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MICROSCOPIC TREATMENT OF THE NUCLEAR REACTION $d(t, n)\alpha$ WITHIN THE ADIABATIC REPRESENTATION

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Abstract. The effective equation for the cluster relative-motion function of the initial and final channels in the nuclear reaction $d+t \to n+\alpha$ is derived within the adiabatic approach. An effective energy-dependent potential is constructed by projecting on the open channels and its asymptotics are studied at large intercluster distances. The asymptotic boundary conditions for the effective system are extracted from the known asymptotics of the microscopic wave function of the five-body system. The connection between these equations and the resonating group model is analysed.

Резюме. Получены эффективные уравнения, описывающие функцию состояния относительного движения во входящем и выходящем каналах ядерной реакции $d+t \to n+\alpha$ в адиабатическом подходе. Построен эффективный, зависящий от энергии, потенциал с помощью проецирования на открытых каналах. Найдены правильные асимптотические граничные условия в терминах асимптотики волновой функции пяти тел. Обсуждается связь с результатами, полученными в модели резонирующих групп.

1. Introduction

In order to avoid serious computational difficulties in the investigation of many-body quantum problems, the extensive use of model reduction is indispensable. In this case it is important to justify the choice of the model and to clarify its relevance with respect to the original complete problem. The purpose of the present paper is to derive within the adiabatic approach [1—3] a model independent equation for the intercluster relative-motion wave function for the reaction $d+t \to n+\alpha$ in the initial and final channels. Moreover, an effective energy-dependent potential, describing the effect of closed channels, turns out to be an important object for modelling in the reduction of the general five-body problem to a given model.

The interest in the $\alpha+t \to n+a$ reaction within this approach stems from the problem of evaluating the probability for a meson sticking to helium [4] in the reaction of muon catalized fusion [5]

$$d+t+\mu \to {}^5\mathrm{He}_{(3/2)^+} + \mu \begin{cases} \xrightarrow{1-\omega_s} {}^4\mathrm{He}+n+\mu \\ \xrightarrow{\omega_s} {}^4\mathrm{He}\mu+n \end{cases} + \mathbf{Q}.$$

In many papers [6—11] devoted to this problem, the many-body and multi-channel nature of the problem prompts to simplify the representations for the channel coupling operators, effective interactions, etc. In [11], using the three-channel model of μ-catalyzed reaction, the channel coupling operators lead to additional energy-dependent interaction in the Faddeev equations. The parameters of this interaction are often extracted by fitting the data on the $^5He^*_{(3/2)+}$ resonance. However, the choice of the channel coupling operators and their parameters may essentially affect the final equations. This explains the interest in determining the effective potentials on the basis of an exact equation for the complete many-body nuclear problem $d+t \to n+\alpha$ and in finding their essential properties that have to be taken into account in the modelling. Besides that the proposed here appoach allows us in each stage of modelling to control the difference between the model expression and the exact one.

The consideration is based on the combination of the adiabatic expansion technique with the Feshbach projection method on open channels [12]. This approach allows the use of local adiabatic representations in the physical regions (see [13, 14]).

In this paper we derive effective equations for the intercluster relative-motion function and calculate their asymptotic behaviour. The asymptotics of the corresponding effective potentials with cluster separation is discussed as well.

The analysis performed here also can be used to trace the connection between the exact equations in the adiabatic representation and the popular resonating-group model [15, 16].

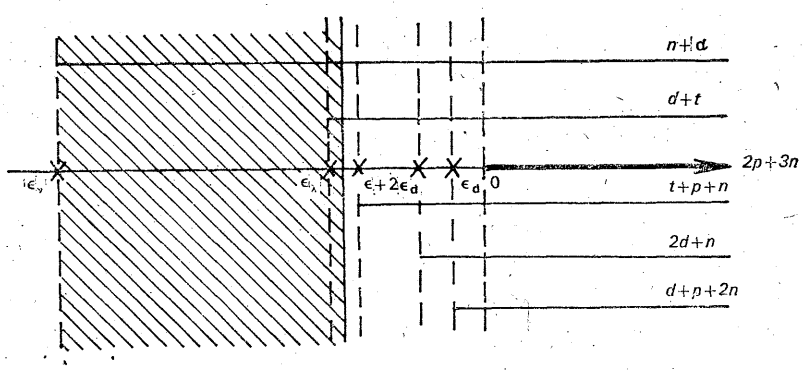
2. Formulation of the problem

Let us consider the $d+t$ system as a system of five spinless particles in the energy region $-28.37\ \text{MeV} \leq E \leq -10\ \text{MeV}$ the energy being measured from the total breakup threshold (see Fig. 1).

Upon separating the centre of mass of the five-body problem, the Hamiltonian in fixed clusterisation a_l (l is the number of clusters) is easily written as

$$H = -\Delta_{x_{a_l}} - \Delta_{y_{a_l}} + \mathbf{V}. \tag{1}$$

Fig. 1. Position of channel threshold in the system $d=t$: $\varepsilon_d = -2.29$ MeV is the deutron binding energy (the channel $d+p+n$ threshold),
$\varepsilon_t = -8.48$ MeV is the tritium binding energy (the channel $t+p+n$ threshold),
$\varepsilon_\lambda = \varepsilon_d + \varepsilon_t = -10.77$ MeV is the channel $d+t$ threshold,
$\varepsilon_\lambda = -28.37$ MeV is the binding energy of ^4He (the channel $n+\alpha$ threshold),
$Q = \varepsilon_\lambda - \varepsilon_\nu = 17.6$ MeV is the energy realized of reaction $d+t \to n+\alpha$



Here the Jacobi coordinates x_{a_l}, y_{a_l} are introduced as follows. $x_{a_l} \in \mathbf{R}^{15-3l}$ is the set of reduced nucleon coordinates with respect to the centre of mass of the a_l-cluster; $y_{a_l} \in \mathbf{R}^{3(l-1)}$ is the set of reduced intercluster coordinates; $\mathbf{V}$ is the sum of all internucleon interactions.

The Hamiltonian in the form (1) corresponds to the representation of the Hilbert space of states $\mathscr{H} = L^2(\mathbf{R}^{12})$ in the chart corresponding to clusterisation a_l in the form:

$$\mathscr{H} = L^2(\mathbf{R}^{3(l-1)}_{y_{a_l}}) \times L^2(\mathbf{R}^{15-3l}_{x_{a_l}}). \tag{2}$$

In the energy region under consideration only two-cluster channels of the reaction ($l=2$) are open; therefore in what follows we shall omit index l, assuming it to be equal to 2.

We introduce the Hilbert bundle corresponding to the problem (see, e. g. [17]) with a base $B^a = \mathbf{R}^3_{y_a}$ and typical fiber $\mathscr{F}_{y_a} = L^2(\mathbf{R}^9_{x_a})$. Then, according to the general scheme [2, 14] the following representation is valid:

$$\Psi(X) = \sum \chi^k_a(y_a)\, \varphi^k_a(x_a;\, y_a). \tag{3}$$

Here $\Psi(X)$ is the total wave function of the system satisfying the Schrödinger equation

$$(H-E)\Psi(X) = 0, \quad X \in \mathbf{R}^{12}. \tag{4}$$

$\chi^k_a(y_a)$ denotes the expansion coefficients of Ψ over the "moving Hilbert frame" $\varphi^k(\cdot, y_a)$.

The functions $\{\varphi^k\}$ are chosen in a special way. First, we devide the full potential $\mathbf{V} = \mathbf{V}(x_a, y_a)$ into two parts

$$\mathbf{V}(x_a, y_a) = V^{\text{int}}_a(x_a) + V^{\text{int}}_a(x_a, y_a), \tag{5}$$

where $V^{\text{int}}_a(x_a)$ involves interactions between particles united in one cluster $V^{\text{ext}}_a(x_a, y_a)$, between particles from different clusters. Then we single out in V^{ext}_a a part depending only on the intercluster coordinate

$$V^{\text{eff}}_a(y_a) = \int |\Phi_a(x_a)|^2\, V^{\text{ext}}_a(x_a, y_a)\, dx_a \tag{6}$$

$$\hat{V}(x_a, y_a) \equiv V^{\text{ext}}_a(x_a, y_a) - V^{\text{eff}}_a(y_a). \tag{7}$$

By $\Phi_a(x_a)$ we denote the ground state function of noninteracting clusters in clusterisation a. Clusterisation corresponding to the $d+t$ channel will be denoted by λ and to $n+\alpha$ channel by ν. Then, $\Phi_\lambda(x_\lambda) = \Phi_d \Phi_t$ corresponds to the deuterium and tritium wave functions and $\Phi_\nu(x_\nu) = \Phi_{^4\text{He}}$ is the wave function of $^4\text{He} \equiv \alpha$. It should be noted that nuclei d, t and ^{4}He have no excited states, which significantly simplifies the analysis within the formulated approach.

In accordance with (5)—(7) the total Hamiltonian can be written as

$$H = (-\Delta_{y_a} + V^{\text{eff}}_a(y_a)) \otimes I + H'_a, \tag{8}$$

where

$$H'_a \equiv h_a \otimes I + \hat{V}(x_a, y_a) \tag{9}$$

$$h_a \equiv -\Delta_{x_a} + V^{\text{int}}_a(x_a). \tag{10}$$

The operators H and H_a' act in the space $\mathscr{H}$, $D(h_a)$ and $R(h_a) \subseteq L^2(\mathbf{R}^9_{x_a})$. According to the Hilbert bundle decomposition one gets

$$H_a' = \int \oplus H_a'(y_a)\, dy_a, \tag{11}$$

where $H_a'(y_a)$ are operators in the fibre $\mathscr{F}_{y_a} \sim L^2(\mathbf{R}^9_{x_a})$ and depend parametrically on the base point $y_a \in B^a$. The operators $H_a'(y_a)$ are selfadjoint in the region $D(H_a') = D(-\Delta_{x_a}) \in \mathscr{F}_{y_a}$, and consequently their eigenfunctions form an orthonormal basis in $\mathscr{F}_{y_a}$, necessary for expansion (3)

$$H_a'(y_a)\varphi_a^k(x_a, y_a) = \varepsilon_a^k(y_a)\varphi_a^k(x_a, y_a). \tag{12}$$

Here the eigenvalues $\varepsilon_a^k(y_a)$ define the energy curves in $\mathbf{R}^3_{y_a}$. It is to be noted that the presence of a continuous spectrum in decomposition (3) forces the continuous sum decomposition.

Substituting (3) into the Schrödinger equation (4) and taking into account (8)—(12) we get an effective system for the coefficients χ:

$$[-(\nabla_{y_a} + A_a)^2 + \operatorname{diag}\{\varepsilon_a^k(y_a)\} + (V_a^{\text{eff}} - E)]\chi(y_a) = 0, \tag{13}$$

where the operator A_a defines the connection form (associated with the Hilbert bundle construction) having matrix elements

$$A_{nm} \equiv \langle \nabla_{y_a} \varphi_a^m, \varphi_a^n \rangle \tag{14}$$

$\langle , \rangle$ being the scalar product in $L^2(\mathbf{R}^9_{x_a})$.

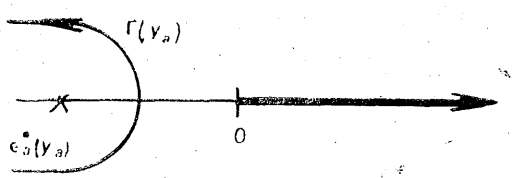
3. Effective equations

The investigation of the system of equations (13) encounters some serious computational difficulties and remains out of the scope of this paper. Therefore, we shall use a generalization of the Feshbach projection method, which allows one to separate the contributions of open and closed (with respect to the energy) channels and to find a more convenient form for the effective equations.

Separating in the spectrum the frame Hamiltonians ($H_a'(y_a)$, the lowest eigenvalue $\varepsilon_a^0(y_a)$ defines the spectral projector

$$P(y_a) = -\frac{1}{2\pi i} \oint \Gamma(y_a)(H_a(y_a) - z)^{-1}\, dz, \tag{15}$$

Fig. 2. The spectrum of the frame Hamiltonian
$H_a'(y_a) = -\Delta_{x_a} + V^{\text{int}}(x_a) + \hat{V}(x_a, y_a)$, $\varepsilon_a(y_a)$ is the eigenvalue of $H_a'(y_a)$



where the contour $\Gamma(y_a)$ is shown in fig. 2. The integration over the base $B^a=\mathbf{R}^3_{y_a}$ gives the operator

$$P_a=\int\oplus P_a(y_a)\,dy_a. \tag{16}$$

One can easily check that P_a is acting in $\mathscr{H}$ according to the rule

$$P_aF=\int F(X)\varphi_a^0(\tilde{x}_a,\ y_a)\,d\tilde{x}_a\varphi_a^0(x_a,\ y_a)=\langle F,\ \varphi_a^0\rangle\,\varphi_a^0. \tag{17}$$

Decomposing again $\mathscr{H}$ into direct sum

$$\mathscr{H}=P_a\mathscr{H}+Q\mathscr{H} \tag{18}$$

$$Q_a\equiv I-P_a \tag{19}$$

we represent the total wave function $\Psi(X)$ in the form

$$\Psi=P_a\Psi+Q_a\Psi=\langle\Psi,\ \varphi_a^0\rangle\,\varphi_a^0+Q_a\Psi. \tag{20}$$

The representation (20) corresponds to the separation of the contributions of open and closed channels in the energy region under consideration (thus, in both clusterizations $d+t$ and $n+\alpha$ only one channel is open in the considered energy region). In this case the adiabatic representation of the form (3) is carried out only on open channels and $\chi_a^0\equiv\langle\Psi,\ \varphi_a^0\rangle$.

Substituting (20) in Eq. (4) and projecting on $P_a\mathscr{H}$ and $Q_a\mathscr{H}$ we get the system

$$\begin{aligned}&\langle H(\varphi_a^0\chi_a^0),\ \varphi_a^0\rangle-E\chi_a^0-\langle HQ_a\Psi,\ \varphi_a^0\rangle=0\\&(Q_aHQ_a-E)Q_a\Psi+Q_aH(\varphi_a^0\chi_a^0)=0.\end{aligned} \tag{21}$$

Eliminating the $Q_a\Psi$ from the system (21) we obtain an equation for the function χ_a^0 describing the relative motion of clusters in clusterization a.

$$\langle H(\varphi_a^0\chi_a^0),\ \varphi_a^0\rangle+W_a(E+i0)\chi_a^0-E\chi_a^0=0 \tag{22}$$

$$W_a(E):\ \chi(y_a)\to\langle HQ_a(Q_aHQ_a-E)^{-1}Q_aH(\chi\varphi_a^0),\ \varphi_a^0\rangle, \tag{23}$$

i. e.

$$\varphi_a^0\,W_a(E)\,\chi(y_a)=P_aHQ_a(Q_aHQ_a-E)^{-1}Q_aHP_a(\chi\varphi_a^0). \tag{24}$$

Define the "connection" on the open and closed channels

$$A_{00}^a\equiv\langle\nabla_{y_a}\varphi_a^0,\ \varphi_a^0\rangle \tag{25}$$

$$(A_0^a)^2\equiv\langle Q_a\nabla_{y_a}\varphi_a^0,\ Q_a\nabla_{y_a}\varphi_a^0\rangle \tag{26}$$

and the covariant derivative

$$D_{0a}\equiv\nabla_{y_a}+A_{00}^a. \tag{27}$$

Then one can easily find that

$$\langle\nabla_{y_a}\varphi_a^0,\ \varphi_a^0\rangle=\nabla_{y_a}A_{00}^a+(A_{00}^a)^2-(A_0^a)^2. \tag{28}$$

Using the properties (12) of the frame Hamiltonian we have

$$\langle H_a'(y_a)\,\varphi_a^0,\ \varphi_a^0\rangle=\varepsilon_a^0(y_a), \tag{29}$$

and now one can rewrite Eq. (12) in the form that is analogous to (13):

$$\{-D_{0a}^2+\varepsilon_a^0(y_a)+V_a^{\text{eff}}(y_a)+(A_0^a)^2+W_a(E)-E\}\chi_a^0(y_a)=0. \tag{30}$$

The equation obtained (30) is equivalent to the original Schrödinger equation (4) since for its derivation we did not use simplifying assumptions. Therefore, Eq. (30) does not remove the difficulties of the many-body problem since the energy-dependent operator (23) $W_a(E)$ contains the many-body resolvent $Q_a(Q_aHQ_a-E)^{-1}Q_a$. However, this approach permits to single out explicitly the most important object for simulation — the energy dependent operator $W_a(E)$.

The operators (23) $W_a(E)$ have a very complicated form defined by the resolvent $Q_a(Q_aHQ_a-E)^{-1}Q_a$ in the subspace $Q_a\mathscr{H}$. A rigorous analysis of their analytical and asymptotic properties is a separate nontrivial problem and we shall restrict our considerations only to some general observations.

At large intercluster distances $|y_a|\to\infty$ the resolvent $G_a(E)\equiv Q_a(Q_aHQ_a-E)^{-1}Q_a$ can be represented in the form [18]

$$G_a(E)_{|y_a|\to\infty}\sim\oint_{\Gamma_a}(-\Delta_{x_a}+V^{\text{int}}(x_a)-z)^{-1}(-\Delta_{y_a}+V_a^{\text{eff}}(y_a)-E+z)^{-1}\,dz, \tag{31}$$

where the contour Γ_a is shown in Fig. 3. The analysis of the three-body problem performed in the above cited papers permits to assume the following, related to Eq. (31) asymptotic behaviour for $W_a(E)$:

$$(W_a(E)\chi_a)_{y_a\to\infty}\sim\Xi/|y_a|^4, \tag{32}$$

where Ξ plays the role of cluster polarizability.

The analytical properties of the operator W_a are determined by the analytical properties of $G_a(E)$. The former operatior is in general case a nonsymmetric one

$$\text{Im}\,W_a(E)=\frac{1}{2i}(W_a(E)-W_a^*(E))=-1/\varphi_a^0\,P_a\mathfrak{H}_a(E)HP_a(\varphi_a^0\cdot), \tag{33}$$

where $\mathfrak{H}_a(E)$ is the resolvent discontinuity on the continuous spectrum

$$\mathfrak{H}_a(E)=-1/(2i)\{G_a(E+i0)-G_a(E-i0)\}. \tag{34}$$

For the asymptotic of the resolvent kernel $G_a(X, X', E)$ we use the representation

$$-1/\pi\,G_a(X, X', E)_{|y_a|\to\infty}\sim 1/(2\pi)^6\int\Psi_0(X)\Psi_0^*(X')\,\delta(P^2-E)\,dP \tag{35}$$

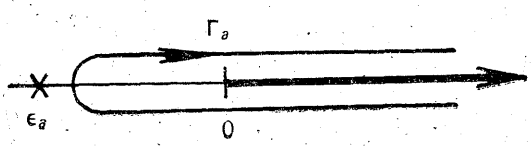
$$+1/(2\pi)^3\int\Psi_b(X)\,\Psi_b^*(X')\,\delta(q_b^2+\varepsilon_b-E)\,dq_b,$$

where Ψ_0 and Ψ_b are the wave functions of the total break-up channel and the $b\neq a$ channel, respectively; ε_b is the channel threshold. One sees that the imaginary part of $W_a(E)$ in the asymptotics $|y_a|\to\infty$ becomes nontrivial at $E\geq\varepsilon_b$, i. e. where another reaction channel is open.

4. Asymptotic boundary conditions

For the correct formulation of Eq. (30) for the relative motion of clusters $\chi_a^0(y_a)$ in clusterization a, asymptotic boundary conditions are needed. For the problem under consideration this procedure is very much simplified in comparison with the general situation since only two-cluster channels are open. The boundary conditions can be calculated from the assymptotic behaviour of the components of the

Fig. 3. Tue spectrum of the operator $-\Delta_{x_a}+V_a^{\text{int}}(x_a)$



Yakubovsky-Faddeev wave function Ψ (see [13, 19]). Since only two-cluster channels are open, besides the plane wave in the initial channel

$$\Psi_p \doteq \Phi_\lambda(x_\lambda) \exp\{ip_\lambda x_\lambda\} C_p, \tag{36}$$

where $\Phi_\lambda(x_\lambda) = \Phi_d \Phi_t$ is the state functions of nuclei d and t, p_λ is the momentum conjugate to y. (In the channel $d+t$: $a=\lambda$), the asymptotics $\Psi(X)$ (except in some special directions) has only the spherical waves in $\mathbf{R}^3$

$$J_a(y_a, E) = \exp\{i\sqrt{E-\varepsilon_a}\} |y_a|^{-1} . C_a;\ a=\lambda,\ \nu. \tag{37}$$

Here $\varepsilon_a = \lim\limits_{\substack{|y| \\ t \to \infty}} \varepsilon_a^0(y_a)$ is the channel threshold, $C_\nu = 1$, C_λ, C_p are the distorted Coulomb factors (see [19] and [13]).

Thus

$$\Psi(X, p_\lambda)_{|X|\to\infty} \tag{38}$$

$$\sim \Phi_\lambda(x_\lambda) e^{ip_\lambda y_\lambda} C_p + \sum F_{a\lambda}(\hat{y}_a, p_\lambda) e^{i\sqrt{E-\varepsilon_a}|y_a|} |y_a|^{-1} . C_a,$$

where $F_{a\lambda}(y_a, p_\lambda)$ are the relevant amplitudes. Since the basis $\{\varphi_a^k\}$ is orthonormal, one can get from (38) the asymptotic behaviour for $\chi_a^0(y_a)$, $|y_a| \to \infty$. Since $\varphi_a^0(x_a, y_a)$ is a function of the bound ground state of the operator $H_a'(y_a)$

$$\varphi_a^0(x_a, y_a)_{|y_a|\to\infty} \sim \Phi_a(x_a), \tag{39}$$

where Φ_a is the ground state function for the noninteracting clusters. Then,

$$\chi_a^0(y_a) = \langle \Psi, \varphi_a^0 \rangle_{|y_a|\to\infty} \sim \int C_p \Phi_\lambda(x_\lambda) e^{i(p_\lambda x_\lambda)} \overline{\Phi_a(x_a)}\, dx_a \tag{40}$$

$$+ \sum_{b=\lambda,\nu} \int F_{b\lambda}(\hat{y}_b, p_\lambda) C_b e^{i\sqrt{E-\varepsilon_b}|y_b|} |y_b|^{-1} \overline{\Phi_a(x_a)}\, dx_a$$

since Φ_a are bound states they are localized (see, e. g. [13, 19] in the region of small $|x_b|$ or in other terms:

$$\Phi_b(x_b)_{|x_b|\to\infty} \sim f^b \exp\{-\sqrt{|\varepsilon_b|}\, |x_b|\} |x_b|^{-1} (1+0(1)). \tag{41}$$

Therefore, the first term in (40) at $a=\nu$ does not contribute to the asymptotics with $|y_a| \to \infty$ since the integrand contains the product $\Phi_\lambda(x_\lambda)\overline{\Phi_\nu(x_\nu)}$. At $a=\lambda$ the exponent can be put of the integration sign. The same takes place in the second term at $a=b$ It remains to estimate the cross terms ($a \neq b$) in the second term in (40). The factor $\overline{\Phi_a(x_a)}$ has a support localized in the region of small $|x_a|$, which means that for $|y_a| \to \infty$ in the leading order of y_a the integrand can be replaced by the matrix C^{ba} that corresponds to the part of orthogonal matrix of the Jacobi coordinate transformation $y_b = C^{ab} y_a - S^{ab} x_a$. Thus in the leading order we have in the $n+\alpha$ channel

$$\chi_\nu^0(y_\nu)_{|y_\nu|\to\infty} \sim F_{\nu\lambda}(\hat{y}_\nu, p_\lambda) e^{i\sqrt{E-\varepsilon_\nu}|y_\nu|} |y_\nu|^{-1} . \int \overline{\Phi_\nu(x_\nu)}\, dx_\nu \tag{42}$$

$$+ F_{\lambda\lambda}(\hat{y}_\lambda, p_\lambda) C_\lambda e^{i\sqrt{E-\varepsilon_\lambda}|C^{\lambda\nu} y_\nu|} |C^{\lambda\nu} y_\nu|^{-1} \int \overline{\Phi_\nu(x_\nu)}\, dx_\nu$$

and in the $d+t$ channel

$$\chi_\lambda^0(y_\lambda)_{|y_\lambda|\to\infty} \sim C_p e^{i(p_\lambda y_\lambda)} + F_{\lambda\lambda}(\hat{y}_\lambda, p_\lambda) C_\lambda e^{i\sqrt{E-\varepsilon_\lambda}|y_\lambda|} . \tag{43}$$

$$|y_\lambda|^{-1} \int \overline{\Phi_\lambda(x_\lambda)}\, dx_\lambda + F_{\nu\lambda}(\hat{y}_\lambda, p_\lambda) e^{i\sqrt{E-\varepsilon_\nu}|C^{\nu\lambda} y_\lambda|} |C^{\nu\lambda} y_\lambda|^{-1} \int \overline{\Phi_\lambda(x_\lambda)}\, dx_\lambda.$$

The expressions (42) and (43) define the asymptotic boundary conditions for Eqs (30) for the function of the relative cluster-motion $\chi_a^0(y_a)$ in the elastic $(a=\lambda)$ and rearrangement $d+t\to n+\alpha$, $(a=\nu)$ channels.

After projection on the open channels one conserves only the term $\varepsilon_a^0(y_a)$ playing the role of an effective potential in Eq. (30). For this term one can easily calculate the asymptotics as $|y_a|\to\infty$. According to (12), $\varepsilon_a^0(y_a)$ is an eigenvalue of the operator $\hat{H_a}(y_a)=h_a+\hat{V}(x_a, y_a)$ at fixed y_a. Let us consider the operator $h_a=-\Delta_{x_a}+V_a^{\text{int}}(x_a)$ as an unperturbed Hamiltonian and the potential $V_a(x_a, y_a)$ defined by formulae (6) and (7) as a perturbation. In the $n+\alpha$ clusterisation the intercluster interaction is rapidly decreasing

$$V_\nu^{\text{ext}}(x_\nu, y_\nu=0(|y_\nu|^{-N}), \quad \forall N>0, \ |y_\nu|\to\infty,$$

and therefore, as $|y_\nu|\to\infty$ and

$$\hat{V}_\nu(x_\nu, y_\nu)=V_\nu^{\text{ext}}(x_\nu, y_\nu)-\int|\Phi_{^4\text{He}}(x_\nu)|^2V_\nu^{\text{ext}}(x_\nu, y_\nu)=o(|y_\nu|^{-N}), \quad N\in\mathbf{N}. \tag{44}$$

On the contrary, in the $d+t$ channel clusterisation the intercluster interaction contains the Coulomb term

$$V_\lambda^{\text{ext}}(x_\lambda, y_\lambda)=\mathbf{B}|x_{pp}|^{-1}+o(|y_\lambda|^{-N}), \quad \forall N>0, \ |y_\lambda|\to\infty,$$

where $\mathbf{B}$ is the reduced charge and x_{pp} is the reduced interproton coordinate. Using the Jacobi mass matrices of transformation, $\tilde{C}$ and $\tilde{S}$, one can express the vector $x_{pp}\in\mathbf{R}^3$ through x_λ, y_λ:

$$x_{pp}=\tilde{C}x_\lambda+\tilde{S}y_\lambda.$$

Then

$$V_\lambda^{\text{ext}}(x_\lambda, y_\lambda)_{|y_\lambda|\to\infty}\sim\mathbf{B}|y_\lambda|^{-1}\left|\tilde{C}\frac{x_\lambda}{|y_\lambda|}+\tilde{S}\hat{y}_\lambda\right|^{-1}.$$

In this case

$$\hat{V}_\lambda(x_\lambda, y_\lambda)_{|y_\lambda|\to\infty}\sim\mathbf{B}|y_\lambda|^{-1}\left\{\left|\tilde{C}\frac{x_\lambda}{|y_\lambda|}+\tilde{S}\hat{y}_\lambda\right|^{-1}-\int|\Phi_\lambda(x_\lambda)|^2\right. \tag{45}$$

$$\left.\left|\tilde{C}\frac{x_\lambda}{|y_\lambda|}+\tilde{S}\hat{y}_\lambda\right|^{-1}dx_\lambda\right\}\sim\mathbf{B}|y_\lambda|^{-1}\left\{\left|\tilde{C}\frac{x_\lambda}{|y_\lambda|}+\tilde{S}\hat{y}_\lambda\right|^{-1}-|\tilde{S}|^{-1}\right\}.$$

The latter step is justified by localization of $\Phi_\lambda(x_\lambda)$ in the region of small $|x_\lambda|$ and by the normalisation $\|\Phi_\lambda(x_\lambda)\|=1$.

Let us represent $\hat{V}_\lambda$ in the form $\hat{V}_\lambda=|y_\lambda|^{-1}V_\lambda$, where

$$V_\lambda\equiv|y_\lambda|\hat{V}_{\lambda|y_\lambda|\to\infty}\sim\mathbf{B}\left\{\left|\tilde{C}\frac{x_\lambda}{|y_\lambda|}+\tilde{S}y_\lambda\right|^{-1}-|\tilde{S}|^{-1}\right\} \tag{46}$$

$$\sim\mathbf{B}\langle y_\lambda, \tilde{C}x_\lambda\rangle/|y_\lambda|\tilde{S}|\tilde{S}|+0(|y_\lambda|^{-2})$$

and consider $|y_\lambda|^{-1}$ as a perturbation parameter for the operator h_a.

Then for the "energy term" $\varepsilon_\lambda(y_\lambda)$ as $|y_\lambda|\to\infty$ from perturbation theory one gets (see, e.g. [19])

$$\varepsilon_\lambda^0(y_\lambda)=\varepsilon_\lambda-\frac{1}{|y_\lambda|^4}\mathbf{B}^2|\tilde{S}|^2\int\frac{|V_{0k}|^2}{k^2-\varepsilon_\lambda}dk(1+o(1)), \tag{47}$$

where ε_λ is the $d+t$ channel threshold obtained from the eigenvalue problem

$h_\lambda \Phi_\lambda = \varepsilon_\lambda \Phi_\lambda$,

the coefficient $|V_{0k}|^2$ being defined by the relations

$$|V_{0k}|^2 = \left| \int \langle \hat{y}_\lambda, \tilde{C} x_\lambda \rangle \Phi_\lambda(x_\lambda) \Phi(k, x_\lambda) dx_\lambda \right|^2. \tag{48}$$

$\Phi(k, x_\lambda)$ denote the eigenfunctions of the continuous spectrum of the operator h_λ

$$(h_\lambda - k^2) \Phi(k, x_\lambda) = 0. \tag{49}$$

For the $n+\alpha$ channel, by virtue of (4), calculations analogous to (47) lead to the asymptotics as $|y_\nu| \to \infty$

$$\varepsilon_\nu^0(y_\nu) = \varepsilon_\nu + o(|y_\nu|^{-N}), \quad \forall N > 0, \tag{50}$$

where ε_ν is the $n+\alpha$ channel threshold.

5. Relation with the RGM

Now let us discuss the relation of the above-described scheme with the resonating group method (RGM). As it is known [15] the RGM utilises the following ansatz for the wave function

$$\Psi_{\mathrm{RGM}} = \mathscr{A}\{\Phi(x_a)\chi_a(y_a)\} \equiv \Phi(x_a)\chi(y_a) \sum_{\mathbf{P}=I} (-1)^P \mathbf{P}\{\Phi(x_a)\chi(y_a)\}, \tag{51}$$

where $\mathscr{A}$ is the antisymmetrization operator in all permutations of the particles $\mathbf{P}$ $(-1)^P$ is the permutation sign, I is the identity permutation and $\{x_a, y_a\}$ is a pair of multidimensional Jacobi coordinates introduced as in Section 1. For $\Phi(x_a)$ we choose the functions modelling the intercluster states. Substituting ansatz (51) into the Schrödinger equation and then projecting on $\Phi(x_a)$ we get the known RGM equation

$$\langle \Phi(x_a) | H - E | \mathscr{A}\{\Phi(x_a)\chi(y_a)\} \rangle = 0 \tag{52}$$

for the function of the relative motion of clusters $\chi(y_a)$.

Now we rewrite Eq. (52) in the form:

$$\{-\Delta_{y_a} + \langle V^{\mathrm{ext}}(x_a, y_a)\Phi(x_a), \Phi(x_a)\rangle \tag{53}$$

$$+ \langle [-\Delta_{x_a} + V^{\mathrm{int}}(x_a)]\Phi(x_a), \Phi(x_a)\rangle - E\}\chi(y_a) + \mathscr{D} = 0,$$

where $V^{\mathrm{int}}(x_a)$ is the interaction between the particles of one cluster, $V^{\mathrm{ext}}(x_a, y_a)$ is the interaction between particles from different clusters $V^{\mathrm{ext}} + V^{\mathrm{int}} = V$, and

$$\mathscr{D} = \langle (H-E) \sum (-1)^{\mathrm{P}} \mathbf{P}\{\Phi(x_a)\chi(y_a)\}, \Phi(x_a)\rangle$$

are the exchange terms generated by the antisymmetrisation operator $\mathscr{A}$. Then if ε_a is the sum of binding energies of non-interacting clusters (channel threshold) corresponding to the state function $\Phi(x_a)$, the following formulae hold

$$[-\Delta_{x_a} + V^{\mathrm{int}}(x_a)]\Phi(x_a) = \varepsilon_a \Phi(x_a) \Rightarrow \langle [-\Delta_{x_a} + V^{\mathrm{int}}(x_a)]\Phi(x_a), \Phi(x_a)\rangle = \varepsilon_a$$

$\langle V^{\mathrm{ext}}(x_a, y_a)\Phi(x_a)\rangle = V^{\mathrm{eff}}(y_a)$ (according to (6))

$V^{\mathrm{eff}}(y_a) + \varepsilon_a = V^{\mathrm{D}}$ being the direct RGM potential.

This enables one to rewrite Eq. (53) in the form:

$$[-\Delta_{y_a}+V^{\text{eff}}(y_a)+\varepsilon_a-E)\chi(y_a)+\mathcal{D}=0. \quad (54)$$

Hence, it is seen that with accuracy up to the exchange terms $\mathcal{D}$, the RGM equation (54) arises from the effective equation of the adiabatic approach (30) after the following simplifications:

i) neglecting the connection A^a_{00} in the covariant derivative D_{0a};

ii) taking for $\varepsilon^0_a(y_a)$ the limiting value ε_a (for $|y_a|\to\infty$) and the corresponding change of $\varphi^0_a(x_a, y_a)$ by $\Phi_a(x_a)$;

iii) neglecting the effect of closed channels appearing as an energy dependent potential $W_a(E)$ and the presence of connection A^0_Q.

The absence of exchange terms in Eq. (30) is caused by the fact that the representation (20) for the total wave function used in the adiabatic approach is not an ansatz solution of the many-body Schrödinger equation but an exact solution. Thus, Eq. (30) is equivalent to the original Schrödinger equation (4). Therefore, one should not beforehand antisymmetrise the wave function Ψ with respect to permutation of nucleons since the total Hamiltonian is always invariant with respect to these permutations. Thus, antisymmetrisation can be performed already after the solution of the Schrödinger equation. This means that all exchange effects are "hidden" in the operator $W_a(E)+(A^a_Q)^2$.

6. Conclusions

Extracting from the experimental data (see [3]) on $d+t\to{}^4\text{He}+n$ reaction the parameters of the energy-dependent model potential $W(E)$ (see [11]) the approach developed here can be used in the muon-catalysed three-channel model to calculate the sticking coefficient ω_s in the concrete formulation of the Faddeev equations for the final $\mu+{}^4\text{He}+n$ channel. The results of these calculations will be published elsewhere.

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