and calendar schedule. Without the completion bond the producer cannot approach investors or banks. The issuing company reads the script, questions experts on the financial success potential, checks the budget, meets the director and sometimes the leading actors. The amount of the insurance is the budget of the movie + 10% for the unbudgeted expenses. The cost of the completion bond is usually not over 6-7%. The representative of the insurance company is always present on the set of the movie.
10. In order to get the money on security of presales, agreements with the authorities of the state about the reliefs and, if necessary, in order to get the missing amount in the budget, the producer can approach the bank, private or institutional investors.
11. The production and post-production phases are carried out.
12. The work with the US distributor about the positioning of the movie, marketing campaign, etc. is carried out.
13. Realization of other rights. (if any are left after the agreement with the local distributor): VOD, DVD, television.
14. Release of the movie.
15. From the proceeds made by the movie, movie theaters take 50% and the rest of the receipts are given to the distributor. The distributor subtracts expenses for the release (P&A), the amount of the minimum guarantee, if it was paid, and its share of the gross. Money left after the payments on a full scale is passed to the producer.

16. Usually, the first to get the money are banks, which gave loans secured on rights, the next are private investors. Often share of the profit is also given to the director and the main stars of the movie. The last one to get the money is the producer.

2. Cooperation in movie value chain

2.1. Bargaining and negotiations

In the motion picture industry the question of box-office revenues allocation among its creators is always a topical one. The investors have to get the profit, since it is their return on investments. The producers get all the residual income, which is left after the exhibitors and the distributors take their shares. The producers pay back the borrowed money to the investors. However, there are a lot of reasons, why sharing contracts may be arranged with those participants of the movie making process, the contribution of who is intangible. In other words, the producers, who by the principles of motion picture industry get the rest of the money after all deductions, start sharing income from the movie with those participants, the contribution of who is very difficult to assess financially. In this paper we are considering predominantly the case of an independent movie production and consider the following players: producers, actors and directors. The model, where the producer is affiliated with the studio works with completely same logic, the difference is only in the names of the cooperation participants: instead of the producer, the distributor will share revenue with actors and director. Going back to our schematic representation of the movie value chain, now we consider the relationship among the players of the first link of the chain.

The question arises: what exact percentage from the income it is needed to set in order for all the parties to be satisfied and not to have objective reasons to decline the solution. It is quite obvious that the larger power is in the hands of producers, since they are the owners of the rights of the movie, and thus they want to arrange a deal in the most favorable way for themselves. At the same time the stars do not want to agree for the terms, which will not satisfy them sufficiently. The conflict situation appears. Instruments of game theoretical modeling come to the aid, and with their help it becomes possible to define the income imputations in different forms. These imputations can be used as a basis of negotiations. Certainly, a huge role play the skills of the producer, studio representative, agents or lawyers of actors and directors to negotiate more favorable for their own side terms and conditions, but the method, which will be described later in the chapter, is suggested to be used only as a base for negotiations of this type. It serves as a tool for decision-making support.

2.2. Cooperative contract as cooperative game

Imputation in cooperative game. Let's consider game in characteristic function form - game $\Gamma =< N, v >$. Here $N = \{1, 2, \ldots, n\}$ - set of players (in our case those are producers, a director and actors, which can claim for a share of movie proceeds). The real-valued function v with the set of players N defined on coalitions $S \subset N$ is called a characteristic function of the n-person game. Here the inequality $v\,(T) + v\,(S) \leq v\,(T \bigcup S)\,,\ v\,(\emptyset) = 0$ holds for any nonintersecting coalitions $T,\ S\ (T \subset N,\ S \subset N)$ (Petrosyan, Zenkevich, 2016, p. 168). This is called a *superadditivity property*, which means that the payoff of the united coalitions is no less than that of the two nonintersecting coalitions, when they act independently. If this inequality is not fulfilled (which means that the united coalition does not bring additional payoffs), then the unifying into coalition is senseless, and it will be more rational for the players to act independently. Let's consider that the issue of imputation choice is modeled by a cooperative game.

Value $v(S)$ is a gain of the coalition S, i.e. that payoff, which the participants can get when working together. In game theory it is supposed that function $v\,(S)\,,\ S \subset N$ has a superadditivity property (Petrosyan, Zenkevich, 2016, p. 168), meaning, how has already beed described earlier, the payoff of the participants, if they work together, should be bigger than the sum of their payoffs in case they work independently. In case of the movie industry this property is always fulfilled, since only in cooperation participants can create the final product (a different matter is that the composition of the participants may vary), and all together they achieve a synergetic effect. From the superadditivity property it follows the inequality $\sum_{i=1}^{k} v(S_i) \leq v(N)$. This implies that the maximum payoff may be achieved only upon participation of all players in the maximal coalition, and there is no such decomposition of the set N that the guaranteed payoff to these coalitions would exceed the payoff of all players acting together $v(N)$. Thus, all participants have a motive to cooperate in confines of the maximal coalition. Let's discuss, what happens in terms of motion picture industry. No participant (producer, director and actor) can create a movie by himself only or in a tandem with another participant. The project will be realized only upon the participation of all three parties. Consequently, we can say that there is an obvious synergetic effect.

Now let's bring in the notion of the payoff *imputation.* The vector $\alpha = (\alpha_1, \ldots,\ \alpha_n)$, which satisfies the conditions

$$\alpha_i \geq v\,(i)\,,\ i \in N, \tag{1}$$

$$\sum_{i=1}^{n} \alpha_i = v(N) \tag{2}$$

where $v\,(i)$ is the value of the characteristic function for a single-element coalition $S = \{i\}$, and α_i is the payoff of the same coalition, is called an imputation (Petrosyan, Zenkevich, 2016, p. 171).

Condition (1) of the imputation is called an *individual rationality condition*, and it implies that in order for a member to decide to participate in a coalition he should receive at least the same amount he could receive if acting alone without support of other players. Condition (2) is called a *collective (or group) rationality condition.* It implies that there are no other imputations of the payoff of $v(N)$, which would bring each player a larger payoff than the considered imputation. Consequently, only if the condition of collective rationality is fulfilled, vector $\alpha = (\alpha_1, \ldots, \alpha_n)$ can be taken as admissible. Therefore, in order for the vector $\alpha = (\alpha_1, \ldots, \alpha_n)$ to be an imputation in a cooperative game $\Gamma =< N, v >$, it is necessary and sufficient that it could be represented as $\alpha_i = v\,(i) + \gamma_i,\ i \in N$, where $\gamma_i \geq 0$ payoffs from cooperation of the player $i \in N$. Meaning that each player should gain more in cooperation, than he would gain by acting alone. If the condition $\sum_{i \in N} v(i) < v(N)$ is fulfilled, the game is called essential. This means that cooperation brings a positive payoff.

Cooperative contract in movie production. In motion picture industry, when the movie is under production the main participants of the process face the problem of income allocation, which they earn from the movie release. As a payoff $v(N)$ we will consider the revenue of the movie excluding the share of the exhibitors (usually 50% of the total box-office), because as it has been discussed in the first chapter of the paper, the exhibitors get their share before the proceeds are distributed among those who actually produced the movie. However, it is important to note that the decision about the receipts allocation is made at the initial stage, when neither the final result nor the success of the movie is known. Thus, the forecasted box-office is considered as payoff. Consequently, the issue of the characteristic function construction arises. It is needed to calculate the value of the characteristi function for the contribution of each participant. At this point we need to look more precisely at the principles of the expected box-office calculations. Analysts look at the movies of the same genre, with the cast of the similar class, and then basing on that data they make their forecasts about the box-office for the new movie. Moreover, they look at the box-offices of the movies with the participation of the certain leading actor and compare them to the movies of the same genre with less renowed actors. Moreover, movies of the director, which pertain to the same genre, as the one under production, are analyzed. All those estimations are taken into consideration, and on their base the forecast of a specific movie is made. In Hollywood there exist various advanced box-office forecasting models, which allow to obtain quite accurate estimates.

In order to demonstrate the mechanisms of the imputation calculations mechanisms we would like to show them on numerical examples. First, let's consider a fabricated example, and later in the paper examine real cases from Hollywood practice. So let's assume that a certain movie "Z" is produced. As we remember, the producer is a person with ultimate responsibility for a movie, meaning that he owns all the rights (if we consider an independent movie production). He is responsible for finding the funding for movie creation, which can come from different sources, including his own assest. Those mechanisms were discussed in the first chapter of the paper. So basically, he either spends his own money, or he has an obligation to the investors to pay back the borrowed money. Thus, we will consider that his contribution will be estimated proportionally to the financing assets he brings to the project, since for the purposes of this model we assume that the producer is in possession of the sum of money, which is enough to make a movie. What is more, the producer later deals with the investors with the money, which he receives as a part of revenue distribution deals, that is why it is also important to make sure that the producer makes enough money to pay back. Moreover, money as a hard asset stresses the high bargaining power of the producer. Now let's move back to the example. The movie's "Z" budget equals to \$20 mln., however, due to high risks of the project, two producers: producer A and producer B, decide to cooperate in order to share risks, and each of them commits \$10 mln. For the leading role a star actor X with worldwide recognition is invited. It is forecasted that his participation may significantly increase the movie's box-office. It is expected that on condition of this actor's participation, the box-office will equal \$200 mln. At the same time without this actor the movie will also be able to become profitable, although not to such an extent: the box-office is forecated to be \$160 mln. So it is possible to say that by his participation actor X is increasing the box-office by \$40 mln. Since the participation of this actor significantly augments the profit-earning capacity of the movie, it will be more advantageous to all of the project participants, if he stays in the project. In other words, this actor obtains a large power to influence his remuneration. He may set a deal for movie earnings participation. Since in our case we consider an A-list star, let's consider that he is able to sign a contract for revenue (and not profit) participation. Here the issue arouses: what share exactly to offer an actor in order for him to have enough motivation to work on the movie, but at the same time not give him a share, which would be too large, in order to guarantee the return on investments to producers. It is also needed to take in account the uncertainty of the box-office, because the forecasts do not have 100% accuracy and in reality they may be far from estimations.

The revenue, which will be divided among the considered participants after the deduction of the exhibitors share (50%) will equal to \$100 mln. or $v(\{1,2,3\}) = 100$. Now let's find out, how much the players will be able to earn, if they do not operate in maximal coalition. Not a single player will be able to create a movie alone, and consequently to gain additional gains. So $v(\{1\}) = v(\{2\}) = 10$ (each producer has his \$10 mln.); $v(\{3\}) = 0$ (the actor does not earn anything). If producers A and B try to make a movie without this actor, they will manage to do it, because they can hire another less famous actor, which will not claim for revenue share. However, the expected box-office of such movie will be smaller and will equal to \$80 mln. or $v(\{1,2\}) = 80$. If any of the producers tries to create a movie on his own, none of them will succeed due to insufficiency of the budget. Thus, the producers will only have their \$10 mln. in hand: $v(\{1,3\}) = 10$; $v(\{2,3\}) = 10$.

The imputation of this cooperative game is an allocation of revenue, gained under cooperation. It is worth stating that the superadditivity property is fulfilled, which assumes that each player in coalition should add some value to this coalition. Now let's move to the definition of income imputation.

2.3. Optimal imputations in cooperative game

Nash bargaining solution. Obviously, for each player the notion of optimality means the maximization of his share of cooperation payoff. However, not a single player can guarantee the maximization of the payoff, since the matter is the division of the common payoffs.

First of all, let's consider bargaining problem (Petrosyan, Zenkevich, 2016, p. 160). It will allow us to define the upper border of pretentions of the weakest participant of cooperation, in our case an actor (or a director), since their contribution to cooperation is intangible and therefore difficult to assess. The thing is that negotiations about the way, how to allocate the proceeds, may last infinitely long, and then end up with no result, if systematical approach is not applied. Thus, the reasonable solution to a dispute would be to invite some independent arbiter, who has an equal attitude towards all the parties, and who would act fairly. If the arbiter is in fact unbiased and fair, then he will probably make a decision, which would suit all players. Nash bargaining solution serves as such arbiter.

To find Nash bargaining solution (NBS) we need to apply Nash function:

$$H(\alpha_1, \ldots, \alpha_n) = \prod_{i=1}^{n} (\alpha_i - v_i). \tag{3}$$

NBS is the solution to the following optimization problem (Petrosyan, Zenkevich, 2016, p. 164):

$$\max H\ \ (\alpha_1, \ldots, \alpha_n) = \max_{\{\alpha_i\}} \prod_{i=1}^{n} (\alpha_i - v_i) \tag{4}$$

under conditions:

$$\alpha_i \geq v\,(i)\,,\ i \in N,$$

$$\sum_{i=1}^{n} \alpha_i = v(N).$$

In other words, the payoff of every coalition from cooperation should be no less than they could earn when working alone. The sum of those payoffs should be equal to the total payoff of the coalition.

It is clear that the result achieved in such a way is Pareto optimal. This means that there is no other division of the game $v(N)$, under which each each player gets more than his share in a specific imputation.

Nash bargaining solution arouses interest, because it has a number of important properties (Mazalov, 2010): efficiency (or Pareto optimality); linearity (under linear transformations optimality remains) and symmetry. The latter property implies the equal status of the players, i.e. if players have the same market (bargaining) power, then the Nash bargaining solution is symmetrical.

In case of the motion picture industry NBS can be applied for calculation of the participation shares in proceeds allocation as a kind of a reference for further negotiations, because it can always be calculated, under any values of the characteristic function. This imputation can be considered as an upper border of the share, to which an actor or a director may claim. The thing is that as has already been said before, the NBS is considered fair, just because it allocates the total payoff from cooperation among players (the income with the exclusion of costs) in equal shares. It does not take into consideration probable inequality of the players. Thus the calculated shares may not satisfy the producers, which would not want to share with the participants that much, since he or she does not contribute to the common affair tangibly, and consequently, it is very hard to estimate the degree of his input contribution to the overall performance of the project. Consequently, it is needed to find such imputations, under which not a single coalition would have objective reasons to decline them.

However, let's first go back to our reference example and calculate a Nash bargaining solution. NBS will be a solution to the following problem:

$$\max(\alpha_1 - 10)\ \ * (\alpha_2 - 10) * \alpha_3$$

under constraints:

$$\alpha_1 + \alpha_2 + \alpha_3 = 100 \tag{5}$$

$$\alpha_1 \geq 10,$$

$$\alpha_2 \geq 10,$$

$$\alpha_3 \geq 0.$$

In order to calculate Nash bargaining solution we need to maximize the product of the gains of each player. In addition, both of the considered producers expect to get no less than what they have contributed, i.e. not less than \$10 mln., actor X also expects to het some reward for his participation in the project. All together they will not be able to get more than the box-office revenue of \$100, which we have forecasted.

By solving (5) problem, we get the following imputation:

$$\alpha_1 = 36,67, \alpha_2 = 36,67, \alpha_3 = 26,67. \tag{6}$$

We can see that the shares are divided in a way, if all the participants were equal. We cannot forget that the producers have invested \$10 mln. each, thus, if we deduct their expenditures from the total payoff, we get the absolutely equal distribution of profit among the producers and the actor, a situation, which certainly cannot happen in real life, because producers, having a higher bargaining power, than actor, will simply not allow him to receive such a large share. However, Nash bargaining solution let us make a conclusion about the maximum of all possible shares, which could theoretically get the weakest player.

Nondominant bargaining solution. Now we need to find such imputations, which would be nondominant, meaning that no coalition would have objective reasons to disagree with these imputations. In other words, the gain of each coalition (both single-element and two-element) of players would be no less than that if they worked independently. Since each participant in any situation gets benefits from the joint activity with other players, they do not have objective reasons to disagree with such imputation. Basically, such distribution is stable in a way that it is disadvantageous for any coalition to separate from other players and distribute a payoff of this smaller separated coalition among its members. There can be many such imputations, and the participants may have different subjective reasons to disagree with a certain earnings imputation. Thus, it is needed to find an solution, which would satisfy all the players. Such impulation may become nondominant bargaining solution. For its definition we need to consider only the set of nondominant imputations.

Mathematically the problem of finding nondominant bargaining solution would look as following:

$$\max_{\alpha_i} \prod_{i=1}^{n} (\alpha_i - v_i)$$

under constraints:

$$\sum_{i=1}^{n} \alpha_i = v(N), \tag{7}$$

$$\alpha(S) \geq v(S), S \subset N,$$

where $\alpha(S) = \sum_{i \in S} \alpha_i$.

This method will again give us the fair division, as in the case of Nash bargaining solution, however, it will take into consideration the inequality of the initial contributions of the players. Nondomination of the imputation means that no coalition has objective reasons to decline such imputation. This gives another reference during negotiations about the final earnings division. Despite the clear advantage of such method over the previous one, its disadvantage resides in the fact that it does not consider the inequality of the bargaining power of players, and the difference is in their abilities to impose their own rules of the game.

Let's move to our example and calculate the nondominant bargaining solution in respect to the producers A and B and the actor X. We need to solve the following problem:

$$\max(\alpha_1 - 10) \ * (\alpha_2 - 10) * \alpha_3$$

under constraints:

$$\alpha_1 + \alpha_2 + \alpha_3 = 100, \tag{8}$$

$$\alpha_1 \geq 10,$$

$$\alpha_2 \geq 10,$$

$$\alpha_3 \geq 0,$$

$$\alpha_1 + \alpha_2 \geq 80,$$

$$\alpha_1 + \alpha_3 \geq 10 \ ,$$

$$\alpha_2 + \alpha_3 \geq 10.$$

We again maximize the product of the players' payoffs, and at the same time not only each participant expects to get not less than he has contributed, but each pair of players also expects to get from cooperation not less than they could earn if working in pair. Since producers A and B can manage to create a movie without the participation of this certain actor, and it will have a drawing capacity of $80 mln., they assume that if they hire the actor X for the leading role, their income will also increase, as otherwise the decision to share proceeds with him would be senseless. At the same time if producers work alone, they will not be able to finance the budget, thus, the movie will not be produced, and they will remain only with their $10 mln. at hand.

After solving the (8) problem we get the following result:

$$\alpha_1 = 40, \alpha_2 = 40, \alpha_3 = 20. \tag{9}$$

Therefore the nondominant bargaining solution of the payoffs shares under cooperation gets the following form: (40%, 40%, 20%). We can see that the revenue again has been divided quite evenly, however, now the inequality of the initial contribution is taken into account, and the producers receive $10 mln. more than in the case of NBS. Nevetherless, such solution would hardly suit the producers, as 20% of the movie box-office is a way too large percentage for investors to give to an actor. Therefore, although this solution is not optimal for solving the considered problem, it allows an actor to roughly estimate, what he can generally claim his pretentions for when cooperating on the movie creation. From this provision the next idea appears, which could serve as a support for defining the payoff imputations.

Maximum and minimum nondominant imputations. With the help of the nondominant imputations calculation (those imputations, under which no coalition of players would have objective reasons to decline them) we can calculate maximum and minimum value of the actor's or director's share from cooperation. Thus, we will get a corridor of the values on the set of nondominant imputations, in which there will be located the share of the weakest of the players. In other words, we will get the pretentions' range of a player with intangible input. In the general case such corridor of feasible changes of the income share can be calculated for any player.

In general case the set of equations for *minimum nondominant imputation (MIN solution)* for player i will look as follows:

$$\min_{\alpha_i}(\alpha_i - v_i)$$

under constraints:

$$\sum_{i \in S}^{n} \alpha_i = v(N), \tag{10}$$

$$\alpha(S) \geq v(S), S \subset N.$$

We minimize the payoff of player i, for which we want to find a corridor of the possible income percentage values. Income of the other players is divided among them proportionally to their contribution to the result of cooperation.

The problem of the search of *maximum nondominant imputation (MAN solution)* for player i will be as follows:

$$\max_{\alpha_i}(\alpha_i - v_i)$$

under constraints:

$$\sum_{i \in S}^{n} \alpha_i = v(N), \tag{11}$$

$$\alpha(S_i) \geq v(S_i), S \subset N.$$

The problem is absolutely similar with the first one with the only difference that now the payoff of the considered player is not minimized, but maximized.

Now let's go back to our example of the movie "Z" production and calculate minimum and maximum nondominant solutions for it.

For calculation of the MIN solution for player 3 (actor X) let's solve the following problem:

$$\min \alpha_3$$

under constraints:

$$\alpha_1 + \alpha_2 + \alpha_3 = 100, \tag{12}$$

$$\alpha_1 \geq 10,$$

$$\alpha_2 \geq 10,$$

$$\alpha_3 \geq 0,$$

$$\alpha_1 + \alpha_2 \geq 80,$$

$$\alpha_1 + \alpha_3 \geq 10 \ ,$$

$$\alpha_2 + \alpha_3 \geq 10.$$

As a result we get the MIN solution:

$$\alpha_1 = 50, \alpha_2 = 50, \alpha_3 = 0. \tag{13}$$

Obviously the minimum value of the actor's X share is 0%, i.e. he does not participate in the revenue distribution and gets only fixed payment from the movie budget. The shares of the producers are equal, as they have financed the budget in equal proportions.

Now let's find the maximum nondominant solution, which would show the shares of the box-office allocation, if the actor was offered the maximum possible percentage.

$$\max \alpha_3$$

under constraints:

$$\alpha_1 + \alpha_2 + \alpha_3 = 100 \tag{14}$$

$$\alpha_1 \geq 10,$$

$$\alpha_2 \geq 10,$$

$$\alpha_3 \geq 0,$$

$$\alpha_1 + \alpha_2 \geq 80,$$

$$\alpha_1 + \alpha_3 \geq 10,$$

$$\alpha_2 + \alpha_3 \geq 10.$$

As a result we get the MAN solution:

$$\alpha_1 = 40, \alpha_2 = 40, \alpha_3 = 20. \tag{15}$$

The computed solution shows that the maximum share, which an actor can claim for, equals 20%. So we have set the borders, within which actor X can negotiate about his percentage. However, they give us only a range of actor's pretentions.

It is quite obvious that 20% is too high, and the producers will not accept such imputation. Generally speaking, one needs to take into account that the participants of our cooperation are unequal in their power of influence on negotiations' result. Generally speaking, producers have a higher bargaining power, than actors, because they are owners the rights for the picture and they possess the unambiguous and easily measurable resource – finances. Creative participants of cooperation, having an intangible input in cooperation, have a lesser extent of influence on the final decision concerning their remuneration, since their contribution is hard to assess numerically. All the methods of revenue distribution, which were considered before, did not take into account the inequality of players' power of influence on their payoff share. Consequently, there is a need of introducing the weights for each of the players, which would allow removing the problem of inequality of the players.

Weighted nondominant bargaining solution. If every player would be assigned with some weight, then is seems that the problem of bargaining power inequality of different players would be offset, and the game would be normalized. So in general case *weighted nondominant bargaining solution* (WNB solution) is the solution to the following problem:

$$\mathrm{max}\mathrm{H}^{\mathrm{w}}\left(\alpha_1,..,\alpha_{\mathrm{n}}\right)=\mathrm{max}_{\{\alpha_{\mathrm{i}}\}}\prod_{\mathrm{i}=1}^{\mathrm{n}}\left(\alpha_{\mathrm{i}}-\nu_{\mathrm{i}}\right)^{w_i}.$$

under constraints:

$$\sum_{i\in S}\alpha_i = v\left(S\right),\ S\subset N, \tag{16}$$

where $w_i \geq 0;;w_1+..+w_n=1$. The vector $w=(w_1,..,w_n)$ is a set of weights w_i of players, where parameter w_i characterizes the power of player i, $i\in N$ in the game.

Shapley index. For the producer the weight is defined by the amount of commited financing sources. With those participants, the contribution of who is intangible, everything is not so obvious. For the definition of the weights of those players two methods can be introduced. First of them is the use of Shapley index. Shapley index is calculated on the basis of Shapley value, which actually can be considered as as a tool for solution of the problem, considered in the paper. The advantage of optimal imputation's definition with the use of Shapley value resides in the fact that such imputation exists in each game and it is unique. Shapley value is calculated as follows:

$$\phi_i\left[v\right]=\sum_{S\backslash i\in S\subset N}\frac{(s-1)!\,(n-s)!}{n!}\left[v\left(S\right)-v\left(S\backslash i\right)\right],\ i\in N, \tag{17}$$

where $s=|S|$.

Shapley value has several properties (Petrosyan, Zenkevich, 2016, p. 182): independence from irrelevant alternatives; independence from repositioning of the numbers of the players and linearity.

On the basis of *Shapley value* it is possible to calculate *Shapley index*, which is often used as a measure of the player's power in a certain game:

$$sh\left[v\right]=\left[sh_1\left(v\right),...,sh_n(v)\right],$$

where

$$sh_i = \frac{\phi_i[v]}{v(N)}, i = \overline{1,n}. \tag{18}$$

Going back to our example, let's calculate the Shapley value according to the above stated formula. We get the imputation: (46,67%, 46,67%, 6,67%). By deviding the obtained shares by the expected value of the movie revenue, we get Shapley index, which we will use as a power of influence of this or that player. Generally, Shapley index is widely used in politics in evaluation of parties influence in parliament. In our case the index will calculate the influence of certain participants of moviemaking with the help of relationships among coalitions, in which a participant is essential towards all winning coalitions. Weights of the participants can be also shown in vector form (0,47; 0,47; 0,07). Using these weights let's calculate the revenue imputation with the formula of weighted nondominant bargaining solution. By solving the problem with Excel functions, we get the solution (47,33%; 47,33%; 5,33%). The revenue is divided in the other way than was obtained with the usage of Nash bargaining solution and nondominant bargaining solution. Now it is taken into account that the producers have more opportunities for exerting pressure on an actor. This is an approximation of what happens in real life. The thing is, no matter how talented and famous an actor is, producers will not allow him to claim for unlimited amount of money. Even in the cases when the role is written specially for a certain actor, and the director does not want to consider anyone else for this role, it is needed to accept that in motion picture industry, just as in any other business, those who possess rights and money, are the ones to dictate the termes.

Expert weighted solution. However, weights of the players can be calculated not only by the method of Shapley index. In fact, with all its advantages, it has one serious disadvantage when applied to movie industry: it is impersonal and does not consider the details of each specific project. In motion picture business there are never two absolutely identical projects, although the main principles of the movie production are always the same. Moreover, when the weights of an actor or a director are estimated, a very important role is played by individual properties of each single person. That is why in this work for defining weights of actors and directors it is suggested to use the questionnaire, questions of which are aimed at estimation of factors, which influence the bargaining power of actors, i.e. their weight. It is supposed that the questionnaire is intended for the movie experts, those people, who are to a large extent aware of the mechanisms of the movie industry, who know this business from the inside and sometimes possess the insider information. The questions are not abstract, but always concern certain movies, certain actors and directors. The answers intend subjective opinions of experts towards the influence on the box-office of such factors as Oscars awards or nominations, the number of those, the experience of an actor at the moment of the movie production, number of financially successful movies with his participation. Moreover, the experts are invited to give a subjective estimation of influence on the bargaining power and consequently on the probability of signing a sharing contract of such factors, as the fact that the movie is a sequel, the established reputation of an actor/ director in terms of the behavior and tendency towards shrinking, the length of the contract with this or that studio/ producer (in case of the franchise, when initially a serie of movies is planned to be created), the diversity of the actor's/ director's areas of activity, for instance, his musical activity, participation in TV series, advertising of famous brands, in other words, everything, which allows to increase the visibility of a person. The most important factor, which is suggested for the experts to estimate in the questionnaire, is the level of participant's ability to increase the final movie's box-office. It is possible to examine the questionnaire more thoroughly in the appendix.

The criteria, in their turn, are devided into groups in order of importance. Therefore, to the most important the following ones were assigned: the ownership of the Oscar award, Oscar's nominations, the length of the career and consequently the experience, the variety of activities, and the fact that the movie is a sequel. To the next in descending order of importance group of factors were assigned: the number of Oscar awards, the number of Oscar's nominations, the fact (or the absence) of previous joint projects of an actor and a director. To the less least important criteria were assigned: the reputation of an actor in terms of his behavioral patterns. Consequently, to each group an importance value was assigned on the scale from 1 to 3. The answers of the experts on each question are converted to the 5-point scale. Then the average values are calculated on each question. At the same time the weight of the each criteria is calculated by division of the importance value of each criteria by the sum of all importance values. In other words, if, for example, the importance value of the question is 3, and the sum of importance values for all questions for the estimation of this actor is 14, then the weight of the criterion will equal 0,21. Then finally the weight of the actor is found by the sum of the products of average values of each question and the corresponding weight of the criterion.

However, due to the fact that the weight is calculated irrespective to other participants of cooperation, it arrives too big for obtaining relevant results. Thus, to calculate the weighted nondominant bargaining solution, we normalize the weight of an actor or a director by multiplying the calculated weight by the largest in history percentage, which has ever been obtained by a participant with a similar degree of involvement to a project. Under the similar degree of involvement to a project it is implied the quantity of the obligations taken by a participant: whether he fulfills only an acting job, or also a producing one; whether the director is only fulfilling his direct job or he also possesses the rights for the script and in addition produces the project. By multiplying the weight, which was obtained in the course of the expert evaluations, by the largest in the history participation share, we can get the final weight, which would be used for actually imputation calculations.

The general form of the set of equations for defining the expert weighted solution have the following form:

$$\max\, H^w\left(\alpha_1, .., \alpha_n\right) = \max_{\{\alpha_i\}} \prod_{i=1}^{n} \left(\alpha_i - v_i\right)^{w_i}.$$

$$\alpha\left(S\right) \geq v\left(S\right),\ S \subset N, \tag{19}$$

where: $\alpha\left(S\right) = \sum_{i \in S} \alpha_i$; $w = (w_1, .., w_n)$; $w_i \geq 0$; ; $w_1 + .. + w_n = 1$.

Let's move to the reference example. Let's suppose that according to the expert questionnaire, the weight of an actor amounted to 0,8. Consequently, by multiplying this coefficient by the maximum revenue share in history, which amounts to 0,2 (the remuneration of Arnold Schwarzenegger for the movie "Terminator 3: Rise of the Machines") we get that the final weight of an actor X equals to 0,16. Let's remind, that weights of the producers are difined proportionally the invested resources. Since in our case both producers provided equal amount of funding, their weight is also equal, and it amounts to 0,42. It is needed to specify, why the value is exactly 0,42. In order for the game to be normalized, the sum of participants' weights should equal 1. Consequently, when we first found the weight of the weakest player, we deducted it from 1, and the remaining sum of weights we divided between the producers proportionally their financial contribution to the project.

Now we need to solve the following problem of non-linear programming:

$$\max{(\alpha_1 - 10)}^{0,42}\} * {(\alpha_2 - 10)}^{0,42} * {\alpha_3}^{0,16} \tag{20}$$

under constraints:

$$\alpha_1 + \alpha_2 + \alpha_3 = 100,$$

$$\alpha_1 \geq 10,$$

$$\alpha_2 \geq 10,$$

$$\alpha_3 \geq 0,$$

$$\alpha_1 + \alpha_2 \geq 80,$$

$$\alpha_1 + \alpha_3 \geq 10\ ,$$

$$\alpha_2 + \alpha_3 \geq 10.$$

By solving this problem in Excel, we get the expert weighted solution:

$$\alpha_1 = 43,6, \alpha_2 = 43,6, \alpha_3 = 12,8. \tag{21}$$

In the percentage format the imputation will look as following: (43,6%; 43,6%; 12,8%). We see that this solution is more fair than all of the considered earlier. From the one hand, it takes into account the contribution of an actor, but not overestimates it, what Nash bargaining solution and nondominant bargaining solution did. From the other hand, it does not give the large overbalance in favor of producers, what showed Shapley value imputation and Shapley index weighted solution. The thing is that the two latter methods substantially underestimated the contribution of an actor, since they did not consider the specificity of projects, and were calculated only on the basis of mathematical repositionings and that utility addition, which actor X could bring to each coalition under condition that his initial input equals zero in money terms. Thus, we can draw a conclusion that the method of revenue imputation finding using the weighted nondominant bargaining solution, when weights are found by expert estimations, gives the most unbiased results from all methods. It should be stressed that the results obtained are not postulated as reference ones. The offered tool set is suggested to be used as a mathematically justified system of support to solution of strategically important ptoblem of revenue distribution among the participants of cooperation.

Let's look at the method of the income imputation finding in a more concise way. We proceed from the assumption that in the receipts allocation may take part following parties: a producer, leading actors and a director. All other participants of the movie making are included in the budget part of the movie, and their payments are fixed. The first step is the definition of the expected box-office and the quantitative forecasts concerning the ability of each of the participant, whose input is intangible, to increase the expected box-office by his participation. This is carried out on the basis on their previous works analysis and the box-office results of the similar genre or storyline movies. Then on the basis of the achieved data the characteristic function is built. After that the imputations are calculated by different methods.

Firstly, it is suggested to calculate the Nash bargaining solution, which shows the absolute maximum of the share, which an actor can claim for. Then the maximum and the minimum nondominant imputations are calculated, which denote the range of pretentions of an actor or a director, which might be accepted by a producer due to the fact that they will not have the objective reasons to decline the imputation. Inside this corridor of values the nondominant bargaining solution is situated, which tries to solve the problem of the as even income imputation among the participants as possible. In order to solve the problem of inequality of power of influence of the participants, it is suggested to bring in the notion of the weighted bargaining solutions, which would consider the bargaining power of an actor and would give more realistic results. Upon that weights are suggested to be calculated by two methods: by calculating Shapley index or by expert questionary. Since for calculating Shapley index the Shapley value itself is needed, it makes sense to check this imputation with respect to relevancy for finding the optimal solution. The imputation, found with the help of Shapley value, and weighted nondominant bargaining solution with the weights-Shapley index, both give realistic results, however, the shares of actors nevetherless remain too big in relation to the shares of producers. This situation can never happen in real life. Therefore, it is suggested to bring in another approach to income imputation definition, namely, expert weighted nondominant bargaining solution. In this case the weights of the producers would be defined proportionally the funding of the project, and the weights of actors and directors would be defined on the basis of expert evaluations. At the same time the value, which was found by consolidation of the expert evaluations, is suggested to be taken as a share of the maximim percentage, which has ever been received by a participant of a movie project with the same level of involvement. This is exactly what will be taken as the weights of actors and directors. Weighted nondominant imputation has an advantage over the other imputations in the sense that it takes into consideration the main characteristic of actors and directors for the motion picture business: their ability to bring in the added value to a movie, which results in increased cash flow generation. However, this method gives a realistic proceeds division, since it considers the larger bargaining power of producers in comparison to creative talent. Thus, a whole spectrum of mathematically justified imputations is presented, and the reliance on them may facilitate negotiations. Although the final decision will anyway to a high extent depend on the skills of layers and the representatives of actors and directors, as well as on the ability of producers to negotiate favorable deals, the author of this work suggests to use the method of weighted nondominant bargaining solution as a base of negotiations, since this solution seems the most relevant in application to the sphere of movie making due to the specific context of the motion picture industry.

2.4. Case studies of producer's and talent revenue-sharing imputation.

In this part of the paper the author would like to show the realization of the methodology and test its applicability of specific examples. Let's consider the creation of three movies: "Inception" (2010), "Alice in Wonderland"(2010) and "Terminator 3: Rise of the Machines" (2003) and try to calculate the income imputations for each of them.

Movie Inception. Let's start with Christopher Nolan's movie "Inception", the main role in which was performed by Leonardo DiCaprio. This movie initially had a very large budget of \$160 mln (IMDb) due to the star cast and massive special effects, consequently, quite a massive payoff was also expected. It is necessary to state here that although the main trajectory of this paper is devoted to the independent movie production, the case of "Inception" is actually the situation, when the producer is affiliated with the studio (i.e. distributor). The thing here is that financing of "Inception" was quite a complicated deal, since the budget was quite large. The The rights for the movie were in the possession of Christopher Nolan, because he wrote the original script, thus, he decided to co-produce this movie with another producer Emma Thomas. The deal with Warner Bros. was arranged, and all the financing was provided by the studio. Later Warner Bros. distributed the movie for the US theatrical release. Now let's try to model the game situation, as if we were in the shoes of people making decisions on the revenue distribution. In the situation considered the following participants claim for the revenue share: the studio (who was the investor of the project), the leading actor, A-lister Leonardo DiCaprio and the director (and at the same time the scriptwriter and the co-producer of the project) Christopher Nolan. Let's call them Player 1, Player 2 and Player 3 correspondingly. All them due to their high power of influence claim for the revenue and not profit participation share.

Let's first build characteristic function, and for that task we need box-office forecast. In order to do that, we need to analyze the movies of the same genre, the same scale and intensity of special effects usage and the same degree of star cast involved, and then we will be able to get the idea of the approximate box-office amount. Let's take an average box-office amount and assume that "Inception" will earn around \$800 mln. However, we should remember that not the entire box-office is distributed, but the revenue after deduction of the exhibitors' share, which is according to the established practice, constitutes 50%. Therefore, the value of the characteristic function of the maximal coalition is \$400 mln., i.e. $v(N) = 400$. No one from the participants will be able to create a movie working independently. Christopher Nolan will not have funding for such a large scale project, the producer does not have the rights for the script, as it is in Nolan's possession, and obviously that DiCaprio will not be able to create a movie on his own. Therefore, the payoffs of Nolan and DiCaprio will be zero ($v(2) = v(3) = 0$). As we have decided earlier in the paper, the producer's power will be estimated proportionally to the investments, he has been able to obtain, since he has obligations to investors. So the producer will only have the money in the amount of \$160 mln. ($v(1) = 160$). If the participants started to team up into different paired coalitions, only a coalition of Nolan and the studio would be able to make a movie. They could assign another actor, who would have a fixed fee and not a sharing contract. Let's assume that without DiCaprio's participation the expected box-office of the movie would be of a smaller amount. For getting a sense of this possible amount let's analyze the filmography of Christopher Nolan, then look at the coefficient of return on investments of these movies and after that let's multiply the budget of "Inception" on average return on investments coefficient. (Actually there are a lot of different means of forecasting the box-office, however, this is not the goal of this paper, thus in this work only simplified versions are represented, moreover, all the forecasts are approximations). So we get that the forecast on "Inception" box-office without DiCaprio's participation in it equals to \$500 mln., and consequently, the considered for the reasons of imputation allocation amount will constitute a half of it and will equal to \$250 mln., $v(1,3) = 250$, $v(1,2) = 160$, $v(2,3) = 0$.

Therefore, after definition of characteristic function we can start calculating the imputations by different methods. With the help of Excel program, we get the following imputations:

1. Nondominant bargaining solution: (240; 80; 80) or in percent format: (60%; 20%; 20%).
2. Minimum nondominant imputation: (400; 0; 0) or in percent format: (100%; 0%; 0%).
3. Maximum nondominant imputation: (160; 0; 240) or in percent format: (40%; 0%; 60%).
4. Shapley value: (255; 50; 95) or in percent format: (64%; 12%; 24%).
5. Shapley index weighted solution (weights of the participants are defined by Shapley index): (313; 30; 57) or in percent format: (78%; 8%; 14%).
6. Expert weighted solution (weights are defined by expert questionary): (306; 38; 55) or in percent format: (76%; 10%; 14%).

However, the calculation of the last imputation is needed to be examined in more detail, as at this moment it makes sense to show the method of weights calculation of a participant on a specific example. In order to find out the power of influence of Leonardo DiCaprio and Christopher Nolan, expert questionary were conducted. In my case they were 40 people, who are somehow connected with the movie industry: current and former managers of the production companies and students of movie universities in Russia and the USA. The questionnaire can be examined in the appendix. After converting the answers to a 5-point scale and their normalization relatively to the significance of these of those factors the following results were obtained: DiCaprio's "weight" constituted 0,8 and Nolan's – 0,77. However, these weights were calculated relative to 1, and we need to normalize them for our game. Consequently, in order to find out the power of influence of Leonardo DiCaprio on the final box-office we calculate his share from the maximum percentage, which has ever been received by an actor in history. Maximum percentage was received by Arnold Schwarzenegger in 2003 for his role of Terminator in the third part of the Terminator franchise, and it constituted 20% (Epstein, 2011) of the movie's gross. Thus, the weight of DiCaprio, which defines his influence on the final box-office is: 0,8*0,2=0,16. For Christopher Nolan the base for the weight computation will be different. The maximum percentage for a director, who is also a scriptwriter and a co-producer, in history was received by James Cameron for the movie "Avatar" (2009) and constituted 30% (deadline.com). Therefore, the final weight of Nolan is: 0,77*0,3=0,23. After calculating the weights of the perticipants, we can compute the weighted nondominant bargaining solution, which is described earlier in the paragraph.

Then we can compare the obtained results with the revenue distribution in real life. The total box-office amounted to \$823 mln., consequently, after deduction of the exhibitor's part, the amount entitled to distribution among the main players was \$411,5 mln., from which Leonardo DiCaprio has taken \$59 mln. and Christopher Nolan – \$69 mln., which is in percentage form was 14% and 17% correspondingly. The distribution, calculated by the weighted nondominant bargaining solution method is close to the real numbers, which gives us a notion that the methodology can be applied to real life cases. Even more than that, we can even say that actually Nolan and DiCaprio could have claimed for a larger percentage than the one, whey have received in reality. It only proves the fact, that producers (especially, if they cooperate with studios for finding the financing, and thus they are binded to their obligations to those huge and powerful parts of media conglomerates, like we have seen in case of "Inception") due to their lagre level of influence can impose their own termes by making weaker participants of the cooperation agree on less favorable conditions.

Movie Alice in Wonderland. Now I would like to move to the approbation of the methodology on another example, namely on Tim Burton's movie, which was released in 2010, "Alice in Wonderland". One if the leading roles in the movie was performed by Johnny Depp. The budget of this movie was even larger than the one in the previous example: \$200 mln (IMDb). All the investments were found in one source - Walt Disney Pictures, meaning that the producer Richard D. Zanuck was binded with an obligation to Walt Disney Pictures and thus, in negotiation model he incorporates the bargaining power of the studio. Let's model this game situation, by solving which we will get the revenue distribution of the motion picture. Just as in our previous example three participants claim for the share of revenues: the producer (and in our model we assume that he incorporates the bargaining power of a studio as an investor), the leading actor Johnny Depp and the director of the movie Tim Burton. For the purpose of our game construction they will be denoted as Player 1, Player 2 and Player 3. All of them claim for a share of revenue, not of net profits. On basis of the previous mutual works of Burton and Depp, as well as on the basis of movies of the similar genre, let's assume that the forecasted amount of revenues after the deduction of exhibitor's share will amount to \$600 mln., i.e. $v(N) = 600$. Now let's find the characteristic function value for each coalition. We assume that players will not be able to make the movie under any circumstances except for the situation of the maximal coalition of three players. The producer will only have the money, but will not be able to shoot the movie, Burton and Depp will not manage without financing as well. Moreover, this game has a peculiarity: Burton and Depp will form a coalition, meaning that their actions will be coordinated, and they will act as a single player. This situation happens, because when Tim Burton was invited to direct the cinematization of Lewis Carroll's novel, as one of his terms he claimed the mandatory participation of Depp in this motion picture. Otherwise, Burton refused to direct "Alice in the Wonderland". Burton and Depp have constituted a great tandem for many years now, which attracts a huge crowd to movie theaters, and a rare Burton's movie does not have Depp in it. The producer had to accept this term, since, firstly, it was only Burton, who they wanted to see as a director, and, secondly, as has been stated earlier, this tandem attracts a lot of people. Therefore, taking into consideration this condition, it is clear that the coalition of Player1 and Player 2 or a coalition of Player 1 and Player 3 is impossible. Thus, the characteristic value for each coalition will look as following:

$$v\,(1) = 200,$$

$$v\,(2) = v\,(3) = 0,$$

$$v\,(1,2) = 200,$$

$$v\,(1,3) = 200,$$

$$v\,(2,3) = 0.$$

Now we can calculate the revenue imputations. With the help of Excel program we obtain the following results:

1. Nondominant bargaining solution: (333; 133; 133) or in percent format: (56%; 22%; 22%).
2. Minimum nondominant imputation: (600; 0; 0) or in percent format: (100%; 0%; 0%).
3. Maximum nondominant imputation: (326; 147; 126) or in percent format: (54%; 25%; 21%).
4. Shapley value: : (333; 133; 133) or in percent format: (56%; 22%; 22%).
5. Shapley index weighted solution (weights of the participants are defined by Shapley index): (422; 89; 89) or in percent format: (70%; 15%; 15%).
6. Expert weighted solution (weights are defined by expert questionary): (444; 72; 84) or in percent format: (72%; 12%; 14%).

In order to calculate the expert weighted solution the weights were calculated by expert questionary. 62 industry insiders answered questions about Tim Burton's and Johnny Depp's powers of influence. According to the evaluations, weight of Johnny Depp (in absolute terms) equaled to 0,9, and weight of Tim Burton – 0,82. In order to normalize these coefficients, we again take the obtained share from the maximum percentage, which was received by the movimaking process participant of the same category. If for Depp the basis remains 20%, the basis for Burton differs from the previous example, since during the production of Alice in Wonderland Burton accomplished functions only of a director. Consequently, his share will now be calculated on the basis of 25%, the largest revenue share obtained by Michael Bay for the movie "Transformers", and it will constitute 0,21. Under these weights we get the imputation (72%; 12%; 14%).

Now let's check, what has happened in reality. The total box-office amounted to $1,02 bln., consequently, $510 mln. were distributed among the participants. Johnny Depp received $40 mln. and Tim Burton - $50 mln. In order to offset the difference in scale, let's look at the percentages. The share of Depp was 8% and Burton got 10%. We can trace that despite the smaller size of the participants' shares of remuneration (than in previous example), the proportion between the shares of actor and director in both cases is the same: the director's share is 1,2 times larger than the actor's one. Moreover, the calculated remuneration share is of the approximately same order, which happened in reality. Therefore, it is again proved that it is possible to effectively calculate participation shares of the participants of cooperation on the basis of the methodology elaborated in this paper.

Movie Terminator 3: Rise of the Machines. The last example, which will demonstrate the suggested methodology of the income allocation, is the third part of the Terminator series "Terminator 3: Rise of the Machines", which has already been mentioned earlier in the paper. The budget of the "Terminator 3" equaled to \$150 mln., which constituted an unbelievably large sum of money for the motion picture budgets of the early 2000s. However, the task of finding such a big sum for financing the movie was not an easy task for the producers. Not a single investor wanted to invest so much money into one project, because the risks were very high. The producer Mario Kassar came up with a solution and decided to raise money from several studios. Three studios agreed to participate in the deal: Warner Bros., which invested \$51,6 mln., Tokio-based company Toho-Towa with \$20 mln. investment and Sony Pictures Entertainment, the share of which in co-financing was the largest - \$77,4 mln. However, all the companies had a mandatory requirement to the producers: Arnold Schwarzenegger should participate in the project. They considered that only the face of Schwarzenegger could draw significant box-office to the movie. Otherwise they refused to provide the funding. This is exactly the reason, why this movie project is of such an interest to us. This term gives Schwarzenegger a huge bargaining power, because he, having a support of the investors, could make almost any claims, concerning his remuneration. Since forecasted box-office promised to be quite big, and also due to the fact that the acquisition of the rights for the franchise and the script development cost the producer a couple douzens million dollars, they did not want to abandon the project and continued the movie production.

So let's proceed to the modeling of the game situation. Since the producer has arranged the deal with three studios for the funding of the movie, and thus he has obligations to all of them, and consequently according to our model, he incorporates the bargaining power of all three, but since their investments were different, for the purposes of more explicit demonstration of the model and more precise results, the author decided to consider 4 players, instead of 2. So instead of saying that the producer acts on behalf of each of the studios, for simplification of labelling we will be naming studios by their names. So 4 players claim for the revenue share: studios Warner Bros., Toho-Towa and Sony Pictures Entertainment, as well as the leading actor Arnold Schwarzenegger. For the purpose of convenience let's denote them as Player 1, Player 2, Player 3 and Players 4. In our case we have an assumption that they will be able to create a movie only in maximal coalition, which consists of all four players, and the characteristic function value in this case will be the expected box-office of the movie with the deduction of the exhibitors' share. On the basis of available data on two previous Terminator movies, let's assume that the expected box-office will constitute \$520 mln., consequently, the revenue of the maximal coalition will constitute \$260 mln. ($v(N) = 260$). It is needed to specify that as a forecasted revenue the amount, larger than actual box-office, which was achieved in reality, was chosen on purpose. Usually, each new movie which is a sequel, is able to draw a larger box-office than the previous serie. "Terminator 2" was an exceptionally successful motion picture, which managed to achieve \$519 mln. in the box-office. So it is logical to presume that the expected revenue of "Terminator 3" was supposed to be at least equal to that of the previous serie of the franchise.

In all other cases of different coalitions combinations, characteristic function value will equal the sum of the disposed funds of the participants of the coalition. Thus, $v(1) = 52$, $v(2) = 20$, $v(3) = 78$, $v(4) = 0$, $v(1,2) = 72$, $v(1,3) = 130$, $v(1,4) = 52$, $v(2,3) = 98$, $v(2,4) = 20$, $v(3,4) = 78$, $v(1,2,3) = 150$, $v(1,2,4) = 72$, $v(1,3,4) = 130$, $v(2,3,4) = 98$.

When characteristic function is defined, we can proceed to imputations calculation. With the help of Excel, we get the following results:

1. Nondominant bargaining solution: (84; 25; 151; 0) or in percent format: (32%; 10%; 58%; 0%).
2. Minimum nondominant imputation: (84; 25; 151; 0) or in percent format: (32%; 10%; 58%; 0%).
3. Maximum nondominant imputation: (52; 20; 78; 110) or in percent format: (20%; 8%; 30%; 42%).
4. Shapley value: (79; 48; 106; 28) or in percent format: (31%; 18%; 41%; 11%)
5. Shapley index weighted solution (weights of the participants are defined by Shapley index): (86; 40; 123; 12) or in percent format: (33%; 15%; 47%; 4%).
6. Expert weighted solution (weights are defined by expert questionary): (77; 30; 115; 38) or in percent format: (30%; 11%; 44%, 15%).

Let's look at the expert weighted solution. Interest here raises the basis of the weight. In all other cases, which were considered earlier in the paragraph, we have taken the share (calculated on the basis of expert questionary) from the maximum percentage, which an actor ever received for his work. As that percentage appeared the share of Schwarzenegger for his work in "Terminator 3" movie. Therefore, an approach, based on historic data is not applicable in current example. We know the maximum share, which Schwarzenegger can get for his participation in this movie (this share was calculated on the basis of maximum nondominant imputation). It equals \$110 mln. or 42% of the revenue. These 42% we will take as a basis for Schwarzenegger's weight calculation in this project, which will define his degree of influence on the final outcome of negotiations. Subsequent to the results of expert questionary (62 experts) Schwarzenegger's weight equals to 0,82, consequently, his normalized weight equals to 0,11. The weights of investors are defined proportionally to the share of their funding of the budget. Thus, the weights of the companies were as following: Warner Bros. – 0,31; Toho-Towa – 0,18; Sony Pictures Entertainment – 0,41. In accordance with such power of influence distribution of the cooperation participants we get expert weighted solution: (30%; 11%; 44%, 15%).

In reality Schwarzenegger for his role in "Terminator 3: Rise of the Machines" received 20% of the movie receipts, which constitutes a much larger amount than practically any other share of income distribution, which are considered in this paper. Generally speaking, the contract, obtained by Schwarzenegger, is still considered one of the best examples of deals ever made by actors in Hollywood. This result was achieved, firstly, due to paramount importance of Schwarzenegger participation in the movie production for investors. Secondly, a significant role is played the high level of professionalism of actor's layers, because only thanks to their negotiation skills Schwarzenegger was able to emerge the winner from the unequal battle with studios and producers. What is more, an effect may be exerted by the poor performance of the movie in the box-oofice: it earned almost \$100 mln. less, than expected. If the assumption, that the expected revenues of the subsequent part of the franchise should be larger than those for the previous one, is right, and the same was presumed by the producers during calculations of different scenarios, it is possible to say that Schwarzenegger did not realized expectations of the producers and investors. In this case his contribution, which was estimated at 20% of the movie's revenues is overestimated. If the methodology, which is suggested by the author, was used, such situation may have been avoided, since mathematically justified recommendations would have clearly showed the overestimation of the actor's contribution.

From studying Table 1, where the consolidated results of methodology approbation on Hollywood cases is presented, it is easily traceable that the Expert weighted solution gives the best results in terms of feasibility and applicability to real life. The fact that it takes into account the considerations of the experts of the industry, and, thus, the bargaining power of the participants is incorporated into the calculations, allows for the most accurate results from all of the methods considered in the paper. Therefore, the expert weighted solution is the solution, recommened to usage by the author.

Table 1: Methodology approbation results

	Inception	**Alice in Wonderland**	**Terminator 3**
Nondominant BS	(60%; 20%; 20%)	(56%; 22%; 22%	(32%; 10%; 58%; 0%)
MIN solution	(100%; 0%; 0%)	(100%; 0%; 0%)	(32%; 10%; 58%; 0%)
MAN solution	(40%; 0%; 60%)	(54%; 25%; 21%)	(20%; 8%; 30%; 42%)
Shapley value	(64%; 12%; 24%)	(56%; 22%; 22%)	(31%; 18%; 41%; 11%)
Shapley index WS	(78%; 8%; 14%)	(70%; 15%; 15%)	(33%; 15%; 47%; 4%)
Expert WS	(76%; 10%; 14%)	(72%; 12%; 14%)	(30%; 11%; 44%, 15%)
Reality	(69%; 14%; 17%)	(82%; 9%; 10%)	(28%; 11%; 41%; 20%)

Source: Compiled by the author

3. Coordination in movie value chain

3.1. The concept of coordination

When the value chain of a movie exists, a question of how to incentivize all the crucial links to act fairly and avoid opportunistic behavior arouses. As has been discussed in the first chapter, there is quite a substantial amount of cheating involved in the value chain. Exhibitors distort the amount of revenue in order to retain a larger lump of it; the distributors creatively increase the amount of their expenses on the books in order to eliminate the net profit, which is to be distributed among the producer and the creative talent. Actually, the weakest party here is the producer, since it is he, who receives all the residuals from the proceeds, therefore, it is in the producer's primary interests to have the contracts in the value chain coordinated. The contracts in such value chains are participation ones. Generally (without the relation to the motion picture industry), such contracts are organized in the following way. A supplier sells a product to a retailer for a specific price and the latter shares a part of the revenue with the supplier. Upon the offer of the purchase price of the supplier, the retailer sets the quantity of the product to order before the demand is actually realized. Depending on the business situation, it is also in the power of a retailer to set the retail price for the product when the order is placed to the supplier, or it can determine the price on the basis of the market price. In these settings, a typical revenue sharing agreement determines a fraction of the supply chain revenue to be kept by the retailer. The proportion, in which the contractors share their revenue in the case of a typical revenue sharing contract, is independent of the amount of the revenue realized (Pasule-Desai, 2012). Under a revenue-sharing contract, a retailer pays a supplier a wholesale price for each unit purchased, plus a percentage of the revenue the retailer generates (Cachon and Lariviere, 2005).

However, movie industry has its own peculiarities. The exhibitor does not pay a distributor a wholesale price for getting the motion picture for exhibition, and the income of both is only the shared revenues from the movie exhibition. In their turn, the distributor and the producer also usually do not have monetary relationships before the final income is actually distributed. The producer usually transferres the rights for movie distribution, and only then receives the income as a share of the final box-office. However, there still are cases (although rare), when the minimum guarantee is paid by the distributor to the producer for obtaining the rights for movie distribution. So basically, there is a kind of a wholesale price, but at the same time they also share revenues from the movie release. Moreover, in case of movie industry there is no supply chain; it is rather a value chain: each link of the chain adds some value on the way of a product from initiation till the end consumer. Thus, in this chapter we will try to adapt the existing coordination models of supply chain sphere to the value chain of motion picture industry environment, and then we will approbate those on numerical examples.

Supply chain coordination. First of all, let's study the supply chain coordination to derive some conclusions for the purpose of our paper: coordination in the movie value chain. However, in order to be more consistent, it makes sense to first look at the bigger picture of supply chain inter-organizational stages.

This framework – C3: cooperation, coordination, collaboration – is very popular for classifying the nature of the relationships inside the chain. *Cooperative relationship* is defined by motivating one of the partners to invest resources or increase profitability of the other partner in the chain. These partnerships usually are more advantageous towards that partner of the chain, who enjoys a greater bargaining power (Munson et al., 1999). Usually, this incentive takes a form of a long-term contract. In this kind of relations the structure and control originates from one partner, but actually both partners experience advantages from the relationships, since they secure business and behavior. Moreover, as contract and financial investments involved have a long-term nature, a particular level of trust is required (Ketchen et al., 2006). By *coordinative relationships*, supply chain tries to gain alignment and fluidity across the chain by informing each chain member of the preferred behavior for each transaction (Arshinder, 2007). Coordination contracts benefited both parties downstream and upstream, although the company with the larger bargaining power enjoyed more advantages. By these contracts downstream party secured the price and the quality level, whereas the upstream party decreased the risk connected with the errors of the downstream partner (Park et al., 2006). *Collaborative relationships* require the established cooperative and coordinative relationships. Thus, a collaborative supply chain is defined as "integration and management of chain organizations and activities through cooperative organizational relationships, effective business processes and high levels of information sharing to create high performing value systems that provide member organizations a sustainable competitive advantage" (Handfield and Nichols, 2002). Collaborative relationships concentrate on constructive disagreement and part from the idea of bargaining power in the intention to create the strongest supply chain. Generally, collaborative chains are defined by the voluntary investment of resources by one chain participant to another chain participant or joint venture in order to reinforce the partnership overall. Such type of relations is considered rather as a long-term investment than a short-term tactic (Ketchen et al., 2008). However, collaborative type of relationship is out of the scope of our paper, and is considered as a possible direction of further research in application to the motion picture industry.

Since the topic of this part of the paper is coordination, a more detailed look should be focused on it. If a company wants to effectively transform the competitive advantage into profitability, it should develop efficient coordination within its boundaries and beyond them (Dyer and Singh, 1998). Basically, coordination between independent companies is crucial in order to achieve flexibility, which is needed to constantly improve logistic processes in response to ever changing external environment. The main problem resides in the method to attain the consistency towards the mutual goal of the partners, since the effectiveness of the chain is dependent on how well the members perform together, and not on how well each member works independently. There are different coordination modes distinguished. The classification is constructed on the concepts of mutuality and focus. The concept of mutuality pertains to unifying efforts of the independent companies (MacNeil, 1980). Mutuality is comprised of complementarity and coherency of actions of the chain links. In its turn, focus refers to putting emphasis on operational and organizational relations. The classification distinguishes four coordination modes: logistics synchronization, information sharing, incentives alignment and collective learning (Simatupang, Wright, Sridharan, 2002). In order to achieve the common goal by integration the actions of various players, the knowledge of coordination is needed. It consists of notion about key drivers of coordination modes, which influence the chain's performance. Let's move to the taxonomy of coordination modes. Reciprocal relations become important in order to make networking within the members of the chain easier. The main issue of supply chain management becomes how to coordinate the members in order to perform all together as a whole to achieve the common goal of chain profitability in unstable market environment (Simatupang, Wright, Sridharan, 2002). Malone and Crowston (1994) identify coordination as management of interrelatedness between performed operations, which air to achieve a goal. In terms of supply chain, coordination may be regarded as a proper combination of a number of objects in order to attain a goal.

Let's consider the concepts, on which the taxonomy is built on. Simatupang, Wright and Sridharan (2002) identify mutuality of coordination as "the underlying values of responsibility among partners with a strong emphasis on sustaining relationship in order to build effective goal attainment". Milgrom and Roberts (1990) state that modern supply chain does not accept incremental adjustments made independently, but rather requires significant and coordinated changes in the comprehensive perspective of business. Complementarity of the stages of the chain will lead to augmentation of the total gains, such as, for example, higher level of sales and lower costs, which may be shared by all the participants of the chain. "When the links of the chain synchronize the decision-making about value creation to coordinate the sharing of the benefits associated with logistics improvement, they are likely to shape complementarity" (Simatupang, Wright, Sridharan, 2002).

The other important dimension of coordination is focus. It can be on either operational or organizational linkages. Linkages exist when the operations, performed by one of the participant of the chain may somehow influence the work or the results of work of another chain participant. Thus, linkages are the liaisons among companies, where joint decisions between chain participants have to be coordinated. Milgrom and Roberts (1990) identify four coordination modes:

1. Logistics synchronization;
2. Information sharing;
3. Incentive alignment;
4. Collective learning.

Every mode exists in different context and stresses different cognitive processes. In our case, case of producer's incentives plan improvement, clearly, from the modes mentioned above, we need to consider the incentive alignment one.

Incentives define how those in charge of decision-making will be rewarded or penalized for the taken actions. Current incentives affect both types of behavior of a chain participant: individual and communication with partners. A conflict of interests may arouse, when the incentives lead to actions, which maximize personal benefits, but at the same time decrease total gain (Clemons and Row, 1993). Simatupang and Sridharan (Simatupang and Sridharan,2002) consider that one of the methods to deal with this conflict of interests is to introduce the incentive schemes that are based on the all-embracing performance, and which include both value creation with regard to the customers and profitability. This coordination mode is known as incentive alignment, and it encourages the behavior of the partners, which would be consistent with customer focus and total profit (Lee, 2000). Companies, which partake the complementarity of business process, will try to solve the issue of incongruity of incentives in reciprocally satisfying ways, drawing on relational contracts, especially is the customer demand is uncertain. These contracts determine such parameters as price, quantity, time and quality (Simchi-Levi et al., 1999).

One of the features of the incentive scheme is that it is offered before the mutual benefits are realized. It is intended to motivate the chain participants to relate their decisions with the profitability of the entire supply chain. A number (or even all) of the reciprocal benefits that follow from better coordination of the chain can be allocated in more incentives. Larger gains from incentives will affect the behavior of decision-makers and make them improve chain performance (Simatupang, Wright, Sridharan, 2002).

Coordination problem in movie value chain. This principle of increasing the motivation of the chain members by means of coordination, which was studied in the sphere of supply chain, perfectly fits the task of incentives alignment in the movie industry, where the value chain is considered. As has been studied in the first chapter of the paper, there are a number of problems, which appear between the links of the value chain, due to possibility of opportunistic behavior. However, it has been noticed by the author of the paper that the situations of the opportunistic behavior may be avoided, if all the chain members were motivated to think in the terms of the benefits of the entire chain and not in terms of their own benefits individually. Let's first formalize the stages of movie creation and delivery to the end consumer in terms of the value chain.

Producer is considered as a manufacturer or a supplier, since he produces the product and initially it is he, who has the rights for the product. Then he transferres (and sometimes sells) the rights for movie distribution to the distributor. If the sale occurs, those usually are presales, which are carried out before the movie is actually ready, and the price (minimum guarantee) is used by a distributor to cover a part of the incurred costs, and basically can be considered as a part of the budget funding. Rights are transferred for a specific time period (usually, between 5 and 15 years), and the distributor has to squeeze everything possible from the movie. Then the distributor makes prints (copies of the movie), which will be then distributed to the movie theaters. He also is responsible for marketing campaign. Those activities are quite costly ones. The distributor arranges contracts with exhibitors – retailers – where the movie will be released. At this point there is no prices. Usually, the distributor works with exhibitors for a long period of time, the relations are already well established, and there is no need for extra insurance. Exhibitors order a specific quantity of copies (they can be adjusted later depending on the demand of the movie), and they get those copies for a specific period of time (which can also be adjusted upon necessity). Then after the movie is released, there is a movement backwards the chain. Firstly, there is revenue split between the exhibitors and the distributors (the percentages have been discussed in the first chapter of the paper), then the rest of the proceeds are split between the distributor and the producer. It can be traced that this chain has some similarities with the supply chain, thus we assume that some of the supply chain principles may be transferred to the topic of our research. As has already been discussed earlier, there are possibilities for cheating and opportunistic behavior to appear in the relationships of the links. This problem can be solved by the tools of mathematic modeling, which will be considered further.

Revenue sharing base model. Firstly, let's study the revenue-sharing contracts coordination, suggested by Cachon and Lariviere (2005), which will explain the principles of the model in general supply chain environment.

The revenue sharing base model has a supplier, who is interacting with a single retailer.

There are two decisions to be made by the retailer in order to forecast the total revenue generated over a single selling period. Those are: the number of units to purchase from a supplier and the retail price. There are two points of view on the method to determine the revenue function. From marketing standpoint (Lilien et al, 1992) the revenue function is derived on the basis of a deterministic demand curve, while the operations point of view (Tsay et al, 1998) reflects the idea that it is derived from stochastic demand with a fixed retail price, i.e. a newsvendor model. The formulation, proposed by Cachon and Lariviere (2005) embraces both of these revenue functions. It illustrates that the revenue sharing contracts coordinate the supply chain, meaning, that the retailer makes decisions concerning quantity and price (supply chain optimal actions) and the total profit of the chain may be arbitrary divided between the firms. Moreover, a single revenue sharing contract can coordinate a supply chain with several noncompeting retailers even if the retailers have different revenue functions.

According to Cachon and Lariviere (2005), revenue-sharing contracts are very effective in a broad variety of supply chains. However, there, of course, are some limitations. Firstly, revenue sharing does not coordinate competing retailers, if each retailer's revenue is dependent on its quantity, its price and the actions of other retailers. Secondly, revenue sharing lays down the administrative burden on the firms. In order to ensure that the revenues are split appropriately, the supplier must control retailer's revenues. These costs are sometimes that significant that gains from coordination might not always cover them. Thirdly, the chain is not coordinated, if the demand is influenced by noncontractable and costly retailer effort.

Now let's move to the supply chain coordination with revenue-sharing contracts. Let $\{q^0, p^0\}$ be a quantity-price pair that maximizes (q, p). We assume that (q, p) is upper semicontinuous in q and p, so $\{q^0, p^0\}$ exists, but it need to be unique. Revenue-sharing contracts achieve supply chain coordination by making the retailer's profit function an affine transformation of the supply chain's profit function; hence, $\{q^0, p^0\}$ maximizes $\pi_r(q, p)$.

Let's consider the set of revenue-sharing contracts with

$$w = \psi c - c_r \tag{22}$$

and $\psi \in (0, 1]$. With those contracts, the firms' profit functions are:

$$\mathit{\Pi}_r\,(q, p) = \psi \mathit{\Pi}(q, p). \tag{23}$$

Furthermore, $\{q^0, p^0\}$ is the retailer's optimal quantity and price; i.e., those contracts coordinate the supply chain (Cachon, Lariviere, 2005).

Given the profit function (23), it follows that $\{q^0, p^0\}$ maximizes the retailer's profit when $\psi > 0$. To obtain (23), substitute $w = \psi c - c_r$ into (1) and simplify. The supplier's profit function follows from (23) and $\pi_s\,(q, p) = \pi_r\,(q, p) - \mathit{\Pi}\,(q, p)\,;\,;\ \psi \leq 1$ ensures and $\pi_s\,(q, p) \geq 0$.

The theorem indicates that ψ is the retailer's share of the supply chain's profit in addition to its share of revenue. Therefore, revenue-sharing contracts coordinate the supply chain and arbitrarily allocate profit. The certain profit split chosen probably depends on the firms' relative bargaining power. With the strengthening of the retailer's bargaining position, one would anticipate ψ increases. As a proxy for bargaining power, each firm may have an outside opportunity profit, $\pi_i > 0$, that the firm requires to include in the relationship; i.e., $\pi_i(q, p) \geq \pi_i$ is required to gain firm i's participation. It is possible to satisfy both firms' requirements when $\pi_r + \pi_s < (q^0, p^0)$, but the feasible range for ψ will be more limited.

Extreme ψ values raise two other issues. First, the retailer's profit function becomes quite flat as $\psi > 0$; while q^0 remains optimal for the retailer, a deviation from q^0 imposes little penalty on the retailer. Second, from (22), the coordinating wholesale price is actually negative when $\psi < c_r/c$. In other words, if the retailer's share of the channel's cost is high, the retailer is already in a low-margin business before the supplier cuts its part of revenue. If the supplier wants to claim a large portion of revenue, he must subsidize the retailer's purchase of product. If one wishes to rule out negative wholesale price, then a positive retailer cost establishes a floor on retailer profit under coordinating contracts.

A prerequisite for coordination is a wholesale price below the supplier's cost of production c_s. The supplier loses money in selling the product and only makes money by participating in the retailer's revenue. Selling below cost is necessary because revenue sharing systematically drops the retailer's marginal revenue curve below the integrated supply chain's. In order to have marginal revenue equal marginal cost at the desired point, the retailer's marginal cost must also be less than the integrated system's.

Given that the set of coordinating contracts is independent of the revenue function, it follows that a single revenue-sharing contract can coordinate the actions of multiple retailers with different revenue functions as long as each retailer's revenue is independent of the other retailer's actions (i.e., they do not compete) and they have the same marginal cost, c_r.

3.2. Coordinating contracts in movie value chain

After studying the general concepts of coordination and examining the principles of coordinating contracts functioning we can conclude that they are applicable to the motion picture industry. There are three players: producer, distributor and exhibitor. The producer gives to the distributor the good (q), which are the rights for the movie ($q = 1$, since they are not quantifiable). There is no price for the movie rights, which the distributor pays to the producer, because their income is the share of the revenue generated by the movie after release: $w_d = 0$. In this model we take the sales period as exogenously specified. Both parties have their costs with c_{pr} being the costs of the producer and c_d - the costs of the distributor. The distributor expects to generate some income for the movie (P), and he needs to estimate it. Then the distributor makes the copies in some predetermined by the arrangement with the exhibitor quantity ($q \geq 0$) and transfers them to the exhibitor. There is again no wholesale price ($w_e = 0$), which the exhibitor could have paid to the distributor for each copy. They only share the total income generated by the exhibitor after releasing this movie. This income is $R\ (p,\ q)$, where q is the quantity of copies, ordered by the exhibitor and p is the income generated by each single copy. The exhibitor has costs c_e.

Coordination is possible, when there is cooperation. Basically, the decisions are made on two stages, and two different cooperation relationships can be distinguished. So it is suggested to introduce sharing contracts on two stages. We will consider them step-by-step. Let's start with the producer-distributor relationships.

Producer and distributor divide the income generated by the movie in some shares with φ being the share of the distributor and $(1 - \varphi)$ being the share of the producer. Basically, they cooperate, since only working together they maximize their income, which they later share. However, it is possible to say that they do not coordinate, because there is no sharing contract between them, since $w_d = 0$. Meaning, they only split the final income in some shares, but there is still room for cheating of the parties (in our case distributor cheats on producer, since he has larger power and more mechanisms to do so), since there is no contract coordination. So the author suggests introducing the price for the movie rights w_d, which would allow to avoid opportunistic behavior, because all the parties are interested in minimization of the costs in order to maximize the total gain, thus they are motivated to act fairly. The price w_d is calculated according to the formula: $w_d = \psi c_{pr} - c_d$.

The second step is the second level of cooperation. We have already coordinated the first stage, but it is possible to move further and coordinate the whole chain. The thing here is that we will consider the producer and the distributor as a single player now, which cooperates with the exhibitors for the realization of the movie. This is done due to the fact that to the time when the distributor makes deals for a certain movie with the exhibitors he already cooperates with the producer. When selling the movie for exhibition the distributor is obliged to later share the revenue with the producer, thus, it is logical to consider them as a single entity at this level of coordination. The costs of the compounded player are $c_{prd} = c_{pr} + c_d$ and the costs of the exhibitor are c_e, which are the costs incurred by the exhibitor in consideration each single copy. Since the costs of the exhibitor are considered per copy, we need to normalize the costs of the producer and the distributor in order to account them per copy as well. Thus, we need to divide the costs of the producer and the distributors by number of copies (q): $c_{prd} = (c_{pr} + c_d)/q$. Since in the US all the major movie theater chains have already switched to the digital equipment, meaning that there is no more need for buying several copies of the movie per theater, if the movie is to be shown on more than one screen. One copy is ordered by the exhibitor per theater, and then it is uploaded to the data server and transmitted to as many screens of the specific theater, as needed. Thus, for calculating the number of copies in our research we take the number of the theaters in a chain, assuming that it will be shown in all of them, since only the huge blockbusters are considered in the paper.

Another major difference with the first stage is that we already know the revenue allocation between the parties: as has been discussed in the first chapter, according to the accepted principles of the industry, the exhibitor retains 50% of the box-office. Thus, $\varphi = 1/2$. Since there is again no price w_e, there is no sharing contract, meaning that there are only share of the revenue allocation, which do not eliminate the possibilities for the parties to cheat. For instance, exhibitor can behave opportunistically towards distributor, thus, depriving him of a part of income. If we introduce the wholesale price for copies w_e, we will coordinate the chain, so this is exactly what is suggested to be done by the author. It is calculated as following: $w_e = \psi c_{prd} - c_e$.

We have considered a basic case of the movie value chain. However, it should be specified that in reality in the vast majority of cases there are several exhibitors involved, since the movie needs to be shown in as many locations, as possible, in order to get the maximum possible revenue.

The calculations of the transfer prices remain the same with the only difference that there augments the number of copies considered in the model, since the number of theaters increases. Thus, the costs of the distributor and the producer are dispersed over a larger number of copies. This scheme is the one, which is widely spread in the US, but even more broadly it is used in Europe, where there are almost no such powerful studios as Hollywood ones.

Another case should be considered as well. This is the case, when the producer is affiliated with the studio. In this case the studio (which is also a distribution company) finances the movie and the producer acts as a manager and has a flat fee instead of participation in the revenues. Basically, what happens is that at some point the rights are transferred to the studio, if the latter finances the whole budget of the movie. This means that the distributor also bears the budget costs. So our chain reduces. This case is to a high extent spread in the US due to such factors as huge movie budgets and the industry domination by majors (six largest movie studios in Hollywood, which have astounding financial capabilities, since they are parts of huge media conglomerates).

So by introducing the sharing contracts in the chain, we achieve the task of coordination. The main result of this is the elimination of the costs of control, because every player is interested in maximizing the revenue of the chain. Otherwise there exist loopholes for opportunistic behavior. Currently, in the situation of the absence of the wholesale price, participants of the movie value chain frequently bloat their costs in order to retain a larger lump of the proceeds, thus diminishing the percentage base for the next players. Coordination is feasible to resolve this problem, and it is applicable to the situation, since the necessary precondition is cooperation, and cooperation does exist in the considered case.

It is needed to be mentioned that in the paper we consider the model, which is centralized in a sense that under exhibitor we consider some specific chain of movie theaters. However, in reality usually distributors work with many exhibitors, meaning, that the chain is decentralized. Such types of chains can also be coordinated, but the calculations are more complicated and larger amounts of data are needed in order to fulfill the task, thus, this is out of the scope of this research paper.

3.3. Case studies

In this part of the research paper the application of the elaborated method will be shown on specific examples. Cases will include movies "Inception" and "Alice in Wonderland" , both of which have already been considered earlier in the paper during the discussion of the cooperation problem.

Movie Alice in Wonderland. Let's start with the movie "Alice in Wonderland". On this example due to the availability of the data the author would like to demonstrate all possible situations of the relationships inside the chain, which will lead to different outcomes. For the purposes of the paper from all the participants involved in this movie creation we will consider the producer Richard D. Zanuk, the distributor Walt Disney Pictures and as an exhibitor we will consider the largest movie theater chain in the USA – Regal Entertainment Group. So basically, what process do we have now: the producer gives the rights for the movie release to Walt Disney Pictures, and it does not get anything in return for giving away the rights. They only agree on some percentage from the final proceeds of the movie. In our model we consider the budget of the movie as producer's costs. Then the distributor creates copies, elaborates and conducts marketing campaign (P&A) and then sets agreement on some number of copies with the exhibitor (Regal in our case). Regal estimates the demand and orders a specific number of copies from the distributor. The distributor does not get any transfer price immediately from the exhibitor. They will later share the revenue of the movie in a predetermined by the industry proportion of 50:50. Since the access to the data about the costs of each theater is limited, we will take industry averages. The average weekly expense of a theater is around $5000 per week. Regal Entertainment Group owns around 558 locations in the US, and they all have adopted the digital technology, which means that they do not use the hard copy prints of the movie, they have it in digital form. Those copies are usually distributed either on hard drive or via Internet and satellite. In either way only one copy per theatre is needed no matter how many screens in a single theatre will show the picture. So the number of copies in our calculations will equal the number of screens of the theatre chain. We assume that every theatre of the chain will order this movie, because this is a loud blockbuster with huge budget and star cast, as well as massive marketing campaign, thus, it is expected that it will be popular with public. Currently there is only a contract stipulating the proportion, in which the revenue will be divided, but there is no coordination of the contracts, meaning that the participants still have the motives to increase their costs on the books, because in this way they will be able to retain a larger lump of the proceeds. So the author suggests to introduce the transfer prices, which will mitigate the adverse effects of working in cooperation.

The costs of the producer equal \$200 mln. ($c_{pr} = 200000000$). The costs of the distributor are the P&A costs. Walt Disney Pictures conducted a huge marketing campaign, which cost \$75 mln. ($c_d = 75000000$). The share of the distributor was in line with industry averages and constituted 80% ($\varphi = 0,8$). With this data at hand we can calculate the possible transfer price w_d, which the distributor could pay the producer for the rights of the movie. With the introduction of this parameter, the contract would be transferred from simply participation contract to sharing contract. According to the formula $w_d = \psi c_{pr} - c_d$ we get $w_d = 0,8 * 200000000 - 75000000 = 85000000$. So it is suggested that the distributor pay \$85 mln. to the producer as a fee for having the opportunity to distribute the movie. Those \$85 mln. could serve as a recoupment of the incurred costs for getting the movie done for the producer.

Now we can move to the second level of coordination, where we will find the transfer price w_e, which is a price per copy, which the distributor can receive from the exhibitor. Importantly, at this point the producer and the distributor are considered as a single entity, because it is essential that they already have contract and work together. Since we are calculating a price per copy, we need to adjust all the costs. It has been stated that the average weekly costs per theatre are \$5000. It is assumed that such blockbuster as “Alice in Wonderland” can be shown by the movie theatre chain in the period of up to 17 weeks, thus the costs per theatre for the whole period of movie screening will equal to \$85000 ($c_e = 85000$). The costs of the distributor and the producer have to also be adjusted, since all the calculations are made per copy. So $c_{prd} = \frac{c_{pr}+c_d}{N}$, where N – number of copies. In our case it looks as following: $c_{prd} = \frac{275000000}{558}$. Using the coordinating formula $w_e = \varphi c_{prd} - c_e$ we get $w_e = 0,5 * \frac{275000000}{558} - 85000 = 161416$. Thus, the suggested price per copy for the movie theatre to pay is \$161416.

We have considered the case in the situation, where there is only one theater chain involved. In this situation the price per copy would be \$161416. However, in reality we have multiple theatre chains, which the distributor is working with, thus, the model should be slightly modified. Let’s consider a situation, when the distributor has made agreements with several US theatre chains. For the purposes of the example let’s say that the distributor have made deals with 5 largest theatre chains in the US. So, apart from Regal Entertainment Group with 558 locations, we will consider AMC Theatres with 346 locations, Cinemark Theatres with 334 locations, Carmike Cinemas with 278 locations and Bow Tie Cinemas with 55 locations.

In this situation although the first step will be exactly the same as in the first case, the second step will be different. The idea now is that the unified costs of the distributor and the producer will be spread over a larger number of copies. Since we consider that each theater chain order the number of copies, which equal the number of the locations, we get that there are 1571 copies in total ($q = 1571$). The theater expenses are different in each chain and in the majority of locations, and they are not publicly available, so for the purposes of the example we will consider the industry averages. The period for which the copy is expected to be on screens is 17 weeks. Then we have:

$$q = 1571,$$

$$c_{e_1} = c_{e_2} = \cdots = c_{e_5} = 5000,$$

$$c_{prd} = 275000000,$$

$$\varphi = 0,5.$$

Then

$$w_e = 0,5 * \frac{275000000}{1571} - 85000 = 2524.$$

In the other words, the transfer price per copy is \$2524. This means that in case the theatres will be purchasing copies, the agreement between the distributor and the exhibitors will be sharing, i.e. no party will have objective reasons to manipulate their costs, because if they increase the costs on the books, they diminish the revenue, from which they have a share. So all the participants are motivated to maximize the possible revenue.

Basically, by introducing this methodology we do not only state that the transfer price should be implemented, but we also explain the way, how it should be calculated.

Movie Inception. Another example, which will demonstrate the methodology, is the movie “Inception”, which has also been considered. This example is interesting, because in this case the producer and the distributor Warner Bros. act like a single entity from the first stage and the problem is a one-step problem. This happens, because the producer is affiliated with the studio (which is also the distributor), meaning that the producer gets the financing of the motion picture in return for the rights for the movie. So basically, in affiliation with the studio he ceases to be the ultimate claimant for all proceeds, generated by the movie. Therefore, the chain is as follows. The producer affiliates with the distributor and the rights are in the possession of the distributor from a certain point. The distributor in return fully finances the production of the picture. So, basically, all the decisions concerning movie (including some creative part corrections) are made by the distributor and the producer works for a flat fee and does not participate in the profit. Then the distributor sets deals with theater chains for the screening of the movie. When they share revenues from the box-office, the exhibitor takes 50% and then the distributor deals with talent, who participated in proceeds, from its 50%. So we have that the costs for production were at the level of \$160 mln. The costs of the distributor for P&A equaled \$100 mln. Thus, the compounded costs equal \$260 mln. Let's assume that the deal for distribution was set with 5 largest movie theatre chains, and the total number of locations equals 1571 theatres. Thus, the number of copies released also equals 1571. The house expense per theatre is taken the same as in previous examples - \$5000 per week per location. The movie was on screens for 16 weeks. Thus, we have the following solution.

$$q = 1571,$$

$$c_{e_1} = c_{e_2} = \cdots = c_{e_5} = 5000,$$

$$c_{prd} = 260000000,$$

$$\varphi = 0,5.$$

Then

$$w_e = 0,5 * \frac{260000000}{1571} - 80000 = 2750.$$

So the transfer price per copy is \$2750 per copy, meaning that the each theatre chain, cooperating with the distributor, has to pay \$2750 per each copy of the movie for the rights to show this movie in their movie theaters. In this case, again, the opportunistic behavior of the parties is eliminated. With the introduction of price per copy, there appears the dependence on the costs, because their value is used for calculation of transfer price. Thus, if a theater starts increasing its costs, then the transfer price will also enlarge, and this is certainly not favorable situation for exhibitors. So all parties are motivated to act fairly.

Peculiarities of the methodology. In this chapter the methodology has been developed, which incentivizes all of the participants of the value chain to maximize the gain of the chain, because unlike in the case of simply participation contracts, under which they share revenue, with the suggested methodology they share profit of the chain. This mitigates the opportunistic behavior in the chain, since the participnats do not have the incentives to increase their costs on the books anymore. With the introduction of transfer price, participants get the goods for a certain amount of money, which is in the direct dependence of the costs: the higher the costs, the higher will the price to pay be. There are no incentives to artificially diminish the costs as well, because the smaller the costs on the books are, the larger will be the sum of money to pay in accordance to the sharing contracts. Therefore, all the participants of the chain are motivated to, firstly, act fairly, and, secondly, to operate efficiently in order to have the costs at the optimal level to stay profitable.

The interesting thing here is that in the international context different schemes of the participants' relationship in the value chain are in practice. All of them have been discussed in the chapter. In the USA due to the large share of the blockbusters in production, which require extremely large investments, the case of the producer affiliation with the studio is more spread. This case was considered on the example of the movie "Inception". In Europe, especially in France and Italy, movies are predominantly independently financed, when the producer usually works with several institutional and private investors in order to finance the budget, therefore, the case, where the producer and the distributor are separate legal entities, is applicable here. This case was considered on the example of the movie "Alice in Wonderland". Basically, most widely spread cases have been considered and the methodology has been adjusted for each of them.

4. Conclusion

The problem of cooperation in the movie value chain has been studied, and the methodology of box-office income allocation has been improved with the adaptation to the motion picture industry environment. Known to the literature methods of optimal revenue imputation have been investigated, which are Nash bargaining solution, the core (set of nondominant imputations), Shapley value and Shapley index. Due to drawbacks of those methods in application to motion picture industry, because of high specificity of the relationship among the parties involved, new approaches to the shares of movie revenue allocation computation have been introduced.

Coordination concept of supply chain has been studied and the possibility to apply it with some modifications to the motion picture industry has been elicited. Several types of value chains in movie industry have been considered for development of the methodology, which are in use in various countries. The peculiarity of the elaborated methodology is the introduction of transfer prices between the links of the chain, which allow transforming the participation contracts between the counteragents to sharing contracts. This innovation would motivate them to work for profit maximization and eliminate incentives for opportunistic behavior, since the transfer price is constructed on the basis of the costs of the participants and their shares in the final allocation of revenues. Therefore, with the introduction of the suggested methodology, the only optimal behavior for the participants becomes the fair one, since with the artificial increase of the costs the transfer price they need to pay will augment, and with the artificial decrease of the costs the amount of money they need to pay according to the sharing contracts will get bigger. Introduction of the transfer prices allows the producer reimburse a part of the costs connected with movie production almost immediately after setting the deal with the distributor, and avoid waiting long time till the theatrical release of the movie. The same logic applies to the distributor, since he is able to recoup a part of his costs with the transfer of copies to the exhibitors without waiting till the movie makes money in the theaters.

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Bibliography of L. A. Petrosyan (Л.А.Петросян) Scientific Papers

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Abstract The bibliography is prepared in connection with the 75th anniversary of professor L.A. Petrosyan - acting Dean of the Faculty of Applied Mathematics and Control Processes at Saint Petersburg State University starting from the year 1975. The bibliography contains the list of papers in scientific journals, one can find there publications in his native country Russia and abroad. The bibliography is useful for students, and specialists in the field of applied mathematics, game theory and operations research. It shows wide range of interests and scientific activity of the author.

Keywords: Game theory, differential game, pursuit game, cooperative stochastic games, subgame consistency, Shapley Value.

The bibliography is published in connection with the seventy fifth anniversary of professor L.A.Petrosyan. Bibliographic data of the scientific papers are presented in the original language.

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