

Spatial market equilibrium in the case of linear transportation costs*

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For citation: Krylatov A. Y., Lonyagina Y. E., Golubev R. I. Spatial market equilibrium in the case of linear transportation costs. *Vestnik of Saint Petersburg University. Applied Mathematics. Computer Science. Control Processes*, 2020, vol. 16, iss. 4, pp. 447–454.
<https://doi.org/10.21638/11701/spbu10.2020.409>

In this article, we study the spatial market equilibrium in the case of fixed demands and supply values, the requirement of equality in regard to overall supply and overall demand, and linear transportation costs. The problem is formulated as a nonlinear optimization program with dual variables reflecting supply and demand prices. It is shown that the unique equilibrium commodity assignment pattern is obtained explicitly via equilibrium prices. Moreover, it is proved that in order to obtain absolute values of equilibrium prices, it is necessary to establish a certain base market price. Therefore, once the base market price is given, then other prices are adjusted according to spatial market equilibrium.

Keywords: spatial market equilibrium, non-linear optimization, multipliers of Lagrange, Karush—Kuhn—Tucker conditions.

1. Introduction. The modern market was formed mainly due to the division of labor. Actually, the market already lost its national and territorial boundaries and turned into a global market for commodities from spatial perspectives. The sale and purchase of commodities can occur at completely different prices, bounded above by the price of demand, and below — by the price of supply. Actual prices depend primarily on the structure of market and transaction costs, which incorporate the transportation costs. There are several structures of the market.

- Perfect competition markets: many small firms with homogeneous products.
- Monopoly: there is only one company on the market that produces unique products.
- Monopolistic competition: there are many small firms on the market whose products are heterogeneous.
- Oligopoly: there are a small number of large firms with homogeneous or heterogeneous products on the market.

For each of the above structures, it is possible to determine such a situation (point) in the market, when neither the buyer nor the seller is interested in changing the current situation. The price at which the product offered on the market corresponds to the demand is called the equilibrium. The market mechanism begins to work, exerting pressure on prices from the lower and upper sides to achieve an equilibrium price. The study of the market, as well as the principles of its functioning and regulatory mechanisms, today seems to be relevant and necessary for understanding the essence of the socio-economic processes that are currently taking place throughout the world.

The first consideration of the spatial price equilibrium problem was made in [1]. The foundations for the study of spatial production, consumption, and trade of commodities

* This work was supported by the Russian Foundation for Basic Research (project N 19-31-90109).
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was given in [2]. Up-to-date there exists a wide range of computational techniques for coping with such kind of problems [3–7]. Comprehensive mathematical models concerning spatial equilibrium are studied in [8–10]. Spatial equilibrium models are commonly exploited to solve the traffic assignment problem [11–14]. Moreover, its applications can be found in energy markets [15, 16] and telecommunication markets [17].

In this paper, we study the spatial market equilibrium in the case of linear transportation costs. The problem is formulated in a form of nonlinear optimization program in Section 2. The supply and demand price functions are assumed to be given as well as the unit transaction cost functions, which are assumed to incorporate the unit transportation costs. The unique equilibrium commodity assignment pattern is obtained explicitly via equilibrium prices in Section 3. Moreover, in Section 3 it is proved that in order to obtain absolute values of equilibrium prices, the market moderator has to establish the basic market price. Conclusions are given in Section 4.

2. Spatial market equilibrium. Consider the set of suppliers M and the set of consumers N , which are associated with commodity production, distribution, and consumption. We denote by s_i the supply of $i \in M$, and by λ_i — the price of a unit of the i th supply, $\lambda = (\lambda_1, \dots, \lambda_m)^T$. By d_j we denote the demand of $j \in N$, and by μ_j — the price of a unit of the j th demand, $\mu = (\mu_1, \dots, \mu_n)^T$. Finally, let $x_{ij} \geq 0$ be the commodity volume between a pair (i, j) , while $c_{ij}(x_{ij})$ is the cost of the transaction of a unit of x_{ij} . Let us also introduce the indicator of market relations:

$$\delta_{ij} = \begin{cases} 1 & \text{for } x_{ij} > 0, \\ 0 & \text{for } x_{ij} = 0, \end{cases} \quad \forall (i, j) \in M \times N.$$

Definition. Commodity assignment pattern x is the *spatial market equilibrium* if and only if

$$\begin{aligned} \lambda_i + c_{ij}(x_{ij}) &= \mu_j & \text{for } x_{ij} > 0, \\ \lambda_i + c_{ij}(x_{ij}) &\geq \mu_j & \text{for } x_{ij} = 0, \end{aligned} \quad \forall (i, j) \in M \times N.$$

Thus, if the sum of the i -th supplier's price and the transaction costs between i and j exceeds the demand price of j -th consumer, then the pair (i, j) will not have any market relations. The commodity assignment pattern x^* such as

$$x^* = \arg \min_x \sum_{i \in M} \sum_{j \in N} \int_0^{x_{ij}} c_{ij}(u) du \quad (1)$$

subject to

$$\sum_{j \in N} x_{ij} = s_i \quad \forall i \in M, \quad (2)$$

$$\sum_{i \in M} x_{ij} = d_j \quad \forall j \in N, \quad (3)$$

$$x_{ij} \geq 0 \quad \forall i, j \in M \times N, \quad (4)$$

under

$$\sum_{i \in M} s_i = \sum_{j \in N} d_j, \quad (5)$$

is proved to be spatial market equilibrium [10, 18].

3. Pricing mechanism in case of linear transaction costs. Within the present paper, we examine spatial market equilibrium in case of linear transaction costs. In other words, we assume that

$$c_{ij}(x_{ij}) = a_{ij} + b_{ij}x_{ij} \quad : \quad a_{ij} \geq 0, b_{ij} > 0 \quad \forall (i, j) \in M \times N.$$

Lemma 1. Assume that demands and supplies are fixed in the spatial market and satisfy the requirement (5). The spatial market equilibrium under linear transaction costs is obtained by the following commodity assignment pattern:

$$x_{ij} = \begin{cases} \frac{\mu_j - \lambda_i - a_{ij}}{b_{ij}}, & \text{if } \mu_j - \lambda_i > a_{ij}, \\ 0, & \text{if } \mu_j - \lambda_i \leq a_{ij}, \end{cases} \quad \forall i \in M, j \in N, \quad (6)$$

where (λ, μ) are such that

$$\begin{pmatrix} B_\mu & -B \\ B^\top & B_\lambda \end{pmatrix} \begin{pmatrix} \mu \\ \lambda \end{pmatrix} = \begin{pmatrix} d_1 + \sum_{i \in M} \frac{a_{i1}\delta_{i1}}{b_{i1}} \\ \vdots \\ d_n + \sum_{i \in M} \frac{a_{in}\delta_{in}}{b_{in}} \\ s_1 + \sum_{j \in N} \frac{a_{1j}\delta_{1j}}{b_{1j}} \\ \vdots \\ s_m + \sum_{j \in N} \frac{a_{mj}\delta_{mj}}{b_{mj}} \end{pmatrix}, \quad (7)$$

while

$$B_\mu = \begin{pmatrix} \sum_{i \in M} \frac{\delta_{i1}}{b_{i1}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sum_{i \in M} \frac{\delta_{in}}{b_{in}} \end{pmatrix}, \quad B_\lambda = \begin{pmatrix} -\sum_{j \in N} \frac{\delta_{1j}}{b_{1j}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -\sum_{j \in N} \frac{\delta_{mj}}{b_{mj}} \end{pmatrix}$$

and

$$B = \begin{pmatrix} \frac{\delta_{11}}{b_{11}} & \cdots & \frac{\delta_{m1}}{b_{m1}} \\ \vdots & \ddots & \vdots \\ \frac{\delta_{1n}}{b_{1n}} & \cdots & \frac{\delta_{mn}}{b_{mn}} \end{pmatrix}.$$

P r o o f. Let us consider Lagrangian of the problem (1)–(4):

$$\begin{aligned} L = & \sum_{i \in M} \sum_{j \in N} \int_0^{x_{ij}} c_{ij}(u) du + \sum_{i \in M} \lambda_i \left(\sum_{j \in N} x_{ij} - s_i \right) + \\ & + \sum_{j \in N} \mu_j \left(-\sum_{i \in M} x_{ij} + d_j \right) + \sum_{i \in M} \sum_{j \in N} \xi_{ij}(-x_{ij}), \end{aligned}$$

where λ_i , μ_j , and $\xi_{ij} \geq 0$ for $i \in M$, $j \in N$ are multipliers of Lagrange, according to Kuhn–Tucker conditions

$$\frac{\partial L}{\partial x_{ij}} = c_{ij}(x_{ij}) + \lambda_i - \mu_j - \xi_{ij} = 0 \quad \forall i \in M, j \in N, \quad (8)$$

$$\frac{\partial L}{\partial \lambda_i} = - \sum_{j \in N} x_{ij} + s_i = 0 \quad \forall i \in M,$$

$$\frac{\partial L}{\partial \mu_j} = - \sum_{i \in M} x_{ij} + d_j = 0 \quad \forall j \in N,$$

$$\xi_{ij}(-x_{ij}) = 0 \quad \forall i \in M, j \in N.$$

Since $\xi_{ij}(-x_{ij}) = 0$ for all $(i, j) \in M \times N$, then

$$\begin{aligned} \text{if } x_{ij} > 0 &\Rightarrow \xi_{ij} = 0, \\ \text{if } x_{ij} = 0 &\Rightarrow \xi_{ij} \geq 0, \end{aligned} \quad \forall (i, j) \in M \times N,$$

hence, due to (8):

$$c_{ij}(x_{ij}) \begin{cases} = \mu_j - \lambda_i, & \text{if } x_{ij} > 0, \\ \geq \mu_j - \lambda_i, & \text{if } x_{ij} = 0, \end{cases} \quad \forall (i, j) \in M \times N,$$

or in case of linear transaction costs:

$$a_{ij} + b_{ij}x_{ij} \begin{cases} = \mu_j - \lambda_i, & \text{if } x_{ij} > 0, \\ \geq \mu_j - \lambda_i, & \text{if } x_{ij} = 0, \end{cases} \quad \forall (i, j) \in M \times N,$$

that leads to

$$\begin{aligned} x_{ij} &= \frac{\mu_j - \lambda_i - a_{ij}}{b_{ij}}, & \text{if } x_{ij} > 0, \\ a_{ij} &\geq \mu_j - \lambda_i, & \text{if } x_{ij} = 0, \end{aligned} \quad \forall (i, j) \in M \times N,$$

consequently, commodity volume between pair (i, j) depends on value of a_{ij} :

$$\begin{aligned} \text{if } a_{ij} &\geq \mu_j - \lambda_i \Rightarrow x_{ij} = 0, \\ \text{if } a_{ij} &< \mu_j - \lambda_i \Rightarrow x_{ij} > 0, \end{aligned} \quad \forall (i, j) \in M \times N,$$

so the expression (6) holds.

Now we substitute x_{ij} into the balance equations and get the following expressions:

$$\sum_{j \in N} \mu_j \frac{\delta_{ij}}{b_{ij}} - \sum_{j \in N} \lambda_i \frac{\delta_{ij}}{b_{ij}} = \sum_{j \in N} \frac{a_{ij} \delta_{ij}}{b_{ij}} + s_i \quad \forall i \in M,$$

$$\sum_{i \in M} \mu_j \frac{\delta_{ij}}{b_{ij}} - \sum_{i \in M} \lambda_i \frac{\delta_{ij}}{b_{ij}} = \sum_{i \in M} \frac{a_{ij} \delta_{ij}}{b_{ij}} + d_j \quad \forall j \in N,$$

that in a matrix form is equivalent to (7).

The lemma is proved.

The next lemma proves that the system of linear equations (7) has infinitely many solutions.

Lemma 2. *The following statements hold:*

- the left-side matrix from (7) is singular, and its rank is $m + n - 1$;
- the system of linear equations (7) is solvable.

Proof.

I. Let us sum rows from the first one to n in the left-side matrix from (7):

$$\left(\sum_{i \in M} \frac{\delta_{i1}}{b_{i1}}; \dots; \sum_{i \in M} \frac{\delta_{in}}{b_{in}}; -\frac{\delta_{11}}{b_{11}} - \dots - \frac{\delta_{1n}}{b_{1n}}; \dots; -\frac{\delta_{m1}}{b_{m1}} - \dots - \frac{\delta_{mn}}{b_{mn}} \right)$$

that is

$$\left(\sum_{i \in M} \frac{\delta_{i1}}{b_{i1}}; \dots; \sum_{i \in M} \frac{\delta_{in}}{b_{in}}; -\sum_{j \in N} \frac{\delta_{1j}}{b_{1j}}; \dots; -\sum_{j \in N} \frac{\delta_{mj}}{b_{mj}} \right)$$

and let us sum rows from $n+1$ to $n+m$ in the left-side matrix from (7):

$$\left(\frac{\delta_{11}}{b_{11}} + \dots + \frac{\delta_{m1}}{b_{m1}}; \dots; \frac{\delta_{1n}}{b_{1n}} + \dots + \frac{\delta_{mn}}{b_{mn}}; -\sum_{j \in N} \frac{\delta_{1j}}{b_{1j}}; \dots - \sum_{j \in N} \frac{\delta_{mj}}{b_{mj}} \right)$$

that is

$$\left(\sum_{i \in M} \frac{\delta_{i1}}{b_{i1}}; \dots; \sum_{i \in M} \frac{\delta_{in}}{b_{in}}; -\sum_{j \in N} \frac{\delta_{1j}}{b_{1j}}; \dots; -\sum_{j \in N} \frac{\delta_{mj}}{b_{mj}} \right).$$

As one can see, obtained rows are equal. Therefore, the left-side matrix from (7) is singular.

Moreover, according to (3), $\sum_{i \in M} x_{ij} = d_j$ for any $j \in N$, then there exists at least one $x_{ij} > 0$ for any $j \in N$. In other words, $\sum_{i \in M} \delta_{ij} \geq 1$ for any $j \in N$ and, consequently, $\sum_{i \in M} \frac{\delta_{ij}}{b_{ij}} > 0$ for any $j \in N$. Thus, it is clear that the first n rows are mutually linearly independent. On the other hand, according to (2), $\sum_{j \in N} x_{ij} = s_i$ for any $i \in M$, then there exists at least one $x_{ij} > 0$ for any $i \in M$. In other words, $\sum_{j \in N} \delta_{ij} \geq 1$ for any $i \in M$ and, consequently, $\sum_{j \in N} \frac{\delta_{ij}}{b_{ij}} > 0$ for any $i \in M$. Thus, it is clear that the rows from $n+1$ to $n+m$ are mutually linear independent. However, the sum of the first n rows is equal to the sum of rows from $n+1$ to $n+m$. Consequently, the Gauss–Seidel method achieves a trapezoid matrix which dimension is $m+n-1$. Therefore, the rank of the left-side matrix from (7) is $m+n-1$.

II. According to the Kronecker–Capelli theorem, for a system to be solvable, it is necessary and sufficient that the rank of the extended matrix of this system be equal to the rank of its main matrix. Construct an extended matrix of the system

$$\bar{A} = \begin{pmatrix} \sum_{i \in M} \frac{\delta_{i1}}{b_{i1}} & \dots & 0 & & & d_1 + \sum_{i \in M} \frac{a_{i1}\delta_{i1}}{b_{i1}} \\ \vdots & \ddots & \vdots & & -B & \vdots \\ 0 & \dots & \sum_{i \in M} \frac{\delta_{in}}{b_{in}} & & & d_n + \sum_{i \in M} \frac{a_{in}\delta_{in}}{b_{in}} \\ & & & -\sum_{j \in N} \frac{\delta_{1j}}{b_{1j}} & \dots & 0 & s_1 + \sum_{j \in N} \frac{a_{1j}\delta_{1j}}{b_{1j}} \\ & B^T & & \vdots & \ddots & \vdots & \vdots \\ & & 0 & \dots & -\sum_{j \in N} \frac{\delta_{mj}}{b_{mj}} & & s_m + \sum_{j \in N} \frac{a_{mj}\delta_{mj}}{b_{mj}} \end{pmatrix}.$$

It is clear that $\text{rank} \bar{A} \leq m + n$ since $\dim \bar{A} = (m + n) \times (m + n + 1)$. On the other hand, according to the first part of the proof, $\text{rank} \bar{A} \geq m + n - 1$. Hence, $m + n - 1 \leq \text{rank} \bar{A} \leq m + n$. Let us sum rows from the first one to n in the matrix \bar{A} :

$$\left(\sum_{i \in M} \frac{\delta_{i1}}{b_{i1}}; \dots; - \sum_{j \in N} \frac{\delta_{1j}}{b_{1j}}; \dots; d_1 + \sum_{i \in M} \frac{a_{i1}\delta_{i1}}{b_{i1}} + \dots + d_n + \sum_{i \in M} \frac{a_{in}\delta_{in}}{b_{in}} \right)$$

that is

$$\left(\sum_{i \in M} \frac{\delta_{i1}}{b_{i1}}; \dots; \sum_{i \in M} \frac{\delta_{in}}{b_{in}}; - \sum_{j \in N} \frac{\delta_{1j}}{b_{1j}}; \dots; - \sum_{j \in N} \frac{\delta_{mj}}{b_{mj}}; \sum_{j \in N} d_j + \sum_{j \in N} \sum_{i \in M} \frac{a_{ij}\delta_{ij}}{b_{ij}} \right)$$

and let us sum rows from $n + 1$ to $n + m$ in the matrix \bar{A} :

$$\left(\sum_{i \in M} \frac{\delta_{i1}}{b_{i1}}; \dots; - \sum_{j \in N} \frac{\delta_{1j}}{b_{1j}}; \dots; s_1 + \sum_{j \in N} \frac{a_{1j}\delta_{1j}}{b_{1j}} + \dots + s_m + \sum_{j \in N} \frac{a_{mj}\delta_{mj}}{b_{mj}} \right)$$

that is

$$\left(\sum_{i \in M} \frac{\delta_{i1}}{b_{i1}}; \dots; \sum_{i \in M} \frac{\delta_{in}}{b_{in}}; - \sum_{j \in N} \frac{\delta_{1j}}{b_{1j}}; \dots; - \sum_{j \in N} \frac{\delta_{mj}}{b_{mj}}; \sum_{i \in M} s_i + \sum_{i \in M} \sum_{j \in N} \frac{a_{ij}\delta_{ij}}{b_{ij}} \right).$$

Since $\sum_{i \in M} s_i = \sum_{j \in N} d_j$, then the sum of the first n rows is equal to the sum of rows from $n + 1$ to $n + m$. Thus, $\text{rank} \bar{A} < m + n$ and, hence, $\text{rank} \bar{A} = m + n - 1$. Therefore, the rank of the extended matrix of the system (7) is equal to the rank of its main matrix.

The lemma is proved.

Proved lemmas lead that the following theorem holds.

Theorem. *In the case of fixed demands and supplies, requirement (5), and linear transaction costs, there is only one independent spatial market price for the equilibrium commodity assignment pattern.*

P r o o f. According to Lemma 1, spatial market equilibrium in case of linear transaction costs is obtained by the unique commodity assignment pattern (6) with demand and supply prices that satisfy the system of linear equations (7). According to Lemma 2, the system of linear equations (7) is solvable with respect to $m + n$ variables (prices), while the rank of the main matrix is $m + n - 1$. Therefore, there is only one independent variable, while all others depend on it.

The theorem is proved.

The conducted study revealed an important conclusion: only one price value is independent value. In other words, once the basic market price is given, then other prices are adjusted according to spatial market equilibrium.

4. Conclusion. In this paper, we study the spatial market equilibrium and examine the case of linear transaction costs. Obtained results show that the dual variables, which are the Lagrange multipliers, reflect supply and demand prices. Thus, when this problem is solved, the equilibrium commodity assignment pattern is obtained as well as the equilibrium prices. Moreover, we find out that to obtain absolute value of equilibrium prices someone (actually the market moderator) has to give the basic market price. Once the basic market price is given, then other prices are adjusted according to spatial market equilibrium.

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Received: October 18, 2020.

Accepted: October 23, 2020.

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Пространственное рыночное равновесие в случае линейных транспортных затрат*

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* Работа выполнена при финансовой поддержке Российского фонда фундаментальных исследований (проект № 19-31-90109).

Для цитирования: Krylatov A. Y., Lonyagina Y. E., Golubev R. I. Spatial market equilibrium in the case of linear transportation costs // Вестник Санкт-Петербургского университета. Прикладная математика. Информатика. Процессы управления. 2020. Т. 16. Вып. 4. С. 447–454. <https://doi.org/10.21638/11701/spbu10.2020.409>

Исследуется пространственное рыночное равновесие в случае фиксированных значений спроса и значений предложения, требования равенства совокупного спроса и совокупного предложения, а также линейных функций затрат на перемещение товаров. Задача сформулирована в виде задачи нелинейной оптимизации с двойственными переменными, отражающими цены спроса и предложения. Показано, что единственное равновесное состояние распределения товаров может быть явно выражено через равновесные цены. Кроме того, выявлено, что для получения абсолютных значений равновесных цен необходимо установить некоторую базовую рыночную цену. Таким образом, доказано, что как только задана базовая рыночная цена, другие цены корректируются в соответствии с пространственным рыночным равновесием.

Ключевые слова: пространственное рыночное равновесие, нелинейная оптимизация, множители Лагранжа, условия Каруша—Куна—Таккера.

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