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WORKING PAPER

A. Loukianova, D. Nurullaev

EVALUATION OF REAL OPTIONS PORTFOLIO FOR INVESTMENT PROJECTS

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Abstract: The goal of this paper is to offer recommendations for evaluating the portfolio of real options in investment projects. The study reviews the real options methodology, analyses real options valuation models, and formulates methodology for valuing real options portfolio.
The proposed approach was tested using the example of a company that produces shale oil. As a result, the mechanism for assessing the portfolio of real options and their application was offered to managers who are engaged in the evaluation of investment projects.

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INTRODUCTION

Contemporary literature considers many approaches to investments valuation. Traditional ones (based on discounted cash flows approach) are often criticized for their drawbacks that do not allow managers to consider all possible risks within an investment and to manage them. To respond to the needs of management to make investment projects valuation more flexible, the real options theory was created as an extension of the financial options theory.

While traditional views claim that managerial decisions are limited in the face of uncertainty and thus organizational inertia dominates, real options theory insists that companies can deal with unpredictable future and may benefit by applying option way of thinking and managing investments under changing conditions (Kogut, 1991; Dixit & Pindyck, 1994).

Real options theory can be defined as a systematic approach that uses different scientific approaches and types of analysis (economic, management, statistical, decision-making, etc.) to apply options theory to valuing real assets, as opposite to financial assets, in uncertain business reality (Mun, 2014).

The main advantage of the real options approach is that it makes decision-making process more flexible and allows to change some aspects of an investment project according to changing environment. However, this approach has some issues connected with its practical implementation. According to financial options theory, investor assesses an option with one underlying asset. Apparently, the world of real investments is different, and some projects can include several real options simultaneously. Another problem is that in the portfolio of investments the correlation of underlying assets is also possible that means that the overall value of a project may not be equal to a simple sum of options.

As a response to these problems, new studies appeared, trying to propose a solution to the issue of options interrelation. One of the pioneers in this area was L. Trigeorgis who wrote several papers devoted to the problem of real options portfolio within the investment projects (Trigeorgis, 1988; 1991; 1993a; 1993b; 2012). The other researchers also tried to establish the approach to make real options theory more applicable to practical cases. They offered some solutions and methodology how to deal with the problem of interdependencies. For example, the scientific work of R. Brosch (2008) made a lot of contribution to the investigation of the problem of the real options portfolio valuation. He accumulated the results of the main previous researches, created the methodology for portfolio valuation, and provided some numerical analysis by applying the methodology proposed.

To sum up, it can be stated that the valuation of real options portfolio is very important topic within real options theory due to its practical applicability to real business cases.

In this paper, the real options analysis (henceforth ROA) is considered from the portfolio point of view. The paper considers relevant articles about this approach and offers the methodology that is based on previous researches and has some practical applications based on simulated or real-life cases. The idea of the paper is to give recommendations to managers that are responsible for investment decisions about some tools that would help them to make valuation process more effective and clear. Thus, formally the *goal of the current paper* is to propose recommendations to managers concerning the valuation of the real options portfolio of investment projects.

The *research hypothesis* is that if the options interact in the project, the value of a portfolio cannot be the sum of individual values of these options. Instead, some synergy effect is obtained, positive or negative.

The paper is organized as follows. The *first section* is devoted to the literature review of the topic. The portfolio aspect of real options is considered with a lot of attention being paid to the problem of interdependencies between options and underlying assets.

The *second section* is devoted to valuation model of real options portfolio. In the first step real options valuation models are considered. The next step is an extension of simple real options valuation methods to the portfolio of real options that have correlations within one project. Finally, in the third step differences between portfolio approach and simple ROA are analyzed and the main advantages of the portfolio approach are emphasized.

The *third section* considers several cases in the context of real options portfolio. In each case the real options portfolio creation is explained. Then the project is valued using the methodology from the second section and at the final stage the value of the project using portfolio approach is compared to the one delivered using simple additivity method. As a result, the formulated approach to portfolio of real options creation and valuation is proposed.

PORTFOLIO APPROACH TO REAL OPTIONS

Definition of Real Option Portfolio

To introduce the portfolio approach of real options theory, it is necessary to define the term of portfolio. Simply defined, portfolio is “a group of investments” (Farlex Financial Dictionary, 2012) or “a particular combination of assets in question” (Neftci, 2015). So, portfolio is the object that consists of several elements that create a portfolio. In terms of real options theory, these assets are real assets and hence real options are written on these assets.

In real options models L. Trigeorgis showed that real options on the same underlying assets can interact, requiring a simultaneous valuation of all options written on the same underlying asset (Brosch, 2008). For example, there are two financial European call options on two different stocks with no correlation. The combined value of these options is a sum of options on each of these stocks. Another example shows what interaction of options means. There is a plant for which an investor has two options: either to defer further investments in the project up to time t or to execute European option to abandon the project in time $t+n$. In terms of ROA, these options correspond to each other because the decision can be made subsequently and hence the value of option to abandon increases because there is an opportunity to defer and see what will happen in the business environment.

So, we can see the obvious interaction between these two options. The combined value of these options will be higher than their separate values, because in isolation the deferral option does not necessarily give opportunity to abandon the project if business conditions are bad, while combining deferring and abandonment options increase time horizon for put European option (abandonment). Only in combination synergy effect from these two options can be obtained.

Since both effects occur simultaneously, it is not possible to value “deferral” and “abandonment” options in isolation. The decision about exercising the first option requires to explicitly take into account the existence of the subsequent option. This relationship is structurally akin to the valuation of compound options (Gesken, 1979) Hence, the arising effect can conveniently be labelled as compoundness. Specifically, L. Trigeorgis (1993a) defines interactions between real options written on the same underlying asset as “intra-project compoundness”. Following the same logic, an analogous effect is identified for several, interdependent underlying assets which he denotes as “inter-project compoundness” (Trigeorgis, 2012, p. 132). Both inter–project and intra–project compoundness must be considered in the context of portfolios of real options. (Brosch, 2008).

Real options as well as assets can be the objects of constraints. Real assets can have financial or operating or other types of interactions. As a result, some assets can be mutually exclusive or complementary ones. This affects the possibility of joint exercise of bundles of real options on different underlying assets. Likewise, the existence of constrained resources, e.g., funds available, influences the feasibility of joint option exercise (Brosch, 2008).

Therefore, portfolios of real options here are defined as combinations of multiple risky assets and multiple real options, written on these assets, that are subject to constraints. Cases with only one underlying asset, or one real option, are special portfolio cases that reduce the scope of portfolio analysis dramatically. In order to seize all possible portfolio effects, it is important to analyze multiple underlying assets with multiple real options simultaneously. The usual "laboratory" setting for real options analysis with one underlying asset and one real option does not provide a framework that is capable of handling realistic decision problems. Thus, it is prone to ignore key portfolio effects with possibly substantial impact on (optimal) option exercise (Brosch, 2008).

To sum up, we can conclude that a portfolio of real options is a complicated phenomenon. Its valuation requires not only the assessment of interacting real options within one option, but also simultaneous valuation of a number of underlying assets with several real options in each asset.

Portfolio approach

A suitable approach for portfolio analysis must include all main factors that can influence the value of a project. From the definition of portfolios of real options it follows directly that portfolio aspects can be attributed to the real assets involved, the real options involved, or constraints. Budget constraints are of special importance because they can have a considerable limiting impact and require a detailed modeling of the investment dynamics. Moreover, the ensuing budget levels over time are state– and path–dependent (Brosch, 2008). Based on these considerations, R. Brosch (2001) categorizes portfolio aspects as follows:

- *interactions at the real options level*. At this level, he defines intra-project compoundness and inter-project compoundness. The first one is about correlations and interdependencies of many real options written on one underlying asset. The last one differs from the first one by taking into account correlations and interdependencies of several real options and several real assets simultaneously. For inter–project compoundness, the correlation between the underlying assets has to be modeled explicitly.
- *interactions at the real asset level*. At the assets level, direct and indirect qualitative interactions are singled out. Direct interactions are those which are inseparably connected to the underlying real assets, such as physical properties or operating synergies. Indirect interactions have their origin outside the strict asset level and are due to constraints, most prominently budget constraints. Both are qualitative in that they are not merely stochastic in nature, but result from

the properties of projects or the specific background of the company (that would be different for another company).

As was mentioned, all interactions take place at one moment of time and, as a result, the separation of value impact of one single aspect is not possible to assess in isolation. All effects must be valued only jointly in order to include synergy effect of combination of real assets and real options within the project. It is the reason why simultaneous modeling approach is required, such as compound option pricing. Also, to handle all interactions, it is important to include all available constraints, such as budget or time, to the simultaneous modeling.

This complex approach allows to formulate the model as one stochastic optimization problem subject to constraints. Interactions are captured through the interplay of constraints as well as state– and path–dependency of investment decisions and cash flows (Brosch, 2008).

Interdependencies of options and assets

Peirson and Bird (1981) prove that interdependence of assets in the portfolio cannot be assessed as two independent ones. They claim that for better quality of analysis deeper research is required. Betge (1995) offers the explanation why some assets can be interdependent. He offers the following classification:

- *direct qualitative interactions*. It means that physical properties of investments cause qualitative interaction between two investment assets. They can be mutually exclusive (e.g. build office or residence in a land), complementary (e.g. build a plant and canalization system) or synergetic (build a residence and a school).
- *indirect qualitative interactions*. This results from constraints included in investment plan. The relationship is indirect because they are not inseparably connected to the investment opportunities considered; at the same time they are qualitative because they do not result from stochastic relationships and cannot be avoided by diversification. These constraints are connected to the firm making investment decisions. Most importantly, indirect qualitative interactions stem from binding capital constraints, i.e., capital rationing.

Meier et al. (2011) discuss a problem of real options portfolio as subject to investments and offers two models for real options portfolios. The first model reflects a standard maximizing value problem. They calculate the value of portfolio as the sum of values of options in the portfolio. Authors only replace net present value (henceforth NPV) of the projects by options values and do not offer something new that could be different from the traditional approach. In the model there is no interdependence between real options and hence model is stated as static. In the second model they use dynamic formulation of a problem. In this model investment decisions take place over time in binominal framework. All projects are related to the same underlying stochastic variable but are independent from one another. Further, only call options, which can be postponed indefinitely, are considered. Once an option is exercised, there are no further options available. Therefore, the options are stand-alone and do not interact. So, due to considering static budget constraints and not taking into account the interdependence between options, their paper does not manage to provide a feasible approach to value the portfolio of interacting options.

The direct qualitative interaction between assets makes a simple additive method inapplicable to the calculation of the value of portfolio. Instead, synergetic effect should be considered. It comes from underlying mean-variance portfolio concept based on systematic risk. NPVs add up if (and only if) the only relevant portfolio aspect is diversification and markets are perfect and complete. Other interdependencies, e.g. due to budget constraints, can cause deviations from additivity (Brosch, 2008).

To sum up, we can conclude that the source and type of interaction matters not only for underlying assets, but also for options. Simple additive method of NPVs violates the real value of a portfolio due to ignoring mathematically significant mean-variance approach in valuing portfolios. The interaction models are actively used in financial options theory and they must be translated to the real options approach.

Real Options Analysis in a Portfolio Context

Many authors state that portfolio of interacting real options must be modeled explicitly. Following this argument, several experts in real options theory made deeper research into the nature of real options. Thus, Trigeorgis (2012) defined two types of options interactions. The first one (when several options are interacting based on the same underlying asset) is called “intra-project” interaction. The second way is interaction of several underlying assets is called “inter-project” interaction.

Additionally, R&D investments are studied by many authors with a lot of attention being paid to options interaction. Due to the exploratory nature of R&D projects which are typically multi–stage, early projects can generate insights about future projects. Moreover, in an R&D pipeline many projects are typically undertaken, but only few make it to a marketable product, so the bundle of projects must be assessed from a portfolio point of view (Brosch, 2008). Childs et al. (1998) considered two mutually exclusive projects, both of which run simultaneously but at the final stage only one could be implemented. Their main conclusion was the idea that in highly correlated short-term projects with low volatility and large capital requirements, sequential development is desirable. Similarly, Childs and Triantis (1999) as well as Lint and Pennings (2002) analyzed the parallel development of two R&D projects that are mutually exclusive. Denardo et al. (2004) proposed a stochastic search algorithm for R&D projects with a sequential compound decision process. Gustafsson and Salo (2005) also focus on project selection subject to budget constraints, which is achieved by explicitly modeling the decision maker's terminal utility (Brosch, 2008).

There is some motivation to the fact why so many research papers are about R&D projects. These types of projects are very risky, hence managers need some hedging flexible strategies to avoid so huge uncertainty. Huchzermeier and Loch (2001) in their research modeled a compound R&D decision problem where there is an option for management to take corrective action as a means of managing the risk involved in R&D projects which may stem from different sources (e.g., market payoffs and project schedules). Going deeper into this topic, Vassolo et al. (2004) revealed that in the biotechnology sector mutually competitive, correlated projects tend to be sub–additive, but that the sharing of resources among firms may create positive spill–overs resulting in super–additivity. Chien (2002) and Kavadias and Loch (2004) also analyzed some specific R&D investments from the standpoint of inter-project correlation. Later, Smith and Thompson (2005) showed that for a risk–averse investor investing into highly correlated projects can be more desirable. To additionally testify to that, Dias (2006) confirmed that positive correlations cause learning and synergy effects for a company. This is opposed to financial options where investors seek for diversification and avoid high correlated securities (Brosch, 2008).

There are a number of other models that analyze different project's interactions. For example, Brown and Davis created an investment model in which a first project can be followed up by one of two mutually exclusive projects. Following the framework designed by Trigeorgis (1993a), Rose (1998) and Bowe and Lee (2004) analyzed and made recommendations for intra–project interactions of options within infrastructure projects. Triantis and Hodder (1990) in their paper considered very popular flexible production system option of switching the production mix among two products over time. Kogut and Kulatilaka (1994) and Huchzermeier and Cohen (1996) made a significant contribution to real options interaction theory by analyzing a global manufacturing network under exchange rate risk. The idea was that there is a switching option between different manufacturing strategies contingent on exchange rate realizations that decreased the level of risk for companies from the manufacturing industries dramatically. Wang and de Neufville (2004) value options inherent in the design of large physical systems, such as hydropower stations, modeled as path–dependent options. They created a model to value options using a stochastic mixed–integer program that can be implemented on commercially available optimization platforms. This idea is a great development in applying theoretical basics of real options interactions to practical cases. Martzoukos et al. (2003) model path–dependent investment problems by considering stochastic switching costs. The goal of this research was to propose the algorithm that could keep track of all possible paths as well as potential early option exercises on each path. Kamrad and Siddique (2004) considered the interactions problem from supply chain management standpoint. They conducted the analysis of the interactions between producers' and suppliers' investment decisions. For example, the research included such important options as order–quantity flexibility, supplier–switching, profit sharing, and supplier reaction options. The source of uncertainty were discretized exchange rate processes. The authors offered solution to the problem using numerical approach through a backward recursion in dynamic programming (Brosch, 2008).

All aforementioned papers analyzed many complex practical and theoretical problems in the aspect of real options interactions, either intra–project or inter–project. The drawback of corresponding articles is that they did not take into account an explicit portfolio perspective on real options. The most notable exception is the approach by Luehrman (1998), however he considered the portfolio perspective from qualitative point of view only. He chose a gardening metaphor where the firm is behaving like a gardener who only picks tomatoes (= exercise options) which are "ripe and perfect" (= at the optimal time). He defined food and bad gardeners like active and passive ones. Active gardener is informed and knows which tomatoes to pick, which ones to leave yet a little while, and which ones to pick even if they are not yet fully ripe, in order to prevent the squirrels (=the competitors) from stealing them. The bad gardener acts in a different way and, as a result, suffers a lot of losses. This metaphoric presentation explicitly discovers the nature of real options portfolio, but does not consider any quantitative framework that could value the options interactions within the project or investment strategy (R. Brosch, 2008).

In the earlier paper of Trigeorgis (1988) and Trigeorgis and Kasanen (1991) portfolio perspective was considered more explicitly. The authors analyzed compound synergetic effects from parallel projects. Additionally, growth options in inter-projects relations were considered by Kester (1984, 1993). At the same time, while Kasanen and Trigeorgis (1993) and Kasanen (1993) put more emphasis on modeling synergies, Mauer and Ott (1995) analyze replacement decisions as follow–up projects (Brosch, 2008).

Management of Portfolios of Real Options

Real options theory states that optimal management decisions must be taken simultaneously, considering all relevant portfolio aspects, to maximize a project's value. R. Brosch (2008) distinguishes two dimensions of managing portfolios of real options:

- *Portfolio design*. Optimal future exercising is supposed, so managers create the maximum value portfolio with this assumption
- *Portfolio execution*. Real options must be exercised equally in order to obtain full value of options

Moreover, portfolio design analyzes which assets and which options should be included in the portfolio of real options. Decision is based on the analysis of elements of portfolio and of total value of different portfolios. The portfolio with a maximum yield should be included to a project. Damisch (2002) also supposes that optimal value of a portfolio can be obtained not only by including new underlying assets or options, but also by changing assumptions about available underlying assets and embedded options (for example, by modifying volatility included or time of exploration).

To sum up, the main idea is that the optimal portfolio management is more about execution of existing design of a portfolio of real options. Assumptions about portfolio theory are based on the idea that options will be exercised in optimal point of time. So, we can conclude that the main challenge for management is to define this optimal time. The information about this point of time should be included to the value of the portfolio in a valuation process, otherwise the real value of portfolio can be violated. Thus, the proposed model gives clear management recommendations about which options should be exercised, and when, suggesting to exercise real options as in the optimal policy (Brosch, 2008).

Generally, all main aspects of future portfolio execution are already included in the portfolio design because managers are supposed to use optimally created structure in future. On the other hand, during portfolio execution unpredicted new information can be revealed that opens new alternatives for the portfolio design. So, this proves again that manager should not strictly stick to one chosen strategy.

RESEARCH METHODOLOGY

General assumptions and features of the model

There are many questions about practical applicability of real options theory. As mentioned before, the idea works well for financial assets, but has some difficulties with real assets. The idea is that the model assumptions should be consistent with the concept of financial assets and financial options. The assumptions for real options portfolio are the same as for single real options (Trigeorgis, 2012; Copeland and Antikarov, 2001). They are the following (Brosch, 2008):

- Capital markets are perfect and complete. In other words, a spanning portfolio is available for each investment opportunity considered.
- The risk–free interest rate is known for all markets. This rate is constant and does not depend on maturity.
- The values of the underlying assets follow discrete binomial processes.

- There are no any dividends on underlying assets.
- Investments are at least in part irreversible, unless corresponding real options exist.
- The decision maker has discretionary decision rights that can be interpreted as real options.
- The universe of all available options can be specified exhaustively.
- Options are proprietary options. There are no agency conflicts.
- All input parameters necessary for the purpose of option valuation are unambiguous and known.

The next step in specifying the model is to define the objective function. Generally, the main goal of a company is to maximize shareholders value, and hence, market value of equity (Copeland et al. 2015). For the problem considered, the objective function can be stated as follows: to maximize the value of portfolio of real options of the project with one underlying asset and several real options.

The portfolio value is calculated as the expected value of the optimal exercise policy of the portfolio of real options. This optimized strategy of exercises corresponds to the investment program in terms of traditional capital budgeting theory (Brosch, 2008).

For any investment project managers can choose the most appropriate mode for specific situation that maximizes project value. In theory the most representative modes for any model are:

- Mode 1. Money for project are not invested
- Mode 2. Money invested in relatively low production capacity to maximum possible one
- Mode 3. Money invested in relatively high production capacity to maximum possible one

So, we have several possible switches between modes:

- 1→1: Investor does not invest to project and stays with zero capacity
- 1→2: Invests into low capacity
- 2→1: Invests into high capacity
- 2→3: Switches from low capacity to high capacity
- 3→2: Decreases production capacity
- 2→1: Closes down the production from low capacity
- 3→1: Closes down the production from high capacity

In other words, the model also assumes that the process can be restated. For example, after closing down a project, the money can be reinvested to rerun the project with high or low capacity. This approach adds more flexibility to managers and reflects real life business cases. It is important to mention that calculations can include additional switching costs that can be either positive or negative (Brosch, 2008).

It is generally presumed that an underlying asset follows stochastic diffusion processes. The idea is that each period the price θ of underlying asset moves up or down with probability *p* and *1-p* respectively.

For valuation purposes we use log-transformed version of binominal numerical analysis described in Trigeorgis (1991). Following standard practice in the real options literature, the gross project value (V_t) is assumed to follow a standard diffusion Wiener process described by the following formula:

$$\frac{dV}{V} = (\alpha - \delta)dt + \sigma dz \qquad (1)$$

where α is actual expected return from a project; δ is return rate shortfall from the equilibrium return of a similar-risk traded financial asset; σ is standard deviation of project value; dz is standard Wiener process. (Trigeorgis, 1993a).

The valuation process looks very similar to the theory of financial options, especially when we try to estimate a price for European call-option. At the same time, valuation of multiple real options on one underlying asset has some specifics that comes from the nature of real options. Trigeorgis (1993a) and Kulatilaka (1995) showed that real options written on the same underlying asset actually interact and simple additivity does not hold anymore. So, valuation of projects with multiple interacting options must be conducted as the whole process of valuing the bundle of real options and underlying asset.

The rationale behind this idea is follows. Real options written on the same underlying asset are connected through this asset. Hence, if any option is exercised, the underlying asset as well as other options are affected. The simplest example of option to abandon proves that. If investor abandons the project, all subsequent options are foregone.

There are more examples when exercising real options affects the value of the whole project. For example, put exercise decision has to take the existence of subsequent options into account because these have a strictly positive value which is forfeited (Brosch, 2008). At the same time, the value of subsequent options is affected by the possibility that the put may be exercised. This complicated nature of interactions demands a new approach, different from traditional ones, like plain–vanilla financial option pricing (Trigeorgis 1991, 1993a; Brosch 2001). Trigeorgis (1993a) uses the model of numerical analysis of options interactions. Through explicit examples he proves that the value of interacting options is not additive, and interactions typically have negative sign. Also, he explains that interactions nature depends on several factors, like type, separation, degree of being in the money, and order of real options. This analysis finally shows that some usual options properties are preserved, e.g., sensitivity to time to maturity or volatility.

In the further paragraph there will be showed the intra-project model of valuation of interacting real options. The model is based on previous researches of Trigeorgis (1993a) and R. Brosch (2008).

Binominal Option pricing for one underlying asset

Real options valuation model

The value of option is determined by discounting certainty-equivalent or risk-neutral expectations of future payoff at a given risk-free interest rate, *r*. Generally speaking, the price for any asset can be found by replacing the actual growth rate, α, with certainty-equivalent rate in the following formula:

$$\hat{\alpha} = \alpha - RP, \qquad (2)$$

where RP is an appropriate risk premium, behaving as if the world were risk neutral. (Hull 2014).

According to the theory RP = Sσ, where S = (α-r)/σ is the asset's market price of risk or reward-to-variability ratio. Given that α=α*- δ, then α-RP= (α*- RP)-δ=r-δ. This is equivalent to a risk-neutral valuation, where the actual drift (α) would be replaced by the risk-neutral equivalent drift, $\hat{\alpha}$=r-δ. For traded assets (in equilibrium) or for those real assets with no systematic risk (e.g., R&D projects, oil exploration, etc), α = r or δ = 0.

Stochastic Processes

The stochastic process of the underlying assets is assumed as discrete binominal process. The model specification is based on the main assumptions of financial options theory. Firstly, the distribution of the stock price converges to the log-normal distribution in the limit, when Δt → 0 (Kwok, 2014, pp. 199 ff.). Secondly, geometric Brownian motions are widely used (Duffie, 2001, p. 88). On the other hand, for model specifications, it is not necessary to converge to a log -normal distribution of the underlying assets (Brosch, 2008).

The simplest and way to present binominal model is to use a standard version of binomial lattice trees. Horizontal dimension reflects time, while the vertical one represents up-down movements of an underlying asset. For the purposes of the model, new conventions are introduced. All variables are extended first by time *t (t=1, 2, …, T),* then by movement scenario *s* *(*s_t *=1, 2, …, S(t)),* and then by actions (if necessary). So, the combination of time moment, scenario, and action (if included) are defined as "system state space". (Brosch, 2008). For example, the value of option in time 3, for scenario up→ up is V(3,1). More examples are in the figure below (Fig. 1).

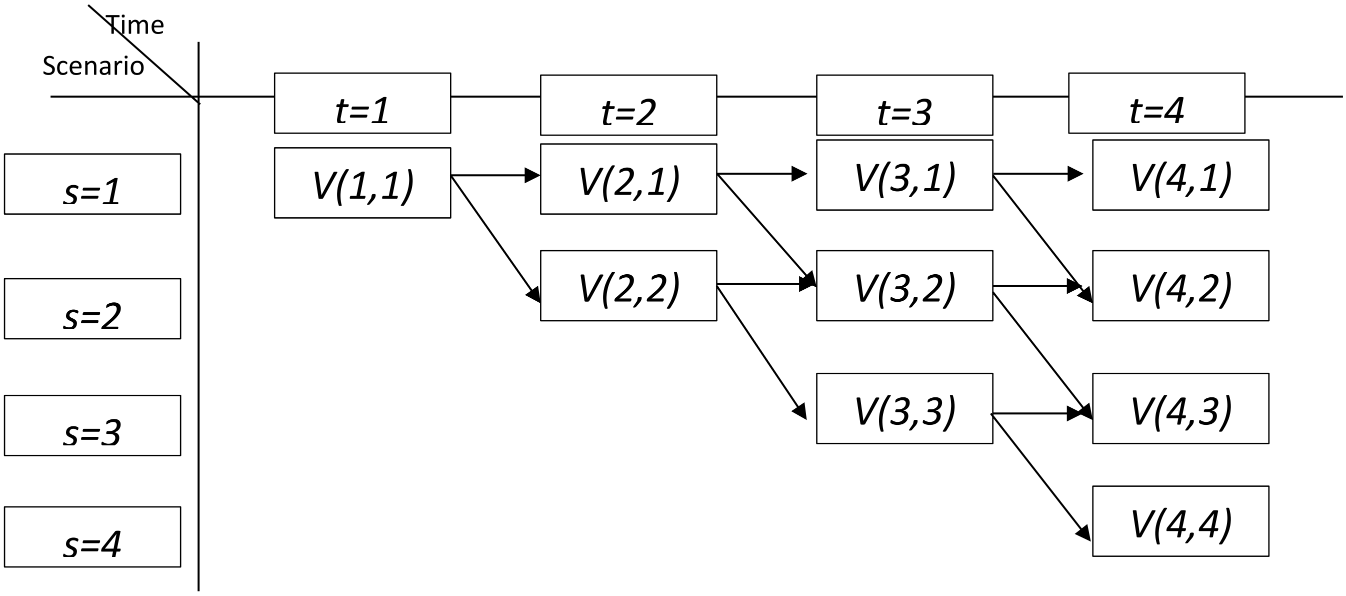

Figure 1. Binominal lattice tree example. (Brosch, 2008)

Binominal Option Pricing Model

The Cox-Ross-Rubinstein (CRR) and R. Brosch models are used to present the basic binominal option pricing model. The notation come s from CRR derivation model, and model specifications and determinations come from R. Brosch (2008) Portfolios of real options analysis. Also, the model developed by R. Brosch is referred to CRR derivations in order to introduce the notation while the content of the CRR model is unchanged.

The binominal model based on several basic assumptions and notations:

- θ is value if underlying asset. The value θ follows a stationary time-discrete multiplicative binominal process;
- There are two factor of underlying asset movement: *u* (up) and *d* (down);
- Factors Up and Down are interrelated such that $u \times d = 1$;
- σ is the volatility of the underlying asset specified by Up and Down factors;

In the limit the process converges to a log–normal distribution of the return of the underlying asset:

$$u = e^{\sigma\sqrt{\Delta t}}\,, and\ d = \frac{1}{u} \tag{3}$$

So, determined Up and Down factors are reasons for the stochastic process of the value of underlying asset, θ. So, based on the table example from previous paragraph, the system state space grows *in t (t=1, 2, …, T)* and *s* (s_t *=1, 2, …, S(t))* as follows in more general term:

$$\theta(t,s) = \theta(1,1) \times u^{t-s} \times d^{s-1} \tag{4}$$

The important part for the model is determining risk-neutral probabilities p and *1-p*. According to ROA theory, the Up scenario appears with probability p and the Down scenario with probability *1-p*. The risk-free rate is calculated as continuously compounded risk-free rate of return:

$$p = \frac{e^{r_f \Delta t} - d}{u - d} \tag{5}$$

Now, the basis for valuing an option with one underlying asset exemplified as European option can be obtained as follows. First, the boundary condition on the option exercise at t = T is imposed. For example, let's assume European Call with exercise price X, the terminal value condition will be defined as follows:

$$V(T,s) = [\theta(T,s) - X, 0], \qquad where\ s = 1,2,\dots,T \tag{6}$$

Finally, the model for binominal option pricing proceeds in backward recursion, discounting probability weighted expected values to the first period when V (1,1):

$$V(t-1,s) = \frac{V(t,s) \times p + V(t,s+1) \times (1-p)}{e^{r_f \Delta t}}, \tag{7}$$

$$where\ t = 2,\dots,T; s = 1,\dots,t$$

Now, we have the final version for option pricing with all specifications and the model to expand these model for valuing the portfolio of real options written on one underlying asset.

Real options portfolio valuation model.

The algorithm for portfolio valuation is based on the general framework developed by Kulatilaka (1988) and Kulatilaka and Trigeorgis (1994). At any moment, the project can either continue operating in the current mode for one more period, receiving a short–term payoff (i.e., current cash flow) plus the long–term option value from optimal future switching, or switch immediately to a new mode by incurring specified switching costs (Brosch, 2008).

For each combination of time and state (t, s), and an underlying asset i, there is an entering mode $a_i(t,s) = 1, 2, \dots, A_i(t,s)\ \forall\ (t,s)$ and a leaving mode $a'_i(t,s) = 1, 2, \dots, A'_i(t,s)\ \forall\ (t,s)$. So, in any moment of time, a manager can take decision about switching from mode $a_i(t,s)$ to $a'_i(t,s)$, or stay the same mode so that $a_i(t,s) = a'_i(t,s)$.

So, we have considered before three modes, and let's introduce them as mode specifications for the model for one underlying asset.

- $a' = 1: waiting\ to\ invest\ (do\ not\ invest\ in\ moment\ of\ time)$
- $a' = 2: invest\ (or\ continue)\ to\ operate\ in\ low\ capacity$
- $a' = 3: invest\ (or\ continue)\ to\ operate\ in\ high\ capacity$

Each switch is associated with a cash flow realization $c(\theta(t,s),a')$. So, in each period cash flow realized from a project depend on the leaving operating mode a'. The amount of cash flows can be any value of function of underlying asset $\theta(t,s)$. The cash flow function can be represented by a positive scalar, drawing on an analogy to levels of capacity:

- $c(\theta(t,s),a') = 0: no\ investments$
- $0 < c(\theta(t,s),a') < 1: invested\ in\ low - capacity$
- $c(\theta(t,s),a') = 1: continue\ with\ low\ capacity$
- $c(\theta(t,s),a') > 1: invested\ in\ high\ capacity$

To make the model more realistic, the switching costs should also be considered as $I(a,a')\forall a,a'$. These costs represent additional investments the company must incur to change the mode, or in terms of ROA it is the exercise price. So, to be consistent with previous assumptions, we assume that not changing the mode does not cause any costs. All other possible variations of investments for switching are presented below:

- $I(a,a') = \infty$: switching is not possible (applicable for European options, when switching is available only for one determined point of time)
- $I(a,a') = 0$: The mode is not changed, or the switching is costless
- $I(a,a') > 0$: Investments occur (analogous of call option)
- $I(a,a') < 0$: Disinvestments occur (selling the resources) is analogous of put option. (Brosch, 2008)

For the model we assume flexibility for switching algorithms, due to more applicability of a such approach. It can handle any structure of time–dependent symmetric or asymmetric switching costs, any cash flow payoff function and any options combinations in terms of type, sequence and option maturity. The switching algorithm is designed to model a bundle of real options that can be interpreted as a joint complex, compound option (Trigeorgis 2012, pp. 185 ff.). So, we come up with the model when the valuation cannot be achieved by valuing each option separately and then adding up the values. The main reason for that is since each switching possibility represents a possible real option exercise and the switching costs matrix introduces an asymmetry. So, deviations from value additivity make the valuation and interpretation of incremental option contributions challenging (Trigeorgis, 1993a).

And continuing with switching logic of the model, the valuation algorithm can be formalized as a backward recursion in a stochastic dynamic programming fashion. So, it means that the algorithm consists double iteration:

1. It moves backward in time, applying the Bellman principle of optimality

2. At each point in time, it iterates over all entering modes, each time choosing the optimal leaving mode.

Speaking about the value of underlying asset, we have determined it as binominal process with probability of U movement *p,* and *(1-p)* for Down. Every decision at any point of time depends on two main factors. First, current cash flow per period which is realized after the switch, as a function of $\theta(t,s)$ and a'. Second, expected value of future net cash flows, starting from the derived leaving mode.

All these assumptions and algorithms are translated into stochastic dynamic program based on backward recursion, which is derived similar in Trigeorgis (2012, p. 185) and R. Brosch (2008):

$$V(T,s,a) = \max_{a'}[c(\theta(T,s),a') - I(T,a,a')] \qquad (8)$$

$s = 1,2,\dots,T;\ a = 1,2,\dots,A$

$$V(t-1,s,a) == \max_{a'}\left[c(\theta(T,s),a') - I(T,a,a') + \frac{V(t,s,a')\times p + V(t,s+1,a')\times(1-p)}{e^{r_f \Delta t}}\right] \qquad (9)$$

$t = 2,3,\dots,T;\ s = 1,2,\dots,t;\ a = 1,2,\dots,A$

where t Time indicator, $t = 1,2,\dots,T$,

s Scenario indicator, *s = 1,2, . . . , t,*

a Entering operating mode, *a = 1, 2, . . . , A*,

a' Leaving operating mode, *a' = 1,2, . . . , A'*,

V Value of portfolio,

c *(·, a')* Cash flow function for operation in mode *a', a' = 1,2, . . . , A'*,

θ Stochastic value of underlying asset,

I Investment or switching costs,

p Risk–neutral probability for up–jump,

r_f Risk–free rate of return.

At every step of the system, for each entering mode, the stochastic programming mode determines the best leaving (or operating) mode, that maximizes the value of a project. Decision based on several inputs:

- Switching costs
- Expected future value given leaving mode.

The expected future value also includes all possible future switching decisions with the leaving mode the new starting point. Summarizing the model, we can determine logic sequence is as follows (Borsch, 2008):

1. Observe the value of the underlying stochastic asset (variable);
2. Choose optimal switching policy;

3. Realize the cash flow in this period for the leaving mode.

The chosen leaving mode then will be entering mode for the next period. This stochastic dynamic programming formulation considers any possible structure of switching costs, including the non–symmetric case. So, in some steps (nodes), staying in the same mode can be optimal decision, because it can prevent a future costly switching back to the earlier mode. This approach represents hysteresis effect: even though immediate switching may appear profitable from a short–term perspective, for dynamic long–term considerations it can be optimal to remain in the original mode (Dixit and Pindyck, 1994; Trigeorgis, 2012).

CASE STUDIES

Description of the project. Project background

The case study considered in the paper is devoted to shale oil extraction projects. There are several reasons why this type of investment project is considered in the paper. First, the shale mining nowadays one of the riskiest projects due to relatively high extraction costs and very volatile oil prices, the underlying asset is perfectly fits to the model assumptions, and it represents the bunch of projects where ROA is applicable and the models can be tested as the prove for general validity if the model.

The principle of shale oil extraction is different from traditional ones. The oil is extracted not from oil lakes, but from shale rock and other low permeability rock formations. It was made possible as technologies improved and the development of horizontal drilling techniques and hydraulic fracturing ("fracking"). Shale extraction has grown rapidly on the middle of XX century.

Despite being unconventional oil resources, shale oil and gas formations can be found around the world. In 2013, the US Energy Information Administration estimated that about 11% of total crude oil, or approximately 345 billion barrels are of shale oil from these formations. The countries with largest amount of technically recoverable shale oil resources include Russia, the United States, China, Argentina, and Libya. That means, that shale oil extraction projects can be realized in all continents, and it becomes more attractive as traditional crude oil resources are decreasing. (Shale Oil, Investopedia)

At the same time, shale mining is not widespread around the world. As it was mentioned before, the main reason for that is high production costs for this type of extraction. There is a trend for decreasing costs for shale extraction, but it never can be as cheap as traditional methods. So, as a result most companies prioritize conventional methods in the extraction, because they are less vulnerable to the oil price fluctuations. And oil prices instability during last 4-5 years made shale mining very risky.

In the following graph (Fig. 2) it is clear how oil prices change and how the cost per barrel also changes. Additionally, this graph proves why assumptions about high risk of shale mining is true. Following the oil prices trend, the number of projects in shale extraction is also constantly changes, especially in the U.S, in the country where these unconventional methods are most developed.

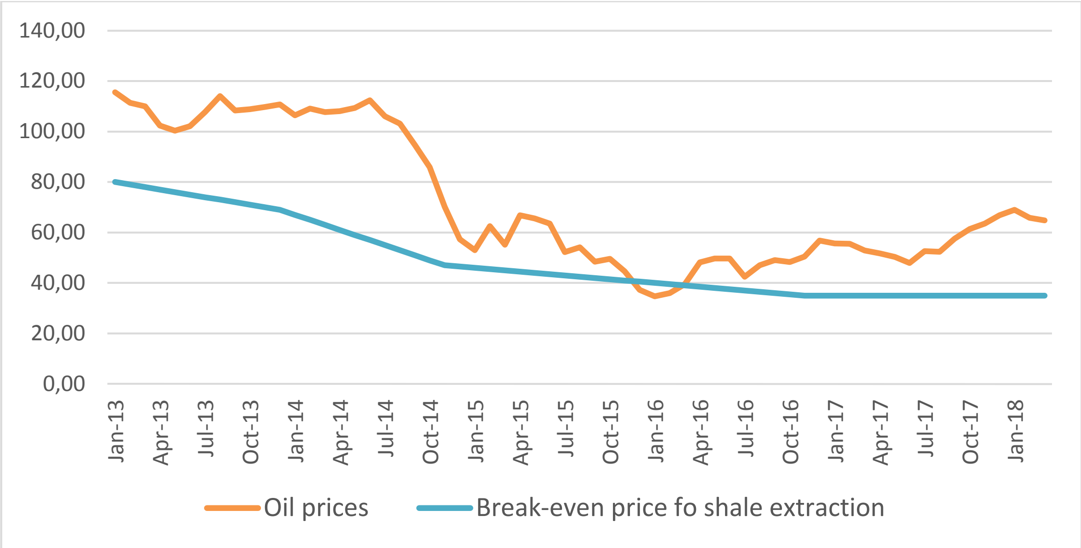

Figure 2. Oil prices and break-even price for extraction. (World Oil, 2018)

Since 2013, the average wellhead break-even price (BEP) for key shale plays has decreased from $80/bbl to $35/bbl. This represents a decrease of over 55%, on average. The wellhead BEP decreased across all key shale plays.

There are several reasons behind the observed drop in BEP. A part of it is attributable to the structural changes, such as improved well performance (which can be measured by improvements in the EUR); and the improved efficiency gains (which can be measured by the effect of lower drilling and completion cost, a result of more effective operations). Another set of drivers behind the falling BEP can be referred to as cyclical changes, which are driven by the industry cycle into which the oil industry entered in 2014, with a plummeting oil price.

We can see, that difference between oil prices and BEP is relatively small that and even in some points the BEP was lower than oil prices. If the trend will continue to more decreasing of oil prices, the shale projects can be even more risky, despite decreasing BEP. So, all these factors ask more flexibility for shale extraction companies in terms of production capacities. And real option approach that considers opportunity to be flexible in the unstable underlying assets case can maximize the value if such projects. If we apply standard NPV calculations, most of shale projects seems unprofitable, but they still operate, that proves another approach, like ROA is necessary for these types of projects. So, in the paragraphs it is clearly proved why ROA, especially portfolio approach for real options is best way to assess these investment projects.

Project inputs for analysis

First, main characteristics of the project will be implemented. The project is medium capacity shale oil extraction project in the North America. This region is selected as base case, because now most of the companies operating on the shale are placed in the US. On the other hand, it does not mean that these projects cannot be implemented in other countries, because the cost of investments and operational costs do not change significantly over the countries due to the same technologies and suppliers.

The shale oil resources are restricted in the capacity. As a rule, typical shale oil field can be classified as small sized according to international classification. A small sized crude oil field contains from 1 to 5 million tons of crude oil per field. Converting it to barrels of oil (crude oil prices are defined in U.S. dollars per barrel), in average as 7.5 barrels per one ton of crude oil (average rate for U. S WTI brand crude oil), we obtain from 7.5 million to 37.5 million of barrel. For our project we will take the average amount of 22.5 million barrels of crude oil. The amount of crude oil in this field is about 3.5 million of barrels.

Considering that in average the lifetime for small sized crude oil field is 15 years, it means that in average it is possible to produce 1.5 million of barrel per year, or 4000 barrels per day. For these purposes the company needs at least 40 oil drilling and extracting machines with average capacity of 100 barrels per day per machine (for contemporary oil wells, the capacity ranges from 90 to 110, so we can take also an average amount.

To sum up, all necessary preliminary information about the project is given in Table 1.

Table 1. Summary of project inputs

Region	North America (U. S)
Crude oil brand	Brent
Shale field classification	Small seized
The amount of oil in the field	22,5 million of barrels
The field lifetime	15 years
Average amount of barrels per day	4000 barrels
Number of extracting machines	40

Several main input factors are obtained. Now, it is necessary to value the amount of investments in U.S. dollars to the project. Construction of oil extraction plant is a multistep project. Though, the investment amount analysis goes by step-by step investments and analysis presented below.

Step 1. Buying the field and all necessary licenses for the extraction. For our analysis we assume that the company already possesses the field and the main problem to make decision whether it is profitable to make investments or not. Also, important to mention, that all preliminary expenses about exploring the amount of oil in the field are not included in the investment analysis because it is common practice for oil companies. More complicated task is to value the amount of money company pays for licensing. The rules for obtaining licenses to explore on state mineral rights vary from state to state. Federal onshore exploration licenses are obtained through oral competitive bidding. So, it means that we cannot put this investment here, but after conducting all investment analysis, because we the company's maximum bid price cannot be higher than the value of the project.

Step 2. Buying all necessary machines and capacities for drilling. The least-expensive rigs are those classified as U.S. small footprint land rigs. U.S. shale-ready rigs tend to cost about $3 million to $5 million more than small footprint rigs. These small rigs are suitable for the project purposes due to small oil capacity in the field. Considering 40 drilling machines for the project we obtain about $120 million for main machines capacities. According to the statistics all surrounding expenses connected to services and other related expenses to run the production takes up to $130 million for small capacity onshore extraction. It means that total amount of investments to the project approximately is $250 million and can take up to 2 years for finishing the construction.

Step 3. Valuing operational costs for the plant. All operation costs are divided to fixed and variable costs. According to the industry specifics the fixed costs are about 15% of calculated average monthly revenue for the project. It means for our project we can assume that monthly fixed costs will be about $1.1 million per month. (Assuming $60 per barrel). Variable costs can be also defined as break-even price. It already demonstrated that the break-even prices for shale onshore extraction is approximately $35 for 2018.

Step 4. Risk-free rate. We need risk-free rate (not WACC) for the project valuation according to the real options valuation model in binominal options pricing. The project take place in North America and the idea is to make the case analysis worldwide applicable, so we can use as risk-free rate international ones. In common world-wide practice as a risk free-rate we can take 1-year T-bills rate of U.S. They vary over time, but most companies take the rate as 1%. For the purposes of the hypothetical project this rate is valid as best fir and avoid 0% risk-free rate.

Now, after introducing all necessary for the proper investment analysis information, we can start to apply all the research method and find maximum value of the project.

Investment analysis of the project

Binominal numerical analysis of the project: No options

First, to make binominal numerical analysis of the project we need to make some historical analysis of the underlying asset. The lifetime of the project is 15 years from start monthly, so to make the calculations close to real figures, we take 15 years retrospective analysis of Brent oil prices. This data is necessary for finding basic factors, like Up and Down factors that need standard deviation of underlying asset. Just to give overall view of oil prices movements, look at the graph below.

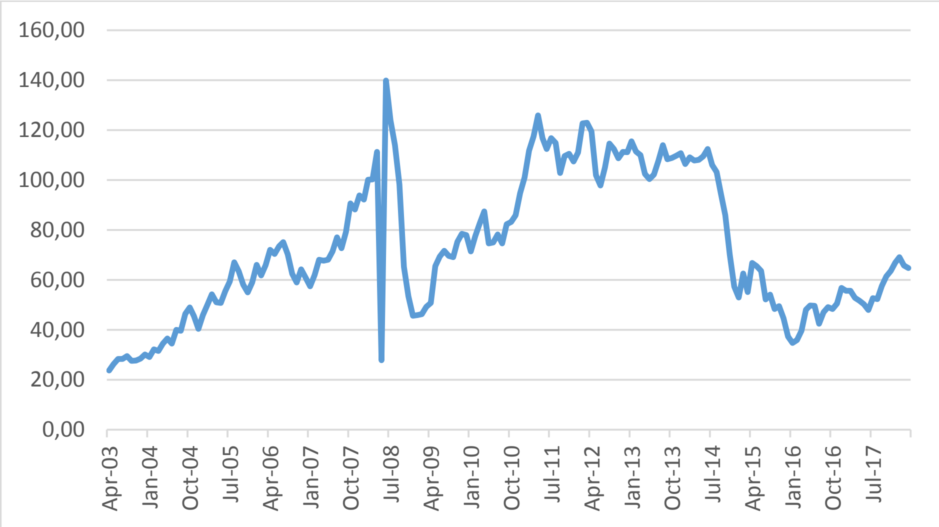

Figure 3. Oil prices dynamics (Investing.com, 2018)

We can see that in some periods we observe high deviations of monthly prices due to different shocks in row oil market. So, these outliers should be removed from the model otherwise we get some extremal (impossible) prices in our binominal price tree due to high standard deviation.

After excluding outliers from the analysis, we obtain some basic factors, like Up and Down, and risk neutral probabilities necessary for the analysis of the project. So, the first step in valuing the project value without any options is to build binominal prices forecast for future 180 months (15 years). To be realistic, we should put some restrictions on maximum and minimum oil prices. In previous 15 years, the maximum oil rice was $139 per barrel and minimum $23.68 per barrel. Nobody knows the future, so to capture 99.99% of possible oil prices values, we put minimum price restriction of $10 per barrel and maximum $150 if ever prices in our model goes to those levels. Applying Up and Down factors and putting restrictions we obtain binominal prices model for underlying asset. (Appendix 1).

The next step is based on forecasted oil prices, to forecast cash flows for every period for each possible state of oil prices. The cash flows are calculated in simplified way, like revenues minus expenses. All revenues come from selling crude oil in the amount of 4000 barrels per day or 120 000 barrels per month times forecasted monthly average oil price. Expenses are variable costs per barrel plus fixed costs. Variable costs are number of barrels sold times cost per barrel.

Summarizing, the cash flows formula is presented below. Results of calculations are in Appendix 2.

$$CF_{it} = Numer\ of\ Barrels \times (Price\ per\ barrel_{it} - Variable\ cost\ per\ barrel) - Fixed\ costs \quad (10)$$

Obtained cash flows for each period and each forecasted oil prices are used further to value the project by discounting cumulated expected cash flows by backward induction to period 0. The method of backward induction and finding the value of the project is described in Section 2. Here, simplified and more understandable way is described using graphical presentation of the valuation model with numerical example.

Risk-free rate = 1%

Risk neutral probabilities: P = 0.55 (Go Up) 1-P = 0.45 (Go Down)

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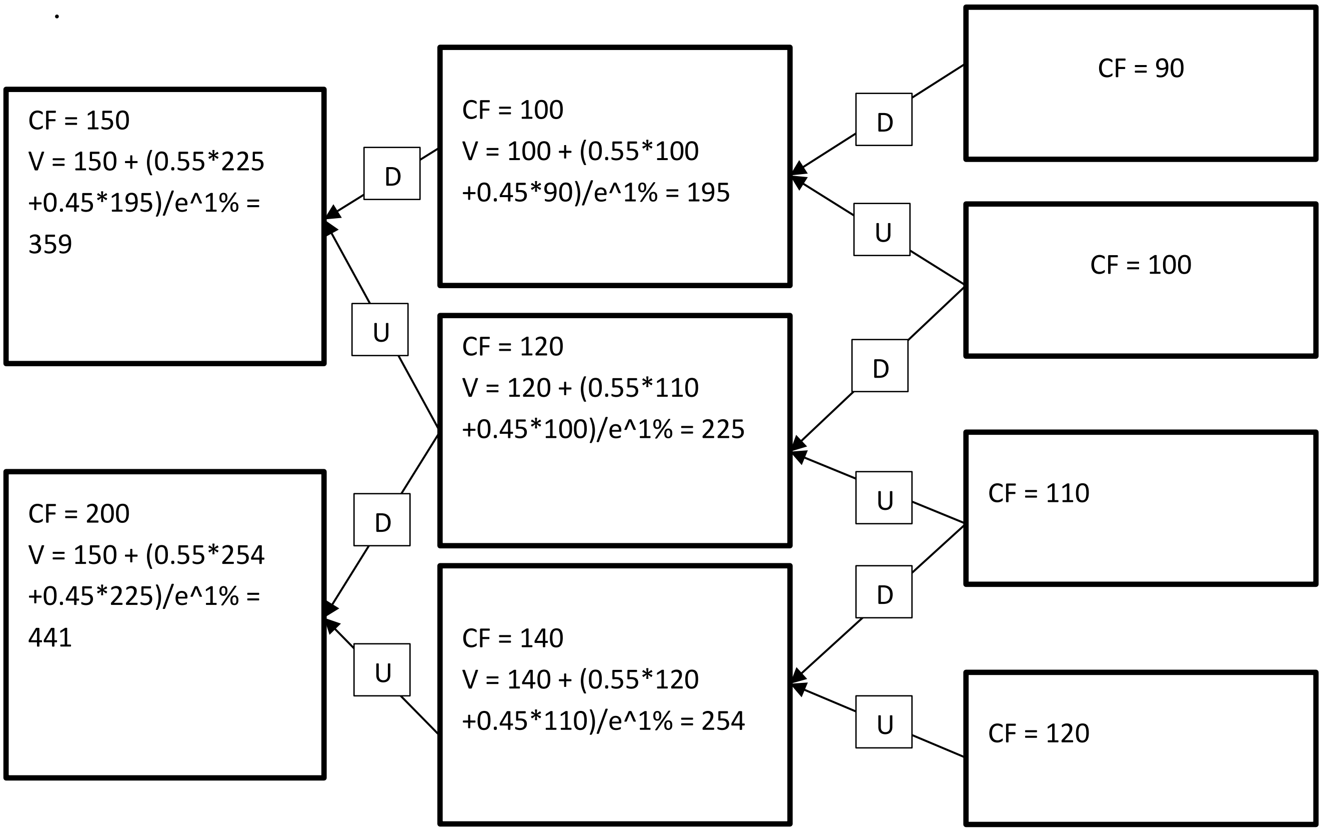

Figure 4. Binominal lattice tree in examples

Discounting by the same logic over all 180 periods we end up with the value of the project in period 0. In the analysis we obtained the value of the project without any options of \$49.25 million. (Details in Appendix 3). So, it means that this is the maximum amount the company can afford to bid for the license for oil drilling in a planned shale. What if other companies offer more? So, here we need more flexibility to maximize the value of the project and overbid competitors and buying license for profitable project. Here we have particular managerial application for the project. Now, let's move further and try to add maximum value to the project by creating the portfolio of options for the project.

Analysis of the project with options

In this paragraph we are going to consider how the value of the project changes with the implementation of different options separately. That will make a base for comparison of the sum options with the value of options and help make conclusions how the value of portfolio is different from the sum and which approach is better.

For crude oil extraction projects, it is possible to implement 4 basic options types:

- Option to defer the project. That means if circumstances on oil market are not appropriate, the project can be deferred to better conditions in future.
- Option to abandon the project. That means that if that crude oil prices in markets such low that make the project unprofitable, the company can close or sell pot the project.
- Option to expand the production capacities. That means that if oil prices are high enough company can consider the expansion of production to near small seized field.
- Option to contract the production capacities. That means that if conditions in crude oil markets are bad bur till make sense to produce with lower capacities, the company can reduce production capacities to decrease fixed costs of the project.

Option to defer. Option to defer the project can be considered if and only if the oil prices are low and cannot allow the producers start a profitable business. In the present conditions oil prices are high enough and even have a trend to increase in future, that means that option to defer the project will not be considered for the valuation of the project. It is better to start still prices are high enough than wait and end up without any projects. So, only remaining three options will be evaluated.

Option to abandon. The option to abandon is an analogue of European put option. If the company anodons the project, it sells out all the assets by terminal value for the moment of selling period. Moreover, company can choose the best period to exercise the option. The terminal value for the end of period 180 is 0, the assets loses the value linearly with lifetime of the project. That means that $250 million loos approximately $1.39 million dollars per month. To calculate the terminal value of the project for time *t* and oil prices in state *i*, look at the formula below.

$$TV_{ti} = 250 - \frac{250}{180} * t \qquad (11)$$

So, it means that the option will be exercised only if cumulative cash flows for time *t* are lower that the terminal value of the project in time *t* and oil prices in state *i*. The idea is formally stated in the formula below.

$$V_{ti} = \max(Cumulative\ CF_{it}; TV) \quad (12)$$

It means that we have the maximization task the aim of which is to find the best exercise time for option to abandon that maximizes the value of the project in time *t = 0*. The graphical results of solving the problem are presented in Appendix 4. Solving the problem, we obtained the optimal exercise time for European put (abandon the project). In time t = 38 the value of the project in time 0 is maximum.

Summarizing, we can state that after 38 months after starting the project, the option must be exercised if Cumulative cash flows are less that TV of assets that equals to $197.6 million + project value of $35 million is $232,6 million. Simply speaking, if oil prices are lower than $55 per barrel of crude oil, it is better to sell out the business and maximize the value of the project. The value of project will increase to $49.4737million, and the value of option is $0.2280 million.

Option to contract. The option to contract is an analogue of European put option. The idea of the option is in opportunity to contract the production capacities of the field in the case of negative cash flows for the period, that will allow to decrease fixed costs twice, but the production capacities will decrease only by 1/4 by capacity per drilling machine from 70% to 90%. At the same time all free capacities can be sold out for its Terminal value. The tricky thing here is that, terminal value + contracted capacity can be even higher not only of cash flows become negative, but also in the case when it is higher than cumulative cash flows in particular period and oil prices period.

After making a number of calculation, we obtained several important moments. Contracting production in average decreases negative cash flow per period by 3/4, in case of positive cash flow, in average they decrease by 1/4.

This problem needs optimization task of finding the best expiration period for the option, that maximizes the project value. Applying a number of programmers, the best expiration period was find. It is equal to 78. Terminal Value of the assets to be sold are equal to $70 million. The option must be exercised if the oil prices in period 78 are less or equal to $57.8 per barrel.

Summarizing the option analysis results, we can conclude that European put option with exercise price of $70 million, and expiration time of 78 month after running the project, has the value of $3.2011 million and increases the value of the project to $52.4467 million. The option must be exercised if crude oil prices are less or equal to $57.8 per barrel.

Option to expand. The option to expand is an analogue of European call option. It allows the company to expand the project to the near field with additional 3.5 million of barrels. It can be possible of and only of oil prices are high enough to cover investments and make the project more profitable.

The expansion is possible not earlier than 5 years after running the project. This is common practice for new fields that are developed. In 5 years the project proves its validity, and additional expansion can be considered.

The expansion assumes of extraction all 3.5 million barrels of oil up to the end of the project. So, it means 0.028 million of barrels per month for 120 months. So, if the option is exercised, the extraction capacities will increase approximately to 0.15 million barrels per month. The expansion asks more investments. For simplicity. We can assume that increase in capacity for 15% per month ask the same rate of initial investment plus additional project fees (approximately 40%) for expansion, this is also common practice for industry. So, total investments are about 37.5 (share of initial investments) + 17.5 (fees) = $54.5 million of investments for expansion in total.

Expansion of the project in average increases the cash flow for the project to 27% per month. So, it means that expansion to the project should at least as profitable as not expanding. Formal description is presented below in formula.

$$Cunulative\ CF\ before\ expansion_{it} \leq Cumulative\ CF\ after\ expansion_{it} - Inv \quad (13)$$

Summarizing, we have European call, with expiration time equals to 60, that allows by investing additional $54.5 million earn additional 27% CF per month. So, the value of option equals to $10.4861 million, and increases the value of project to $59.7318 million. It is profitable to exercise the option if oil prices are more than $76 per barrel.

All the summary about options values are presented in the table below.

Table 2. Summary of options analysis.

Option	Type	Exercise price	Expiration time	Value, million
Abandon	European put	$232.6 million	38	$0.228
Contract	European put	$70 million	78	$3.201
Expand	European call	$55 million	60	$10.486

We can conclude, that all options add some value to the portfolio. It means that all of them should be included to the portfolio to maximize the value of project. The most influence has the option to expand, the least one, option to abandon. Anyway, all of them add significant value and cannot be ignored.

Real options portfolio analysis

The basic part of the analysis is conducted. Now, it is a time to value what is the value of the portfolio of options. In the project we have 3 existing possible options with known characteristics: type, expiration type, amount of investments. The aim of creating the portfolio of options is to maximize the value of the project. So, the model based on the assumption that combining the options in a portfolio gives the value that is not simple sum their individual values. The main purpose here is to allocate the options on the time line so, that they will create positive synergy and maximize the value of options.

Moreover, different combinations of portfolio will be tested, because there is a possibility that portfolio of 2 options will add more synergy than group of 3. Having 3 options, following possible combinations are possible:

- Abandon & Contract
- Abandon & Expand
- Expand & Contract
- Abandon & Contract & Expand.

First step, is analyze their interactions. Look at the Appendix 6 to know whether these options interact or not. If they do not, there is no reason even to make further analysis, we just can sum up their values. Otherwise, we need to apply portfolio approach. Second step, all the options will be place on the time line according to their expiration times for individual valuations. Then, expiration times will be changed (considering all the possible restrictions), in order to find other possible value maximizing combination, if it does exist. So, then we can conclude whether chosen expiration times for individual value maximizing also best choice for portfolio or not.

Let's consider the first portfolio: Abandon & Contract. We can see clear interaction of the options. That means that we apply portfolio approach. (Appendix 6). Placing them with the same expiration rimes gives us the negative synergy of \$0.1804 million. And, actually it is expectable. First, we consider abandoning the project, and only after that, contracting. At least, it looks illogical, because any manager first would consider contracting the expansion capacities, and then, if situation even worse, to close. So, logically, the abandon option should be exercised after contracting. We already know, that maximum value we obtain, if and only if we exercise the option do contract in the period 78. So, option to abandon should be considered after this time. The logic stands true, if the option to abandon exercised next period after the option to contract, the value of portfolio is maximum. The synergy still negative, but less than previous one and equals to \$0.1654 million. We already have first evidence, that the value of portfolio is not equal to the sum of individuals values, that proves significant interaction between options. The next portfolio to be considered is Abandon & Expand. They also interact. It means we cannot sum up their individual values. (Appendix 6) After conducting a number of analysis to find out the best placement of expiration period on project lifetime, it was clarified that original expiration periods give the maximum value for a portfolio. The value of the project reaches to $59.9068 million and the value of the portfolio is $10.6612 million which is $0.0530 million less than the sum of these options. We can see significant increase in the value of the project, approximately by 25%, which can be crucial in bidding process.

The final two option portfolio is Expand & Contract option. The same logic applied here, and the same results about expiration time of options are obtained as in the previous portfolio. (Appendix 6). The value of the project reaches to $64.2827 million and the value of the portfolio is $15.0371 million which is $1.3498 million more than the sum of these options. And here we get positive synergy from combining two options. Total value of the project from these actions increased by 28.4%. So, we already have three proves that the value of portfolio cannot be considered as the sum of the values individually, and their combination can provide either positive or negative synergy, and this has to be considered to reflect actual value of the portfolio.

As we have a task to maximize the value of the project, also combination of all three options in a portfolio must be considered. It is expectable, that if options interact 2 by2, so the portfolio of all 3 options also well interact. (Appendix 6) After applying the maximization techniques, we obtained that optimal expiration period for combination should be the following:

- Abandon and Contract must be placed as they were in Abandon & Contract portfolio
- Expand expiration time is equal to its original one.

So, the following results are obtained. The value of the project reaches $64.3575 million, and the value of portfolio is 15.1187, that $1.1966 million more than the simple sum of their values. Here we get the maximum value of the project which 28.8% higher than its original value. So, it means that the hypothesis is held, and the value of portfolio is not a sum of individual values, but their interaction influences the value of portfolio significantly.

Analyzing the results of the analysis presented in the table below, we can make several very important conclusions about the value of real options portfolio.

Table 3. Options portfolio analysis summary.

Portfolio	Value of Portfolio, million	Sum of options values, million	Difference, million
Contract & Abandon	$3.264	$3.429	$-0.1654
Abandon & Expand	$10.661	$10.714	$-0.053
Expand & Contract	$15.037	$13.687	$1.350
Expand & Abandon & Contract	$15.112	$13.915	$1.197

First, the difference between the sum of the options values and the portfolio values prove that interaction (overlapping) of the options influences significantly to the value of the portfolio. So, the hypothesis telling that the value of interacting options is not the simple sum of them holds.

Second, based on the analysis of previous paragraph, the value of the portfolio also can be violated by changing the expiration time of options. It explains that the nature if interaction also matters.

Third, we actually obtained better value for our project that allows to the company increase the bid price. At the same time, in both cases of negative or positive synergy, the value of portfolio must be considered not just as a sum. If we have negative synergy, it means we can overvalue the project by simply summing up the value and bid wrong price. If we have positive synergy, we can undervalue the project and bid lower price then we can afford and probably lose the auction for the field.

CONCLUSION

The goal of given research was to propose recommendations of valuation of real options portfolio for investment projects. The idea is that real options analysis is not always based on a single option, but on many options instead. Besides, these options can interact. The interaction can influence the value of the project, so the idea is to recommend some tools that would allow managers to make analysis of real options portfolio valid and effective.

The literature review showed that real options analysis is a complicated process that can be tackled by different approaches. Analyzing the real option portfolio is even more complicated. The reviewed researches proved that existing interdependencies of options in the portfolio influence the value of the project. So, if any analyst can see any interdependence of options in the portfolio, he should understand that summing up their individual values is not applicable.

As was mentioned, the problem of portfolio valuation is not new and already has been already considered by other scientists. In other words, some approaches already exist. The idea of the current paper was to combine the best features of them and offer the understandable methodology for the portfolio valuation. The methodology is introduced as a step-by-step guide. First, the general assumptions and the features of the model are formulated. Then, binominal option pricing model is presented. The binominal approach is the base for analysis in the current research. And finally, the model was expanded to the valuation of the portfolio of options. The model is presented as stochastic dynamic programming approach that maximizes the value of a project by choosing the best leaving mode for every step. The advantage of the model is that it seeks not only for short-term profitability, but also considers long–term perspectives, that makes the model strategically optimizing.

Finally, the methodology was applied to the simulated case study. However, to make the case very close to real life, the prototype of real project was implemented. The prototype is the shale oil extraction project. All the inputs for the project are very close to real numbers but averaged due to limited access to real project information. Moreover, the crude oil as underlying asset was implemented to the model on the basis of real historic data. Based on conducted analysis, following results for real options portfolio analysis are obtained:

1. The interaction of options influences the value of project significantly. The analysis results extensively showed if there is some interaction of real options in the project, the value of the portfolio can never be the sum of this options. The value of portfolio can be either positive or negative, depending on the type of interaction.
2. Allocation of real options throughout the lifetime of the project matters. When we place options together in one project as a portfolio, the value of the project changes. Moreover, the expiration time that maximizes values of an individual option does not necessarily maximize the value of portfolio of options.
3. Ignoring the interaction of options can lead to mistakes in valuing the project. That means that if the portfolio approach is ignored, the value of a project can be overvalued or undervalued. Consequently, this result can lead to either accepting unprofitable project or rejecting the project with high potential.
4. The problem of value maximization needs to be solved by optimal combination of real options in a portfolio and efficient allocation of them within time-line of the project. Managers should develop flexible thinking. To put it differently, they should not stick to a given inputs for options but try to maximize the value changing inputs instead.

The main contribution of the paper is the managerial applicability of complicated theoretical model to practical cases. First of all, the methodology for valuing the portfolio of options was developed in the simplified form that makes the model understandable for managers without strong mathematical background. If any manager is considering the multi-optional project with interacting options in it he/she can apply the corresponding methodology, Secondly, the investment analysis was implemented for shale oil extraction project. These types of projects are widespread all over the world due to shale mining boom in the world. At the same time, due to high cost of production and volatility of crude oil prices in world markets, shale mining is still a high risky project that needs more flexibility to maximize the project value. This value can be used by managers to bid in an auction for the license for oil extraction in a particular field. By applying the portfolio analysis that reflects the real value of project, flexible managers can obtain the realistic value of the project that will allow them to bid optimal price in comparison with the other managers. It means that they will not overvalue or undervalue the project.

One possible limitation of the model is that it is not universal for all multi-optional projects. If some of general assumptions of the model are not met, this would mean that the binominal valuation model is not applicable. In most cases this is the biggest challenge for ROA. There are many businesses where underlying asset prices are regulated, or they do not follow stochastic process. Another limitation can be complexity of model for too many periods. Too volatile underlying asset prices can lead to extreme values of forecasted prices and make the model inapplicable. To overcome this limitation, it is necessary to apply other models.

Overall, the paper makes conclusions that can be interesting both for scholars and business practitioners. From the scientific side, the study provided an attempt to apply the theoretical models to the case that is very close to practical conditions. From the managerial side, the paper simplified complicated model and made is accessible for most of managers who are interested in applying real options analysis.

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APPENDICES

Appendix 1. The binominal prices model for underlying asset.

Example.

Metrics	Input		Calculated Factor	Result
Number of periods	180		Up	1,01
Avg Monthly St Dev (excl outliers)	0,9%		Down	0,99

Prices												
0	1	2	3	4	5	6	7	8	9	10	11	12
60	59,5	59,0	58,5	58,0	57,4	57,0	56,5	56,0	55,5	55,0	54,5	54,1
	60,5	60,0	59,5	59,0	58,5	58,0	57,4	57,0	56,5	56,0	55,5	55,0
		61,1	60,5	60,0	59,5	59,0	58,5	58,0	57,4	57,0	56,5	56,0
			61,6	61,1	60,5	60,0	59,5	59,0	58,5	58,0	57,4	57,0
				62,1	61,6	61,1	60,5	60,0	59,5	59,0	58,5	58,0
					62,7	62,1	61,6	61,1	60,5	60,0	59,5	59,0
						63,2	62,7	62,1	61,6	61,1	60,5	60,0
							63,8	63,2	62,7	62,1	61,6	61,1
								64,3	63,8	63,2	62,7	62,1
									64,9	64,3	63,8	63,2
										65,4	64,9	64,3
											66,0	65,4
												66,6

Appendix 2. Cash flows for each period and each possible value of oil prices.

Example.

Forecasted CF for period, mln $												
0	1	2	3	4	5	6	7	8	9	10	11	12
-250,0	3,0	3,0	2,9	2,9	2,8	2,7	2,7	2,6	2,6	2,5	2,4	2,4
	3,2	3,1	3,0	3,0	2,9	2,9	2,8	2,7	2,7	2,6	2,6	2,5
		3,2	3,2	3,1	3,0	3,0	2,9	2,9	2,8	2,7	2,7	2,6
			3,3	3,2	3,2	3,1	3,0	3,0	2,9	2,9	2,8	2,7
				3,4	3,3	3,2	3,2	3,1	3,0	3,0	2,9	2,9
					3,4	3,4	3,3	3,2	3,2	3,1	3,0	3,0
						3,5	3,4	3,4	3,3	3,2	3,2	3,1
							3,6	3,5	3,4	3,4	3,3	3,2
								3,6	3,6	3,5	3,4	3,4
									3,7	3,6	3,6	3,5
										3,8	3,7	3,6
											3,8	3,8
												3,9

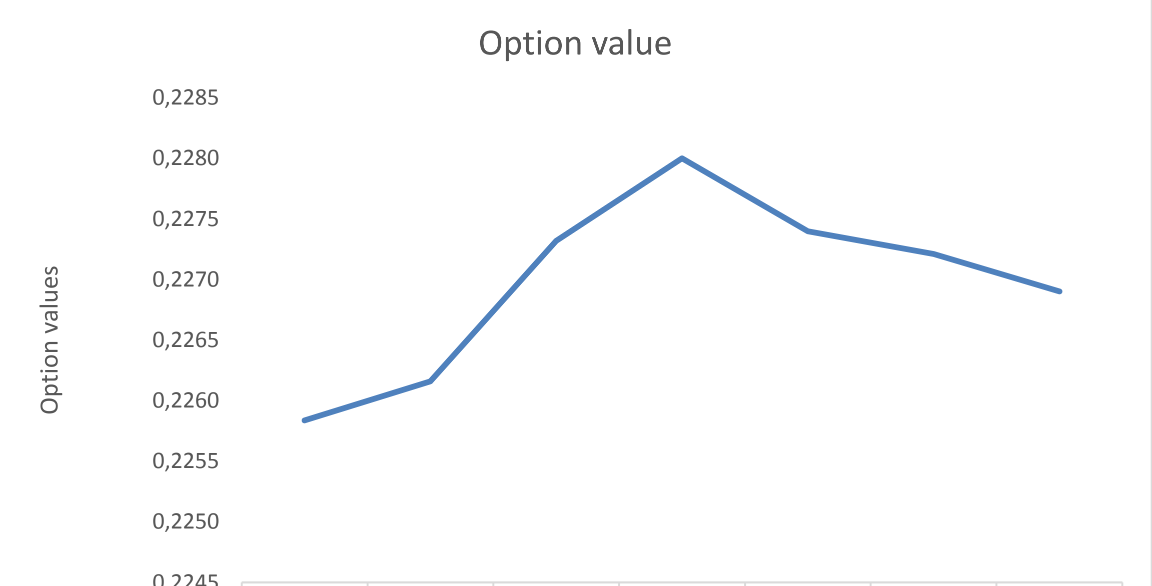
Appendix 3. The value of the project.

Example.

Calculated Factor	Result
Up	1,01
Down	0,99
P	0,56
1-P	0,44

Analysis input	Amount
Investments, $ mln	250
Monthly fixed cost, $	1,1
Variable costs per barrel, $	25
Barrels per month (avrg cap), mln	0,12
Monthly risk-free rate	1%

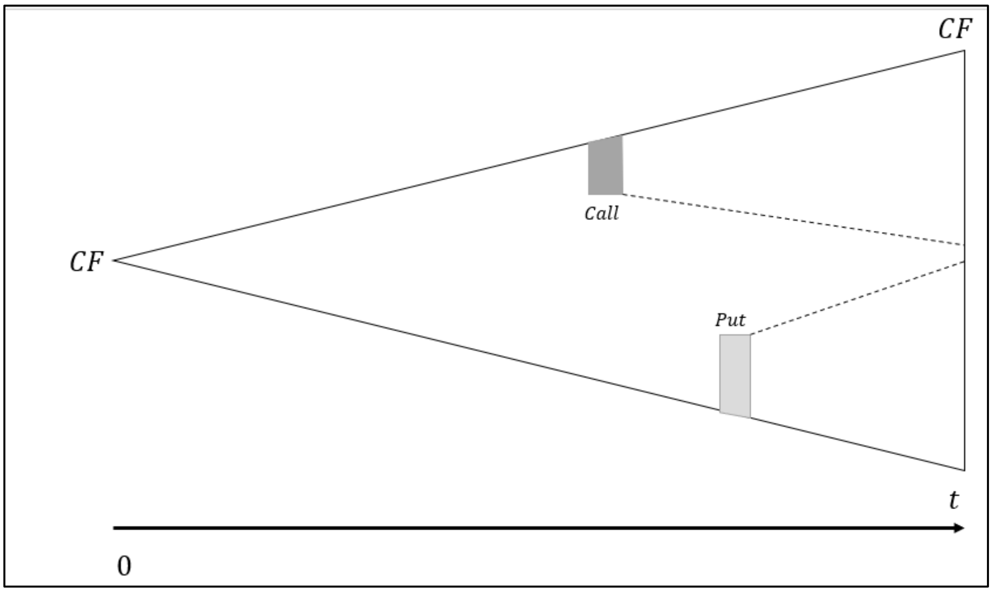
Appendix 4. Finding optimal exercise time for option to Abandon.



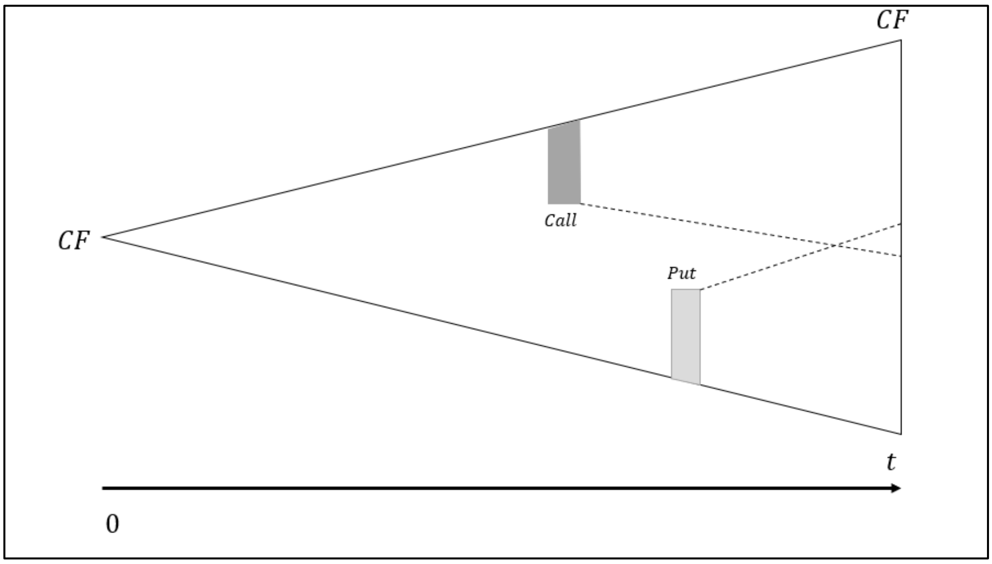
	24	30	36	38	40	42	48
Option value	0,2258	0,2262	0,2273	0,2280	0,2274	0,2272	0,2269

Appendix 5. Example of not interacting and interacting options

1) Not interacting options

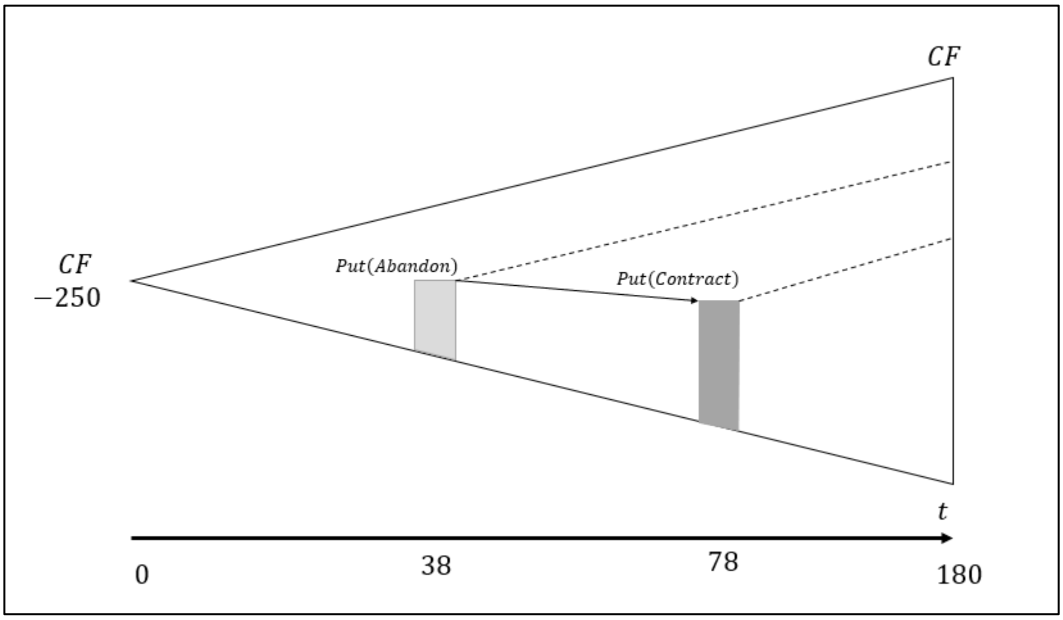


2) Interacting options.

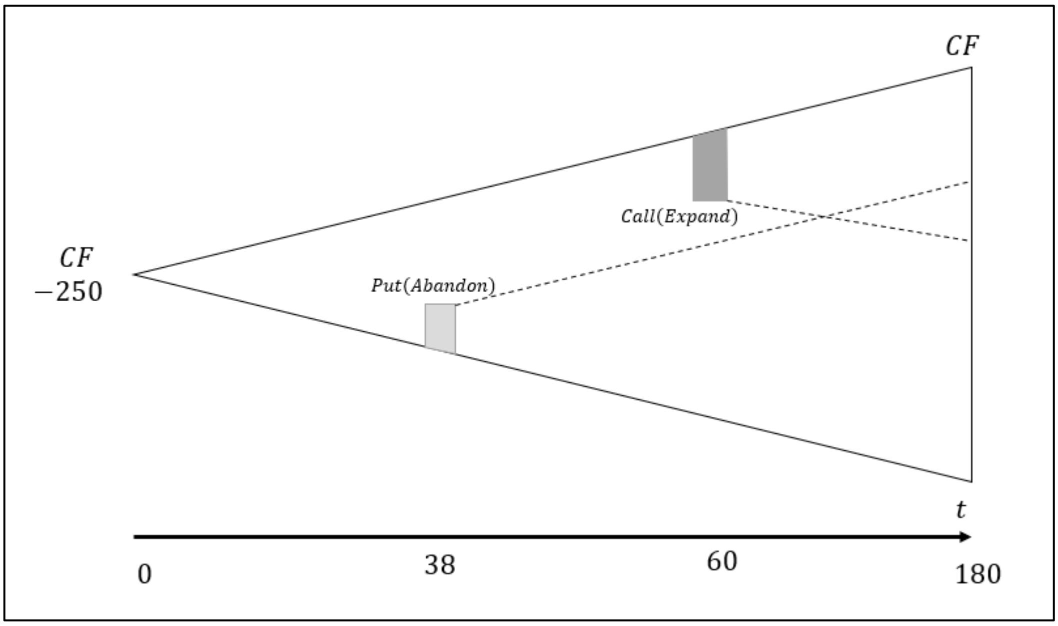


Appendix 6. Simplified presentation of options interaction analysis for the project.

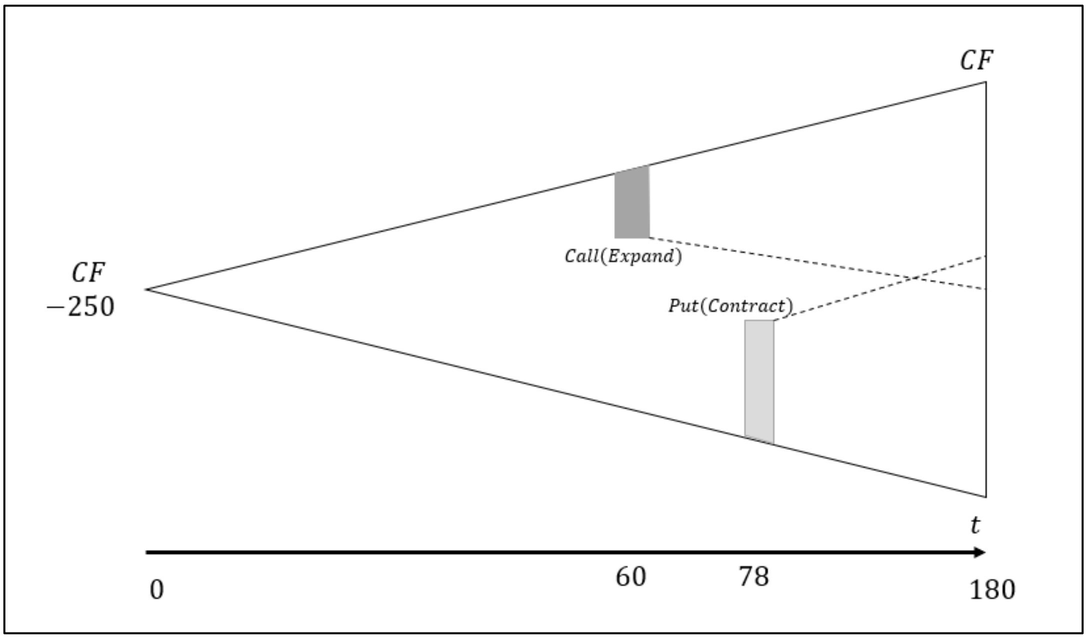
1) Abandon & Contract



2) Abandon & Expand



3) Expand & Contract



4) Abandon & Expand & Contract