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Logarithmic violation of scaling in strongly anisotropic turbulent transfer of a passive vector field

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Inertial-range asymptotic behavior of a vector (e.g., magnetic) field, passively advected by a strongly anisotropic turbulent flow, is studied by means of the field-theoretic renormalization group and the operator product expansion. The advecting velocity field is Gaussian, not correlated in time, with the pair correlation function of the form $\propto \delta(t-t')/k_{\perp}^{d-1+\xi}$, where $k_{\perp} = |\mathbf{k}_{\perp}|$ and $\mathbf{k}_{\perp}$ is the component of the wave vector, perpendicular to the distinguished direction ("direction of the flow")—the d-dimensional generalization of the ensemble introduced by Avellaneda and Majda [Commun. Math. Phys. **131**, 381 (1990)]. The stochastic advection-diffusion equation for the transverse (divergence-free) vector field includes, as special cases, the kinematic dynamo model for magnetohydrodynamic turbulence and the linearized Navier-Stokes equation. In contrast to the well-known isotropic Kraichnan's model, where various correlation functions exhibit anomalous scaling behavior with infinite sets of anomalous exponents, here the dependence on the integral turbulence scale L has a logarithmic behavior: Instead of powerlike corrections to ordinary scaling, determined by naive (canonical) dimensions, the anomalies manifest themselves as polynomials of logarithms of L. The key point is that the matrices of scaling dimensions of the relevant families of composite operators appear nilpotent and cannot be diagonalized. The detailed proof of this fact is given for the correlation functions of arbitrary order.

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I. INTRODUCTION

Much attention has been attracted to the problem of intermittency and anomalous scaling in developed magnetohydrodynamic (MHD) turbulence; see, e.g., Refs. [1–8] and references therein. It has long been known that in the so-called Alfvénic regime, the MHD turbulence demonstrates behavior similar to that of the usual fully developed fluid turbulence: a cascade of energy from the infrared range towards smaller scales, where the dissipation effects dominate, and self-similar (scaling) behavior of the energy spectra in the intermediate (inertial) range. Moreover, the intermittent character of the fluctuations in the MHD turbulence is much more strongly pronounced than in ordinary turbulent fluids.

The solar wind provides a kind of appropriate "wind tunnel" in which different approaches and models of the MHD turbulence can be tested [3–7]. In solar flares, highly energetic and anisotropic large-scale motions coexist with small-scale coherent structures, finally responsible for the dissipation. Thus modeling the way the energy is redistributed, transferred along the spectra and eventually dissipated, is a difficult task. The intermittency strongly modifies the scaling behavior of the higher-order correlation functions, leading to anomalous scaling, described by infinite sets of independent "anomalous exponents."

A simplified description of the situation was proposed in Ref. [2]: The large-scale field $B_i^0 = n_i B^0$ dominates the dynamics in the distinguished direction $\mathbf{n}$, while the activity in the perpendicular plane is described as nearly two dimensional. This picture allows for precise numerical simulations, which show that turbulent fluctuations organize in rare coherent structures separated by narrow current sheets. On the other hand, the observations and simulations show that the scaling behavior in the solar wind is closer to the anomalous scaling in the three-dimensional fully developed hydrodynamic turbulence rather than to simple Iroshnikov-Kraichnan scaling suggested by two-dimensional picture with the inverse energy cascade; see, e.g., the discussion in Ref. [3]. Thus further analysis of more realistic three-dimensional models is welcome.

Two main simplifications of the full-scale model are possible here. First, the magnetic field can be taken as *passive*, that is, not to affect the dynamics of the velocity field. This approximation is valid when the gradients of the magnetic fields are not too large. What is more, the renormalization group analysis shows that such a "kinematic regime" can indeed describe the possible infrared (IR) behavior of the full-scale model [9].

Second, description of the fluid turbulence remains itself a difficult task. Once the feedback of the magnetic field is neglected, the velocity can be modelled by statistical ensembles with prescribed properties.

In spite of their relative simplicity, the models of passive fields, advected by such "synthetic" velocity ensembles, reproduce many of the anomalous features of genuine turbulent mass or heat transport. At the same time, they admit a detailed analytical treatment. Most remarkable progress was achieved for Kraichnan's "rapid-change model," where the correlation function of the velocity is taken in the powerlike form $\langle vv\rangle \propto \delta(t-t')k^{-d-\xi}$, where k is the wave number, d is the dimension of space, and ξ is an arbitrary exponent. There, for a passive *scalar* field (temperature or density of an impurity), the existence of anomalous scaling was established on the basis of a microscopic dynamic model [10]; the corresponding exponents were calculated within controlled approximations [11] and, eventually, within systematic perturbation expansions in a formal small parameter ξ [12]. Detailed review of the theoretical research on the passive scalar problem and the bibliography can be found in Ref. [13].

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