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Graduate School of Management
Faculty of Applied Mathematics & Control Processes
THE INTERNATIONAL SOCIETY OF DYNAMIC GAMES
(Russian Chapter)

GAME THEORY AND MANAGEMENT

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Game Theory and Management

GTM2011

June 27-29, 2011, St. Petersburg, Russia

ABSTRACTS

Edited by Leon A. Petrosyan and Nikolay A. Zenkevich

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The collection contains abstracts of papers accepted for the Fifth International Conference Game Theory and Management (June 27–29, 2011, St. Petersburg University, St. Petersburg, Russia). The presented abstracts belong to the field of game theory and its applications to management.

The abstract volume may be recommended for researches and post-graduate students of management, economic and applied mathematics departments.

Computer design: Tatyana Grigorova and Andrew Zyatchin

ТЕОРИЯ ИГР И МЕНЕДЖМЕНТ. Сб. тезисов 5-ой международной конференции по теории игр и менеджменту / Под ред. Л.А. Петросяна и Н.А. Зенкевича. – СПб.: Высшая школа менеджмента СПбГУ, 2011. – 268 с.

Сборник содержит тезисы докладов участников 5-ой международной конференции по теории игр и менеджменту (27–29 июня 2011 года, Высшая школа менеджмента, Санкт-Петербургский государственный университет, Санкт-Петербург, Россия). Представленные тезисы относятся к теории игр и её приложениям в менеджменте.

Тезисы представляют интерес для научных работников, аспирантов и студентов старших курсов университетов, специализирующихся по менеджменту, экономике и прикладной математике.

Компьютерная верстка: Т.В. Григорова и А.В. Зятчин

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Vladimir V. Mazalov	Institute of Applied Mathematical Research, Karelian Research Center RAS, Russia
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Shmuel Zamir	The Hebrew University, Israel

Plenary Video Presentation

Martin Shubik	Yale University, USA

Tutorial

Georges Zaccour	GERAD, HEC Montreal, Canada

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WELCOME ADDRESS

We are pleased to welcome you at the Fifth International Conference on Game Theory and Management (GTM2011) which is held in St. Petersburg University and organized by the Graduate School of Management (GSOM) in collaboration with the Faculty of Applied Mathematics & Control Processes and the International Society of Dynamic Games (Russian Chapter).

The Conference is designed to support further development of dialogue between fundamental game theory research and advanced studies in management. Such collaboration had already proved to be very fruitful, and has been manifested in the last two decades by Nobel Prizes in Economics awarded to John Nash, John Harsanyi, Reinhard Selten, Robert Aumann, Eric Maskin, Roger Myerson and few other leading scholars in game theory. In its applications to management topics game theory contributed in very significant way to enhancement of our understanding of the most complex issues in competitive strategy, industrial organization and operations management, to name a few areas.

Needless to say that Game Theory and Management is very natural area to be developed in the multidisciplinary environment of St. Petersburg University which is the oldest (est. 1724) Russian classical research University. This Conference was initiated in 2006 at SPbU as part of the strategic partnership of its GSOM and the Faculty of Applied Mathematics & Control Processes, both internationally recognized centers of research and teaching.

We would like to express our gratitude to the Conference's key speakers – distinguished scholars with path-breaking contributions to economic theory, game theory and management – for accepting our invitations. We would also like to thank all the participants who have generously provided their research papers for this event. We are pleased that this Conference has already become a tradition and wish all the success and solid worldwide recognition.

Co-chairs GTM2011

Professor Valery S. Katkalo,	Professor Leon A. Petrosyan
Vice-Rector,	Dean, Faculty of Applied Mathematics &
St. Petersburg University	Control Processes

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WELCOME

On behalf of the Organizing and Program Committees of GTM2011, it gives us much pleasure to welcome you to the International Conference on Game Theory and Management in the Graduate School of Management and Faculty of Applied Mathematics & Control Processes of St. Petersburg University. This conference is the fifth of the St. Petersburg master-plan conferences on Game Theory and Management, the first one of which took place also in this city fife years before. It is an innovated edition as to investigate the trend and provide a unique platform for synergy among business and financial systems, on one hand and industrial systems, on the other, in game-theoretic support of national economies in the recent process of globalization. Mathematical and especially game-theoretic modeling the globalized systemic structure of the world of the future, and managing its conduct towards common benefits is becoming a primary goal today.

This conference held in new millennium is not unique as the Fifth International Conference on Game Theory and Management since parallel to the conferences GTM2007, GTM2008, GTM2009 and GTM2010 other international workshops on Dynamic Games and Management were held worldwide. Because of the importance of the topic we hope that other international and national events dedicated to it will follow. Starting our activity in this direction fife years before we had in mind that St. Petersburg University was the first university in the former Soviet Union where game theory was included in the program as obligatory course and the first place in Russia where Graduate School of Management and Faculty of Applied Mathematics were established.

The present volume contains abstracts accepted for the Fifth International Conference on Game Theory and Management, held in St. Petersburg, June 27-29, 2011. As editors of the Volume V of Contributions to Game Theory and Management we invite the participants to present their full papers for the publication in this Volume. By arrangements with the editors of the international periodical Game Theory and Applications the conference may recommend the most interesting papers for publication in this journal.

St. Petersburg is especially appropriate as a venue for this meeting, being "window to Europe" and thus bridging the cultures of East and West, North and South.

Acknowledgements. The Program and Organizing Committees thanks all people without whose help this conference would not have been possible: the invited speakers, the authors of papers, all of the members of Program Committee for referring

papers, the staffs of Graduate School of Management and Faculty of Applied Mathematics & Control Processes.

We would like to thank Maria Dorokhina, Margarita Gladkova, Anna Tur, Tatyana Grigorova and Andrew Zyatchin for their effective efforts in preparing the conference.

We thank them all.

Leon A. Petrosyan, GTM2011 Program Committee
Nikolay A. Zenkevich, GTM2011 Organizing Committee

Computation of Stationary Nash Equilibria in General-Sum Discounted Stochastic Games

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Keywords: *Algorithmic game theory, Stochastic games, Markov games, Nash equilibria*

Numerous economic problems in the field of capital accumulation, advertising, pricing, marketing channels, macroeconomics, warfare, resource economics and pollution can be modeled as general-sum discounted stochastic games. In these environmments where every economic agent tries to maximize its cumulative profit Nash equilibrium is generally recognized as the optimal solution concept. In Nash equilibrium each agent's policy is the best-response to the other agents' policies. Thus no agent can gain from unilateral deviation.

In this paper we present a replicator dynamics based algorithm that allows to calculate stationary Nash equilibria of general-sum discounted stochastic games with a given accuracy. The experiments have shown that with the use of our algorithm much higher percentage of general-sum discounted stochastic games could be solved than with the use of the existing methods: nonlinear optimization [2] and stochastic tracing procedure [3][1]. The developed algorithm is based on the approach we first proposed for multi-agent reinforcement learning [1]. We also extend theoretical basis for our approach.

Four nonlinear optimization algorithms: CONOPT, KNITRO, MINOS, SNOPT are compared with stochastic tracing procedure (TP)[2] and the developed Nash-RD algorithm.

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[1] To the best of our knowledge there are no other approaches to solve general-sum discounted stochastic games.

[2] We are infinitely grateful to P. Jean-Jacques Herings and Ronald Peeters who were so kind as to render their original stochastic tracing procedure.

The percentage of games for which we managed to find Nash equilibria with the use of the above approaches with given accuracy $\varepsilon = 0.001$ (relative accuracy $\varepsilon = 10^{-5}\%$) is presented in the corresponding columns of table 1. The percentage is calculated for 100 games of each class that differs in the number of states, agents and actions. The games are generated with uniformly distributed payoffs from interval $[-100, 100]$ and transition probabilities. Discount factor $\gamma = 0.9$. As it can be seen from the Table 1, the developed algorithm showed the best results for all game classes. The main reason is that nonlinear optimizers are inclined to get stuck in local optima, whereas only global optima constitute Nash equilibria. As for stochastic tracing procedure, if we had set internal parameter to less than 10^{-8}, we would very probably have got solutions to higher percentage of stochastic games with given accuracy $\varepsilon = 10^{-3}$ but there were some games (``–'' in the table) whose processing has taken us 5 hours already[1].

Table 1: Results of Experiments

States	Agents	Actions	CONOPT	KNITRO	MINOS	NOPT	TP	Nash-RD
2	2	2	58%	65%	66%	67%	83%	100%
2	2	3	39%	39%	41%	46%	86%	98%
2	2	5	16%	30%	19%	20%	79%	90%
2	2	7	12%	18%	12%	12%	67%	93%
2	2	10	8%	10%	5%	2%	–	90%
2	3	2	44%	47%	51%	43%	82%	92%
2	3	3	22%	33%	28%	27%	81%	92%
2	3	5	21%	25%	17%	13%	–	90%
2	3	7	7%	13%	5%	5%	–	92%
2	5	2	34%	44%	27%	39%	82%	93%
2	5	3	20%	26%	11%	21%	–	94%
2	7	2	21%	31%	15%	33%	–	87%
5	2	2	36%	37%	41%	40%	83%	100%
5	2	3	17%	15%	15%	20%	59%	97%
5	2	5	1%	5%	2%	1%	44%	91%
5	2	7	1%	7%	0%	0%	–	82%
5	3	2	18%	20%	11%	12%	77%	85%
5	3	3	2%	4%	4%	6%	66%	79%
5	5	2	9%	13%	9%	8%	–	72%
10	2	2	12%	16%	24%	23%	68%	100%
10	2	3	2%	3%	3%	1%	35%	98%
10	3	2	5%	7%	1%	1%	70%	82%

[1] Intel Celeron, 1.50GHz, 504 MB of RAM.

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Nash Equilibrium Scenarios for Russian Automotive Market Development

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Keywords: *Car industry, Extensive form of the game, Nash equilibrium, Pareto equilibrium*

In this paper the problem of equilibrium scenarios for Russian automotive market development and its substantial interpretation is considered. Scenarios of present situation development are examined on the basis of a game theory model, early applied in [1].

For this purpose retrospective analysis of car industry's state of affairs in Russian Federation was conducted from interested parties' points of view. To create a model of interested parties' interaction on this market we defined three players: Russian government, foreign and Russian automotive companies. Next step in analysis is determination of players' strategies. There are a lot of studies have been written about ways of Russian car industry development. On the basis of these papers main players' strategies have been conducted. There are different regulation measures for government, ways of penetration Russian market for foreign companies and actions of native carmakers.

By virtue of defined strategies an extensive three-step form of dynamic game was built [3]. Each tree path determines the scenario that leads to particular outcome with different for each player payoffs. Quantitative appraisal of these payoffs was identified by the method of expert evaluations. For this purpose raw information from interviews and surveys was collected. Multiplicity of information sources ensures relevant results of data gathering.

Nash and Pareto equilibrium scenarios of the game were found. Besides interpretation of results from individual point of view, there are also offered possible ways of cooperation that lead to equilibrium outcomes alteration.

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The A-Serial Cost Sharing Rule

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Keywords: *Cost sharing rule, Axiomatic characterization, Independence of higher demands.*

Cost sharing problems arise in many real life situations, which include the production of private goods by a group of people using a jointly owned production facility, the allocation of overhead expenses of a company among its divisions or charging individuals for any service that may be provided for groups of people. In all such situations each of the individuals involved demand a quantity of the output and the cost of producing the whole demanded amount, input, has to be paid by the individuals. A cost sharing rule associates each demand profile with a sharing of the overall cost.

Cost sharing problems have received a great attention and many cost sharing rules have been proposed so far. When the returns to scale of the production technology are constant, that is, when the cost function is linear the average cost sharing method is widely accepted as the most reasonable rule. However, when the returns to scale are not constant the choice of an appropriate mechanism is far from obvious. In case of increasing returns to scale the serial cost sharing rule proposed by Moulin and Shenker (1992) is especially interesting given its appealing properties. On the other hand, when the returns to scale are decreasing several rules have been proposed as counterpart of the serial cost sharing rule, see for instance de Frutos (2009) and Albizuri and Zarzuelo (2010). Nevertheless, the cost function may be piecewise concave and piecewise convex. In such a situation the aforementioned rules loose most of their appealing properties. Several generalizations of the serial cost sharing rule may fit in such a situation, see for instance Albizuri (2009) and Albizuri (2010). In this work we take one step beyond and propose the family of a-serial cost sharing rules which generalizes the α-serial cost sharing rule (Albizuri, 2010), which in turn generalizes the dual serial cost sharing rule

(Albizuri and Zarzuelo, 2010), the self-dual serial cost sharing rule (Albizuri, 2009), and the serial cost sharing rule (Moulin and Shenker, 1992).

The characterization of cost sharing rules by means of reasonable sets of properties is a main topic in the literature since it helps on deciding which rule is more appropriate in a given context, see for instance Moulin and Shenker (1994) and Hougaard and Østerdal (2009). The serial cost sharing rule of Moulin and Shenker (1992) is characterized by means of two properties. The first one, Anonymity, states that the labeling of the agents should not affect the cost sharing and is a basic property any cost sharing rule should satisfy. The second one, the so called Independence of higher demands requires that the payoff of an agent does not depend on demands which are higher than his own. The α-serial cost sharing rule of Albizuri (2010) is characterized by means of three properties, the aforementioned Anonimity, a-Independence of higher demands, and scale invariance. The a-Independence of higher demands property is a modification of the Independence of higher demands and is based on the a function that maps positive real numbers to positive real numbers. Finally, the Scale invariance property states that the scale by which the good is measured should not affect agents' payoffs. When a-Independence of higher demands and Scale invariance are considered together the a function has to be linear, moreover $a(t)=\alpha(t)$ for every positive real number t.

A main feature of our a-serial cost sharing rule is the fact that the function a may not be linear. Therefore, the family of α-serial cost sharing rules is included in our family. The a-serial cost sharing rule is characterized by means of two properties, Anonimity and a-Independence of higher demands.

Finally the new family of cost sharing rules is illustrated by means of an example.

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Towards a Sustainable and Time-Consistent Use of the Forest

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Keywords: *Carbon sequestration, Sustainability, Forest, Time consistency, Transfers.*

In this paper we model the role of the World's Forests as a major carbon sink and consider the impact of forest depletion on the accumulation of CO2 in the atmosphere. Forest exploitation brings economic revenues in the form of timber and agricultural use of deforested land, while at the same time exacerbates the problem of excessive accumulation of CO2 in the atmosphere. In this paper we consider two types of agents: Forest owners who exploit the forest and draw economic revenues from it, and Non-Forest owners who suffer the externality of having a decreasing forest stock. We retrieve the cooperative solution for this game and implement the economic transfers that allow us to reach a sustainable and time-consistent use of the forest.

Game Theoretic Modeling of Corruption in Hierarchical Control Systems

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Keywords: *Hierarchical games, Corruption, Sustainable management.*

Static game theoretic models of corruption in two-level and three-level hierarchical control systems based on the concept of sustainable management [1] are considered. A model of administrative (compulsive) corruption in a control system of the principal-agent type has the form

$$g_0(q,u,b) \to \max, 0 \le q \le \overline{q} \le 1;$$
$$g_1(q,u,b) \to \max, 0 \le u \le 1-q \le 1, 0 \le b \le \overline{b} \le 1;$$

$$\frac{\partial g_0}{\partial u} \ge 0, \frac{\partial g_0}{\partial b} \ge 0, \frac{\partial g_0}{\partial q} \le 0, \frac{\partial g_1}{\partial u} \ge 0, \frac{\partial g_1}{\partial b} \le 0, \frac{\partial g_1}{\partial q} \le 0;$$

q is a quota (compulsion control variable); $\overline{q}$ is a maximal admissible quota; q_0 is a legally established quota; u is an action of the agent; $0 \le u \le a$ is the condition of homeostasis; b is a bribe; $\overline{b}$ is a maximal admissible bribe.

A function $q(b)$ describes bribery if it does not increase on [0,1] and $\exists b_0 > 0: \quad q(b_0) < q_0$. Then, a connivance takes place if $q(0) = q_0$ and an extortion takes place if $q(0) > q_0$. The bribe-taker's behavior is characterized by tractability and greed. The parameter of tractability is a quantity $q_{\min} = \min\limits_{0 \le b \le 1} q(b)$, and the parameter of greed is a quantity $b_{\min} : q(b_{\min}) = q_{\min}$.

The following model is investigated as an example:

$$g_0(q,u,b) = bf(u) \to \max, \quad 0 \le q \le \overline{q} \le 1;$$
$$g_1(q,u,b) = (1-b)f(u) \to \max, \quad 0 \le u \le 1-q, \quad 0 \le b \le \overline{b} \le 1,$$

where $f(u)$ is a production function. As far as $f(u)$ does not decrease then the optimal agent's action is $u^* = 1-q$ and the problem of compulsion is reduced to the Germeyer game Γ_2 in the form

$$g_0(q,b) = bf(1-q) \to \max, \quad 0 \le q \le \overline{q} \le 1;$$
$$g_1(q,b) = (1-b)f(1-q) \to \max, \quad 0 \le b \le \overline{b} \le 1$$

with the coincident interests on q and antagonistic interests on b [3].

Assume that $f(u) = \sqrt{u}$ then the optimal strategy of the principal has the form

$$\tilde{q}^*(b) = \begin{cases} 0, & b = 1-\varepsilon-\sqrt{1-\overline{q}}, \\ \overline{q}, & otherwise. \end{cases}$$

So, $q_{\min} = 0$ (maximal tractability), $b_{\min} = 1-\varepsilon-\sqrt{1-\overline{q}}$. For example, if $\overline{b} = \overline{q} = 1/2$ then $\tilde{q}^*(b) = \begin{cases} 0, & b = 1-\varepsilon-\sqrt{2}/2, \\ 1/2, & otherwise. \end{cases}$ Therefore, for a little bribe $b \cong 0.15 - \varepsilon$ the bribe-taker is ready to take away the restrictions of homeostasis completely.

An application of models of the described type to the problems of real estate development [2] is considered. Compulsion and impulsion in the three-level hierarchical systems as well as their dynamic versions are also presented.

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On The Application of Π-Strategy When Pursuer Is Slower Evader

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Keywords: *Differential game, Pursuer, Evader, Strategy, Parallel pursuit, Polyhedron, Phase constraint*

It will be considered the simple motion differential game when the maximal speed ρ of Pursuer P is less than the maximal speed σ of Evader Q but there is the phase constraint $Q \in K$ where K is connected subset of $\mathbb{R}^d$ being a union of finite number of r-dimensional simplexes ($r \le d-1$)called briefly quasiedron. (We will denote this game Γ .)

Let x be the radius-vector of P and y be the same for Q. Put $z = x - y$ and $\xi = z / \mid z \mid$. Consider the equation

$$\lambda^2 - 2\langle \xi, v\rangle\lambda - \alpha^2 + v^2 = 0,$$

where $\langle \xi, v\rangle$ is the scalar production of the vectors v and ξ. It has the positive root $\lambda_G(z,v) = \langle \xi, v\rangle + \sqrt{D_G(z,v)}$ if and only if

$$D_G(z,v) := \langle \xi, v\rangle^2 + \alpha^2 - \mid v \mid^2 \ge 0.(1)$$

Definition. The function

$$u_G(z,v) = v - \lambda_G(z,v)\xi$$

defined on the region (1) is called *the strategy of parallel pursuit* in the G-game or briefly Π_G *-strategy*.

Π_G -strategy was introduced in [6] for the case $\rho > \sigma$ and applied to solve the problem of R.Isaacs [4] about the game with the ball as survival zone in essential general situation where a survival zone was arbitrary convex set (see also [7]). Later Π_G -strategy for the case $\rho = \sigma$ was used to solve the simple motion game with several number [8] and arbitrary family [12] of pursuers.

In [1] it was given another application of Π_G -strategy for the case $\rho < \sigma$ when Evader should move along the boundary of convex subset of the plane $\mathbb{R}^2$ with at list one angle point. (About other applications of Π_G -strategy see [1, 5, 12].) In report the new application of Π_G -strategy will be exposed for the same case $\rho < \sigma$.

Consider the pursuit problem is a phase constraint $Q \in K$ where K is a connected subset of $\mathbb{R}^d$ being a union of a finite number of r -dimensional symplexes, $r \leq d-1$. Remind that r -dimensional symplex is a convex hull of $r+1$ affine independent points of $\mathbb{R}^d$.

Take some unit vector $n \in \mathbb{S}^d$. Let Σ be one of the symplexes constituting K (such relation will be written as $\Sigma \sqsubseteq K$) and $[\Sigma]$ is the affine hull of Σ. Obviously $max_{v\in[\Sigma]}\langle n,v\rangle = 1$ if n is directed in parallel to $[\Sigma]$, otherwise $max_{v\in[\Sigma]}\langle n,v\rangle < 1$ all the more $max_{v\in\Sigma}\langle n,v\rangle < 1$. Thus if n is not parallel to any of planes $[\Sigma], \Sigma \sqsubseteq K$ then

$$max_{\Sigma} max_{v\in\Sigma}\langle n,v\rangle < 1.$$

This implies

$$\delta := \min_{n\in\mathbb{S}^d} max_{\Sigma} max_{v\in\Sigma}\langle n,v\rangle < 1.(2)$$

Note that δ can be calculated be linear programming.

Theorem. If $\rho \,/\, \sigma > \delta$ then in the game $Gamma$ pursuer is able to win.

P r o o f is provided using Π_G -strategy twice – first 'to catch' the projection of the point Q to subspace $\langle \hat{n}, x = 0\rangle$ and afterwards approaching to Q in parallel way to the vector $\hat{n}$. ($\hat{n}$ is denoted the value of n supplying the minimum in (2).

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The Provision of Relative Performance Feedback Information: an Experimental Analysis of Performance and Happiness*

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Keywords: *Relative performance, Feedback, Piece-rate, Flat-rate, Happiness*

Performance appraisals have become standard practice in organizations. Since the early 1980's, between seventy-four and eighty-nine percent of American businesses have used them, see Murphy and Cleveland (1991). Informing agents about how well they are performing relative to their peers, in other words, by providing workers with relative performance feedback information, is a common way in which performance appraisals are implemented. Given its widespread use, it is important to understand the consequences of providing relative performance feedback information. Managerial economics and social psychology has devoted quite a lot of attention to the study of performance appraisals (see Bretz et al., 1992, and Levy et al., 2004, for reviews). Research on Economics, however, has paid little attention to relative performance feedback information and to all of its potential effects.

This paper empirically studies the provision of relative performance feedback information under both piece-rate and flat-rate incentives on two important measures: agents' *performance* and agents' *affective response*. Affective response includes measures of agents' emotional state, such as happiness (subjective well-being or experienced utility), arousal (or motivation) and feeling of dominance. In addition, given

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that in practice the relative performance feedback information is rarely provided just once, we investigate the *dynamic effects* of its provision on both, performance and affective response. Furthermore, to understand if men and women react differently to relative performance information, we analyze the gender differences. By exploring all of these dimensions, we provide a comprehensive empirical analysis of the effects of the provision of relative performance feedback information, which, so far, have not been addressed.

Our study makes two important contributions. First, we go beyond the study of the effect of relative performance feedback information on individual performance by also analyzing the effect on individuals' affective response. Second, we compare the effect of relative performance feedback under two commonly used incentive schemes, piece-rate and flat-rate, allowing us to disentangle between the effect of relative income feedback and the relative performance feedback on both, performance and affective response.

We propose a controlled laboratory setup, where subjects perform a real effort task and are rewarded according to piece-rate or flat-rate incentives. There are four working periods. Between periods, the control subjects are provided with their absolute performance, while the treated subjects are provided with their absolute performance and with the average performance in the session. Once feedback is provided, both control and treated subjects' affective response is elicited, that is, they are asked to rate their happiness, arousal and dominance levels. See Figure 1 for a graphical description of the experiment.

With respect to performance, under piece-rate incentives, consistent with the mainstream of previous findings, we find that the provision of relative performance feedback information had a strong and positive effect on individual performance, even after controlling for individual characteristics, such as ability. Those subjects who received relative performance feedback information increased their performance by 17 percent compared to those who did not. With regard to the dynamic effects of providing relative performance feedback information, we find that in all periods, the treated subjects outperform the untreated, although the effect becomes weaker over time. In addition, the *content* of the feedback information (i.e., positive (negative) feedback when agents are informed that they are performing above (below) the average) does not affect subjects' subsequent performance differently, since all subjects increase their performance. This is consistent with a theoretical framework in which individuals get extra utility when performing better than others and get disutility when performing worse than others, i.e. individuals have competitive preferences. Finally, we find a strong gender difference in the reaction to the treatment, such that solely male subjects drive the overall treatment effect. This is a new and interesting result that adds to the recent literature on gender differences in relevant economic environments. In particular, this finding shows that gender differences are important not only in competitive or tournament-like environments, but also when competition is rather symbolic.

With respect to the affective response, under piece-rate incentives, we find that the provision of relative performance feedback information had strong effects on both happiness and dominance levels. Contrary to the findings on performance, we show that the treatment had very different effects on those who are receiving positive versus negative feedback. We find that receiving positive (negative) feedback affects subjects' happiness and dominance levels positively (negatively), such that when we only consider the overall treatment effect, the opposite signs cancel out. With respect to the happiness, the relative feedback leads to an increase in the gap (or inequality) of subjects' happiness, between those performing above and below the group average, by 8 percentage points. With respect to the dominance levels, the treatment leads to an increase in the inequality of subjects' feeling of dominance between those performing above and below the group average, by 6 percentage points. Moreover, the inequality in both happiness and dominance increases over time with the cumulative information. Finally, unlike the effect on performance, we find no gender differences in the affective response to the feedback treatment.

To disentangle the relative performance from the relative income effect, we replicate our experiment under flat-rate incentives. Regarding individual performance, we find that individuals react when relative performance information has consequences in terms of relative income (i.e. strong effect under piece-rate but insignificant effect under flat-rate incentives). Interestingly, we still find large gender difference in performance; while men continue to react positively to the relative performance feedback, where the magnitudes are much smaller than under piece-rate, women react negatively to it. In addition, we do not see any effect of relative performance feedback information on affective response under flat-rate incentives. These findings are important in two ways. First, relative performance feedback affects individuals' emotional state (increasing inequality in happiness and in the feeling of dominance) only when this information has consequences in terms of relative income (i.e., under piece-rate). Second, it also rules out the concern for potential experimental "demand" effects in the elicitation of subjects' affective response (i.e., subjects react to the information because they feel they are expected to by the experimentalist). Given the inequality in happiness and feeling of dominance is only found under piece-rate incentives and not under flat-rate incentives, this effect is less likely to be driven purely by experimental demand effects.

The findings from this paper have important implications for understanding whether or not an organization would choose to provide relative performance feedback information. We have shown that there are very strong effects on performance, when performance is rewarded, and this implies strong incentives for an organization to employ this mechanism, especially because its implementation has a negligible cost. However, we have gone beyond conventional thinking on this issue and we have highlighted that this mechanism, while being very effective on increasing performance, has important consequences on individuals' affective state. Overall, any organization, only after understanding *all* the consequences of providing agents with relative performance feedback information can evaluate the appropriateness of such a policy.

Diffusion of New Product in Social Networks

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Consider a social network consisting of n agents which form a finite set $N = \{1,\ldots,n\}$. Consider the diffusion of a new product on the market characterized by the network.

The following states of the network are possible: $(0,n)$, $(1,n-1)$, ... , $(n,0)$. Pair $(i,n-i)$, $i = \overline{0,n}$, is a state of the network, where i is a number of active agents and $(n-i)$ is a number of susceptible agents. The active agent has the product, the susceptible agent doesn't have the product. The network can move from one state to another.

Suppose there is some factor of influence on the network agents. For example, it could be advertising of the product. Denote the level of advertising as $\lambda = f(c)$, where c is the amount of money invested in advertising of the product.

Denote the probability of transition from susceptible to active as $p = p(\lambda, A)$, where A is a number of active agents.

Consider the case where the reverse transition isn't possible.

The diffusion process can be described using the Markov chain with the finite state space $\{(0,n),(1,n-1),\ldots,(n,0)\}$ and matrix of transition probabilities:

$$\Pi = \begin{pmatrix} C_n^0 p^0 (1-p)^n & C_n^1 p^1 (1-p)^{n-1} & \dots & C_n^n p^n (1-p)^0 \\ 0 & C_{n-1}^0 p^0 (1-p)^n & \dots & C_{n-1}^{n-1} p^{n-1} (1-p)^0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

Let $f(\cdot)$ and $p(\lambda, A)$ be known. Suppose there is one firm on the market and its task is to maximize profits in the long run; i.e. profits of the firm is $b(f, c, A, r)$, where $r > r_0$ is a price of the product, r_0 is a cost price of the product.

The strategy of the firm is $\{(r_j, c_j)\}$, $j = 1, 2, \dots$, where (r_j, c_j) is a strategy of the firm at stage j, $j = 1, 2, \dots$

The optimal strategy of the firm, i.e. the strategy maximizing the total profit of the firm, is found.

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Charitable Asymmetric Bidders[*]

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Keywords: *All-pay auctions, Charity, Externalities, JEL Classification*

Introduction

Fundraising activities for charitable purposes have become increasingly popular. One reason is the growing number of non-governmental organization with humanitarian or social purposes. Another one is the decrease of government participation in culture, education and related activities. The purpose of these associations are either the development and promotion of culture or aid and humanitarian services. Even in France, a country without any fundraising tradition, some organizations began to appear, such as the French Association of Fundraiser[1] in 2007.

Commonly used mechanisms to raise money are voluntary contributions, lotteries and auctions. Even though most of the fundraisers still use voluntary contributions[2], auctions are increasingly used. Indeed, for some special events or particular situations, auctions provide a particular atmosphere. The popularity of auctions for charity purposes can also be observed by the increase in internet sites

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[1] http://www.fundraisers.fr/

[2] There is further evidence of this phenomenon on the Internet with the emergence of sites such as http://www.JustGive.org.

offering the sale of objects and donating a part of their proceeds to charity. Well-known examples include Yahoo! and Giving Works of eBay. Many others have been created, such as the Pass It On Celebrity Charity Auction[1] in 2003, where celebrities donated objects whose sale revenue contributed to a "charity of the month". We can also cite cMarket Charitable Auctions Online[2] created in 2002 and selected as a charity vehicle by more than 930 organizations.

In charity auctions, bidders make their bid decisions taking into account two parameters: Their valuation for the item sold and their altruism or sensitivity to the charity purpose. In this paper we consider valuations drawn with the same distribution in an independent private values model. Then, we introduce asymmetry in the altruism parameter with complete information. As in Bulow et al. (1999) and Wasser (2008), this framework has the advantage of avoiding the complexity and the narrow results of asymmetric auctions with incomplete information. In the usual asymmetric auction literature, valuations are drawn from different distributions. Changing these distributions could change the ranking of the revenue among different auction designs (for example, see Krishna (2002)). Maskin and Riley (2000), de Fru- tos (2000) and Cantillon (2008) succeed in determining the revenue ranking between first-price and second-price auctions under some conditions that the distributions should satisfy. Consequently, in this literature, distributions of the bidders' value are crucial elements.

The purpose of this paper is to determine which of the two auction designs - all-pay auction or first-price auction - is better at raising money for charity when bidders are asymmetric in their altruism parameters with complete information and values are drawn in a independent private values model. As in the case with symmetric bidders (Goeree et al., 2005, Engers and McManus, 2007) we conclude that the all-pay auction is better than the first-price auction.These results show that different auction designs are better for different environments. Indeed, in a complete information framework Bos (2009) shows that when the asymmetry among bidders is strong enough, the ranking of revenues is reversed. In particular, winner-pay auctions outperform all-pay auctions.

Our result confirms the one of Goeree et al. (2005) and indicates that all-pay auctions should be considered seriously to raise money for charity purposes. Yet to the best of our knowledge, all-pay auctions have never been implemented in real life for

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[1] http://www.passitonline.org/

[2] http://www.cmarket.com/

charity purposes. However, it seems easy to do it. For example, every bidder could buy a number of tickets simultaneously as in a raffle. Contrary to a raffle, though, the winner will be the buyer with the highest number of tickets in hand.

This paper is closest the spirit to Bulow et al. (1999). They compare first-price and second-price auctions in an independent private signals model with common values and two bidders. The signals are drawn from the uniform distribution and some parameters, that could be interpreted as altruism parameter to the charity purpose, are asymmetric under complete information. Although they apply this framework to toeholds and takeovers, it is well suited for charity. In their paper, they determine that when these parameters are asymmetric and small enough, the revenue ranking could be reverse so that the first-price outperforms the second-price auction. Unlike them, we compare first-price to all-pay auctions in an independent private values model. The only other papers on asymmetric auctions with this kind of externalities are de Frutos (2000) and Wasser (2008). de Frutos (2000) compares first-price and second-price auctions with altruism parameters equal to 1/2 and bidders' values drawn from different distribution. Her framework is quite different to ours as she does not investigate all-pay auctions and the asymmetry concerns bidders values and not altruism parameters. However, dividing our all-pay auction by 1 minus the bidder's altruism parameter leads to study the all-pay auction in her framework with uniform distributions[1] Thus, in a technical way, our papers are connected. Wasser (2008) investigates k + 1-price winner-pay auctions with asymmetry on the altruistic parameters. Yet, he does not compare the expected revenue among the auction design but focuses on the performance of auctions as mechanisms for partnership dissolution. Thus our papers are complements as they are related thanks to the existence and uniqueness of the first-price auction but differs on economic problems raised and results.

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Game-Theoretical Models of non-Anonymous Multi-Threshold Behavior

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Keywords: *Threshold models of behavior, Nash's equilibrium, Linear programming problem*

There are several mathematical models describing the agent's behavior of conformity, i.e. an agent follows the behavior of the others. Among them there is a bunch of models that have been investigated since [0] was published by M. Granovetter. They are called threshold models of collective behavior. These models are developed for situations where an agent has two alternatives one of which she chooses according to her utility function (so called binary decisions). This utility function depends upon the number or the proportion of the others that chose particular alternative. If this number or proportion is greater than the threshold that characterizes the agent, she chooses the same alternative. Otherwise she chooses the opposite one. This is done in order to maximize her utility.

The classic example is the decision to join a riot or not. Here the agent's threshold is the proportion of the group he would have to see join before he would do also. The cost of joining a riot declines as the riot size increases, since the probability of being arrested decreases as the riot size increases. Each agent has her own level of cost or threshold she is willing to pay for taking part in a riot. According to this behavior it is important to investigate the size of the riot in equilibrium.

There are other of binary-choice situations where threshold models can be applied [1]: diffusion of innovations, rumors and diseases, strikes, voting, educational attainment, leaving social situations, migration and experimental social psychology. Many of these applications have been developing recently.

In the proposed work, the game-theoretical model of a binary-choice threshold behavior is considered (see also [0,0]). This generalized version includes the influence matrix, describing the level of influence among the agents and multi threshold behavior, where decision of agents may alter according to two or more thresholds. For example, a company would choose to be involved on a particular market if there are enough other companies are also present (first threshold). However, if the number of companies on the market exceeds the upper level (second threshold), then the company quits. One can include more than one threshold into a model if there are reasons for the utility function to be altered.

Let's $N = \{1,\dots,n\}$ be a set of agents and describe the threshold models of collective behavior as a strategic form game $G = (\{X_i\}_{i\in N}, \{u_i(\cdot)\}_{i\in N}, N)$, where the set of binary strategies of a player i is $X_i = \{0,1\}$, and her utility function $u_i : \prod_{i\in N} X_i \to \mathbb{R}$ is defined below.

Define a binary strategy of a player i as $x_i \in \{0;1\}$, where choice "1" means, that the player is active, whereas choice "0" – the player is inactive. Define strategy profile, i.e. vector of all strategies, as $x = (x_1,\dots,x_n) \in X = \prod_{i\in N} X_i$. Conditions of the game for player i is described with the help of the following $n-1$ dimensional vector of strategies:

$$x_{-i} = (x_1,\dots,x_{i-1},x_{i+1},\dots,x_n) \in X_{-i} = \prod_{i\neq j} X_j \ .$$

To define conformity behavior let's consider social pressure as an external condition for the player. Social pressure is the summary action of other players with consideration of trust or, more commonly, influence among players. Level of influence of player j at player i is denoted by $t_{ij} \in [0,1]$. In order to normalize influence suppose that $\sum_{i\neq j} t_{ij} = 1$. Thus the conditions for player i can be defined as $\sum^{i}(x_{-i}) = \sum_{j\neq i} t_{ij} x_j$. This value can be considered as social pressure. Matrix $T = \{t_{ij}\}_{i,j\in N}$ is named influence matrix. Diagonal elements of this matrix are defined below.

The Anonymous binary behavior is characterized by the fact that the player doesn't differentiate its opponents and uniformly subjected by their influence, i.e. $t_{ij} = 1 / (n-1), i \neq j$.

Suppose that the behavior of player i is defined by the following utility function:

$$u_j\left(x_j, \sum{}^i(x_{-i})\right) = \left(\sum_{j \neq i} t_{ij} x_j - \theta_i\right) x_i,$$

where θ_i is some non-negative number $0 \leq \theta_i \leq 1$. If social pressure $\sum{}^i(x_{-i})$ is greater than θ_i, then the player gains more profit of being active, if it is less than θ_i, then the player gains more profit of being inactive. The player is indifferent under the (neutral) condition $\sum_{j \neq i} t_{ij} x_j = \theta_i$. Number θ_i characterizes the threshold of "switching" the player from inactivity to activity. We denote it as autonomy of agent. Autonomy can be appropriately interpreted as a negative influence of the player in a reflective sense. That's why self-influence we denote with negative threshold value $t_{ii} = -\theta_i \ \forall i \in N$.

In similar way utility function for multi-thresholds game can be defined.

Let's consider Nash's equilibrium of this game. Nash's equilibrium in pure strategies ([4]) for game G is strategy profile x_N, if and only if for all $i \in N$ and $x_i \in X_i$ the following inequality is valid:

$$u_i\left(x_i^N, \Sigma^i\left(x_{-i}^N\right)\right) \geq u_i\left(x_i, \Sigma^i\left(x_{-i}^N\right)\right).$$

It was proved that in multi-threshold models the search of Nash's equilibrium is equivalent (necessary and sufficient) to certain (knapsack type) linear programming solving. This method is more optimized than an exhaustive search.

For the anonymous behavior game, with one threshold per player, the structure of Nash's equilibrium is very simple. Let's arrange thresholds by their value in increasing order. The structure of Nash equilibrium is as follows. All the payers whose thresholds are on the left of the value of common action $\sum_i x_i$ are active, the rest are inactive. The structure for non-anonymous game is more sophisticated, but can be described with the help of special algorithm of ordering defined in the proposed work.

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МАТЕМАТИЧЕСКАЯ ТЕОРИЯ ИГР И ЕЕ ПРИЛОЖЕНИЯ

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Advertising Plan for a Licensed Brand in a Complementary Business with Stochastic Advertising Effect

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Keywords: *Stochastic differential game, Stackelberg equilibrium, Advertising, Licensing*

We consider a brand owner who decides to enter a fashion licensing contract in a complementary business in order to introduce a new type of product with his brand on it.

It may be the case of a famous fashion brand owner who creates a perfume with his brand logo on it. We assume that the advertising effects of the licensor in terms of brand sustainability are well known by himself such that we may consider them as a deterministic function of advertising effort.

On the contrary he does not know how well the licensee's advertising campaign will work, so that we represent its effect on the brand goodwill as a random variable.

In the licensing agreement, the licensor has a twofold objective: on one hand he wants to maximise his own profits coming from the licensing contract, on the other hand he wants the advertising campaign to improve the final value of the brand goodwill.

We formulate a Stackelberg differential game where the owner of the brand acts as the leader, while the licensee is the follower. We assume that the effects of the advertising campaign of the licensee may be affected by some uncertainty and we compare the optimal advertising strategies we have found in both deterministic and stochastic scenarios.

Combining Incentive Schemes with Mechanisms of Counter Planning and Plan Adjustment

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Keywords: *Counter planning, Early replanning, Principal-agent problem, Moral hazard*

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A typical planning mechanism met in business is centralized, i.e. the plan is set by a principal to her agents on the basis of historical data, current circumstances, and corporate strategy (see, for instance, Lewis & Sappington, 1997). A disadvantage of such approach is that detailed private information available to agents is missed. A counter planning mechanism where agents are asked for their plans provides an alternative.

Counter planning mechanisms for individual employees were introduced in the late 60th of the XX century in Soviet economy to motivate employees' "enhanced obligations". The responsibility for the plan quality in the process of counter planning is supported by a system of penalties. Now, fees for deviations from the planned consumption level are typical in wholesale energy and natural gas contracts, but are less common within organizations.

Counter planning mechanisms were first studied from the game-theoretic point of view by Burkov (1977). Linear penalties were shown to be enough to achieve any specific desired plan stringency (the probability of plan underfulfilment), which is determined by the ratio $\pi_O/(\pi_U + \pi_O)$, where π_U and π_O are the penalty rates for plan underfulfilment and overfulfilment respectively. Surprisingly, in the linear case the principal need not even know the probability distribution of output to implement the first best. In last decades the counter planning mechanisms were implemented in several industries and demonstrated their applicability and high effectiveness.

Unfortunately, the classic theory fails to explain the absolute values of the optimal penalty rates. In this paper we equip the model of counter planning with agent's planning costs and efforts and immerse it into the moral hazard framework. The aim of the analysis is to develop the policy recommendations on penalty strength. The policy must be simple enough to be used in management consulting projects under time pressure and lack of statistics.

Additional information arrives in the process of plan execution by an agent. When agent's output expectations change during the planning period the principal is interested in plan adjustment. To motivate timely re-planning requests the agent is faced with another system of re-planning penalties of smaller strength, as compared to the plan failure penalties (the idea of early replanning from Burkov, 1977).

From the point of view of the principal-agent theory the counter planning mechanism belongs to the class of hidden-action models. Alike adverse selection framework, agent's private information is not related to his performance. Instead, agent knows the probability distribution of output z given his productive action y, and environment $\theta \in \{\theta_L, \theta_H\}$ (the cumulative probability function is denoted by $F(z, y, \theta)$). Initially, the agent knows $p := \text{Prob}(\theta = \theta_L)$ and can resolve uncertainty about the value of θ with probability $\Delta \in [0, 1]$ at cost $cp(\Delta)$ (Dowd, McGonigle, & Djatej, 2010). Then the agent reports plan x to the principal. After that the agent chooses productive effort y and incurs cost c(y). Then he gets know the exact value of θ and reports adjusted plan x'. Finally, output z is realized.

Efforts y and Δ are not observed by the principal. The principal instead builds the incentive scheme $\sigma(z, x, x')$ for the agent basing on initial plan x, adjusted plan x', and output z.

The payoff of the principal is

$$\Phi(x, x', z) = H(z) - \sigma(x, x', z) - \lambda_1(x, x') - \lambda_2(x', z),$$

where $H(\cdot)$ is the profit function, $\lambda_1(\cdot)$ are plan adjustment expenses, and $\lambda_2(\cdot)$ are losses from the plan failure. Typical incentive scheme is combined from constant payment σ_0, bonus $\sigma_1(z)$, plan adjustment penalties $-\pi_1(x' - x)$, and plan failure penalties $-\pi_1(z - x')$.

Accordingly, the agent's payoff is

$$f(\Delta, x, y, x', z) = u(\sigma(x, x', z)) - c(y) - c_p(\Delta),$$

where $u(\cdot)$ is the utility of money (strictly concave for risk-averse agent).

Analogous to the simplest moral hazard model (Harris and Raviv, 1977) no problem arises in the case of a risk neutral agent, when $u(\sigma) = \sigma$. The optimal combined mechanism replicates principal's profits and costs to an agent, while constant payment $\sigma 0$ is chosen to fulfill individual rationality. In the more realistic situation of a risk-averse agent (including the important case of guaranteed payment constraints) a number of biases arise from principal's efforts to maximally secure an agent. We perform the detailed analysis of these biases to justify the following extensively used policy recommendations:

- When $H(z)$ is monotone, the incentive function $\sigma(z, x, x')$ is also monotone in z.
- Any desired plan stringency can be implemented by the principal.
- Plan failure penalties $\pi_1(z - x')$ never exceed $\lambda_2(\cdot)$ and increase when $\lambda_2(\cdot)$ increase.
- $\pi_1(z - x')$ increase when planning costs increase.
- Productive and planning efforts are complementary.
- Plan adjustment penalties increase when plan adjustment expenses $\lambda_1(\cdot)$ increase.

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Management Games: Implementing Advanced Robust Incentive Schemes

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We consider the case when each agent can estimate the probability of different output levels and the minimum payment he needs to agree with the certain level of output (and, thus, the efforts).

Contemporary contracts theory (see for example, Laffont & Martimort, 2002) offers quite good incite for solution of adverse-selection and moral hazard problems for principal-agent(s) setting. At the same time, these results are based on the proposition that cost functions of agents are known by agents themselves and a principal except for some parameters, which are private information of agents (and are not observable by the principal). These functions are quite difficult to identify in real life in order to implement the effective incentive scheme in practice. In the theory of control in organizations (see, for example, Goubko, Novikov, 2008) a number of results were obtained, allowing to build incentive schemes in which plans (contracts) for agents are determined on the basis of information they report to the principal. Sometimes this information has quite simple structure. To solve the considered problem, we combine two basic mechanisms: transfer prices mechanism and counter planning mechanism. Initial game-theoretic study of these mechanisms was performed by Burkov (1977, 1989), and then extended by Novikov (2007).

For the experimental study we design a management game to be played by students. Each participant (of the total number N) receives its own randomly generated profile represented in following way:

output increase	y1	…	ym
minimal cost to increase	c1	…	cm
probability of output increase	p1	…	pm

such that for any participant: $y_0 = 0$, $c_0 = 0$, $p_0 = 100\%$, $\forall i \in [1, m-1]$

$y_{i+1} > y_i$, $(c_{i+1} - c_i) / (c_i - c_{i-1}) \geq (y_{i+1} - y_i) / (y_i - y_{i-1})$, $p_{i+1} \leq p_i$.

The principal announces the mechanism (how this data will be utilized in order to create the incentive scheme for each participant $j \in N$):

$$\sigma^j(x^j, z^j) = \sigma(x^j) - \begin{cases} \pi^+ * (z^j - x^j), \, z^j > x^j \\ \pi^- * (x^j - z^j), \, z^j < x^j \end{cases},$$

where x^j – the planned output increase to be chosen by agent j, $z^j = \max(x^j, y_i^j)$, $i : \max\limits_{k \in [1,m]} p_k^i \geq r$ – the final output increase, implemented by a participant taken in account the external probability r. The first part of the incentive scheme is a uniform rule $\sigma(x^j)$ where values x^j and $\sigma(x^j)$ are determined from the table:

desired output increase	x1	…	xm
incentive to be paid	σ_1	…	σ_m

The table is constructed by the principal basing on the information provided by participants. Two approaches are used during the experiments. The first one is the linear approach – $\sigma_i = \lambda x_i$, where λ is the "transfer" price. The second one is more complicated, which utilizes the solution of an adverse selection problem. The second part of the incentive scheme is intended to stimulate agents to choose such x^j, that the probability of its overfulfuilment is greater or equal than the desired plan stringency P (which is also determined by the principal) – $p(x^j) \geq P$. According to Burkov (1977)

$$P = \pi^+ / (\pi^+ + \pi^-).$$

After receiving this rule the participants report to the principal their private information (possibly, untruthfully).

The goal of the principal is to choose $x = (x_1, x_m)$, $\sigma(x^j)$, π^+ to maximize

$$u^0 = E\sum_{j\in N}(z^j - \sigma^j(z^j, x^j)).$$

The goal of each participant is to maximize his utility function $u^j = E(\sigma^j(z^j, x^j) - c^j)$. Materialization of incentives in case of students is realized through the dependency of scoring points they receive during study from their games' payoffs.

In our report we analyze the results of this management game played by students.

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Collective Punishment and the Evolution of Preferences

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Keywords: *Collective action, Punishment, Preference dynamics, Cooperation.*

We present an overlapping generation dynamic model of intentioned cultural transmission of preferences to analyze which are the sources of cooperation (and efficiency) in a non-repeated incomplete contract scenario.

Trust and trustworthiness play an essential role in the development of the economic activity, particularly in order to achieve efficiency (see Arrow (1972), Putnam (2000), Tabellini (2008), Algan and Cahuc (2007), etc…). But, what are in turn, the determinants of trust? Why the level of trust is so different among societies? Is there any relation between the levels of trust and institutions?

To analyze this problem in a theoretical framework, we analyzed, in a previous work (Olcina and Calabuig, 2010), an extended trust game with a costly punishment phase in an intergenerational dynamic setting.

In fact, many real life economic situations are trust games with a team of investors. Moreover, punishment itself is also a team decision problem. The workers' capacity for punishment in this situation is endogenous, depending on the number of workers adhering to this activity. Therefore, in addition to institutions, it seems crucial for the effectiveness of punishment, the ability of different groups to overcome the collective action problem that is at the core of almost any form of collective punishment.

Why and how do different societies succeed in solving this coordination problem?

And what is the relation, if any, with the strength of the punishment institutions?

We present an overlapping generation dynamic model of intentioned and costly cultural transmission of preferences in order to analyze this team trust game with collective punishment. We are interested in the influence of punishment institutions on the long-run distribution of preferences and behaviour, especially in the collective punishment coordination problem.

Our main results are the following. Trust evolves only if there is enough punishment capacity in the society and individuals are able to solve the collective action problem faced in the punishment phase. There is some kind of strategic complementarity. In particular, strong punishing institutions facilitate the solution to the coordination problem in the punishment stage and vice versa.

Different institutions generate different behaviors and what is, probably, more important, they influence the long run incentives to socialize on particular preferences that affects norms of negative reciprocity (punishment).

The only long-run outcomes of our dynamics are the Fully Cooperative Culture and the Non-Cooperative Culture. The fully cooperative culture is only feasible for high values of the institutional capacity of punishment. Its basin of attraction is larger, the higher is the institutional capacity of punishment, the higher is the degree of peer pressure and the smaller is the individual cost of collective punishment. However, uniqueness is not achieved. Our model shows hysteresis: initial conditions matter, because they can lead the society to a different steady state.

A Stochastic Competitive R&D Race Where "Winner Takes All"

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Keywords: *R&D race, Nash equilibria, Global optimality, Resource allocation.*

Introduction.

Increased global competition faced by today's companies has strongly affected the business cycles as well as the life cycles of products and underlined the advantage of being the first one to bring out an innovation. In many markets such as technology and pharmaceutical markets, the speed of innovation determines who primarily gets the market share. Consequently, firms involved in such markets have to continuously consider the nontrivial problem of selecting R&D projects to invest in and the amount of investment in each of the selected projects. In this work, we formulate this problem as a race among several agents who compete over the development of a certain product, project or process. Each firm decides whether to enter the race and if so, how much to invest in the development. The investments of the firms are assumed to be one-time payments made at the beginning of the planning horizon and cannot be retrieved regardless of the actual length of the project. The competition is characterized as a winner-takes-all mechanism, since the revenue of the project is acquired by the firm that completes the project first while all other firms earn nothing. The completion time of the project for each firm is assumed to be random with an exponential distribution whose rate parameter is proportional to the investment of the firm. These completion times are assumed to be independent across firms. The costs and revenues are subject to continuous discounting. The technological and marketing efficiencies of a firm differentiate it from the other firms. The main contributions of this work are the explicit representations of a unique Nash equilibrium and of a unique globally optimal solution as functions of the system parameters.

Nash Equilibrium.

The computation of the unique Nash equilibrium involves ranking of the firms according to the product of their two parameters (specifically their technological and marketing efficiencies) and computing a breakpoint for each firm. A firm invests a positive amount in a given project if and only if the market interest rate is below its respective breakpoint. Hence, two consecutive breakpoints that capture the market interest rate between them determine the set of firms that invest positive amounts in the Nash equilibrium. Our analysis provides closed-form expressions for these amounts and their utilities.

The simple representation of the Nash equilibrium also leads to a variety of insights regarding the sensitivity of the market activities, the utilities and the expenditures in the equilibrium to changes in firm-specific and environmental parameters. Among these, we show that the equilibrium expenditure of a firm is bounded above by 25% of its revenue from the project, independently of all other parameters of the problem. Our analysis also suggests that the regulation of the interest rate by a centralized decision maker (e.g., offering of loans at reduced interest rates by a governmental agency) does have a certain impact over the set of firms actively involved in the race. In particular, such actions can always assure that two leading firms invest positively. The criterion for positively investing becomes tighter in the presence of competition for all firms but the leading one, whose breakpoint is the same whether or not there is competition.

Global Optimum.

The problem of maximizing the sum of the firms' utilities, which we refer to as global utility, always has a solution in which at most one firm invests a positive amount. In other words, a globally optimal solution consists of either lack of activity or a monopoly. We provide an explicit representation for the optimal investment amount of the optimal monopoly and the resulting utility. Our analysis indicates that the globally optimal solution exhibits concentration of effort even when full diversification is present in the unique Nash equilibrium. The criterion to rank the firms for constructing the Nash equilibrium is different from the one for choosing the optimal monopoly. Consequently, the Nash equilibrium can be substantially different from the globally optimal solution. We illustrate through an example that the unique monopoly of a globally optimal solution does not necessarily invest a positive amount in the unique Nash equilibrium. In contrast to the globally optimal solution, the equilibrium solution can induce a full competition in the market. As a means to compare the equilibrium and globally optimal solutions, we explore the efficiency of the system, which we define as the ratio of the utility of the unique Nash equilibrium to the globally optimal utility.

Extensions.

Our results apply to a rent-seeking model that is fundamental in the public choice literature and to an instance of the well-known Cournot model in economics. These results can be partially extended to the problem where the dependence of the completion time rates on the investments is nonlinear.

On Voluntariness of Nash Equilibrium*

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Keywords: *Nash equilibrium, Rational expectations, Free will, Determinism*

In this note we discuss a psychological trait that under certain conditions can be associated to the notion of Nash equilibrium: that of voluntariness. To what extent is the equilibrium outcome of a game what players wanted it to be? The exposition is initially framed in the simplest possible setting, one-shot bimatrix games with payoffs known to the players. It is assumed that rationality is exhaustively captured by players' optimizing behaviour in terms of best reply to an opponent's *expected* strategy. The mechanism by which expectations are formed is not assumed to be an ingredient of rationality, i.e. expectations may or may not be rational, consistent or inconsistent. It is shown that the set of outcomes arising from these assumptions may contain a Nash equilibrium even when expectations are inconsistent. We term *involuntary* such an equilibrium. When we remove the assumption of knowledge of the opponent's payoffs, the performance of a learning algorithm in presence of two pure-strategy equilibria, one voluntary one involuntary, shows - surprisingly - the prevalence of the latter. Our discussion dispels a common misconception present in the literature - the identification of Nash equilibrium to consistent alignment of expectations. Besides consequences in econometrics, specifically in the sort of statistical inference used to x-ray agents' psychologies, the question involves deeper epistemology aspects. The possibility of involuntary equilibria is akin to whether Knowledge could be assimilated to 'Justified True Belief', a thesis denied in the classical paradox of [5]. Furthermore, involuntary equilibria provide substantive support to the compatibilist view in the debate over free will vs determinism [3].

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Towards an Economic Theory of Consciousness

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Keywords: *Self-control, Amnesia, Delusion, Etiology*

We apply the methodology of information economics in an intra-person multiple-self setting to model the relation between one's economic well being and state of consciousness, including amnesia and delusion, that may underpin some categories of mental (dis-)order. We posit the notion of semi-conscious choice in which the individual habituates, through long-term intra-personal interactions, into being "strategically" forgetful or delusional to enhance the motivation of one's future selves in making decisions. We endogenize the individual's habituated state of recall error proneness in equilibrium from his attitude towards intertemporal discounting, including present bias. We further study the ex ante welfare of the decision maker in relation to his equilibrated state of consciousness and oer it as an economic measure of his mental well being.

The Impact of Market Size on Technological Heterogeneity

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Keywords: *Heterogeneity, International Trade, Market size, Equilibrium*

In our model the economy consists of two countries, "Home" (H) and "Foreign" (F), one production factor - labour - and one differentiated sector including continuum of varieties or brands. In our notation,

L is the total population in the economy;

$s \in [0, 1]$ is the share of population in country H (so $1-s$ is the share of population in country F);

N^H and N^F are masses of firms in country H and country F correspondingly;

$c^H = c(F^H)$ and $c^F = c(F^F)$ are the marginal costs in country H and country F (we assume that $c^{'}(F) < 0$);

F^H and F^F are fixed costs in countries H and F ;

w^H and w^F are wages in countries H and F, normalized as $w^H = w$ and $w^F = 1$.

Each consumer's total utility in country $k = H, F$ from consuming domestic varieties and varieties imported from $l = F, H$ is

$$U^k = \sum_{i=1}^{N^k} u(x_i^{kk}) + \sum_{j=1}^{N^l} u(x_j^{lk}),$$

where $u(\cdot)$ is an increasing and concave elementary utility function, $x_i^{Hk} \geq 0$ is the individual consumption in country $k \in \{H, F\}$ of a commodity produced by $i-th$ firm in country H. Similarly, x_j^{Fk} is the consumption in country $k \in \{H, F\}$ of the commodity produced by $j-th$ firm in country F.

Two budget constraints in countries H and F are, respectively,

$$\sum_{i=1}^{N^H} p_i^{HH} x_i^{HH} + \sum_{j=1}^{N^F} p_j^{FH} x_j^{FH} \leq w + \Pi^H,$$
$$\sum_{i=1}^{N^H} p_i^{HF} x_i^{HF} + \sum_{j=1}^{N^F} p_j^{FF} x_j^{FF} \leq 1 + \Pi^F,$$

where p_i^{Hk} is the price for , x_i^{Hk}, while p_j^{Fk} is the price for x_j^{Fk}, whereas Π^H and Π^F are the total profits (actually this term is excessive because $\Pi^H = \Pi^F = 0$ at the equilibrium due to free-entry condition).

The profit maximization by i-th producer in country H means

$$sL(p_i^{HH}(x_i^{HH}) - wc(F^H))x_i^{HH} + (1-s)L(p_i^{HF}(x_i^{HF}) - wc(F^H))x_i^{HF} - wF^H \to \max_{x_i^{HH},\, x_i^{HF},\, F^H},$$

while similarly program of j-th producer in country F is

$$sL(p_j^{FH}(x_j^{FH}) - c(F^F))x_j^{FH} + (1-s)L(p_j^{FF}(x_j^{FF}) - c(F^F))x_j^{FF} - F^F \to \max_{x_j^{FH},\, x_j^{FF},\, F^F}.$$

Here $p_i^{Hk}\left(x_i^{Hk}\right)$ is the inverse demand function for the commodity produced by i-th firm in country H for consumption in country $k \in \{H, F\}$, and $p_j^{Fk}\left(x_j^{Fk}\right)$ is it counterpart in F.

The *symmetric equilibrium* can be characterized through several equations. First, using the maximization of x^{kl} by consumers and producers,

$$\frac{u'(x^{HH})}{u'(x^{FH})} = \frac{wc^H}{c^F} \cdot \frac{1 - r_u(x^{FH})}{1 - r_u(x^{HH})}, \qquad \frac{u'(x^{FF})}{u'(x^{HF})} = \frac{c^F}{c^H w} \cdot \frac{1 - r_u(x^{HF})}{1 - r_u(x^{FF})}, \tag{1}$$

where, as in [1],

$$r_u(x) = -\frac{u''(x)x}{u'(x)}.$$

Second, free entry or zero-profit conditions entail

$$\frac{sr_u(x^{HH})x^{HH}}{1 - r_u(x^{HH})} + \frac{(1-s)r_u(x^{HF})x^{HF}}{1 - r_u(x^{HF})} = \frac{F^H}{c^H L}, \qquad \frac{sr_u(x^{FH})x^{FH}}{1 - r_u(x^{FH})} + \frac{(1-s)r_u(x^{FF})x^{FF}}{1 - r_u(x^{FF})} = \frac{F^F}{c^F L}. \tag{2}$$

Third, producer's cost optimization implies

$$c^{'}\left(F^{H}\right)\left(sLx^{HH}+\left(1-s\right)Lx^{HF}\right)=-1,\quad c^{'}(F^{F})(sLx^{FH}+(1-s)Lx^{FF})=-1, \tag{3}$$

Fourth, the balance in labour markets give us

$$N^{H}(F^{H}+c^{H}sLx^{HH}+c^{H}(1-s)Lx^{HF})=sL, \tag{4}$$

$$N^{F}(F^{F}+c^{F}sLx^{FH}+c^{F}(1-s)Lx^{FF})=\left(1-s\right)L. \tag{5}$$

Finally, consumer's budget constraints in the symmetric equilibrium take the form

$$N^{H}p^{HH}x^{HH}+N^{F}p^{FH}x^{FH}=w,\qquad N^{H}p^{HF}x^{HF}+N^{F}p^{FF}x^{FF}=1, \tag{6}$$

where

$$p^{HH}=\frac{wc^{H}}{1-r_{u}(x^{HH})},\qquad p^{HF}=\frac{wc^{H}}{1-r_{u}(x^{HF})},\qquad p^{FF}=\frac{c^{F}}{1-r_{u}(x^{FF})},\qquad p^{FH}=\frac{c^{F}}{1-r_{u}(x^{FH})}. \tag{7}$$

Thus, the symmetric equilibrium is determined by the system (1)-(7).

Our goal is to find the relative market size (s) affects all market outcomes: investments, prices, outputs, etc. So far only the results for some particular utility functions and technologies are obtained. In particular, under CES-function, when the elasticity of marginal costs is monotone with respect to fixed costs, then in equilibrium $F^{H}=F^{F}$, i.e. *the market size does not affect the technology used*, which is important for trade theory.

Our preliminary results show that this result need not hold true in general case.

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On Models of Information Management in Social Networks

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Keywords: *social network, Nash equilibrium, information management.*

In recent years, considerable research interest was focused on social networks – for example, authors of the survey [1] or the monograph [3] demonstrate the large number of studies on the formation of networks and their dynamics.

Term social network describes social structure, consisting of elements or agents (the actors - individual or collective: individuals, families, groups, organizations, for example) and a set of relations between them (such as dating, friendship, cooperation, communication and etc.). Social network as mathematical object can be defined as a finite weighted graph, where weights of edges depend on interaction of agents.

In the modeling of social networks it is usually assumed that the main characteristic of its element (his opinion on any issue or contamination in spreads of epidemics and etc.) changes in time according to some specified law on the basis of characteristics of neighboring elements. In these models a network element is assumed to be passive. Let's call them models of the first type.

In some models a network element chooses the characteristics (e.g, to act or not) on the basis of their abilities and interests. In such models, in contrast with models of the first type, network element is active and is considered to have its own interests (formalized in the form of the objective function) and freedom of choice. These ones are models of the second type.

This work is devoted to investigation of an example of a combined model and takes into account both types of models: let's characterize each element by some parameter that can vary under the influence of other agents and which can influence the governing body - the center. At the same time, each agent has an objective function and chooses his action from the set of feasible actions.

Formally speaking we suppose that there is a finite set of agents, N = {1, 2, ..., n}, each of which could be characterized by a parameter – type – ri (where 0 <ri <1), the objective function fi and selects the action xi (where xi is a nonnegative real number). In this example we use the following objective function of i-th agent:

$$f_i(x_1,...,x_n) = x_i(x_1 + ... + x_n - 1) - \frac{x_i^2}{r_i}. \tag{1}$$

Interpretation of (1) is the following: agents are making actions xi to make some joint action that is successful (increase the objective function of an agent) if the amount of action exceeds a certain threshold, which could be taken equal 1. If the action was successful, the gain of the agent (the first part of the objective function) depends on his action: the greater action the greater gain. On the other hand, action of the agent, by itself, decrease its objective function (second part of the objective function), which depends on the type ri – the larger its type the less are his costs to make action (for example, it can be explained if we consider type as experience or skill of agent).

Structure of equilibrium of this game is described in [4]. It was shown that a set of actions (0, 0, ..., 0) to be always a Nash equilibrium, and while the condition

$$\sum_i \frac{r_i}{2 - r_i} > 1 \tag{2}$$

is true there is also a non-zero equilibrium.

Now let us consider external center, which seeks to control the types of agents to bring them into the most desirable for him Nash equilibrium and assume that the goal of the center is to find the set of network states (sets of types of agents), in which his objective function reaches its maximum value.

Some possible options for the objective functions of the center are discussed in details in [2]. In this work we choose the following objective function:

$$F = -\sum_j x_j. \tag{3}$$

It describes the situation when any non-zero actions of agents for the center are undesirable, and its objective is to minimize their total actions. An example would be the reaction of security systems to the actions of intruders.

In this model any situation when actions of agents are zero actions is beneficial for the center. It means that it is optimal for him to find the way to exclude a non-zero Nash equilibrium by making it impossible for agents (the models of influence on the types of agents in social networks are described, for example, in [1, 2, 4]). In this case, a non-zero Nash equilibrium does not exist for non-compliance with condition (2).

So, by now we obtained a condition on the types of agents, under which the center achieves its goals. Under this condition, the system does not have any nonzero Nash equilibrium and, thus, the only equilibrium is zero, in which the objective function of the center reaches its maximum value.

Let's consider the possibilities to implement the control by changing the initial opinion (types) of the agent. Being in a social network, agents communicate to exchange views. Agents in the network affect each other, and the degree of influence given by the direct trust matrix A of dimension n×n, where aij ≥ 0 means the degree of confidence in the i-th agent's j-th agent.

Here we may speak about influence, and about trust, and assume that these two concepts are opposite in the following sense: the expression «degree of confidence i-th agent is equal to the j-th aij » identity within the meaning of the expression «degree of influence of the j-th agent on the i-th equal to aij».

Exchange of views leads to exchange of types of agents according to the types of agents, which he currently trusts. In the case of agent interaction lasts long enough, their types, stabilize (stabilization condition is described in detail in [1]) and is fully determined by initial types and the resulting matrix of influence

$$A^{\infty} = \lim_{\tau \to \infty} (A)^{\tau}.$$

If we define the initial vector of types r0 types vector r becomes (in the absence of any effects of the center) as follows:

$$r = A^{\infty} r^{0}.$$

If the center's information management characterized by the vector u, thus reducing their types in the initial time (from (2) it is clear that the center goal would be to reduce the types of agents) and the result vector ru = (r1, ..., rn) takes the form :

$$r^{u} = A^{\infty}(r^{0} - u). \qquad (4)$$

Thus, the question of the existence of an information management center, providing the absence of non-zero equilibrium, is reduced to the existence of control u, allows the simultaneous fulfillment of relations

$$\sum_i \frac{r_i}{2 - r_i} \leq 1 \tag{5}$$

and (4). In general, the analytical verification of the existence of such management is equivalent to solving an optimization problem of minimizing the left side (5) (under appropriate restrictions on the vector u).

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Selfish Routing Problem in Wireless Network

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Keywords: *Wireless network, Selfish routing, SNR, Nash equilibrium*

The problem of selfish routing in wireless network is considered. The problem is presented as a game, where players are mobile connection devices, which choose radio stations to connect to the network. Strategies in the game are probabilities, which players use to choose stations. Players act selfish. Each device chooses a station trying to maximize its ratio “signal/noise” (SNR), which depends on: 1) the distance between the device and the station, 2) connections of all devices on the station. In this model signal is inversely as the square of his distance to chosen station, and noise is a sum of all signals at the station and some constant noise (white noise). Nash equilibrium in this model is an object of the research.

As an example consider the 1-dimensional game with 2 players moving on interval $[0,1]$. Two identical radio stations (“0” and “1”) with white noise levels c are situated at the corresponding ends of the interval. At each time moment first player's coordinate is some $x \in [0.1]$, second is some $y \in [0,1]$. Each player in each situation (x, y) must determine which of two stations is better to connect. Players define following strategies: p is a probability that first players chooses the station 0, q is a probability that second players chooses the station 0. For station 1 probabilities are correspondingly $1-p$ and $1-q$. Payoff function for each player is an expectation of his SNR:

$$H^1_{x,y}(p,q) = \frac{pq}{1 + \dfrac{x^2}{y^2} + cx^2} + \frac{p(1-q)}{1+cx^2} + \frac{(1-p)q}{1+c(1-x)^2} + \frac{(1-p)(1-q)}{1 + \dfrac{(1-x)^2}{(1-y)^2} + c(1-x)^2},$$

$$H^2_{x,y}(p,q) = \frac{pq}{1 + \dfrac{y^2}{x^2} + cy^2} + \frac{p(1-q)}{1+c(1-y)^2} + \frac{(1-p)q}{1+cy^2} + \frac{(1-p)(1-q)}{1 + \dfrac{(1-y)^2}{(1-x)^2} + c(1-y)^2}.$$

How to Play the Games? Nash versus Berge Behavior Rules

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Keywords: *Berge Equilibrium, Moral preferences, Mutual support, Social Dilemmas, Situations.*

Although experimental evidence supports the predictions of standard economic theory about the outcomes of social interactions in several competitive situations (Davis and Holt, 1993), it usually does not corroborate the theory in cases of cooperative situations. In two-player zero sum games, when there is a Nash equilibrium, subjects usually play according to that strategy (Lieberman, 1960) and even exploit the non-optimal responses of their partner in order to maximize their benefits (Kahan and Goehring, 2007). Conversely, in mixed-motive game experiments, subjects often cooperate more than they should. Despite the wide disparity in the experimental results and protocols on the prisoner's dilemma (PD), meta-analysis shows that, on average, about 50% of subjects cooperate (Colman, 1995; Ledyard, 1995; Sally, 1995). Similar anomalies related to Nash predictions have been documented in studies of the chicken game (CG), where cooperation is the dominant outcome observed (Rapoport and Chammah, 1965), and in trust or investment games, where trust and reciprocity are often more significant than predicted by subgame perfect equilibrium (see e.g. Berg et al., 1995; Bolle, 1998).

Fischbacher et al. (2001) and Kurzban and House (2005) observe that in public good experiments, not all subjects play in the same fashion. In their experiments, more than 50% and up to 65% of subjects contribute on condition that others do the same, less than 30% are pure free riders, and the rest adopt a mix of these two behaviors. This illustrates the heterogeneity of behavior rules: individuals have types and are likely to adapt their behavior to their immediate environment (see also Boone et al., 1999; Brandts and Schra, 2001; Keser and Winden, 2000; Fehr and Fischbacher, 2005).

Experiments also show that individuals may change behavior according to the situation of their play. For example, Zan and Hildebrandt (2005) found in a school experiment that children adopt different behavior rules according to the type of game they are playing, with cooperative games involving more reciprocal interactions than competitive ones. Players may also adopt different behavior rules according to their life situations and to the society in which they live. Henrich et al. (2004, 2010) for example, observe that fairness and mutually beneficial transactions are more frequent in integrated societies. Their interpretation is that subjects conform less to Nash predictions, as if they were being motivated by social norms reflecting moral preferences. This conclusion is confirmed by other experimental studies such as Engelmann and Normann (2010). These authors have found that levels of contribution in a minimum-effort game vary across countries and, more interestingly, between natives, former immigrants, and new immigrants within the same country.

The formal literature often considers moral preferences in games as other-regarding payoff transformations, a concept first mooted by Edgeworth (1893). The key to this approach is the assumption that a player's utility is a twofold function, related to individual welfare and to the welfare of the other. Individuals are found to care about how payoffs are allocated, depending on the partner, the game situation, and how the allocation is made. To redefine the utility function in this way allows for rationalizing behaviors. The Nash behavior rule is safe and players are supposed to choose the actions that maximize their individual redefined utility, given that others do the same. However, to include moral preferences in the utility function leads us to assume that moral agents are concerned only with outcomes rather than with actions, which is a peculiar interpretation of moral preferences. As Vanberg (2008: 608) contends, "moral principles, standards of fairness, justice,[...] are codes of conduct that require persons to act in fair, just, or ethical ways. They tell them not to steal, not to lie, to keep promises, etc. They are typically concerned not so much with what a person wants to achieve but with how she seeks to achieve what she wants". In other words, moral preferences are also preferences for acting morally, following a moral rule of conduct.

Our principal motivation in this paper is to examine whether complementary behavior rules and equilibrium may be intertwined with the Nash rule, leaving the payoff matrix unchanged. In line with Pruit and Kimmel (1977), we confer on individuals the capacity to adapt their behavior rules to the situation. In some games, such as zero-sum situations where self-oriented maximization is sufficient to drive action, the Nash rule would tend to be adopted. In others, such as games involving collective action, complementary rules embedding moral preferences would potentially drive the action. This situational perspective has analogies with the rule-following behavior approach proposed by Vanberg (2008), and is more generally inspired by the situational approach in social psychology, according to which personality is construed not as a generalized or a contextual tendency but as a set of "If ...then" contingencies that spawn behaviour of the "If situation X, then behavior Y" type (Mischel and Shoda, 1995, 1999).

We focus on how to play usual game situations such as PD, CG or trust games and posit one possible complementary behavior rule to Nash and its associated equilibrium concept. Keeping the utilitarian perspective and building on the experimental observation that the majority of subjects are reciprocal in public good games, we focus on specific forms of reciprocity that may well illustrate collective decision-making. Our behavioral hypothesis is that choice in many interactive situations requires that each player make the welfare of the other a key feature of his or her reasoning. Individuals would choose to play this way because this is a common rule-following behavior that improves social welfare in many situations. This is mutual support and leads us to posit that in some game situations, individuals care about the welfare of others if they believe that others reciprocate. Real life examples are numerous and are related to savoir vivre, a set of rules of conduct such as respect for others, politeness or courtesy.

To examine mutual support in social interactions, we exploit an old concept, the Berge equilibrium. We think this concept is appropriate for two main reasons. The first is that mutual support is a possible interpretation of Berge equilibrium. Playing under Berge rules, agents choose the strategy that maximizes the welfare of others. The second is that, theoretically, the Berge behavior rule and Berge equilibrium are good complements for Nash: Berge equilibrium is defined in a non-cooperative game theoretical setting but is not a refinement of Nash; it explains some cooperative situations while Nash explains many competitive ones; and it has some common theoretical properties with Nash, making it particularly appropriate and easy to handle in a type-based perspective.

Multilevel N Leader – M Follower Decision Making Models with Genetic Algorithms and Applications

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Keywords: *Hierarchical multi-level games, Stackelberg-Nash strategy, Genetic algorithm*

In this work we deal with hierarchical games with one (and in general N) players acting as leader(s) in a two level leader-follower model where the rest of players play a noncooperative game and react to the optimal decision taken by the leader(s). When we deal with more that one leader, we assume that the leaders also play a noncooperative game between themselves.

The leader(s) takes into account the followers' best reply and solve an optimization problem (a Nash equilibrium problem). In this model the uniqueness of the Nash equilibrium of the follower players has been supposed.

We present a computational methodology to reach a Stackelberg - Nash solution for the hierarchical game via genetic algorithm (GA).

The idea of the Stackelberg-Nash GA is to bring together genetic algorithms and the leader-follower strategy in order to process a genetic algorithm to build the solution. The follower players make their decisions simultaneously at each step of the evolutionary process, solving a Nash equilibrium problem.

This methodology is moreover extended to a hierarchical L-level (or multi level) game in which more than a leader hierarchy (and so strategy strenght indeed) could be modelled, a very common situation in field such as engineering, economics and social problems.

Applications in simple mathematical functions would emphasize the algorithm capability of explore and describe variuos phenomena with a good agreement between results precision and computation time.

International Law Regimes Modification: Game-Theoretic Approach

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Keywords: *Game theory in International Relations, Classical Games in International Relations, Game theory in Law, Bargaining, Enforcement in International Law, Applied Game theory in USSR and Russia*

At the Cold War epoch the game theory used to be a corner stone of the global deterrence policy. Nowadays the shift towards multipolarity in international relations makes this method of research of IR actual again. In the article the evolution of applications of games theory in IR including modern period is given as well as principal causes on why the game theory hasn't received a wide circulation in Russian and Soviet international studies. A list of classical games which are used to analyse modification of international regimes is provided. Finally, application of game-theoretic approach to international regimes is illustrated in detail by global ecological issues case.

Solutions for One-Stage Bidding Game with Incomplete Information

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Keywords: *Bidding game, Incomplete information, Optimal strategy*

We investigate a model of one-stage bidding between two stockmarket agents where one unit of risky asset (share) is traded. Before bidding starts a chance move determines the "state of nature" and therefore the liquidation value of a share. This value is equal to the integer positive m (the state H) with probability p and 0 (the state L) with probability $1 - p$. Player 1 (insider) is informed about the "state of nature" , Player 2 is not. Both players know probabilities of a chance move. Player 2 knows that Player 1 is an insider.

Both players propose simultaneously their bids, t.e. they post their prices for a share. Any integer bids are admissible, but the reasonable bids are $0,1,\ldots,m-1$. The player who posts the larger price buys one share from his opponent for this price. If the bids are equal, no transaction occurs. The described model of one-stage bidding is reduced to zero-sum game $G_1^m(p)$ with lack of information on the side of Player 2. It is the one-stage case of n-stage bidding game $G_n^m(p)$ with incomplete information investigated in Domansky (2007). For the first time this model with arbitrary admissible bids was introduced in De Meyer, Saley (2002).

The bidding game $G_\infty^m(p)$ of unlimited duration was solved in Domansky (2007). But the solution of n-stage games $G_n^m(p)$ is an open problem. It is obtained for the case of three admissible bids only ($m = 3$), see Kreps (2009). We give the complete solution for the one-stage bidding games $G_1^m(p)$ with arbitrary integer m and with any probability $p \in (0,1)$ of the high share price.

It follows from the theory of games with incomplete information that the value $V_1^m(p)$ of the game $G_1^m(p)$ is a continuous concave piecewise linear function over $[0,1]$ with a finite numbers of domains of linearity. The optimal strategy of the uninformed Player 2 is constant over any linearity domain.

Proposition 1. *The set of peak points of value function $V_1^m(p)$ over the interval $(0,1)$ is the unification $P^m \cup Q^m \cup S^m \cup T^m$, where*

a) $P^m = \{p_1,\dots,p_{m-1}\}$, $0 < p_1 < \dots < p_{m-1}$ *:*

$$1 - p_1 = \frac{m-1}{m}, \quad 1 - p_2 = \frac{m-2}{m-1}, \qquad 1 - p_k = (1 - p_{k-2})\frac{m-k}{m-k+1}.$$

b) $Q^m = \{q_0,\dots,q_{m-2}\}$, $1 > q_0 > \dots > q_{m-2} = p_{m-1}$ *:*

$$1 - q_0 = \frac{1}{m}, \quad 1 - q_1 = \frac{1}{m-1}, \qquad 1 - q_k = \frac{1 - p_{k-1}}{m-k}.$$

c) $S^m = \{s_2,\dots,s_{m-2}\}$, *where* $p_2 < s_2 < p_3,\dots,p_{m-2} < s_{m-2} < p_{m-1}$.

d) $T^m = \{t_1,\dots,t_{m-3}\}$ *,where* $q_1 > t_1 > q_2,\dots,q_{m-3} > t_{m-3} > q_{m-2} = p_{m-1}$.

For any peak points p the inequality

$$V_1^m(p) \le m \cdot p(1-p)(1)$$

holds and for any $p \in P^m \cup Q^m$ it turns to be the equality.

Remark 1. *As the value of one-stage bidding game with arbitrary bids is equal to $m \cdot p(1-p)$, see De Meyer, Saley (2002), the inequality (1) implies that this value exceeds the value of one-stage bidding game with discrete bids. For points $p \in P^m \cup Q^m$ the value $V_1^m(p)$ coincides with the value of one-stage bidding game with arbitrary bids.*

Corollary 1. *As the set $P^m \cup Q^m$ is everywhere dense over $[0,1]$, then*

$$\lim_{m\to\infty} V_1^m(p) \,/\, m = p(1-p).$$

Here we restrict ourselves to a description of solutions of game $G_1^m(p)$ for $p \in (0, p_{m-1} = q_{m-2})$, the left part of the interval (0.1). The solution for the right part $(q_{m-2}, 1)$ is analogues and (non strictly speaking) mirror-like with respect to the point $p = p_{m-1} = q_{m-2}$.

Obviously, if the share price is zero (the state L) that Player 1 (insider) posts the zero bid for any probability p. Thus, the problem is to describe the optimal strategy of Player 1 for the high share price (the state H) and the optimal strategy of the uninformed Player 2.

The following Proposition 2 describes the supports of optimal strategies over the intervals of linearity of value function $V_1^m(p)$. For Player 1 it is the case of state H.

Proposition 2. *For* $0 < p < p_1 = 1 / m$ *Player 2 uses the bid* 0. *Player 1 uses the bid* 1.

For $p_1 < p < p_2 = 1 / (m-1)$ *Player 2 uses the bids* 0 *and* 1. *Player 2 uses the bids* 0, 1 *and* 2 *for* $p = p_2 = 1 / (m-1)$.

For $p_2 < p < s_2$ *Player 2 uses the bids* 0 *and* 2.

For the double interval $p_1 < p < s_2$ *Player 1 uses the bids* 1 *and* 2.

For $s_k < p < p_{k+1}$, $k = 2,\dots,m-2$ *Player 2 uses the bids* $0,2,3,\dots,(k+1)$, *if the number* k *is even and Player 2 uses the bids* $0,1,3,\dots,(k+1)$ *if* k *is odd. Player 2 uses the bids* $0,1,2,3,\dots,(k+1)$ *for* $p = p_l$.

For $p_k < p < s_k$, $k = 3,\dots,m-2$ *Player 2 uses the bids* $0,1,3,\dots,k$ *if the number* k *is odd and Player 2 uses the bids* $0,2,3,\dots,k$ *if* k *is even.*

For the double interval $k,1 \cup k,2$ *Player 1 uses the bids* $0,1,2,,\dots,k$.

For any $k = 3,\dots,m-2$ there are two linearity intervals over $(0, p_{m-1})$ where the maximal bids of the Player 2 optimal strategy is k. One interval has the form (s_l, p_{l+1}) and we denote it $k,1$. The other interval has the form (p_{l+1}, s_{l+1}) and we denote it $k,2$. We write $2,1$ and $2,2$ for the intervals (p_1, p_2) and (p_2, s_2).

Consider the best reply of Player 1 to the optimal strategy of Player 2 for $p \in k,i$, $i = 1,2$. Let $v_{k,i}^H$ ($v_{k,i}^L$) be the corresponding payoff of Player 1 for the state H (for the state L).

The following Theorem provides the recurrent description of value function $V_1^m(p)$ for any linearity domain.

Theorem. *For the interval* k,i, $k = 3,\ldots,m-2$, $i = 1,2$

$$V_1^m(p) = v_{k,i}^L(1-p) + v_{k,i}^H p,$$

where payoffs $v_{k,i}^H$ *and* $v_{k,i}^L$ *are given by the recurrent formulas*

$$v_{k,i}^H = \frac{(m-k)^2}{v_{k-1,i+1}^H}, \quad v_{k,i}^L = (v_{k-1,i+1}^L - k)\left(\frac{m-k}{v_{k-1,i+1}^H}\right) + k,$$

with the initial payoffs $v_{2,1}^L = 1/(m-1)$, $v_{2,1}^H = m-2$, $v_{2,2}^L = 2/(m-1)$, $v_{2,2}^H = (m-2)^2/(m-1)$.

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Decomposition of Distributions $\mathbb{R}^n$ and Models of Multistage Bidding

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Keywords: *Multistage bidding, Repeated game, Incomplete information.*

1. We investigate the model of multistage bidding where n types of risky assets (shares) are traded. Two agents take part in the bidding having different information on liquidation prices of the traded assets. These prices are random values. They are determined by the initial chance move for the whole period of bidding according to a known to both players probability distribution p over $\mathbb{R}^n$. Player 1 knows prices of shares. Player 2 does not have this information but he knows that Player 1 is an insider.

At each step of bidding both players make simultaneously their bids, i.e. they post their prices for each type of shares. The player who posts the larger price for a share of given type buys one share of this type from his opponent for this price. Any integer bids are admissible. Players aim to maximize the values of their final portfolios, i.e. money plus obtained shares evaluated by their liquidation prices. For the first time the model with arbitrary bids is introduced by De Meyer and Moussa Saley (2002).

The described model of multistage bidding is reduced to the zero-sum repeated game with lack of information on one side. Here we give solutions to such games of infinite duration $G_\infty(p)$ with arbitrary distributions p with finite variances. To describe the optimal strategy of Player 1 we construct the symmetric representation of n-dimensional probability distributions $p \in \Theta(y)$ with a fixed expectation vector y as convex combinations of extreme points of the set $\Theta(y)$, i.e. distributions with not more than $(n+1)$-point supports. This is sufficient to give such representation for the set $\Theta(0)$.

This representation is a straight generalization of the analogous representation for distributions over two-dimensional integer lattice obtained in Domansky and Kreps (2010).

We construct optimal strategies of Player 1 for bidding games with arbitrary distributions $p \in \Theta(x)$ as convex combinations of his optimal strategies for games with distributions having not more than $(n+1)$-point supports, i.e. the elements of the symmetric representation of the initial distribution.

For each type of assets optimal strategies of Player 2 reproduce his optimal strategies for games with one asset, see Domansky and Kreps (2009).

2. Let Δ^0 be the set of $(n+1)$-point sets $(x^1,\ldots,x^{n+1})$, $x^i \neq 0$, such that the point 0 belongs to the simplex $\Delta(x^1,\ldots,x^{n+1})$. Let $\mathrm{Int}\Delta^0$ be the set of $(n+1)$-point sets from Δ^0 such that the point 0 belongs to the interior of the simplex, and let $\partial^k\Delta^0$, $k=1,\ldots,n$ be the set of $(n+1)$-point sets from Δ^0 such that the point 0 belongs to the $(n-k)$-face of the simplex.

For $x \in \mathbb{R}^n$, $x \neq 0$, let $\Delta^0(x)$, $\mathrm{Int}\Delta^0(x)$ and $\partial^k\Delta^0(x)$ be the sets of n-point sets $(x^1,\ldots,x^n)$ such that the set $(x^1,\ldots,x^n,x)$ belongs Δ^0, $\mathrm{Int}\Delta^0$ and $\partial^k\Delta^0$ correspondingly.

For a distribution $p \in \Theta(0)$ and a point $x \in \mathbb{R}^n$, $x \neq 0$ set

$$\Phi(p,x) = \int_{\mathrm{Int}\Delta^0(x)} \det[x^1,\ldots,x^n]p(dx^1)\ldots p(dx^n)$$

$$+\sum_{k=1}^{n-1}\frac{1}{k+1}\int_{\partial^k\Delta^0(x)} \det[x^1,\ldots,x^n]p(dx^1)\ldots p(dx^n).(1)$$

Theorem 1. *For any distribution $p \in \Theta(0)$ the amount $\Phi(p,x)$ does not depend on x, i.e. this is an invariant $\Phi(p)$ of the distribution $p \in \Theta(0,0)$.*

This fact is the base for constructing the symmetric representation of probability distributions over $\mathbb{R}^n$ with fixed expectations as convex combinations of distributions with not more than $(n+1)$-point supports and with the same expectations.

Theorem 2. *Any distribution* $p \in \Theta(0)$ *has the following representation as a convex combination of distributions with not more than* $(n+1)$*-point supports:*

$$p = p(0)\cdot\delta^0 + \int_{\mathrm{Int}\Delta^0} \frac{\sum_{i=1}^{n+1} \det[x^{i+1},\ldots,x^{i+n}]}{\Phi(p)} \cdot p^0_{(x^1,\ldots,x^{n+1})} \cdot p(dx^1)\ldots p(dx^{n+1})$$

$$+\sum_{k=1}^{n-1}\frac{1}{k+1}\int_{\partial^k\Delta^0} \frac{\sum_{i=1}^{n+1} \det[x^{i+1},\ldots,x^{i+n}]}{\Phi(p)} \cdot p^0_{(x^1,\ldots,x^{n+1})} \cdot p(dx^1)\ldots p(dx^{n+1}),$$

where $p(0)$ *is the atom of distribution* p *at* 0, *and* $\Phi(p)$ *is given by (1). All arithmetical operations with subscripts are fulfilled modulo* $n+1$.

3. Now we construct optimal strategies for Player 1 making use of the developed above decomposition for the initial distribution p.

a) If the state chosen by chance move is 0, then Player 1 stops the game.

b) Let the state chosen by chance move be $x \neq 0$. Player 1 chooses a number of points by means of lottery with probabilities

$$P(n) = \frac{\mathrm{Int}\Phi(p,x)}{\Phi(p)}; \quad P(n-k) = \frac{\partial^k\Phi(p,x)}{\Phi(p)}.$$

c) If a number of points n is chosen, then Player 1 chooses a set of n points $(x^1,\ldots,x^n) \in \mathrm{Int}\Delta^0(x)$ by means of lottery with probabilities

$$\alpha_{(x^1,\ldots,x^n)}(p)\cdot p(dx^1)\ldots p(dx^n) = \frac{\det[x^1,\ldots,x^n]}{\Phi(p)} \cdot p(dx^1)\ldots p(dx^n).$$

and further plays his optimal strategy $\sigma^*(\cdot \mid x)$ for the state x in the $(n+1)$-point game $G(p^0_{x^1,\ldots,x^n,x})$.

d) If a number of points $n-k$ is chosen, then Player 1 chooses a hyperplane h^{n-k} containing the point x by means of lottery with probabilities

$$\frac{\int_{\partial^k\Delta^0(x,h^{n-k})} \det[x^1,\ldots,x^n]p(dx^1)\ldots p(dx^n)}{\partial^k\Phi(p,x)}.$$

e) If a hyperplane h^{n-k} is chosen, then Player 1 chooses a set of $n-k$ points $(x^1,\ldots,x^{n-k}) \in h^{n-k}$ by means of lottery with probabilities

$$\frac{\det[x^1,\dots,x^{n-k}]p(dx^1)\dots p(dx^{n-k})}{\int_{\mathrm{Int}\Delta^0(x,h^{n-k})}\det[x^1,\dots,x^{n-k}]p(dx^1)\dots p(dx^{n-k})},$$

and further plays his optimal strategy $\sigma^*(\cdot \mid x)$ for the state x in the $(n-k+1)$-point game $G(p^0_{x^1,\dots,x^{n-k},x})$.

As the optimal strategies σ^* ensure Player 1 the gains equal to one half of the sum of component variances in the $(n-k+1)$-point games and as the sum of component variances is a linear function over $\Theta(0)\cap M^2$, where M^2 is the class of distributions with finite second moments, we obtain the following result:

Theorem 3. *For any distribution $p\in\Theta(0)\cap M^2$ the compound strategy depicted above ensures Player 1 the gain $\frac{1}{2}\sum_{i=1}^{n}D_p[x_i]$ in the game $G_\infty(p)$.*

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On The Computation of Individual Penalties in a Problem of Minimizing the Total Penalty for Late/Early Completion of Jobs on A Single Machine

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Keywords: *Scheduling, Shapley value, Average per capita formula, Nucleolus.*

In this paper we give a technology to solve the problem of scheduling several jobs on a single machine with late/early penalties and a common due date. This requires at least two algorithms belonging to discrete optimization and game theory, respectively; a third algorithm for solving a sequence of linear optimization problems would solve better the problem. It is well known that the problem of scheduling n jobs on a single machine with a common due date can be solved in the most simple case by the algorithm due to J.J.Kanet (1981). Given such a problem by a list of completion times with the sum smaller than, or equal to the due date, the total deviation from the due date will be minimized for every coalition of customers, generating a cooperative TU game. Now, the total worth of the penalties, given by the worth of the grand coalition, should be divided "fairly" to the customers, providing the individual penalties. An efficient value, like the Shapley value would give the individual penalties; however, if the number of jobs is large, the Shapley value should be computed by an algorithm due to the author, based upon a so called Average per capita formula, which allows a parallel computation, (Dragan,1992). If we still consider that the results provided are unfair, we can use an algorithm providing the nucleolus of the game, which is asking for solving a sequence of no more than n linear optimization problems. We give a four customers example how the three stages of the technology, and the corresponding algorithms will work. The algorithm for computing the nucleolus may be the Kopelowitz algorithm (1967), or the algorithm due to the author (1981). If the scheduling problem is more complex, then other algorithm may be needed to generate the scheduling game.

On One-Concavity of Data Cost Games

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Keywords*: Data cost game, 1-concavity, Nucleolus, Core*

This paper broadens the game theoretic approach to the data sharing situation initiated by Dehez and Tellone (2010). The origin of their mathematical study is the data and cost sharing problem faced by the European chemical industry. Following the regulation imposed by the European Commission under the acronym "REACH" (Registration, Evaluation, Authorization and restriction of CHemical substances), manufacturers and importers are required to collect safety information on the properties of their chemical substances. There are about 30,000 substances and an average of 100 parameters for each substance. Chemical firms are required to register the information in a central database run by the European Chemicals Agency (ECHA). By 2018, this regulation program REACH requires submission of a detailed analysis of the chemical substances produced or imported. Chemical firms are encouraged to cooperate by sharing the data they have collected over the past. To implement this data sharing problem, a compensation mechanism is needed.

This data sharing problem can be specified as follows. A finite group of firms agrees to undertake a joint venture that requires the combination of various complementary inputs held by some of them. These inputs are non-rival but excludable goods, i.e., public goods with exclusion such as knowledge, data or information, patents or copyrights (the consumption of which by individuals can be controlled, measured, and subjected to payment or other contractual limitations). In what follows we use the common term data to cover generically these goods. Each firm owns a subset of data. No a priori restrictions are imposed on the individual data sets. In addition, with each type of data there is associated its replacement cost, e.g., the present cost of duplicating the data (or the cost of developing alternative technologies). Because these public goods are already available, their costs are sunk. In summary, the data sharing situation involves a finite group of agents, data sets owned by individual agents, as well as a discrete list of costs of data.

In the setting of cooperative attitudes by chemical firms, the main question arises how to compensate the firms for the data they contribute to share. The design of a compensation mechanism, however, is fully equivalent to the selection among existing solution concepts in the mathematical field called cooperative game theory. In fact, the solution part of cooperative game theory aims at solving any allocation problem by proposing rules based on certain fairness properties. For that purpose, the data and cost sharing situation needs to be interpreted as a mathematical model called a cooperative game by specifying its fundamental characteristic cost function. We adopt Dehez and Tellone's game theoretic model in which the cost associated to any non-empty group of agents is simply the sum of costs of the missing data, i.e., the total cost of data it does not own. In this framework, no cost are charged to the whole group of agents. The so-called data cost games are therefore compensation games to which standard cost allocation rules can be applied, such as the Shapley value, the nucleolus, the core and so on. The determination of these game theoretic solution concepts may be strongly simplified whenever the underlying characteristic cost function satisfies, by chance, one or another appealing property. The main purpose of this paper is to reveal the so-called 1-concavity property for the class of data cost games. The impact of the 1-concavity property is fundamental for the uniform determination of particular solution concepts like the core and the nucleolus.

The Game Theoretical Effects of Border Tax Adjustments

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Keywords: *Game Theory, Border tax, Adjustment, International environmental agreements, Competitiveness, Carbon leakage.*

Carbon leakage and competitiveness concerns are some of the main reasons why a global environmental agreement is lacking to fight climate change. Many studies discussed the adoption of a border tax adjustment (BTA) to allow countries that would like to implement a carbon tax to level the playing field with imports. The big issue with these CGE type papers is that the other country is not allowed to react by adopting itself a carbon tax to avoid being punished with the BTA. The model proposed in this paper looks at the optimization of two different governments and their respective firms. A BTA parameter is modelled which takes a value between 0 and 1 and represents the possible value of the BTA depending on both countries environmental policy allowing countries to have different policies. Optimal tax rates and quantities are derived for many potential scenarios. For given parameter values and a BTA parameter above 0.25 we have found it optimal for both countries to tax where the two taxes reach equality at 0.72. In addition, a BTA parameter of 0.5 yields the highest total welfare. This finding could give significant insight on the role the WTO with respect to BTA.

On Bilateral Barters

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Keywords: *Exchange markets, Bilateral barters and cometitive equilibrium, Management of energy and resources*

We study a economy in which a fixed finite set I of economic agents exchange commodity bundles or contingent claims, in order to maximize their own utilities. Important instances which are relevant to management of energy and resources comprise exchange of natural resources or user-rights to such, transfers of fish quotas, production allowances, pollution permits, or rights to water use. The exchange of contingent claims, in which case, what comes into focus is mutual insurance or security exchange, is also an important economy of that type.

In that setting, it is well known that when transferals are costless and multilateral trades can be arranged at no expense, and when some technical conditions are fulfilled, voluntary trade will lead inevitably to Pareto optimality. However, when transferals are costly and multilateral trades difficult or impossible, optimality is problematical. In that case, this paper asks: *Can equilibrium - whence Pareto efficiency - obtain merely via repeated, bilateral exchange?*[1] And, most importantly: *may traders dispense with optimization?* Moreover, while exchange still remains in swing, *can parties proceed without announcement of prices*? That is, might market equilibrium - and the attending prices - emerge as *final* outcomes, identifiable only *after* all desirable transactions are completed?

We propose a repeated bilateral barters process between owners of commodity bundles or contingent claims, where the focus is on feasible, free, but constrained exchanges, driven by differences in margins. Presuming transferable utility, we provide sufficient conditions for convergence to market equilibrium. It facilitates transactions

[1] Several studies consider links between pairwise, t-wise, and overall Pareto optimality; see e.g. Feldman (1973), Madden (1975), Goldman and Starr (1982), Fisher (1989).

that some parties make strictly feasible choices - and have continuously differentiable objectives. In quite common, most convenient cases each constraint set is polyhedral.

In addressing these issues the paper relates to production games and price-supported core solutions (Flå m (to appear), Osborne and Rubinstein (1994)). Unlike most studies, it plays down the importance of agents' experience, their information, rationality and skills. Rather, since real parties often are short on some of such desirable requisites - and because the goods exchanged are diverse and divisible - what we propose is an agent-based, dynamic (but fictitious) experiment, driven by myopic traders (Smith (1982), Tesfatsion and Judd (2006)). Presumably, trade proceeds until no two agents can improve their joint lot. When stable, the resulting process provides some micro-foundation for market equilibrium.

Numerous studies have offered strategic underpinning of such equilibrium. Many deal, however, with anonymous trading posts, or merely with two-sided markets - or with indivisible goods, to be transferred from sellers to buyers. One line of inquiry let agents meet pairwise, maybe randomly, to bargain. Other approaches evoke auctions (Osborne and Rubinstein (1990), Roth and Sotomayor (1990) and Rubinstein and Wolinsky (1990)).

In contrast, the market we consider isn't composed of pure buyers and sellers; diverse parties may demand some goods but offer others. Goods are divisible and numerous. Encounters could occur periodically, even in fixed order. Bargaining isn't precluded, but never made explicit. Similarly, although permitted, no auction, market maker, or price mechanism need ever come into play. Also, at no intermediate stage must choice be efficient, optimal or stable somehow.[1] And finally, agents can come with general feasible sets and non-smooth objectives.

[1] For these and related issues, see Madden (1975), Shapley and Shubik (1977), Saari (1985), Osborne and Rubinstein (1990), Roth and Sotomayor (1990), Ghosal and Morelli (2004), Gintis (2006).

Sustainability of Cooperation over Time with non-Linear Incentive Equilibrium Strategies

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Keywords: *Differential games, Incentive equilibrium strategies, Credibility*

In the literature several approaches have been proposed to ensure the sustainability over time of an agreement reached at the starting date of a differential game. One of the approaches appropriate for two-player differential games is to support the cooperative solution by incentive strategies, [2], [3]. Incentive strategies are functions of the possible deviation of the other player and recommend to each player to implement his part of the agreement whenever the other player is doing so. The equilibrium incentive approach allows embodying the cooperative solution with an equilibrium property. Therefore, by definition each player will find individually rational to stick to his part in the coordinated solution. One important property that should be checked is the credibility of these incentive strategies, [3], [4]. These strategies are credible if each player will implement his incentive strategy and not the coordinated solution if he observes that the other one has deviated from the agreement. Recently some papers, [5], [6], have provided conditions to check for the credibility of incentive strategies for the class of linear-state and linear-quadratic differential games. To preserve the special structures of the games the analyses have been restricted to linear incentive strategies that are not always credible.

The focus of this paper is the characterization of non-linear incentive equilibrium strategies for two-player differential games. The aim of the study is to check if the definition of less restricted incentive strategies in terms of the permitted deviation from the coordinated solution facilitates the credibility and implementation of these strategies. To this end, we consider a class of incentive strategies that are defined as non-linear functions of the control variables of both players and the current value of the state variable. We show that it is possible to chose the incentive strategy functions in such a manner that, in the long run, the optimal state path is arbitrary close to the corresponding cooperative state trajectory.

The non-linearity of the incentive strategies does not usually allow the analytical characterization of the feasible set which ensures their credibility. We resort to numerical algorithms to carry out this analysis. We illustrate the use of these strategies on a well-known example drawn from environmental economics, [1].

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Market Power in Markets with Price-Fee Competition and Costly Infrastructure

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Keywords: *Cooperative Games, Assignment Games, Market Power.*

We analyze price-fee (or non-linear pricing) competition on a market for a homogenous good. The good is produced against constant marginal costs and supplied through an endogenous infrastructure, where bilateral links are costly to build. For a given infrastructure, we define stable market outcomes that take into account the endogenous expansion of the existing infrastructure. In every such outcome, each buyer exclusively trades with one supplier, the one with whom he can reach maximal bilateral welfare. The price equals marginal costs and the nonnegative fees are bounded from above by the buyer threat to switch suppliers, taking into account building costs for nonexisting links. The maximal stable fees also arise as equilibrium outcomes of a negotiation model that naturally extends oligopolistic price competition. Then, competition in prices and fees is Pareto efficient, because the suppliers capture the deadweight loss from traditional price competition. Depending upon the parameter values, the consumer is either worse of or equally well off compared to traditional price competition.

Model Differential Game with Two Pursuers and One Evader

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Keywords: *Pursuit-evasion differential game, Linear dynamics, Value function, Optimal feedback control*

A model differential game [1] with two pursuers and one evader is studied. Three inertial objects moves in the straight line. The dynamics descriptions for pursuers P_1 and P_2 are

$$\begin{array}{ll} \ddot{z}_{P_1} = a_{P_1}, & \ddot{z}_{P_2} = a_{P_2}, \\ \dot{a}_{P_1} = (u_1 - a_{P_1}) \,/\, l_{P_1}, & \dot{a}_{P_2} = (u_2 - a_{P_2}) \,/\, l_{P_2}, \\ |\, u_1 \,| \le \mu_1, & |\, u_2 \,| \le \mu_2, \\ a_{P_1}(t_0) = 0, & a_{P_2}(t_0) = 0. \end{array} \tag{1}$$

Here, z_{P_1} and z_{P_2} are the geometric coordinates of the pursuers, a_{P_1} and a_{P_2} are their accelerations generated by the controls u_1 and u_2. The time constants l_{P_1} and l_{P_2} define how fast the controls affect the systems.

The dynamics of the evader E is similar:

$$\begin{array}{ll} \ddot{z}_E = a_E, & \dot{a}_E = (v - a_E) \,/\, l_E, \\ |\, v \,| \le \nu, & a_E(t_0) = 0. \end{array} \tag{2}$$

Let us fix some instants T_1 and T_2. At the instant T_1, the miss of the first pursuer with the respect to the evader is computed, and at the instant T_2, the miss of the second one is computed:

$$r_{P_1,E}(T_1) = | z_E(T_1) - z_{P_1,E}(T_1) |, \quad r_{P_2,E}(T_2) = | z_E(T_2) - z_{P_2,E}(T_2) | . \tag{3}$$

Assume that the pursuers act in coordination. This means that we can join them into one player P (which will be called the *first player*). This player governs the vector control $u = (u_1, u_2)$. The evader is counted as the *second player*. The result miss is the following value:

$$\varphi = \min\{r_{P_1,E}(T_1), r_{P_2,E}(T_2)\}. \tag{4}$$

At any instant t, all players know exact values of all state coordinates z_{P_1}, $\dot{z}_{P_1}$, a_{P_1}, z_{P_2}, $\dot{z}_{P_2}$, a_{P_2}, z_E, $\dot{z}_E$, a_E. The first player choosing its feedback control minimizes the miss φ, the second one maximizes it.

Relations (1)—(4) define a standard antagonistic differential game. One needs to construct the value function of this game and optimal strategies of the players.

Now, let us describe a practical problem, whose reasonable simplification gives model game (1)—(4). Suppose that two pursuing objects attacks the evading one on collision courses. They can be rockets or aircrafts in the horizontal plane. A nominal motion of the first pursuer is chosen such that at the instan $T_1 \neq T_2$ t T_1 the exact capture occurs. In the same way, a nominal motion of the second pursuer is chosen (the capture is at the instant T_2). But indeed, the real positions of the objects differ from the nominal ones. Moreover, the evader using its control can change its trajectory in comparison with the nominal one (but not principally, without sharp turns). Correcting coordinated efforts of the pursuers are computed during the process by the feedback method to minimize the result miss, which is the minimum of absolute values of deviations at the instants T_1 and T_2 from the first and second pursuers, respectively, to the evader. The passage from the original non-linear dynamics to a dynamics, which is linearized with the respect to the nominal motions, gives [2] the problem under considerations.

In the paper [3], a case of ``stronger'' pursuers is considered and analytically methods are applied to the problem of solvability set construction in the game with zero result miss. For $T_1 = T_2$, an exact solution is obtained; if , then some upper approximation for the set is given. In general case, the exact analytical solution cannot be got, in the authors opinion.

A previous paper [4] of the authors is dedicated to a numerical investigation of the problem in two marginal cases (of ``strong'' pursuers and ``weak'' pursuers). This work deals with cases of other ratio of players' capabilities.

The numerical study is based on algorithms and programs for solving linear differential games worked out in the Institute of Mathematics and Mechanics (Ural Branch of Russian Academy of Sciences, Ekaterinburg, Russia). The central procedure is the backward constructing level sets (Lebesgue sets) of the value function. Optimal strategies of the players are constructed by some processing of the level sets.

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Network Formation Games in Mixed-flow Models

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In mixed-flow network formation model (possible applications include urban transportation network), a player can choose to create one-way links, two-way links or not to create any link. As far as we know, this mixed-flow model was proposed by Gao[1] for the first time. On the basis of one-way and two-way flow network model [2-8], the existence of Nash networks in mixed-flow model can generally be guaranteed, but the number of Nash networks in mixed-flow network far exceeds the former two models'. For example, the number of Nash networks in these three types of network with 4 nodes is 58, 128 and 810.

We study the existence of Nash network and compile corresponding programs. If we take the four types of the simplest equilibrium network structure (two-way link, one-way link with bilateral cost, minimal cycle and multiple nodes) as the basic elements of the mixed-flow equilibrium network, any mixed-flow equilibrium network with finite nodes would be composed of these four elements. At the same time, the role of cycle is equivalent to two-way links'.

In this paper, we give a specific example with regard to the nonexistence of Nash network in mixed-flow network with 5 nodes under the conditions of B&G function [2] to prove that Nash network in mixed-flow network with homogeneous profits and heterogeneous link costs doesn't always exist. Moreover, we give a specific example on the nonexistence of Nash network in mixed-flow network with negative profits.

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A Stackelberg Tariff Game between Provider, Primary and Secondary User

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Keywords: *Wireless networks, Resource management, Stackelberg game, Tariff*

Bandwidth resource management is an important issue in wireless networks, such as WiMAX, UMTC, etc. The quality of service (QoS) reached by a user, which transmits a signal, depends very strongly on the value of spectrum band provided by a spectrum holder. From economic point of view, holders and users form a spectrum market, within which spectrum bands are considered as a resource flowing from owners to consumers regulated by market mechanisms. Users are able to rent the spectrum band purchased from the holders. Thus, for holders it is important to find the optimal price to sell the bandwidth spectrum, whereas users try to find balance between using the bandwidth resource for its own needs and renting unused bandwidth capacity for profit gain.

In this paper, we introduce the following hierarchical Stackelberg game with three players: a spectrum holder (provider), a primary and a secondary user. The Provider assigns the price for bandwidth resource to maximize its own profit. The primary user buys a license for using frequency bandwidth from a provider to transmit signals. The primary user can also earn some extra money giving unused frequencies capacity for rent to the secondary user charging him by assigned tariff for interference so that the quality of the primary user transmission was not worse than a threshold value. The secondary user can either buy the frequency bandwidth or he can choose the service of the other provider based on comparing of the suggested QoS and prices which is set for the network access.

The provider's payoff π_{prov} is the profit obtained by selling the bandwidth resource to the primary user:

$$\pi_{prov}(C_W) = C_W W,$$

where W is the spectrum bands bought by the primary user and C_W is its price assigned by the provider.

The primary user allows the secondary user to use the spectrum band so that his throughput would be not worse than a threshold value $\overline{\pi}_P$, i.e. the presence of the secondary user does not have to spoil the transmission's quality for the primary user. The payoff of the primary user π_P is his throughput plus profit he gets by giving access to the network for the secondary user minus how much it costs to buy the bandwidth resource. We assume that the primary user charges the secondary user proportional to interference power:

$$\pi_P(W, C_S) = W\log\left(1 + \frac{h_{PP}P_P}{\sigma^2 W + g_{PS}P_S}\right) - C_W W + C_S g_{PS} P_S,$$

where h_{PP} and g_{PS} are the fading channel gains, σ^2 is the background noise, P_P is the total power employed by the primary user, P_S is the power employed by the secondary user and C_S is the tariff for access to the network to the secondary user. Let us assume that $P_S \in [0, \overline{P}_S]$, where $\overline{P}_S$ is the total power employed by the secondary user.

The secondary user payoff is his throughput minus expenses, i.e.

$$\pi_S(P_S) = W\log\left(1 + \frac{h_{SS}P_S}{\sigma^2 W + g_{SP}P_P}\right) - C_S g_{PS} P_S,$$

where h_{SS} and g_{SP} are the fading channel gains. This QoS has to be at least $\overline{\pi}_S$ which is a QoS the secondary user can gains from the other provider.

For this three level Stackelberg game the optimal strategies of the players are found in closed form. Numerical modelling demonstrates how the equilibrium strategies and corresponding payoffs depend of network parameters is performed.

Games and Inventory Management with Stochastic Demand

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Keywords: *Nash equilibrium, Non-cooperative game, Distributor*

In this paper stochastic $\left\langle y,\ R \right\rangle$-models of material stocks operating control are treated. We consider two types of oligopoly of N firms (distributors): quantitative and price competition. We assume that each firm has a warehouse. Demand for goods which are in stock is stochastic. Distributors are considered as players in two level non-cooperative game. At the high level optimal solutions of distributors about quantities of supply or selling prices are based on rational solution at low level and form Nash equilibrium. We describe the quantitative competition in the context of model of Cournot and price competition in context of modified model of Bertrand. Thus at the low level each player i chooses internal strategy $\left(y_i,\ R_i \right)$ as an optimal reaction to competitive player's strategies which are called external, $i = 1,\dots,N$. Here y_i is order quantity, R_i is maximal (threshold) inventory level of distributor. Optimal strategy $\left(y_i^*,\ R_i^* \right)$ player i appears to be a value on border of the set of internal strategies or it is the solution of a system of transcendental equations. The iterative method is developed to solve this system. Necessary and sufficient conditions for existence of equilibrium in pure strategies are formulated.

Evaluation of Services Quality Provided by Mobile Operators Under Competition

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Keywords: *Quality evaluation, Quality inclination, Willingness to pay, Uniform distribution, Triangular distribution, Exponential distribution, Two-stage game, Nash equilibrium, Optimal quality differentiation.*

In this paper three game-theoretical models of quality level choice under competition are analyzed. We consider the case when firms are assumed to produce homogeneous products (or services) differentiated by quality on some industrial market. The models are an extension of the basic game-theoretical model of quality choice under competition described in [10].

The game-theoretical models are presented as dynamic games which consist of the following stages:

At first, each firm i chooses its product (or service) quality levels s_i;

At the second stage firms compete in price p_i. Assume that both consumers and companies know the quality level s_i.

Each consumer buys at most one unit of the good. Consumers differ in their willingness to pay for quality level s, which is described by the parameter $\theta \in [0, b]$. This parameter is called "taste for quality". The utility of a consumer with a willingness to pay for quality θ when buying a product (or service) of quality s at a price p is equal to:

$$U_\theta(p) = \begin{cases} \theta s - p, & p \le \theta s \\ 0, & p > \theta s \end{cases}.$$

In this paper we suggest three models which is differ in the distribution of consumers' willingness to pay for quality θ: the situations with uniform, symmetric triangular and exponential distribution are analyzed. The investigated industrial market is considered to be partially covered.

The payoff function of the firm i which produces the product (provides a service) of quality s_i, where $s_i \in [\underline{s}, \bar{s}]$, is the following:

$$R_i(p_1, p_2, s_1, s_2) = p_i(s_1, s_2) D_i(p_1, p_2, s_1, s_2),\ i = 1,2,$$

where $p_i(s_1, s_2)$ is the price of the product (or service) of the firm i, $D_i(p_1, p_2, s_1, s_2)$ – the demand function for the product (or service) of quality s_i, which is specified.

The problem of equilibrium estimation is solved using backward induction. The strong Nash equilibrium in the investigated game was obtained in the explicit form which allowed us to evaluate prices, market shares of companies and revenues in the equilibrium.

The theoretical models are described for the situation when there are two players on the market and this model is extended for the oligopoly.

Three suggested models cover various possible market conditions:

There is same number of consumers who are ready to pay differently for higher quality level (uniform distribution).

The majority of consumers will choose average quality level and a minority are willing to pay for the critical quality levels (symmetric triangular distribution).

The majority of consumers have the willingness to buy products (or services) with the critical level of quality (exponential distribution). In this paper the situation when consumers are eager to buy the lowest level of quality is considered, but it may be extended to the situation when consumers are willing to pay for the highest level of quality.

As well as theoretical investigation the paper includes empirical part.

For the empirical study the mobile industry is analyzed and the comparative analysis of the two markets is done. Here the Russian and Portuguese markets are chosen with 5 and 3 mobile operators correspondingly.

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Social Capital. A Game Theoretic Approach

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Keywords: *Social capital, Communication situation, Myerson value.*

Individual social capital refers to the network of relations that an actor posses conferring him the ability that, for our proposal, we will consider of social or economic kind, of obtaining advantages and benefits. The approach we present here suppose a set of actors whose interests are modelled by a cooperative game. A priori, in such a situation, each player has the possibility of directly communicate with all the others. However, as time goes by, typically, there is a tendency to stick the existing linkages so that, at a fixed instant, the remaining links among players will be given by a graph. The social or economic possibilities of different actors change, then, from the original situation to this last one, in which there are restrictions in the communication. As a consequence of the relations that they still conserve, some of them will improve their control, influence and power and thus their economic possibilities. On the other hand, those actors that have broken a lot of links with the others will be, foreseeable, in a worse position, which will affect negatively their expected results.

Our proposal is to consider that variation as a measure of individual social capital. Formally, we will define the social capital for a player in a communication situation, as the difference between the economic result for this player (measured as his Shapley value) in the graph restricted game and in the original one. This measure satisfies several relevant properties to be considered as a social capital one.

Static Coalitional Multi-Criteria Model Provided by the Example of Innovative Projects

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Keywords: *Coalitional game, PMS-vector, Compromise solution*

Static coalitional multi-criteria game model in mixed strategies implies finding a compromise route (one or several) on a network graph, which would "suit" all participants-players, which provide movement along the existing routes under condition of their coalitional interaction. Positive and negative recommendations for maintaining motion at a particular route are players' strategies. Their payoffs are determined according to the recommendations. Since we have the presence of coalitional partitions between the players, it is necessary to determine the "optimal" coalitional partition.

The problem is demonstrated with the example of the interaction between participants of several innovative projects.

Optimization of Coalition Structures in Tree-like Hierarchical Games

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Keywords: *Hierarchichical games, Shapley value, Coalitions structure*

Considered a two-level hierarchical coalition game. On the first level - the distribution center on the second level - production centers. Distribution center sends goods to production centers, each production center has own technological matrix. Also production centers have the opportunity to join a coalition.

Suggested that the technological matrix of the centers of production depends on the of coalition it belongs. The main task of this game is to find the best payoff of manufacturing facilities and distribution center and optimal coalition structure.

Also considered case with two distribution centers.

Proposed the agorithm for finding players' best payoff and optimal coalition structure

The Evolutionary Model of Influenza in Risk-Group

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Keywords: *Evolutionary game, Vaccination problem, Replicator dynamics, Epidemic process, SEIR model*

The main purpose of this work is constructing evolutionary model of the influenza epidemic evolution in one risk-group in the urban population, focusing on SEIR model. In current model we select one risk group and divide it into four subpopulations Susceptible (S), Latent (E), Infected (I), and Resistant (R) individuals. In well-mixed population individuals from the each group randomly interact, hence during season epidemic Susceptible individuals become Latent or Infected and then become Resistant. Latent individuals we define as individuals, which are already infected, but who may not have clinical symptoms, moreover Latent individuals can transmit infection to Susceptible individuals. Consequently we consider evolution of the epidemic process as a sequential changing states from Susceptible individuals to Latent or Infected individuals and finally to Resistant.

One of the most important problem is the protection of population during annual influenza epidemic season. To protect population there exists a system of the preventions that reduce sickness rate. One of the most effective procedures to avoid the epidemic is vaccination. The influenza epidemic is a fast spreading process, involving the large part of total population. However, in addition to influenza, other forms of respiratory viral diseases circulate in the population, and individuals, vaccinated against the influenza can be infected by one of these forms, which reduces the effectiveness of vaccination. Hence the society should focus on the organization of preventive measures. Especially we have to focus on some risk-groups from the total population, because vaccination of total population is not effective and very expensive. Previous research proofed that about 70% of population should be vaccinated to avoid epidemic of influenza before the epidemic season.

In this paper we take into account social aspect of influenza epidemic and choose vaccination as a control action in the risk-group. In the model we consider two situation and estimate state changing in the risk-group during epidemic of influenza: in the Situation 1 all individuals in the risk-group are unvaccinated and in Situation 2 total amount of individuals in risk-group are vaccinated. Vaccination procedures occur before season epidemic of influenza begins, to avoid repeated infection of vaccinated individuals. It is necessary to take into account regulation of individual's immune system after vaccination, because failing health after vaccination not allow to resist against another viruses. Unfortunately influenza vaccines are effective only for one season owing to mutation of pathogens and waning immunity. Influenza epidemic continues until there are no more newly infected individuals.

Dynamics of risk-group states in evolutionary game, describing an epidemic process is constructed, based on social aspect of vaccination and infection expenses. Numerical simulation are also presented in the paper.

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The Optimization Model of Encashment Process in The ATM Network

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Keywords: *ATM network, Optimal routes, Vehicle Routing Problem with Time Windows, Statistical methods, Upload value, Cash withdrawal.*

The optimization problem of encashment process in the ATM network can be considered as the prediction problem of ATM refusal and construction optimal routes for the money collector teams. We focus on the modified algorithms for the Vehicle Routing Problem with Time Windows and statistical methods to compile the requests from the ATM network for processing center. Moreover we estimate the upload value using statistical analysis of cash withdrawal data. A numerical example is considered.

In this work we consider a problem in which a set of geographically dispersed ATMs with known requirements must be served with a fleet of money collector teams stationed in the depot in such a way as to minimize some distribution objective. This problem is combined with the problem of composition service requests from the ATM network. We assume that the money collector teams are identical with the equal capacity and must start and finish their routes at the depot within a certain time interval.

To enhance the operating efficiency of the ATM we propose to analyze the upload value – the sum of money uploaded in ATM while its servicing. The statistical analysis of cash withdrawal by one client for one operation helps to estimate the upload value and moreover the face value and amount of each bank note.

Thereby we define the necessity of servicing each ATM and predict the future requests for the money collector teams, based on the statistical data and restrictions. The optimal routes to load ATMs depend on the current requests and predictable requests.

The purpose of this work is to optimize encashment process in the ATM network in case of the prediction of ATM refusal. To solve the problem we base on some modified algorithms for the Vehicle Routing Problem with Time Windows which imply that there are time windows at the stops for servicing ATMs and time windows for each route. Each ATM is visited only once by exactly one collectors team within a given time interval, all routes start and end at the depot also within a certain time interval, and the total demands of all ATMs on one particular route must not exceed the capacity of the money collectors vehicle. A numerical example is considered.

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Stable Families of Coalitions for Network Resource Allocation Problems

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Keywords: *Cooperative games, Coalitions, Network LP, Superaditivity, Graphs*

Introduction

This research is based on results proving the existence of a non-empty K -core, that is, the set of allocations acceptable for the family K of all feasible coalitions, for the case when this family is a set of subtrees of a tree. A wide range of real situations in resource management, which include optimal water, gas and electricity allocation problems can be modeled using this class of games. The present research is pursuing two goals: 1. optimality and 2. stability. Firstly, we suggest to players to unify their resources and then we optimize the total payoff using some standard LP technique. The same unification and optimization can be done for any coalition of players, not only for the total one. However, players may object unification of resources. It may happen when a feasible coalition can guarantee a better result for every coalitionist. Here we obtain some stability conditions which ensure that this cannot happen for some family K. Such families were characterized in Boros et al. (1997) as Berge's normal hypergraphs. Thus, we obtain a solution which is optimal and stable. From practical point of view, we suggest a distribution of profit that would cause no conflict between players.

Rationale

Optimal performance of multi-agent economic systems is a pivotal problem of the modern economics. Performance quality of such systems can be described by a set of numerical characteristics reflecting the way these systems operate. These characteristics could include profit, cost, gross margin, quantity or number of produced commodities, and will be referred to henceforth as Objective Functions (OF). The primal managerial task is to optimize (maximize or minimize) these OF. Without loosing the generality, the maximization formulation will be considered.

There are two possible approaches to selecting the optimal strategy. The first one (henceforth 'collective' approach) is used when an overall system performance is considered and individual agents' performances are not the main focus of the research. In this case, the most common tool employed for the system optimization is the non-linear or linear programming, usually integer or mixed. The second approach implies optimization considered from each of the agents' point of view (henceforth referred to as 'individual'). With this approach, the system optimization tool is the game theoretic methods. The fist approach (overall optimization) is equivalent to the second approach (game) when all agents form one large coalition. The value of overall maximized OF is not less than the sums of the individual OFs obtained by individual players or coalitions as solutions of a game. It is tempting to suggest that agents form one coalition and then divide the total payoff between them. However, some players (or coalitions of players) might be better off pursuing strategies different from the strategy formulated for the overall optimization for 'one large coalition' game. Moreover, players or coalitions of players, which are better off playing separately, exist in the general case.

This paper describes quite a wide range of systems for which the non-conflicting distribution of optimal 'collective' OFs between a system's agents is possible. These systems belong to the class of the so-called network systems and are characterized by the existence of carriers connecting individual players who are located in nodes. The performance of such systems can be optimized using the network LP methods, containing a set of very specific constraints, related to the capacity of the carriers and the existence of paths between different nodes. A wide range of real economic systems (water supply networks, electricity and gas supply networks, telecommunications, etc.) can be described as such network systems, which makes the results of the reported research highly demanded by managerial practices in these areas. The fundamental result employed in this paper states that for such systems there exists a non-conflicting redistribution of total income between the players.

Mathematical formulation

We consider the situation when a group of agents I that has an access to the set of limited resources R has a goal to optimize some objective function. P is the set of products (industries). We ask the following question: what optimal cooperation strategy, or coalition structure, should be chosen by the agents in order to optimize their economic outputs. The linear case appears when for each player $i \in I$ his strategies x_i are the shares of his activities in industry (product) $p \in P$. Let

$$A = \|a_{rp}\|$$

be a matrix representing the amount of resource $r \in R$ needed for producing a unit of product $p \in P$, and

$$b^i = (b_1^i, b_2^i, ..., b_{|R|}^i)$$

be a vector of resources available for player *i*.

Then we can formulate the production strategy choice for each player as a LP problem:

Maximize the revenue function $f(x) = \sum_{p \in P} c_p x_p$

Subject to $Ax \le b,\ x \ge 0$, where *b* is a vector of available resources.

This formulation can be settled for each player $i \in I$ or coalition $K \subseteq I$. Respectively, we replace *b* by b^i or b^K. For each coalition *K* vector b^K is additively defined:

$$b^K = \sum_{i \in K} b^i$$

The corresponding LP solution will be denoted x^*.

Let us define function *v* on the set of coalitions as:

$$v(K) = Cx^*(b^K)$$

This function is superadditive, that is, for each two disjoint coalitions K_1 and K_2

$$v(K_1 + K_2) \ge v(K_1) + v(K_2)$$

This result was proven by Gomory and Johnson (1973) and further developed by Blair and Jeroslow (1977) and Schrijver (1980). It is referred in the literature as superadditive (or subadditive) duality. We can combine it with BGV theorem (see Boros, Gurvich, Vasin (1997) and the originating paper of Gurvich and Vasin (1977)) stating that family of coalitions K. $\subseteq 2^I$ is stable (that is the K -core $C(v,$ K.) is not empty for any superadditive characteristic function v) if and only if K is a normal hypergraph, according to Berge (1970).

Concluding remarks

Let us consider a network LP optimization problem for an acyclic graph. Referring to the results mentioned in Section 3, we can state that a family of coalitions is stable if and only if it is a normal hypergraph. In particular any family subtrees of a tree form a stable family of coalitions. Thus, there exists an allocation corresponding to the optimal LP solution, which is stable. It means that there is a distribution of the total income among players which is acceptable for all feasible coalitions.

In nutshell, this paper suggests a new approach to optimization and fair distribution of total payoff among players. This method is based on integration of two fundamental results, which are

1. Superadditive duality for LP optimization, and
2. The BGV theorem claiming that a family of coalitions K is stable (that is the K -core $C(v, \text{K.})$ is not empty for any superadditive characteristic function v) if and only if K is a normal hypergraph.

Then BGV theorem can be applied for acyclic network systems. The following two assumptions are crucial: 1. **acyclicity** of corresponding graph and 2. **superadditivity** of the optimal value. It allows us to conclude that some ‘natural’ families of coalitions are stable, that is, admit a non-empty core. Therefore, it is possible to conclude that a fair distribution of payoff among the payers is possible. Thus, the existence of an optimal and stable solution for such class of games is proven. It should be also mentioned that the "superadditive duality" holds not only for LP but for integer programming as well. Hence, the same analysis of stability is applicable for these, more general, models.

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Leadership and Peer Effects in an Organization

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Keywords: *Socialization, Organizational culture, Peer effects, Preference dynamics*

Consider an organization or Corporation composed by a Principal (or a leader) and by a finite group of agents (or followers). The Principal has some ideal organization composition or vector of preferred actions one for each agent and can invest in costly socialization trying to instil this "corporate culture" in all the agents of the organization. Each agent has as well her ideal action. When an agent makes a decision each period her behavior is driven by two competing motives: she wants her behavior to agree with her personal ideal action and she wants also her behavior to be as close as possible to the average behavior of her peers. Ideal actions or preferences evolve over time. There are two sources of preference (and therefore, action) change. On the one hand, there exists a costly corporate socialization effort exerted by the Principal trying to transform the ideal action of each agent into his own ideal action. On the other hand, each agent's ideal action changes in the direction of actual behavior (self-persuasion or cognitive dissonance). We are interested in the long-run outcomes of this situation and in particular in the ability of the Principal to fully instil the corporate culture in the members of the organization.

The D.W.K. Yeung Condition for Cooperative Differential Games with Nontransferable Payoffs

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Keywords: *Pareto-Optimality, Time-consistency, Payoff Distribution Procedure*

Consider a two-person non-zero-sum differential game $\Gamma(x_0,[t_0,+\infty))$ starting from the initial state $x_0 \in \mathbb{R}^2$ at moment $t_0 \in \mathbb{R}^1$ and duration $[t_0,+\infty)$. The motion equations have the form

$$\dot{x} = f(x,u_1,u_2),\ x \in \mathbb{R}^2,\ u_i \in U_i,\ i = 1,2,\ t \in [t_0,+\infty)$$
$$x(t_0) = x_0$$

Here $u_i \in U_i$ is the control variable of player i and U_i is a compact set.

The payoff of player i is given by:

$$K_i(x_0[t_0,+\infty),u_1(\cdot),u_2(\cdot)) = \int_{t_0}^{+\infty} e^{-\rho(t-t_0)} g_i(x(t),u_1(t),u_2(t))dt,\ g_i \geq 0,\ i = 1,2$$

where ρ is the discount rate. Suppose that payoffs are nontransferable.

If players agree to cooperate, they play an open-loop strategy pair $u^*(t) = (u_1^*(t),u_2^*(t))$ which generates a Pareto-optimal payoff vector.

The stringent requirement for solutions of cooperative differential games is time-consistency. Time-consistency of cooperative solution guaranties optimality of chosen optimality principle along the optimal trajectory. Time-consistent solution to nontransferable payoffs requires 1) Pareto optimality throughout the game horizon, 2) individual rationality throughout the game horizon.

For time-consistency of solution the notion of Payoff Distribution Procedure (PDP) was introduced.

Suppose that player may use irrational acts acts to extort additional gains if later circumstances allow. Consider the case where the cooperative scheme has proceeded up to time t and some players behave irrationally leading to the dissolution of the scheme. A condition under which even if irrational behaviors appear later in the game the concerned player would still be performing better under the cooperative scheme is the irrational behavior proof condition (Yeung, 2006), which also called the D.W.K. Yeung condition.

The D.W.K Yeung condition for the differential game with infinite duration $\Gamma(x_0,[t_0,+\infty))$ is described by following inequality:

$$w_i(x_0,[t_0,+\infty)) \le \int_{t_0}^{t} e^{-\rho(\tau-t_0)}\beta_i(\tau)d\tau + e^{-\rho(\tau-t_0)}w_i(x^*,[t,+\infty)),$$

where x^* is the cooperative optimal trajectory, $w_i(x,[t,+\infty))$ is noncooperative payoff and $\beta_i(\tau)$ is time-consistent payoff distribution procedure.

It is proved that if the PDP proposed in [1] is used the D.W.K Yeung condition for any Pareto-optimal solution in the cooperative differential game with nontransferable payoffs is satisfied.

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Heterogeneous 2×2 Matrix Games

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Keywords: *Non-cooperative games, Nash Equilibria, Heterogenous Population*

Introduction

The objective of this paper is to characterize the Nash equilibria, NE, of 2×2 games where each player can be of wo different types, I and II, knows the type of her opponent, but ignores her own. Let us describe informally in three steps the approach followed in this paper.

Players interact bilaterally giving rise to the following kind of encounters or states of nature: $(I,I),(II,I),(I,II)$ and (II,II) . which occur under certain probabilities denoted by $P(I,I),P(II,I),P(I,II)$ and $P(II,II)$.

A two-person strategic game with two actions and complete information, the *surface game*[1], is assumed to be played in every encounter. The presence of two different types allows players to condition their strategies on the type of opponent she might face. For instance, if each player has two actions to choose then the same or a different action in front of players of type I and players of type II could be played. Of course players may also randomize between actions. The payoff of a player to be maximized will be the expected payoff conditioned on the type of her opponent.

With these ingredients at hand, in this paper we propose a way to play a 2×2 matrix game within this set-up. Then we solve it by using the NE solution concept. We find that: (i) Every NE in the surface game can be extended to heterogenous game. Moreover, games that have only one mixed strategy profile and games with only one NE in pure strategies do not generate additional NE equilibrium strategy profiles. (ii) Games with common interest, coordination and anticoordination games, do always

[1] See Binmore and Samuelson (2001)

generate additional NE. We characterize all the NE strategy profiles. We also characterize the NE strategy profiles for degenerate games.

Summarizing, the solution of heterogeneous 2×2 matrix games is a superset of the NE of that game. The new NE equilibria, if any, are equilibria which depend on the given probability distribution over the type of the players and the payoffs of the surface game.

The paper is organized as follows. Section 2 recalls a brief and usual classification of 2×2 games according to the number and kind of NE they have. Section 3 contains the probabilistic modelling of the heterogenous game and some rules that will govern player's behaviour within this set-up. In Section 4, the NE strategy profiles for heterogenous games are determined. Section 5 contains a discussion of the results and some examples. Some ideas and futures developments conclude.

Dynamic Model of Lobbying in Social Networks*

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Keywords: *Social Network, Information Impact, Game Theory*

Introduction

Consider the social network built by n agents. An opinion of the i-th agent in the time t defined by the $x_i^t \in [0,1]$, $i \in N = \{1,2,...,n\}$, $t = 0,1,2,...$. An information impact of agent i to agent j defined by value $a_{ij} \geq 0$, $i,j \in N$. The impact matrix $A = \| a_{ij} \|_{N \times N}$ is non-negative and stochastic by rows: . The agents have initial beliefs $x^0 = (x_i^0)_{i \in N}$. At each step the agent i changes her opinion in view of opinions of the other agents:

$$x_i^t = \sum_{j \in N} a_{ij} x_j^{t-1}, t = 1,2,...; i \in N. \tag{1}$$

Interactions of the agents repeat until they have the common opinions [2]:

$$x = A^{\infty} x^0, where A^{\infty} = \lim_{t \to \infty} (A)^t. \tag{2}$$

There are two players who can lobby in the social network. The lobbying $u_{t,i}^k$ of the player $k \in \{1,2\}$ at the step t is the affecting on the agent's i opinion: $x_i^t = x_i^{t-1} - u_{t-1,i}^k$, $u_{t-1,i}^k \in [-1,1]$; $i \in N$; $t = 1,2,3,...$. The overall price for lobby is $C_{k \in \{1,2\}}(u) = \sum_{t=0}^{\infty} \sum_{i=1}^{n} \delta^t (u_{t,i}^{k \in \{1,2\}})^2$.

The payoff function is $H_k(u) = M_k(x) - C_k(u)$, $k \in \{1,2\}$. The player aims to maximize her payoff.

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Optimal Double-Stopping Problem on Trajectories

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Keywords: *Optimal stopping, Urn sampling, Ballot problem*

We consider following optimal double-stopping problem on trajectories. Suppose that we have an urn containing m minus balls and p plus balls. We draw from the urn sequentially one at a time without replacement. The value -1 is attached to minus ball and value $+1$ to plus ball. Determine sequence $Z_0 = 0$, $Z_n = \sum_{k=1}^{n} X_k$, $1 \leq n \leq m + p$, where X_k is the value of the ball chosen at the k-th draw. Each time a ball is drawn, we observe the value of the ball and decide either to stop or continue drawing. The problem of this paper is to stop with maximum probability at the first on the minimum of the trajectory formed by $\{Z_n\}_{n=0}^{m+p}$ and then on the maximum. We find the optimal stopping rule in this problem, i.e. we derive the optimal stopping times (σ^*, τ^*) such that

$$\begin{aligned} &P\{Z_{\sigma^*} = \min_{0 \leq n \leq m+p} Z_n, Z_{\tau^*} = \max_{0 \leq l \leq m+p} Z_l\} \\ &= \sup_{(\sigma,\tau) \in C, \sigma < \tau} P\{Z_\sigma = \min_{0 \leq n \leq m+p} Z_n, Z_\tau = \max_{0 \leq l \leq m+p} Z_l\}, \end{aligned}$$

where C is the class of all double-stopping times.

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Stackelberg Strategies for Dynamic Games with Energy Players Having Different Time Durations

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Keywords: *Stackelberg, Linear quadratic, Different time durations*

We consider a system that consists of a major electrical power producer player (Public Power Corporation – PPC) playing in infinite time horizon, and minor players (power producers and consumers) remaining in the system for finite time durations, which time durations are overlapping. We study how they interact among themselves (horizontal interaction), and with the major player respectively (vertical interaction), via their decisions/strategies. We study a deterministic LQ version of the problem in discrete time. In our previous work we employed the Nash equilibrium and we studied the behavior of the system. In this paper we use the Stackelberg equilibrium with the long-term players in the role of the Leader.

The Game-Theoretical Model of Cost Allocation in Communication Network

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Keywords: *Differential search game, Information set, Mixed strategies*

The network $G(X, D)$ is considered, here X is the set of vertexes $\{x_1, ..., x_n\}$, D is the corresponding set of edges.

The cost matrix $C = \{c_{i,j}\}$ is also given.

Denote by $S = \{s_l, l = \overline{1,k}\}$ the set of so called "warehouses" and by $T = \{t_l, l = \overline{1,k}\}$ the set of so called "shops".

Consider players A_i as pairs $(s_i, t_i), i = 1, ..., k$. The strategy set of each player A_i is the set of all possible routes connecting s_i and t_i. Denote this set by $\mathrm{A}_i = \{\alpha_i\}$.

Consider the situation $\alpha = (\alpha_1, ..., \alpha_k), \alpha_i \in \mathrm{A}_i$. The total cost in this situation is equal to $\sum\limits_{(i,j) \in L \subset D} c_{ij}$, where L is the subset of edges from the routes connecting "warehouses" and "shops" in situation α.

The cooperative game setting is considered. To compute the value of characteristic function for each coalition $M \subset A = \{A_1, ..., A_k\}$ it is necessary to find an optimal communication plan minimizing the payoff of coalition M.

As an optimality principle the Shapley value is chosen. The example with 40 nodes and 3 players is computed.

The Least Square Values for Games with Restricted Cooperation

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Keywords: *The Shapley value, Restricted cooperation, Least square values, Consistency*

The traditional assumption in cooperative game theory is that every coalition is feasible and can be formed in a game. However, in many real life situations not every group of players has the opportunity to cooperate and to collect their own payoff. We say that we deal with cooperative games with restricted cooperation when (may be) not all coalitions can be formed.

Each solution for games with restricted cooperation is a generalization of some solution for classic TU games. This paper deals with one more generalization of the Shapley value.

It is well-known that the Shapley value for a classic TU game can be considered as the solution of an optimization problem. Such a problem also can be formulated and solved in a case of restricted cooperation and it is possible to consider the result as a solution for the class of games with restricted cooperation.

We describe a class of solutions which can be defined by a similar way, and we give axiomatization of all these solutions, including one for the new modification of the Shapley value.

For this purpose we define a new property with the name "set consistency". A solution satisfies this property if this solution has no changes when we add one coalition to the collection of feasible coalitions (and define the characteristic function for this coalition as the sum of the solution values for elements of the coalition).

The main fact is that:

If some solution for games with restricted cooperation

1. satisfies the set consistency property and
2. coincides with some least square value for games with full cooperation

then this solution is the least square value for each game with restricted cooperation.

Web Values and the Average Web Value for Cycle-Free Directed Graph Games

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Keywords: *TU game, Cooperation structure, Cycle-free directed graph, Tree and sink values.*

For arbitrary cycle-free directed graph games web values are introduced axiomatically and their explicit formula representation is provided. The web values may be considered as natural extensions of the tree and sink values studied in A. Khmelnitskaya and D. Talman (2010) "Tree-type values for cycle-free directed graph games" in case when management in a directed network is located at any level. Any anti-chain of a given digraph may be chosen as a management team. The main property for the web value is that every player being a successor of the management team receives the worth of this player together with all his successors minus what these successors receive and every player being a predecessor of the management team receives the worth of this player together with all his predecessors minus what these predecessors receive while every manager receives the worth of himself together with all of his successors and predecessors minus what these successors and predecessors receive. Simple recursive algorithms for calculation the web values are provided. Afterwards we define the average web value as the average of web values relevant to all feasible management teams in a given digraph. The efficiency of web values and the average web value is studied.

Matchings with Simplest Semiorder Preference Relations

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Keywords: *Matchings, College Admission Problem, Semi-orders*

Matching problem with preferences was first stated in the Gale and Shapley's pioneering paper. [4]. Let us briefly describe it. There are two groups of agents: students and colleges, and students should be admitted to colleges. Agents on each side of the market have preferences over agents on the opposite side. Furthermore, each college can admit more than one student and has quota (maximum number of seats).The following question arises: does such matching exist, that is stable, i.e. all agents will agree to follow this matching according to their preferences?

Gale and Shapley introduced the following definition of matching's stability:

1. Individual rationality. No student should be admitted to the college, if she prefers staying unmatched to studying in this college; the same rule applies to colleges.
2. Pairwise stability. Does not exist college-student pair such that simultaneously: student prefers this college to those that she has under matching, and college admitted other student under matching, which is less preferred that this student.
3. Empty seats. Does not exist college-student pair such that simultaneously: student prefers this college to those, which she has under matching; and college finds this student acceptable and has empty seats.

Gale and Shapley [4] made following assumptions about agents' preferences:

1. preferences of students over colleges are linear orders;
2. preferences of universities are linear orders over individual applicants;
3. preferences over sets of students are responsive to the preferences over individuals.

They proved that in case of such preferences set of stable matchings is not empty. Furthermore, constructive proof was proposed: so called "Deferred Acceptance Algorithm", which always find such a stable matching, was described.

It was shown, that in case of such strict preferences all stable matchings are Pareto-efficient (taking into account preferences of both sides of the market).

Much further research was based on this pioneering paper.

In this paper we're especially interested in the models, allowing indifferences in preference profile. One of the first papers, investigated matching problem with indifferences in preference profile, was [1]. In this paper, preferences of universities over individual applicants are considered as weak orders. This problem statement follows real-life example: preferences of municipal schools in American cities. [1] analyzed situation in Boston and NYC school districts. Mechanism, which was used in this districts, did not produce stable matching of applicants and schools, so it was in some sense unfair. Authors proposed the following admission procedure: first indifferences in schools' preferences are broken randomly; second, deferred acceptance mechanism is applied to the admission problem with strict preferences.

Two new definitions of stability were introduced: weak stability and strong stability. Existence of strongly stable matching is not guaranteed , while existence of weakly stable matching is proved for weak order college preferences.

However, [3] show that matching, produced by the procedure described above, may not be Pareto-efficient. For the matching problem, where preferences of applicants over universities are strict and universities have weak order preferences over individual applicants, Erdil and Ergin propose algorithm, which provides Pareto-efficient stable matching. This algorithm is based on so-called Stable Improvement Cycles.

In this paper, we introduce many-to-one matching problem, in which students have linear order preferences over colleges, and colleges have simplest semi-order preferences over students [2].

Let us show en example of simplest semi-order preferences. Assume college b and three applicants a_1,a_2,a_3. The following preference relation over these students is an example of simplest semi-order:

$a_1 \approx a_2$, $a_2 \approx a_3$ but $a_1 > a_3$

Below we will briefly describe main results of our paper.

Observation 1. Stable matching in case of such preferences always exists.

It's easy to show, as indifferences in preference profile can be relaxed to form linear order preference. We know from [4] that in this case stable matching exists. Such matching will also be weakly stable under original simplest semi-order preferences.

Theorem 1. For each matching, which is weakly stable in problem with simplest semi-order preferences there exists relaxation of indifferences to linear order form, such that matching remains stable.

We introduce modified (compared to [3]) definition of Stable Improvement Cycle

Definition 1. Stable Improvement Cycle in matching μconsists of distinct students such that

1. Each student is admitted to a college under μ
2. Each student prefers college of his "cycle neighbor" to his own college under μ

   Let A_b be the set of all students, who prefer college b to their colleges under μ. Then D_b will denote the set of most preferred students in A_b. (Note that students which are "half of step" below the best ones in simplest semi-order preference, are not included in $D_{b)}$
3. Each student belongs to D_b of his cycle neighbor's college.

Theorem 2. Weakly stable matching μ is Pareto-dominated by another stable matching (according to students' preferences) iff it admits a Stable Improvement Cycle.

So, this theorem allows us to extend results of the weak order case to the wider class of problems.

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Analysis of a Repeated Three-Person Game with Finite Number of Strategies for Players

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Keywords: *Repeated game of three players, Finite number of strategies, Behavior types*

Consider the following repeated three-person game with finite number of strategies. Let Player 1 (P1), Player 2 (P2), and Player 3 (P3) have l, m and n strategies, respectively. Let f_{ijk}, g_{ijk} and h_{ijk} be payoffs of P1, P2 and P3, respectively, for a strategy triple (i,j,k), where $i \in L = \{1,\dots,l\}$, $j \in M = \{1,\dots m\}$ and $k \in N = \{1,\dots n\}$.

Let the players choose their strategies sequentially in rounds 1,2,…. Assume that in each round P1 and P2 act in the class of mixed strategies. A collection $\vec{p} = (p_1, p_2, \dots, p_l)$, where $p_i \geq 0$, $\sum_{i=1}^{l} p_i = 1$, is a mixed strategy of P1, and a collection $\vec{q} = (q_1, q_2, \dots, q_m)$, where $q_j \geq 0$, $\sum_{j=1}^{m} q_j = 1$, is a mixed strategy of P2. So, the $(l-1)$-dimensional simplex S_{l-1} and $(m-1)$- dimensional simplex S_{m-1} are the sets of mixed strategies of P1 and P2, respectively. Assume also, that P3 uses only pure strategies from the set N.

Let the set $S = S_{l-1} \times S_{m-1} \times N$ be a state space of the repeated game. At a state $(\vec{p}_t, \vec{q}_{t,} k_t) \in S$ the expected payoff for P1 is defined by formula $f(\vec{p}_t, \vec{q}_t, k_t) = \sum_{i=1}^{l} \sum_{j=1}^{m} p_{i,t} q_{j,t} f_{ijk_t}$. The expected payoffs of P2 and P3 are defined by means of replacement of the symbol f by symbols g and h, respectively.

Consider non-cooperative dynamics. Assume that in each round t the players know the current state $(\vec{p}_t, \vec{q}_t, k_t)$ and choose a state $(\vec{p}_{t+1}, \vec{q}_{t+1,} k_{t+1})$ for next round from the following set

$U_{\alpha,\beta}(\vec{p}_t, \vec{q}_t) = \{(\vec{p}, \vec{q}) \in S_{l-1} \times S_{m-1} : \left| p_{i,t} - p_i \right| \leq \alpha, \left| q_{j,t} - q_j \right| \leq \beta, i \in L, j \in M\}$,

where α and β are positive, sufficiently small numbers. The transition $(\vec{p}_t, \vec{q}_t, k_t) \Rightarrow (\vec{p}_{t+1}, \vec{q}_{t+1}, k_{t+1})$ is realized under keeping the principle of non-decrease of players' payoffs during the game. The proposed approach is the following one.

Let $(\vec{p}_t, \vec{q}_t, k_t)$ be a current state in the game and ${k_{t+1}}^* \in N$ be a fixed strategy of P3. Consider two problems.

Problem 1. Find $\vec{p}^1$ maximizing the function $f(\vec{p}, \vec{q}_t, {k_{t+1}}^*)$ over the set $U_{\alpha,\beta}(\vec{p}_t, \vec{q}_t)$ under the condition $g(\vec{p}, \vec{q}_t, {k_{t+1}}^*) \geq g(\vec{p}_t, \vec{q}_t, {k_{t+1}}^*)$.

Problem 2. Find $\vec{q}^2$ maximizing the function $g(\vec{p}_t, \vec{q}, {k_{t+1}}^*)$ over the set $U_{\alpha,\beta}(\vec{p}_t, \vec{q}_t)$ under the condition $f(\vec{p}_t, \vec{q}, {k_{t+1}}^*) \geq f(\vec{p}_t, \vec{q}_t, {k_{t+1}}^*)$.

Problems 1 and 2 have solutions.

Consider the auxiliary bimatrix game (A^*, B^*) with the matrices

$$A^* = \begin{pmatrix} f(\vec{p}_t, \vec{q}_t, {k_{t+1}}^*) & f(\vec{p}_t, \vec{q}^2, {k_{t+1}}^*) \\ f(\vec{p}^1, \vec{q}_t, {k_{t+1}}^*) & f(\vec{p}^1, \vec{q}^2, {k_{t+1}}^*) \end{pmatrix}$$

$$B^* = \begin{pmatrix} g(\vec{p}_t, \vec{q}_t, {k_{t+1}}^*) & g(\vec{p}_t, \vec{q}^2, {k_{t+1}}^*) \\ g(\vec{p}^1, \vec{q}_t, {k_{t+1}}^*) & g(\vec{p}^1, \vec{q}^2, {k_{t+1}}^*) \end{pmatrix}$$

In this bimatrix game strategy 1 is repeating $\vec{p}_t$ for P1 and repeating $\vec{q}_t$ for P2, and strategy 2 is switching from $\vec{p}_t$ to $\vec{p}^1$ for P1 and switching from $\vec{q}_t$ to $\vec{q}^2$ for P2. To obtain $(\vec{p}_{t+1}, \vec{q}_{t+1}, k_{t+1})$ the players find Nash equilibria. It is proved that the bimatrix game (A^*, B^*) has at least one Nash equilibrium in the class of pure strategies.

Thus, the pair $(\vec{p}_{t+1}, \vec{q}_{t+1})$ is determined; it depends on ${k_{t+1}}^*$. After that the strategy k_{t+1} of P3 is chosen from the condition $h(\vec{p}_{t+1}, \vec{q}_{t+1}, k_{t+1}) - h(\vec{p}_t, \vec{q}_t, k_t) \geq 0$. If such strategy k_{t+1} is unique, then one chooses strategy giving the maximum to the left-hand side. Thereby the dynamics of the considered repeated game is completely determined.

Besides mentioned above local criteria of players quite often there exist also global criteria evaluating the quality of the process wholly. And dynamics constructed above not always leads to a state which optimizes global criteria. It appears that the using so-called behavior types for players (normal, altruistic, aggressive and paradoxical ones) can be effective in this situation.

An illustrating example when each of three players has only two strategies was considered. The corresponding trajectories were calculated.

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Do Experts Matter? Evidence from a Cascade Experiment

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Keywords: *Information cascades, Learning, Bayesian choice, Experiment, Experts.*

Bikhchandani, Hirshleifer and Welsh (1992) showed that herding can be rational. Suppose have exogenous queue of agents making their Bayesian choices using their private signals and observable choices of other agents. At a certain round it will be rational for one of them to discard his or her own signal and follow the majority (the herd). This time point is a start of rational information cascade: agents ignore their signals according to the Bayes updating. Anderson and Holt (1997) presented first experimental results of cascades emerging in the laboratory.

Bikhchandani et al (1992) stated that agents with high precision signals ("fashion leaders") can break or reverse a cascade. Bayesian updating makes their opinion more influential than aggregated choice of "regular" agents. We present experimental results that try to capture this influence in a laboratory setting.

We use basic design by Anderson and Holt (1997) with some modification. First (or control) session is a standard observational learning (cascade) game of the binary choice. Agents guess between two urns A and B containing white and black balls in different proportions. We have a 9-person queue with the signal precision of 0.6. In the second session we randomly picked one of the participants and presented him or her as an "expert" with 0.9 signal precision.

Our results show that highly informed agents (or "experts") tend to provoke a cascade: there were only 25% of cascade rounds in the first session and 73% in the second (expert) session. At the same time we had the same rate of errors in both session which can be explained by the high rate of incorrect cascades in the expert session. Econometric analysis using the logit error function proved that agents who made their choices before experts were more likely to follow Bayesian decision model, although we had no single cascade in expert session before expert's choice (fig.1). Cascades provoked by experts were stable and "deep" (depth is a difference between A and B choices).

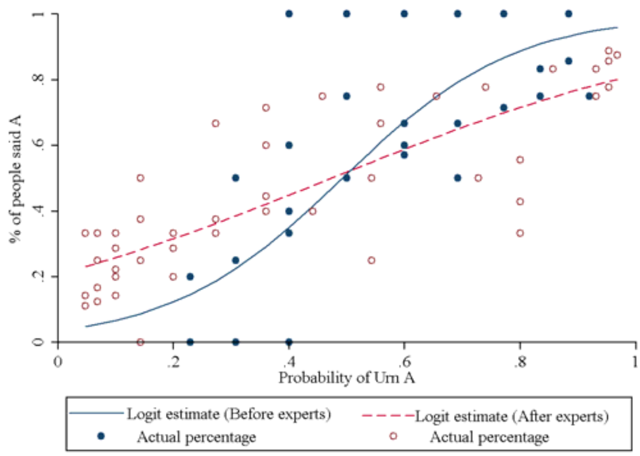

Fig.1. Logit Estimation of Errors

Our results fit other experimental papers in the field. Those are first experiments (Anderson and Holt, 1997), experiments with different payoffs (Anderson, 2000), institutional design experiments (Hung и Plott, 2001), market applications (Kurbler and Weizsacker, 2003), net structures (Choi et al, 2004) and some others. Alevy et al (2007) showed that professionals usually trust their private signals than others' choice. We confirm experimenta results by Goeree et al (2007) that explain instability of cascades and their locality; authors prove that experimental results fit quantal response equilibrium (QRE) models. Lat result is confirmed by (Choi et al, 2004). Ziegelmeyer et al (2010) introduce two types of agents with high and low signal precisions. Low-informed agents tend to equilibrium choices, and high-informed "experts" usually deviate from the equilibrium and don't break cascade (no more than 15% of the game rounds). Sasaki conducted an experiment with declining and rising precisions. The first design participants tend to make more right choices.

Also our experiment showed that Bayesian choice can't be the best and even effective strategy in the real world due to irrationality of real people. These results fit meta-study presented by Weizsacker (2008).

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Journals in Game Theory

GAMES AND ECONOMIC BEHAVIOR

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Stochastic Cooperative Games Application Concerning the Realization of Large-Scale Investment Projects

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Keywords: *Stochastic cooperative games, Cooperative game with transferable utilities, Characteristic functions of cooperative game, Value at risk (VaR), Large-scale investment projects.*

Regularities of initiation and following development of large-scaleinvestment projects researches are becoming more and more important in the terms of modern economic situation. Such projects are quite often characterized with rather diverse participants' structure concerning both scales, and organizational and legal forms.

For the past years projects of government and public private partnership as well as that of intergovernmental kind, supposing various and multi-faceted investors' involvement, have become of a special importance. In so called "classical" investment researches attention is firstly paid to evaluation problems likewise risk factors and uncertainties taking into account as soon as they do exist at all the stages of significant investment projects realization.

At the same time, cooperative effects, inevitably arising during origination and following development of investors' coalitions, appear to be of a particular interest, especially in situations when coalitionists have differences not only in organizational and property characteristics, but also in objectively owned economical interests systems.

In this case mathematical models and methods, that give us an opportunity to analyze regularities of significant investors' coalitions appearance, seem to be of current importance.

When solving such problems, cooperative game theory implementation is becoming efficient and profitable enough. Within the primary case the situation analyzing investors consolidation possibility aiming realization of some large-scaleinvestment project can be described with classical cooperative game with transferable utility (I, v), in which

$v(i)$ – incomes, that individual investors $i \in 1..m$ can rely on, provided they can act on their own;

$v(S)$ – various participants' coalitions $(S \subset 2^I)$ incomes.

In the situation under consideration we need full ("large") coalition of all potential participants ($I = \{1..m\}$) to be formed to implement the project, and $v(I)$ will be a utility (income) received by results of the project.

Among the "fundamental" disadvantages of this extremely abstract, limited and primitive model a prerequisite concerning a possibility of individual participants (as well as their various coalitions) incomes representation as determinate values. An assumption about these incomes being stochastic variables $\tilde{v}(S\}$ with some known distribution densities seem to be more plausible and thus more attractive. So we come to a requirement of classical cooperative games modification and taking into account the characteristic functions values randomness. Thereby a stochastic cooperative game we will understand in the sense of pair of sets $\Gamma = (I, \tilde{v})$, where:

$I = \{1..m\}$ – set of participants;

$\tilde{v}(S)$ – random variables, that have known distribution densities $p_{\tilde{v}(s)}(x)$, they are interpreted as income (utility, payments) of corresponding coalitions $S \subset I$.

Within this approach the concept of "sharing" becomes an interesting development. Particularly, we can consider the vector $x \in R^m$ as sharing in stochastic cooperative game, if this vector meets the condition of individual

$$\mathbf{P}\{x_i \geq \tilde{v}(i)\} \geq \alpha \text{ (or } x_i \geq F^{-1}_{\tilde{v}(i)}(\alpha) = v_\alpha(i)\text{)} \tag{1}$$

and group

$$\mathbf{P}\{\sum_{i=1}^{m} x_i \leq \tilde{v}(I)\} \geq \alpha \text{ (or } \sum_{i=1}^{m} x_i \leq F^{-1}_{\tilde{v}(I)}(\alpha) = v_\alpha(I)\text{)} \tag{2}$$

players' rationality with probability α.

It is important to emphasize that $v_\alpha(\{S\})$, defined from the condition $\mathbf{P}\{\tilde{v}(S) \leq x\} = \alpha$, is α–fractile random variable distribution $\tilde{v}(S)$ – or, in risk management terms, VaR of income of coalition S

$$v_\alpha(S) = F^{-1}_{\tilde{v}(S)}(\alpha).$$

It is necessary to pay attention to the fact that transition from "classical" determined cooperative games to their stochastic variant opens the entire row of interesting and perspective directions for research. For example, let's take a notice of the characteristic functions of cooperative games construction problems.

For stochastic cooperative games, as well as for determined ones, the concept of superadditivity may be considered. Under strict superadditivity we understand the game, in which for $\forall \alpha$ an inequation is fulfilled:

$$v_{\alpha}(S \cup T) \geq v_{\alpha}(S) + v_{\alpha}(T). \qquad (3)$$

The game is not strict superadditive (almost superadditive), if the condition (3) is true for all α, begun from some α'.

Whereas in case of determined cooperative games the effect of collateral payment is seen when we compare the amount of income $v(S) + v(T)$ of two different coalitions S and T, that are received separately, with the income $v(S \cup T)$ of coalition $S \cup T$, that arises from their association, in the case of stochastic cooperative games we deal with more complicated and versatile situation.

Certainly, in stochastic cooperative game, as in analogue of determined game, the effect of collateral payment also can be generated by the transition (when the participants of coalitions S and T are associating in the only one coalition $S \cup T$) to some new value of characteristic function $\tilde{v}(S \cup T)$.

However if we simply add the incomes of coalitions S and T, that in determined game corresponds to insignificant cooperative game, we should take into account that VaR of stochastic amount of their incomes $\tilde{v}(S) + \tilde{v}(T)$, in general case, will not be equal the amount of VaR incomes

$$v_{\alpha}(S + T) \neq v_{\alpha}(S) + v_{\alpha}(T). \qquad (4)$$

In addition, we should emphasize that both cases, in which coalitions' revenues are independent variables, and cases conceding possible correlation between them deserve special consideration.

All listed stochastic cooperative games' properties can be widely used in the procedures of building characteristic functions. Indeed, the problem of determination of the potential coalitions' income often seems to be hard-to-solve or unsolvable on practice, but the problem of finding probabilistic characteristics of variables' sums with known cumulative distribution curves has standard solution.

Reflexive Partitioning Equilibria

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Keywords: *Bounded rationality, Reflexive ranks, Collective behavior, Mutual awareness.*

Consider the set $N = \{1, 2, \ldots, n\}$ consisting of n *agents*. The agent i chooses its action $x_i \in \Re^1$. The vector $x = (x_1, x_2, \ldots, x_n) \in \Re^n$ of the agents' actions called the *game situation* determines their gains defined by the *goal functions* $F_i(x)$, where $F_i(\cdot)$: $\Re^n \to \Re^1$. The rationality of the agent behaviour consists in striving to maximization of its goal function by choosing its own action: $x_i \in BR_i(x_{-i}) = \mathrm{Arg}\max\limits_{y \in \Re^1} F_i(y, x_{-i})$, $i \in N$, where $x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n) \in \Re^{n-1}$ is the *game environment* for the i-th agent, $BR_i(\cdot)$ is the *best response* of this agent, $i \in N$. Let us suppose that the functions $F_i(\cdot)$ are such that for any agent in any game environment there exists a unique best response.

Define $\aleph = \{N_0, N_1, \ldots, N_m\}$ as the partitioning of the agents set N, where N_i is the set of agents with i-th rank of reflexion, $i = \overline{0, m}$, m is the maximum rank of reflexion (Korepanov and Novikov, 2011). Let us call $\aleph$ by the *reflexive partition*. Let us suppose that the agent with some rank of reflexion k reliably knows the sets of agents with all lower ranks k' (where $k' < k - 1$) and certainly considers all agents with equal or higher ranks ($k'' \geq k$) as possessing some ranks (for example, the rank less by unity than its own rank (i.e. $k - 1$)). It corresponds to the assumption about that the agent excludes existence of agents with the same or higher rank of reflexion that its own one. At that the agent can incorrectly estimate the sets of agents with ($k - 1$)-th, k-th, and higher ranks of reflexion.

Let the vector x^0 of «initial» actions of the agents (or common priors as the probability distribution over $\Re^n$) be given as common knowledge among the agents. The behavior of the agents with k-th rank of reflexion depends on his awareness structure. Let $\aleph_{jk}$ be the *subjective reflexive partition* – this is the notions of the agent j with k-th rank of reflexion about the partition of all agents to reflexion ranks: $\aleph_{jk} = (\underbrace{N_0, N_1, \ldots, N_{k-2}, N_{k-1} \cup N_k \cup \ldots \cup N_m \setminus \{j\}}_{k}, \{j\}, \underbrace{\varnothing, \ldots, \varnothing}_{m-k-1}), j \in N_k$ (*). Let us note that within the framework of this expression the agent possessing k-th rank of reflexion has the right notions about the reflexion ranks of all the agents with strictly lesser ranks of reflexion. The *awareness structure* is determined by the aggregate of the reflexion partitions of all the agents. Rationality of the reflexive agent consists in choosing his action as his best response under subjective reflexive partition.

Thus, within the framework of the proposed *reflexive model of collective behavior*, the vector sequence of the agents' actions is uniquely defined by specifying the tuple $(N, \{F_i(\cdot)\}_{i \in N}, \aleph, x^0)$ consisting of the set of agents N, their goal functions $\{F_i(\cdot)\}_{i \in N}$, and the reflexive partition $\aleph$. This concept generalizes models of strategic reflexion (Chkhartishvili and Novikov, 2004), cognitive hierarchies and k-level models of collective behavior (see the survey in Wright and Leyton-Brown, 2010).

Consider the "game" model with homogeneous agents and aggregated influence of the game environment to the gain of each of them. Let:

1) all the agents from the set N have equal goal functions ($F_i(\cdot) = f(\cdot)$, $i \in N$);

2) the goal function of i-th agent depends on its action x_i (at that it is continuous and concave with respect to this variable) and on the *aggregated situation* $Q(x)$, where $Q(\cdot)$: $\Re^n \to \Re^1$ is the symmetric function of its arguments;

3) the agents make decisions once and simultaneously;

4) the initial vector of actions x^0 and the reflexive partition $\aleph$ are fixed.

The agents with the zero rank of reflexion will choose the actions $x_i = \arg \max\limits_{y \in \Re^1} f(y, Q(y, x^0_{-i}))$, $i \in N_0$. The agents with the first rank of reflexion will choose the actions $x1_j = \arg \max\limits_{y \in \Re^1} f(y, Q(y, x_{-j}))$, $j \in N_1$. The agents with the second rank of reflexion in accordance with expression (*) will choose the actions

$x2_j = \arg\max_{y \in \Re^1} f(y, Q(y, x_{l \in N_0}, x1_{l \in N_1 \cup N_2 \setminus \{j\}}))$, $j \in N_2$. And so on. The agents with the m-th rank of reflexion will choose the actions $xm_j = \arg\max_{y \in \Re^1} f(y, Q(y, x_{l \in N_0}, x1_{l \in N_1}, \ldots, x[m-1]_{l \in N_{m-1} \cup N_m \setminus \{j\}}))$, $j \in N_m$.

The vector $x^*(\aleph) = (x_{l \in N_0}, x1_{l \in N_1}, x2_{l \in N_2}, \ldots, xm_{l \in N_m})$ of agents' actions, which may be referred to as the *reflexive partitioning equilibria*, depends on the reflexive partition $\aleph$. Hence, modifying the reflexive partition one can modify the actions of the agents, i.e. realizes the *reflexive control* (see also the game-theory models of the reflexive control in (Chkhartishvili and Novikov, 2004)). Examples of applications of this concept of equilibrium to economical, financial and organizational problems is given in (Korepanov and Novikov, 2011).

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Dynamic Advertising Competition on Fashion Market

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Keywords: *Differential game, Oligopoly dynamic competition, Profit maximization, Advertising strategy, Nash equilibrium*

This paper presents the model of oligopolistic competition by advertising, as advertising is considered to be a main marketing instrument in modern industrial markets.

Since markets usually change with time, it is reasonable to investigate competition as a dynamic process. This gives an opportunity to analyze interdependencies between competitors' strategies and market changes. We research fashion market, which is a high-competitive oligopoly market and thus, the advertising strategy is essential for surviving on it.

We use a game theory approach and rather, a differential game approach, to model the competition on fashion market. Proposed by Gary M. Erickson in 2009 (Erickson 2009), the presented model has been successfully modified and adopted to fashion market.

The model accepts main specifications of fashion market. And the first specification requires that the demand of the firm should decrease proportionally to the demand of the whole market. Then the second specification asks for advertising the products of the firm only for the potential customers.

The model implies that there are n firms (players) on the market. It is assumed that each firm on the market has its maximum sales potential N_i. The firms use advertising $a_i(t),\, i = 1,2,\dots n,$ as a strategic instrument with an effectiveness β_i to find new customers and increase their sales $s_i(t)$. Players want to maximize their discounted profit V_i over an infinite horizon:

$$\max_{a_i > 0} \int_0^\infty e^{-r_i t} (q_i s_i - a_i^2)\, dt,$$

where q_i is a unit contribution and r_i is a discount rate of firm i. The cost of advertising is intended to be quadratic.

The model considers the dynamic demand as:

$$\dot{s} = \beta_i \alpha_i \sqrt{N_i - s_i} - \rho \sum_{j=1}^{n} s_j, \quad i = 1, 2 \ldots n,$$

where ρ is a decay rate.

This model is a modification of the Vidale-Wolfe model and, as it was already said above, of the Erickson model as well.

Using the models for V_i and $\dot{s}_i$, we set the dynamic differential game:

$$\max_{a_i > 0} V_i = \max_{a_i > 0} \int_0^\infty e^{-r_i t} (q_i s_i - a_i^2) dt, \, i = 1, 2, \ldots n,$$

with the restrictions

$$\dot{s}_i = \beta_i a_i \sqrt{N_i - s_i} - \upsilon \sum_{j=1}^{n} s_j, \, i = 1, 2, \ldots, n.$$

The Nash equilibrium and the expression for the optimal advertising strategies have been found for this problem:

$$a_i = \frac{\beta_i}{2} D_i \sqrt{N_i - s_i},$$

where

$$D_i = \frac{2(-r_i + \sqrt{r_i^2 + q_i \beta_i^2})}{\beta_i^2}.$$

The further analysis of the solution, as well as the comparative analysis for the case of the symmetric competition has been performed.

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Stable coalitional solution under environmental constraints

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Keywords: *Nash equilinrium, Time-consistency, Coalitional game, Differential game*

A game-theoretic model of territorial environmental production is studied. The process is modeled as cooperative differential game. In this paper the problem of allocation over time common cooperative benefit incurred by coalitions of firms in a coalitional game is considered. Coalitional solution is defined as follows: the Nash equilibrium in the game played by coalitions is computed and then the value of each coalition is allocated according to some given mechanism between its members. In this paper the mechanism for allocation over time of total individual benefit so that the initial agreement remains valid for the whole duration of the game is proposed. We proved that the coalitional total stock of accumulated pollution is strictly less then the pollution under Nash equilibrium for the whole duration of the game. The numerical example is given.

Modeling the Modernization Process of Innovative Engineering Enterprise

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Keywords: *Game theory applications, Industrial organization, Risk and uncertainty*

The issues relevant to the most dynamic sector of native engineering innovative high-tech machinery enterprises are considered. Such enterprises are typical for industry of machine tools and specialized equipment production, for military-industrial complex. They are capable to respond quickly on the demands of consumption, to produce an equipment competitive on the local and world market.

To operate effectively in the market of innovative equipment, the continuous modernization of enterprise production is an inevitable process involving significant risks. The only possible base for development of integrated modernization projects, for searching the most effective ways to reduce the intra-costs is investigations that determine the relationship between the type of machinery enterprise and management system structure. Enterprise management needs effective tools not only to assess the current efficiency of production, but also allows determine the direction of its continuous improvement. The challenge is to ensure coherent changes in the production management system and productive assets, i.e. precise technologies and technical equipment of production.

The main topic under discussion in the paper is the essential features of modernization process in high-tech machinery enterprises producing an experimental and series products on the same equipment. The formalized description and results of economic and mathematical modeling are presented. The direct-costing technique is selected as a basis for modeling.

The model under consideration reflects the following features of the high-tech machinery enterprises producing an experimental and series products on the same equipment.

1) There exist two parallel manufacturing flows of commercial products and experimental prototypes. The same metal-cutting equipment with numerical control that produce both flows of products reduces costs and improves the controllability, but also increases risks associated with uncertainties of the experimental design. It seems reasonable to use high-speed performance of management systems as the feature for such enterprises. That have to be considered in the process of reorganization of production management and the formation of motivation system of production workers and managerial staff.

2) The manufacturing cycles of innovative products are longer than the lifetime of modernization projects. Standard technique for risk analysis of investment project on the basis of the breakeven point is oriented on the base period less in duration then the period of the life of the investment project.

For high-tech machinery enterprises is common to work on forward contracts, fixing production volumes and prices of products for quite a long time (3-4 years). At the same time, inflation expectations cause the increase in cost of basic production factors. Rising prices for energy, raw materials, wages, utilities invariably leads to a limitation of profits.

An efficient tool at the disposal of production managers is to reduce standard variable costs at the expense of upgrading and providing its administrative modernization.

In the paper the technology determine the optimal date of purchase of equipment and design rationale for its composition, based on the coefficients of equivalence. The market competitiveness of machine-building enterprise depends on the level of productive capacity. However, the market competitiveness is mainly determined by the efficiency of the government and management for the production system and the whole enterprise.

The proposed model allows us to consider two aspects of performance management systems.

1) Priority measures to modernize the management in comparison with purely technological improvements in the productive capacity of the enterprise

2) The comparison of two schemes of management organization on engineering enterprises. The scheme based on the separation of ownership and control is described in terms of game formalism. One consequence may be effects similar to an agency conflict.

Model comparison is made with the scheme of charging the costs to top management on fixed costs. Payment for top managers of state corporations set up this way.

The presented results are used extensively when working with students and training specialists in the field of production modernization of high-tech innovative engineering plants and preparing the management reserve.

Tax Auditing Using Statistical Information About Taxpayers

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Keywords: *Tax auditing, Tax evasion, Statistical information, Income distribution, Optimal audit strategies, Optimal budget*

A model of tax auditing [1] in assumption, that tax authorities have some statistical information about the distribution of income among population, is considered.

As in previous models [1, 4] there is an assumption in the model that the true tax liability i of each taxpayer takes its value from finite set $M = \{m_0, m_1, \ldots, m_{N-1}\}$, where $0 \leq m_0 < m_1 < \ldots < m_{N-1} = 1$. The tax liability is a non-dimensional relative value, which can be defined as some possible payment of one taxpayer, measured in money units. Dividing the range of possible tax payments on N intervals, we make a correspondence between each interval and some group of taxpayers. Along with i the reported tax liability $r(i)$ is considered. This function and its argument take values from the set M.

If the evasion was revealed the taxpayer must pay the level of his evasion and penalty $(1+\pi)(i-r)h$, where money coefficient h and marginal penalty rate π assumed to be constants. There are different levels of rationality: risk averse, risk neutral and disposed to risk. First, let's consider risk neutral players and investigate tax evasions of the groups of taxpayers with the same level of income.

The tax authorities assumed to get some statistical information, which can be considered as an indicator of the tax evasion of a taxpayer, who declared some level of r. The information, mentioned above, is called a signal s, as it was called in [4]. Signal s can take two values $s \in \{d, u\}$, where u – is an information that the true income level is higher then declared, and d – is an absence of this information.

A taxpayer's strategy is to make a decision to evade or not to evade, i. e. to declare his income level less or equal to his true level of income ($r(i) < i$ or $r(i) = i$). A tax authority's strategy is to choose the audit probability $P(A \mid r = m_{l-1})$, where A is event, that the tax audit passed.

A tax authority's net income R consists of taxation T_{preA} (taxpayers' payments corresponding to their declared tax liability), taxes T_{postA} on the evasion level and penalties P (as the audit results) less total audit cost C :

$$R = T_{preA} + T_{postA} + P - C.$$

The following results were obtained for the risk neutral taxpayers:

1. The **theorem 1** about the optimal audit strategies for each income-level group, based on the condition

$$\begin{aligned} &P(A \mid r = m_0) \geq \frac{1}{1+\pi}, \\ &P(A \mid r = m_1) \geq \frac{1}{1+\pi}, \\ &\ldots\ldots\ldots\ldots\ldots \\ &P(A \mid r = m_l) \geq \frac{1}{1+\pi}, \\ &\ldots\ldots\ldots\ldots\ldots \\ &P(A \mid r = m_{N-2}) \geq \frac{1}{1+\pi}. \end{aligned} \tag{3}$$

2. The **theorem 2** about the optimal audit strategies with consideration of the signals, based on the condition

$$P(A \mid r = m_l, s = u) \geq \frac{\lambda}{1+\pi} \cdot \frac{1}{1 + (\lambda - 1)P\{s = u \mid r = m_l\}}, \tag{4}$$

where λ is the coefficient of trust to the signal, defined as

$$\lambda = \frac{P(A \mid r = m_l, s = u)}{P(A \mid r = m_l, s = d)}.$$

3. The **theorem 3** about the optimal budget $B_l = k_l P(A \mid r = m_l)c$ for tax auditing in the l -th income group (where c is the cost of one audit).

This model also can be considered for other levels of rationality of the players.

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ВЕСТНИК САНКТ–ПЕТЕРБУРГСКОГО УНИВЕРСИТЕТА

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On The Consistency of Equilibria in Multicriteria Extensive Games

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Keywords: *Extensive games, Multicriteria games, Equilibria, Time-consistensy*

We consider finite n-person extensive games with perfect (or incomplete) information [2, 3] where the player`s payoff is given by a vector instead of a scalar (so-called multicriteria extensive games).

Pure strategy combinations $\varphi = (\varphi_1, \ldots, \varphi_n) \in \prod_{j=1}^{n} \Phi_j$ provide to each player i "payoffs" given by an $r(i)$-vector valued function $H_i : \prod_{j=1}^{n} \Phi_j \to R^{r(i)}$, i.e. player i takes $r(i)$ criteria into account. We denote by $MG^P(n, K, r(1), \ldots, r(n))$ the class of all finite n-person multicriteria extensive games with perfect information (on the game tree K).

For all $x, y \in R^t$ we will use the notation $y > x$ if and only if $y_i > x_i$ for all $i \in \{1, \ldots, t\}$. The strategy profile $\hat{\varphi} = (\hat{\varphi}_1, ..., \hat{\varphi}_n)$ is called (weak) equilibrium [4, 1] in multicriteria game $\Gamma \in MG^P(n, K, r(1), \ldots, r(n))$, iff

$$\forall i \in N \,\overline{\exists} \varphi_i \in \Phi_i : H_i(\varphi_i, \widehat{\varphi}_{-i}) > H_i(\widehat{\varphi}_i, \widehat{\varphi}_{-i})$$

The set of all equilibriums in multicriteria game Γ denote by ME(Γ).

Decomposition of Γ at the position x onto subgame Γ_x and factor-game Γ_D generates corresponding decomposition of pure strategies $\varphi_i \to (\varphi_i^D, \varphi_i^x), i \in N = \{1,...,n\}$ and mixed strategies as well.

As was established in [2] for n-person finite extensive unicriterium games if we find the Nash equilibrum φ^x in the subgame Γ_x and the Nash equilibrum φ^D in the corresponding factor-game $\Gamma_D(\varphi^x)$, the composite strategy profile $\varphi = \{(\varphi_i^D, \varphi_i^x)\}_{i=1}^n$ forms the Nash equilibrium in Γ. This fact, in particular, allows to use the backwards induction procedure to construct subgame perfect equilibrium in unicriterium multistage game.

Unfortunately, this basic result is not valid for equilibria in multicriteria extensive games (with perfect or incomplete information), and thus one can not use the backwards induction procedure (in the direct way) to construct subgame perfect equlibriums in multictiteria game $\Gamma \in MG^P(n, K, r(1),\ldots, r(n))$.

However, we prove that the set $SPME(\Gamma)$ of all subgame perfect equilibriums (in pure strategies) in multicriteria game $\Gamma \in MG^P(n, K, r(1),\ldots, r(n))$ in nonempty and propose the technique to construct this set of equilibriums.

In addition, we prove that the set $ME(\Gamma)$ in mixed strategies in a finite multictiteria n-person extensive game Γ with incomplete information satisfies the time-consistensy property [3].

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A Game-Theoretic Approach for Assignment of Items Weights in The Test

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Keywords: *Testing, Cooperative game, Item weight*

Some problem of test design is a problem of appointment of items weights [1]. In many tests the items weights are equal. As the complexity of the test structure increases, it becomes necessary to determine the difficulty level of the test and its parts. It is especially important if we want to compare students who had different tests or if a time for solving of all test tasks is insufficiently.

Definition. Let $I = \{1, 2, \ldots, n\}$ be a set of test items. A complexity of the test I is a nonnegative function that assigns a number $v(K)$ to any aggregate K ($K \subseteq I$) of test items and possesses the following properties.

1. $v(\varnothing) = 0$

2. If $K' \subset K$ then $v(K') < v(K)$.

The number $v(K)$ is called the complexity of aggregate K. If a student can solve all tasks of a subset $K \subseteq I$ then we say that he has knowledge level K or he has test skill K.

Thus, the complexity function v should be defined on 2^n subsets of set I, and it is necessary to consider a incomplete and inconsistent understanding of intuitive complexity. In game theory the complexity function v is called the characteristic function of a cooperative game $\Gamma = < I, v >$ that characteristic function $v(K)$ possessing monotonic property [3].

In practice, the function v is additive and the complexity of any aggregate of tasks K is $v(K) = \sum_{i \in K} \phi_i$, where ϕ_i is complexity (weight) of the i-th test item.

An item weight can be found after a testing of a large group of pupils. The weight ϕ_i of item i is a proportion N_i / N, in which the number N_i is the number of students solved the item i of test and N is the total number of pupils participated in testing [1]. But such approach stimulates nonconventional methods of training because the less pupils solved a task the more weight get the task. However it is quite possible that time expended on preparation a student for solution of a rare task is short and the teacher begins to prepare students for a solution of rare tasks breaking logic of a teaching course.

In this work, we first define complexity function v and after we define items weights of the test I.

The learning process usually can be represented as a tree of learning. In this graph vertices are knowledge levels and edges are training ways of student. Thus, complexity function v can be defined from structure of a training course and from experience or program of learning.

Let $v(K)$ be a time for a pupil preparation to solve all items of aggregate $K \subseteq I$. Then weight ϕ_i of the i-th item of the test I can be find by the formula

$$\phi_i = \sum_{i \notin K} p_K \cdot (v(K \cup \{i\}) - v(K)), \tag{1}$$

where $\{p_K\}$ is a probability distribution on set all subset $\{K\}$ that don't contain the i-th item.

The difference $v(K \cup \{i\}) - v(K)$ in the formula (1) is time for getting skill i if he can solve all tasks for subset $K \subseteq I$. Hence ϕ_i is the expected time to prepare a student for the i-th item for various knowledge levels.

If all sequences of training have equals probabilities then $p_K = k! \cdot (n-k-1)! / n!$ $k = | K |$, and ϕ_i becomes the expected time of training of the student. (We understand an arbitrary permutation of numbers $1, 2, \ldots, n$ as the program of training.).

If all knowledge levels have equals probabilities then $p_K = 1 / 2^{n-1}$ and ϕ_i becomes the expected time of training of the student. In this case the number ϕ_i is called the weight of Banzafa in the test.

Example. We assume that test items $I = \{1, 2, \ldots, n\}$ are ordered so that it is impossible to get skill i without getting skills with smaller numbers. Or the student can't solve item i if he can't solve item $i-1$. Suppose that the time for getting skill i is equal to a_i. Thus $a_i = v(\{1, 2, \ldots, i\}) - v(\{1, 2, \ldots, i-1\})$. Hence, the time for getting skill K is defined by the time for training the student for getting skill with the greatest number from subset K that is

$$v(K) = \sum_{i=1}^{m} a_i \, , \; m = \max K \qquad (2)$$

Now we have a theorem, which allows us to find the items weights for the test $I = \{1, 2, \ldots, n\}$ with characteristic function (2).

Theorem. In a cooperative game $\Gamma = < I, v >$ with characteristic function (2) the Shapley value $\phi = (\phi_1, \phi_2, \ldots, \phi_n)$ has the following coordinates

$$\phi_i = \sum_{k=1}^{i} \frac{a_k}{n-k+1} \, , \; i = \overline{1, n} \, ,$$

And coordinates of the Banzhaf vector $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ of the game are the following

$$\sigma_i = \sum_{k=1}^{i} \frac{a_k}{2^{n-k}} \, , \; i = \overline{1, n} \, .$$

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Uncertainty Aversion and a Theory of Incomplete Contract

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Keywords: *Uncertainty aversion, Strategic uncertainty, Coalition-formation, Stability, Core-criterion*

This paper is to provide a theoretical foundation of incomplete contract in an extensive game of multi-agent interaction. It aims to explain why rational agents may agree upon incomplete contracts even though it is costless to sign a complete one. It is argued that an incomplete contract creates strategic uncertainty. If agents' attitudes toward uncertainty are not neutral, then an incomplete contract as final solution can be the consequence of common knowledge of rationality. This paper assumes that all agents are uncertainty averse in a sense of Gilboa and Schmeidler (1989); and that agents can form coalitions as part of strategic play. All these are embedded into the equilibrium solution concept.

Multi-Agent Interaction in the Dynamic Options Model

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Keywords: *Multi-Agent Interaction, Equilibrium, Portfolio Optimization, Compromise Solution, Bellman Dynamic Programming Method*

A multi-agent interaction model on the derivatives market is formalized. The market state is influenced by the controls u_i^t that are being chosen at the moment $t \in T = \{0,1,2,...\}$ by the investors $i = 1,...,|I|$. Agents $i = 1,...,|I|$ act on the market by diversifying their portfolios with stocks and bonds at the moment $t \in T = \{0,1,2,...\}$. The scheme based on the Bellman dynamic programming method for finding the guaranteed payoff of the investor considering issuing a call option on stocks is proposed. Numerical example is given.

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Hierarchical Game Model for Sugar Production

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Keywords: *Sugar production model, PMS vector, Nash Equilibrium*

Two level game theoretical model for sugar production is considered. On the first level we have one player A (company responsible for sugar production) on the second level the players $B(i)$, $i = 1 \ldots n$ representing sugar production facilities. The players B can be divided on two subsets $M(1)$ and $M(2)$. $M(1)$ - the set of players producing sugar for consumption and the set $M(2)$ of players producing sugar for fuel production. The production costs of each facility is given (dependent of the belonging to one of the sets $M(1)$ or $M(2)$). The profit of each player B and the player A also depends from the fact for what purpose the sugar is produced. The constructed $n+1$ person game gives the possibility to investigate *Nash Equilibrium*, in a factor 3 person game were $M(1)$, $M(2)$ and A are considered as players. Having *NE* for three person game one can compute *PMS-vector* in the previously defined $n+1$ person game. This gives the possibility of optimizing the partition of the set $B = (B(1) \ldots B(n))$ on two subsets $M(1)$ and $M(2)$.

Centrality in Weighted Social Networks.
A Game Theoretic Approach

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Keywords: *Weighted networks, Centrality, probabilistic Myerson value, Generalized probabilistic Myerson value*

This communication deals with a key issue in network analysis: the problem of centrality of nodes in networks. Most of the work on this topic is about the case in which links are dichotomous. In the present work, we consider the case of networks in which ties are not just either present or absent, but have some weight attached to them. Following Granovetter (1973) we will interpret these weights as a function of duration, intimacy or intensity of the relations. Moreover, we will suppose that actors in the network are simultaneously players in a cooperative TU game representing the interests that motivate their interactions. Our approach is then a game theoretic one. We will propose as centrality measure for each node its probabilistic Myerson value (Calvo et al., 1999 and Gómez et al., 2008) assuming that the game is a symmetric one and thus, no a priori differences among players exist. We will prove that this measure satisfies some relevant properties to be considered as a centrality one.

Chinese Auctions

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Keywords: *Blotto game, Chinese Auction, Budget constraint.*

A Chinese Auction is one of the most popular mechanisms at charity or other fund-raising events. In a Chinese Auction, bidders buy lottery tickets, which are essentially chances to win items. Bidders may buy as many tickets as they like, and bid them on any item(s) they want by placing them in a basket or other container in front of the item(s) they are trying to win. At the conclusion of bidding, the winning ticket is drawn from the tickets bid on each item, and the item is given to the owner of that ticket.

We consider a model where K bidders are competing for N items in the Chinese Auction. Bidders have to decide how to allocate their budgets across all the items. The main assumption of this paper is that the winner for each item is determined stochastically.

We analyze four situations: bidders can have given, or costly budgets and aim to maximize the total prize, or maximize a chance to win the majority.

We find a unique pure-strategy equilibrium of the game in all four situations. It turns out that the players allocate their budgets in the same proportion and each player competes in each contest in the Nash equilibria. Moreover, the individual equilibrium strategy depends on the contests' values and the individual budget, but it is independent from the budgets of all other players. The equilibria have a monotonic property: a player with higher budget has higher chance to win each item.

We consider also a situation when individual budgets are private information. It turns out that there exists a unique monotonic Bayesian equilibrium.

There are many applications of our model, such as R&D, arm races, military conflicts, simultaneous rent-seeking activities, and so on.

Mechanisms of Environmental Incentive Regulation: Why Ecological Policies in Transition and Developing Economies are not Effective?

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Keywords: *Mechanisms, Environment, Regulation, Game, Incentives, Contract*

An important part of the global environment stabilization problem relates to ensuring effective ecological regulation in transitional and developing economies. Usually researchers try to explain modest results of economic policy in Russia and other transition economies, in particular of environmental incentive regulation, by presence of inherited behavior and institutions, and also by conflicts between new formal and old informal institutes. The paper demonstrates another possibility: "new" economies may possess purely economic features which force serious distinctions in results of work of institutional mechanisms which have shown good results in developed countries. A model is built as a game between a politician (a regulator who may be interested or uninterested in rents of firms) and two types of firms (one of them is more interested in increasing pollution levels).

Laffont (2000) has investigated a model, in which firms-monopolists have the following cost function:

$$C(\theta,d) = \theta(K-d)\,,$$

where $K > 0$ – is a constant common for all firms, $\theta > 0$ is a cost parameter which is a private information for a firm (type of firm), $d > 0$ – is a pollution level permitted for a firm of that type (determined by the regulator or chosen by the firm from a menu of contracts offered by the regulator). If there are two types of firms, $\underline{\theta} < \bar{\theta}$, then, in

Laffont's case, firm $\underline{\theta}$ is always efficient and receives information rent. Matveenko (2010) introduced a more general cost function:

$$C(\theta, d) = \kappa(\theta) - \theta d \ ,$$

where $\kappa(\theta) > 0$. With two types of firms, it is natural to consider

$$\tilde{K} = \frac{\kappa(\overline{\theta}) - \kappa(\underline{\theta})}{\overline{\theta} - \underline{\theta}}$$

as an index of relative economic efficiency.

Let S be a social value of a project; $V(d)$ is a social value of a harm from pollution, and $V'(.) > 0, V''(.) > 0$. Welfare of consumers equals

$$S - V(d) - (1 + \lambda)t \ .$$

The factor $1 + \lambda$ may be interpreted as return factor, which characterizes the gain of using in other projects means which the society loses making transfer t.

With the generalized cost function, information rent is received by a relatively efficient firm. For "small" values of $\tilde{K}$, the firm of type $\overline{\theta}$ proves to be relatively efficient firm, while for "high" value of $\tilde{K}$ – the firm of type $\underline{\theta}$ does. For "intermediate" values $\tilde{K}$ no type of firm is able to capture a rent. The concepts of "small" and "large" $\tilde{K}$ are made precise in dependence on which type of regulator is in power and forms the menu of contracts. Three types of regulators are under consideration, they differ by objective functions: the social maximizer, interested majority, or uninterested majority. One denotes by α^* a frequency of "interested" in society in those periods when "interested" are in power; v is a frequency of firms of type $\underline{\theta}$ in economy.

It is proved in Matveenko, 2010 that under some conditions interested majority in power ought to solve what mechanism to use: separating (a menu of contracts) or pooling (a uniting contract for both types of firms).

In the present paper another situation is considered: the society defines a kind of mechanism, while the ruling majority makes solution under this mechanism. Under assumption of small $\Delta\theta$, the mechanisms are compared from the point of view of social welfare.

We show that under conditions which seem to be typical for developing and transitional economies, namely for "small" $\tilde{K}$ (when firms-polluters of type $\overline{\theta}$ gain from extending their possibilities to pollute) and under condition

$$\frac{v}{\alpha^*} < 1 + \lambda < \frac{1 - v}{\alpha^*}$$

(which means that firms of type $\overline{\theta}$ are usual in the economy), the pooling (that is non-market) mechanism is preferable for society.

It appears that if under similar conditions the choice of mechanism is executed not by the society but by the interested majority, the separating (i. e. in the more degree market) mechanism would be picked out.

For "large" $\tilde{K}$ (when "green" firms are more effective, what is typical for developed countries) the separating (market) mechanism is preferable for society, and the interested majority picks out this mechanism as well.

The model shows that even the "standard" regulation institutions, which proved to be good in developed countries, may give absolutely other, unexpected results in those economies, in which the firms producing relatively high direct or indirect ecological damage thanks to that possess large relatively economical effectiveness and occupy an essential part of economy. This case is typical for developing and transitional economies.

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Bargaining Models and Mechanism Design

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The talk is devoted to the bargaining models and the analysis of its design which satisfies some proofness conditions. The bargaining process is based on some definitions: must be determined

– the participants (players)

– the sequence of the moves

– the payoffs of players

– horizon of bargaining process

– fairness principles.

We start from the cake division problem and discuss different positional and rational models to solve it. We introduce the bargaining model of sequential random offers in which the arbitrator generates some variants of the decision and the players choose one of them. We consider different versions of this approach in cake division problem and related problems.

Arbitration procedures also apply to Labor-Manager interactions in wage rate problem. We consider traditional and new arbitration procedures and compare the decisions of conventional, final-offer arbitration and their combination. Some of arbitration procedures use the multiple arbitration. In this case the solution is accepted via voting or lottery mechanism.

Another popular bargaining model is the seller-buyer interactions. We formulate the multistep bargaining model in which the players use the Bayesian strategies and can change it in each step. The equilibrium is derived for different probability distributions for reservation prices.

We discuss some new approaches to the construction of the tender's design. The players $\{1, 2, ..., n\}$ submit the projects which are characterized by vectors $\{x^1, ..., x^n\}$

from some feasible set S in space R^m. The arbitrator considers the bids and chooses one of the projects using a stochastic procedure. The winner k receives a gain $h_k(x^k)$ which depends on the parameters of the project. So, the payoffs in this non-cooperative game are

$$H_k(x^1,...,x^n) = h_k(x_k)\mu(S_k), \quad k = 1,...,n,$$

where $\cup_k S_k$ is the Voronoi diagram of set S. Nash equilibrium in the problem is derived.

The bargaining process finally ends with an agreement. We analyze some conditions for stability of this agreement and time consistency for the dynamic systems.

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On a Discrete Arbitration Procedure with Nonuniform Distribution

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Keywords: *Game, Arbitrator, Equilibrium*

We consider a non-cooperative zero-sum game in which two players L and M (Labour and Manager) have a dispute on an improvement in the wage rate. Player L makes an offer x, and player M - an offer y; x and y are arbitrary real numbers. If $x \le y$, there is no conflict, and the players agree on a payoff equal to $(x+y)/2$. If, otherwise, $x > y$, the parties call in the arbitrator A. He use the final offer arbitration procedure [1]. Denote the arbitration settlement by z. The payoff in this game has a form: $H(x,y) = EH_z(x,y)$, where

$$H_z(x,y) = \begin{cases} \dfrac{x+y}{2}, & if \quad x \le y \\ x, & if \quad x > y, |x-z| < |y-z| \\ y, & if \quad x > y, |x-z| > |y-z| \\ z, & if \quad x > y, |x-z| = |y-z|. \end{cases} \tag{1}$$

Let $-\infty < y \le 0 \le x < +\infty$ and z is a discrete random variable. In the papers [2]-[3] for the cases where z is distributed with equal probabilities in the points -1 and 1 or $-1, 0$ and 1, respectively, the equilibrium in the game is found among mixed strategies. Denote $f(x)$ and $g(y)$ the mixed strategies of the players L and M, respectively. We have:

$$f(x) \ge 0, \int_0^{+\infty} f(x)dx = 1, \quad g(y) \ge 0, \int_{-\infty}^{0} g(y)dy = 1.$$

By symmetry, it follows that the value of the game is equal to zero, and the optimum strategies must be symmetric in respect to axes of ordinate, i.e. $g(y) = f(-y)$. It therefore suffice to build an optimal strategy for one of the players, e.g. L.

Let the arbitrator chooses one of three numbers: $-1,1,0$ - with the probabilities $\frac{1-p}{2}$, p, $\frac{1-p}{2}$ $(0<p<1)$, respectively. The case $p=1$, $p=0$ and $p=\frac{1}{3}$ were analysed above.

Theorem 1. *If* $p \in [p_0, 1)$, *where* p *is the root of the equation* $p^4+8p^3+4p^2+4p-1=0$ *from the interval* $\left(3-2\sqrt{2}, \frac{1}{5}\right)$, *then for the player* L *the optimal strategy is*

$$f(x)=\begin{cases} 0, & \text{if} \quad 0 \le x < c, \\ \dfrac{1+p}{4p}\dfrac{\sqrt{c}}{\sqrt{x^3}}, & \text{if} \quad c < x < c+2, \\ 0, & \text{if} \quad c+2 < x < +\infty, \end{cases} \tag{2}$$

where $c=\frac{(1-p)^2}{2p}$.

Let the arbitrator chooses one of four numbers: $-3,1,1,3$ - with probabilities $\frac{1}{2}-p, p, p, \frac{1}{2}-p$, respectively. It is obviouse, that $0 \le p \le \frac{1}{2}$. The case $p=\frac{1}{2}$ was analysed above and the case $p=0$ gives the same result.

Theorem 2. *If* $p \in \left[p_0, \frac{1}{2}\right)$, *where* p *is the root of the equation* $32p^5+16p^4+24p^3+8p^2-8p+1=0$ *from the interval* $\left(\frac{1}{\sqrt{35}}, \frac{1}{5}\right)$, *then for the player* L *the optimal strategy is*

$$f(x)=\begin{cases} 0, & \text{if} \quad 0 \le x < c, \\ \dfrac{c+1}{4p\sqrt{(x+1)^3}}, & \text{if} \quad c < x < c+2, \\ \dfrac{c+3}{4p\sqrt{(x-1)^3}}, & \text{if} \quad c+2 < x < c+4, \\ 0, & \text{if} \quad c+4 < x < +\infty, \end{cases} \tag{3}$$

where $c=\frac{\sqrt{p^2+1}}{p}-2$.

Let the arbitrator chooses the number 0 with probability p, and chooses the numbers $-n$ and n with equal probabilities $\left(\frac{1-p}{3-p}\right)^n$ $(n \in N, 0 < p < 1)$. We have:

$$p + 2\sum_{n=1}^{\infty}\left(\frac{1-p}{3-p}\right)^n = 1.$$

Theorem 3. *If* $p \in [p_0, 1)$, *where* p *is the root of the equation* $p^5 + p^4 - 8p^3 - 6p^2 - 5p + 1 = 0$ *from the interval* $\left(\frac{1}{7}, \frac{1}{6}\right)$, *then for the player* L *the optimal strategy is the strategy (2).*

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Numerical Approximations of the Information Sets in a Simple Search Game on the Plane

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Keywords: *Differential search game, Information set, Mixed strategies*

The zero-sum differential search game between pursuer P and evader E in the class of mixed strategies with finite spectrum is considered([1], [2], [3]). The dynamic of the game is described by the following differential equation

$$P:\dot{x} = u, \|u\| \le \alpha,\; x(0) = x_0, \|x_0\| = r + l, x, u \in R^2,$$
$$E:\dot{y} = v, \|v\| \le \beta,\; y(0) = y_0, \|y_0\| \le r, \beta \le \alpha, y, v \in R^2,$$

where $r > 0$ is the radius of the uncertainty disk of the initial disposition of Player E and the quantities r , l , α , and β are parameters of the game. The detection set S(x) of player P is the disk of radius l centered at the position of the pursuer. Each player has only a priory information on the initial position of the other. Player P tries to maximize the probability of detection of the evader using mixed strategies. The mixed strategies of the pursuer determined with help of the an auxiliary game with a team of pursuers. In this paper for the game with a team of pursuers we construct the information set ([4], [5], [6]) for the position of the evader and study properties and numerical approximation problems of this information set.

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The Set of Nash Equilibria in a Differential Game Using Chance Moves in Limite Number of Points

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Keywords: *Equilibrium, Strategy, Distribution.*

A non-zero-sum differential game is considered. A method of back-coming inductiona is used to find the situations of absolute equilibrium by Nash. Each of the elements of the set, from which a player chooses, has different probability.

МАТЕМАТИЧЕСКАЯ ТЕОРИЯ ИГР И ЕЕ ПРИЛОЖЕНИЯ

Выходит ежеквартально с января 2009 года

Efficient Enumeration of Complete Simple Games

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Keywords: *complete simple game, distributive lattice, pre-ordering.*

This algorithm does not use linear programming, being its computational complexity analysis easier than the ones which use it. Moreover, the enumeration is made (pre-)ordering the complete games in a lattice structure, which may be interesting for the verification of other properties; for instance, the level of a game inside the lattice, its distance to the greatest and least element, or the distance between two different games.

Finally, we also compare this algorithm with the used in [FM08], from the point of view of CPU time, and the maximum number of players for which is able to obtain in the practice all complete games.

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Sequential Equilibria of Games with Infinite Sets of Types and Actions

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We define a general concept of sequential equilibrium for dynamic multi-stage games in which players have infinite type sets and infinite action sets. A finite approximation of an infinite game is defined by taking a finite partition of each player's type space and a finite subset of each player's action space at each period. Sequential equilibria are then defined as limits of approximate sequential equilibria of a net of such finite approximations.

Any game has a nonempty set of sequential equilibria.

Journals in Game Theory

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Proportional Solutions to Games with Restricted Cooperation

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Keywords: *Claim problem, Proportional method, Proportional nucleolus, Weighted entropy*

A *TU–cooperative game with restricted cooperation* is a quadruple (N, A, c, v), where N is a finite set of agents, A is a collection of coalitions of agents, c is a positive real number (the amount of resourses to be divided by agents), $v = \{v(T)\}_{T\in A}$, where $v(T) > 0$ is a claim of coalition T. We assume that A covers N. A *solution* F is a map that associates to any game (N, A, c, v) a subset of the set $\{\{y_i\}_{i\in N} : y_i \geq 0, \sum_{i\in N} y_i = c\}$. We denote $y(S) = \sum_{i\in S} y_i$.

If $A = \{\{i\} : i \in N\}$ then a *claim problem* arises. Claim methods and their axiomatic justifications are described in surveys [1] and [3].

In this paper we consider generalizations of the Proportional method for claim problems.

Let $X \subset R^n$, $f_1,\dots,f_k$ be functions defined on X. For $z \in X$, let π be a permutation of $\{1,\dots,n\}$ such that $f_{\pi(i)}(z) \leq f_{\pi(i+1)}(z)$, $\theta(z) = \{f_{\pi(i)}(z)\}_{i=1}^n$. Then $y \in X$ belongs to the *nucleolus with respect to* $f_1,\dots,f_k$ *on* X iff

$$\theta(y) \geq_{lex} \theta(z) \quad forall \quad z \in X.$$

We consider the following generalized methods.

1. A vector $y = \{y_i\}_{i\in N}$ belongs to the *proportional solution* of (N, A, c, v) iff there exists $\alpha > 0$ such that $y(T) = \alpha v(T)$ for all $T \in A$, $y_i \geq 0$ for all $i \in N$, $\sum_{i\in N} y_i = c$.

2. A vector $y = \{y_i\}_{i\in N}$ belongs to the *weakly proportional solution* of (N, A, c, v) iff $y_i \geq 0$ for all $i \in N$, $\sum_{i\in N} y_i = c$, $y(S) / v(S) = y(Q) / v(Q)$ for $S, Q \in A$ with $S \cap Q = \varnothing$.

3. A vector $y = \{y_i\}_{i\in N}$ belongs to the *proportional nucleolus* of (N, A, c, v) iff y belongs to the nucleolus w.r.t. $\{f_T\}_{T\in A}$ with $f_T(z) = z(T) / v(T)$ on the set $\{z = \{z_i\}_{i\in N} : z_i \geq 0, \sum_{i\in N} z_i = c\}$.

4. Let G be a class of strictly increasing continuos functions g defined on $(0, +\infty)$ such that $g(1) = 0$, $g(t) \to +\infty$ as $t \to +\infty$, and $\lim_{x\to 0} \int_a^x g(t)dt < +\infty$ for each $a > 0$.

A vector $y = \{y_i\}_{i\in N}$ belongs to the g*–solution* of (N, A, c, v) if y minimizes $\sum_{S\in A} \int_{v(S)}^{z(S)} g(t / v(S))dt$ on the set $\{z = \{z_i\}_{i\in N} : z_i \geq 0, \sum_{i\in N} z_i = c\}$.

Examples.

1.Let $g(t) = \ln t$, then $\int_{v(S)}^{z(S)} g(t / v(S))dt = z(S)(\ln(z(S) / v(S)) - 1) + v(S)$ and the g–solution is the weighted entropy solution [2].

2.Let $g(t) = t^q - 1$, where $q > 0$, then we obtain the minimization problem for $\sum_{S\in A} [\frac{z(S)^{q+1}}{v(S)^q} - (q+1)z(S)]$ that was considered in [4].

For each A, $c > 0$, v with $v(T) > 0$, the proportional nucleolus and the g–solution of (N, A, c, v) are nonempty and define uniquely $y(T)$ for each $T \in A$.

The proportional solution of (N, A, c, v) is nonempty for all $c > 0$, v with $v(T) > 0$ iff A is a minimal covering of N.

Necessary and sufficient condition on A that ensures nonemptyness of the A–weakly proportional solution for all $c > 0$, v with $v(T) > 0$ was obtained in [2].

The g–solution is contained in the proportional solution of (N, A, c, v) for all $c > 0$, v with $v(T) > 0$ iff A is a partition of N.

For each $i \in N$, denote $A_i = \{T \in A : i \in T\}$.

Theorem 1. *Let $g \in G$. The g–solution of (N, A, c, v) is contained in the weakly proportional solution of (N, A, c, v) for all $c > 0$, v iff $A = \cup_{i=1}^{k} B^i$, where each B^i is contained in a partition of N;*

$Q \in B^i$, $S \in B^j$, and $i \neq j$ imply $Q \cap S \neq \varnothing$;

for each $i \in N$, $Q \in A_i$ $S \in A$ with $Q \cap S = \varnothing$, there exists $j \in N$ such that $A_j = A_i \cup \{S\} \setminus \{Q\}$.

Theorem 2. *The proportional nucleolus of (N, A, c, v) is contained in the weakly proportional solution of (N, A, c, v) for all $c > 0$, v iff $A = \cup_{i=1}^{k} B^i$, where* each B^i is contained in a partition of N;

$Q \in B^i$, $S \in B^j$, and $i \neq j$ imply $Q \cap S \neq \varnothing$;

if $M \subset \{1, \ldots, k\}$, $\cap_{i \in M} S_i \neq \varnothing$ for some $S_i \in B^i$, then $\cap_{i \in M} Q_i \neq \varnothing$ for all $Q_i \in B^i$, $i \in M$.

A collection of coalitions A is *totally mixed at* N if $A = \cup_{i=1}^{k} P^i$, where P^i are partitions of N and for each collection $\{S_i\}_{i=1}^{k}$, $S_i \in P^i$, we have $\cap_{i=1}^{k} S_i \neq \varnothing$.

Theorem 3. *Let $g \in G$. The g–solution of (N, A, c, v) coincides with the weakly proportional solution of (N, A, c, v) for all $c > 0$, v iff A is totally mixed at N.*

Theorem 4. *Let $A = \cup_{i=1}^{k} B^i$, where $k \geq 2$,*

each B^i is contained in a partition of N;

$Q \in B^i$, $S \in B^j$, and $i \neq j$ imply $Q \cap S \neq \varnothing$;

each $i \in N$ belongs to m coalitions in A, $2 \leq m \leq k$;

if we take any m collections B^{j_1},... B^{j_m}, then for each collection $\{S_{j_t}\}_{t=1}^m$ with $S_{j_t} \in B^{j_t}$, we have $\cap_{t=1}^m S_{j_t} \neq \varnothing$.

Then the proportional nucleolus of (N, A, c, v) *coincides with* g*–solution of* (N, A, c, v) *for all* $g \in G$, $c > 0$, v *and is contained in the weakly proportional solution.*

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The Dynamic Interaction between Entrepreneurship and the Public Sector Efficiency

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Keywords: *Occupational Choice, Public Sector Efficiency, Entrepreneurship, Cultural Transmission, Social Preferences.*

Entrepreneurship is widely viewed as a key aspect of economic dynamism as it determines productivity, innovation and employment and, then, economic growth. In this paper we focus in two of the main determinants of the level of entrepreneurship in a society: the regulatory framework (related with the public sector performance) and the distribution of preferences in the society and its intergenerational transmission, what is also called the culture of a society.

It is often suggested that there is a relationship between the public sector performance and the entrepreneurship level and it is many times argued that the best thing a Government can do to promote entrepreneurship is to create an efficient context in which entrepreneurship and other forms of individual initiative can flourish. As the OECD-Eurostat Entrepreneurship Indicator Programme (EIP 2009) indicates entrepreneurship requires a good, clear and enforceable regulatory framework, e.g. property rights, institutions for resolving disputes, protection of contractual partners, etc.

Many authors have argued that the prevailing social norms, attitudes or culture of a particular region have played a critical role determining its development. Several institutional, political or economic outcomes are difficult to explain just in terms of economic incentives. Recent empirical research supports the theoretical proposition of a positive correlation between risk attitudes and the decision to become an entrepreneur (see, e.g., Cramer, Hartog, Jonker, and Van Praag, 2002; Caliendo, Fossen, and Kritikos, 2006). In addition, Domen, Falk, Fuman (2006), De la Paola (2010) and Leuermanm

Necker (2010) report evidence that a crucial determinant of entrepreneurship, willingness to take risk, is transmitted from parents to children.

There exists also another strong indirect indicator of this parental transmission. Recent empirical work has shown that self-employment is correlated across generations, so that the children of the self-employed are themselves more likely to be self-employed (Colombier and Masclet 2008; Hundley 2006, among others). Therefore, not only preferences but also behaviour shows a high intergenerational correlation.

In this context, the efficiency of the public sector as a key factor for entrepreneurship will influence not only the decision of risk takers to become entrepreneurs but also the incentives of parents to transmit risk-taking preferences. Therefore, there is a two-way feedback between changes in the preference distribution of the population and changes in the distribution of behaviour in the population.

We present an overlapping generation model with intentional and costly cultural transmission of preferences in order to analyze the interaction between entrepreneurship and the efficiency of the public sector. Two important features of our model are that on the one hand, each adult makes an occupational choice between becoming an entrepreneur, a civil servant or a routine producer. And, on the other hand, all the adults´ population in each generation makes a decision on taxes by majority voting. This collective decision on a proportional tax affects the net profits obtained by the entrepreneurs and the wage of the civil servants.

If an individual decides to become an entrepreneur and start a risky project, the expected results will depend crucially on the level of effort exerted by the civil servants. This level of effort captures in the model the level of efficiency of the public sector and can have other alternative interpretations as, for instance, the level of corruption in the public sector. If civil servants work hard (or alternatively there are low levels of corruption), the expected profits for the entrepreneur are the highest. Conversely, if they exert low effort (or there are high levels of corruption), the project may likely fail.

We want to analyze in a dynamic setting this interaction when risk preferences are heterogeneous in the population and endogenous. In the initial condition of the society there are individuals with preferences for risk-taking that might choose to become entrepreneurs under the appropriate circumstances. But, there is also an initial proportion of risk averse individuals that will never choose to become an entrepreneur. Cultural (preference) transmission combines direct (parental) transmission and oblique transmission (Cavalli-Sforza and Feldman, 1981; Boyd and Richerson, 1985). Each adult has one offspring and has to make a costly decision regarding his/her child education. Parents are altruistic and care about their offspring's well-being. The expected utility their children will obtain in the labour market depends on their preferences. As a consequence, parents try to transmit the more valuable preferences through socialization process taking into account their own expectations (Bisin and Verdier, 1998, 2000, 2001). If this parental transmission does not succeed then children acquire risk-taking preferences from the social environment.

We characterize the long run behaviour of this society, that is, the stable steady states of the cultural transmission dynamics. Our framework shows how and why different cultures can arise in the long run. Namely, three very different cultures are possible. A first one with a large share of risk takers in the population, a high level of entrepreneurship and a level of taxes just enough to implement high effort by officials (that is, an efficient public sector). A second one in which there is a majority of risk averse individuals in the population. But, although there is a significant proportion of risk- taking individuals, there are no entrepreneurs at all because of the threat of confiscatory taxes on the entrepreneurs' profits. In this society individuals work on the traditional (and less risky) sector and officials obtain a very low wage. And finally, there is a last equilibrium culture also with a majority of risk averse individuals and with a high share of officials and routine producers but still a low positive level of entrepreneurship. In this culture the majority votes for confiscatory taxes, which implies that the entrepreneurs' profits are just non-monetary and the wages paid to the officials are high. Our analysis shows how the culture that eventually prevails in the long run depends on the parameters of the model and also on the initial condition (the history) of the society. In this sense, our model can contribute to account for the differences observed among countries (Mediterranean vs. Nordic) or within countries (north vs. south of Italy or Spain) concerning the dynamics of the productive structure and the public sector efficiency.

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Principles of Stable Cooperation in Stochastic Games

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Keywords: *Cooperative stochastic game, Time consistency, Subgame consistency, Payoff distribution procedure, Strategic stability, Irrational behaviour proof condition*

Stochastic games [1] with the finite sets of players, game elements and the finite sets of players' strategies in each game element are considered in the work. The game element is defined by the game in the normal form. We use the mathematical expectation of the player's payoff as a measure of the player's payoff in the stochastic game. The duration of the game is a discrete random variable and it is supposed to have geometric distribution. The players are also supposed to use stationary strategies in the game, i.e. a player's choice of the strategy at each game stage does not depend on the stage number and history but depends on the game element which is realized at this stage.

In the paper the cooperative version of the described above stochastic game is examined. We find the cooperative decision [2,3], i.e. such a strategy-profile (or strategy-profiles if there are more than one) in pure stationary strategies in a stochastic game when using them all players cooperating receive the maximal total expected payoff (in this strategy class). The characteristic function (maxmin in pure stationary strategies) is defined for imputation deriving. Before the game starts players can find the components of the chosen imputation (Shapley value, core, nucleolus, etc.) if they know the game parameters. Players know that if they want to receive the components of the chosen imputation they have to use the cooperative decision (some strategy-profile in pure stationary strategies).

A stochastic game is a dynamic game. If during the game process players get into game elements at each stage and realizing the cooperative decision get the payoffs according to their payoff function determined within the game in the normal form of this stage then in most stochastic games the expected sum of player's payoffs is not equal to the expected value of the imputation component derived at the beginning of the game. It means the breach of the first condition of stable cooperation, the breach of subgame consistency (time consistency) of the cooperative agreement. The failure of this condition can lead to the coalition breakup. In 1977 L.A. Petrosyan introduced the conception of time consistency for the class of differential games [4]. In 1979 he proposed the procedure of players' payoffs redistribution, i.e. payoff distribution procedure (PDP)[5]. Redistributing the players' payoffs at each game stage, i.e. in each game element realized in stochastic game it can be obtained that the expected payoff of any player in the game be equal to the expected value of the corresponding imputation component. Using PDP the regularization of the initial stochastic game is realized.

The second principle of stable cooperation is strategic stability that guarantees the existence of Nash equilibrium in the regularized stochastic game in the class of behaviour strategies with the players' payoffs which they expect to receive as a result of the cooperative agreement (see [6]).

The third principle of stable cooperation is irrational behaviour proof condition [7]. This condition guarantees that if cooperation breaks down for some reason at some game stage and after that the player plays independently then he will totally receive not less than if he has played independently from the beginning of the game.

In the work three principles of stable cooperation (see [6]) are formulated for the defined above stochastic games. The procedure of stochastic game regularization is introduced and sufficient conditions for the second and third principles of stable cooperation are obtained. All theoretical results are demonstrated with the numerical example.

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An Axiomatic Characterization of a Proportional Excess Invariant Solution for NTU Games

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Keywords: *Cooperative NTU games, Proportional excess, Proportional excess invariant solution.*

In (Pechersky, 2010) a proportional excess invariant solution (i.e. solution, which is invariant with respect to proportional excess) for NTU games was defined as a generalization of the proportional excess invariant value for positive TU defined by E.Yanovskaya (2004). We call this solution proportional invariant for short.

Firstly recall the definition of the proportional excess for NTU games. It was defined axiomatically by author (see, for example, Pechersky, 2007). Let N be a set of players. Let CG_+^N denotes the space of NTU games possessing the following properties for every $S \subseteq N$, where N is the finite set of players:

(a) $V(S)$ is positively generated, i.e. $V(S) = (V(S) \cap R_+^S) - R_+^S$ and $V_+(S) = V(S) \cap R_+^S$ is a compact set, and every ray $L_x = \{\lambda x : \lambda \geq 0\}$, $x \neq O = (0,\ldots,0)$ does not intersect the boundary of $V(S)$ more than once;

(b) O is an interior point of the set $V\,\hat{}\,(S) = V(S) \times R^{N \setminus S}$.

(Of course, all $V(S)$ are comprehensive).

Let v be a positive TU game, i. e. $v(S) > 0$ for every S. Then the corresponding NTU game $V \in CG_+^N$ can be defined by

$$V(S) = \{x \in R_+^S : x(S) \leq v(S)\} - R_+^S .$$

For $S \subset N$ a set $V\ (S) \subset R^S$ will be called a game subset, if it satisfies (a) and (b). The space consisting of all game subsets satisfying (a) and (b) will be denoted by CG_{N+}^S.

Let $V \in CG_+^N$ be an arbitrary game. Then the proportional excess $h_S : CG_{N+}^S \times R_+^N \to R$ is defined by

$$h_S(V,x) = 1\ /\ \gamma(V(S), x_S),$$

where $\gamma(W,y) = inf\ \{\lambda > 0 : y \in W\ \}$ is the gauge (or Minkowski gauge) function.

If $V \in CG_{N+}^S$ corresponds to a positive TU game v, then $h_S(V,x) = v(S)\ /\ x(S)$.

A solution ψ (not necessarily single-valued) on CG_{N+} is called proportional invariant if for every two games $V, W \in CG_{N+}$ and every $x \in \partial V_+(N), y \in \partial W_+(N)$, the equalities

$$h_S(V,x) = h_S(W,y) \text{ for every } S \subset N$$

imply

$$x \in \psi(N,V) \Leftrightarrow y \in \psi(N,W).$$

Theorem 1 *(Pechersky, 2010). There is a proportional invariant solution on* CG_{N+}. *For every game* $V \in CG_{N+}$ *it can be defined as the solution of the system*

$$x \in \partial V_+(N),$$

$$\sum_{S:S \ni i} h_S(V,x) = \sum_{S:S \ni j} h_S(V,x) \quad \text{for all } i, j \in N\ . \tag{1}$$

We call such a solution p.i.-solution. If we replace system (1) by the following one

$$\sum_{S:S \ni i} c(s) h_S(V,x) = \sum_{S:S \ni j} c(s) h_S(V,x) \quad \text{for all } i, j \in N \tag{2}$$

for some non-negative numbers $c(s) \geq 0,\ s \leq n-1,\ n = |\ N\ |$, then the corresponding solution we shall call p.i.(c)-solution.

The following properties follow immediately from the definition and the existence theorem.

1) If $V \in CG_{N+}$ corresponds to a positive TU game v, then $\left|\psi(V)\right| = 1$, and $\psi(V) = \psi(v)$, where ψ is the corresponding proportional invariant TU value.

2) P.I.-solution ψ (and p.i.(c)-solution, too) possesses efficiency, anonymity and positive homogeneity properties.

Let us introduce an operation on CG_{N+}. We call it the directional addition, and define it as follows. Let $A, B \in CG^S_{N+}$. Then for every $x \in R^S_+$ there are exactly two points $y \in \partial A$ and $z \in \partial B$ such that $y = \lambda_x x$ and $z = \mu_x x$ for some positive numbers λ_x and μ_x. Then the directional sum of A and B, denoted by $A \otimes_d B$, is defined as follows:

$$A \otimes_d B = \text{comp}\ \{\cup_x \left(\lambda_x + \mu_x\right) x\},$$

where $\text{comp}\, F$ denotes the comprehensive hull of a set F.

Let now $V, W \in CG_{N+}$ with $V(N) = W(N)$. Define the game $V \otimes_d W$ as follows:

$$(V \otimes_d W)(N) = V(N) = W(N),$$

$$(V \otimes_d W)(S) = V(S) \otimes_d W(S) \text{ for every } S \subsetneq N.$$

Clearly $V \otimes_d W \in CG_{N+}$.

It is not difficult to prove that

$h_S(V \otimes_d W, x) = h_S(V, x) + h_S(W, x)$ for every $S \subsetneq N$.

Proposition 1. *Let $V, W \in CG_{N+}$ with $V(N) = W(N)$, and $x \in \psi(V) \cap \psi(W)$. Then $x \in \psi(V \otimes_d W)$.*

We call this property restricted directional linearity (similarly to restricted linearity in TU case).

Theorem 2. *A proportional excess invariant solution ψ on CG_{N+} possesses efficiency, anonymity, positive homogeneity and restricted directional linearity if and only if there is a system of nonnegative numbers $c(s) \geq 0,\ s \leq n-1,\ n = \mid N \mid$ such that ψ is p.i.(c)-solution.*

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Extending the Condorcet Jury Theorem to a General Dependent Jury

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We investigate necessary and sufficient conditions for the existence of Bayesian-Nash equilibria that satisfy the *Condorcet Jury Theorem* (*CJT*). In the Bayesian game G_n among n jurors, we allow for arbitrary distribution on the types of jurors. In particular, any kind of dependency is possible. If each juror i has a "constant strategy", σ^i (that is, a strategy that is independent of the size $n \geq i$ of the jury), such that $\sigma = (\sigma^1, \sigma^2, ..., \sigma^n ...)$ satisfies the *CJT*, then by McLennan (1998) there exists a Bayesian-Nash equilibrium that also satisfies the *CJT*. We translate the *CJT* condition on sequences of constant strategies into the following problem:

(**) For a given sequence of binary random variables $X = (X^1, X^2, ..., X^n, ...)$ with joint distribution P, does the distribution P satisfy the asymptotic part of the *CJT*?

We provide sufficient conditions and two general (distinct) necessary conditions for (**) We give a complete solution to this problem when *X* is a sequence of exchangeable binary random variables.

N-Person Transportation Problem with Non-Overlapping Paths

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Keywords: *Network game, Shortest path problem, Characteristic function, Shapley value.*

Network game in which a finite number of players wish to reach the same point "A" but their paths cannot intersect is considered. Each edge of the network has its transportation cost. The main task of this game is to find the best way to reach point "A" for each player in case of full or partial cooperation of players.

The algorithm is proposed for finding players paths, which lead to a situation where the total cost of transportation for all players will be close to minimum.

On that basis the cooperative game model is constructed. The characteristic function is computed using the proposed algorithm. For some given networks the Shaply Value is calculated. It turns out that the proposed algorithm gives also transportation strategies which form a Nash equilibrium in corresponding noncooperative game.

Global Emission Ceiling versus International Cap and Trade: What is the Most Efficient System when Countries Act non-Cooperatively?

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Keywords: *Climate change, International cap and trade system, National emission quotas, Global emission ceiling, Social Nash equilibrium, Normalized equilibrium, Quasi-variational inequalities*

Since more than fifteen years, countries have been engaged in a process of reduction of Greenhouse gases (GHG) emissions in order to limit the extent of global warming. The first step to the international control of GHG has been reached with the signature of the Kyoto Protocol (1997). During theses negotiations, individual emission reduction targets, or put differently individual emission quotas, have been assigned to 41 developed countries. (Targets were defined with respect to the reference year 1990 and had to be reached during the first period of commitment 2008-2012.) Moreover, the principle of the international trading of emission permits has been adopted.

In spite of this attempt to solve the climate change problem, it is now clear that countries have not yet achieved full cooperation toward its resolution. Firstly, one of the two biggest GHG emitters, the United States, has not ratified the treaty. The second one, China, was not submitted to emission reduction during the first period of implementation and Russia has ratified the treaty at a later date (in 2004). In addition, participating countries have agreed on emission reduction targets that were far from matching scientific recommendations, because they were not stringent enough. More recently, the Copenhagen (2009) submit's objective was to bring all countries involved in the climate change problem – including the USA, China and developing countries – together around a table and to decide on an international timetable for reducing further GHG emissions during the next decades. But, discussions did not lead to real progress. Actually, many countries were reluctant to accept individually constraining emission quotas. Nevertheless, in order to avoid the worst consequences of global warming, it seems that a common view has emerged about the necessity the limit the rise of temperature to 2 degrees C at the horizon of 2050. In a sense, most countries have then agreed on a global emission reduction effort – or emission cap – but have failed to decide on how this global burden should be shared between countries.

Motivated by this last observation, this paper raises the question of whether a system that would consist in setting a global emission cap and letting countries directly choose their emission levels (under this common constraint) can outperform, from a welfare perspective, an international cap and trade system where countries have to decide on national emission quotas.

In the same vein as Helm (2003), we assume that national governments behave non cooperatively under any circumstances. This approach may be criticized since it looks a bit extreme but it does not seem to be at odd with what has been observed in successive climate negotiations. We therefore consider a static partial equilibrium model that has the following features. In each heterogeneous country (or group of countries), polluting firms use a technology that involves a single input, namely, emissions. Emissions generate profits at home but aggregate emissions are a public bad since they are damaging to all countries. In this context, two general scenarios are envisioned. First, countries directly and non cooperatively choose the amount of emissions (business as usual scenario, BAU). Next, we assume that an international cap and trade system (ICT), such as the one designed during the Kyoto protocol, is implemented. This system consists of the choice, by each country, of a national emission quota and of the possibility to trade emission permits on a competitive market.

Comparing these two scenarios, Helm (2003) finds ambiguous results both in terms of aggregate emissions and welfare. He notably shows that the ICT system may fail to lower aggregate GHG emissions (with respect to the BAU scenario). In fact, some governments (the less concerned by the environment) have the incentive to choose an emission quota higher than the emission level they would have chosen in the absence of market. The reason is that they perceive their influence on the equilibrium market price. In this situation, emissions trading may be unanimously preferred by countries since the market allows for efficiency gains. In the opposite case where the ICT succeeds in lowering emissions, some countries may lose from emission trading and consequently may not approve this system.

This paper adds to this analysis the opportunity for countries – for now on players – to adopt a global emission cap (GEC) that puts a (potentially binding) ceiling on aggregate emissions. Thus we will have two different regimes in each scenario depending on whether the global cap is binding. Note that a priori, countries can choose between the two scenarios and between the two regimes.

To start with, we consider a game with general quadratic payoffs functionals and only two players. The players differ with respect to both their environmental concerns and their technology. The first player, who is the most aware about environmental issues and who owns the most efficient technology, can be identified as a group of industrialized countries (such as the European Union). The second player thus represents a group of developing countries.

The first part of the analysis is devoted to the calculation of non cooperative solutions in each scenario. In a social Nash equilibrium (also called coupled constraint Nash equilibrium, see for instance Krawczyk, 2005), each player chooses her emission level (BAU) – or her emission allowance (ICT) – taking as given the strategy of the other player and the constraint imposed by the global ceiling. When this ceiling is non binding, there exists a unique social Nash equilibrium. The aggregate welfare attained under the ICT is higher than the one obtained in the BAU whereas aggregate emissions are identical. When the emission cap is binding, there exist in general an infinity of Nash equilibria. We therefore proceed to a selection by the normalized equilibrium (introduced by Rosen, 1965, see also Krawczyk, 2005, who motivates the use of this selection in environmental games) and solve the problem using the quasi-variational inequalities theory (see notably Morgan and Romaniello, 2006). For any level of the emission cap, the normalized equilibrium is uniquely determined and both global and individual welfare can be defined as functions of the cap.

Within both scenarios, the emphasis is then on the effect of the setting of the global ceiling on welfare. Since countries can choose between the two regimes, our first concern is with the acceptability of the GEC. In the BAU scenario, we can show that there exists a non-empty range of values for which setting a binding ceiling improves all players' welfare. Therefore, this regime allows the world economy to reduce GHG emissions and translates into welfare gains (by comparison with the other regime, without any pollution control).

When the trading of emission permits is allowed that is, in the ICT scenario, we can also find a non empty set of values such that imposing a binding emission cap is beneficial to the player representing developed countries. Developing countries' welfare may increase or decrease as a consequence of the introduction of the cap. Assuming that environmental concerns are low, even for the first player, then the other player also benefits from the GEC.

More importantly, we can identify a non empty set of values, for the global ceiling, for which the GEC system alone allows the world economy to reach a level of aggregate welfare (respectively of aggregate emissions) higher than (respectively lower than) the level resulting from the ITC system alone. In other words, from an aggregate perspective, the GEC outperforms the ITC system. In addition, it is worth mentioning that the GEC has the advantage of circumventing the critical question of how the initial allocation of permits between countries should be determined (this difficulty explaining why international negotiations fail to achieve an efficient agreement) since countries only need to agree with each other on the global emission ceiling to be imposed. They do not have to engage in binding individual quota and can choose freely their emission level under the constraint imposed (but accepted) by the ceiling. This clearly echoes what has been observed in Copenhagen.

An incoming development of this work consists in extending the analysis to a larger number of groups of countries. Considering at least three groups of countries allows us to capture the heterogeneity of countries that participate to international climate negotiations. For instance, it is interesting to introduce a third group, say Russia and the United States, who owns an efficient technology but has low environmental concerns and see how the results are modified in the extended framework. Another natural extension of this analysis is to account for the intertemporal dimension of the climate change problem. This can be done by assessing the issue raised by the present paper in a differential game.

On the Extreme Points of Polytopes of Some Subclasses of Big Boss Games

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Keywords: *Cooperative game, Big boss game, (0-1)-form, Extreme points, Shapley value, consensus value*

One of cooperative game theory problems is the characterization of extreme directions of polyhedral cones of various classes of games and description the behavior of solution concepts defined on these cones ([4]). One of important is the cone of big boss games ([2]), which we denote by K_1^N. A game $\nu \in G^N = \{g : 2^N \to R : \nu(\oslash) = 0\}$, where $N = \{1,\dots,n\}$ and $n \geq 2$, is called a *big boss game* with player 1 as big boss if: ν is monotonic, $\nu(S) = 0$ if $S \not\ni 1$ (boss property), $\nu(N) - \nu(N \setminus S) \geq \sum_{i \in S} M_i(\nu)$ for each $S \subseteq N \setminus 1$ (union property), where $M_i(\nu) = \nu(N) - \nu(N \setminus i)$. These games have been used for numerous applications and are extensively studied. Moreover, all results received for monotonic clan games ([3]) are applicable also to games in K_1^N. In some studies a big boss game is defined without the monotonicity condition. Since the most solution concepts satisfy on K_1^N the relative invariance with respect to strategic equivalence, we will consider a big boss games in (0-1)-form. At normalization the cone K_1^N will be transformed to $(2^{n-1} - 2)-$dimensional polytope P_1^N which can be described by its extreme points. Denote $\Omega_1^N = \{S \in 2^N : S \not\ni 1 \; or \mid S \mid = 1\}$. Next proposition provides the system of non redundant conditions for P_1^N. Let $\nu \in G^N$. Then $\nu \in P_1^N$ iff the following conditions hold

(i) $\nu(S) = 0$ for $S \in \Omega_1^N$, $\nu(N) = 1$,

(ii) $\nu(S_1) \le \nu(S_2)$ if $1 \in S_1 \subset S_2 \subseteq N$, $|S_2| = |S_1| + 1$, $|S_2| \ne n - 1$,

(iii) $\nu(S) - \sum_{i \in N \setminus S} \nu(N \setminus i) \le |S| + 1 - n$ if $1 \in S \subset N$, $|S| \le n - 2$.

None of these conditions is implied by the others.

Denote by $ext(P)$ a set of all extreme points of polytope P. Clearly, the simple games belonging to P_1^N are its integer extreme points. In particular, $u_T \in ext(P_1^N)$, where u_T is unanimity game, $T = \{1, i\}$, $i \in N \setminus 1$. Since players 1 and i are symmetric in ν_T, we have the big boss game with two "bosses". For $n \le 4$ the set $ext(P_1^n)$ contains integer games only. We have calculated all extreme points of P_1^5 and divided the non integers into seven equivalence classes. The extension of results for P_1^5 allowed us to receive the description of some types of extreme points of general polytope P_1^N and explicit formulas for corresponding consensus and Shapley values, denoted by K and Sh respectively. Let $\nu_k^n \in G^N$ is determined for all $k \in \{2, \ldots, n-1\}$ by

$$\nu_k^n(S) = 0 \textit{ if } S \in \Omega_1^N \textit{ or } |S| \le k - 1, \ \nu_k^n(S) = 1 \textit{ otherwise.} \tag{1}$$

Then $\nu_k^n \in ext(P_1^N)$ and

$$K_1(\nu_k^n) = \frac{n-k+2}{2n}, \ Sh_1(\nu_k^n) = \frac{n-k+1}{n}, \tag{2}$$

$$K_i(\nu_k^n) = \frac{n+k-2}{2n(n-1)}, \ Sh_i(\nu_k^n) = \frac{k-1}{n(n-1)} \ \textit{ for } \ i \in N \setminus 1. \tag{3}$$

Similar results are received for some non integer games in $ext(P_1^N)$.

The core $C(\nu)$ of each game $\nu \in P_1^N$ is determined by marginal vector only ([2]), so all games in P_1^N having identical marginals have the same core. If $C(\nu)$ is a singleton, i.e. $C(\nu) = \{x^c\}$, then the bargaining set, kernel and lexicore coincide with x^c. Moreover, x^c coincides with nucleolus, $\tau-$value, $AL-$value and any core selector. Thus, x^c should reflect many principles of fairness. However, for games with zero $M_i(\nu)$, $i \in N \setminus 1$, we have $x^c = (1,0,\ldots,0)$. According to x^c player 1 (boss) gets the whole surplus from cooperation that ignores the productive role of other players. Such games are in particular ν_k^n determined by (1) and all games in their convex hull and also all games in G^N having corresponding (0-1)-form. At the same time, by (2) and (3) we obtain different consensus and Shapley values. For example,

$$K(\nu_2^6) = (\frac{1}{2},\frac{1}{10},\ldots,\frac{1}{10}),\ K(\nu_5^6) = (\frac{1}{4},\frac{3}{20},\ldots,\frac{3}{20}),$$

$$Sh(\nu_2^6) = (\frac{5}{6},\frac{1}{30},\ldots,\frac{1}{30}),\ Sh(\nu_5^6) = (\frac{1}{3},\frac{2}{15},\ldots,\frac{2}{15}).$$

Thus, for $\nu \in co(\{\nu_k^n\}_{k=2}^{n-1})$ the consensus and Shapley values prescribes a rather natural outcomes.

We name a game $\nu \in P_1^N$ l-*symmetric* if each pair of powerless players $i,j \in N \setminus 1$ is symmetric in ν. Let LSP_1^N be a set of all l-symmetric games. The set $ext(LSP_1^n)$ can be described by means of binary vectors. Let $\nu \in G^N$ and let $x = (x_2,\ldots,x_{n-2})$ be 0,1-valued vector. Then $\nu \in ext(LSP_1^N)$ iff it has one of following representations:

(i) there is such $k \in \{2,\ldots,n-1\}$ that $\nu = \nu_k^n$, where ν_k^n determined by (1),

(ii) $\nu(S) = 0$ for $S \in \Omega_1^N$, $\nu(N) = 1$, $\nu(N \setminus j) = \dfrac{n-2}{n-1}$ for $j \in N \setminus 1$, $\nu(S) = f(\mid S \mid)$ otherwise, where $f(i)$ is determined for all $i \in \{1,\ldots,n-2\}$ by

$$f(1) = 0,\ f(i) = f(i-1)\ if\ x_i = 0\ \ and\ \ f(i) = \frac{i-1}{n-1}\ if\ x_i = 1.$$

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Behavioral and Structural Remedies in Merger Control

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Keywords: *Merger control, Behavioral remedies, Structural remedies, Effciency*

The issue of the research

Merger control is one of the ongoing issues of the competition policy. Nowadays merger control is exercised according to the “rule of reason”, that is the decision should be made taking into account both positive and negative consequences of the merger. In the last decade the reduction in the bans on mergers and simultaneous increasing of nominal approvals could be seen in this area of competition policy. It means that Competition Authority approves the merger on condition that the merging companies meet the requirements to reestablish the broken competitive conditions while retaining efficiency gains form the merger. Such requirements (conditions) were called remedies.

These remedies could be divided into two groups: behavioral and structural. The former group comprises regulations which limit the behavior of the companies after the merger. The latter includes the requirements of the partial divesture of assets of the merging companies before the merger takes place.

Competition Authorities of the USA and the EU prefer to impose structural remedies which are preferential for the number of reasons while in Russia Competition Authority mainly relies on behavioral remedies regardless of the fact that legislative framework for application of structural remedies was established in 2006.

The growing number of situations where remedies were applied to mergers led to the necessity to increase efficiency of this competition policy tool. This is proved by publicizing both the results of empirical studies of competition policy and the findings of research papers based on formal methods of investigating interaction between competition authorities and merging companies when the remedy was applied.

Brief summary of academic context

Researchers pay special attention to structural remedies due to the fact that Competition Authorities of the USA and the EU prefer to rely on them. Medvedev (2004) devises a model describing interaction between competition authorities which impose the requirements of the divesture of assets and the merging companies on the basis of Cournot model. Rey (2000), Gonzalez (2003) analyze the impact of information asymmetry on the efficiency of remedy applications. The notion of remedy design as an incentive contract is discussed by Werden, Froeb, Tschantz (2005). Special attention should be paid to the research of Cosnita, Tropeano, (2005) with the primary objective to work out an incentive contract which would make the companies to disclose information about actual efficiency gains form the merger.

The aim of the research

The aim of this research paper is to study the possible opportunist behavior of the merging companies under conditions when competition authorities can choose type of remedies with the help of game theory methods.

The theoretical framework

In order to devise a model the following concepts were used: static games, dynamic games, the theory of incentive contracts, oligopoly theory (Cournot model).

Main findings

This paper contributes to the economic analysis of merger control by taking into account the institutional features of the environment: 1) transparency, which determines the probability of punishment merged firms and 2) ability of the competition authority to enforcement requirements. The firms interact on the basis of Cournot model before and after merger. The objective of Competition Authority is consumer surplus. We propose a model that shows how these conditions can affect the choice between behavioral and structure remedy. The first stage we consider the case of symmetric information about the efficiency gains from merger. Further, we introduce the assumption that the merger's efficiency gains are private information of the merging partners and the competition authorities do not observe the magnitude of it. We analyze the probability of merger control error depending on options selected parametres.

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Discrete-Time Bioresource Management Problem with Many Players

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Keywords: *Dynamic games, Bioresource management problem, Time-consistency*

Model with logarithmic payoffs

Let n players (countries or fishing firms) exploit the fish stock during infinite time horizon. The dynamics of the fishery is described by the equation

$$x_{t+1} = (\varepsilon x_t - \sum_{i=1}^{n} u_{it})^{\alpha}, x_0 = x, \quad (1)$$

where $x_t \geq 0$ – size of population at a time t, $\varepsilon \in (0,1)$ – natural death rate, $\alpha \in (0,1)$ – natural birth rate, $u_{it} \geq 0$ – the catch of player i, $i = 1,\ldots,n$.

We suppose that the utility function of country i is logarithmic. Then the players' net revenues over infinite time horizon are:

$$J_i = \sum_{t=0}^{\infty} \delta^t \ln(u_{it}), \quad (2)$$

where $0 < \delta < 1$ – the common discount factor for countries.

The characteristic function for cooperative game is constructed in two unusual forms. In the first model players outside the coalition K switch to their Nash strategies, which were determined for initial non-cooperative game. So it is the case when players have no information about coalition formation. In the second model players outside the coalition K determine new Nash strategies in the game with $N \setminus K$ players. This case corresponds to the situation when players know that coalition K is formed.

For both cases we constructed the Shapley value and time-consisted imputation distribution procedure. Also we proved that the Yeung's condition and each step rational behavior condition are fulfilled.

Liner programming problems

First we show that the C-core in our problem is not empty for every $t \geq 0$. And in addition the each step rational behavior condition is fulfilled

$$\xi_i(t) - \delta\xi_i(t+1) \geq V_i(x_t) - \delta V_i(x_{t+1}).$$

The obtained solution gives the imputation distribution procedure identical for all players:

$$\beta_i(t) = \frac{1}{1-a}(\ln x_t - \delta \ln x_{t+1}) + B_i + B_\xi. \quad (3)$$

And if we want to move from the "center" of C-core, we can solve linear programming problem

$$\begin{cases} \xi_i(t) \geq V_i(x_t), \\ \sum\limits_{i \in K} \xi_i(t) \geq V_K(x_t), \\ \sum\limits_{i \in N} \xi_i(t) = V_N(x_t), \\ \xi_i(t) - \delta x i_i(t+1) \geq V_i(x_t) - \delta V_i(x_{t+1}). \end{cases} \quad (4)$$

subject to any functional depending on time.

For example we consider the following functional

$$\sum_{t=0}^{T} \beta_1(t) \to \max.$$

It can be interpreted as: among our homogeneous players there is a leader (player 1) whom should be payed the biggest share of the cooperative payoff.

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Uncertainty Aversion and Backward Induction

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Keywords: *Centipede game, Backward induction, Uncertainty aversion, Subgame perfection, Choquet expected utility*

The centipede game has become a benchmark both for the empirical adequacy and the theoretical consistency of game theoretic concepts. In any Nash equilibrium – and thus in every equilibrium refinement – the first player chooses `Down' immediately; in the unique subgame-perfect equilibrium the players choose `Down' everywhere.

Empirically, experimental evidence suggests that players do not act in this way (see, e.g., McKelvey and Palfrey (1992). Theoretically, subgame-perfection applies equilibrium arguments, that hold for rational players, off the equilibrium path. This is consistent only under the assumption that deviations from rational play are not evidence of non-rationality, e.g. because rational players might tremble (Selten1975). This aspect has led to a controversial debate about backward induction (see, e.g, Basu (1988), Reny (1993), Aumann (1995), Binmore (1996), Aumann (1996), Halpern (2001).

McKelvey and Palfrey (1992) are able to interpret experimental evidence in the sense of Kreps, Milgrom, Roberts and Wilson (1982, henceforth KMRW). In their model, the structure of the game is not mutual knowledge. Instead there is a small probability of being matched with an `altruistic' opponent who always plays `Across'. McKelvey and Palfreay show that, as a consequence, it is indeed rational to play `Across' early in the game.

There are two arguments against this way of interpreting the experimental evidence. First, if taken as an explanation of evidence rather than an equilibrium effect, it relies on the actual existence of such altruists in the subject pool. The second, formulated by Selten (1991) in the context of the KMRW approach to the finitely repeated prisoner's dilemma, is that the analysis proceeds by changing the game, and not by analysing the same game in which the paradox arises. However, both criticisms do not apply if the players are assumed to know the game, but lack mutual knowledge of rationality, as suggested by Milgrom and Roberts (1982, p. 303). If the rational players believe that non-rational opponents always play `Across', the analysis of McKelvey and Palfray is indeed an explanation of the actual evidence in the original game.

Still, this approach to modelling lack of mutual knowledge of rationality leads to conceptual difficulties:

First, there is no reason why rational players should hold this specific belief about opponents that they do not consider to be rational. If the Bayesian-Nash equilibrium is identified with rational play, then any deviation must be considered non-rational. Therefore, not only is the specification of the belief that non-rational players always play `Across' ad hoc, in the absence of a theory of non-rationality there is no basis for specifying any particular belief.

Secondly, this also holds in particular for the uniform distribution as a model of complete ignorance. There is no reason why a non-rational player should be assumed to choose all his strategies with equal probability. In addition, there is the well-known problem that a uniform probability depends on the description of the space of uncertainty: For instance, if a state is split into two sub-states, the combined probability of the two sub-states under the uniform distribution is higher than the probability of the original state. As a consequence, the description of the state space would be pivotal for the equilibrium.

Thirdly, and more fundamentally, if the Bayesian-Nash equilibrium is identified with rational play, then any deviation must be considered non-rational. This problem is related to, but different from the first: Not only need the players not have a particular belief about non-rational opponents, according to the rationality concept they must not have any particular belief. This consistency requirement follows from an identification of Bayesian-Nash equilibrium with rational play, because this implicitly defines all other strategies as non-rational.

Finally, the analysis of games under incomplete information on the basis of the Bayesian-Nash equilibrium assumes that the types of a player correspond to a consistent hierarchy of beliefs about the underlying uncertainty (Harsanyi 1967). This leads to the usual infinite regress. Thus in this analysis the rational player not only believes that a `non-rational' opponent always plays `Across', but also believes that the non-rational opponent believes a rational player to believe this, ... ad infinitum. But this means that a rational player must believe that his non-rational opponent has an infinite and consistent hierarchy of beliefs. This, of course, is at odds with the interpretation of this opponent as non-rational. It is for this reason that McKelvey and Palfrey refer to structural uncertainty and `altruistic' types.

Nevertheless, the KMRW approach has been extremely useful in helping to understand strategic interaction, particularly in industrial organization (e.g. Kreps and Wilson (1982), Milgrom and Roberts (1982), and, as in McKelvey and Palfrey (1992), in experimental game theory.

Our model is in the same spirit as KMRW (1982) and McKelvey and Palfrey (1992). We postulate that rationality is not mutual knowledge, i.e. an opponent may or may not be rational. We replace the assumption that players have a specific belief about non-rational play with the assumption that players are genuinely uncertain about the way non-rational opponents play. When facing uncertainty, players maximise Choquet expected utility (Schmeidler1989, henceforth CEU). According to CEU, players act in face of uncertainty as if they maximise subjective expected utility. However, in contrast to a situation in which players face risk, players' beliefs do not have to be additive, i.e. the `probabilities' that the players use to weigh consequences do not have to add to 1.

Our contribution is to define an equilibrium concept that extends subgame perfection to a game with genuine uncertainty due to lack of mutual knowledge of rationality. Thus we do not need to make any assumption about the behaviour of non-rational players, and we can avoid modelling them as types. Instead, we can make an assumption about the rational players' attitude towards uncertainty. We assume that they are uncertainty averse, but only boundedly so. We show that this results in an equilibrium in the centipede game in which rational players play `Across' early in the game and `Down' late in the game, in spite of their uncertainty aversion. Moreover, it is subgame-perfect in the sense that decisions are optimal at every node in the game.

Our result is due to an interaction between the game-theoretic definition of strategy as a contingent plan and the players' attitude towards uncertainty. In calculating expected utilities, a player who is uncertainty averse will use `probability weights' that do not add up to 1, and a `probability residual' (the difference between the sum of the weights and 1) that he will allocate to the worst outcome. As long as the degree of uncertainty aversion is bounded, however, every strategy of the non-rational opponent will enter the calculation with some positive weight, however small. Since a strategy is a contingent plan, it specifies an action – `Across' or `Down' – after every history of the game, even those that are excluded by the strategy itself (because it specifies `Down' very early).

Mathematical Models for Decision Making with Ordered Outcomes

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Keywords: *Decision making problem, Games with ordered outcomes in the normal form, Games on graphs with chance moves, Extension of order to the set of probability measures.*

Since such models have several chance mechanisms (mixed strategies in games in the normal form; chance moves in games on graphs; probability distributions on the set of states in games against nature etc.) then, to evaluate chosen strategies, we need to construct for given order relation its extension to the set of probability measures. In our work we use so called the canonical extension of order relation which was introduced by the author in 1976 (see [1]). The canonical extension is based on the fact that any order relation ω on a set A is approximated by the set $C(\omega)$ consisting of all isotonic functions from the ordered set $< A, \omega >$ into R , that is, the following equivalence holds:

$$a_1 \leq^{\omega} a_2 \Leftrightarrow (\forall f \in C(\omega)) f(a_1) \leq f(a_2).$$

We now define the canonical extension of ω to the set of probability measures by the formula

$$\mu_1 \leq^{\omega} \mu_2 \Leftrightarrow (\forall f \in C(\omega)) \overline{f}(\mu_1) \leq \overline{f}(\mu_2),$$

where $\overline{f}$ is the usual extension of the function f to the set of probability measures: $\overline{f}(\mu) = (f, \mu)$ (the right part is the standard scalar product).

In our previous work [2, 3] conditions for existence of equilibrium points and Nash equilibrium in mixed strategies for games with ordered outcomes were indicated and also certain methods for their finding were proposed. In this report, we consider analogous problems for games on graphs with chance moves and many-criterion optimization models with partial ordered set of criteria.

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Pervasion of Change Acceptance: a Combined Planned Behaviour and Game Theoretic Approach

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Keywords: *Behavior, Change, Equilibrium, Evolution, Game, Group, Incentive, Intention, Norms, Stability*

The question of why and how innovation can be pervasive in a given environment has already been addressed at different levels. For instance, in the field of marketing, deciding on when putting a new and innovative product on the market is a classical issue. Of course, reasons pertaining to price or to the degree of innovation are the first ones that need to be taken into account, but they may not be the only ones: thus the belief that the customer has about the added value of this new product, as well as his / her belief that he / she will know how to use it, or the pressure of social norms like fashion may have an important impact.

Likewise behavioral theories have been concerned already for a long time with the issue of organizational change within a company or an administration. The problem can turn out to be crucial if acceptance by those who are concerned by the change is a requirement for that change to be effective and efficient.

Now the question is: what are the conditions for acceptance to prevail?

Formulated in such terms, this question should ring a bell in the head of game theorists, for whom the possibly related issue of cooperation is a core entry point to Game Theory: indeed if all accept, change will prevail, while if a sufficient number of parties concerned do not accept the change, the latter will not occur.

Several attempts have already been made to combine approaches based on Attribution Theory, beliefs and intentions with experiments of the Prisoner's Dilemma (Kelley & Stahelski) to analyze how psychological factors can trigger, or on then opposite, prevent cooperation.

In his Theory of Planned Behavior Ajzen analyzes behavioral intentions with respect to three parameters referring to a given individual:

- the effect of a given behavior on his / her personal position
- the perception of social pressure
- the perception of the ease or difficulty to adopt the behavior under consideration

Focusing on organizational change requiring that a minimum of parties concerned accept such change, the present paper proposes to combine the Ajzen's Theory of Planned Behavior (TPB) with game theoretic tools, in order to analyze the conditions for which change can successfully prevail.

More precisely, the paper will consider different assessments made by the individuals of the three TPB parameters defining behavioral intentions, i.e. attitude toward behavior, subjective norms and perceived behavioral control.

In a first part, the paper will determine the conditions for change to prevail on the elementary case of static two player games associated with five types of group psychology: hostile, neutral or favorable to change, harmonious (i.e. in which consensus is what matters the most), and evolving.

In a second part, through considering a group comprised of N individuals able change to their minds, the paper will propose an evolutionary game theoretic model that will determine the conditions under which the group psychological features can evolve toward acceptance of the proposed organizational change.

Leaning on the results of this second part, and especially on the cases where evolution does not lead to a general acceptance of the organizational change, a third and last part will analyze how introduction of incentives may change the picture.

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Necessary and Sufficient Conditions for the Existence of Inclusion Map of Game with Preference Relations into Game with Payoff Functions

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Keywords: *Game with preference relations, Strict homomorphism, Inclusion map, Game with payoff functions*

Let K be a class of games with preference relations, $\mathcal{K}$ be a class of games with payoff functions. We consider game $G \in \mathrm{K}$ in the form

$$G = \langle (X_i)_{i \in N}, A, (\rho_i)_{i \in N}, F \rangle$$

where $N = \{1, \ldots, n\}$ is a set of players, X_i is a set of *strategies* of player i, A is a set of *outcomes*, $\rho_i \subseteq A^2$ is a preference relation of player $i (i \in N)$ and realization function F is a mapping of set of *situations* $X = X_1 \times \ldots \times X_n$ in set of outcomes A.

Game $\Gamma \in \mathcal{K}$ is given in the normal form

$$\Gamma = \langle (X_i)_{i \in N}, (\lambda_i)_{i \in N} \rangle$$

where λ_i is a payoff function of player i.

The concept of homomorphism from game G into game Γ is introduced by analogy with concept of homomorphism for games of class K ([1]).

Homomorphism from game G into game Γ can be define as a n-tuple of mappings $f = (\psi_i)_{i \in N}$, where $\psi_i : A \to \mathbb{R}$, such that for each $i \in N$ the following condition

$$a_1 \overset{\rho_i}{\lesssim} a_2 \Rightarrow \psi_i(a_1) \le \psi_i(a_2)$$

holds;

homomorphism will be strict if and only if for each $i \in N$ the condition

$$a_1 \overset{\rho_i}{<} a_2 \Rightarrow \psi_i(a_1) < \psi_i(a_2)$$

is satisfied;

homomorphism will be reciprocal if and only if for each $i \in N$ the following condition

$$a_1 \overset{\rho_i}{\lesssim} a_2 \Leftrightarrow \psi_i(a_1) \leq \psi_i(a_2)$$

holds.

Strict homomorphism f from game G into some game Γ is called *inclusion map* of game G into class $\mathcal{K}$.

Reciprocal homomorphism f from game G into some game Γ is called *isomorphic* inclusion map of game G into class $\mathcal{K}$.

Consider several elementary properties of inclusion map.

1. If there exists isomorphic inclusion map of game G into game Γ then game G is game with linear and transitive preference structure.
2. Any inclusion map of game with linear preference structure into game with payoff functions is isomorphic.

The main result of this paper is the following theorem.

There exists inclusion map of finite game G into class $\mathcal{K}$ if and only if for each $i\left(i \in N\right)$ preference relation ρ_i is aciclic under its symmetric part ρ_i^s.

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Corporate Conflicts Influence on the Personnel Management Efficiency in Russia (Oil Companies Example)

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Keywords: *Agency problem, Management efficiency, Employees' efficiency estimation models, Labour quality, Mergers, Management opportunistic behavior, Employees' motivation and remuneration.*

Corporate conflicts influence on the efficiency of personnel management at any level: the board of directors, top management, middle management, linear management and workers. This is explained by the existence of agency problems in the relationship as between owners and managers and between managers and their subordinates. There are various schemes of remuneration and bonuses of employees, allowing to solve this problem. Moreover, market for corporate control is a way to encourage managers to do their best for their shareholders.

In modern literature on corporate governance and finance, the effect of agency problem on the business results is considered in several directions.

On the basis of existing theoretical structures, a game-theory model was constructed. It allows to trace the relationship between shareholders, managers and other enterprenees of the company which is involved in the corporate conflict on the basis of the real Russian oil company example. Also it allows to estimate the possibility and costs of monitoring the work results, the incentive contracts effectiveness and shareholder wealth.

The model is based on choosing between two premium plans that top-management offer for their workers at the time of business reorganization: one of the plans depends on profit, the other does not. One department of the company is profitable, while the other is not. There are several plans of re-organization of non-profit business, but financial crisis interferes in the affairs of the company. During this period top-management begins to rebuild the system of staff motivation, both for managers and workers.

The problem is very urgent, because scientists and businessmen discus the effectiveness of various remuneration schemes for several centuries and solve this problem in different ways. It is rather interesting to learn why are such companies as Shell, BP and others guided only by the quality of work and abandon the idea to put rewards of their workers in dependence on the profit?

The model presented here demonstrates an attempt to answer this question. Consider its basic assumptions:

1. game is played by 4 players: Nature, Managers, Workers and Shareholders;
2. game is finite and lasts for 2 periods;
3. all players maximize their outputs;
4. it is a game with complete and imperfect information, players take desisions independently, without entering into an agreement with each other.

Model can be briefly described as follows.

In the zero-period nature decides whether business would be profitable or not. In our model, the success of business does not always depend on the efforts of managers and workers.

In the first period, managers decide which bonus plan they could choose for their employees: a plan, which depends on profit or b plan which depends only on quality.

The next move is made by workers: they can work well and as the result the company would get profit, but they may work badly, and the company would get losses. Workers themselves may be of two types, “good” and “bad”. Company’s profit does not always depend on their effort and performance.

At last shareholders enter the game, they already know how much profit was earned during the period. They have 2 strategies:

1. to pay annual bonuses to managers and workers, spending all earned profit;
2. not to pay bonuses, to reinvest part of profit in business development with a promise to pay bonuses later.

In this case we can consider two options: one with "fair" shareholders, who would fulfill their promise concerning bonuses and a model with shareholders who can cheat their employees.

In the second period game is repeated and ends with the progress of shareholders, but in the second period they may still dismiss the management team (3rd strategy). It is important to note that managers may not always be aware of what would be shareholders actions under certain circumstances.

Ability to change the management team, which is a necessary condition for solving the agency problem from the viewpoint of shareholders, may limit the possibility of using incentive schemes for workers by managers. This effect must be taken into account as a constraint while developing and estimating different business strategies for the whole business and its parts including crisis times.

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One Model for Airline Route Formation

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Keywords: *Competition model, Network formation, Coalition, Cooperative solution.*

A competition model on airline market is considered. Each airline attracts passengers only by ticket price on flights which it operates between the cities from a given finite set. It is proposed two types of airlines behavior: noncooperative and cooperative scenarios.

In noncooperative scenario each airline aims to maximize its profit in a route network, and as a solution concept both Nash equilibrium and Pareto solution are considered.

In cooperative scenario also two cases are considered. In the first case two fixed airlines cooperate only on a route which connects their hubs to maximize their total profit on the route, and on other routes their behavior as well as behavior of other airlines remains noncooperative. The second case differs from the first only by objective function. In this case two fixed airlines maximize their total demand function on the route but such behavior may significantly reduce airline profits. Here we refer to cooperative theory and choose the solution concept (core, the Shapley value).

All considered cases are illustrated with numerical examples.

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A Competition in the Market Logistics

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Keywords: *Market logistics, Game-theoretical model, Point of equilibrium*

Consider the logistics market with three firms transporting goods for the customers. Each firm define their own pricing scheme(let firms 1 and 3 service customers in turn, firm 2 services all customers together without queue). Customers choose firm trying to minimize net value of service casualties. The game-theoretic approach used to find optimal behavior of customers considered as players.

Define the non-antagonistic game in normal form: $\Gamma =< N, \{p_i^j\}_{i\in N}, \{H_i\}_{i\in N} >$, where

$N = \{1,\dots,n\}$ - set of players,

$\{p_i^{(j)}\}_{i\in N}$ - set of strategies, $p_i^{(j)} \in [0,1]$, $j = 1,2,3$,

$\{H_i\}_{i\in N}$ - set of payoff functions.

$$H_i = -(p_i^{(1)}Q_{1i} + (1 - p_i^{(1)} - p_i^{(3)})Q_{2i} + p_i^{(3)}Q_{3i}) = -(p_i^{(1)}(Q_{1i} - Q_{2i}) + p_i^{(3)}(Q_{3i} - Q_{2i}) + Q_{2i}),$$

where $p_i^{(1)}$ is the probability of player i choose firm 1, $p_i^{(3)}$ - is the probability of player i choose firm 3, $p_i^{(2)} = 1 - p_i^{(1)} - p_i^{(3)}$ - is the probability of player i choose firm 2. We consider the casualty functions below: $h_i = -H_i$, $i = 1,\dots,n$.

Define customer specific loss of waiting service r .

$Q_{1i} = r(t_i^{(11)} + t_i^{(12)}) + c_1$ - player i expected loss for firm 1's service, where $t_i^{(11)}$ - mean time of waiting service by firm 1, $t_i^{(12)}$ - mean time of service by firm 1, c_1 - fixed cost of firm 1's customer order fulfillment.

$Q_{2i} = (r + c_{22})t_i^{(22)} + c_{21}$ - player i expected loss for firm 2's service, where $t_i^{(22)}$ - mean time of service by firm 2, c_{21} - fixed cost of firm 2's customer order fulfillment, c_{22} - cost of firm 2's unit service time.

$Q_{3i} = rt_i^{(31)} + (r + c_{32})t_i^{(32)}$ - player i expected loss for firm 3's service, where $t_i^{(31)}$ - mean time of waiting service by firm 3, $t_i^{(32)}$ - mean time of service by firm 3, c_{32} - cost of firm 3's unit service time.

Firms' service times are independent exponential distributed random variables with density functions $f_1(t) = \frac{1}{\mu_1} e^{\frac{1}{\mu_1}t}$, $f_2(t) = \frac{1}{\mu_2} e^{\frac{1}{\mu_2}t}$, $f_3(t) = \frac{1}{\mu_2} e^{\frac{1}{\mu_3}t}$, $(t > 0)$ respectively.

Customers choose only one of three logistic firms. There are k_1 customers on service in the firm 1($k_1 - 1$ of them are in the queue) and k_3 customers on service in the firm 3($k_3 - 1$ of them are in the queue).

Theorem 1 *There exists a unique point of equilibrium* $(p_1,\dots,p_n)$, $i = 1,\dots,n$ *in the game above defined as follows:*

1) $p_i = (1,0,0), i = 1,\dots,n$, if:

$$\begin{cases} \mu_1 r((k_1 + 1) + \frac{1}{2}(n-1)) + c_1 < \mu_2(r + c_{22}) + c_{21} \\ \mu_1 r((k_1 + 1) + \frac{1}{2}(n-1)) + c_1 < \mu_3(r(k_3 + 1) + c_{32}) \end{cases}$$

2) $p_i = (0,1,0), i = 1,\dots,n$, if:

$$\begin{cases} \mu_2(r + c_{22}) + c_{21} < \mu_3(r(k_3 + 1) + c_{32}) \\ \mu_2(r + c_{22}) + c_{21} < \mu_1(r(k_1 + 1)) + c_1 \end{cases}$$

3) $p_i = (0,0,1)$, $i = 1,\ldots,n$, if:

$$\begin{cases} \mu_3(r(k_3+1) + \frac{1}{2}r(n-1) + c_{32}) < \mu_1(r(k_1+1)) + c_1 \\ \mu_3(r(k_3+1) + \frac{1}{2}r(n-1) + c_{32}) < \mu_2(r + c_{22}) + c_{21} \end{cases}$$

4) $p_i = (\frac{\mu_2(r + c_{22}) + c_{21} - \mu_1 r(k_1+1) - c_1}{\frac{1}{2}\mu_1 r(n-1)}, 1 - \frac{\mu_2(r + c_{22}) + c_{21} - \mu_1 r(k_1+1) - c_1}{\frac{1}{2}\mu_1 r(n-1)}, 0)$

$i = 1,\ldots,n$, if:

$$\begin{cases} \mu_1 r((k_1+1) + \frac{1}{2}(n-1)) + c_1 \le \mu_3(r(k_3+1) + c_{32}) \\ \mu_1(r(k_1+1)) + c_1 \le \mu_2(r + c_{22}) + c_{21} \le \mu_1 r((k_1+1) + \frac{1}{2}(n-1)) + c_1 \end{cases}$$

5)

$p_i = (0, 1 - \frac{\mu_2(r + c_{22}) + c_{21} - \mu_3(r(k_3+1) + c_{32})}{\frac{1}{2}\mu_3 r(n-1)}, \frac{\mu_2(r + c_{22}) + c_{21} - \mu_3(r_3(k_3+1) + c_{32})}{\frac{1}{2}\mu_3 r(n-1)})$

$i = 1,\ldots,n$, if:

$$\begin{cases} \mu_3(r(k_3+1) + \frac{1}{2}r(n-1) + c_{32}) \le \mu_1(r(k_1+1)) + c_1 \\ \mu_3(r(k_3+1) + c_{32}) \le \mu_2(r + c_{22}) + c_{21} \le \mu_3(r(k_3+1)r + \frac{1}{2}r(n-1) + c_{32}) \end{cases}$$

6)

$p_i = (\frac{\mu_3(r(k_3+1) + \frac{1}{2}r(n-1) + c_{32}) - \mu_1 r(k_1+1) - c_1}{\frac{1}{2}(n-1)(\mu_1 r + \mu_3 r)}, 0, 1 - \frac{\mu_2(r + c_{22}) + c_{21} - \mu_3(r(k_3+1) + c_{32})}{\frac{1}{2}\mu_3 r(n-1)})$

$i = 1,\ldots,n$, if:

$$\begin{cases} \left[\begin{array}{l} \mu_1 r((k_1+1) + \frac{1}{2}(n-1)) + c_1 \le \mu_2(r + c_{22}) + c_{21} \\ \mu_3(r(k_3+2) + \frac{1}{2}r(n-1) + c_{32}) \le \mu_2(r + c_{22}) + c_{21} \end{array} \right. \\ \mu_1(r(k_1+1)) + c_1 \le \mu_3(r(k_3+1) + \frac{1}{2}r(n-1) + c_{32}) \\ \mu_3(r(k_3+1) + c_{32}) \le \mu_1 r((k_1+1) + \frac{1}{2}(n-1)) + c_1 \end{cases}$$

7)

$$p_i = (\frac{\mu_2(r + c_{22}) + c_{21} - \mu_1 r(k_1 + 1) - c_1}{\frac{1}{2}\mu_1 r(n-1)}, \frac{\mu_2(r + c_{22}) + c_{21} - \mu_3(r(k_3 + 1) + c_{32})}{\frac{1}{2}\mu_3 r(n-1)},$$
$$1 - \frac{\mu_2(r + c_{22}) + c_{21} - \mu_1 r(k_1 + 1) - c_1}{\frac{1}{2}\mu_1 r(n-1)} - \frac{\mu_2(r + c_{22}) + c_{21} - \mu_3(r(k_3 + 1) + c_{32})}{\frac{1}{2}\mu_3 r(n-1)})$$

$i = 1,\ldots,n$, if:

$$\begin{cases} \mu_1(r(k_1 + 1)) + c_1 \le \mu_2(r + c_{22}) + c_{21} \le \mu_1 r((k_1 + 1) + \frac{1}{2}(n-1)) + c_1) \\ \mu_3(r(k_3 + 1) + c_{32}) \le \mu_2(r + c_{22}) + c_{21} \le \mu_3(r(k_3 + 1) + \frac{1}{2}r(n-1) + c_{32}) \\ \mu_1(r(k_1 + 1)) + c_1 \le \mu_3(r(k_3 + 1) + \frac{1}{2}r(n-1) + c_{32}) \\ \mu_3(r(k_3 + 1) + c_{32}) \le \mu_1 r((k_1 + 1) + \frac{1}{2}(n-1)) + c_1 \end{cases}$$

Strategies of customers optimal behavior under competition in the logistics market are found.

Showcase Showdown Game with Sequential Steps

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Keywords: *Showcase Showdown, Sequential game, Optimal stopping, Threshold strategy.*

We consider a non-cooperative n-person optimal stopping game related with the popular TV game "The price is right". In this game each of players in turn spins the wheel once or twice attaining some total score and then waits for the results of the succeeding players' spins. The object of the game is to have the highest score, from one or two spins, without going over a given upper limit. The variant of this game has been analyzed in ([1]), ([6]), ([2]) and in the article ([3]), where each player makes the decision without information about behavior of the other players.

We consider a sequential variant of this game, in which each next player has information about the results and actions of the other players, who have chosen their actions before. As the upper limit is taken 1, and the players observe the random numbers in $[0,1]$. Each player chooses one or two random numbers. After the first step a player decides to choose the number or to continue the game. The player makes his decision knowing what the previous players have done. The objective of a player is to get the maximal number of scores which is not exceeded the level 1. If the scores of all players exceed 1 then the winner is the player whose score is closest to 1.

In the article ([3]) the method based on the dynamic programming theory wore used for the construction the optimal solution for the n-person game. The dynamic programming approach for optimal stopping problem have been considered in ([4]), ([5]).

We use the dynamic programming approach to construct the final formula allowing to find the optimal thresholds for the first player in the n-person game. The solution of the following equation gives the optimal threshold u maximizing the probability of the winning for the first player in the n-person sequential game:

$$u^{2n-2} = \frac{1 - u^{2n-1}}{2n - 1}.$$

Numerical results of the optimal strategies of the first player for different n are also given. Additionally we describe the optimal behavior of the other players of the game.

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Multinomial Logit Analysis and Competitive Behavior in the Market

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Keywords: *Multinomial logit analysis, Hotelling's duopoly model on the plane, Nash equilibrium, prices*

Classical Hotelling's duopoly model [Hotelling, 1929] studies players' competitive behavior in a linear market. The price and distance from a customer to a market participant have an impact on customer decision which player's product to buy. Hotelling found equilibrium prices for market participants. Since then a large number of researches have been made on Hotelling's model and its different modifications mostly for the case of two players' game.

We study a case of n players on the plane with distance presented by euclidean metric. Multinomial logit analysis is applied to model customer demand.

Let assume, that each customer i faces n alternatives and wants to maximize the utility received from buying a product of player j. Each alternative is characterized by price p_j and distance to a customer. The utility function depends on product characteristics and has a random part.

Following multinomial logit model assumptions [McFadden, 1973], the probability of choosing player j by customer in (x, y) is equal to

$$P_j(x,y) = \frac{e^{-\beta_1 p_j - \beta_2 \sqrt{(x-x_j)^2 + (y-y_j)^2}}}{\sum_{i=1}^{n} e^{-\beta_1 p_i - \beta_2 \sqrt{(x-x_i)^2 + (y-y_i)^2}}}, \ \beta_1, \beta_2 \geq 0 .$$

Player's competitive behavior in the market is examined using Nash equilibrium. In that case, equilibrium in price satisfies conditions

$$\begin{cases} \dfrac{\partial H_j}{\partial p_j} = S_j + p_j \dfrac{\partial S_j}{\partial p_j} = 0, \ \mathrm{j} = 1 \dots n-1 \\ \dfrac{\partial H_n}{\partial p_n} = \pi - \sum_{i=1}^{n-1} S_i + p_n \sum_{i=1}^{n-1} \dfrac{\partial S_i}{\partial p_i} = 0 \end{cases},$$

where S_j is a market share for player j.

We present computational results for modeling players competitive behavior in a real market with $n = 4$ and $n = 16$.

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On Locally-Optimizing Pursuit Strategies

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Keywords: *Directionally differentiable payoff, Approaching several evaders, Smooth upper approximations of payoff*

Let $z_P(t) \in \mathbb{R}^{n_P}$ and $z_e(t) \in \mathbb{R}^{n_e}$ obey the equations

$$\begin{aligned} \dot{z}_P(t) &= f_p(z_P(t), u_P(t)), \quad z_p(0) = z_p^0, \\ \dot{z}_e(t) &= f_e(z_e(t), u_e(t)), \quad z_e(0) = z_e^0, \; e \in E = \{E_1, \ldots, E_{N_E}\}, \end{aligned} \tag{1}$$

where $t \geq 0$, $u_P(t) \in \mathrm{U}_P \subset \mathbb{R}^{m_P}$, $u_e(t) \in \mathrm{U}_e \subset \mathbb{R}^{m_e}$, U_P and U_e are compact sets, $f_P : \mathbb{R}^{n_P} \times \mathrm{U}_P \to \mathbb{R}^{n_P}$ and $f_e : \mathbb{R}^{n_e} \times \mathrm{U}_e \to \mathbb{R}^{n_e}$, z_P^0 and z_e^0 are initial positions of the players. Let $z = (z_P, z_E) \in Z = \mathbb{R}^N$, $N = n_P + n_{E_1} + \ldots + n_{E_{N_E}}$, and

$$\dot{z}(t) = f(z(t), u_P(t), u_E(t)), \; z(0) = z^0, \tag{2}$$

where $u_E = (u_{E_1}, \ldots, u_{E_{N_E}})$, $f_E(z_E, u_E) = (f_{E_1}(z_{E_1}, u_{E_1}), \ldots, f_{E_{N_E}}(z_{E_{N_E}}, u_{E_{N_E}}))$, $z^0 = (z_P^0, z_E^0)$, $f(z, u_P, u_E) = (f_P(z_E, u_P), f_E(z_E, u_E))$. We assume that f is jointly continuous and locally Lipschitz with respect to z, and satisfies the extendability condition; see, e.g., [1].

Let $\mathcal{K} : Z \to \mathbb{R}^+$ be a directionally differentiable function that evaluates a given state, and P strive to get a lowest value of $\mathcal{K}$ along trajectories of (1) by an instant $\tau \geq 0$. We define locally-optimizing strategies $U_P^l : Z \to U_P$ and

$U_e^l : Z \to U_e$ as those functions that meet the following steepest descent/ascent conditions; see, e.g., [2,3,4],

$$\begin{aligned} f_P(z_P, U_P^l(z)) &\in \mathrm{Arg} \min_{v_P \in f_P(z_P, \mathrm{U}_P)} \partial_{v_P} \mathcal{K}(z), \\ f_e(z_e, U_e^l(z)) &\in \mathrm{Arg} \max_{v_e \in f_e(z_e, \mathrm{U}_e)} \partial_{v_e} \mathcal{K}(z), \, e \in E. \end{aligned} \tag{3}$$

For given duration, initial state $z^0 \in Z$, partition Δ of $[0,\tau]$ and pursuit strategy $\mathcal{U}_P^l$, consider a differential inclusion

$$\dot{z}(t) \in f(z(t_i), \mathrm{U}_P^l(z(t_i)), \mathrm{U}_E), \, z(0) = z^0, \tag{4}$$

where $\Delta = \{t_0, t_1, \ldots t_n\}$, $\mathrm{U}_E = (\mathrm{U}_{E_1}, \ldots, \mathrm{U}_{E_{N_E}})$, $t_0 = 0$, $t_n = \tau$, $t_i \le t < t_{i+1}$, $i = 0,1,\ldots,n-1$. Let $Z_P(z^0, \mathcal{U}_P^l, \Delta)$ be a set of continuous functions $[0,\tau] \to Z$ that are absolutely continuous and meet (4) for almost all $t \in (0,\tau)$; see, e.g., [1]. Let us evaluate $\mathcal{K}$ by the instant $t = \tau > 0$. Since $\mathcal{K}$ is directionally differentiable,

$$\mathcal{K}(z(t_{i+1})) - \mathcal{K}(z(t_i)) = \partial_{v_i} \mathcal{K}(z(t_i)) \delta t_i + o(\delta t_i) \tag{5}$$

where $t_{i+1} = t_i + \delta t_i$, $z(t_{i+1}) = z(t_i) + v_i \delta t_i$, $v_i \in f(z(t_i), \mathcal{U}_P(z(t_i)), U_E)$, $i = 0,1,\ldots,n-1$. From (5) we obtain that

$$\mathcal{K}(z(t_n)) - \mathcal{K}(z(t_0)) = \sum_{i=0}^{n-1} \partial_{v_i} \mathcal{K}(z(t_i)) \delta t_i + o(|\Delta|) \tag{6}$$

and $\partial_{v_i} \mathcal{K}(z(t_i)) \le \partial_{v_i^l} \mathcal{K}(z(t_i))$ where $v_i^l = f(z(t_i), \mathcal{U}_P^l(z(t_i)), \mathcal{U}_E^l(z(t_i)))$, $i = 0,1,\ldots,n-1$. Thus we have

$$\mathcal{K}(z(t_n)) - \mathcal{K}(z(t_0)) \le \sum_{i=0}^{n-1} \partial_{v_i^l} \mathcal{K}(z(t_i)) \delta t_i + o(|\Delta|). \tag{7}$$

Let U_P^l and U_e^l be uniquely defined with use of (3), $k^l(t) = \partial_{\dot{z}^l(t)} \mathcal{K}(z^l(t))$, where

$$\dot{z}^l(t) = f(z^l(t), \mathcal{U}_P^l(z^l(t)), \mathcal{U}_E^l(z^l(t))), \, z^l(0) = z^0.$$

Theorem 1 *If k^l is continuous on $[0,\tau]$ and $\int_0^\tau k^l(t) dt < 0$ then $\mathcal{K}(z^l(\tau)) < \mathcal{K}(z^0)$.*

Thus, under the assumptions of the theorem, P guarantees a decrease of the initial value of $\mathcal{K}$ by $t = \tau$ with use of $\mathcal{U}_P^l$. When τ is not given, P could terminate the game at the initial instant, or at an instant when k^l changes its sign from minus to plus.

We apply the approach to several games with

$$\mathcal{K}(z) = \max_{e \in E} \rho_z(e, P)$$

and its smooth upper approximations, where $\rho_z(e, P)$ is Euclidian distance from P to e at the state z, $e \in E$. Plane kinematics of the players is described by transition equations for wheeled robots with less control variables than degrees of freedom; see, e.g. [5]. The conditions (3) do not allow to define U_P^l and U_e^l properly even for those simple cases, and some additional assumptions are made.

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Stable Cooperation in Differential Game of Resource Extraction

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Keywords: *Differential games, Random duration, Resource extraction, Stable cooperation, Time-consistency, Random duration, Weibull distribution*

The differential game of non-renewable resource extraction on the base of the classical cake-sharing model is examined. The main focus of the paper is an application of the stable cooperation concept which includes time-consistency of the cooperative agreement (Petrosyan, 1977), strategic stability (Petrosyan, Zenkevich, 2009) and irrational behavior proofness (Yeung, 2006) to the game of resource exploitation. At first I investigate the time-consistency, strategic stability and irrational behavior proofness for the classical model with n symmetric players. Then I consider one modification of the game with elements of stochastic framework, in the sense that the terminal time T is a random value in order to increase the realness of the modeling. I suggest modified concept of the time-consistency (Marin-Solano, Shevkoplyas, 2010) and Yeung's condition which is suitable for the problem with random duration of the game. It is analytically proved that the results for games with random duration cover the results for deterministic games and games with constant discounting. Then the cooperative model of the resource extraction game is examined for time-consistency, strategic stability and irrational behavior proofness under condition of Weibull distribution for the random terminal time T (Shevkoplyas, 2009; Shevkoplyas, 2010b). Accordingly to Weibull distribution shape parameter, the life circle of the non-renewable resource extraction such as an "infant" stage, "adult" stage and "aged" stage is described. The egalitarian solution is given and analyzed on cooperative stability conditions for all three stages of the game.Finally, I consider another modifications of the differential game of resource extraction and research the time-consistency problem for new formulations.

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Complete Solution of a Pursuit-Evasion Differential Game with Hybrid Evader Dynamics

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Keywords: *Pursuit-evasion game, Capture zone, Hybrid evader dynamics*

A pursuit-evasion game with perfect state information, bounded controls and prescribed duration, representing the mathematical model of an aerial interception between two objects (a pursuer and an evader) is considered. The velocities and the bounds of the lateral acceleration commands of both objects are constant. The cost function of the game is the miss distance, to be minimized by the pursuer and maximized by the evader. The dynamics of each object is approximated by a first-order transfer function. Thus, their dynamic modes are defined at any time by the maximal lateral acceleration $a_{p_{\max}}\,(a_{e_{\max}})$ and the time constant $\tau_p(\tau_e)$. The evader has two possible dynamic modes (e.g. aerodynamic and thrust vector controls). The aerodynamic control mode provides higher lateral acceleration, but the thrust vector control mode is faster. The pursuer has only a single dynamic mode.

The game with fixed evader and pursuer dynamic modes was solved in [1]. This game has a saddle point solution in feedback strategies. The solution of the game is based on its scalarization by introducing a new state variable, called the zero-effort miss distance (ZEM). This leads to the decomposition of the game space into two regions of different optimal strategies. In the regular region, which can be covered by candidate optimal trajectories, both players use their maximal lateral acceleration with the sign of the ZEM and the value of the game depends on the initial conditions. In the other (singular) region, the optimal strategies are arbitrary, subject to the control constraints, and the value of the game is constant. If the pursuer has advantage both in maneuverability and agility (acceleration rate), i.e.

$$\begin{aligned} &a_{p_{\max}} > (a_{e_{\max}}); \\ &a_{p_{\max}} / \tau_p > (a_{e_{\max}}) / \tau_e \end{aligned} \tag{1}$$

the game value is zero and the closure of the singular zone becomes the capture zone, the set of all initial conditions, where by using optimal strategies point capture (zero miss distance) is guaranteed. Since most of the realistic initial conditions are in the singular zone, the capture zone size can serve as figure of merit. In the differential game with hybrid evader dynamics, it is assumed that for both evader dynamic modes, the capture conditions (1) are satisfied. During this game the evader can change its dynamics once. The order of the evader dynamic modes and the timing of the switch constitute the evader dynamics schedule, which becomes its additional control. The pursuer knows the two dynamic modes of the evader, but not the current one. In order to ensure this uncertainty, the initial lateral acceleration of the evader a_e0 has to satisfy the condition

$$a_{e0} < \min\{a_{e1_{\max}}, a_{e2_{\max}}\} \tag{2}$$

In earlier studies, one-sided robust optimal control problems, namely the robust pursuit of a hybrid evader [2, 3] and the robust evasion by a hybrid evader [4], were investigated separately. Moreover, in [2] and [4] it was assumed that the initial lateral acceleration of each player is zero. The present paper extends the investigation by solving a zero-sum differential game (a two-sided optimization problem) with non zero lateral acceleration of the evader.

Not having information on the current dynamic mode of the evader, the pursuer selects the center of the convex hull of the uncertainty set as a new aiming point. This allows formulating a new game of lower dimension (two instead of four) with perfect knowledge of the aiming point. Based on the solution of this game, the upper value of the original game is obtained and. the respective capture conditions are established. Subject to these conditions, there exists a non-empty set of initial positions, for which the upper game value is zero. This set is the smallest capture zone guaranteed by the hybrid evader.

In order to obtain the lower game value, an auxiliary game is introduced. This game is solved subject to the assumption of perfect information; i. e. the pursuer knows also the dynamic schedule of the hybrid evader, which is the worst case for the evader. Due to the hybrid evader dynamics, the auxiliary game has impulsive dynamics. Based on the solution of this game, the largest capture zone guaranteed by an optimally controlled pursuer is obtained and the set of initial positions, for which the lower game value of the original game is non-zero, is constructed.

It is shown that in the original differential game, the upper and the lower game values are, in general, different, meaning the game has no saddle point. As a rule, the reason of such phenomenon is the non-separability of a zero-sum game. In the game, considered in this presentation, the saddle point does not exist because of the non-symmetric information structure: the evader knows perfectly both the state and the parameters of the game, while the pursuer does not know the current mode of the evader dynamics. Illustrative examples are presented.

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Wisdom in Tian Ji's Horse Racing Strategy

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Keywords: *Tian Ji's horse racing strategy, Eulerian number, Game theory.*

In the modern world today, the survival of the fittest holds for general people, but the survival of the cunning is possible for intellectuals. Nowadays, with the development of society, every company faces more intense competition, especially for weaker and smaller enterprises. The situation applies to developing nations all over the world too. Every nation, every country, strong or weak, vies for a foothold in every arena of economy, sports, arts and so on. All aims to be the strongest and the best, but in actuality, the strong possesses much more advantages and better talents than the growing multitude of developing nations.

Despite the fact, even the strongest of warriors has his Achilles' heel. In such a competitive world whereby almost everything can be rivaled in, it is necessary to plan the order of engagement or competition to overcome the stronger or to claim a greater victory. In economics, companies and government agencies allocate resources according to the needs and requirements of the situation, often giving the minimum budget for the success of a project to maximize profit. In another way, it is a way of conserving resources to cope with greater threats. On a personal level, when playing a group game, leader must choose his players in sequence. The sequence of the group game should be calculated properly, especially to encounter a stronger competitor. This procedure is very similar to the Tian Ji's horse racing strategy, an interesting legendary Chinese story. In this paper, the generalized Tian Ji's horse racing strategy is analyzed mathematically with the aim of finding general clues which have the potential to be applied in the modern world today.

In ancient China, there was an era known as "Warring States Period" (403 BC — 221 BC) during which China was not a unified empire but divided by independent Seven Warring States with conflicting interests, one of which was Qi State located in eastern China. From 356 BC to 320 BC, the ruler of Qi State was Tian Yin-Qi (378 BC

— 320 BC), King Wei of Qi. The story of the Tian Ji's horse racing strategy, which is well-known and popular in China today, was originally recorded in the biography of Sun Bin (? — 316 BC), as a military strategist in Qi State ruled by King Wei of Qi:

General Tian Ji, a high-ranked army commander in Qi State, frequently bet heavily on horse races with King Wei of Qi. Observing that their horses, divided into three different speed classes, were well-matched, Sun Bin then advised Tian Ji, "Go ahead and stake heavily! I shall see that you win." Taking Sun Bin at his word, Tian Ji bet a thousand gold pieces with the King. Just as the race was to start, Sun Bin counseled Tian Ji, "Pit your slow horse against the King's fast horse, your fast horse against the King's medium horse, and your medium horse against the King's slow horse." When all three horse races were finished, although Tian Ji lost the first race, his horses prevailed in the next two, in the end getting a thousand gold pieces from the King.

Amazedly, the victorious strategy (as did Tian Ji after following Sun Bin's advice) was remarkable to be solved 2300 years long before operations research and game theory were invented. This was only one way that Tian Ji could claim a victory over the King. All other ways would present Tian Ji with loss. Naturally, the Sun Bin's victorious advice, named the Tian Ji's horse racing strategy, may be extended to the scenario that Tian Ji and the King would race horses with arbitrary N different speed classes.

This paper discusses the formulation of determining the winning, drawing and losing probabilities of the generalized Tian Ji's horse racing strategy for any given racing horse numbers. Based on the Eulerian number, the way of calculating the combination of having M wins in N-horse racing is so straightforward, thereby enabling us to find the probability of winning an entire game by having more wins than losses. It is worth to mention to this end that that the larger N results in the higher winning probability. Philosophically, it is typically the epitome of winning in numbers.

The Tian Ji's horse racing strategy is an interesting legendary Chinese story, which gives valuable insights to intellectuals that nothing is absolutely certain and thus nothing is impossible. Studying the theory of the Tian Ji's horse racing strategy is very beneficial to society. In sports, this could be used in matching competitors in group games, such as, tennis, football, table tennis, badminton, etc. In economics, it could be used to allocate minimum resources to suitable tasks for optimizing profits. In engineering logistics, the results could be applied to arrange and transport goods with limited vehicles or manpower.

On Some Properties of The [0,1]-Nucleolus in Cooperative TU-Games

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Keywords: *TU-game, The prenucleolus,The simplified modified nucleolus, The modified nucleolus (the modiclus).*

In our work we describe a new solution concept of a cooperative TU-game, called the [0,1]-nucleolus. It is based on the ideas of the nucleolus and the simplified modified nucleolus. The [0,1]-nucleolus takes into account both the constructive power and the blocking power of a coalition with all possible ratios between the powers.

In the paper we show that this solution satisfies the following properties: nonemptiness (NE), covariance property (COV), anonymity (AN), Pareto optimality (PO), reasonableness (RE) and dummy player (DUM). Moreover, the [0,1]-nucleolus satisfies the individual rationality property (IR) for a class of 0-monotonic games and the single valued property (SIVA) for the class of constant-sum games.

Also in the paper we investigate connection between the [0,1]-nucleolus and some well-known solutions of cooperative TU-games such as the Shapley value, the prenucleolus, the simplified modified nucleolus, the modiclus.

Modelling Hardware, Proprietary, Free and Pirated Software Competition

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Keywords: *Mixed Oligopoly, Open Source, Computer Piracy, Cournot Equilibrium*

At the moment all the software users are choosing between the three options: to buy licenses and use the commercial proprietary software (e. g. Microsoft Windows as an operating system, Microsoft Office as an office suite, etc.); to use free or open source software (e. g. Linux, OpenOffice, etc.); and to use illegal (pirated) copies of proprietary software without buying licenses. These three options correspond to the following three types of software market players: profit-maximizers (for example, Microsoft); non-for-profit players (for example, Linux team); pirates.

Correspondingly, software developers try to determine the optimal way of value capture: to sell the licenses for the use of their products or to distribute the products for free and to collect income from sales of complementary products or additional services.

Such a market structure requires new approaches to research methodology as well as to business development methodology.

In this work we explain the equilibrium structure of the market of hardware, proprietary and free software, and illegal copies of proprietary software. We propose a simple model of market interactions between hardware vendors, proprietary and free software developers, and pirates. We consider two hardware suppliers, Intel and AMD, both maximizing profits forming a traditional duopoly, while proprietary software supplier, Microsoft (which sells licenses for Windows operating system), pirates (they sell illegal copies of Windows operating system), and the community of free software developers (distributing Linux operating system), form a mixed oligopoly, in which only the first two parties maximize their profits.

The model is used to calculate the optimal pricing strategies and market shares for all the products.

We consider that the bundle of hardware (a PC) with operating system is sold in the market. There are Intel-based and AMD-based PCs in the market, both of which can be sold with legal or illegal Windows, or Linux.

Thus, the user selects one of 6 products: Intel / AMD-based PC running legal / illegal Windows or Linux.

A user appraises legal Windows greater than illegal Windows, and illegal Windows greater than Linux. A consumer values Intel-based PC greater than AMD-based PC.

The demand functions for the combined products are linear, and the user will buy the bundled product (a PC with an operating system) if and only if the consumer value of this product for the given user exceeds its price.

Pirates have no fixed costs, and all the variable costs (for all the market players) are close to zero.

Hardware and software manufacturers do not conspire and do not co-operate in other ways. Each manufacturer makes pricing decisions based on available market information on the prices of other players' products (i. e., Cournot situation is considered).

When making pricing decisions each manufacturer considers that other players do not react to the change of the price by this manufacturer, i.e. cross price elasticities are equal to zero.

PC assemblers and sellers form the market of a perfect competition, and do not affect the price of the bundled product (a PC with operating system), unlike manufacturers of CPUs and the proprietary software.

We use the following designations: $q_{\max}$ – PC market capacity; P_I and P_A – the maximal possible prices for Intel / AMD-based PC; P_{I+W}, P_{I+PW}, $_{PA+W+M}$, P_{A+W}, and P_{A+PW} – maximal possible prices for Intel / AMD-based PC running legal / illegal Windows; p_I and p_A – prices for Intel and AMD CPUs set by manufacturers; p_W – Windows license price set by Microsoft; p_{PW} – price for illegal copy of Windows set by the integrated pirating agent; q_{I+W}, q_{I+PW}, q_{I+L}, q_{A+W}, q_{A+PW}, and q_{A+L} – demand on the products; f_I ; f_A and f_M – fixed costs; π_I, π_A, π_M and π_P – profits of Intel, AMD, Microsoft and the integrated pirating agent.

In the formulated assumptions the model of hardware and software manufacturers' interactions looks as follows:

$$q_j(p) = q_{\max}\left(1 - \frac{p}{P_j}\right), \; j \in \{I+L, A+L, I+W, A+W, I+PW, A+PW\};$$

$$q_I = q_{I+W} + q_{I+PW} + q_{I+L}, \quad q_A = q_{A+W} + q_{A+PW} + q_{A+L};$$

$$q_W = q_{I+W} + q_{A+W}, \quad q_L = q_{I+L} + q_{A+L}, \quad q_{PW} = q_{I+PW} + q_{A+PW}.$$

$$\pi_I = q_I p_I - f_I, \quad \pi_A = q_A p_A - f_A, \; \pi_M = q_W p_W - f_W, \; \pi_{PW} = q_{PW} p_{PW}.$$

Writing the first-order conditions of profit maximum for each seller, we can easily find the reaction functions, and if we consider Cournot situation where eaach market player makes pricing decisions assuming the prices of other players' products are constant, and all cross price elasticities are equal to zero. Thus at the Cournot equilibrium (CE) we have (assuming all the $P_j = P$):

$$p_I^{CE} = 8P / 40, \; p_A^{CE} = 4P / 40, \; p_W^{CE} = 18P / 40, \; p_{PW}^{CE} = 9P / 40,$$

$$q_{I+W}^{CE} = 14q_{\max} / 40, \; q_{A+W}^{CE} = q_{A+PW}^{CE} = q_{A+L}^{CE} = 4q_{\max} / 40,$$

$$q_{I+PW}^{CE} = q_{I+L}^{CE} = 5q_{\max} / 40, \; \pi_I^{CE} = 192Pq_{\max} / 1600 - f_I, \; \pi_A^{CE} = 48Pq_{\max} / 1600 - f_A,$$

$$\pi_M^{CE} = 324Pq_{\max} / 1600 - f_I, \; \pi_P^{CE} = 81Pq_{\max} / 1600.$$

We can see that the price of the most expensive product (legal copy of Windows operating system) at the Cournot equilibrium is 4.5 times greater than the price of the cheapest product (AMD CPU), illegal copies of proprietary software are two times cheaper than legal copies of the same software, the profit of Microsoft is approximately 70% greater than the profit of Intel, and nearly 7.5 times greater than the profit of AMD, while integrated profit of pirates is approximately 70% greater than the profit of AMD. The number of Intel users is two times greater than the number of AMD users, and 33% greater than the number of Windows users; the market share of Windows illegal copies is two times smaller than the market share of corresponding legal copies; the market share of Linux is the same as of illegal Windows.

If we compare this model with the model of the hardware / software market without piracy presented in [13] where the user could select Intel / AMD-based PC running legal Windows or Linux and the market players have the following profits:

$$\pi_I^{CE} = 8Pq_{\max} / 49 - f_I, \; \pi_A^{CE} = 2Pq_{\max} / 49 - f_A,$$

$$\pi_M^{CE} = 9Pq_{\max} / 49 - f_I,$$

we can see the strong network effect of piracy which tends to the increase of all the market players' incomes: the income of Intel increases by 16%, the income of AMD increases by 4%, and the income of Microsoft increases by 36%.

According to IDC, at the moment in the USA the software market is divided between proprietary, free and pirated software (45, 22,5, 22,5% correspondingly), and it shows that the US software market is at the equilibrium state while the other regional market are not.

On Sensitivity of Macroeconomic Models to Control Restrictions[1]

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Keywords: *Macroeconomic models, Sensitivity to restrictions, Parallel computations*

Macroeconomic models of production are under consideration. Statistic data for gross product and production costs are given in discrete instants on a period of time. Macroeconomic models have the form of two ordinary differential equations depending on an investment policy and a cost policy. Rates of taxation, refunding rates and the currency exchange course imply some natural restrictions on velocities of investments and production costs. The restrictions are unknown. We consider parameterized restrictions at the same discrete instants as the given statistical data.

The problem of reconstructions of motions of the model closest to the statistic data is solved. The parallel computations are applied to study sensitivity of macroeconomic models to the parameterized restrictions.

Consider macroeconomic models with dynamics of the form

$$\frac{dx_1}{dt} = f_1(x_1, x_2, u_1), \qquad \frac{dx_2}{dt} = f_2(x_1, x_2, u_2), \tag{1}$$

where $t \in [0, T]$, x_1 and x_2 denote the gross product and the production cost, respectively; the investment policy $u_1(t)$ and production cost policy $u_2(t)$ can be considered as controls; $f_1(\cdot)$, $f_2(\cdot)$ are given functions.

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The statistic data for the gross product and the production costs are known for discrete instants $0 = t_0 < t_1 < \ldots < t_N = T$:

$$(x_1^*(t_0), x_2^*(t_0)), (x_1^*(t_1), x_2^*(t_1)), \ldots, (x_1^*(t_N), x_2^*(t_N)). \qquad (2)$$

Economical reasons provide restrictions on values of the controls:

$$u_1(t) \in U_1(t) \subset U_1, \qquad u_2(t) \in U_2(t) \subset U_2, \qquad (3)$$

the symbols $U_1 \subset R$, $U_2 \subset R$ denote unknown compact sets.

The problem is to reconstruct motions $x_1(t), x_2(t)$ of (1) closest to the statistic data (2).

We consider the following optimal control problem for the macroeconomic system (1), (3). We assume that the admissible controls $u_1(t)$, $u_2(t)$ are elements of the set

$$\mathbf{U_T} = \{\forall u(\cdot) = (u_1(\cdot), u_2(\cdot)) : [0,T] \mapsto U_1 \times U_2 \text{are measurable}\}.$$

The pay-off functional

$$I(x_1(\cdot), x_2(\cdot), u_1(\cdot), u_2(\cdot)) = \int_0^T \left[(x_1^*(t) - x_1(t))^2 + (x_2^*(t) - x_2(t))^2 + \varepsilon \frac{(u_1)^2(\cdot) + (u_2)^2(\cdot)}{2} \right] dt \qquad (4)$$

is to minimize on the set $\mathbf{U_T}$ and (3). Here $x_1^*(t)$, $x_2^*(t)$ are linear interpolations of the statistic data (2), $\varepsilon > 0$ is a regularizing parameter.

We introduce parameters $\{a_1^i < a_2^i, b_1^i < b_2^i\} \in R$, $i = 0, \ldots, N-1$ and consider the sets $U_1(t)$ and $U_2(t)$ of the form:

$$U_1(t) = [a_1^i, a_2^i], \quad U_2(t) = [b_1^i, b_2^i], \qquad t \in [t_i, t_{i+1}], \quad i = 0, \ldots, N-1. \quad (5)$$

Optimal trajectories $x_1(\cdot), x_2(\cdot)$ of the optimal control problem (1), (3), (4) ,(5):

$$x_1(t) = x_1(t; a_1(\cdot), a_2(\cdot), b_1(\cdot), b_2(\cdot)), \qquad x_2(t) = x_2(t; a_1(\cdot), a_2(\cdot), b_1(\cdot), b_2(\cdot))$$

solve the best reconstruction problem for fixed parameters $\{a_1^i < a_2^i, b_1^i < b_2^i\}$.

Varying parameters $\{a_1^i < a_2^i, b_1^i < b_2^i\}$, we can provide evaluation the optimal result (4), (5) and estimate the sensitivity of the macroeconomic model to the parameterized restrictions.

To study the influence of restrictions on solution of the best reconstruction problem we apply new effective tools of parallel computations and the numerical methods for solving optimal control problems [1].

Results of simulations are presented. We nave compared the obtained solutions with results of the previous researches [2], [3].

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Applications of Stochastic Hybrid Systems in Finance

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Keywords: Hybrid *Systems, Financial Modelling, Lévy process, Bubbles management.*

Introduction: stochastic hybrid systems

A general construction of a hybrid system is based on stochastic differential equations driven by Brownian motions and Poisson random measures [1, 2]. Therefore a hybrid system can be considered as an extension of a classical time-driven system with discrete events causing a change in its dynamic behavior. A time-driven system may be defined by a differential equation

$$\dot{x} = f\left(x, u, t\right),$$

where $x \in \mathbb{R}^n$ denotes the state variable $u \in \mathbb{R}^m$ is a parameter, t denotes the time and $f : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \to \mathbb{R}$ is a smooth enough map. A stochastic hybrid system can be constructed from the time driven systems by letting the map f depend on a stochastic process on a discrete state space M representing the switching between the distinct qualitative behaviors or distinct time-drive systems. A Markov chain on M provides a clear example of a stochastic process modeling such switching. The random fluctuations in the evaluation of the asset prices may be a consequence of uncertainty regarding the initial condition, as well as of the noise generated by either a Brownian motion or a Lévy process. [3] Using recent techniques from Optimal Control of such dynamic systems, one proposes tools for the determination of thresholds enabling bubble detection and management. Furthermore, such techniques can also be used in the context of portfolio optimization, where the quantity to be optimized can be taken as, among other examples, the agent's utility from such portfolio or the transaction costs involved on its management.

An extension of Merton's consumption-investment problem

Optimal consumption investment decision consists of determining the proportion to consume and the amount of wealth to allocate between stocks and a risk-free asset so as to maximize expected lifetime utility. The consumption-investment rules of Merton serve to determine the amounts c_t that will be consumed and the fraction of wealth π_t that will be invested in a stock portfolio. The objective is

$$\max E\left[\int_0^T e^{-\delta s} u\left(c_s\right) ds + e^{-\delta T} u(V_T)\right],$$

where u is a known utility function, δ is the subjective discount rate and V_t is the wealth at time t that wealth evolves according to the following stochastic differential equation

$$dV_t = \left[\left(r + \pi_t\left(\mu - r\right)\right)V_t - c_t\right]dt + V_t \pi_t \sigma dV_t \ ,$$

where r is the risk-free rate, $\left(\mu, \sigma\right)$ are the expected return and volatility of the stock market and dV_t is the increment of the Brownian motion, i.e., the stochastic term of the SDE. Many variations of the problem have been explored, but most do not lead to a simple closed-form solution, hence some research has continued to extend and generalize the model. In this study we aim to extend the problem by modeling the evolution of the wealth with a Stochastic Hybrid System.

Management of financial bubbles

A financial bubble is a period of rapid expansion in which the prices of securities rise far above their intrinsic value followed by a market slowdown [4]. It is very important to detect and manage the progress of such bubbles to be able to avoid extreme bursts. An interesting point of view involves the concept of homotopy, i.e., finding conditions for a bubble, formalized on a appropriate topological space, to be homotopic to another. We use the jump component in the Lévy process to model sudden decreases in the price process, just after a bubble burst. Using Stochastic Hybrid Systems our goal is to model financial bubbles and to developing a method to contract a bubble to one point (or shrink them) as soon as possible.

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Competitive Tenders with a Jury

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Keywords: *Game-theoretic tender's model, N-person game, Jury, Project, Equilibrium*

We consider n -person non-zero sum game. The players $\{1,2,...,n\}$ present the projects which are characterized by vectors $\{x^1,...,x^n\}$ from some feasible set S in space R^m. The members of Jury consider the bids and choose one of the projects using a stochastic procedure. Let $a \in R^m$ is a random vector with some distribution function $\mu(x_1,...,x_m)$ which is known to the bidders. The Jury chooses the project x^k which is closest to the point a. The winner k receives a gain $h_k(x^k)$ which depends on the parameters of the project. The components of vector a can be independent or correlated random variables. Independent case corresponds the model when the members of the Jury are independent experts. The game-theoretic model of the problem is presented and the equilibrium in two-dimensional models is derived.

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Journals in Game Theory

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SPRINGER

The Stronger Player Paradox

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Keywords: *Strategic equilibrium, Extended imputations, VN-M non-discriminatory solutions, Stronger player, Paradox.*

An application of the author´s systemic concept of strategic-equilibrium for n-person cooperative games is made to construct strategic-stability outcomes for the 3-person general sum cooperative game with transferable utility. No appeals are made to von Neumann and Morgenstern(vN-M) domination concept. The constructive approach is initially focused on triangular games. This allows us to illustrate the gradual emergence of stages that require specific strategic responses from the players and the coalitions. It shows also a gradual process with increasing levels of player´s understanding of the game structure and its possible outcomes.

In a first stage, previous to considering the formation of the grand coalition, the players make claims and demand admissible utility transfers from each other, based on their bargaining alternatives. The strategic-equilibrium of the game is given in generic form, to be readily computed in specific cases. This is a conditional systemic equilibrium that allows us to see as "one", different mutually exclusive but interrelated possible realities. It anticipates possible developments of interrelated subsystems of game outcomes. It may be given as an extended-imputation associated with a covering collection of subsets of N or as a matrix of payoffs conditioned to the formation of the corresponding covering coalitions.

In a second stage, players in the coalitions of the cover collection realize they can use and accept their maximum sustainable claims, given by the extended imputation that defines the strategic equilibrium of the games, as payoffs taken after a cooperative division agreement is made; and then, as disagreement payoffs or "dividends" if they conformed into a syndicate. If a syndicate forms, the members may divide the corresponding characteristic function value of the coalition, according to those prescribed by the strategic equilibrium. Subsequently, once the cooperative coalitional agreement is accepted, they may become a syndicate to dispute with the excluded player his incremental contribution if the grand coalition is to form. A syndicate internal division rate β specifies how the syndicated members are going to split the amount they will possibly get in bargaining with the excluded player. The amount to be gained from the marginal contribution of the excluded player depends on a syndicate external division rate α that must be agreed with. Thus, the conditional strategic-equilibrium evolves into a conditional equilibrium system of possible outcomes. Since all the variables and parameters are constructed explicitly, a general strategic-equilibrium solution is tentatively given.

The explicit formulas for the possible outcomes allow us to obtain graphs to be analyzed. A remarkable result emerges: In developing the binding agreements on splitting rates, if the internal division parameter β is agreed previous to initiate the process of bargaining with the excluded player, the resulting outcomes of our strategic-equilibrium description conform a vN-M "objective" non-discriminatory solution. If on the contrary the internal division rate β is defined after the external bargaining takes place (always a source of conflict, and disagreement that eventually may require arbitration: Like asking for the price of an already consumed commodity and considering it to be excessively high). These resulting outcomes do not constitute a vN-M solution. Such findings point out the importance of the sequential order in which binding agreements may develop and the remarkable characteristic of vN-M solutions to be sensible to such sources of instability.

The vN-M non-discriminatory solutions emerge naturally as omniscient views of alternative realities. These again, allow us to see as "one "several mutually exclusive but interrelated possibilities. Furthermore, for all non-symmetric games, an even-more surprising result emerges: of the alternative developments of the game, departing from the strategic-equilibrium based description of possible outcomes and in all vN-M non-discriminatory solutions for the general sum-3-person cooperative game, one of the three possible directions the game may develop, (under the same behavior standards) dominates the other two.

The dominating branch of the vN-M solutions corresponds to the conformation of the syndicate of the weaker players against the stronger one (at the coalitional bargaining level): After reaching a bargaining equilibrium, the players realize that the largest incremental contribution is made by the stronger player. So, there is more to be divided between the syndicate and the excluded player, and consequently the weaker players if syndicated enter a scenario where there is more to be gained. This characteristic condition inherent to vN-M non-discriminatory solutions of non-symmetric general sum games, here is referred to as THE STRONGER PLAYER PARADOX.

РОССИЙСКИЙ ЖУРНАЛ МЕНЕДЖМЕНТА

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A Strategic Equilibrium for n-person Cooperative Games

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Keywords: *Strategic-Equilibrium, Extended-Imputations, Bargaining Alternatives, Strategic Dependency, Balanced Collections, Alternative for Matrices, Utility-Transfer Analysis.*

This paper, a fundamental strategic equilibrium is identified and mathematically characterized for all n-person cooperative games with transferable utility. Based on von Neumann and Morgenstern´s (vN-M) detached extended imputations role in relation to the stable set solution of a cooperative game, necessary and sufficient conditions establishing existence of structural and strategic equilibriums are given. These equilibriums generalize the symmetric (objective) vN-M solutions for all general-sum n-person cooperative games. The mathematical characterization of the equilibriums for these games is accomplished in terms of linearly-balanced covering collection structures, admissible utility transfers, strategic independence of players and two basic theorems of the alternative for matrices: The Fredholm alternative form of the fundamental theorem of linear algebra, and the Farkas lemma. Among the many possible strategic equilibriums identified for the general cooperative game, the existence of a fundamental coalitionally rational strategic equilibrium for every game is established. It is shown to always exist and to constitute a von Neumann and Morgenstern un-dominated system of interrelated extended imputations. The fundamental strategic equilibrium is also shown to be not necessarily a solution but an attractor-like system from which all rational solutions to the cooperative game may emerge. Heuristic procedures based on the linear programming characterization of the strategic equilibrium of a game are given for its computation and also as key tool to generate, among several, vN-M non-discriminatory solutions**.** Examples are given to show the explicative capacity for other existing solution concepts and the implications for sharpening existing and developing new ones such as the here introduced as *the strategic-equilibrium core* of the cooperative game.

Robust Optimization of Stochastic Inventory Control Problems

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Keywords: *Robust Optimization, Min-Max Game, Dynamic Programming.*

Introduction

Robust optimization is a specific methodology for handling uncertain data. Kouvelis and Yu define robustness as *"an effective way to structure uncertainty and make decision in the presence of it"*. There are several ways of incorporating robustness into a problem. Among them we use the minimax approach -also known as absolute robust decision- formalized by Nilim and El Ghaoui and Iyengar for robust dynamic programming problems. The minimax approach defines a game between the controller and Nature. In the context of inventory management, the controller's aim is to maximize the expected profit, whereas Nature tries is to minimize this objective function and acts upon observing the controller choice. The optimal robust solution designates the solution which has the highest result after minimization by Nature and is referred as the robust counterpart of the classical problem. Nilim and El Ghaoui and Iyengar simultaneously studied robust stochastic dynamic programs and showed some important properties. In both models, nature selects the transition probability distribution within an uncertainty set defined for every state, stage and the controller's action pair. Under certain assumptions, they both established that the robust counterparts of classical dynamic programming problems satisfies the Bellman equation and therefore can be solved within nearly the same (polynomial) time if the uncertainty set has some particular optimization properties. Later several authors proposed robust policies that are more moderate and experimentally more adaptive to perturbations. Paschalidis showed that introducing a subset of the uncertainty set into the model rather than the whole uncertainty set performs better than the absolute robust approach, whereas Xu and Mannor modelled and investigated nested uncertainty sets.

Model

We study robust counterparts of general control problems from queueing and inventory theory in this paper and investigate certain properties of optimal policies. The problems we consider are a set of discrete-time expected revenue maximization problems. As an inventory example, we use the single-resource multiple class inventory problem (also known as single-leg airline revenue management, Lautenbacher and Stidham). This is a prototype example where a specified amount of inventory is sold within a fixed horizon to different classes of customers. The dynamic programming solution of the problem maximizes the expected total profit by considering the remaining inventory for each stage and decides whether to admit or reject an arriving demand depending on the customer class. When a class-i customer is admitted to the system the total inventory is decreased by one and a reward R_i is obtained. Our main model is based on the absolute robust approach but later we extend the results to other robust approaches proposed in the literature (Xu and Mannor).

Main Results

We show that a set of structural results for the value function such as increasingness, concavity and supermodularity propagates to the robust counterpart of the classical problem. The main conclusion of our work is that the optimal policy is still threshold type. Moreover, we also show that the monotonicity properties that establish how the optimal thresholds change over time continue to hold. Our approach is general enough to cover a number of well-known models from the literature (Lautenbacher and Stidham, Lippman) and we discuss the impacts of robustness for these examples.

A Model of the Two Stage Market

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Keywords: *Forward market, Cournot competition, oligopoly, arbirtageurs, Subgame perfect equilibrium.*

Forward market is a known instrument for reduction of large producers' market power. This opportunity is of special interest in context of electricity markets development. Bushnell (2005) considers a symmetric oligopoly and a two-stage market with Cournot competition at the spot market and no arbitrage condition. For a constant marginal cost, he shows that introduction of the forward market with known forward positions reduces the market power as well as increasing of the number of producers from n to n^2. His model assumes a special order of consumption (individuals with higher reservation prices buy at the forward market) and equal forward and spot prices. However, the real markets do not meet these assumptions. The actual spot prices are typically less than the forward prices, but sometimes essentially exceed them.

The present paper aims to study a dynamic oligopoly model with a random outcome at the spot market. In this case no arbitrage condition means that forward price p^f is equal to the mathematical expectation of spot price p^s. At the first stage producers $a \in A$ set volumes q_a^f supplied at the forward market, $\sum_{a \in A} q_a^f \overset{def}{=} q^f$. At the second stage those consumers who decide to bid at the forward market determine demand function $D^f(p)$. Each consumer b is characterized by reserve price r_b and risk-aversion parameter $\lambda_b \in [\lambda_{\min}, \lambda_{\max}]$, $\lambda_{\min} < 0 < \lambda_{\max}$, and aims to maximize his utility function $U_b = U(\Delta, \lambda_b)$, where $\Delta = r_b - p$, $(\ln U(\Delta, \lambda))''_{\Delta\lambda} \leq 0$. The forward price p^f proceeds from the balance $q^f = D^f(p^f)$. At the third stage the producers meet the residual demand at the Cournot auction using correlated mixed strategies dependent on an observable random factor. Price p_1 occurs with probability w, and price p_2 - with probability $1 - w$. We determine rational strategies of the agents in this game according to the subgame perfect equilibrium concept.

Proposition 1. a) Consumers with reserve prices $p_1 < r_b < p^f$, as well as risk neutral ($\lambda_b = 0$) or preferring ($\lambda_b < 0$) consumers with $p^f < r_b < p_2$, buy the good only at the spot market under low price p_1. b) For $p^f < r_b < p_2$ there is a threshold $\lambda(r)$ such that consumers with $\lambda_b > \lambda(r)$ buy the good at the forward market, and those with $\lambda_b < \lambda(r)$ act as in case a); $\lambda(r)$ decreases from $\lambda_{\max}$ till 0 in this interval. c) For $r_b > p_2$, risk preferring consumers buy the good at the spot market and risk averse consumers – at the forward market.

Proposition 2. Consider a symmetric oligopoly with n firms and fixed marginal cost c. Let the share α of risk preferring consumers be fixed for $r_b > p_2$.Then the SPE prices and production volumes meet the following relations: $p_1 = p^* - \dfrac{q^f}{d(n+1)}$,

$q^{s1} = nd(p_1 - c) = nd(\Delta^* - \dfrac{q^f}{d(n+1)})$, $p_2 = p^* > p_1$, $q^{s2} = n\alpha d\Delta^* < q^{s1}$,

$q^f = \dfrac{nd\Delta^*(n+1)(n+1-2w)}{w(n^2+1)}$, where p^* is a Cournot price for one-stage auction, $\Delta^* = p^* - c$, $D^f(p) = \overline{D} - dp$.

The ratio $\dfrac{p^f - c}{p^* - c}$ is very close to Bushnell's value $\dfrac{n+1}{n^2+1}$. The SPE exists iff $w \in [w_1(n,\alpha), w_2(n,\alpha)]$, the bounds of the interval for different n and α are as follows:

n \ α	0		0.1		0.2		0.3		0.4	
	w_1	w_2	w_1	w_2	w_1	w_2	w_1	w_2	w_1	w_2
2	0.667	0.667	0.74	0.772	0.81	0.837	0.874	0.9	0.944	0.963
3	0.75	0.75	0.828	0.864	0.909	0.941	0.987	1	-	-
4	0.8	0.8	0.88	0.917	0.968	1	-	-	-	-
5	0.833	0.833	0.92	0.94	-	-	-	-	-	-
6	0.857	0.857	0.948	0.976	-	-	-	-	-	-
7	0.875	0.875	0.966	0.995	-	-	-	-	-	-
8	0.889	0.889	0.984	1	-	-	-	-	-	-
9	0.9	0.9	0.99	1	-	-	-	-	-	-
10	0.909	0.909	-	-	-	-	-	-	-	-

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Nash Equilibria in an Advertising Game with Damping Interference Effects

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Keywords: *Non-cooperative game theory; Advertising; Nash Equilibrium.*

Two firms compete for demand from the same group of customers, as they provide a homogeneous market with substitutable products (goods or services). The two manufacturers want to maximize their profits. Each firm may advertise its brand, obtaining a double effect: positive on its own brand and negative on the competitor's one. The manufacturers decide their advertising strategies simultaneously and know the consequences of their actions on their products demands, and finally on their profits.

We set the analysis in the natural framework of non-cooperative game theory under complete information, where Nash equilibria are the most important descriptions of the firms' behaviors.

A fundamental feature of the model is the assumption that the advertising effect on demand is mediated by the goodwill variable, similarly as in the dynamic model proposed by Nerlove and Arrow [5] which has been the basis for an important stream of literature (see [2, Section 3.5]). We believe that the idea of using the intermediate variable goodwill is particularly useful when representing the effect on the demand of several and simultaneous advertising actions, as is the case here and has been in [8].

In various known models (see e.g. [3, 6]), interference of one manufacturer is represented as a negative term added to the competitor's advertising effect on goodwill. Here we assume a different viewpoint and represent interference as a damping factor multiplied to the competitor's virtual goodwill; this representation is analogous to the interference term in the dynamic model of Leitmann and Schmitendorf [4], but recalls also some multiplicative representations of joint advertising actions (see [1, 7]).

The multiplicative interference representation of the present model is essentially different from the additive one, because here the interference effect of one manufacturer's action is modulated by the other manufacturer's action, whereas in the additive model the interference effect of one manufacturer's action is invariant with respect to the other manufacturer's action. Therefore, we present a stronger form of interaction between manufacturers, which seems more realistic.

We discuss the existence and features of the Nash equilibria of the game.

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On Nash Equilibrium (Semi-)Uniqueness Results for Smooth Aggregative Games

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Keywords: *Aggregative game, Nash equilibrium, (semi-)uniqueness, Hahn conditions, homogeneous Cournot oligopoly.*

The motivation for this contribution is the following result in [1] for a homogeneous Cournot oligopoly game:

Theorem 1. *Consider a homogeneous Cournot oligopoly game where:*

a. *no oligopolist has a capacity constraint;*

b. p *is decreasing and continuous;*

c. $v \in]0, +\infty[$*;*

d. $p(y) = 0 \ (y \geq v)$*;*

e. $p \restriction [0, v[$ *is twice continuously differentiable;*

f. *every* c^i *is twice continuously differentiable;*

g. *for every* i : $Dc^i(x^i) > 0 (x^i > 0)$*;*

h. *for every* i *and* $y \in [0, v[$ *there exists* $\alpha < 0$ *such that* $Dp(y) - \mathrm{D}^2\mathrm{c} \leq \alpha(\mathrm{x}^i \geq 0)$*;*

i. *for each Cournot equilibrium* n

$$\sum_{k \in \{j | n^j > 0\}} - \frac{D^2 p(\Sigma_l n^l) n^l + Dp(\Sigma_l n^l)}{Dp(\Sigma_l n^l) - D^2 c^k(n^k)} < 1. \tag{1}$$

Then:

1. *There exists at most one Cournot equilibrium.*

2. *If for every* i *and* $(x^i, y) \in X^i \times [0, v[$ *with* $x^i < y$ *also* $x^i D_2 p(y) + Dp(y) \leq 0$ *holds, then there exists a unique Cournot equilibrium.*◊

We are especially interested in statement 1, i.e. in semi-uniqueness as proving existence in 2 is routine. The proof of 1 in [1] is based on a detailed analysis of the global properties of the *marginal reductions* $t^i : \mathbb{R}_+ \times [0, v[\to \mathbb{R}$, defined by

$$t^i(x^i, y) := Dp(y)x^i + p(y) - Dc^i(x^i),$$

by means of the *virtual backward best reply correspondences* $b^i : [0, v[\multimap \mathbb{R}$ *defined by*

$$b^i(y) := \{x^i \in X^i \mid t^i(x^i, y) = 0\}.$$

Concerning the nominator and denominator of (1) note that

$$Dp(y) - D^2 c^i(x^i) = D_1 t^i(x^i, y), \quad D^2 p(y)x^i + Dp(y) = D_2 t^i(x^i, y).$$

As far as we know, Theorem 1(1) still is a strongest to date in the sense that there is no other result in the literature that not only implies but also substantially improves it. Theorem 1 improves upon a similar result in [2] by not excluding degenerate equilibria.2 The proof given in [1] is much more elementary than the proof in [2] which deals with Cournot equilibria as the solution of a complementarity problem to which differential topological fixed point index theory is applied. In fact the approach in [1] is a refinement of that in [3].

We not only will generalize Theorem 1 to a class of aggregative games, but also will obtain results that substantially improve this theorem intrinsically.

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Robust Set-Valued Prediction in Games

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Game theory has transformed economics and greatly influenced other social and behavioral sciences. The central solution concept used in applications is that of Nash equilibrium. Yet Nash equilibria can be fragile and Nash equilibrium play does not generally follow from assumptions of rationality or of evolution. It is here argued that an exploration of methods for robust set-valued prediction in games is called for, and some such approaches and avenues for future research are discussed.

Recent Advances on Ellipsoidal Cooperative Games

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Keywords: *Game theory, Uncertainty, Ellipsoids, Core.*

This paper is motivated by recent advances in gene-environment networks under uncertainty, the collaboration of Kyoto Protocol being one example here. In fact, the genes and further items in these regulatory networks and related dynamical systems are regarded in a generalized way and viewed as actors (players).

In this study, we present a new model class of cooperative games under ellipsoidal uncertainty, a class of transferable utility games where the worth of each coalition is an ellipsoid instead of a real number. The important issue of whether and why individuals and organizations choose to cooperate (or not) when faced with uncertainty on outcomes or costs has generated a productive line of research in recent years. Crises in the world and the need that players act in the directed manner of, e.g., the joint implemention of the Kyoto Protocol, has brought the necessity of collaboration into public awareness.

Cooperative game theory has been enriched in the last recent years with several models which provide decision-making support in collaborative situations under uncertainty. Such models are generalizations of the classical model regarding the type of coalition values. Thus, the characteristic functions are not real-valued as in classical case - meaning that payoffs to coalitions of players are known with certainty - but they capture the uncertainty on the outcome of cooperation in its different forms: stochastic uncertainty, fuzzy uncertainty, interval uncertainty, ellipsoidal uncertainty. Involving of certainty into cooperative games is motivated by the real world where noise in observation and experimental design, incomplete information and further vagueness in preference structures and decision making play an important role. This causes a great mathematical challenge which was first approached and well understood in the case of interval-valued uncertainty. Since the interval calculus is not able to represent the player's mutual dependencies, similarities and possible affinities to collaborate, we introduce ellipsoids associated with clusters of players which are considered to share such kinds of correlations. We briefly explain the background in clustering theory. For an example, we look an economic situation leading to an “ellipsoid glove game”. We present benefits and difficulties of the ellipsoidal concepts, ways of how to reduce it to the interval concept, we discuss structural frontiers and challenges.

The ellipsoidal core has been recently introduced by Weber, Branzei and Alparslan Gök to answer the important question “How to deal with reward/cost sharing problems under ellipsoidal uncertainty?”. Further, we deal with the ellipsoidal core for cooperative ellipsoidal games. Here, we study properties of this solution concept, relate it with the interval core for cooperative games.

In the aforementioned studies, the research started with fully deterministic models, in the form of networks, their dynamics and optimization. Since, however, the real world is characterized by uncertainty in the form of noise in the observations and a lack of knowledge about how the items of the models interact, the authors left the deterministic real-valuedness of the models and turn to interval-valued models. In fact, the given (data) and predicted values of the biological and entire environmental information can hardly be identified by single scalar numbers, but they can be easily hosted in some confidence intervals. The same can be said for the levels of interactions of all these items. When aggregating all the intervals of data vectors, state vectors or vectors of parameters using Cartesian products, we obtain (confidence) parallelepipeds. Those parallelepipeds and intervals usually come from a perspective where functional dependencies among any two of the errors made in the measurements of the gene-environmental levels are not taken into account explicitly.

Ellipsoids are a suitable data structure whenever data are affected by uncertainty and there are some correlations between the items under consideration. In the real world, noise in observation and experimental design, incomplete information, vagueness in preference structures and decision making are common sources of uncertainty, besides technological and market uncertainty. It is often easy to forecast ranges of values for uncertain data. Nevertheless, the representation of data uncertainty in terms of ellipsoids is more suitable than the error intervals of single variables since ellipsoids are directly related to covariance matrices whose characteristic functions are interval-valued, and present conditions for the nonemptiness of the elipsoidal core of a cooperative ellipsoidal game.

The ellipsoidal core catches the ellipsoidal uncertainty on coalition values with the players' individual payoffs, i.e., the payoffs for each player belongs to an ellipsoid. Ellipsoids go beyond intervals and their Cartesian products such as cubes or parallelepipeds. Specifically, intervals mean parallelism with no correlations included, while ellipsoids allow including correlations among players from the financial (monetary) viewpoint. Since the inclusion of correlations inserts more information into the model of ellipsoidal games and into the ellipsoidal core concept we could call it a regularization plus an increase of accuracy.

As our examples, we study the Ellipsoid Glove Game, the Ellipsoid Police Game and the Ellipsoid Kyoto Game.

Finally, we indicate some topics for further research regarding the ellipsoidal core such as: to find conditions guaranteeing the non-emptiness of the interval core of a cooperative ellipsoidal game, to design efficient algorithms for determining ellipsoidal core elements, to characterize the ellipsoidal core on suitable classes of cooperative ellipsoidal games. We invite the interested reader to participate in this exciting new research programme and its modern applications.

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Belief Distorted Nash Equilibria – when Beliefs about Future Create Future

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Keywords: *Multi stage and repeated games, Games with continuum of players, N-player dynamic games, Nash equilibrium, Belief-distorted Nash equilibrium, Subjective equilibrium, Self-verification of beliefs, Common ecosystem, Cournot oligopoly, Competitive equilibrium, Minority game, Prisoner's dilemma, Stock exchange*

In the paper we examine discrete time dynamic games in which the state variable changes in response to players' decisions via a certain statistic of a profile of players decisions while the players form some expectations about the future values of these two global variables based on their history.

It is not assumed, that a player knows the actual game – the other players' strategy sets, payoff functions, or even the number of players (and, consequently, his own marginal influence on the global variables). Beliefs are either set theoretic (regarded as possible realizations) or probabilistic.

In such a case Nash equilibrium is not an obvious solution and the concept of belief distorted Nash equilibrium (BDNE) is introduced to replace it.

First, we assume that instead of maximizing their actual payoffs given strategies of the other players, at each stage players maximize their anticipated or expected payoffs given beliefs about influence of their current decision on the global variables (pre-BDNE). To complete the notion of BDNE, we add an assumption that along the profile it is impossible to falsify beliefs.

Note that along a BDNE the future behaviour of the global variables is caused mainly by players' beliefs that they are going to behave this way.

The most illustrative for the phenomenon we want to focus on is the ozone hole problem caused by emission of fluorocarbonates (CFCs). After discovering the problem, ecologists suggested to decrease the emission of CFCs, among others by stopping using deodorants containing them. Making such a decision seemed highly unreasonable for each player since his influence on the global emission of CFCs and, consequently, the ozone layer is negligible. However, ecologists made people believe that they are not negligible. People reduced, among others, usage of deodorants containing CFCs, which decreased the global emission and the ozone hole stopped expanding and, as it is claimed now, it started to shrink. Whatever the mechanism is, the belief "my decision not to use deodorants containing freones will cause the ozone hole to shrink" can be verified by the empirical evidence.

The concepts introduced in this paper will be presented and compared with different concepts of equilibria – Nash and subjective equilibria – using the following examples.

1. A simple ecosystem constituting a common property of its users. We assume that the number of users is large and that every player may have problems with assessing his/her marginal influence on the aggregate extraction and, consequently, the future trajectory of the state of the resource.

2. A repeated minority game being a modification of the El Farol problem. There are players who choose each time whether to stay at home or to go to the bar. If the bar is overcrowded, then it is better to stay at home, the less it is crowded the better it is to go.

3. A model of a market describing either Cournot oligopoly or competitive market (considering these two cases as one model is not a coincidence). Players may have problems with assessing their actual share in the market and, therefore, their actual influence on prices.

4. A repeated prisoners dilemma. At each stage each of two players assesses possible future reactions of the other player to his/her decision to cooperate or defect at this stage.

5. A simplified model of a stock exchange with real price formation system. At each stage players form some expectations about future prices and possibility to buy or sell at those prices.

Differential Game of Pollution Control with Overlapping Generations

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Keywords: *Bidding Nash equilibrium,Cooperative solution, Overlapping generations, Age-structuredgame*

We consider a stable (not necessarily stationary) age-structured population modeled by the McKendrick partial differential equation. In constrast to the resource extraction model of Jorgensen and Yeung (1999), where new generations appear at discrete time steps, we assume that at each instant in time a new generation enters the game. Further the mortality as well as the fertility rate of the model are constant over time and exogenous (implying a stable population). The maximal length of life of one cohort equals omega (can be assured by an assumption, see e.g. Anita (2000)).

The cohorts maximize their lifetime utility by chosing the optimal emission rate (i.e. age- and time-dependent control) over their life time. The emissions are aggregated over time and cohorts (i.e. time-dependent state). The objective functions then consists of three components. (i) Utility from the emissions (e.g. production), (ii) disutility from emissions (e.g. pollution), (iii) altruism. (i) and (ii) are standard in economics. (iii) goes back to Barro and Becker (1989) and includes the idea that also the utility of the progenies has to be included in the objective functional. The last motive is very important since otherwise all players behave without any care about the time after the own life.

The resulting model looks quite similar to Shevkoplyas and Kostyunin (2010) (see also Breton et al. (2005)), but works considerably different and has a different interpretation. The duration of the game is no longer random, the differential game evolves over time, but includes overlapping generations and an altruistic motive is included.

We calculate the open-loop Nash equilibrium for the differential game and provide economic interpretations for the derived expressions. On the other hand we deal with the cooperative solution for the differential game resulting in an age-specific optimal control model (solved by the corresponding maximum principle presented in Brokate (1985) or Feichtinger et al. (2003)). By comparing the outcome of both solutions we are able to figure out the relevant differences and provide important economic interpretations. Finally numerical simulations show the solution for both outcomes over time and over the life-time of different cohorts. We are able to illustrate the difference in the long-term behavior of the model compared to the model without an altruistic motive.

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Consistent Subsolutions of the Least Core

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Keywords: *Least core, Prekernel, Consistency*

The least core, a well-known solution concept in TU game setting, satisfies many properties used in axiomatizations of TU games solutions: it is efficient, anonymous, covariant, possesses shift-invariance, max-invariance, and the reconfirmation property. However it is not consistent. In view of the prenucleolus – a consistent solution – is contained in the least core, the latter may contain other consistent subsolutions. Since on the class of two-person games the least core coincides with the prekernel, any such a consistent subsolution is a subsolution of the intersection of the least core (LC) with the prekernel (PK) as well. We present and characterize the largest consistent subsolution of the least core. Denote it by LC_{cons}.

Its axiomatic characterization is obtained with the help of that for the intersection of the core with the prekernel for the class of balanced games, and with the shift-invariance property of both the least core and the prekernel.

Theorem 1 *The LC_{cons} solution is the maximal under inclusion solution for the class of all TU games that satisfies efficiency, the equal treatment property, covariance, shift-invariance, and consistency on the class of balanced games.*

Let us give a representation of solution LC_{cons} for every game (N,v). For this give some notation. For an arbitrary TU game (N,v) and its efficient payoff vector x denote

$$\mathcal{S}_1(v,x) = \arg\max_{S\subset N}(v(S) - x(S))\,;$$

$$I_1(v,x) = \left\{(i,j) \mid s_{ij}(x) = \max_{i',j'\in N} s_{i',j'}(x)\right\},$$

where

$$s_{ij}(x) = \max_{S: i \in S, j \notin S} (v(S) - x(S)) .$$

For $j > 1$ $I_j(v,x) = \left\{ (i,j) \mid s_{ij}(x) = \max_{(i',j' \notin \bigcup_{l=1}^{j-1} I_l(x))} s_{i'j'(x)} \right\}$;

$$\mathcal{S}^{ij}(v,x) = \arg \max_{S: i \in S, j \notin S} (v(S) - x(S)) ;$$

$$\mathcal{E}_k(v,x) = \bigcup_{(i,j) \in I_k(v,x)} \mathcal{S}^{ij}(v,x) ; \tag{1}$$

Evidently, $\mathcal{E}_1(v,x) = \mathcal{S}_1(v,x)$.

$\mathcal{T}_k(v,x)$ – the partition of N , generated by $\mathcal{E}_k(N,v)$, $k = 1,...,l$.

Theorem 2 *A payoff vector* $x \in LC_{cons}(N,v)$ *if and only if the collections* $\mathcal{E}_j(v,x)$ *are weakly balanced on* $T \in \mathcal{T}_{j-1}(v,x)$ *, for* $j = 1,...,l$.

With the help of Theorem 2 it is easy to show that the *lexicographic prekernel* (PK_{lex}) [1], another subsolution of the intersection of the least core with the prekernel, is contained in the LC_{cons}.

The LC_{cons} contains other consistent subsolutions. The solutions $\sigma_k, k = 1,2,3...$ are not consistent. Denote by $LC_{cons}, \eta_2, ..., \eta_k$ their largest consistent subsolutions. It is clear that for every game (N,v)

$$LC_{cons}(N,v) \supset \eta_2(N,v) \supset ... \supset PN(N,v).$$

The sequence of consistent solutions $LC_{cons}, \eta_2, ..., \eta_2, ...$ possesses the same properties as the sequence of k -prekernels $PK = PK_2 = PK_3 \supset PK_4 \supset .. \supset PN$ except for maximality under inclusion [2]. Therefore, the last result is the inclusion

$$\eta_k(N,v) \subset PK_{k+1}(N,v) \ for \ every (N,v).$$

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Subgame Consistent Solution for Random-Horizon Cooperative Dynamic Games

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Keywords: *Cooperative dynamic games, Random horizon, Subgame consistency*

Cooperative games suggest the possibility of socially optimal and group efficient solutions to decision problems involving strategic action. In cooperative dynamic games, a stringent condition – that of *subgame consistency* – is required for a dynamically stable cooperative solution. In particular, under a subgame consistent cooperative solution an extension of the solution policy to a subgame starting at a later time with a state brought about by prior optimal behavior will remain optimal. Dynamic consistency ensures that as the game proceeds players are guided by the same optimality principle at each instant of time, and hence do not possess incentives to deviate from the previously adopted optimal behavior. A rigorous framework for the study of subgame-consistent solutions in cooperative stochastic differential games (which are in continuous-time) was established in Yeung and Petrosyan (2004 and 2006). A generalized theorem was developed for the derivation of an analytically tractable "payoff distribution procedure" leading to dynamically consistent solutions.

In this paper, we extend subgame consistent solutions to discrete-time dynamic cooperative games with random horizon.

Probabilistic Voting Equilibria under Non-Risk-Neutral Candidates

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Keywords: *Public choice, Electoral competition, Probabilistic voting, Formal models in political science, Risk preference*

In this paper I analyze the effects of candidate risk preference on equilibrium in the probabilistic voting model. It is assumed that the candidates have preferences other than the maximization of the expected number of votes or the probability of win maximization. I derive the comparative statics for two voters and one-dimensional policy space. Each voter cares about both the policy platform and the identity of the candidate. It is shown that an increase in the value of exactly one vote causes each candidate to choose a position closer to that of its partisan voter. Numeric computation of equilibria show that these results can be generalized to three or more voters.

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Abstracts

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