# Passive advection of a vector field: Anisotropy, finite correlation time, exact solution, and logarithmic corrections to ordinary scaling

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In this work we study the generalization of the problem considered in [Phys. Rev. E **91**, 013002 (2015)] to the case of *finite* correlation time of the environment (velocity) field. The model describes a vector (e.g., magnetic) field, passively advected by a strongly anisotropic turbulent flow. Inertial-range asymptotic behavior is studied by means of the field theoretic renormalization group and the operator product expansion. The advecting velocity field is Gaussian, with finite correlation time and preassigned pair correlation function. Due to the presence of distinguished direction **n**, all the multiloop diagrams in this model vanish, so that the results obtained are exact. The inertial-range behavior of the model is described by two regimes (the limits of vanishing or infinite correlation time) that correspond to the two nontrivial fixed points of the RG equations. Their stability depends on the relation between the exponents in the energy spectrum $\mathcal{E} \propto k_{\perp}^{1-\xi}$ and the dispersion law $\omega \propto k_{\perp}^{2-\eta}$. In contrast to the well-known isotropic Kraichnan's model, where various correlation functions exhibit anomalous scaling behavior with infinite sets of anomalous exponents, here the corrections to ordinary scaling are polynomials of logarithms of the integral turbulence scale $L$.

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## I. INTRODUCTION

Over decades much attention has been paid to the problem of intermittency and anomalous scaling in fully developed turbulence. Both the natural experiments and numerical simulations suggest that the violation of the classical Kolmogorov-Obukhov theory [1] is even more strongly pronounced for a advected field than for the velocity field itself; see, e.g., Refs. [2,3] and references therein. At the same time, the problem of passive advection appears to be easier tractable theoretically. Although the theoretical description of the fluid turbulence on the basis of the stochastic Navier-Stokes (NS) equations remains essentially an open problem, considerable progress has been achieved in understanding passive advection by random "synthetic" velocity fields. The most remarkable progress has been achieved for the so-called Kraichnan's rapid-change model [4], in which the velocity field is modeled by a Gaussian ensemble, not correlated in time, with zero mean and pair correlation function of the form

$$\langle v_i(x)v_j(x')\rangle = \delta(t-t')D_0 \int_{k>m} \frac{d\mathbf{k}}{(2\pi)^d} P_{ij}(\mathbf{k}) \frac{1}{k^{d+\xi}}\, e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}. \tag{1.1}$$

Here $P_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j/k^2$ is the transverse projector, $k \equiv |\mathbf{k}|$, $D_0 > 0$ is an amplitude factor, $d$ is the dimensionality of the $\mathbf{x}$ space, and $0 < \xi < 2$ is a parameter with the real ("Kolmogorov") value $\xi = 4/3$. The anomalous exponents have been calculated on the basis of a microscopic model and within regular expansions in formal small parameters [3].

A passively advected field may be chosen both scalar and vector; the latter case corresponds to the magnetohydrodynamic (MHD) turbulence. From the experimental point of view it is a special problem, closely related to the processes taking place in the solar corona, e.g., with solar wind; for a detailed discussion see Refs. [5–7] and references therein.

In solar flares, highly energetic and anisotropic large-scale motions coexist with small-scale coherent structures, finally responsible for the dissipation. A simplified description of the situation was proposed in Ref. [6]: the large-scale field $B_i^0 = n_i B^0$ dominates the dynamics in the distinguished direction **n**, while the activity in the perpendicular plane is described as nearly two-dimensional.

The observations and simulations show that the scaling behavior in the solar wind is closer to the anomalous scaling of the three-dimensional fully developed hydrodynamic turbulence, rather than to the simple Iroshnikov-Kraichnan scaling suggested by the two-dimensional picture with the inverse energy cascade [7]. Thus, further analysis of more realistic three-dimensional models is welcome.

One of the possibilities to make the original Kraichnan's model (1.1) anisotropic is to replace the ordinary transverse projector with the tensor quantity $T_{ij}(\mathbf{k})$, which contains a fixed unit vector **n**:

$$T_{ij}(\mathbf{k}) = a(\psi)P_{ij}(\mathbf{k}) + b(\psi)n_s n_l P_{is}(\mathbf{k})P_{jl}(\mathbf{k}), \tag{1.2}$$

where $a(\psi)$ and $b(\psi)$ are some functions of $\psi$, the angle between the vectors **n** and **k**; see, e.g., Refs. [8–10]. This formulation of the problem corresponds to the small-scale anisotropy and contains an isotropic model as a special case, if $a(\psi) = 1$ and $b(\psi) = 0$.

Another possibility is the "strongly anisotropic" model that does not contain an isotropic one as a special case and is obtained by introducing the velocity field $\boldsymbol{v}$ having preferred direction **n**:

$$\boldsymbol{v}(t,\mathbf{x}) = \mathbf{n} \times v(t,\, \mathbf{x}_{\perp}). \tag{1.3}$$

In this paper, we consider a more realistic model with *finite* (and not small) correlation time. For this purpose the correlation function (1.1) has to be modified, and instead

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