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HIV incidence in Russia: SIR epidemic model-based analysis

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The problem of predicting the incidence rate of the human immunodeficiency virus (HIV) in Russia is considered. The official morbidity levels were taken as initial data; for numerical modelling, the SIR model was applied to take into account the birth rate, mortality, as well as chemoprophylaxis and isolation of a group of infected but not epidemically dangerous patients. The search for the coefficients of the model is examined in detail using gradient descent with an auxiliary system applied. Various scenarios of the epidemic development are estimated, depending on the percentage of the number of patients who are undergoing therapy. The consequences of achieving the goals of the UNAIDS strategy 90–90–90 (90 % people who are aware of their status, 90 % among them are on HIV treatment and 90 % among them are virally suppressed) are described. It is shown that upon reaching the target levels of involvement of patients in anti-epidemic measures, the number of infected people can be kept within 1 % of the total population with a further decrease.

Keywords: HIV, math modeling, predicting, incidence.

Introduction. The Human Immunodeficiency Virus (HIV) infection is a disease that causes a gradual decrease in the general immunity of a person, as a result the body loses its ability to resist pathogenic bacteria. The last stage in the development of HIV infection, in which the destruction of vital systems of the body occurs, is Acquired Immunodeficiency Syndrome (AIDS).

The scale of the HIV epidemic is relatively small in Russia, the number of infected reaches 1.3 million people [1], but it keeps growing [2]. At the same time, the annual incidence is very high — in 2018, about 100 thousand new cases were detected. In terms of the number of new infections to the population, Russia holds a leading position in the world.

Cases of complete cure for HIV infection after stem cell transplantation are known [3], but they are single, prohibitively expensive and thus can be neglected, assuming that HIV infection is almost incurable in Russia. Without supportive therapy, on average after 5–10 years, HIV infection leads to the development of AIDS and death.

Therapy, which prolongs the life and working capacity of an HIV-infected individual, is expensive, and the spread of HIV entails serious economic losses. Thus, an important task is to control the spread of HIV infection. Large funds are allocated for the study of epidemic mechanisms and methods of counteraction, the development of a vaccine and drugs. Another task is to reduce the number of new cases, i. e. infection prevention. One more goal to achieve is to support individuals already infected with HIV: prolonging and improving their quality of life.

The life expectancy of HIV infected patients varies considerably, depending on many factors: age, lifestyle, emotional state, region of residence, history of chronic diseases, etc. The average life expectancy (with no curation) is about 9–11 years from the moment of infection. In case of treatment and required regime refusal, life expectancy is reduced to 2–5 years. There are also known cases when people have lived with the virus for quite a long time — more than 40 years from the moment of infection.

This paper analyzes the key features of HIV infection and disease statistics in Russia for the further use of this information for the epidemic course prediction. Much attention is paid to the peak of the epidemic — the moment of the maximum number of infected people.

Literature review. One of the most famous works on mathematical modeling of the spread of diseases [4], describes the dynamics of the epidemiological process using a system of differential equations, the solutions of which characterize the dynamics of changes in the number of subgroups in the population in question. Kermak and McKendrick introduced one of the simplest models of the dynamics of the epidemic — the SIR model, which considers three groups of individuals: susceptible to the disease $S(t)$ (Suspectible), infected $I(t)$ (Infected) and $R(t)$ (Removed) dropped out from the group due to recovery or death.

The analytical solution of the SIR model, as well as its modification taking into account fertility and mortality, are considered in [5].

There are many models examining the division of a population into a larger number of different groups of individuals, depending on the stage of the disease, the presence of immunity, etc.

In addition to the three groups of SIR models the article [6] discusses the MSEIR disease spread model, which introduces a group of individuals with passive immunity from birth (M) and a group of individuals in the latent stage of infection (E), when the virus has already entered the body, but the disease has not yet begun to manifest, and the individual is not able to spread the disease.

In [7] the SIR model of the dynamics of the epidemic, various modifications of the model: taking into account fertility and mortality, the virulence of the pathogen, the latent phase of the disease are considered. The types of anti-epidemic measures are also described with examples of their application and optimization. Among other things, measures are being taken to control the HIV epidemic, such as chemoprophylaxis and isolation of patients.

A lot of existing HIV models consider heterogeneous models, dividing populations into different risk groups. In this case, basically two stages are considered — susceptibility to the disease and infection. One of such models was considered in [8]. In this model, there are three risk groups: the core of the infection (people most at risk of infection due to risky behavior), the bridge group (people in contact with both the nuclear group and the rest of the population) and the main population (non-core people or bridge group).

The main problem of such models is a large number of uncertain parameters, the errors in the determination of which significantly affect the simulation result.

The problem formulation. A feature of HIV statistics is the lack of accuracy in the coefficients of the equations. We propose a simple mathematical model with small amount of coefficients and focus on the most accurate determination of parameters.

For analysis, we take statistics on the incidence of HIV infection in Russia for 2008–2018. Earlier data are less accurate due to the low degree of awareness of HIV infection in that period and may adversely affect the accuracy of the resulting model.

HIV statistics in Russia for 2008–2018 is presented in the following form:

Year	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
HIV, %	0,31	0,33	0,36	0,37	0,41	0,45	0,50	0,54	0,60	0,64	0,69

The main goal of this work is to develop a forecast for the disease spread.

Coefficients identification. Let us consider the simplest SIR model first:

$$\begin{cases} \dot S(t) = -\beta I(t)S(t), \\ \dot I(t) = \beta I(t)S(t) - \gamma I(t), \\ \dot R(t) = \gamma I(t), \end{cases}$$

$$S(t)+I(t)+R(t) = N > 0,\ S(t_0) = S_0 > 0,\ I(t_0) = I_0 > 0,\ R(t_0) = R_0 \geqslant 0,\ \beta > 0,\ \gamma > 0.$$

In order to consider the shares of the total number of individuals, we take $N = 1$. Thus, each of the values will correspond not to the size of the group, but to its share of the total number.

Since the number of people who died from AIDS at 2008–2018 is known, one can calculate

$$\gamma = \frac{R(2018) - R(2008)}{\int_{2008}^{2018} I(t)dt} \approx 0.05.$$

Since $1/\gamma \approx 20$ in SIR model is the average life expectancy, this value reflects the fact that approximately half of the patients receive therapy and this does not lead to a decrease in their life expectancy.

For a qualitative assessment of the epidemiological situation, it is necessary to have sufficiently accurately estimated parameters of the model under consideration. To find them, statistical data on the disease are used. In the SIR model, we are interested in the coefficient β. To obtain the model coefficients for a number of measurements, we solve the following problem.

Let k be the number of available measurements of the number of people infected with HIV in Russia by year. Thus, we have k values $I_1, I_2, \ldots, I_i, \ldots, I_k$.

For different β, I_0, the system will have different solutions. It is necessary to select such values of these parameters so that the error in the solution obtained, when comparing with real statistics is minimal, so that

$$\sum_{i=i}^{k} [I(t_i) - I_i]^2 \to 0,$$

where $I(t_i)$ is the solution of the system for $I(t)$ for fixed β, I_0.

Using a grid of the values of β and I_0, solving the corresponding systems numerically, we obtain approximate value for available data: $\beta = 0.13$.

More accurate approximation can be achieved using local minimum search methods [9].

Consider the first two equations of the system. Rewrite the system in the form

$$\begin{cases} \dot S = -\beta IS = f_1(S, I, \beta), \\ \dot I = \beta IS - \gamma I = f_2(S, I, \beta) \end{cases}$$

with initial values $S(0) = S_0,\ I(0) = I_0$.

The method is based on minimisation of the function

$$\Phi(\beta) = \frac{1}{2}\sum_{i=1}^{11}\left(I(t_i,\beta) - I_i\right)^2.$$

Gradient equations have the form

$$\frac{d\beta}{d\tau} = \frac{\partial\Phi(\beta)}{\partial\beta} = -\sum_{i=1}^{10}(I(t_i,\beta) - I_i)y_{11}(t_i)$$

with auxiliary equation

$$\begin{cases} \dot{y}_1 = -IS - \beta I y_1 - \beta S y_2, \\ \dot{y}_2 = IS + \beta I y_1 + (\beta S - \gamma) y_2, \end{cases}$$

$$y_1(0) = y_2(0) = 0.$$

As the initial data, we use the previously obtained approximation $\beta \approx 0.13$.

The solution of the Cauchy problem for gradient equation with respect to the system approaches a certain local minimum, in which β take optimal value (Figure 1).

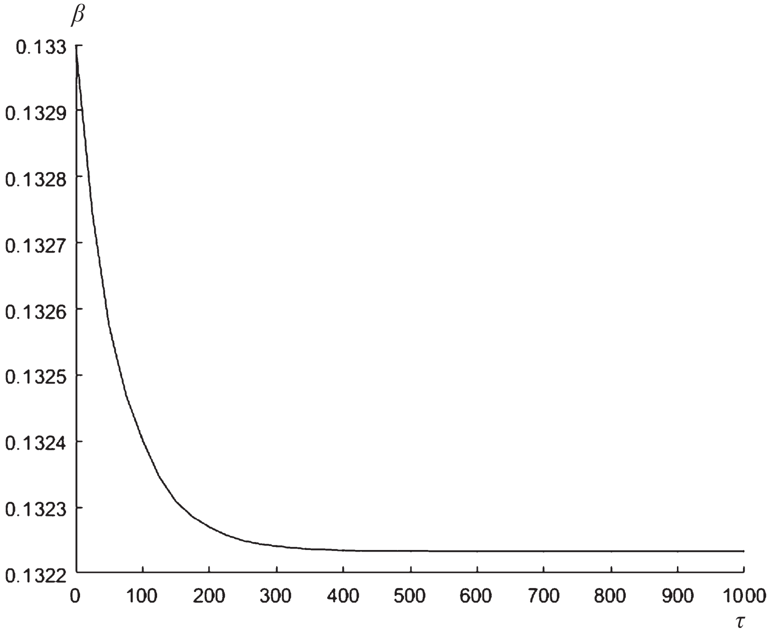

Figure 1. Solution of the Cauchy problem

For different initial conditions, the solution can converge to different local minima. Therefore, a preliminary search for the approximate global minimum is necessary.

As a result, we obtain a more accurate value of the coefficient $\beta = 0.1322$ (in this problem it is not local but global minimum).

SIR model with fertility and mortality rate, chemoprophylaxis and isolation. More accurate prediction of the epidemiological situation requires taking into account additional factors. One of these factors is the demographic situation in the country:

there is a generational change that affects the size of a particular group of individuals in mathematical models. Thus, fertility and mortality rates should be take into account.

For HIV infection, unlike many diseases, a vertical transmission mechanism is characteristic: infected mothers are more likely to have already infected children. Thus, the influx of individuals into the group of susceptible individuals will occur both at the expense of the susceptible and at the expense of a certain proportion of the infected. The inflow and outflow in the group of retired individuals for HIV infection does not occur, i. e. for this disease, this group characterizes the number of individuals who died from the disease. The outflow from the other two groups added to consideration should take into account the death of people from all the other factors.

In addition to β and γ, the following coefficients are also present in the new model: $\alpha > 0, \mu > 0$ — fertility and mortality rate in the country (average value of the number of newborns per year per individual), $\varphi_1 > 0$ — the probability to give birth a healthy child for an infected mother. According to statistics, the average probability is $\varphi_1 \approx 0.7$ [10].

To consider the epidemiological situation in Russia, we take the average value of these coefficients over the past 11 years: $\alpha = 0.01272$, $\mu = 0.01344$.

Chemoprophylaxis in the general case is the prevention of the development of the disease in the early stages of infection. In the case of HIV infection, there are two areas of chemoprophylaxis.

1. Chemoprophylaxis of parenteral and sexual infection. Such methods have been developed primarily for prophylaxis in health workers who have been exposed to infected blood on the mucous membrane or who have been injured by an HIV-contaminated tool. The effectiveness of the use has been proven, as a result the risk of infection is significantly reduced. The onset of chemoprophylaxis is considered inconsistent if more than 72 hours have passed since the infection. It can also be used for sexually transmitted infections or for transfusion of blood infected with HIV.

2. Decrease in probability of giving birth to an infected child in a HIV-infected mother. This method of prevention on average reduces the risk of infection with a vertical transmission mechanism to 8 %. Thus, under chemoprophylaxis $\varphi_2 = 0.92$.

For HIV infection, the following interpretation of isolation is relevant. A part of infected individuals after the diagnosis of the disease changes their lifestyle, limiting themselves in actions that can cause infection of others. Thus, they move into a group of isolated individuals I_{is} who are not able to spread the infection. In this group the coefficient, characterizing the probability of having a healthy child will be higher due to the use of chemoprophylaxis.

Consider the coefficient ω, which characterizes the probability of transition from infected group to a group of isolated ones. By increasing this probability, the course of the epidemic can be positively affected. The main factors for this value to depend on are timely diagnosis of the disease and the conscious behaviour of infected individuals, in which they limit themselves to actions dangerous for the environment.

Launched in 2014, the UNAIDS Fast-Track strategy outlined plans to step up the HIV response in low- and middle-income countries to meet the SDG 3 target to end AIDS by 2030 [11]. The strategy acknowledges that, without rapid scale-up, the HIV epidemic will continue to outrun the response. To prevent this, it underlines the need to reduce new HIV infections and AIDS related deaths by 90 % by 2030, compared to 2010 levels. To achieve this, the Fast-Track strategy sets out targets for prevention and treatment, known as the 90–90–90 targets (90 % people who are aware of their status, 90 % among them are on HIV treatment and 90 % among them are virally suppressed, the latter we will refer to as isolated, the product still gives only 73 % of all infected population). The Russian HIV 2020 programm should have achieved the same results up to 2020 [12], but it is evident that it fails since only 42 % of infected are on HIV treatment and less than 25 % are annually tested. This gives ω not exceed 0.1.

The transition diagram for this model is presented in Figure 2.

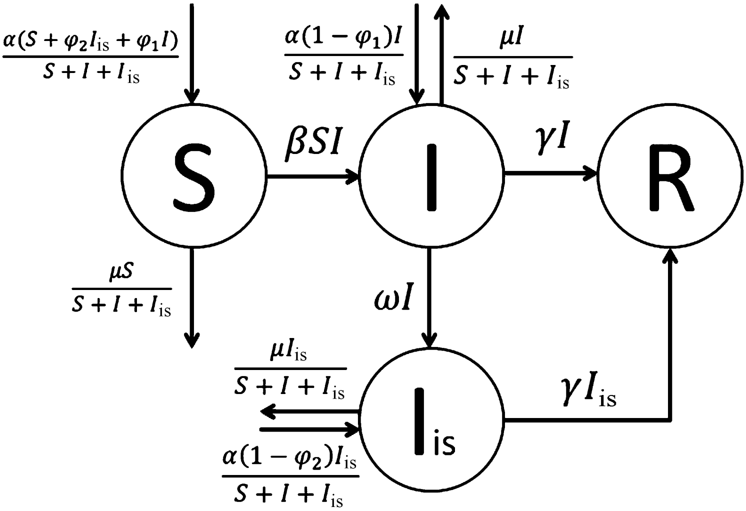

Figure 2. Transition diagram for a model with isolation, chemoprophylaxis, and birth and death rates

The model with isolation of patients, chemoprophylaxis and fertility and mortality rates will be the following:

$$\begin{cases} \dfrac{dS(t)}{dt} = -\beta I(t)S(t) + \dfrac{\alpha(S(t)+\varphi_2 I_{is}(t)+\varphi_1 I(t)) - \mu S(t)}{S(t)+I(t)+I_{is}(t)}, \\ \dfrac{dI(t)}{dt} = \beta I(t)S(t) - \gamma I(t) + \dfrac{\alpha(1-\varphi_1)I(t) - \mu I(t)}{S(t)+I(t)+I_{is}(t)} - \omega I(t), \\ \dfrac{dI_{is}(t)}{dt} = \omega I(t) - \gamma I_{is}(t) + \dfrac{\alpha(1-\varphi_2)I_{is}(t) - \mu I_{is}(t)}{S(t)+I(t)+I_{is}(t)}, \\ \dfrac{dR(t)}{dt} = \gamma(I(t)+I_{is}(t)). \end{cases}$$

Using the technique developed above, the most probable $\beta = 0.21$. The simulation shows almost linear increase of the total number of infected up to 6 % of population to the end of the century.

Let us suppose more realistic scenario of increasing ω in the nearest future according to HIV 2020 programm. Figure 3 shows graphs of the development of the epidemic for various values of ω. It is clear that even double increase ω to 0.2 leads to a rapid decline in the epidemic up to late 2020th.

Conclusion. Based on the results of the research that has been carried out it is possible to describe three potential epidemic development scenarios.

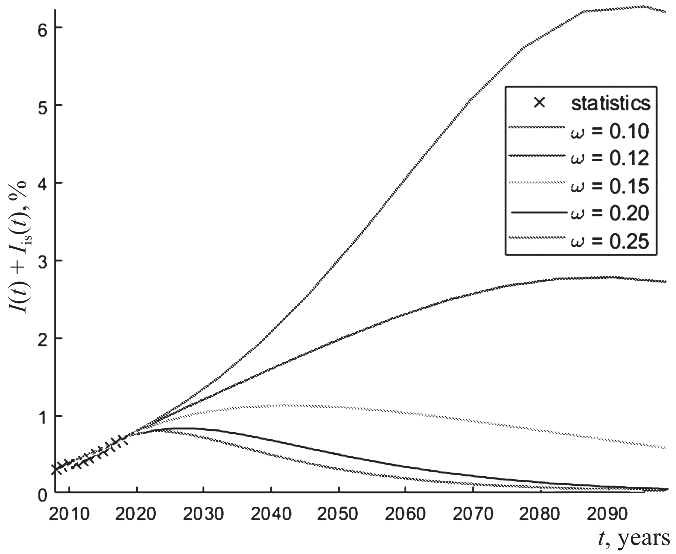

Figure 3. The development of the epidemic with different values of ω

In case the existing epidemic factors keep influencing (e. g. having only 40 % infected on HIV treatment), the country will face a large-scale epidemic with the total number of infected reaching 6 % by the end of the century.

In case of an increase of the amount of infected on HIV treatment up to 50–60 %, a gradual decrease of the total number of infected reaching 1.0–1.5 %. by 2040th can be expected.

In case the goals of the UNAIDS Fast-Track strategy are achieved, the epidemic level will start downsizing having reached its peak value not exceeding 1 % in the beginning of 2020th.

References

1. Beyrer C., Wirtz A. L., O'Hara G., Léon N., Kazatchkine M. The expanding epidemic of HIV-1 in the Russian Federation. *PLoS Med*, 2017, vol. 14(11), pp. 1002462. https://doi.org/10.1371/journal.pmed.1002462

2. Gosudarstvennyy doklad "O sostoyanii sanitarno-epidemiologicheskogo blagopoluchiya naseleniya v Rossiyskoy Federatsii v 2018 godu" [State report "On the state of the sanitary-epidemiological well-being of the population in the Russian Federation in 2018"]. *Russian Federal Service for Surveillance on Consumer Rights Protection and Human Wellbeing (Rospotrebnadzor)*, 2018, pp. 116–120. (In Russian)

3. Gupta R. K., Abdul-Jawad S., McCoy L. E. et al. HIV-1 remission following CCR5Δ 32/Δ32 haematopoietic stem-cell transplantation. *Nature*, 2019, vol. 568, pp. 244–248.

4. Kermack W. O., McKendrick A. G. A Contribution to the mathematical theory of epidemics. *Proceedings of the Royal Society*, 1927, pp. 700–721.

5. Harko T., Lobo F. S. N., Mak M. K. Exact analytical solutions of the Susceptible-Infected-Recovered (SIR) epidemic model and of the SIR model with equal death and birth rates. *Applied Mathematics and Computation*, 2014, no. 236, pp. 184–194.

6. Hethcote H. The mathematics of infectious diseases. *SIAM Review*, 2000, no. 42(4), pp. 599–653.

7. Kolesin I. D., Zhitkova E. M. *Matematicheskiye modeli epidemiy* [*Mathematical models of epidemics*]. Saint Petersburg, Solo Publ., 2004, 90 p. (In Russian)

8. Nosova E. A. Models of control and spread of HIV-infection. *Mathematical biology and bioinformatics*, 2012, vol. 7.2, pp. 632–675. https://doi.org/10.17537/2012.7.632

9. Babadzanjanz L. K., Boyle J. A., Sarkissian D. R., Zhu J. Parameter identification for oscillating chemical reactions modelled by systems of ordinary differential equations. *Journal of Computational Methods in Sciences and Engineering*, 2003, vol. 3, no. 2, pp. 223–232.

10. *World Health Organization.* Available at: https://www.who.int/news-room/fact-sheets/detail/hiv-aids (accessed: August 10, 2019).

11. *UNAIDS. Ambitious treatment targets: writing the final chapter of the AIDS epidemic.* Geneva, UNAIDS Publ., 2014. Available at: https://www.unaids.org/sites/default/files/media_asset/JC2670_UNAIDS_Treatment_Targets_en.pdf (accessed: August 10, 2019).

12. *Rasporyazheniye Pravitel'stva RF ot 20 oktyabrya 2016 g., no. 2203-r "O Gosudarstvennoy strategii protivodeystviya rasprostraneniyu VICH-infektsii v RF na period do 2020 g. i dal'neyshuyu perspektivu"* [*Decree of the Government of the Russian Federation of October 20, 2016, no. 2203-r "On the state strategy for counteracting the spread of HIV Infection in the Russian Federation for the period until 2020 and the future"*]. Available at: http://static.government.ru/media/files/cbS7AH8vWirXO6xv7C2mySn1JeqDIvKA.pdf (accessed: August 10, 2019). (In Russian)

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Анализ эпидемической SIR-модели распространения заболеваемости ВИЧ в России

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Рассматривается задача прогнозирования уровня заболеваемости вирусом иммунодефицита человека (ВИЧ) в России. В качестве исходных данных приняты официальные уровни заболеваемости, для численного моделирования применена SIR-модель с учетом рождаемости, смертности, а также химиопрофилактики и выделения группы зараженных, но эпидемически не опасных больных. Решена задача наиболее точного поиска коэффициентов модели при помощи градиентного спуска с использованием вспомогательной системы. Проанализированы разные сценарии развития эпидемии в зависимости от процентного отношения числа больных, которые проходят терапию. Описаны последствия достижения целей стратегии 90–90–90 UNAIDS (90 % больных, знающих о своем заболевании, из которых 90 % проходят терапию, для которых она эффективна в 90 % случаях). Показано, что при достижении целевых уровней вовлеченности заболевших в противоэпидемические мероприятия, количество инфицированных удастся удержать в пределах 1 % от общего числа населения с дальнейшим снижением.

Ключевые слова: ВИЧ, эпидемия, математическое моделирование, прогноз.

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