

# Game Theory and Applications

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### Preface

#### The Game Problem of Soft Landing ..... 1–22

*J. Albus, A. Meystel, A. Chikrii, A. Belousov and A. Kozlov*

##### Abstract

The game problem of pursuit of a controlled object, moving in horizontal plane, by another moving object, moving in three-dimensional space, is treated here. Therewith the horizontal plane stands for a state constraint for the pursuer, who is allowed to move only in the upper half-space. The dynamics of the players model the motions of different-type objects in a medium, characterized by friction. The goal of the pursuer is to achieve a simultaneous coincidence of geometric coordinates and velocities at some finite instant.

The initial phase states are described and the sufficient conditions on parameters of the conflict-controlled object are developed, for which the soft landing may be performed in a finite time. To do this the game problem is reduced to the equivalent problem of control. Then on the basis of in-depth analysis of the attainable set the control of the pursuer in explicit (analytic) form is constructed, which furnish solution to the original problem. On the initial stage velocities of the players are levelled and on the concluding one the soft landing is performed. The duration of each stage may be evaluated in advance. In conclusion some results of the soft landing simulation are presented.

#### A Dual Approach to Compromise Values ..... 23–32

*J.M. Bilbao, E. Lebrón, A. Jiménez-Losada and S.H. Tijs*

##### Abstract

The aim of this paper is to analyse the *Tijs value* as a solution concept for cost allocation problems. Given a cost game  $c$ , we define its *dual  $\tau$ -value* as  $\tau(c^*)$ , if the dual game  $c^*$  is quasi-balanced. Then we show that the dual  $\tau$ -value coincides, for a wide class of cost games, with the *alternate cost avoided (ACA)* allocation proposed in the 1930's by the Tennessee Valley Authority. It turns out that the center of the imputation set, the egalitarian nonseparable contribution and the *ACA* allocation are collinear.

#### Randomized Optimal Stopping Rules for a Class of Stopping Games ..... 33–43

*V. Domansky*

##### Abstract

We consider games of stopping for Markov chain in the formulation introduced by Dynkin (1969). Two players observe a Markov sequence and may stop it at any stage. When the chain is stopped the game ends and Player 1 receives from Player 2 the sum depending on the player who stopped the chain and on its current state.

We describe solutions for a class of stopping games with a countable state space. Payoffs are nonnegative and payoff is zero if Player 1 stops the chain but not Player

2. Estimations of value are obtained with use of randomized stopping times. The qualitative characteristics of solutions are determined with the "limiting" behavior of payoffs.

The explicit solutions are given for the case when  $p(i, i + 1) = 1$ .

## **Multi-objective Linear Production Games ..... 44–54**

*F.R. Fernández, M.A. Hinojosa and J. Puerto*

### **Abstract**

The goal of this paper is to explore the solutions concepts for multi-objective linear production games. Several stability conditions can be defined since one can have various interpretations of an improvement within the multicriteria framework. We present different solution concepts and explore the relationships among them. These concepts generalize the classic ideas of solution for scalar games and can be considered under different preference structures. The special features of the linear production problem make possible to obtain rewards allocations of the core among the players. These allocations are related with optimal solutions of special class of scalar linear production games.

## **Tchebycheff Systems and Analysis of Games Versus Nature on the Unit Square ..... 55–70**

*A.Y. Golubin*

### **Abstract**

A class of antagonistic games in which a finite number of moment constraints are imposed on the strategies of the second player (nature) is considered. A relationship between properties of this class and the part of moment theory concerning Tchebycheff systems of functions is disclosed and studied. Using the notions of moment theory, we introduce a condition of the Tchebycheff complement existence that extends the definition of generalized convex game. Under the condition, the finiteness of optimal strategies spectra is proved and the estimates for the numbers of the spectrum points are found. A criterion for the existence of a Tchebycheff complement for the game with power moment constraints is established in the form of a factorization of the pay-off function derivative. A sufficient condition for the solution uniqueness is obtained. The paper is finished with two examples illustrating the results.

## **Simple Dynamics from Complex Behavior in Duopoly Pricing Games .. 71–79**

*S. Huang, K. Anderson and Y. Yang*

### **Abstract**

We study a class of dynamic pricing duopoly games that model the type of environment in which e-commerce will be carried out in not so distant future. Under Markov settings these games can be solved via backward induction. The solution is found to display very complex patterns as a function of the discount factor, due to bifurcation phenomena in the discrete map induced by backward induction. However, it is possible to define an effective but simpler dynamics that retains the

optimality of the original game. We further show that this effective dynamics can be sustained by steady self-confirming equilibria. Our results (1) set limits on what learning algorithms based on Markov assumptions can achieve and (2) imply that learning in this kind of game should not be focused on the exact reaction functions, but rather on achieving optimal net present values with the realized time series of prices.

<b>A Study to Obtain a Realistic Game Solution in an Air Combat Game .....</b>	<b>80–89</b>
<i>F. Imado and J. Shinar</i>	

#### **Abstract**

A method to obtain the optimal solution of a practical complex problem, formulated as a zero-sum dynamic game is presented and applied to an example of an air-to-air interception in a horizontal plane. The principle of the method, called "method of bounds", is to start with a pair of suboptimal feedback strategies, depending on some parameters, for both players. Next, by solving numerically a set of one-sided optimal control problems for each player, against the set of parameters of the opponent the optimal values of the parameters are found. If the difference between the outcomes of the two parameter optimization problems is small, the feedback strategies using the optimal parameters represent an approximation of the saddle point strategies of the game. The application of the method for the selected minimum time air-to-air interception example showed a very good accuracy.

<b>Conflict and Co-operation in Fisheries: A Game Theory Approach .....</b>	<b>90–106</b>
<i>M. Lindroos and V. Kaitala</i>	

#### **Abstract**

We review multi-agent game theoretic models of international fisheries management. A special interest is in analysing under what circumstances international conflicts may arise and how can these conflicts be avoided and solved. The central model framework that we use is cooperative game theory. We use this framework to study the possibilities, conditions and stability of cooperation in high seas fisheries. The cooperative game theory framework is constructed using the traditional differential games in fisheries economics. This general theoretical framework has been applied to the cases of Norwegian spring-spawning herring and Northern Atlantic bluefin tuna where discrete time and age-structured population dynamics model as a basis for cooperative game models is used. We show that external and internal stability of cooperation depends on several factors including technological efficiency and harvesting costs. Identifying these factors is crucial for any successful cooperative arrangement that would be negotiated for fisheries in the future. Furthermore, the problem of uncertainty may be avoided by simple adaptive modifications to the full cooperation strategy. This result is extremely important in the case of spring-spawning herring since natural variations in the stock have been high. Overall, we stress the importance of sustainable international cooperation of marine fish stocks.

<b>Guidance Law Scheduling</b> .....	<b>107–114</b>
<i>S. Le Ménéec</i>	

#### Abstract

We are developing an Artificial Intelligence application to switch missile guidance laws. This software simulates a pursuer using Proportional Navigation except when the state of the game is reaching the barrier of the differential game, then Expert System rules commute the guidance law to optimal guidance law to keep the state of the game in the capture zone. This study is part of work dedicated to Artificial Intelligence Methods for Missile Guidance and Control Systems which is an EUCLID Contrat (No 99/EF 15.02/006).

<b>Non-symmetric Solution of an Arbitration Game</b> .....	<b>115–120</b>
<i>V.V. Mazalov and A.A. Zabelin</i>	

#### Abstract

The following arbitration scheme is considered. Two players  $L$  and  $M$  interpreted as Labour and Management, respectively, are negotiating on a wage rate. Player  $L$  makes an offer  $x$ , and player  $M$  – an offer  $y$ . We shall assume that  $x$  and  $y$  are arbitrary real numbers. If  $x \leq y$ , there is no conflict, and the players agree on a payoff equal to e.g.  $(x + y)/2$ . If otherwise  $x > y$ , the parties call in the arbitrator ( $A$ ) to settle the dispute. In this paper we consider the *final-offer* arbitration scheme. We shall consider a case where the arbitrator's solution is random and concentrates in two points  $a_1$  and  $a_2$ .

<b>Nash Equilibria Refinements for Multistage and Repeated Games</b> .....	<b>121–130</b>
<i>L.A. Petrosjan</i>	

#### Abstract

Multistage game  $G$  with simultaneous games  $\Gamma(\cdot)$  played on each stage is considered. The definition of outcome, path in tree-graph and history are introduced. The new class of Nash Solutions based on the possibilities of punishment for the deviation on first stages of  $G$  are defined. It is shown that the outcomes under these solutions dominate the classical subgame perfect Nash outcomes. For infinite multistage games  $G$  the regularization procedure is introduced which enables to construct a strong Nash Equilibrium (coalition-proof) in such class of games.

<b>Discretization, Nonlinear and Bilevel Programming in Pursuit–evasion Games</b> .....	<b>131–146</b>
<i>T. Raivio and H. Ehtamo</i>	

#### Abstract

In this paper we deal with approaches to solve pursuit-evasion games by discretization, nonlinear programming and bilevel programming. We first explore different discretization approaches for ordinary optimal control problems. Then, we describe a method to decompose the solution of pursuit-evasion games into optimal control problems that can be solved using discretization and nonlinear programming. Yet another approach is to discretize the game itself at the outset and construct a feasible direction bilevel programming method that makes use of the special structure of the problem. We discuss the performance of the discretization approach in the light of numerical examples, and present some applications.

<b>Random Assignment and Uncertain Employment in Optimal Stopping of Markov Processes .....</b>	<b>147–157</b>
<i>D. Ramsey and K. Szajowski</i>	

#### Abstract

A mathematical model of competitive stopping of a Markov chain is considered. Two decision makers observe goods or applicants for a post sequentially. Each object is characterized by some random variable. The aim of the players is to obtain the most profitable object. The rules of game are as follows. An object can be accepted only at the moment of its appearance. At each moment  $n$  one object is presented. When both players want to accept the same object, then some rule assigning priority of acceptance to one of the players is applied. Any accepted object is available with some given probability. If such an object is not available, it is lost to both players forever. The solution of such games is found by solving a system of nested Bellman Equations. An example related to the secretary problem is given.

<b>Optimal Stopping Games by Equal-weight Players for Poisson-arriving Offers .....</b>	<b>158–169</b>
<i>M. Sakaguchi</i>	

#### Abstract

Players I and II must make a decision to accept (A) or reject (R) an offered job at each offer presentation. The offers arrive during time interval  $[0, T]$  as a Poisson process with rate  $\lambda$ . The offered jobs have random sizes being i.i.d. random variables from a uniform distribution on  $[0, 1]$ . Whenever an offer with size  $x$  arrives it is presented to both players simultaneously, and players must choose either A or R. If the player's choice-pair is A–R or R–A then the player who chooses A gets  $x$  dropping out from the game thereafter, and the other player continues his (or her) one-person game. If the choice-pair is A–A the the lottery is used to the effect that A–R or R–A is enforced to the players with equal probability  $\frac{1}{2}$ . If the choice-pair is R–R, then the current sample  $x$  is rejected and the game passes on the time when a new job arrives next. Players who can not accept any offer until time  $T$  gets a reward of zero. Each player aims to maximize his expected reward. We shall also discuss about the bilateral-move version of the game.

<b>Intermediation and the Poor Property Rights Protection .....</b>	<b>170–174</b>
<i>D. Shaposhnik</i>	

#### Abstract

The paper develops a model of equilibrium on a particular market. Producers and buyers are assumed to have no market power and to act as price-takers. Therefore market is characterized by standard neoclassical demand and supply functions. There are also  $n$  agents called middlemen who buy the commodity from producers and sell it to the customers. So the initial market is separated into the wholesale market and retail one, where the middlemen act one against the other. The case when the middlemen undertake no costs connected with transportation, storing, maintenance of shops, etc. is considered. The imperfect case with incomplete property rights protection is investigated. The environment is taken to be characterized

by a parameter  $\epsilon$ , which has the following sense. If someone is ready to spend the amount  $A$  to force middleman  $i$  out of business, this middleman must pay  $A$  in order to protect herself completely from the aggression. The symmetric Nash-equilibrium is computed in this game.

## **Asymptotics for Singularly Perturbed Differential Games . . . . 175–196**

*N.N. Subbotina*

### **Abstract**

Singularly perturbed differential games with "fast" and "slow" motions and the Bolza type payoff functionals are considered. Sufficient conditions are obtained to converge the value functions of the games to the value function of the limit unperturbed game as the singularity parameter tends to zero. The concept of minimax solutions to Hamilton-Jacobi-Isaacs equations and reduction technique approach are developed to the nonautonomous, nonlinear problems. The constructed limit unperturbed games are defined in phase subspace of "slow" motions under augmented admissible controls of players.