ST. PETERSBURG UNIVERSITY
THE INTERNATIONAL SOCIETY OF DYNAMIC GAMES
(Russian Chapter)

GAME THEORY AND MANAGEMENT

The Seventh International Conference Game Theory and Management

GTM2013

June 26-28, 2013, St. Petersburg, Russia

ABSTRACTS

Edited by Leon A. Petrosyan and Nikolay A. Zenkevich

St. Petersburg University
St. Petersburg
2013

УДК 518.9, 517.9, 681.3.07

GAME THEORY AND MANAGEMENT. Collected abstracts of papers presented on the Seventh International Conference Game Theory and Management / Editors Leon A. Petrosyan and Nikolay A. Zenkevich. – SPb.: Graduate School of Management SPbU, 2013. – 274 p.

The collection contains abstracts of papers accepted for the Seventh International Conference Game Theory and Management (June 26–28, 2013, St. Petersburg University, St. Petersburg, Russia). The presented abstracts belong to the field of game theory and its applications to management.

The abstract volume may be recommended for researches and post-graduate students of management, economic and applied mathematics departments.

Computer design: Tatyana Grigorova and Andrey Zyatchin

ТЕОРИЯ ИГР И МЕНЕДЖМЕНТ. Сборник тезисов 7-ой международной конференции по теории игр и менеджменту / Под ред. Л.А. Петросяна и Н.А. Зенкевича. – СПб.: Высшая школа менеджмента СПбГУ, 2013. – 274 с.

Сборник содержит тезисы докладов участников 7-ой международной конференции по теории игр и менеджменту (26–28 июня 2013 года, Высшая школа менеджмента, Санкт-Петербургский государственный университет, Санкт-Петербург, Россия). Представленные тезисы относятся к теории игр и её приложениям в менеджменте.

Тезисы представляют интерес для научных работников, аспирантов и студентов старших курсов университетов, специализирующихся по менеджменту, экономике и прикладной математике.

Компьютерная верстка: Т.В. Григорова и А.В. Зятчин

CONTENTS

Plenary Speakers

Finn Kydland	University of California, Santa Barbara (USA)
Bernard De Meyer	Université Paris 1, Panthéon-Sorbonne (France)
Burkhard Monien	Paderborn University (Germany)
Leon Petrosyan	St. Petersburg University (Russia)

Organizing Committee

Leon Petrosyan – co-chair	FAM, CGT, St. Petersburg University, Russia
Sergei Kouchtch– co-chair	GSOM, St. Petersburg University, Russia
Nikolay Zenkevich – deputy	GSOM, CGT, St. Petersburg University, Russia
Olga Alkanova	GSOM, St. Petersburg University, Russia
Margarita Gladkova	GSOM, St. Petersburg University, Russia
Andrey Zyatchin	GSOM, St. Petersburg University, Russia

International Program Committee

Eitan Altman	INRIA, France
Jesus Mario Bilbao	University of Seville, Spain
Andrea Di Liddo	University of Foggia, Italy
Irinel Dragan	University of Texas, USA
Hongway Gao	Qingdao University, China
Andrey Garnaev	St. Petersburg University, Russia
Sergiu Hart	Hebrew University, Israel
Steffen Jшrgensen	University of Southern Denmark, Denmark
Ehud Kalai	Northwestern University, USA
Vladimir Mazalov	Institute of Applied Mathematical Research, Russia
Shigeo Muto	Tokyo Institute of Technology, Japan
Richard Staelin	Duke University, USA
Krzysztof Szajowski	Institute of Mathematics and Computer Science, WUT
Myrna Wooders	University of Warwick, UK
David W.K.Yeung	Hong Kong Baptist University, Hong-Kong
Georges Zaccour	HEC, Canada
Nikolay Zenkevich	Saint-Petersburg University, Russia
Paul Zipkin	Duke University, USA

WELCOME ADDRESS

We are pleased to welcome you at the Seventh International Conference on Game Theory and Management (GTM2013) which is held in St. Petersburg University and organized by the Graduate School of Management (GSOM) in collaboration with the Faculty of Applied Mathematics & Control Processes and the International Society of Dynamic Games (Russian Chapter).

The Conference is designed to support further development of dialogue between fundamental game theory research and advanced studies in management. Such collaboration had already proved to be very fruitful, and has been manifested in the last two decades by Nobel Prizes in Economics awarded to John Nash, John Harsanyi, Reinhard Selten, Robert Aumann, Eric Maskin, Roger Myerson, Lloid Shapley, Alvin Roth and few other leading scholars in game theory. In its applications to management topics game theory contributed in very significant way to enhancement of our understanding of the most complex issues in competitive strategy, industrial organization and operations management, to name a few areas.

Needless to say that Game Theory and Management is very natural area to be developed in the multidisciplinary environment of St. Petersburg University which is the oldest (est. 1724) Russian classical research University. This Conference was initiated in 2006 at SPbU as part of the strategic partnership of its GSOM and the Faculty of Applied Mathematics & Control Processes, both internationally recognized centers of research and teaching.

We would like to express our gratitude to the Conference's key speakers – distinguished scholars with path-breaking contributions to economic theory, game theory and management – for accepting our invitations. We would also like to thank all the participants who have generously provided their research papers for this event. We are pleased that this Conference has already become a tradition and wish all the success and solid worldwide recognition.

Co-chairs GTM2013

Professor Sergei P. Kouchtch	Professor Leon A. Petrosyan
Vice-Dean, Graduate School	Dean, Faculty of Applied
of Management	Mathematics & Control Processes

St. Petersburg State University

WELCOME

On behalf of the Organizing and Program Committees of GTM2013, it gives us much pleasure to welcome you to the International Conference on Game Theory and Management in the Graduate School of Management and Faculty of Applied Mathematics & Control Processes of St. Petersburg University. This conference is the seventh of the St. Petersburg master-plan conferences on Game Theory and Management, the first one of which took place also in this city six years before. It is an innovated edition as to investigate the trend and provide a unique platform for synergy among business and financial systems, on one hand and industrial systems, on the other, in game-theoretic support of national economies in the recent process of globalization. Mathematical and especially game-theoretic modeling the globalized systemic structure of the world of the future, and managing its conduct towards common benefits is becoming a primary goal today.

This conference held in new millennium is not unique as the Seventh International Conference on Game Theory and Management since parallel to the conferences GTM2007, GTM2008, GTM2009, GTM2010, GTM2011 and GTM2012 other international workshops on Dynamic Games and Management were held worldwide. Because of the importance of the topic we hope that other international and national events dedicated to it will follow. Starting our activity in this direction six years before we had in mind that St. Petersburg University was the first university in the former Soviet Union where game theory was included in the program as obligatory course and the first place in Russia where Graduate School of Management and Faculty of Applied Mathematics were established.

The present volume contains abstracts accepted for the Seventh International Conference on Game Theory and Management, held in St. Petersburg, June 26-28, 2013. As editors of the Volume VII of Contributions to Game Theory and Management we invite the participants to present their full papers for the publication in this Volume. By arrangements with the editors of the international periodical Game Theory and Applications the conference may recommend the most interesting papers for publication in this journal.

St. Petersburg is especially appropriate as a venue for this meeting, being “window to Europe” and thus bridging the cultures of East and West, North and South.

Acknowledgements. The Program and Organizing Committees thanks all people without whose help this conference would not have been possible: the invited speakers, the authors of papers, all of the members of Program Committee for referring papers, the staffs of Graduate School of Management and Faculty of Applied Mathematics & Control Processes.

We would like to thank Maria Dorokhina, Margarita Gladkova, Elena Lebedkina, Artem Sedakov, Anna Tur, Tatyana Grigorova and Andrew Zyatchin for their effective efforts in preparing the conference.

We thank them all.

Leon A. Petrosyan, GTM2013 Program Committee

Nikolay A. Zenkevich, GTM2013 Organizing Committee

One Guaranteed Search Problem on Trees

Tatiana Abramovskaya

St.Petersburg State University,
Faculty of Mathematics and Mechanics,
Universitetsky prospekt, 28, Peterhof, St. Petersburg, 198504, Russia
tanya.abramovskaya@gmail.com

Keywords: *Guaranteed Search, ε -capture, Search Numbers*

Graph searching is a game in which a team of searchers is aiming at capturing an evader hidden in a graph. The trajectories of the players are continuous functions taking values in the graph. The evader can move arbitrary fast from his current position to another by following the paths (connected any two points in the graph either a vertex or an interior point of the edge) as long as he does not cross a set in the graph that is under control of the searchers. The evader has perfect knowledge of the position and further moves of the searchers. A set on the graph is contaminated if it may harbor the evader, and it remains cleared as long as every path from it to any contaminated point is guarded by at least one searcher. Otherwise recontamination occurs. The evader is caught if the whole graph is simultaneously cleared. The search number of the graph is a minimum number of the searchers that are able to catch the evader.

There are many variants of graph searching, depending on the parameters of the search such as the special powers of the searchers, the visibility of the evader or his ability for counteraction etc. [Fomin and Thilikos, 2008] Tre are several very basic ideas that work for all of them such as “divide and rule” principle (that are closely related to path- and tree–decomposition) [Golovach et al., 2000]. On the contrary, a seemingly obvious intention to forbid the recontamination fail in formalizations that we are interested in.

We consider ε –capture that is graph searching game with an invisible evader, where the condition of the capture is as follows. The evader is caught by a pursuer if they are in a distance less than or equal to a given nonnegative radius of capture ε . We focus on the case of trees not only because this class of graphs is extremely important but also the problem in general is very hard.

There is a sequence of examples disproving a hypothesis that on any planar graph one can guarantee the capture with the smaller radius by adding an extra searcher to a current team [Abramovskaya and Petrov, 2011]. It is known that this hypothesis is not true even for trees. We say that the tree T is non–degenerate if for each positive radius of capture one additional searcher ensures the capture with the smaller radius of capture. We will discuss several significant results on ε –search problem on trees that are in close connection with the hypothesis mentioned above. One of them is as follows: the set of all non–degenerate trees is open and dense in the set of all trees of given combinatorial scheme.

References

[1] Abramovskaya, T. V. and N. N. Petrov (2011). Arbitrarily large jumps of the Golovach function for trees. Vestnik St. Petersburg University. Mathematics, 44(1), 59-65.

[2] Golovach, P. A., N. N. Petrov and F. V. Fomin (2000). Search in Graphs. Proceedings of the Steklov Institute of Mathematics, 1, S90–S103.

[3] Fomin, F. V. and D. M. Thilikos (2008). An annotated bibliography on guaranteed graph searching. Theoretical Computer Science, 399(3), 236-245.

Dynamic Stability of Strategic Alliances

Georgii Aleksandrov[1] **and Nikolay Zenkevich**[2]

[1]*Bonn University, Germany*
alexandrov121@gmail.com

[2]*St. Petersburg University, Russia*
zenkevich@gsom.pu.ru

Keywords: *Dynamic Stability Strategic Alliance, Joint Venture*

This paper summarizes previous researches and proposes an approach of strategic alliances dynamic stability on the basis of time consistency problem. In contrast with classical dynamic game theory, where the problem of cooperative solution finding is considered, in this paper the inverse problem is solved: testing the dynamic stability of existing cooperative solution. To prove this approach, the empirical analysis of the relationship between dynamic stability and the lifetime of strategic alliances is conducted. The analysis suggests that dynamic stability has significant effect on average strategic alliances' lifetime.

Periodicals in Game Theory

GAME THEORY AND APPLICATIONS

Volumes 1–16

Edited by
Leon A. Petrosyan & Vladimir V. Mazalov

NOVA SCIENCE

The Spectrum Value for Coalitional Games

Mikel Álvarez-Mozos[1]**, Ziv Hellman**[2] **and Eyal Winter**[3]

[1]*University of Barcelona, Spain*
mikel.alvarez@ub.edu

[2]*University of Tel Aviv, Israel*
ziv.hellman@mail.huji.ac.il

[3]*Hebrew University of Jerusalem, Israel*
eyal.winter@mail.huji.ac.il

Keywords: *Shapley Value, Political Spectrum, Restricted Cooperation, Axiomatic Characterization*

Assuming a 'spectrum' or ordering on the players of a coalitional game, as in a political spectrum in a parliamentary situation, we consider a variation of the Shapley value in which coalitions may only be formed if they are connected with respect to the spectrum. This results in a naturally asymmetric power index in which positioning along the spectrum is critical. We present both a characterization of this value by means of properties and combinatoric formulae for calculating it. In simple majority games, the greatest power accrues to 'moderate' players who are located neither at the extremes of the spectrum nor in its center. In supermajority games, power increasingly accrues towards the extremes, and in unaninimity games all power is held by the players at the extreme of the spectrum.

The Shapley value [4] has for decades been one of the main indices used in the literature for measuring the relative power of players in voting situations. This value, however, does not take into account the ideology of the agents which is of key importance in political situations. Consider for example a parliamentary situation in which there are n players with the same number of votes and a simple majority of them is required to form a government. A straightforward application of the Shapley value grants each player 1/n, using symmetry considerations. In real-life parliaments, however, it is intuitively clear to all observers that not all members have equal power. It is highly unusual to see, for example an extreme right party joining an extreme left party in a coalition without any center parties also included in the coalition to bridge political differences between them. As the above discussion indicates, part of the problem is that the standard Shapley approach assumes that all possible permutations of the players be used in forming coalitions. That means that even highly unlikely coalitions, such as those formed by an extreme left party joining with an extreme right party while by-passing all the parties in between, including their most natural political allies, must be counted equally along every other coalition. Different approaches have been proposed in the literature to study situations in which not all coalitions are feasible or equally likely. In many papers, the problem is tackled by considering some structure on the set of players to describe the way in which players can form coalitions.

Coalitional games together with these kind of structures are usually denoted games with restricted cooperation. We propose here an intuitive way to modify the Shapley value by taking the political spectrum explicitly into account. The incorporation of the ideological positions of the agents for the study of the power distribution of a decision making body was first done in [1]. In that work agents' political positions are given as points in a high dimensional Euclidean space and a probability distribution on the set of all permutations is inferred from them. Then, a modification of the Shapley value is proposed based on two properties, namely that an ordering and its reverse ordering should have the same probability and that the removal of a subset of agents should not affect the probabilities assigned to the relative orderings of the remaining agents. As we show, our approach is much simpler, first because we can allow the positions to be one-dimensional and second, since we consider only a set of admissible permutations and assign the same probability to all of them. With regard to the properties of the [1] value, the value introduced here shares the first of those properties but not the second one. [5] also considered that the political positions of the agents should be taken into account and he proposed an asymmetric generalization of the Shapley value. This modification of the original Shapley value was also considered in [2] to study the optimal ideological position of candidates. Our proposal is based on very similar ideas; however, computing our value is much simpler and allows for a characterization of the value by means of a set of properties. In this work we assume that there exists a spectrum, from 'left to right' according to which the players are ordered linearly. We then impose the condition that as coalitions are formed a la Shapley, they must be connected with respect to the spectrum. Hence, we propose a novel way to generalize the Shapley value to games with restricted cooperation in which the restrictions arise from the position of the agents in a one-dimensional spectrum. This leads to an interesting new value that may shed light on relative power values in situations in which there is a natural ordering of the players.

References

[1] Owen, G. (1972). Political Games. Naval Research Logistics Quarterly, 18:345–355.

[2] Owen, G. and Shapley, L. (1989). Optimal Location of Candidates in Ideological Space. International Journal of [3] Game Theory, 18:339–356.

[3] Shapley, L. (1953). A value for n-person games. In Kuhn, H. and Tucker, A., editors, Contributions to the Theory of Games, Vol. II. Princeton University Press.

[4] Shapley, L. (1977). A comparison of power indices and a nonsymetric generalization. Technical Report P-5872, Rand Corporation.

Universal Nash Equilibrium for n-player Differential Games

Yurii Averboukh

Institut of Mathematics nd Mechanics UrB RAS
averboukh@gmail.com

Keywords: *Nash Equilibrium, Differential Games, Krasovskii-Subbotin Extremal Shift Rule*

The talk is devoted to universal Nash equilibrium for n-player differential game with weakly coupled dynamics. We assume that the dynamics of the i-th variable are described by

$$\dot{x}_i = f_i(t, x_1, \ldots, x_n, u_i), \quad t \in [0,T], \quad x_i \in \mathbb{R}^d, \quad u_i \in P_i. \tag{1}$$

Here u_i is a control of player i. The purpose of the i-th is to maximize the terminal payoff $\sigma_i(x_1(T), \ldots, x_n(T))$.

We assume that sets P_i are compact, functions f_i and σ_i are continuous, in addition assume that function f_i are Lipschitz continuous with respect to the phase variables and satisfy the sublinear growth condition with respect to $x_1, \ldots, x_n$.

Denote by x and by u the vectors $(x_1, \ldots, x_n)$ and $(u_1, \ldots, u_n)$ respectively. Further, let $f(t,x,u)$ be a vector $(f_1(t,x,u_1), \ldots, f_n(t,x,u_n))$.

Denote

$$\mathcal{U}_i = \left\{u_i : \left[0,T\right] \to P_i measurable\right\}, \quad \mathcal{U} = \{u : [0,T] \to P_1 \times \ldots \times P_n measurable\}.$$

If $u \in \mathcal{U}$, (t^0, x^0) is an initial position then denote by $x(\cdot, t^0, x^0, u)$ the solution of initial value problem

$$\frac{dx(t)}{dt} = f(t, x, u(t)), \quad x(t^0) = x^0.$$

We assume that the players use the control with guide strategies first proposed by Krasovskii and Subbotin for zero sum games [1]. The control with guide strategy of player i is a triple $U_i = (u_i(t,x,w), \psi_i(t^+,t,x,w_i), \psi_i(t^0,x^0))$. Here the function u_i forms the control of player i. It depends on current position of the game (t,x) and on the i-th player's guide w_i. The value of the function ψ_i is a state of the i-th player's guide at time $t_=$ under condition that at time t state of the system is x, state of the i-th player's guide is w_i. The function χ_i initialize the guide of player i.

The control is formed in the following way. Let (t^0,x^0) be an initial position, and let $\Delta = \{t^k\}_{k=0}^r$ be a partition of the interval $[t^0,T]$. Further, assume that player j ($j \neq i$) chooses his control $u_j \in \mathcal{U}_j$. Denote by x^k the state of the system at time t^k, also denote by w_i^k the state of the i-th player's guide at time t^k. The motion generated by the strategy of player i U_i and the controls of other players u_j, $j \neq i$ satisfy the differential equations

$$\frac{dx[t]}{dt} = f(t,x,u), \qquad x(t^0) = x^0$$

with $u_i(t) = u(t_k, x[t_k], w_i^k)$ for $t \in [t^k, t^{k+1})$, here w_i^k is a state of the i-th player's guide at time t^k, $w_i^{k+1} = \psi_i(t^{k+1}, t^k, x^k, w_i^k)$, $w_i^0 = \chi_i(t^0,x^0)$.

Let $\bar{U}$ is a collection of control with guide strategies $U_1,\ldots,U_n$. We shall consider two cases.

• All player use the control U_i and get the same partition Δ. Denote the corresponding motion by $x[\cdot, t_0, x_0, \bar{U}, \Delta]$.

• All players except j-th one use the control with guide strategies U_i, and get the same partition Δ, when player j use the control $u_j \in \mathcal{U}_j$. This control can be also formed stepwise. Denote the corresponding motion by $x[\cdot, t^0, x^0, \bar{U} \mid u_j, \Delta]$.

Definition 1 *We shall say that the collection of control with guide strategies* $\bar{U}^*$ *is a Control with Guide Nash Equilibrium on a set* $G \subset [0,T] \times \mathbb{R}^{dn}$ *if for all* $(t^0,x^0) \in G$ *the following inequality holds for all* $j \in \overline{1,n}$

$$\lim_{\delta\downarrow 0}\sup\{\sigma_j(x[T,t^0,x^0,\bar{U}\mid u_j,\Delta]): d(\Delta)\le\delta, u_j\in\mathcal{U}_j\}$$

$$\le \lim_{\delta\downarrow 0}\inf\{\sigma_j(x[T,t^0,x^0,\bar{U},\Delta]): d(\Delta)\le\delta\}.$$

The main result of the talk is Theorems 1 and 2. In order to formulate them let us introduce the following notions. For $u_j^*\in P_j$ put

$$\mathrm{S}ol^{[j]}(t^0,x^0;u_j^*)=\mathrm{cl}\{x(\cdot,t^0,x^0,u^1,\ldots,u_j^*,\ldots,u_n): u_i\in\mathcal{U}_i,\ \ i\ne j\}.$$

Also,

$$\mathrm{S}ol(t^0,x^0)=\mathrm{cl}\{x(\cdot,t^0,x^0,u): u\in\mathcal{U}\}.$$

Theorem 1 *Let a multivalued function* $S:[0,T]\times\mathbb{R}^{dn}\tilde{A}\mathbb{R}^n$ *satisfy the following conditions*

- $S(T,x)=\{\sigma_1(x),\ldots,\sigma_n(x)\}$;
- for all $(t,x)\in[0,T]\times\mathbb{R}^{dn}$, $t_+\in[t,T]$, $J\in S(t,x)$, $j\in\overline{1,n}$ and $u_j\in P_j$ there exist a motion $y^{[j]}(\cdot)\in\mathrm{S}ol^{[j]}(t,x;u_j)$ and $J'\in S(t_+,y^{[j]}(t_+))$ such that $J_j\ge J_j$;
- for all $(t,x)\in[0,T]\times\mathbb{R}^{dn}$, $t_+\in[t,T]$, $J\in S(t,x)$ there exists a motion $y^c(\cdot)\in\mathrm{S}ol^{[j]}(t,x;u_j)$ such that $J\in S(t_+,y^c(t_+))$.

Then for each compact $G\subset[0,T]\times\mathbb{R}^{dn}$ there exists a Control with Guide Nash Equilibrium on G. The corresponding Nash equilibrium payoff of player i at the position $(t^0,x^0)\in G$ is an element of the $S(t^0,x^0)$.

The construction of the Nash equilibrium strategies is based on Krasovskii-Subbotin extremal shift rule [1].

Theorem 2 *There exists an upper semicontinuous multivalued function* $S:[0,T]\times\mathbb{R}^{dn}\tilde{A}\mathbb{R}^2$ *with nonempty images satisfying conditions of Theorem (1).*

References

[1] Krasovskii N.N., Subbotin A.I., Game-Theoretical Control Problems. Springer, New York (1988)

Pursuit-Evasion Game on the 1-skeleton of Regular Polyhedrons

Abdulla Azamov[1], Atamurat Kuchkarov[2] and Azamat Holboyev[3]

[1,2,3]Institute of Mathematics at the Uzbekistan National University, Uzbekistan, Tashkent
[1]abdulla.azamov@gmail.com
[2]kuchkarov1@yandex.ru
[3]Azamat_liverpool@mail.ru

Keywords: *Pursuit, Evasion, Game, Catch, Graph*

Let $G(K)$ denote the graph consisting of 1-skeleton of a regular polyhedron K in Euclidian space. The team of Pursuers $P_1, P_2, \dots, P_m$ and one Evader Q play Pursuit-Evasion differential game moving along edges of the graph $G(K)$. All points have equal maximal speeds. Let $P_1(t), P_2(t), \dots, P_m(t)$ and $Q(t)$ be a current position of the participants of the game. The aim of the team of Pursuers is to reach the equality $P_i(t) = Q(t)$ for some $i = 1,2,\dots,m$ at some moment t, $t \geq 0$, independently of initial positions of points while the aim of Evader is opposite, i.e. to hold the condition $P_i(t) \neq Q(t)$ for all $i = 1,2,\dots,m$ and t, $t \geq 0$, when a process of pursuit-evasion begins from some initial positions. Other statements of the pursuit problems on the graphs were considered in [1,2]

The formulated game can be strongly formulated based on the approach of [4] or [5] (see also [3]).

Obviously, if m is great enough then the team of Pursuers can win the game. The least value of m, that m Pursuers win the game will be denoted $v(K)$.

Theorem 1. $v(\Delta_n) = 2$ for Δ_n that is the regular n-dimensional simplex $\Delta_n = \left\{ x \in R^{n+1} \mid x_0 + x_1 + \dots + x_n = 1, x_i \geq 0, i = 0,1,\dots,n \right\}$ $(n \geq 3)$.

Theorem 2. $v(\Omega_n) = 2$ for Ω_n that is the regular n-dimensional octahedron

$$\Omega_n = \left\{ x \in R^n \mid \left| x_1 \right| + \left| x_2 \right| + \ldots + \left| x_n \right| \le 1 \right\} \ \left(n \ge 2 \right).$$

Theorem 3. $v\left(K_n\right) = \left[\frac{n}{2}\right] + 1$ for K_n that is the regular n-dimensional cube

$$K_n = \left\{ x \in R^n \mid \left| x_i \right| \le 1, i = 1, 2, \ldots, n \right\} \ (n \ge 2).$$

Theorem 4. $v\left(G\right) = 2$, if G is a regular dodecahedron or icosahedron in R^3.

References

[1] Abramovskaya T.V., N.N.Petrov (2012). Theory of guaranteed search on the graphs. Differential equations and processes of controls, No2, pp. 9-65.

[2] Aigner, M., M.Fromme (1984). A game of cops and robbers. Discrete Appl, Math., No 8, pp. 1-12.

[3] Azamov, A.A. (1986). On an alternative for pursuit-evasion games in an infinite time interval. Journal of Applied Mathematics and Mechanics. Vol. 50, Issue 4, pp. 428-432.

[4] Krasovskii, N.N. (1968). Theory of motion control. Nauka, Moscow.

[5] Petrosyan, L.A. (1993). Differential games of pursuit. World Scientific, Singapore.

On Gaming Situations that Simulate Participation in the Competition for a Job

Abdulla Azamov[1] **and Abduhakim Mamanazarov**[2]

Tashkent branch of Moscow State University
22, Amir Temur str, Tashkent, 100060, Uzbekistan
[1]*abdulla.azamov@gmail.com*
[2]*abduhakim_bazarovich@mail.ru*

The main task of the present stage of development of the Republic of Uzbekistan is to ensure macroeconomic stability, sustainable and balanced growth of the economy, the continuation of structural reforms, modernization and renewal of leading sectors of the economy through the increase of attracted investment. It is natural that in the current circumstances, «only the nation can declare itself that has among its priorities the continuous increase of investments in human capital, training of educated and intellectually developed generation, which, in today's world, is the most important value and a decisive force in the achievement of democratic development, modernization and renovation», – said the President of the Republic of Uzbekistan Islam Karimov in his speech at the opening of the international conference» Preparing an educated and intellectually developed generation as an essential condition for sustainable development and modernization of the country» [1, 90]. Successive investments in human capital will, among others, facilitate high-quality education of contemporary youth. Young professionals of today must not only have expertise in the issues of the modern labor market, but also be able to cooperate with different people. Game theory is of interest as a theoretical foundation for finding optimal strategies for this cooperation.

Analysis of the basic principles of game theory was suggested by John von Neumann in 1928. In his paper, Neumann focuses on the logical foundations of quantum mechanics. In 1944, von Neumann and Morgenstern published their work «The Theory of Games and Economic Behavior», which marked the beginning of the development of mathematical methods for the study of games.

In Russian Federation, the study of game theory and the development of game-theoretical problems began in Leningrad State University in the mid-fifties of the XX century under the leadership of Professor N.N.Vorobeva. It was he and his close associates who established the first school of the theory of games. Since then, L.A.Petrosyan, N.A.Zenkevich, E.V.Shevkoplyas and others have been regarded as globally recognized experts in the field of game theory.

Game theory is a method of studying the best strategies in games, by which any process is meant where the parties are struggling to observe their interests. In the interpretation of the Israeli scholar Aumann game theory served as the scientific basis for building a strategy of economic development of the state. The game refers to the process which involves two or more parties who struggling for the realization of their interests. Each side has its own purpose and uses some strategy that can lead to the victory or loss depending on the behavior of others, taking into account the views of other participants and their possible actions. It is in economics that game theory is mostly applied; however, it also used in n other social sciences such as sociology, political science, psychology, ethics, and others. In economics, it is applicable not only to the solution of general economic problems, but also to the analysis of strategic business problems, development of organizational structures and incentive systems. Game theory is most closely associated with social and economic phenomena occurring in the concrete individual country and the world at large. It is based on the desire of people to achieve their goals. Therefore, only experts who are familiar with game theory can successfully manage staff of any organization.

The theoretical basis for finding the optimal solutions in uncertain conditions is the theory of games. Game is a process model of functioning of system elements, in which the actions of its members occur according to certain rules called strategies. The basic rule of the theory of games is that any party of the system is of the same intellectual capacity as the operating party and makes every effort to achieve its own goals. If management decisions are made by one person and their results do not depend on the actions of others, the theory of optimal management and optimization can be successfully used as mathematical modeling.

At the same time, in many real-world processes decision-making takes place at sufficiently large time intervals, where it is necessary to take into account the results of previous decisions at every current time and to develop appropriate management based on this. It is therefore necessary to ascertain that the appropriate process models can be dynamic games that take into account the fact that the decision-making process can lead to conflicts and it is important to model it for a sufficiently long time interval. Modeling and decision-making using game theory is one of the most controversial issues in modern management science. Game theory as a tool of management instills the idea that it can display a unique solution with the best possible result for the players. In its scientific use game theory focuses on the logical forecast of the behavior of the parties which would be rational for all. It is always necessary to keep in mind that even if conditionally management decision is made by a single person, it is impossible to guarantee that the end result will not depend on the actions of others. Therefore, dynamic and differential games can prove useful models of many situations.

Another nuance of the application of game-theoretical models is the principle of optimality. This basic idea of the principle is that decisions made by managers are reached by means of algorithms which are hard to define, where an optimal solution is chosen from the many emerging options. Two most important properties that will inevitably affect long-term decision should also be mentioned here. These properties were defined by professor L.A.Petrosyan: «The first property is the need to assess the quality of the decision against several criteria. The second one is different assessment of outcomes by different parties involved in the development of solutions» [2, 8]. At the same time, we must be kept in mind that the world is changing and possible steps for each player mainly depend on different factors.

Game theory has significantly raised the current level of understanding of decision-making processes and can open wide horizons and patterns of decision-making in management. The development of management solutions involves the use of a certain amount of information. Depending on the completeness of initial information, management decisions can be reached under certainty, risk and uncertainty. Game theory is seen as the most economically viable element of organization theory and is a very complex area of knowledge. After all, each game has a goal or the end state sought by the players who choose the direction of actions allowed by the rules of the game. In some cases, the purpose of the game is to reach the goal with maximum efficiency. Decisions are made in conditions of certainty, when the leader knows the outcomes of each of the alternative solutions.

Games can be classified based on the number of players, the number of strategies, the nature of the interaction of the players, the nature of the gain, the number of moves, the volume of information available, etc. Depending on the number of players there can be games of two and n players. Games can be finite and infinite according to the number of strategies. If all the players in the game have a finite number of possible strategies, it is called finite. If at least one of the players has an infinite number of possible strategies, the game is called infinite.

Depending on the nature of interaction, games can be:

1) non-cooperative: the players are not allowed to enter into an agreement to form a coalition;

2) cooperative: players can form coalitions.

Depending on the nature of gains, games can be divided into games with zero sum and non-zero sum. Matrix game is the ultimate game of two players with a zero-sum, in which the player's gains are expressed. For matrix games it is proved that any of them has a solution and it can easily be found by reducing the game to a linear programming problem. Games with one participant are games with no competition. The participant plays for score or for a goal. Competitive game is a game where there is a final state, which is sought by each player, but not everyone can achieve it. Bimatrix game is the ultimate game of two players with non-zero sum, in which the gains of each person are expressed by the matrices separately for the appropriate person. Continuous game is a game in which the payoff function of each player is continuous, depending on the strategies. If the function of gains is convex, the game is called a convex game. For such games, acceptable methods of solutions are developed that allow finding the optimal pure strategy for one player and the probabilities of application of clean optimal strategies of another player. Such thematic areas as strategic behavior, competition, cooperation, risk and uncertainty are central to the theory of games, and are directly linked with the managerial tasks.

Therefore, the progress of economic development has proved the fruitfulness of application of methods of games. In fact, game theory is a theory of decision making, applicable to competitive situations and common analytical approach to modeling of social situations in which knowledge, possible actions and motivations of the actors, as well the effects and results of these actions are outlined in details.

As society develops, more areas of application of the theory of games are emerging in a variety of ways. It is often necessary to introduce into practice new types of games. Competition for jobs is an example of such types. Suppose that some "firm" (e.g., admission to a university, selection of candidates for the national team in one or more sports, participation in bids, etc.) announced a competition to fill n number of vacancies (to simplify, we assume that all jobs are equal), and *m* number of applicants are competing for the vacancy. If $m \leq n$, the situation is not cooperative, that is no game is possible. If $m > n$, participants enter into a game (competition) with a "win" or a "lose" result. This kind of game is close to the arbitrary schemes, but it has some peculiarities as well. For example, originally, the game can be non-cooperative, but in this case, the participants can form coalitions, and if there are several stages in the competition, the game becomes a multi-step one with a changing number of participants and the changing dynamics of coalitions. In a particular case, when after the next round we come to a situation of $m \leq n$, we get a cooperative game. When taking into account the quality indicators of the competitors, "the firm" may also be included in the game as a player as the party that seeks to maximize totals of those who have passed the competition. The paper will describe the mathematical model of competition in the form of a dynamic multi-stage game with a dynamic number of rounds and the players and the structure of coalitions, and an optimization algorithm for the "firm" will be suggested.

References

[1] R. B. Myerson (1977). Graphs and cooperation in games. Math. Oper. Res., 2, 255-229.

[2] J.Weibull. Evolutionary Game Theory. Cambridge, MA, MIT Press, 1995.

[3] Л.А.Петросян, С.И.Мамкина. Игры с переменным коалиционным разбинием // Вестник СПбГУ (2004), сер. 1, вып. 3, с. 60-69.

Coordinating a Three-echelon Supply Chain with Uncertain Demand and Random Yield in Production

Sudarshan Bardhan[1] **and Bibhas Chandra Giri**[2]

[1,2]*Department of Mathematics Jadavpur University*
Kolkata - 700032, India.
[1]*dashu 87@yahoo.co.in*
[2]*bcgiri.jumath@gmail.com*

Keywords: *Supply Chain Management, Three Layer Supply Chain, Demand Uncertainty, Random Yield in Production*

One of the major objectives of modern supply chain management is to deal with the growing decentralization among the involved entities and hence minimizing the double marginalization effect inside the chain, especially when the end-customers' demand is not deterministic. Also, to improve the performance of the members as well as total chain in terms of accumulating more profit by the members of the chain, some kind of coordination mechanism is necessary. To address these issues, a three layer supply chain with one raw-material supplier, one manufacturer and one retailer trading over a single item over a single time period is developed and studied in this paper. The market demand faced by the retailer is assumed to be of completely stochastic nature with positive support. The productions of both the raw-material supplier and the manufacturer are assumed to be subject to random yield in the sense that they always produce quantities lesser than they initially plan to. The problem is to maximize profit(s) under different scenario. At the beginning, the centralized model is developed and solved where all the entities act together so that expected profit of the total chain may be maximized. Although impossible to ensure the existence and practice of such a business strategy in real life scenario, the solutions obtained solving the centralized model are set as the benchmark. Next, a very simple and practical scenario, called decentralized model, is considered where each of the acting entities aims at maximizing its own expected profit, without looking at others' interest or even at total chain's performance, and the model is solved by the Nash game strategy. It is shown that the performance of the total chain is lesser than that in the centralized system, which is due the double marginalization effect. Aiming at establishing coordination among the entities so that the total expected profit of the chain may reach the benchmark case and also all the entities may be better paid off, we try to implement different coordination mechanisms on the model. We see that neither the revenue sharing nor the buyback contract coordinates the chain. We then start with a combination of buyback contract and sales rebate and penalty contract between the manufacturer and the retailer; it is seen that the above said contract coordinates the chain, but splits the total expected profit of the chain between the manufacturer and the retailer (and both are better paid off too) and leaves the raw-material supplier with zero expected profit. The contract is then modified by incorporating a two part tariff contract in between the raw-material supplier and upstream entity (entities) which serves our purpose. Different probability distributions are studied in two numerical examples to verify the applicability and robustness of the model.

Journals in Game Theory

INTERNATIONAL JOURNAL OF GAME THEORY

Editor
Shmuel Zamir

SPRINGER

Codes of Conduct and Bad Reputation

Juan Block

Washington University in St. Louis,
1 Brookings Dr., Campus Box 1208, St. Louis, MO
63130-4899, USA.
juanblock@wustl.edu

Keywords: *Reputation, Self-referential Games, Stackelberg*

We study bad reputation games from the perspective of self-referentiality. In self-referential games players have the possibility of understanding opponents' intentions, and this can mitigate the problem of bad reputation. We characterize the probabilities of the Stackelberg type required to overcome a bad reputation problem when there is a possibility that intentions can be observed directly. The complementarity between direct and indirect observation of opponents' intentions is shown to be qualitatively different from games where all agents are long-lived.

Most models of reputation establish that uncertainty about the long-run player's type implies a beneficial reputation effect as long as long-run player can exploit some commitment power. By way of contrast, Ely and Välimäki (2003) propose a novel example where reputation may be bad. With a bad commitment type, they find that in equilibrium the entire surplus is lost. Along the same line, an example which captures all features of their model is mitral valve surgeries. There are two types of surgeries, valve repair and valve replacement. Both surgical procedures are equally successful when appropriately conducted. The right type of surgery is known only by the surgeon. Think of the case where there are two broad categories: good and bad physicians. The bad physician always does valve replacement but the good one performs the necessary surgery. Since this example is analogous to theirs, the model predicts none of physicians would perform a surgery. In reality, however, we still observe the coexistence of bad and good physicians involve in long-run relationships with patients.*

Humans are rarely perfect liars. They reveal -- even unintentionally -- their states of mind either through micro expressions (facial gestures, body posture, etc.) or

*In the US more than 40,000 mitral valve operations are performed every year.

tools such as pen-drumming.* In an evolutionary setting, Levine and Pesendorfer (2007) examine self-referential strategies which have the ability to recognize each other in the context of two player symmetric games. Intuitively, players have the chance of discerning whether opponents conform to a rule of behavior. To generalize this idea, Block and Levine (2012) define self-referential games in which players are able to understand opponents' intentions about chosen strategies by receiving informative signals. The self-referential nature of these games is characterized by the fact that players choose how they will play the game depending on signals they observe, and the choice of such strategies indeed determines the likelihood of those signals. (See also A. Kalai, E. Kalai, Lehrer and Samet (2010).†)

In this paper we analyze bad reputation games from the perspective of self-referentiality in order to study the connection between the two ideas mentioned at the beginning. We show how the possibility of observing opponents' intentions restricts the bad reputation effect, and we identify conditions under which such restriction applies for both weak and strong sources of information about intentions.

In our model, myopic players play against a long-lived opponent whose life span is stochastic. We show that the possibility of fathoming other players' plan of strategies and the occasional renewal of the long-run player mitigate the bad reputation effect. That is, it is generally better to have myopic players with permanent uncertainty about types for the long-run player because this weakens how informative public histories full of bad signals are. Moreover, we are interested in showing how information about opponents' intentions may complement renewal. We characterize conditions on the self-referential game and on the relative likelihood of commitment types to assure that the bad reputation effect will not arise.

Bad reputation games exhibit distinctive features from the bulk of the literature starting with the classic works of Kreps and Wilson (1982) and Milgrom and Roberts (1982). We model bad reputation games borrowing the set-up developed by Ely, Fudenberg and Levine (2008) who give the first characterization of the limit between good and bad reputation identifying properties of this class of games. A long-lived opponent plays against a sequence of different myopic players. Participation of short-run

---

*Tools might be used for undercovering intentions as it is commonly seen in Poker games, players wear caps and sunglasses to masquerade a royal flush. Yet such accessories sometimes end up being evidence of a winning hand.

†They study commitment devices which work very similarly to code-of-conduct in two-player games, however, they consider very precise signals.

players takes place if friendly actions are likely to be played. Since long-run player's actions are imperfectly observed, friendly actions may generate bad signals that can be interpreted as evidence of unfriendly actions. In addition, there exist temptation actions which result in good signals more often but they may be unfriendly. A numerous amount of bad signals points to an unfriendly type, hence a patient normal long-run player eventually chooses temptation actions. In contrast to Ely, Fudenberg and Levine (2008) we focus on costly temptation actions, that is, it is costly to play actions that are more likely to generate good signals. While they find general conditions under which reputation is bad, their characterization does not include all possible commitment type priors. In contrast, we identify the whole set of priors where bad reputation is overcome.

Our main departure from their setting is that the long-run player might be replaced every period. Short-run players remain ignorant about the long-lived player's type since they can observe neither the renewal nor the type. For instance, during a valve operation surgeons may shift every other couple of hours but the patient only knows his primary care surgeon. The possibility of switching types has been previously studied in reputation models but not in the class of bad reputation games. Renewal has a dual effect. First, since the long-run player is likely to be gone by tomorrow he has less incentives to play friendly and harvests the fruits of reputation. Second short-run players are unaware whether a replacement has occurred, this implies that a very unlucky history of bad signals weighs less frequently in the updating of prior probabilities. The latter is prevalent in reputation games, as in Ekmekci, Gossner and Wilson (2012). In Mailath and Samuelson (2001), the long-run player does not lean on a favorable history because players are uncertain whether he might have been replaced by a bad type. In contrast, in our model, the possibility of being renewed partially ``cleans up" a history plenty of bad signals. The reason why renewal may restore the short-run players' beliefs about the long-run player's type is similar to that in Liu and Skrzypacz (2009), consider short-run players having bounded recall. (See also Liu (2011).) Most of the literature on switching types is concerned about equilibrium dynamics (see for example, Hölmstrom (1999), Phelan (2006) and Wiseman (2008)).

We first consider perfect information about opponents' intentions, and show that the precommitment friendly action outcome can be induced by a self-referential Nash equilibrium. In this self-referential equilibrium the long-lived player behaves friendly. We should stress that neither the level of patience nor the probability of being replaced of the long-lived player change any of the qualitative results. Intuitively, self-referential signals are so informative that they implicitly reveal the long-run player's type. Then, we can recover the good equilibrium in Ely and Välimäki's example if there is a bad commitment type.

When this source of information is weak, the bad reputation effect still arises but under more restrictive conditions. In this case, our main result is that self-referentiality strongly complements the probability of renewal of the long-run player. Adherence to the code-of-conduct relies on two critical forces: The long-run player is rewarded for playing friendly actions and is immediately punished for potentially playing a temptation action. When constructing the code-of-conduct in this proof we encounter two new difficulties in the study of self-referential games. One difficulty is that when there is a sequence of myopic players against a long-lived opponent, repetition does not complement self-referentiality as in the case of games with only long-run players. In addition, Block and Levine (2012) study a gift-giving game example with overlapping generations where inter-period connection facilitates the transmission of information.[3] Unlike that environment, here short-run players are not directly connected and therefore the negative externality generated when a myopic player decides not to participate in the game is still present.

[3] See Johnson, Levine and Pesendorfer (2001).

Intergenerational Solidarity within a Closing Pension Fund

Tim Boonen[1], Anja De Waegenaere[2] and Henk Norde[3]

Tilburg University
Netherlands
[1]*t.j.boonen@uvt.nl*
[2]*a.m.b.dewaegenaere@uvt.nl*
[3]*h.norde@uvt.nl*

Keywords*: Cooperative Bargaining, Cooperative Bargaining, Time-consistent Nash bargaining solution*

It is well-known that pension fund participants can benefit from intergenerational risk-sharing, but that different generations have different incentives. This paper models intergenerational solidarity in closing funded pension plans as a bargaining process between generations. We consider generations which each are represented by members of the pension fund's board. We model pension funds that differ in the degree of risk-sharing, as well as in the extent to which pension rights are guaranteed. The degree of risk-sharing is determined by the number of generations over which risk can be shared. For each type of pension fund, we determine the pension fund policy that follows from a bargaining process between the representatives of the different generations. We then compare the welfare effects of each type of pension fund.

In the presentation, we propose a stable pension plan design of a closing pension fund. Pension funds invest the capital accrued by the working participants and pay pension benefits to the retirees. Collective defined contribution and defined benefit pension plans inherently induce intergenerational solidarity. Intergenerational solidarity may be beneficial to risk-averse individuals because it allows for redistribution of risk between generations (see, e.g., Gollier, 2008; Cui et al, 2009). However, redistributions of pension contributions and of risk should be sufficiently attractive for all generations. These generations may have different, possibly conflicting, objectives due to, e.g., a longer investment horizon. Therefore, as argued by Molenaar et al. (2008), age differentiation in pension funds would be desirable. We consider a setting in which different generations have representatives in the pension fund's board. The pension fund strategy is determined as the outcome of a bargaining process between these representatives. As some generations might be overrepresented in the board, we allow for heterogeneous bargaining powers of the generations. We compare the welfare effects for the different generations in a number of pension plan designs that differ in the extent of intergenerational risk-sharing as well as in the extent to which pension rights are "guaranteed" (e.g., defined benefit versus collective defined contribution plans). In each case, the pension fund's board can affect the payoffs to the different generations via its choice of investment policy and payout policy.

Our modeling approach is similar in spirit to Gollier (2008), but it differs in terms of the objective. Gollier (2008) considers a hypothetical social planner who aims at maximizing the discounted sum of the expected utility over the generations. One drawback of using the objective of a social planner to determine pension plan policy is that it can yield outcomes that are not individually rational for some generations, i.e., some generations may be better off by using a closed account without intergenerational risk-sharing. It also does not time into account that generations might differ in size and life expectancy.

In the presentation, we propose a pension benefit strategy based on a time-consistent Nash bargaining solution. This is a solution inspired by regular Nash bargaining solutions (Nash, 1950) in a dynamic context. The Nash bargaining solution is characterized by Nash (1950) and provides insight in the bargaining process of the generations. Time consistency implies that generations take into account that the pension contract will be renegotiated every period. In every period, the negotiations lead to a Nash bargaining solution. The disagreement point is time-varying and determined as in cooperative bankruptcy games. As leaving the pension fund is generally not optional in pension funds, a generation can only claim a minimal amount of the assets of the fund. In this outside option, generations can invest these assets in a closed account. This outside option is generally not Pareto efficient.

Finally, we quantify the welfare effects in a realistic situation. We show that the pension benefit strategy differs quite heavily from the strategy according to a social planner as in Gollier (2008). Particularly, the oldest generations strongly benefit in the time-consistent Nash bargaining solution, which is due to higher bargaining power and a lower life expectancy.

References

[1] Cui, J., F. de Jong, and E. Ponds (2009). Intergenerational risk sharing within funded pension schemes. Journal of Pension Economics and Finance 10, 1–29.

[2] Gollier, C. (2008). Intergenerational risk-sharing and risk-taking of a pension fund. Journal of Public Economics 92, 1463–1485.

[3] Molenaar, R., R. Munsters, and E. Ponds (2008). Towards age-differentiation in funded collective pensions. Work in progress.

[4] Nash, J. F. (1950). The bargaining problem. Econometrica 18, 155–162

Investments in R&D under Trade Liberalization: Monopolistic Competition Model[*]

Igor Bykadorov[1], Sergey Kokovin[2] and Evgeny Zhelobodko[3]

[1,2,3]Sobolev Institute of Mathematics SB RAS
Novosibirsk
[1,2,3]NRU Novosibirsk State University
Novosibirsk
[1,2,3]NRU Higher School of Economics
St. Petersburg

Motivation, topic. Empirics on international trade shows that: (i) there are noticeable cross-countries differences in productivity and quality; (ii) firms operating in larger markets have lower markups (Syverson-2007); (iii) firms are larger in larger markets (Campbell-Hopenhayn-2005); (iv) larger economies export higher volumes of each good, a wider set of goods, and higher-quality goods (Hummels-Klenow-2005). Various explanations exist in the literature. We apply “monopolistic competition” approach. It suggests that a larger market motivates more R&D investment by a firm because of economies of scale.

Background literature and our goal. (i) Monopolistically-Competitive (MC) models were introduced by Dixit & Stiglitz (1977), for trade Krugman (1979). (ii) MC model with specific utility was recently generalized to any utilities with variable elasticity: Zhelodobko, Kokovin, Parenti & Thisse (2012); the firm size was found to positively respond to the market size under all increasingly-elastic demands. (iii) Oligopolistic choice of technology in quasi-linear partial-equilibrium setting was studied by Vives (2008): firm's R&D investment was found to increase with the market size, while the number of varieties can increase or decrease. Our goal is to combine the choice of technology like in Vives - with general monopolistic competition like in Zhelodobko, Kokovin, Parenti & Thisse. The reason is that variable elasticity is crucial for the R&D effects, while general equilibrium is more relevant for trade than partial equilibrium. Our

[*] We gratefully acknowledge the financial support from the Russian government grant No 11.G34.31.0059, Russian Foundation for Basic Research grant No 12-06-00174-a, and an individual grant 11-5231 from the Economics Education and Research Consortium (EERC, with funds provided by the Eurasia Foundation, the US Agency for International Development, The World Bank, the Global Development Network and the Government of Sweden).

first results in Bykadorov, Kokovin, Zhelobodko (2012) relate to a closed economy, now they are extended to trade, that required a new model.

Monopolistic Competition trade model

Assumptions are typical: (i) Increasing returns to scale in (identical) firms; marginal costs $c(f)$ depend on investment cost f. (ii) Each firm i produces one "variety" as a price-maker, but its demand $x_i\left(p_i, p_j, ...\right)$ is influenced by other varieties. (ii) Each demand function results from additive utility function $U = \int_{i \le N} u\left(x_i\right) di.$ (iv) Number of firms is big enough to ignore firm's influence on the whole industry/economy and free entry drives all profits to zero. (v) Labor supply/demand in each country is balanced, trade is balanced.

Goods and consumers. There are N^H firms (=varieties) in Home country, N^F firms (=varieties) in Foreign country, L_H identical consumers in Home country, L_F identical consumers in Foreign country. World population is $L = L_H + L_F$, countries' shares are $s = L_H / L, \quad \left(1 - s\right) = L_F / L.$ Consumer's elementary utility function is $u(x)$, and $x(i) \equiv x_i \quad x(\cdot) : [0, N^H + N^F] \to R_+$ denotes the demand for i-th variety. Each consumer sells a unit of labor (numararie) and chooses an (infinite-dimensional) consumption vector $x(\cdot)$ in utility-maximization as follows

in country $K \in \{H, F\}$: $\begin{cases} \int_0^{N^H} u\left(x_i^{HK}\right) di + \int_0^{N^F} u\left(x_i^{FK}\right) di \quad \to \max\limits_{x^{HK}, x^{FK} \ge 0} \\ \int_0^{N^H} p_i^{HK} x_i^{HK} di + \int_0^{N^F} p_i^{FK} x_i^{FK} di \le w^K, \end{cases}$

where p_i^{MK} is the price of variety i produced in country $M \in \{H, F\}$ and consumed in K, while w^K are wages normalized as $w^H = w$ and $w^F = 1.$

Using a Lagrange multiplier λ_K in country K and FOC, the inverse demand for i-th variety is

$$p_i^{*MK}\left(x_i^{MK}, \lambda_K\right) = \frac{u'\left(x_i^{MK}\right)}{\lambda_K}.$$

Producers: Any i-th firm in country $K \in \{H, F\}$ knows both inverse-demand functions $p_i^{KK}\left(x_i^{KK}\right) = p_i^{*KK}\left(x_i^{KK}, \lambda_K\right)$ and $p_i^{KM}\left(x_i^{KM}\right) = p_i^{*KM}\left(x_i^{KM}, \lambda_M\right)$, and chooses its output $Q_i^K = L_K x_i^{KK} + \tau L_M x_i^{KM}$ where $K \neq M$ and $\tau > 1$ is the iceberg trade-cost coefficient. Total cost, measured in numararie is $C\left(Q_i^K\right) = w^K \cdot \left(c\left(f_i^K\right) Q_i^K + f_i^K\right)$, where marginal cost $c\left(\cdot\right)$ depends on investment f_i^K, and decreases: $c' < 0$. A firm chooses $\left(x_i, f_i^K\right)$ to maximize its profit:

$$\pi_i^K = p_i^{KK}\left(x_i^{KK}\right) L_K x_i^{KK} + p_i^{KM}\left(x_i^{KM}\right) L_M x_i^{KM} - C\left(Q_i^K\right), \quad K \neq M.$$

FOC w.r.t. f_i^K yield $c'\left(f^K\right) Q^K = -1$ (we drop index i further because of symmetry).

Equilibrium is a bundle $\left(x^{HH}, x^{FH}, x^{HF}, x^{FF}, f^H, f^F, N^H, N^F, w\right)$ satisfying all FOC, SOC and budgets, free entry and labor balances.

Main Results. We find that in any situations: (i) Changing trade cost has the opposite impacts on number of firms (diversity) and productivity: when diversity increases productivity goes down. (ii) Under "Home-Market Effect" (disproportionally many firms in the bigger country), the country size also yields such opposite impacts: where diversity is bigger – productivity is smaller.

Other effects like market integration/globalization (decreasing trade cost τ) – depend on the Arrow-Pratt measure of concavity defined for any function g: $r_g\left(z\right) = -\dfrac{z g''\left(z\right)}{g'\left(z\right)}$.

All markets are classified into those with increasingly-elastic demands (r_u' >0) or decreasingly-elastic demands. Namely, in special case of small cost $\tau \approx 1, s > 0.5$ we prove that:

(i) Elasticities of R&D investments and outputs w.r.t. τ have the same sign as r_u'. Moreover, investments and outputs are higher in the bigger country: $f^H > f^F, Q^H > Q^F \Leftrightarrow r_u'\left(x\right) > 0$.

(ii) Elasticities of masses of firms w.r.t. τ has the opposite sign to r_u'. Moreover, the mass of firms is smaller in the bigger country: $N^H < N^F \Leftrightarrow r_u'\left(x\right) > 0$,

(iii) Elasticity of the bigger country's wage w w.r.t. τ is positive. Thereby, the bigger country has higher wages.

References

[1]. Bykadorov I., Kokovin S., Zhelobodko E. (2012) "Investments in productivity and quality under trade liberalization: monopolistic competition model". – Contributions to game theory and management. Vol. V. Collected papers presented on the Fifth International Conference Game Theory and Management (GTM2011). – SPb.: Graduate School of Management SpbU. – P. 61-72.

[2]. Campbell J.R., Hopenhayn H.A. (2005) "Market size matters", Journal of Industrial Economics, V.LIII, p.1-25.

[3]. Dixit A., Stiglitz J. (1977) "Monopolistic Competition and Optimum Product Diversity", American Economic Review, V. 67, N. 3, p. 297-308.

[4]. Hummels D., Klenow P.T. (2005) "The Variety and Quality of a Nation's Exports", The American Economic Review, V.95, N.3, p. 704-723.

[5]. Krugman P.R. (1979) "Increasing returns, monopolistic competition, and international trade", Journal of International Economics, V.9, p. 469-479.

[6]. Syverson C. (2007) "Prices, spatial competition, and heterogeneous producers: an empirical test", Journal of Industrial Economics, V. LV, p. 197-222.

[7]. Vives X. (2008) "Innovation and competitive pressure", The Journal of Industrial Economics, V. 56, I. 3, p. 419–469.

[8]. Zhelobodko E., Kokovin S., Parenti M., Thisse J.-F. (2012) "Monopolistic Competition: Beyond the Constant Elasticity of Substitution", Econometrica, Vol. 80, No. 6. – P. 2765–2784.

Rewarding Idleness*

Andrea Canidio[1] **and Thomas Gall**[2]

[1]Central European University
Hungary
acanidio@gmail.com
[2]University of Bonn
Germany
tgall@uni-bonn.de

Abstract *Market wages reflect expected productivity by using signals of past performance and past experience. These signals are generated at least partially on the job and create incentives for agents to choose high-profile and highly visible tasks. If agents have private information about the profitability of different tasks, firms may wish to prevent over-investment in visible tasks by increasing their opportunity costs. Firms can do so, for instance, by using employee perks. Heterogeneity in employee types induces substantial diversity in organizational and contractual choices, particularly regarding the extent to which conspicuous activities are tolerated or encouraged, the use of employee perks, and contingent wages.*
Keywords*: Multitask Agency, Employee Perks, Career Concerns, Organizational Heterogeneity*

Many studies have documented the existence of within-industry heterogeneity in organizational choice, corporate infrastructure, and contractual choice [see, e.g.,][for a survey]Gibbons2010. This heterogeneity is particularly visible in attitudes towards idleness at work as embodied in corporate investments in perks. Employees at Google have at their disposal a wide variety of on site services and sports facilities, such as tennis courts, a climbing wall, free catering in high-profile restaurants and cafeterias, and various entertainment facilities (such as table football). Additionally, they are allowed to use one workday per week for personal projects.†

However, even within the same industry the provision of employee perks is by no means homogeneous. Google's founders state:

*We are grateful for comments from Andy Newman and the seminar participants at BU, CEU, Université de Lausanne, and Stony Brook Game Theory Meeting 2011. All remaining errors are, of course, our own.

†Microsoft and Yahoo! also provide access to substantial perks, including free cafeterias, a game room, massage services, or lake access. Blizzard Entertainment is supposed to outfit its employees with digital equipment for its online game World of Warcraft.

We provide many unusual benefits for our employees, [...] We believe it is easy to be penny wise and pound foolish with respect to benefits that can save employees considerable time and improve their health and productivity.[*]

In contrast, Chad Little, a former Apple employee, states the following:

[At Apple] The cafe costs, [...] Every floor has a vending machine, which also costs [...] The gym also isn't free, [...] I recall one person asked Steve why these benefits were so low, and the main response was 'it's my job to make your stock go up so you can afford these things.'[†]

This paper argues that workers' task choice is distorted by their career concerns, and firms' contractual and organizational choices react to this distortion. If employees are heterogeneous in their career concerns, companies that are very similar in technology and employee characteristics may nevertheless choose substantially different organizations, giving rise to within-industry organizational heterogeneity.

We focus specifically on the determinants of firms investments in employee perks rewarding idleness. Idleness can be desirable for a principal if, because of career concerns, employees have an incentive to over-invest in complex, visible tasks that generate signals about the agent's ability. This issue is of particular concern in creative professions, if the agent has private information about the profitability of different tasks, that is, the agent has expert knowledge.

To balance their employees' bias toward visible tasks, firms may distort their organizational investments toward employee perks that are complementary to idleness. That is, employee perks that seem to encourage idleness are actually meant do so. However, for agents who derive high value from generating signals on the job, the reward for idleness necessary to balance incentives may be very costly. In this case the optimal organizational form looks substantially different, and encourages agents to work on conspicuous projects while discouraging idleness, though it occasionally results in inefficient task choices. Therefore small differences in the strength of employees' career concerns can generate substantial variety in firms' organizational choices and in the extent to which they tolerate and reward idleness.

PerlowPorter2009 provide empirical support for the relevance of career concerns on employees' tasks choice. They report on a four-years experiment at several

---

[*] Larry Page and Sergey Brin, *Letter from the Founders: "An Owner's Manual" for Google's Shareholders*, accessed from http://investor.google.com/corporate/2004/ipo-founders-letter.html

[†] Retrieved from http://www.quora.com/Apple-Inc-2/What-is-the-internal-culture-like-at-Apple

offices of the Boston Consulting Group, where ``people believe that a 24/7 work ethic is essential for getting ahead, so they work 60-plus hours a week and are slaves of their BlackBerry."* The treatment consisted of forcing people to take time off. Each member of the treatment teams had to leave the office without access to email or BlackBerry for a period of either one full day or one evening per week, depending on the version of the treatment. The paper describes at length the strong resistance toward the project from the consultants, who would have preferred to continue working. The effect of the treatment was that participants reported ``more open communication, increased learning and development, and a better product delivered to the client.† That is, incentives to generate signals appear to have determined working behavior and task choice, and may affect output.

This reasoning applies to areas other than corporate organizational choice. For example, some health care plans in the United States explicitly reward physicians for inactivity by way of bonuses, fee withholds, and expanded capitation [see][]orentlicher1996paying.‡ In addition to contracts that reward inactivity, other forms (e.g., capitation or fee-for-service) are also widely used, generating substantial contractual heterogeneity. The argument also extends to cases where agents, instead of remaining idle, may pursue other productive tasks that do not generate any signal about the agents' abilities. Interpreting teaching as a routine task, academia seems a case in point; universities have substantial organizational heterogeneity with respect to incentives for teaching and research.

To formalize our argument we use a principal-agent model. The agents' productivities are unknown, but their expected values are publicly observable. An agent chooses to perform one of two tasks. One task is routine and its outcome is independent of the agent's productivity; it may be interpreted as idleness. The other task is complex, its outcome is uncertain, and its probability of success depends on the agent's productivity. Its expected return is known only to the agent and may exceed or fall short of the profit of the routine task. Hence, the visible task can be interpreted as starting a

---

*PerlowPorter2009 page 1.

†Ibid. page 4.

‡Physicians may be motivated by their reputation among patients to over-prescribe treatments in the hope of increasing future revenue, a form of career concern. Bonuses, fee withholds, and expanded capitation work roughly as follows: if the total cost of treatments prescribed by a physician falls short of the prespecified amount, the physician receives a bonus payment.

new project, initiating a merger, or launching a marketing campaign. The task is visible: its outcome is publicly observable and generates a signal about the agent's productivity.

Principals invest in two types of corporate infrastructure: productive perks (e.g., large office space or a powerful computer) that are complementary to the visible task, and employee perks (e.g., a swimming pool or a free cafeteria) that are complementary to idleness. Labor contracts need to respect limited liability and fall into one of two regimes: flexible contracts induce the agent to choose a task conditional on the tasks' expected profitabilities, while rigid contracts induce the agent to choose a specific task independently of its profitability.

Because an agent lives for two periods, choosing the visible task when young affects the agent's expected productivity and payoff when old. That is, agents have career concerns, which are stronger the less informative the prior belief about their productivity is. In the labor market equilibrium the type of contract offered and the level and composition of corporate infrastructure depend on the market value and the strength of the agent's career concerns. Higher market value affects organizational choice because being able to generate signals on the job is part of an agent's compensation. Stronger career concerns increase the cost of satisfying incentive compatibility in a flexible contract. Hence, all old agents (who have no career concerns) obtain flexible contracts that maximize expected output. For young agents (who have career concerns), satisfying incentive compatibility may require the composition of corporate investments to be distorted. Given a young and an old agents of equal productivity who both obtain flexible contracts, employee perks are higher and productive perks are lower for the young agent than for the old agent.

Career concerns generate heterogeneity in contractual and organizational choice for the young. Young agents who have high market value, high expected productivity, and thus relatively low career concerns (“proven talents”) receive flexible contracts. These contracts implement the profit-maximizing task choice and efficient investment. Young agents of intermediate expected productivity (”high potentials”) have intermediate market value and derive high value from generating a signal. For these agents, a flexible contract that rewards idleness to balance incentives is very costly to implement. They receive rigid contracts that implement the visible task regardless of its return. This regime corresponds to organizations with strong emphasis on long working hours, where idleness is discouraged. Finally, agents with low expected productivity and low market value but strong career concerns (”hidden gems”) receive flexible contracts.

Low market value and limited liability cause corporate investment to be distorted downwards, particularly for productive perks. The value of generating a signal may be high enough for these agents, such that using rigid instead of flexible contracts increases aggregate surplus. However, limited liability makes it impossible to compensate the principal for the loss in expected profit caused by switching to a rigid contract. Because the different regimes are determined by cut-off productivity levels, corporate investment in perks is discontinuous in employees' expected productivity.

References

[1] Gibbons, R.: 2010, `Inside organizations: Pricing, Politics, and Path Dependence'. Annual Review of Economics 2, 337--365.

[2] Orentlicher, D.: 1996, `Paying physicians more to do less: financial incentives to limit care'. University of Richmond Law Review 30.

[3] Perlow, L. A. and J. L. Porter: 2009, `Making Time Off Predictable and Required'. Harvard Business Review 87(10), 102--109.

ВЕСТНИК САНКТ–ПЕТЕРБУРГСКОГО УНИВЕРСИТЕТА

Серия 8
МЕНЕДЖМЕНТ

Ответственный редактор серии:
Ю.Е. Благов

Издательство Высшей школы менеджмента Санкт-Петербургского университета

A Coopetitive Game Theory Approach to Asymmetric R&D Alliances

David Carfi[1] **and Fabrizio Lanzafame**[2]

[1]*Mathematics Department*
University of California Riverside, USA
davidcarfi71@yahoo.it

[2]*SEAM Department*
University of Messina, Italy
fabrizio.lanzafame@virgilio.it

Abstract. *The paper proposes a mathematical model of coopetitive game that analyzes general asymmetric R&D alliances. The coopetitive point of view, which considers both collaboration and competition together, allows to analyze the functioning of alliances that arise between small and large firms. Starting from the economic models developed in managerial doctrine and from the model of coopetitive game introduced by David Carfi, we adopt an analysis paying attention to some of the most debated questions and some of the topics not yet covered in the literature. A mathematical model of coopetitive game is particularly suitable for exploring a complex type of asymmetric R&D alliances. We propose a formal coopetitive approach, where the coopetitive variable of the model is a real variable; a cooperative effort is required even though partners are potentially competitors in the marketplace and shape the common payoff space that they create. To maximize profit we have suggested; first of all, a complete Pareto analysis (David Carfi), secondly - to share conveniently and fairly the utilities - we propose a Kalai Smorodinsky solution of the bargaining decision problem, in which the decisional constraint is the Pareto boundary of maximum collective utility.*
Keywords: *R&D alliances, coopetitive games, Kalai-Smorodinsky solutions, win-win solutions*

Introduction

We propose a mathematical model to determine possible suitable behaviors (actions) of the partners involved during their strategic interactions, from both the non-cooperative and cooperative points of view. More in details, we apply the complete analysis of a coopetitive differentiable game (Carfi, 2009). To do so, we refer to a specific case: the R&D strategies of a well-established firm.

Each firm competes in the market with all the other firms, but it may also choose to collaborate with the other firms on the market by adopting an innovative technology.

The idea of coopetitive game is already used, in a mostly intuitive and non-formalized way, in Strategic Management Studies (see for example Brandenburgher and Nalebuff).

The idea.A coopetitive game is a game in which two or more players (participants) can interact cooperatively and non-cooperatively at the same time. Even Brandenburger and Nalebuff, creators of coopetition, did not dene, precisely, a quantitative way to implement coopetition in the Game Theory context.

The problem to implement the notion of coopetition in Game Theory is summarized in the following question:

how do, in normal form games, cooperative and non-cooperative interactions can live together simultaneously, in a Brandenburger-Nalebuff sense?

In order to explain the above question, consider a classic two-player normal-form gain game $G=(f,>)$ – such a game is a pair in which f is a vector valued function defined on a Cartesian product $E * F$ with values in the Euclidean plane and $>$ is the natural strict sup-order of the Euclidean plane itself (the sup-order is indicating that the game, with payoff function f, is a gain game and not a loss game). Let E and F be the strategy sets of the two players in the game G. The two players can choose the respective strategies x in E and y in F cooperatively (exchanging information and making binding agreements); not-cooperatively (not exchanging information or exchanging information but without possibility to make binding agreements).

The above two behavioral ways are mutually exclusive, at least in normal-form games: the two ways cannot be adopted simultaneously in the model of normal-form game (without using convex probability mixtures, but this is not the way suggested by Brandenburger and Nalebuff in their approach); there is no room, in the classic normal form game model, for a simultaneous (non-probabilistic) employment of the two behavioral extremes cooperation and non-cooperation.

Towards a possible solution. David Carfi has proposed a manner to pass this impasse, according to the idea of coopetition in the sense of Brandenburger and Nalebuff. In a Carfi's coopetitive game model, the players of the game have their respective strategy-sets (in which they can choose cooperatively or not cooperatively); there is a common strategy set C containing other strategies (possibly of different type with respect to those in the respective classic strategy sets) that must be chosen cooperatively; the strategy set C can also be structured as a Cartesian product (similarly to the strategy space of normal form games), but in any case the strategies belonging to this new set C must be chosen cooperatively.

The Model

In this model we have three players that interact together.

We assume that:

- our first player is a Large Firm (LF) that, in order to develop new products, decides to cooperate with a Small Firm (the second player), operating in the same sector;
- in order to achieve cooperation the first and the second players constitute a Research Joint Venture (RJV) which is our third player;
- the Large Firm determines the amount x of production to buy from the RJV; its revenues are given by the difference between the sale price and the purchase price of x quantity of production;
- the first player pays the sunk costs C' of the RJV and funding research of small firm ($a''y$);
- revenues for the second player are equal to $p''z$, where p'' is the product price and z is the quantity of production. Costs of the second player are represented by investment for research and they are equal to y.
- the Research Joint Venture revenues are calculated as:

$$p'x + p(z - x)$$

where $p'x$ is profit from selling x at price p' and

$$p(z - x)$$

represents profit from selling $z - x$ at price $p > p'$. Lastly, the costs faced by the RJV are equal to

$$cz - p''z$$

where cz is the cost to produce the quantity z and $p''z$ is the payment received from the small firm for the product z.

- C' is a positive constant for the RJV because the sunk costs are paid by the first player.

To clarify the model we resume the principal features of our game:

1) the real number x is the production decided by the first player to buy from research joint venture (RJV), and it is assumed in [0,1];

2) the real number y represents money for research invested by the second player and it is assumed in [0,1];

3) the real number z is the production decided by both players I and II (together) of the RJV, with constraint [0,4];

4) the payoff function of the first player is:

$$f_1(x,y,z) = (p - p')x - C' - a''y$$

where:

- $(p - p')x$ is profit from selling x at price p in the market and buying x at price p' from RJV;
- C' represents the sunk costs invested by the first player to RJV;
- $a''y$ is extra-payment for research y to the second player ($a'' > 1$);

5) the payoff function of the second player is:

$$f_2(x,y,z) = p''z + a''y - y$$

where:

- $p''z$ is the payment received by RJV for the product z;
- $a''y$ is extra-payment for research y ($a'' > 1$) received by the first player (LF);

6) the payoff function of the third player is:

$$f_3(x,y,z) = p'x + p(z - x) - cz - p''z + C'$$

where:

- $p'x$ is the profit from selling x at price p';
- $p(z - x)$ represents the profit from selling $z - x$ at price $p > p'$;
- cz is the cost to produce a quantity z;
- $p''z$ is the payment received by RJV for the product z, where $p > c + p''$.

The Effectiveness of Altruistic Lobbying: a Model Study

Pavel Chebotarev, Zoya Lezina, Anton Loginov and Yana Tsodikova

Institute of Control Sciences of the Russian Academy of Sciences, Moscow Russia

Keywords: *Lobbying, Altruism, Egoism, Stochastic Environment, Voting, Majority, Social Dynamics, Social Attitudes*

1.Introduction

Altruistic lobbying is lobbying in the public interest or in the interest of the least protected part of the society. This "or" is the point: it is necessary to bear in mind that an altruist has a wide range of strategies, from behaving in the interest of the society as a whole to the support of the most disadvantaged ones. How can we compare the effectiveness of such strategies? The second question is: "Given a strategy, is it possible to estimate the optimal number of participants choosing it?" Finally, do the answers to these questions depend on the level of well-being in the society? For example, can we say that the poorer the society, the more important is to focus on the support of the poorest? We answer these questions within the framework of the model of social dynamics determined by voting in a stochastic environment [1–5].

Consider a society consisting of n members (agents); each of them is characterized by a real scalar interpreted as the utility (or capital) level; a negative value is naturally interpreted as a debt. Let the initial distribution of capital be given. A proposal of the environment is a vector of increments of individual capitals. Let these increments be random variables; in the simplest case, we assume that they are independent and identically distributed with mean μ and standard deviation σ. The environment generates a series of proposals each of which is put to the vote. Every agent takes part in the voting and has one vote. With some voting procedure, the profile of votes is converted into a collective decision: the proposal is approved or rejected. The approved proposals are implemented: the participants get the capital increments specified in the proposal. Considering a large series of votes, one can explore the dynamics of the vector of agents' capital: in different environments, with different social attitudes, and with various voting procedures. An interesting version of the model is that with ruining: the agents whose capital values become negative are ruined.

2. A study of altruistic lobbying

Consider a society consisting of two categories of agents: selfish and altruistic ones. An egoist votes for a proposal if and only if this proposal increases his/her capital.

As noted in the introduction, the altruists have a wide range of strategies. Consider the following type of strategy. We order all agents by the increase of the current capital value. Since the capital increments are real-valued random variables, we can assume that all capitals are different. Let an integer $n_0 \leq n$ be fixed. For the current proposal, one calculates the total capital increment of n_0 poorest agents. An altruist supports the proposal if and only if this total is positive, no matter what happens to his/her own capital. If $n_0 = n$, then the altruist supports those proposals that enrich the society as a whole. When n_0 is smaller, then she supports more or less numerous "lower" stratum of the society.

Let K be the same initial capital of all the agents. Assume that the di. As a criterion of the effectiveness of this strategy we consider the relative number of participants that "survive" until the end of the game. Another important parameter is the number (percentage) of altruists in the society.

We study the following question: "How the effectiveness of altruists' strategy depends on their proportion and the threshold n_0 at various parameters μ and σ?" In other words, "What is the optimal proportion of altruists in the society and what is their optimal strategy in a favorable ($\mu > 0$), neutral ($\mu = 0$), and unfavorable ($\mu < 0$) environment?"

Some simulation results are as follows. Let the society consist of $n = 100$ agents; the number of altruists, m, varies. The experiments are conducted at $\sigma = 12$; game lasts 500 steps. On the two horizontal axes in Fig.1, we have the number of altruists, m, and their strategy parameter n_0; along the vertical axis, the relative number of agents at the end of the game. In the diagrams, μ and the initial capital K of the agents are also shown. Some results are as follows.

1. In a neutral environment ($\mu = 0$) and with the support of the poorest agents (n_0 is small), the number of altruists should be small, or they waste their influence,

resulting in ruining players outside the "support screen". At a high n_0, we have saturation with the increase of the number of altruists. With $K = 40$, the optimum is reached at $n_0 = 50$ and $m > 40$ (at high values of m, saturation holds). In other words, altruists should support the lower half of the community. With the extension of the support screen, the effectiveness of the altruistic strategy declines.

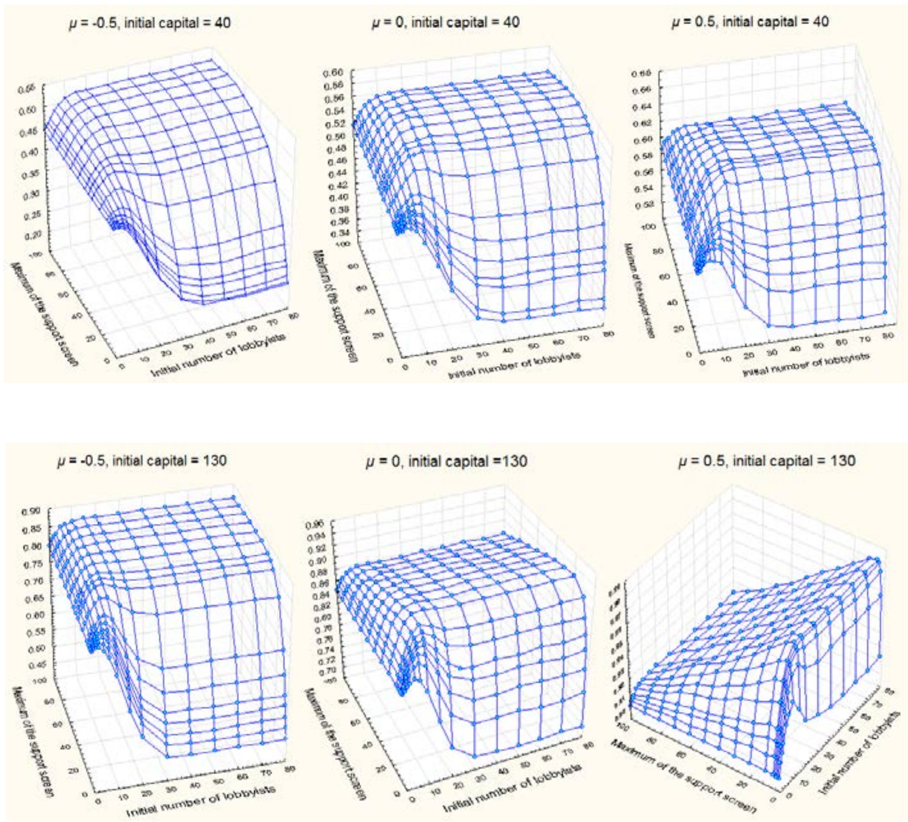


Fig. 1.

2. In an unfavorable environment ($\mu = -0.5$) and a high initial capital ($K = 130$), the picture is very similar. The only significant difference is that there is no pronounced effect of reducing the effectiveness of the strategy with the increase of the right border of the support screen. A maximum is reached at $n_0 = 30$ and $m = 10$, but it does not change significantly with an increase of these parameters.

3. In an unfavorable environment ($\mu = -0.5$) and a lower initial capital ($K = 40$), the maximum is reached at $n_0 = 80$ and $m = 60$. This is a very interesting conclusion: in a really harsh environment, we have to support everyone!

4. Finally, in a supportive environment with a high initial capital ($\mu = 0.5$, $K = 130$), the picture changes dramatically: we should support a thin layer of the poorest; the others will "resurface themselves".

The main conclusion is simple: in rich and affluent societies, support of the poorest is effective. The poorer is the society, the more "wasteful" is the support of the poorest. If you focus on this support, then the zone of trouble grows. A more detailed analysis of the results reveals some more subtle phenomena as well

References

[1] V.I. Borzenko, Z.M. Lezina, A.K. Loginov, Ya.Yu. Tsodikova, and P.Yu. Chebotarev. Strategies of voting in stochastic environment: Egoism and collectivism // Automation and Remote Control, 2006, V. 67. No. 2. P. 311–328.

[2] P. Yu. Chebotarev. Analytical expression of the expected values of capital at voting in the stochastic environment // Automation and Remote Control, 2006. V. 67. No. 3. P. 480–492.

[3] P.Yu. Chebotarev, A.K. Loginov, Ya.Yu. Tsodikova, Z.M. Lezina, V.I. Borzenko. Snowball of cooperation and snowball communism // Proceedings of the Fourth International Conference on Control Sciences. Moscow, 2009. P. 687–699 (in Russian).

[4] P. Yu. Chebotarev, A. K. Loginov, Ya. Yu. Tsodikova, Z. M. Lezina, V. I. Borzenko. Analysis of collectivism and egoism phenomena within the context of social welfare // Automation and Remote Control, 2010. V. 71. No. 6. P. 1196–1207.

[5] P.Yu. Chebotarev, A.K. Loginov, Ya.Yu. Tsodikova, Z.M. Lezina, V.I. Borzenko. Voting in a stochastic environment: The case of two groups // Automation and Remote Control, 2011. V. 72. No. 7. P. 1537–1547.

A Weighted Deegan-Packel Social Capital Index

Michela Chessa[1] **and Anna Khmelnitskaya**[2]

[1]*University of Milan, Italy*
chessa.michela@gmail.com

[2]*Saint-Petersburg State University, Russia*
a.khmelnitskaya@math.utwente.nl

Keywords: *Power Indices, Social Capital Index, Deegan-Packel Index, Shapley Value, Myerson Value*

After Parliament elections the situation when different parties that could obtain a positive number of sits have to decide which coalitions to join to create a majority is quite standard on the political podium. In real life it happens quite often that different majorities are feasible and several parties may choose among a number of alternatives. In such cases it is important to have a tool that may allow the parties to evaluate correctly their actual position under different possibilities and, therefore, to make the better decision concerning their own choice. On the other hand the same tool may be used by an external agent who wants to predict which coalition is more likely to form the majority. This problem is usually modeled from the game theoretical point of view as a simple cooperative game (often a weighted majority game is used for this purpose). A solution to such a game is called a power index. Several power indices are proposed in the literature in order to take into account different features of possible situations; for instance, indices emphasizing the importance of the ordering in the majority formation process (Shapley and Shubik, 1954), the possibility to form different majorities (Banzhaf, 1965; Coleman, 1971), the role played by parties in the majorities with minimal number of agents (Deegan and Packel, 1978; Holler, 1982).

However, for creation of the majority not only the ability of parties to create winning coalitions is important. Also bilateral communication abilities of the parties, or in other words their social network relations, are of great importance. So, the social capital that highlights the importance of social network relations in social affairs needs to be evaluated. Assuming that interests of individuals (players) and their coalitions are represented by means of a cooperative game with transferable utility and players' personal bilateral relations (social network) are introduced via an undirected communication graph, González-Arnagüena, Khmelnitskaya, Manuel and del Pozo (2011, yet unpublished) introduced a numerical measure of an individual social capital as an excess of the player's payoff given by the Myerson value (Myerson, 1977) over that given by the Shapley value (Shapley, 1953). This difference provides a tool for revealing the influence of the player's social network relations to the outcome of the game.

Besides, the evaluation of individual social capital of political parties also needs to involve into account the asymmetry of players represented via their personal weights. Simple example of a hypothetical parliament consisting of 99 members and composed by three parties that have correspondingly 49, 49 and 1 seats clearly demonstrates the importance of the inclusion of individual weights into the determination of the individual social capital. Indeed, while all these tree parties play equal roles in the creation of winning coalitions (any two-party coalition in this example is winning) the social evaluation and social respect of these parties is much different. However, the replacement of the Shapley and Myerson values in the definition of the social capital index by their weighted analogs is senseless, first, because the weighted Shapley value is not monotonic with respect to players' weights, and second, because of the strict increase of computational complexity.

In this paper we propose to evaluate the individual social capital through the difference of the restricted weighted Deegan-Packel index over the weighted Deegan-Packel index. Using the similar ideas to that staying behind the definitions of the weighted Shapley value and Deegan-Packel index, we define the weighted Deegan-Packel index of a player as a sum of the player's power shares in all minimal winning coalitions the player belongs to while the total power is equally shared among all minimal winning coalitions and within each minimal winning coalition it is distributed proportionally to players' weights. Taking into account also the definition of the Myerson value, we define the restricted weighted Deegan-Packel similarly, but now via not all minimal winning coalitions but only via feasible ones. The new social capital index includes already information concerning the parties' ability to create winning coalitions, the communication graph structure and also the individual weights. Moreover, some preliminary computations allow us to observe that the new social capital index is possible to compute for essentially bigger communities than the old one.

Central Asian Gas in Eurasian Power Game

Onur Cobanli

Humboldt-Universitat zu Berlin Berlin
onur.cobanli@hu-berlin.de

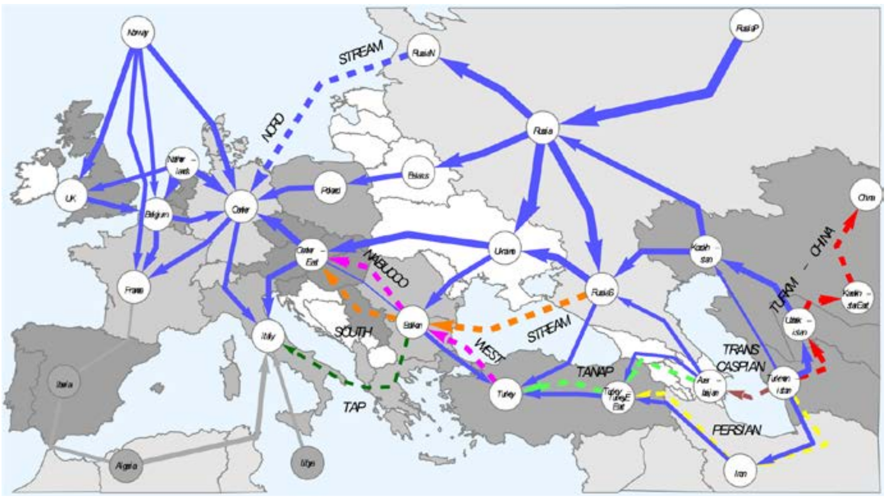
Keywords: *Bargaining Power, Network, Trade Links, Natural Gas, Caspian Sea, China*

The dissolution of Soviet Union in 1991 bore three new sovereign states in Central Asia: Kazakhstan, Turkmenistan and Uzbekistan. They host 27.8 tcm of proven conventional natural gas reserves, which is 13.3% of the world total. However, being landlocked pipelines carrying Central Asian gas to distant markets have to cross multiple countries with different strategic interests. By virtue of the Soviet gas pipeline network the Central Asian countries rely on Russia to transport their gas westwards to European markets and Turkey. In the beginning of nineties the USA initiated the offshore Trans Caspian pipeline through Caspian Sea in order to mitigate the newly sovereign Central Asian countries' dependence on Russia and to diversify imports of Southeast Europe and Turkey. Following the American example, in the last two decades different stakeholders in the region broached several pipeline projects which compete for transit of Central Asian gas to the West. However, in 2007 the Central Asian countries endorsed the Turkmenistan-China pipeline heading to the East in order to access the rapidly growing Chinese market. In this paper I question the Central Asian countries' choice of gravitating eastwards to China instead of westwards to Europe and Turkey, and I study which pipeline project heading westwards benefit the Central Asian countries at most.

The Central Asian countries have a number of pipeline options to diversify their transport routes as well as export markets. While there is only the Turkmenistan- China pipeline to reach eastwards, three routes extend from Central Asia to the West: via Caspian Sea, via Iran, and via Russia (see the Figure 1). However, each route has its own peculiar obstacle. Towards the East in order to reach China Turkoman gas has to travel long distances through two transit countries: Uzbekistan and Kazakhstan, which are potential suppliers as well. Towards the West the route via Caspian Sea is blocked due to legal disputes over Caspian Sea's status and demarcation. The route via Russia strengthens Russia’s dominance on transport of Central Asian gas and exacerbates the Central Asian countries’ deadlock. The West’s protests and political uncertainties make investment in the route via Iran unlikely any time soon.

A pipeline can increase the Central Asian countries’ (bargaining) power in two ways: it intensifies demand competition for their gas by linking them to new consumer markets, and it enhances transit competition for their gas by introducing a new transport route to their export markets or increasing capacities of existing transport routes. Players can act strategically in order to shape the pipeline network according totheir benefit. By endorsing a pipeline project a player can increase demand and/or transit competition for particular supplies and thus, can forestall investment in alternative pipeline projects which lessen its power. Moreover, a player can transfer a part of its power to other players in order to prevent their investment in a pipeline which weakens its power or in order to convince them for investment in a pipeline which subsides their power. If a player gains from a pipeline project, it can use the option to invest in the project as a bargaining chip in negotiations of gas prices, tariffs etc. with other players in order to derive a larger share of surplus (power) from the cooperation.

Figure 1: The Network



Those pipelines under construction or planning, which we consider in detail are dashed: Nord Stream in Blue, South Stream in Orange, and Nabucco-West in Magenta, Trans Adriatic (TAP) in Dark Green, Trans Anatolian (TANAP) in Light Green, Trans Caspian in Pink, the Persian in Yellow and the Turkmenistan-China in Red. White circles represent regions where we have a major transit node, which is linked to local production, local customers and local LNG regasification plants if there is any (the nodes are not shown separately). Solid arrows represent the main pipelines as existing in 2009. Grey nodes and pipelines are taken into account for but not associated with a region in our analysis.

In order to analyze the impact of the pipeline projects on the power structure of the Eurasian gas trade, I modify the disaggregated quantitative model introduced in Hubert & Cobanli (2012) slightly. I shift the focus of the model from the EU towards Central Asia by introducing non-European consumer markets and China which were left out in the referred paper. The cooperative game among the players are represented in value function form which captures essential economic features of the Eurasian gas trade, especially the architecture of the pipeline network. The Shapley value, which is referred as the power of the player as well, allocates the surplus from cooperation within the players by taking interdependencies among the players into account. Since introduction of a new pipeline alters the pipeline network, the value function and thus, the Shapley value of the players change accordingly. The difference between a player's pre- and post-project Shapley values gives the pipeline's impact on its power.

My results confirm the Central Asian countries decision to uphold the Turkmenistan- China pipeline instead of the western pipeline projects aiming European and Turkish markets. However, for the main supplier in the region, Turkmenistan, gravitating to the West via the Caspian route is the most profitable option to diversify its transport routes and export markets. Surprisingly, carrying Central Asian gas further to European markets via the Nabucco-West pipeline or the TAP brings nothing to the Central Asian suppliers due to the number of transit countries on the route. If Azerbaijani gas is carried through the Southern Corridor to European markets, the leverage accruing to the European regions from additional Central Asian supplies is insufficient for an European investment in the Trans-Caspian pipeline. The South Stream pipeline benefits European consumers much more than the Southern Corridor, and contrary to European concerns, it cannot hinder investment in the route via Caspian Sea. Turkmenistan has enough spare production capacity to serve Western and Chinese demand simultaneously. Thus, there is no demand competition between the West and the East.

Power in Game Theory

Rodolfo Coelho Prates

Universidade Positivo
Brazil
raphacz@hotmail.com

Abstract *The main aim of this paper is to discuss power in game theory, in order to model the asymmetries of forces among the players of the game. The starting point is that games are strategic interactions between rational individuals in a social environment and the players don´t have equal forces. Game theory has increased hugely in the last number of years, including several new branches; however, the concept of power in game theory has not been explored to any great extent. Indeed, power is a broad concept that has no clear definition. This paper formalize a model taking account the asymmetric forces between players. It is given some examples, using some well-known games, to illustrate this relation involving power. As a result, this paper presents an approach in two scientific areas still distant, sociology and game theory.*
Keywords: *Game Theory, Power Force*

Some kinds of words or concepts don´t have a precisely meaning. Power is one of these words. It is said that the president of a nation has power, while the civilian people don´t. In other hand, in a democratic environment, the coalition of people is a source of power, because is exactly the coalition that will decide who will be the governor. The director of a company has power, while the works don´t have, but one of these works could be a leader of union, and he could be as powerful as the director. A teacher has power over his students, because he can assign grade to students. All know that in recent times Europe and the United States have been losing of its economic power to China.

In fact, power is an abstract noun as love. It is impossible to see, to touch or to measure it, but it´s possible to feel it. Nye (2011) says that is very common compare power with other things, like energy (physics) or money (economy). Both these comparison are mistaken. It´s impossible compare power with energy due this one can be measured, and also with money, because money is liquid and interchangeable.

To understand the "nature" of this word, different theorists and sciences, including sociology, psychology, politics and economics, have been developing different or complementary concepts of power and its relation to society. Power, in fact, is a broad concept, involves different sciences and has a long historical approach. Hobbes, for instance, was one the first philosophers that analyzed the mean of power in the modern ages – 17th Century. These ideas were published in the Leviathan. To him, power is the ability to secure well-being or personal advantage to obtain, in the future, some apparent good.

After Hobbes, a huge number of understandings were developed, but the meaning is the same: the asymmetries of forces. It is clear that power has different view and that differs according with time and space. Power, in fact, is an important concept within the society, and arises from the interaction between people. In this sense, power could be thought like different views, including sociological, anthropological, international politics, economic, for instance.

In these senses, the aim of this paper is to discuss power in game theory, in order to model the asymmetries of forces among the players of the game. The starting point is that games are strategic interactions between rational individuals in a social environment and the players don´t have equal forces. Game theory has increased hugely in the last number of years, including several new branches; however, the concept of power in game theory has not been explored to any great extent. Indeed, power is a broad concept that has no clear definition. This paper formalize a model taking account the asymmetric forces between players. Beside this introduction, this paper discusses two specific topics, the concepts about power and the presents a model of game that include power, after that is presented the conclusions.

References

[1] Dahl, R. A., The concept of power, Behavioral Science, 1957.
[2] Foucault, M. The history of sexuality. New York: Pantheon Books, 1978.
[3] Galbraith, J. K. The Anatomy of Power, John Kenneth Galbraith, Boston: Houghton Mifflin, 1983.
[4] Hobbes, T., (editor: MacPherson, Crawford Brow,) Leviathan, or The Matter, Forme & Power of a Common-wealth Ecclesiastical and Civill, London, 1968, (original edition: London, 1651).
[5] Nye, J. S. Soft Power, Hard Power and Leadership, 2006. Retrieved: March, 15,2013, from http://www.hks.harvard.edu/netgov/files/talks/docs/11_06_06_seminar_Nye_HP_SP_Leadership.pdf
[6] Nye, J. S. The future of power. Public Affairs/Perseus Book Group, 2011.
[7] Osborne, M., Rubinstein, A. A Course in Game Theory. Cambridge, MA: MIT Press, 1994.
[8] Weber, M.: Economy and Society. Bedminster Press, New York, 1968.
[9] Wiese, H. Applying cooperative game theory to power relations. Quality and Quantity, V. 43, N.r 4, p. 519–533, 2009.
[10] Williamson, O. E., "Transaction cost economics: an overview (pdf)" (in Peter G. Klein and Michael E. Sykuta (eds) The Elgar Companion to Transaction Cost Economic, 2010.

Dynamic Contests with Bankruptcy

Luis Corchon

Carlos III University
Spain
luis.corchon@gmail.com

Keywords: *Contests Dynamics Bankruptcy*

The theory of contests analyzes conflicts where agents spend resources in order to obtain a prize. But the theory never explained what happens if an agent spends repeatedly resources and never gets the prize. It is likely that this agent, sooner or later, will be bankrupted and will face liquidation. In this paper we analyze a two period contest in which agents may become bankrupt at the end of the first period. A bankrupt agent is not present in the second period of the game. We prove that a subgame perfect equilibrium in pure strategies exists for almost all parameter configurations and show that bankruptcy may happen in equilibrium. In particular, we distinguish between bold strategies (risky) strategies where the agent might be bankrupt in the second period and a prudent (safe) strategy where she is never bankrupted. We prove that poor and impatient agents have incentives to play bold strategies. Finally, we provide historical examples of contests in which bankruptcy played a paramount role.

Love Game - Two Sided Matching

Suresh Deman

Centre for Economics & Finance, United Kingdom
s_deman2000@yahoo.co.uk

Keywords: *Matching Information Preferences*

Recently Prof Shapley won his long overdue Nobel Prize in Economics Science on Gale-Shapley algorithm in matching theory (American Mathematical Monthly (1962). Since the publication of above seminal paper the matching frame work has received a great deal of attention of many great economist and mathematicians all over the world.

A matching, as the name suggests, is the pairing up of elements from two sets. The 'marriage market' seeks to match men and women. The 'admissions market' tries to allocate students to schools or colleges. The matching framework even applies to unusual things, such as kidney exchange, where kidney donors are matched with recipients for transplant surgery. Many of these markets existed in the US during the decade of 1950s, but functioned inefficiently. This was either due to imperfect credit market, bounded rationality and asymmetric information. However, mostly these markets also functioned badly because of a failure of coordination between the two sides of the transaction, and the underlying rules that encouraged gaming of the system. For example, the market for medical residents was established in 1952, in which the hospitals competed with each other to offer residency positions to medical students. Initially, there were very few students and a very few rules. The intense competition for them led hospitals to make offers even as much as two years in advance. Students, on the other hand, benefited from waiting as long as possible, in case a better hospital happened to offer them a position. This led to market failure as both the students and hospitals remained unmatched. The hospitals made fewer offers than they had positions open, and students routinely strategized their applications for residencies and efforts to streamline the process proved useless. Prof Shapley proposed a mechanism that was guaranteed to produce a stable matching. It had the added benefit of encouraging truth-telling by both parties meaning thereby revealing their true preferences.

The original case made by Gale and Shapley described a marriage market, but it was immediately clear that their proposed solution the Deferred Acceptance (or DA) Algorithm would apply equally well to other situations, like public school matching, college admission problems, speed dating, matching kidneys, etc. This algorithm was successfully implemented in matching US medical school graduates with hospital residency programs. A more dynamic version of the same study in the job markets where employees and employers are in the constant search for hiring, firing, moving in, moving out according to their expectation rankings have lead to the Nobel prize in Economics (2012).

In subsequent papers on marriage markets it has been shown that the first mover will get the best match and should the player failed to reveal their true preferences it would lead to inefficient outcome. I proposed a dating game and show that both the players are better off (a Pareto superior outcome) if they do not truly reveal theory preferences.

Decompositions of Bbivariate Distributions, Areas of Random Triangles, and Bidding Games

Victor Domansky[1] **and Victoria Kreps**[2]

SPbEMI RAS
Russia
[1]*doman@emi.nw.ru*
[2]*vita_kreps@mail.ru*

Keywords: *Biddiing Games, Incomplete Information, Bivariate Distributions, Brownian Motion*

We consider models of multistage bidding between two agents for shares of two type risky assets. Liquidation prices of shares are integer random variables with distributions $\mathbf{p}$ over $\mathbb{Z}^2$, known to both agents. Before the bidding starts a chance move determines the liquidation prices of shares for the whole period of bidding. Player 1 is informed on these prices, Player 2 is not. Player 2 knows that Player 1 is an insider.

At each subsequent step both players simultaneously propose their bids for each type of assets. The maximal bid wins and one share of corresponding type is transacted at this price. Only integer bids are admissible. Each player aims to maximize the value of his final portfolio (money plus liquidation value of obtained shares).

These n-stage models are reduced to repeated games $G_n(p)$ with incomplete information of Player 2 with countable state and action spaces $\mathbb{Z}^2$ correspondingly. This is a discrete bivariate analog of the model introduced in De Meyer B., Moussa Saley H. (2002) *On the Strategic Origin of Brownian Motion in Finance.* Int. J. of Game Theory, 31, 285-319.

If share prices have finite variances, then values $V_n(p)$ of n-stage games are bounded from above.This makes reasonable considering games $G_\infty(p)$ with unlimited beforehand number of steps. We cannot solve n-stage games but we solve infinite games. We begin with constructing solutions for "elementary" games with two and three states.

We construct symmetric representations of probability distributions p over $\mathbb{R}^2$ with given mean values as probability mixtures of distributions with the same mean values and with not more than three-point supports. The probabilities of three-point distributions are proportional to areas of corresponding triangles. These decompositions allow us to construct optimal strategies of Player 1 for infinite games $G_\infty(p)$ in general case as combinations of his optimal strategies for "elementary" games. As a pattern for imitating we take the symmetric representation of univariate distributions.

Decomposition of bivariate probability distributions. W.l.o.g. we construct this decomposition for the set $\Theta(0,0)$ of centered distributions. Let $p^0_{z_1,z_2}$ and $p^0_{z_1,z_2,z_3}$ be probability distributions over $\mathbb{R}^2$ with zero mean values and with supports $\{z_1,z_2\}$ and $\{z_1,z_2,z_3\}$ respectively.

Sets of triangles. Consider a set of non-ordered triples (z_1,z_2,z_3)

$$\Delta^0=\{(z_1,z_2,z_3),z_i\neq(0,0):(0,0)\in\Delta(z_1,z_2,z_3)\}.$$

We denote $\mathrm{Int}\Delta^0=\{(z_1,z_2,z_3),z_i\neq(0,0):(0,0)\in\mathrm{Int}\Delta(z_1,z_2,z_3)\}$ and $\partial\Delta^0=\{(z_1,z_2,z_3),z_i\neq(0,0):(0,0)\in\partial\Delta(z_1,z_2,z_3)\}$. If $(z_1,z_2,z_3)\in\partial\Delta^0$, then there is an index i such that $\det[z_i,z_{i+1}]=0$. In this case $\arg z_{i+1}=\arg z_i+\pi(\mathrm{mod}\ 2)$, the point $(0,0)\in[z_i,z_{i+1}]$ and the distribution $p^0_{z_1,z_2,z_3}$ degenerates into the distribution $p^0_{z_i,z_{i+1}}$ with the support $\{z_i,z_{i+1}\}$.

Sets of complementary couples. For $\psi\in[0,2\pi)$, let R_ψ be the half-line $R_\psi=\{z:\arg z=\psi)\}$. With each $\psi\in[0,2\pi)$ we associate three sets of non-ordered couples (z_1,z_2)

$$\Delta^0(\psi)=\{(z_1,z_2),z_i\neq(0,0):\forall z\in R_\psi\quad(z_1,z_2,z)\in\Delta^0\}.$$

We denote $\mathrm{Int}\Delta^0(\psi)=\{(z_1,z_2),z_i\neq(0,0):\forall z\in R_\psi\quad(z_1,z_2,z)\in\mathrm{Int}\Delta^0\}$ and $\partial\Delta^0(\psi)=\{(z_1,z_2),z_i\neq(0,0):\forall z\in R_\psi\quad(z_1,z_2,z)\in\partial\Delta^0\}$. We take, that points (z_1,z_2) are indexed counterclockwise.

Constructing the principal invariant. Now we introduce the value that is a bivariate analog of the quantity $\int_{t=0}^{\infty}t\cdot p(dt)$, for our symmetric representations of distributions over $\mathbb{R}^2$. Set

$$\Phi(p,\psi)=(\int_{\mathrm{Int}\Delta^0(\psi)}+\frac{1}{2}\int_{\partial\Delta^0(\psi)})\det[z_1,z_2]p(dz_1)p(dz_2).$$

Theorem 1. *For any distribution* $p \in \Theta(0,0)$ *the quantity* $\Phi(p,\psi)$ *does not depend on* ψ*, i.e. this is an invariant* $\Phi(p)$ *of the distribution* $p \in \Theta(0,0)$.

Remark. For any distribution $p \in \Theta(0,0)$ the quantity $\Phi(p,\psi)$ has the following invariant representation:

$$\Phi(p) = \frac{(\int_{\mathrm{Int}\Delta^0} + 1/2\int_{\partial\Delta^0})\sum_{j=1}^{3}\det[z_j, z_{j+1}]p(dz_1)p(dz_2)p(dz_3)}{1-p(0,0)}.$$

Let μ_p be the probability measure over Δ^0, such that $\mu_p(\{\prod_{j=1}^{3}dz_j\}) \propto \prod_{j=1}^{3}p(dz_j)$, for $(z_1,z_2,z_3) \in \mathrm{Int}\Delta^0$; $\mu_p(\{\prod_{j=1}^{3}dz_j\}) \propto \frac{1}{2}\prod_{j=1}^{3}p(dz_j)$, for $(z_1,z_2,z_3) \in \partial\Delta^0$.

Corollary. The expectation of the area $S(z_1,z_2,z_3)$ of a $\Delta(z_1,z_2,z_3)$ by the measure μ_p is given with the formula

$$\mathbb{E}_{\mu_p}[S(z_1,z_2,z_3)] = \mathbb{E}_p[S(z_1,z_2,z_3) \mid (z_1,z_2,z_3) \in \Delta^0] = \frac{\Phi(p)(1-p(0,0))}{2\mathbb{P}_p[\Delta^0]},$$

where $\mathbb{P}_p[\Delta^0]$ is the proportionality coefficient determining the measure μ_p.

The main decomposition theorem.

Theorem 2. *Any distribution* $p \in \Theta(0,0)$ *has the following symmetric decomposition into a convex combination of distributions with not more than three-point supports:*

$$p = p(0,0)\cdot\delta^0 + (1-p(0,0))\int_{\Delta^0}\frac{S(z_1,z_2,z_3)}{\mathbb{E}_{\mu_p}[S(z_1,z_2,z_3)]}p^0_{z_1,z_2,z_3}\mu_p(\{\prod_{j=1}^{3}dz_j\}).$$

Here the term

$$\int_{\partial\Delta^0}\frac{S(z_1,z_2,z_3)}{\mathbb{E}_{\mu_p}[S(z_1,z_2,z_3)]}p^0_{z_1,z_2,z_3}\mu_p(\{\prod_{j=1}^{3}dz_j\})$$

$$= \sum_{\Psi(p)}\frac{\partial\Phi(p,\psi)}{\Phi(p)}\int_{R_\psi}\int_{R_{\psi+\pi}}\frac{r_1+r_2}{\int_{R_{\psi+\pi}}tp(dt)}p^0_{(r_1,\psi),(r_2,\psi+\pi)}p(dr_2)p(dr_1),$$

where $\Psi(p)$ is a not more than countable set of values of polar angles ψ, such that the measures $p(R_\psi)$ and $p(R_{\psi+\pi})$ of the half-lines R_ψ and $R_{\psi+\pi}$ are more than zero.

Conditional probabilities and optimal strategies for Player 1. We regard coefficients of decomposition as probabilities of distributions with two-, and three-point supports in the symmetric representations of distributions $p \in \Theta(0,0)$. This allows to calculate the conditional probability of any extreme distribution given the point z in its support:

$$P_p\{p^0_{z_1,z_2,z} \mid z\} = \frac{\det[z_1, z_2]\, p(z_1)\, p(z_2)}{\Phi(p)},$$

$$P_p\{p^0_{kw,-lw} \mid z = kw\} = \frac{\partial\Phi(p,w)}{\Phi(p)} \cdot \frac{l \cdot p(-lw)}{\sum_{t=1}^{\infty} t \cdot p(-tw)}.$$

Now we construct the optimal strategy for Player 1 making use of the obtained decomposition for the initial distribution p.

a) If the state chosen by chance move is $(0,0)$, then Player 1 stops the game.

b) Let the state chosen by chance move be $z = kw$ or $z = -lw$, where $k, l \in \mathbb{N}$ and $w \in W$. For definiteness let be $z = kw$. Then Player 1 chooses either a point $z_2 = -lw$ or a pair of points z_2, z_3 by means of lottery with corresponding conditional probabilities.

c) Player 1 plays the optimal strategy $\sigma^*(\cdot \mid z)$ for the state $z = kw$ in the chosen two- or three-point game.

This study was partially supported by the Russian Foundation for Basic Research, projects 10-06-00348-a and 13-01-00462-a.

A Matrix Approach to an Efficient Myerson Value on Union Stable Structures

Hua Dong, Hao Sun[1], Genjiu Xu

Department of Applied Mathematics, Northwestern Polytechnical University
Xi'an, China
[1]hsun@nwpu.edu.cn

Abstract *Hamiache proposed a matrix approach to communication situations. In this work, a generalization of this matrix approach to union stable structures is illustrated. First we define an efficient Myerson value and show a characterization of this new value. Next, two new families of values are introduced axiomatically by associated consistency property.*
Keywords: *Union Stable Structure, Efficient Myerson Value, Associated Consistency, Matrix Approach*

A situation in which a finite set of players can obtain certain payoffs by cooperation can be described by a cooperative game with transferable utility, shortly TU-game, being a pair consisting of a finite set of players and a characteristic function on the set of coalitions of players assigning a worth to each coalition of players. In practice, since the cooperation restrictions exist, only some subgroup of players can form a coalition. One way to describe the structure of partial cooperation in the context of cooperative games is to specify sets of feasible coalitions. Algaba, et al [1] considered union stable systems as such sets. A union stable system of two intersecting feasible coalitions is also feasible, which can be interpreted as the follows: players who are common members of two feasible coalitions are able to act as intermediaries to elicit cooperation among all the players in either of these coalitions, and so their union should be a feasible coalition. And a TU game with a union stable system is called a union stable structure. Besides, the union stable structure is a generalization of games with communication structure and games with permission structure, which are respectively proposed by Myerson and Gilles.

Hamiache presented a matrix approach to construct extensions of the Shapley value on the games with coalition structures and communication structures. This paper aims to generalize this matrix approach to union stable structures, a generalized communication structures. Note that the partition system proposed by Bilbao is the same to the union stable system we discuss here, if we ignore whether or not the empty set is feasible. What worth mention here is that the hypergraph communication situation proposed by Nouweland is also belongs to union stable system.

Definition 1 A matrix A is called a row (column)–coalition matrix if its rows (column) are indexed by coalitions $S \subseteq N$ in the lexicographic order. And a row-coalition matrix $A=[a]_{S,T}$ is called row-inessential or inessential if $A=[a]_{S,T}=\sum_{i\in S} a_{i,T}$ for all $S \subseteq N$.

Definition 2 A union stable system is a pair $(N,\mathcal{F})$ with $\mathcal{F} \subseteq 2^N$ verifying that $\{i\} \in \mathcal{F}$ for all $i \in N$ and for all $S,T \in \mathcal{F}$ with $S \cap T \neq \varnothing$, $S \cup T \in \mathcal{F}$ holds.

Definition 3 Let $\mathcal{E} \subseteq 2^N$ be a set system and $S \subseteq N$. A set $T \subseteq S$ is called a $\mathcal{E}-$component of S if it is satisfied that $T \in \mathcal{E}$ and there exists no $T^{'} \in \mathcal{E}$ such that $T \subsetneq T^{'} \subseteq S$.

Proposition 1 The set system $\mathcal{E} \subseteq 2^N$ is union stable if and only if for any $S \subseteq N \setminus \varnothing$ with $C_{\mathcal{E}}(S) \neq \varnothing$, $\mathcal{E}-$components of S is a partition of S.

In order to give the formal definition of the efficient Myerson value, two matrices closely related to union stable structures are constructed.

Let us define a $\{0,1\}$-square matrix P of order 2^n-1, which is related to union stable structure $(N,v,\mathcal{F})$. So that for all $S,T \in 2^N \setminus \{\varnothing\}$,

$$P[S,T]=\begin{cases}1, & \text{if } T \in C_{\mathcal{F}}(S)\\ 0, & \text{otherwise}\end{cases}$$

Next, we shall make a slight modification of the matrix, and define the matrix Q as follows,

$$Q[S,T]=\begin{cases}1 & \text{if } \quad T=\underline{S}\\ 1 & \text{if } \quad T \in C_{\mathcal{F}}(S \setminus \underline{S})\\ 0 & \text{otherwise}\end{cases}$$

Lemma 2 Given a union stable structure $(N,v,\mathcal{F})$ and the intermediate game (β,v^{β}) in which $\beta=\{B_1,B_2,...,B_r\}$, the vector of weights are $w=(b_1,b_2,...,b_r)$, $b_l=|\beta_l|$ for all $l \in \{1,2,...,r\}$. Then for all $B_l \in \beta$ and all players $i \in B_l$,

$$(M_L \cdot (Q-P)\cdot v)[\{i\}]=Sh_i(N,(Q-P)\cdot v)=\frac{1}{|B_l|}(Sh^{w}_{B_l}(\beta,v^{\beta})-v(B_l))$$

Theorem 3 The efficient Myerson value is the unique value on US^N that satisfies efficiency, additivity, average fairness, point anonymity, redundancy equality and superfluous property.

Also the axioms of theorem 1 are logically independent as shown by the six alternative solutions.

Lemma 4 $P \cdot P = P$ and $Q \cdot Q = Q$.

Lemma 5 $P \cdot M_L = M_L$ and $Q \cdot M_L = M_L$, where M_L is the coefficient matrix corresponding to the inessential game (N, v_L).

Given a union stable structure $(N, v, \mathcal{F})$, the similarity matrix is $F(Y) = I + aY\ (Y = P, Q)$, and $a \in \mathbb{R}, a \neq -1$, define its associated game $v_\lambda^{F(Y)}$ by the following:

$$v_\lambda^{F(Y)} = (I - \frac{a}{1+a}Y) \cdot M_\lambda \cdot (I + aY) \cdot v =$$

$$= [M_\lambda + aM_\lambda \cdot Y - \frac{a}{1+a}Y \cdot M_\lambda - \frac{a^2}{1+a}Y \cdot M_\lambda \cdot Y] \cdot v \tag{1}$$

For any given parameter $a \in \mathbb{R}$, $a \neq -1$, the detailed expression of the limit game is

$$v_L^{F(Y)} = (I - \frac{a}{1+a}Y) \cdot M_L \cdot (I + aY) \cdot v = \frac{1}{1+a} M_L \cdot v + \frac{a}{1+a} M_L \cdot Y \cdot v.$$

In the following, we provide the definition of two families of values for union stable structures and make a list of the axioms that will be used to characterize these two families of value.

Definition 4 Given a union stable structure $(N, v, \mathcal{F}) \in US^N$, for any $a \in \mathbb{R}, a \neq -1$, we define two types of solutions $\phi^{F(Y)}$, where $Y \in \{P, Q\}$

$$\phi^{F(Y)}(N, v, \mathcal{F}) = \frac{1}{1+a} Sh + \frac{a}{1+a} \rho ,$$

where $\rho = \varphi, \eta$ if $Y = P, Q$ respectively.

Inessential game property For all inessential games (N, v) with union stable systems $(N, \mathcal{F})$, the solution verifies $\phi_i(N, v, \mathcal{F}) = v(\{i\})$ for all $i \in N$.

Associated consistency For all games with union stable systems $(N,v,\mathcal{F})$ and the corresponding associated game $(N,v_{\lambda}^{F(Y)},\mathcal{F})$ with $0<\lambda<2/n$, we have $\phi(N,v,\mathcal{F})=\phi(N,v_{\lambda}^{F(Y)},\mathcal{F})$.

Continuity For all convergent sequence of games with union stable systems $\{(N,v_m,\mathcal{F})\}_{m=1}^{\infty}$, the limit of which is game $(N,v_L,\mathcal{F})$, then $\lim_{m\to\infty}\phi(N,v_m,\mathcal{F})=\phi(N,v_L,\mathcal{F})$.

Theorem 6 For any given $a\in\mathbb{R},a\neq-1$, there is a unique value $\phi^{F(Y)}(N,v,\mathcal{F})(Y\in\{P,Q\})$ on US^N verifying inessential game property, associated consistency and continuity.

Next we show the relationship existing among these four types of values, Myerson value, efficient Myerson value, two families of values $\phi^{F(Q)}$ and $\phi^{F(P)}$. For any given $a\in\mathbb{R},a\neq-1$, two families of values $\phi^{F(Q)}$, $\phi^{F(P)}$ and the Shapley value are linked by the relationship of "similarity" are obvious.

Proposition 7 For the square matrix $Y=P,Q$ defined in first page, we have

$$(I-M_L\cdot(I-Y))\cdot(I-\frac{a}{1+a}Y)\cdot(I+aY)\cdot(I+M_L\cdot(I-Y))=I$$

$$(I-M_L\cdot(I-Y))\cdot(I-\frac{a}{1+a}Y)\cdot M_L\cdot(I+aY)\cdot(I+M_L\cdot(I-Y))=M_L\cdot Y$$

The Inverse Problem for Binomial Semivalues of Cooperative TU games

Irinel Dragan

University of Texas
United States
dragan@uta.edu

Keywords: *Inverse Problem for Semivalues, Binomial Semivalues, Banzhaf Value*

In an earlier paper (Dragan, 2005), we introduced the Inverse Problem for the Semivalues of Cooperative TU games as follows: given a Cooperative TU game, a weight vector, and a Semivalue associated with this weight vector, find out the set of all games for which the given Semivalue associated with the given weight vector is obtained for all games in the set. A class of Semivalues, which generalizes the Banzhaf value, and it is called the Binomial Semivalues, has been introduced in an earlier paper by A.Puente (2000). In this case, the weight vector depends on one parameter, so that the results of our previous paper are greatly simplified. In the new paper we derive the solution of the Inverse problem without using the previous solution, following a somehow different path. The solution is illustrated by the Banzhaf value which is obtained for the value one of the parameter.

Consistency and the Core of Multi-Sided Assignment Markets

Francesc Llerena[1], Marina Nunez[2] and Carles Rafels[3]

[1]*University Rovira i Virgili, Spain*
francesc.llerena@urv.cat

[2,3]*University of Barcelona, Spain*
[2]*mnunez@ub.edu*
[3]*crafels@ub.edu*

Keywords: *Multi-Sided Assignment Games, Core, Consistency, Axiomatization*

A two-sided assignment market (or game) (Shapley and Shubik, 1972) is defined by two finite and disjoint sets, usually representing buyers and sellers, a valuation matrix that gives the profit of every partnership formed by a pair of agents from opposite sectors and a vector of reservation values that represent the profit of an individual when remaining alone. Since utility is assumed to be fully transferable, the core of the assignment game consists of the set of payoff vectors that efficiently allocate the profit achieved by an optimal matching of buyers to sellers in such a way that no individual or mixed-pair can improve upon. When utility is not transferable and the only data is the strict preference list of each agent over the set of agents of the opposite sector, the core of a two-sided matching market (Gale and Shapley, 1962) consists of those matchings that are individually rational and pairwise stable, which means that no pair of a buyer and a seller would prefer to be matched together than keeping their actual partners. Although both models are apparently distinct, almost parallel properties hold for the cores of the two markets, such as non-emptiness, the lattice structure and the polarization of interest between both sides of the market.

Axiomatizations of the core of two-sided markets, both with or without transferable utility, appear in the recent literature. Sasaki (1995) provides a first axiomatization of the core of the assignment game by means of consistency, continuity, couple rationality, individual rationality, Pareto optimality and weak pairwise monotonicity. Toda (2003) gives another axiomatization, by adapting to the assignment game the axioms that characterize the core of the coalitional games in Peleg (1986), that are (Davis and Maschler) consistency, individual rationality, Pareto optimality and superadditivity.Two additional axiomatizations for the core of assignment games are also provided by Toda (2005) by means of Pareto optimality, (submarket) consistency, pairwise monotonicity and individual monotonicity (that can be substituted by population monotonicity). It is observed (Toda, 2006) that the cores of the discrete two-sided matching markets are characterized by axioms that almost overlap with the axioms characterizing the core of the two-sided assignment games. In this setting, Toda characterizes the core by means of weak unanimity, population monotonicity, and Maskin monotonicity (that can be substituted by a consistency axiom). And the same axioms characterize the core of one-to-many matching markets, that is, when agents on one side are allowed to establish partnerships with more than one agent on the opposite side. However, as far as we know, no attempt has been made to axiomatically characterize the core of multisided markets, those where more than two sectors exist and partnerships are formed with exactly one agent of each different sector. In this paper, we focus in the transferable utility setting and provide two axiomatizations for the core of the multi-sided assignment game.

The difficulty in extending the axiomatizations of the core of two-sided assignment games to the multi-sided case, lie in the fact that multi-sided assignment games may have an empty core. These means that most of the monotonicity properties used to characterize the core in the two-sided case cannot be straightforwardly extended. We may have a multi-sided assignment game with a non-empty core but when raising the worth of one matrix entry, or new players entering the market, the resulting market may have no core elements. As for the consistency axioms, it was already noted by Owen (1992) that the Davis and Maschler (1965) reduced game of an assignment game at a core allocation, may not be an assignment game. However, this reduced game can be replaced with the derived game, since they both have the same core. In fact, this derived game is also used in Toda (2003). On the other hand, Toda's (2005) consistency axiom can be easily generalized to the multi-sided case. Therefore, we give a first characterization of the core on the domain of balanced multisided assignment games by means of non-emptiness and the aforementioned consistency axioms: derived consistency and submarket consistency. As a consequence, we also obtain a new axiomatization of the core of the two-sided assignment game only using the two consistency axioms.

Finally, on the general domain of multi-sided assignment games, we give another axiomatic characterization of the core in terms of three axioms: unanimity, individual anti-monotonicity and derived consistency. This individual anti-monotonicity axiom is a weaker form of the anti-monotonicity property introduced by Keiding (1986) to characterize the core on the class of cooperative games with non-transferable utility, and it is also used by Toda (2003) to characterize the core of two-sided assignment games with non-linear utility functions.

ВЕСТНИК САНКТ–ПЕТЕРБУРГСКОГО УНИВЕРСИТЕТА

Серия 10
ПРИКЛАДНАЯ МАТЕМАТИКА, ИНФОРМАТИКА, ПРОЦЕССЫ УПРАВЛЕНИЯ

Ответственный редактор серии:
Л.А. Петросян
Зам. ответственного редактора:
Д.А. Овсянников
С.В. Чистяков

Издательство Санкт-Петербургского университета

Service Quality Level Choice on Oligopoly Competition

Margarita Gladkova[1], Nikolay Zenkevich[2] and Maria Kazantseva

[1,2]Graduate School of Management, St. Petersburg State University
[1]gladkova@gsom.pu.ru,
[2] zenkevich@gsom.pu.ru
[3]Vienna University of Economics and Business
[3]lyssia@mail.ru

Keywords: *Quality Choice, Willingness to Pay, Exponential Distribution, Two-Stage Game, Nash Equilibrium, Optimal Quality Differentiation, Mobile Service Market, Fitness Industry*

The growth of the complexity of external environment and conditions of business management, namely, high development of information and communication technologies and competition boost, predefines the identification of new sources of development of companies' competitive abilities and ways to increase management effectiveness. This fact causes the necessity of innovative approaches to strategic decision-making, instruments and tools that help them to reach the leading position in mid-term and long-term perspective. One of the instruments that allow increasing company's competitiveness is the improvement of the service quality. Contemporary approaches to company management are based on the analysis of the adding value framework. The value of the service for the consumer is highly defined by its quality. Therefore, the problem of quality level choice under competition is a very important element of the strategic management. The appropriate choice of service quality level and price provides a company with necessary conditions to maintain high competitiveness and stable development. The evolution of theoretical and applied methodology of quality management under competition is not possible without studying the consumer satisfaction and companies' strategic interaction in the market. Therefore, research objectives are:

- analysis of consumer satisfaction with the service,
- development of game-theoretical models of service providers' interaction,
- definition of the strategy of service quality level choice,

- development of practical recommendations for Russian companies to implement the strategy.

The goal of the research is to develop theoretical basis (models) and practical methods of the service quality level evaluation and choice which is made by the service provider.

The survey was conducted in St. Petersburg and defined consumer preferences and satisfaction in two different industries: mobile service market and fitness industry. The survey and game-theoretical analysis of St. Petersburg industrial market allowed finding current and equilibrium service quality levels.

The models allowed evaluating the change in market shares for mobile operators and Fitness clubs of St. Petersburg.

МАТЕМАТИЧЕСКАЯ ТЕОРИЯ ИГР И ЕЕ ПРИЛОЖЕНИЯ

Ответственный редактор:
Л.А. Петросян
Зам. ответственного редактора:
В.В. Мазалов
Ответственный секретарь:
Н.А. Зенкевич

Выходит ежеквартально с января 2009 года

A Problem of Purpose Resource Use in the Two-Level Control Systems *

Olga Gorbaneva[1] **and Gennady Ougolnitsky**[2]

[1,2]Southern Federal University
Russia
[1]gorbaneva@mail.ru
[2]ougoln@mail.ru

Keywords*: Hierarchical Games, Resource Allocation,Ttop Level, Bottom Level*

We consider a two-level control system that consists of one element in top level and one element in bottom level. The top level has some resource the amount of which is assumed to be equal to one. The top level assigns a part of the resource to the bottom level (a purpose use) while the other part it keeps for the non-purpose use. In turn, the bottom level assigns a part of the received resource for the purpose use while the other part it keeps for the personal non-purpose use. Both levels take their parts of payoff from the purpose use of resource. The model is built as a game of two players where a Stackelberg equilibrium is sought. The payoff functions of both players include two summands: a payoff from the personal non-purpose activity and a part of payoff from the purpose activity of the whole system. So, the payoff functions have the form:

$$g_1(u_1,u_2) = a_1(1-u_1,u_2) + b(u_1,u_2)\cdot c(u_1,u_2) \to \max_{u_1};$$

$$g_2(u_1,u_2) = a_2(u_1,1-u_2) + b(u_1,u_2)\cdot c(u_1,u_2) \to \max_{u_2}.$$

subject to

$$0 \le u_i \le 1, i = 1,2,$$

and the following conditions for a, b, and c are satisfied:

$$a_i \ge 0; \frac{\partial a_i}{\partial u_i} \le 0, \frac{\partial a_i}{\partial u_{j\ne i}} \ge 0, i = 1,2, b_i \ge 0; \frac{\partial b_i}{\partial u_i} \ge 0, i = 1,2, \frac{\partial c}{\partial u_i} \ge 0, i = 1,2.$$

Here index 1 relates to the characteristics of the top level, index 2 to the characteristics of the bottom level respectively; u_i is a part of resource assigned by the

*This work was supported by the Russian Foundation for Fundamental Researches under grants No.12-01-00017 and 12-01-31287.

i-th level for the purpose use (respectively, the rest $1-u_i$ is assigned for the personal non-purpose use); g_i is the i-th level payoff function; a_i is the i-th level personal payoff from the non-purpose activity; b_i is a part of the payoff from the purpose activity of the whole system received by the i-th level; c is a payoff from the purpose activity of the whole system.

Functions a_i and c can be linear, power, exponential or logarithmic ones of the variables u_1 and u_2 are cumulative ones in the sense

$$a_1 = a_1(1-u_1), a_2 = a_2(u_1(1-u_2)), c = c(u_1 u_2).$$

The relations $a_1 = a_1(1-u_1), a_2 = a_2(u_1(1-u_2))$ reflect a hierarchical structure of the considered system. A payoff of the top level from the non-purpose activity does not depend on the part of resource assigned by the bottom level to the purpose activity. In the same time, a payoff from the non-purpose activity of the bottom level depends on the part of resource received from the top level.

The following classes of functions b_i are considered:

1) a uniform one when both players take the equal parts in the payoff from the purpose activity:

$$b_i = \frac{1}{2}, i = 1,2$$

2) a proportional one when the parts of common payoff received by both players are proportional to their assignments in the purpose activity:

$$b_1 = \frac{u_1}{u_1+u_2}, b_2 = \frac{u_2}{u_1+u_2},$$

In general case, $b_1 + b_2 \le 1$, i.e. a situation is possible when the common payoff from the purpose activity is not distributed completely between the players.

In the paper it is supposed that the payoff is distributed completely:

$$b_1 + b_2 = 1$$

This model is a Stackelberg game with two players. The strategy of the i-th player is a part u_i of his resource assigned to the purpose activity. The top level player makes the first move, i.e. he chooses u_1, and informs the player of the bottom level about the choice. After that the second player chooses the optimal value of u_2.

The research aim is to study how the game solution depends on the relation of the functions a_1, a_2, b_1, b_2, c.

Consider as an example the parameterization $a_1(u_1,u_2) = a_1(1-u_1)$, $a_2(u_1,u_2) = a_2 u_1(1-u_2)$, $c(u_1,u_2) = (u_1 u_2)^k$ where c is a power production function.

Let $0 < k < 1$. (power production function).

Then $g_1(u_1,u_2) = a_1(1-u_1) + b_1(u_1 u_2)^k$, $g_2(u_1,u_2) = a_1 u_1(1-u_2) + b_2(u_1 u_2)^k$ . .

The optimal strategy of the second player is equal to

$$u_2^* = \frac{(\frac{a_2}{kb_2})^{\frac{1}{k-1}}}{u_1}.$$

The first player maximizes his payoff function

$$g_1(u_1,u_2^*) = b_1(\frac{a_2}{kb_2})^{\frac{k}{k-1}} + a_1(1-u_1)$$

As the function g_1 decreases on u_1 then $u_1^* = 0$.

Thus, in the majority of the considered models in the economic setting (impulsion) the leader behaves egoistically, i.e. assigns all the resource to the non-purpose use ($u_1^* = 0$). For the choice of the top level player of a strategy either compulsion (explicit constraint in the form $u_1 \geq \hat{u}_1$) or the volunteer choice of a positive strategy by some non-economic incentives is required.

Multicriteria Coalitional Model of Decision-making over the Set of Projects with Constant Matrix of Payoffs in the Noncooperative Game

Xeniya Grigorieva[1] **and Pauline Shaikina**[2]

[1,2]St-Petersburg State University, Russia
[1]kseniya196247@mail.ru
[2]pearl-girl5@mail.ru

Keywords: *Coalitional Game, PMS-vector, Compromise Solution, Multicriteria Model*

Let be N the set of players and M the set of projects. The multicriteria coalitional model of decision-making over the set of projects is formalized as family of games with different fixed coalitional partitions for each project that required the adoption of a positive or negative decision by each of the players. The players' strategies are decisions about each of the project. The vector-function of payoffs for each player is defined on the set situations in the initial noncooperative game. Players forms coalitions in order to obtain higher income. Thus, for each project a multicriteria coalitional game is defined. In each multicriteria coalitional game it is required to find in some sense optimal solution. We reduce the multicriteria coalitional game to a coalitional game with scalar payoffs. Solving successively each of the coalitional games, we get the set of optimal n-tuples for all coalitional games. It is required to find a compromise solution for the choice of a project, i. e. it is required to find a compromise coalitional partition. As an optimality principles are accepted generalized PMS-vector and its modifications, and compromise solution.

Optimal Control of SIR Epidemic Model with Virus Mutations

Elena Gubar [1] and Quanyan Zhu[2]

[1]*St.Petersburg State University, Russia*
alyona.gubar@gmail.com

[2]*University of Illinois at Urbana-Champaign, USA*
zhu31@illinois.edu

Keywords: *Evolutionary Game Dynamics, Virus Mutations, SIR Epidemic Model*

An epidemic of infectious disease occurs when virus population undergoes genetic mutations or new species of viruses are introduced into host population, and the host immunity to that change in the virus population is suddenly reduced below certain threshold. Hence, the epidemic modeling should take into account not only the population dynamics of the host population but also those of the viruses. In traditional epidemiological models, differential equations are used to capture the dynamic evolution of different classes of host populations. In particular, the susceptible (S) is the class of people who are not infected; the infected (I) is the class of people having the disease; the removed or recovered (R) represents deceased or immune people. The commonly used SIR model is used to describe the population migrations between these three classes of models. In order to capture the interdependencies between virus and host populations, we establish a system framework that combines the SIR model with evolutionary models that describe virus mutations.

Mathematical model of virus infection in a population can be used to study those factors, which influence the epidemic growth for improving existing treatment and evaluating new effective prevention measures and treatment. In earlier research in the literature, it has been shown that during epidemic season, influenza virus can mutate, and during the epidemic season, several types of influenza virus circulate in human population. Different mutations of the influenza virus affect human beings with different intensities, and the epidemics evolve depending on the virus type and its intensity. Since viruses are able to adapt to hostile environment, medical treatments may invoke mechanism of mutations in virus population. In our model, we consider two types of viruses with different strains and fitness functions. Both types of viruses spread in urban populations, and hence during the epidemic process, different parts of population will be infected. Characteristics of the two viruses initiate different characteristics of survivability and adaptability in human population over time. Thus, in this work, we focus on imitation dynamics dynamics to describe the evolution of mutation process within the virus population. Imitation dynamics allow us to include stochastic mechanism of virus adaptation in human population under the influence of intensive treatment. Various medical treatments (pharmacological products, quarantine policies, etc.) of the preventions can be applied to reduce the infection rates and the number of the infected in the population and eventually protect the entire population.

Different from the work done in the past, this paper considers a coupled system framework composed of the SIR epidemic model and the evolutionary dynamic model for virus mutation. This framework is motivated by the fact that the epidemic spread of the virus can facilitate the virus mutation, strengthening the virulence of the virus, which will in turn expedite the spread and worsen the epidemics. The SIR epidemic model with virus mutations can capture this complex interactions between the virus and the host, and allows us to explain more complex phenomena through analysis. We couple together two dynamic processes, i.e., the evolution of virus mutation and the epidemic process in human population as one dynamical system. Complex problem is formulated as an optimal control problem for epidemic process in human population, and virus mutation process is described by imitation dynamics.

References

[1] V. Capasso, Mathematical Structures of Epidemic Systems, Lecture Notes in Biomathematics, vol. 97, Springer, 1993 (2nd printing 2008)

[2] Conn M. (ed.), Handbook of Models for Human Aging, Elsevier Academic Press, London, (2006).

[3] Kermack W. O. and McKendrick A. G., ``A contribution to the mathematical theory of epidemics," Proc. of the Royal Society, ser. A. V. 115, no. A771, pp. 700--721, (1927).

[4] Wickwire K. H. A note on the optimal control of carrier-borne epidemics, J. Appl. Probl., no. 12, pp.565--346, (1975).

[5] H. Behncke, Optimal control of deterministic epidemics, Optim. Control Appl. Meth., no. 21, pp. 269--285, (2000).

[6] P. J. Francis, Optimal tax/subsidy combinations for the season, Journal of Economic Dynamic & Control, no. 28, pp. 2037--2054, (2004).

[7] K.R. Rohloff and T. Ba S ar, ``Deterministic and stochastic models for the detection of random constant scanning worms," ACM Transactions on Modeling and Computer Simulation (ACM TOMACS), vol. 18, no. 2, pp. 8:1-8:24, (2008).

[8] Khouzani M. H. R., Sarkar S., and Altman E. , ``Dispatch then stop: Optimal dissemination of security patches in mobile wireless networks," Proc. of 48th IEEE Conference on Decisions and Control (CDC), pp. 2354--2359, (2010).
[9] Omic J. , Orda A. and Van Mieghem P. , ``Protecting Against Network Infections: A Game Theoretic Perspective," Proc. 28th IEEE Conference on Computer Communications (INFOCOM, April 19-25, 2009; Rio de Janeiro, Brazil)
[10] Zhu Q., Bushnell L. , and Ba S ar T., ``Game-theoretic analysis of node capture and cloning attack with multiple attackers in wireless sensor networks," Proc. of the 51st IEEE Conference on Decision and Control (CDC'12, Dec 10-13, 2012; Maui, Hawaii).
[11] Bonhoeffer S., May R. M., Shaw G. M., and Nowak M. A., ``Virus dynamics and drug therapy," Proc. Natl. Acad. Sci. USA, vol. 94, pp. 6971-6976, (1997).
[12] Gubar E., Zhitkova E., Fotina L., Nikitina I. Two Models of the Influenza Epidemic. Contributions to game theory and management, vol. 5, pp. 107--120, (2012).
[13] Weibull J. , Evolutionary Game Theory, The M.I.T. Press, Cambridge, MA, (1995).
[14] Taylor, P. D. and Jonker, L. B., ``Evolutionarily stable strategies and game dynamics," Math. Biosci., vol. 40, pp.145--156, (1978).
[15] Khatri S., Rael R., and Hyman J., ``The Role of Network Topology on the Initial Growth Rate of Influenza Epidemic". Technical report BU-1643-M, (2003).
[16] Khouzani M. H. R., Sarkar S. and Altman E., ``Optimal control of epidemic evolution," in Proceedings of the 30th International Conference on Computer Communications (INFOCOM), pp. 1683--1691, (2011).
[17] Pontryagin L.S., Boltyanskii V.G. , Gamkrelidze R.V. and Mishchenko E.F., The Mathematical Theory of Optimal Processes, Interscience, (1962).
[18] Sandholm W. H., Dokumaci E. and Franchetti F. Dynamo: Diagrams for Evolutionary Game Dynamics, version 0.2.5. Available at http://www.ssc.wisc.edu/ whs/dynamo, (2010).

Interaction Between Social Groups in Extended SIR Model

Elena Gubar[1] **and Ekaterina Zhitkova**[2]

St.Petersburg State University,
Faculty of Applied Mathematics and Control Processes,
Universitetskiy pr. 35, St.Petersburg, 198504, Russia
fax:+7 (812) 428 71 59
http://www.apmath.spbu.ru,
[1]alyona.gubar@gmail.com,
[2]zhitkovakaterina@mail.ru

Keywords: *SIR Model, Vaccination Problem, Evolutionary Game, Optimal Control, Epidemic Process*

Every year epidemic disease occur when epidemic threshold has been passed in human population, what means that the number of infected is over than definite fixed value. Epidemic diseases in human population can be described by classical SIR model, where total population is divided into three subgroups: Susceptible, Infected and Recovered. Susceptible is group where people are not infected, Infected is a group of people having the disease, and Recovered is group, where all members have immunity to the disease.

During the years many medical methods such as preventive measures, intensive treatment, etc. were developed to protect entire population during annual epidemics. Different from the previous works, in this paper, total population is ranged to different social groups such as children, retired, professional, and students. Those groups possess various incomes and risks of complication of diseases, and also speed of the infection spreading. Let us suppose that every person may choose one or couple of medical methods such as preventive measures, quarantine, and intensive treatment, to avoid the disease. Moreover we suppose that chosen decisions may depend on the individuum's social group. In our model we consider two possible decisions such as vaccination or intensive treatment.

In current research we present extended SIR model, that includes the relationship between social groups and establish vaccination company as a optimal control program that influence on the SIR model and allow to reduce quantity of infected during epidemics. This paper estimates how the social position influences on the human decision about methods of treatment and how external factors impact to the spreading of epidemic in different social groups.

References

[1] Gillespie, D. T. Exact stochastic simulation of coupled chemical reactions. J. Phys. Chem. 81, 2340пїS 2361, 1977.

[2] Fu F. Rosenbloom D.I., Wang L. and M.A.Nowak. Imitetion dynamics of vaccination behaviour on social networks.Proceedings of the Royal Society. Proc. R.Soc. B (2011) 278, p-p. 42-49, 2010.

[3] Kermack W.O., Mc Kendrick A.G. A contribution to the mathematical theory of epidemics Proceedings of the Royal Society. 1927, Ser. A. V. 115, No. A771. p.700-721.

[4] Pontryagin L.S., V.G. Boltyanskii, R.V. Gamkrelidze et E.F. Mishchenko, The Mathematical Theory of Optimal Processes, Interscience, 1962.

[5] Sandholm W. H., E. Dokumaci, and F. Franchetti. Dynamo: Diagrams for Evolutionary Game Dynamics, version 0.2.5. http://www.ssc.wisc.edu/ whs/dynamo, 2010.

[6] Weibull J. Evolutionary Game Theory --- Cambridge, MA: The M.I.T.Press, 1995.

[7] Vincent T.L. and Brown J.S. Evolutionary Game Theory, Natural Selection, and Darwinian Dynamics --- Cambridge University Press, 2005.

[9] Bulo S. R., Bomze I. M. Infection and immunization: a new class of evolutionary game dynamics. Games and Economic Behavior 71(1), 2011. P. 193--211.

Stochastic Bankruptcy Games

Helga Habis[1] **and P. Jean-Jacques Herings**[2]

[1]*Institute of Economics, Hungarian Academy of Sciences, and Department of Microeconomics, Corvinus*
University of Budapest, F˝ovám tér 8., 1093, Budapest, Hungary.
helga.habis@uni-corvinus.hu

[2]*Department of Economics, Maastricht University, P.O. Box 616, 6200 MD, Maastricht, The Netherlands.*
P.Herings@maastrichtuniversity.nl

Keywords: *Transferable Utility Games, Uncertainty, Weak Sequential Core, Bankruptcy Games*

The classical bankruptcy problem originating in the Talmud has drawn the attention of game theorists a long time ago. In the original Talmudic version, there is a dying man who has three wives. Upon his death his estate is to be divided among the wives who have claims over the estate, where it is assumed that the sum of these claims exceeds the worth of the estate. The question is how this division or allocation should be done, based on the claims of the players. The game theoretic relevance of this issue was first noticed by O'Neill (1982), who has transformed the original problem into a cooperative game.

The entire literature on bankruptcy games assumes that the value of the estate, as well as the value of the claims, are deterministic. However, prior to the bankruptcy situation, typically these values are stochastic as they depend on currently unknown future market values. Nevertheless, coalitions of players can form prior to the resolution of uncertainty and they can discuss divisions of the estate conditional on its value and conditional on their claims. In this paper we therefore introduce the class of stochastic bankruptcy games.

A stochastic bankruptcy game is a special case of a transferable utility game with uncertainty, or briefly TUU-game. A TUU-game is a two-period cooperative game. In period 0, agents may decide to cooperate or not, facing uncertainty about the state of nature in period 1. In period 1, one state of nature materializes and a state-dependent TU-game is played. When applied to bankruptcy games, we have that the worth of the estate and the claims of the agents (wives) my depend on the state of nature; i.e. in each state of nature a different bankruptcy game may be played.

In a classical, static cooperative game it is implicitly assumed that the players can make fully binding agreements on the allocation of payoffs. When such an assumption is made in our stochastic setting, then coalitions can make fully binding state-contingent allocations of payoffs in period 0, and the game becomes formally equivalent to a non-transferable utility game.

We, on the other hand, study the case where agents cannot make such fully binding agreements. Instead, agents will not stick to their agreements concerning the future if after the resolution of uncertainty, they are better off when deviating. Hence, we only allow for self-enforcing agreements.

An agreement is said to be self-enforcing if there is no coalition of players with a credible deviation in some period. All deviations by singleton coalitions that lead to higher utility are credible. More generally, credible deviations are inductively defined by the requirement that a deviation is credible in some period if there is no further credible deviation by any sub-coalition, now or in the future. In static transferable utility games, the set of deviations and the set of credible deviations coincide (Ray, 1989).

In our setting, credibility plays a crucial role and leads to the concept of the Weak Sequential Core, introduced by Kranich, L., A. Perea, and H. Peters (2005) for finite deterministic sequences of TU-games, by Predtetchinski, A., P. J. J. Herings, and A. Perea (2006) for two-period exchange economies with incomplete markets, and by Habis, H., and P. J. J. Herings (2011) for TUU-games. Moreover, Habis, H., and P. J. J. Herings (2011) give a characterization of the Weak Sequential Core, and show its non-emptiness if all the state-contingent games played in period 1 are convex. Since bankruptcy games are convex, the Weak Sequential Core of stochastic bankruptcy games is non-empty.

Similarly to core-compatibility in a static problem, we use the Weak Sequential Core to test the robustness to uncertainty of the most important allocation rules suggested in the literature; the Proportional rule, the Adjusted Proportional rule, the Constrained Equal Awards rule, the Constrained Equal Losses rule, and the Talmud rule. It is well-known that all rules are stable in a deterministic bankruptcy game, being an element of the classical core. In a stochastic setting, however, each of them is unstable, with the exception of the Constrained Equal Awards rule.

Communication and Coordination in Social Networks: Action as Communication Device.

Jia-Ping Huang[1], Maurice Koster[2] and Ines Lindner[3]

[1,3]*Department of Econometrics and OR, VU University Amsterdam, The Netherlands*
[2]*Amsterdam School of Economics, University of Amsterdam, The Netherlands*

Keywords: *Communication and Coordination, Social Network, Action as Communication Device, Perfect Bayesian Equilibrium*

We consider how the structure of the underlying social network can affect the way people make decisions about collective action. Suppose there are n senior vice-presidents at a company, each of whom must decide whether to propose replacing the unpopular CEO at the next day's board meeting. It would be disastrous to do so without sufficient support from the others. Each person has a personal threshold which represents her willingness to participate. A threshold of μ means "I will show up for the protest if I am sure that at least μ people in total (including myself) will protest."

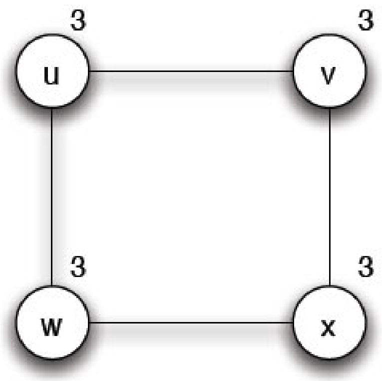


Fig. 1

The links in the social network represent ties of trust. Every player is informed about the network (who is linked to whom), but - due to the risky nature of the protest - is only informed about the threshold of her neighbors. Given a network and a threshold distribution, how should we reason about what is likely to happen? Consider the following 4 players example in Figure 1. which is taken from Chwe [1]. Assume each agent's threshold is $\mu = 3$. 1914b.jpg Each node in the network has a threshold for participation, but only knows the threshold of itself and its neighbors. Consider the situation from u 's perspective (it is symmetric for all nodes). She knows that each v and w have a threshold of 3 such that the three of them could safely protest. She also knows, however, that v and w don't know each others threshold, and so they can't engage in the same reasoning that she can. Chwe [1] argues that it is not safe for u to protest for the following reason. Since u doesn't know x 's threshold it could be very high, say 5, if x would never - under any circumstances - join the protest. In that case w, seeing neighbors with thresholds of 3 and 5, would not join the protest. The same holds for v. Anticipating this behavior u does not initiate protest. Interestingly, all players have threshold 3 and know that there is sufficient support in the network but no one can be sure that the others know that. We are interested in the game when it is played sequentially. Here, the action of u can represent a communication device. Since w doesn't know v's threshold, and u knows that, the initiation of protest could signal to w that the threshold of v is not higher than 3.

Now assume x has threshold 5 such that he never joins the protest. Interestingly, if we now decrease u's threshold collective action of the remaining 3 players is less likely to happen. Note that v and w are not linked such that they must rely on the signal of u to draw conclusions about each other. Fix u's threshold at $\mu_u = 2$. The problem is that the action of u is not a reliable signal anymore as u is already satisfied with participation of just one of her neighbors. This is how v could reason: "u would also revolt if I am led to believe that w's threshold is at most 3. If this were the case u will also revolt when μ_w is 4. Hence, I need not belief that $\mu_w \leq 3$ and u can revolt only when $\mu_w = 2$. But then it is in my interest to join if u revolts ". Note that this cycling vanishes if we make the link between u and v one-directed such that v can observe u but not the other way around. Here, there is no way u can trick v into action. Hence by restricting communication player v is more likely to join the protest.

We analyze these games with the framework of a sequential binary decision game with incomplete and imperfect information. The equilibrium concept under consideration is that of a perfect Bayesian equilibrium. We characterize the equilibria for the games arising from general classes of networks. In particular, we are interested in characterizing games with a full collective action equilibrium. This research contributes to the broad literature on diffusion of behavior and coordination in networks (for an overview see e.g. Chapter 7-9 in Jackson [3]). It can be seen as extension of the work of Chwe [1] in the sense that players communicate with each other not only by direct contact, but also indirectly through the actions they have taken. By adding this kind of implicit communication, we show that the condition of achieving collective action may differ from the model without implicit communication as in [1] and [2].

References

[1] Chwe, M. S.-Y. (1999), Structure and strategy in collective action, American Journal of Sociology, 105(1), 128--156.

[2] Chwe, M. S.-Y. (2000), Communication and coordination in social networks, Review of Economic Studies, 67, 1--16.

[3] Jackson, M.O. (2010), Social and Economic Networks, Princeton University Press.

Equilibrium in Secure Strategies in the Bertrand-Edgeworth Duopoly Model

Mikhail Iskakov[1] and Alexey Iskakov[2]

Trapeznikov Institute of Control Sciences RAS
Russia
mih_iskakov@mail.ru[1]
isk_alex@mail.ru[2]

Keywords: *Bertrand-Edgeworth Duopoly, Equilibrium in Secure Strategies, Capacity Constraints*

We consider a model of price setting duopolists with capacity constraints originated in papers of Bertrand (1883) and Edgeworth (1925). We consider the market for some homogeneous product with a continuous strictly decreasing consumer's demand function $D(p)$ here are two firms in the industry $i = 1, 2$ each with a limited amount of productive capacity Si such that $D(0) > S_1 + S_2$ Firms choose prices pi and play non-cooperatively. The firm quoting the lower price serves the entire market up to its capacity and the residual demand is met by the other firm. All consumers are identical and choose the lower available price on a first-come-first-serve basis. Following Shubik (1959) and Beckmann (1965) we assume in our analysis that the residual demand to the firm quoting the higher price is a proportion of total demand at that price. If duopolists set the same prices firms share the market in proportion to their capacities. Formally we define the payoff functions of players to be:

$$u_1(p_1, p_2) = \begin{cases} p_1 min\{S_1, D(p_1)\}, p_1 < p_2 \\ p_1 min\left\{S_1, \dfrac{S_1}{S_1 + S_2} D(p_1)\right\}, p_1 = p_2 \\ p_1 min\left\{S_1, \dfrac{D(p_1)}{D(p_2)} \max\{0, D(p_2) - S_2\}\right\}, p_1 > p_2 \end{cases}.$$

The concept of Equilibrium in Secure Strategies (EinSS) was proposed in (Iskakov M. (2005) and Iskakov M., Iskakov A. (2012)) as the generalization of Nash equilibrium. Below we provide principal definitions of EinSS for the game $G = (p_i^{\bullet}, u_i^{\bullet},\ \ i \in N)$. The concept of equilibrium is based on the notion of threat and on the notion of secure strategy.

Definition 1. **A threat** of player j to player i at strategy profile s is a pair of strategy profiles $\{p,(p^{'}_{j}, p_{-j})\}$ such that $u_j(p^{'}_{j}, p_{-j}) > u_j(p)$ and $u_i(p^{'}_{j}, p_{-j}) < u_i(p)$ The strategy profile s is said **to pose a threat** from player j to player i.

Definition 2. A strategy p_i of player i is **a secure strategy** for player i at given strategies p_{-i} of all other players if profile s poses no threats to player i. A strategy profile p is **a secure profile** if all strategies are secure.

In other words a threat means that it is profitable for one player to worsen the situation of another. A secure profile is one where no one gains from worsening the situation of other players.

Definition 3. **A secure deviation** of player i with respect to p is a strategy p'_i such that $u_i(p^{'}_{i}, p_{-i}) > u_i(p)$ and $u_i(p^{'}_{i}, p'_{j}, p_{-ij}) \geq u_i(p)$ for any threat $\{(p^{'}_{i},(p'_{j}, p^{'}_{j}, p_{-ij})\}$ of player $j \neq i$ to player i.

There are two conditions in the definition. In the first place a secure deviation increases the profit of the player. In the second place his gain at a secure deviation covers losses which could appear from retaliatory threats of other players. It is important to note that secure deviation does not necessarily mean deviation into secure profile. We assume that the player with incentive to maximize his or her profit securely will look for secure deviations.

Definition 4. A secure strategy profile is an **Equilibrium in Secure Strategies (EinSS)** if no player has a secure deviation.

It is well known that the model of Bertrand-Edgeworth may not possess Nash equilibrium (see e.g. d'Aspremont and Gabszewicz (1980)). Our aim is to solve original problem in terms of the equilibrium in secure strategies. It coincides with the Nash Equilibrium when Nash Equilibrium exists and takes into account the intention of players to maximize their profit under the condition of security against the actions of other players. This approach reflects the natural logic of behavior of players in this model. We will show that in some of these cases there is an EinSS. However for some (big enough) capacities EinSS does not exist either. We prove the following basic result:

Proposition. Let the receipt function $pD(p)$ be strictly concave and reach its maximum at p_M. Then in the game of Bertrand-Edgeworth with the mentioned above payoff functions there is an $EinSS(p^*; p^*)$, where $D(p^*) = S_1 + S_2$, if and only if

$$\begin{cases} arg \max\limits_{p>0} \left\{ pD(p) - S_1 \right\} \le p^* \\ arg \max\limits_{p>0} \left\{ pD(p) - S_2 \right\} \le p^* \end{cases}.$$

If $p^* \ge p_M$ it is a Nash equilibrium. There are no other EinSS in the game.

References

[1] Bertrand J. (1883), Review of Cournot's 'Rechercher sur la theoric mathematique de la richesse', Journal des Savants, 499-508.

[2] Edgeworth F.M. (1925), Papers Relating to Political Economy I, (London: Macmillan).

[3] Iskakov M.B. (2005), Equilibrium in Safe Strategies, Automation and Remote Control, 66, 3, 465-478.

[4] Iskakov M., Iskakov A. (2012), Solution of the Hotelling's game in secure strategies, Economics Letters, 117, 115-118.

[5] d'Aspremont C., Gabszewicz J. (1980), On Quasi-Monopoliies, (CORE Discussion Paper, 8011, Universite Catholique de Louvain).

Resource Monotonic Allocations and the Core in Joint Investment Problems

Josep Izquierdo[1] **and Carlos Rafels**[2]

[1,2]*University of Barcelona, Spain*
[1]*jizquierdoa@ub.edu*
[2]*crafels@ub.edu*

Keywords: *Cooperative Game, Core, Resource Monotonicity, Coalition Monotonicity*

A joint investment cooperative problem is described as the one where agents are endowed with some amount of input (resources, labor, capital,...) so that they can pool them and obtain some amount of output through a technology (production function) with increasing returns to scale. This is a simple but interesting model of a one input-one output production system, already quoted by Lemaire (1984,1991), Mas-Colell et al. (1986) and Izquierdo and Rafels (2001).

The increasing returns provide incentives to cooperate and the core of this cooperative situation turns out to be always nonempty. That is, we can found a feasible and efficient allocation of the total output obtained such that no subcoalition of agents can block it upon. The proportional distribution with respect to the amount of input contributed arises as a natural distribution within the core.

In this paper we address the study of the behavior of the core of this model when some agents vary their contributions of the amount of input; we study resource monotonicity properties of the core.

The monotonicity of the core has been already studied in a general cooperative game theory framework by Young (1985). This author states the incompatibility between the requirement for a solution of lying in the core of the game (core selection) and the fact that the solution increases the payoff to the members of some coalition whenever the worth of this coalition has increased while the worth of other coalitions remain the same (coalitional monotonicity). Meggido (1974) and Calleja et al. (2009) have studied monotonicity properties with respect to the worth of the total coalition. Other authors have examined monotonicity in restricted domains like convex games (Hokari, 2000) or veto balanced games (Arin and Feltkamp, 2005).

We first concentrate on the resource monotonicity of the core when only one agent increases his or her contribution. In this case, the core behaves monotonically, that is, any core allocation of the initial problem can be represented in the core of the new problem so that the payoff to that agent increases. Surprisingly, this property does not hold when two or more agents increase their contributions at the same time. As a consequence, we determine a necessary and sufficient condition for an allocation in the core of the problem to fulfill resource monotonicity.

Furthermore, we also study a necessary and sufficient condition that guarantees that not only the payoff of the agents that have increased their contribution increases, but also guarantees that the payoff of the rest of agents will not be lowered; we call this property strong resource monotonicity. This analysis is important in dynamic frameworks where an agreement on a previous core allocation serves as “staquo quo” for negotiating modifications on the contributions of players. Finally, we state a sufficient condition on the production function that ensures that, for any core allocation and any increasing of the contribution of agents, the strong resource monotonicty property holds.

Comparison of the χ -value and the Shapley-value from an Economic Point of View with Respect to Fair Distribution of Collectively Earned Profits

Susanne Jene[1] **and Stephan Zelewski**[2]

[1,2]University of Duisburg-Essen, Germany
[1]susanne.jene@pim.uni-due.de
[2]stephan.zelewski@pim.uni-due.de

Keywords: *Cooperative Game Theory, Shapley-Value, Chi-Value, Fairness*

Most scientific publications on the subject of solution concepts for cooperation problems only analyze these concepts from a mere mathematical game theoretic point of view.But it is often disregarded whether these mathematical solution concepts can be put into economic practice. One example of a possible use of such solution concepts for cooperation problems in economic practice is the fair distribution of collectively earned profits in a cooperation of legally independent corporations. This contribution yields to analyze whether solution concepts of cooperative game theory are able to solve thispractical problem not only in a formally correct but also in an economically adequate manner.

In the first part of this contribution two mathematical solution concepts of cooperative game theory are compared from formal point of view. The first of these two cooperative solution concepts is the widely known Shapley value that can be described as a rather "classic" solution concept of cooperative game theory. The second is a younger, more innovative solution concept called χ -value. These two cooperative solution concepts are compared relating to the conditions and assumptions they are based on and the characteristics of their resulting solutions, e.g. the stability of the solution. This comparison is made from a mostly game-theoretic point of view.

In the second part a practical example is used to illustrate the calculation of these two solution concepts to analyze them from an economic perspective afterwards. Therefore criteria for a successful use of game theoretic solution concepts applied on distribution problems in economic practice are introduced. These criteria concern for example information requirements. Special attention is drawn to the fairness aspect because a solution concept can only be successfully used in practice if all business partners accept a solution for a distribution problem as fair.

The findings of this contribution are of threefold kind. Firstly, the variety of solution concepts and their results is shown by the comparison of the Shapley value and the χ - value. Secondly, with the help of the practical example it is revealed which information requirements and which other criteria have to be fulfilled to use the presented solution concepts of game theory in economic practice. Thirdly, it is analyzed whether one of the focused solution concepts is likely to be put in practice successfully.

Extensive Partial Cooperative Game with Multi-Coalition Structure

Ji Zhihong[1]**, Gao Hongwei**[2] **and Hu Jinsong**[3]

China Qingdao University, China
[1]gaohongwei@qdu.edu.cn
[2]gaosai@public.qd.sd.cn
[3]cmgta2007@163.com

Keywords: *Multi-Coalition, Partial Cooperative Game, Cooperative Function, Back-Ward Induction*

We dene the new cooperative function and get the extensive partial cooperative game with multi-coalition structure. By backward induction we build the concept of solution in the partial cooperative game and get the corresponding optimal path. The model in this paper overcomes the limitation in the classic game models in which only simple coalition structure can be formed at any node on the game tree.

Journals in Game Theory

INTERNATIONAL GAME THEORY REVIEW

WORLD SCIENTIFIC

Bargaining in Cooperative Games

Dominik Karos

Department of Economics and Statistics, Saarland University
Germany
doka@mx.uni-saarland.de

Keywords: *TU Games, Payoff Configurations, Bargaining*

We consider transferable utility games and introduce a new method to allocate payoffs to players. In general, there are two possibilities to do so. The first method is to find an allocation under the presumption that the grand coalition forms, the second one is to specify an allocation for each partition of the player set. The first one includes classical solutions as the Core [5] or the Shapley value [7]. Solutions of the second type contain for instance the Kernel [4] or the Bargaining set [1]. We will follow the tradition of the latter one.

Our motivation is the following: Consider a player i in a player set N and assume that there is a partition of the set $N \setminus \{i\}$. If i joins any coalition S of this partition or stays alone, it has two effects. First, he is making a marginal contribution to the value of the coalition he joins. Second, he has opportunity costs as he is not joining the other coalitions. The first effect can easily be calculated by the characteristic function of the game. However, for the second one we need some work to do. A vector x which specifies for each player his payoff in each partition is called a payoff configuration. Hence, if a payoff configuration is given, we know for each player in each coalition in each partition his marginal contribution and opportunity costs. If we consider now a coalition S of worth $v(S)$, we are facing a bargaining problem [6]: Players in S have to allocate the value $v(S)$ among them and use a disagreement point to do so. The disagreement point itself depends on marginal contribution and opportunity costs. It is not guaranteed that the sum of disagreement points is less than the coalitions value, hence, in some case we consider a bankruptcy problem [3] rather than a bargaining problem.

We see that each payoff configuration specifies a set of bargaining problems, namely one bargaining problem in each coalition of each partition. The solutions of these bargaining problems are payoff configurations on their own. Hence, bargaining leads to a payoff configuration, and a payoff configuration leads to a new bargaining problem. The natural question which arises is the following:

Is there a payoff configuration x *such that* x *is the solution of the bargaining problem defined by* x *?*

Obviously, the answer depends on the bargaining solution we chose to solve the bargaining problems. Therefore, we call such x stable with respect to the respective bargaining solution. The interpretation is straightforward: Given a stable payoff configuration x, players will not re-bargain their payoffs because re-bargaining would lead to x. We show that for all bargaining solutions which satisfy very weak assumptions such a payoff configuration exists. We analyze the properties of these payoff configurations depending on the properties of the underlying bargaining solution.

After we have found a stable payoff configuration x we can transform the original game into a hedonic game with externalities [2]. In the last part of the paper we investigate whether or not we can find stable partitions in these games. We will show that under weak conditions we can guarantee the existence of a partition into individually and internally stable coalitions.

References

[1] R. Aumann and M. Maschler, The Bargaining Set for Cooperative Games, Advances in Game Theory, 1964, pp. 443-476.

[2] F. Bloch and B. Dutta, Formation of Networks and Coalition, Handbook of Social Economics, 2011, pp. 729-779.

[3] I. Curiel and M. Maschler and S. Tijs, Bankruptcy Games, Mathematical Methods of Operations Research 31 (1987), 143-159.

[4] M. Davis and M. Maschler, The Kernel of a Cooperative Game, Naval Research Logistics Quarterly 12 (1965), 223-259.

[5] J. von Neuman and O. Morgenstern, Theory of Games and Economic Behavior, Sixtieth-Anniversary Edition, Princeton University Press, 2004.

[6] J. Nash, The Bargaining Problem, Econometrica 18 (1950), 155-162.

[7] L. Shapley, A Value for n-Person Games, Contributions to the Theory of Games, 1953, pp. 307-317.

The Axiomatization of the Prenucleolus for Some Classes of Veto-Balanced Games

Ilya Katsev[1], Rene van Den Brink[2] and Gerard van der Laan[3]

[1]*St. Petersburg Institute for Economics and Mathematics, Russian Academy of Sciences, Tchaikovsky st. 1, 191187 St. Petersburg, Russia*
katsev@yandex.ru

[2,3]*VU University Amsterdam, Netherlands*
[2]*j.r.vanden.brink@vu.nl*
[3]*g.vander.laan@vu.nl*

Keywords: *Prenucleolus, Veto Game, Sobolev Theorem, Consistency, Peer-Group Game*

This talk postulates a natural question: what weaker analogous of Sobolev theorem (1975) can be proved? This theorem (improved by Orshan at 1994) characterized the prenucleolus by three properties: ETP (equal treatment property), COV (covariance) and CONS (consistency). Two firsts looks very natural and probably cannot be removed.

So we are interesting in weaker variants of Davis-Mashler consistency property. There is an evident way to make such property: to create some restriction on players which can be removed from the game. But what kind of restriction can we consider? This talk proposes the next answer: we can remove a player if after it the game remains a member of some class of games.

For example, if we consider the class of veto-rich games, then the veto-player cannot be removed, because the resulted game will be outside of this class.

In this talk we will prove the Sobolev theorem for the class of veto-balanced games, also we will show that for some subclasses of this class the proposition of theorem is not true and discuss the question what analogues of this theorem can be proved.

Overconfidence, Imperfect Competition, and Evolution

Karen Khachatryan

Department of Economics, Stockholm School of Economics,
Box 6501, Sveavagen 65, S-113 83 Stockholm,Sweden.
karen.khachatryan@hhs.se

Keywords: *Overconfidence, Imperfect Competition, Product Differentiation, Evolution, Market Selection*

This study explores whether market competition between firms owned and run by managers favors overconfident managers. We study this question in a linear duopoly setting with differentiated products. The main result is that when there is complete information about the competitor's type, evolutionary market selection forces will always favor a positive degree of managerial overconfidence. This result is robust to both the form of the strategic interaction and the nature of product differentiation. We also study the case of incomplete information about the competitor's type under quantity competition and show that evolutionary forces may still favor overconfident managers if market selection is driven by relative rather than absolute profit performance.

The Average Covering Tree Value for Directed Graph Games

Anna Khmelnitskaya[1], Ozer Selcuk[2] and Dolf Talman[3]

[1]*Saint-Petersburg State University, Russia*
a.khmelnitskaya@math.utwente.nl

[2,3]*Tilburg University, Netherlands*
[2]*ozeeer@gmail.com*
[3]*talman@uvt.nl*

Keywords: *TU game, Directed Communication Structure, Marginal Contribution Vector, Myerson Value, Average Tree Solution, Stability*

In classical cooperative game theory it is assumed that any coalition of players may form. However, in many practical situations the collection of feasible coalitions that can be formed is restricted by some social, economical, hierarchical, or technical structure. The study of games with transferable utility (TU) and limited cooperation represented by means of undirected communication graphs was initiated by Myerson. In an undirected communication graph on the set of players, a link between two players is interpreted as the players' ability to communicate bilaterally with each other and therefore only connected coalitions are feasible. For such games, Myerson introduces the Myerson value which is equal to the Shapley value of the induced restricted game.

In this paper we consider communication structures introduced by means of directed graphs (digraphs) which represent partial orderings of the players. For a directed link in an arbitrary digraph there are two possible different basic interpretations. One interpretation is that a link is directed to indicate which player has initiated the communication but at the same time it represents a fully developed communication link where players are able to communicate in both directions with each other. In such a case it is natural to assume that there is no subordination of players and to focus on component efficient values. Another interpretation of a directed link assumes that a directed link represents the only one-way communication situation. In this case we have again different possibilities for the interpretation of a directed link. The first option is when the communication between players is supposed to be possible only along the directed paths in the digraph, for example a flow situation. Another option is to assume that the digraph represents the subordination of players such that after each player any of his subordinates may follow as long as this does not hurt the total subordination among all players prescribed by the digraph. An example of such a situation is a sequencing problem when the tasks that have to be performed are not necessarily linearly ordered but the ordering of the tasks is represented by some directed graph.Suppose at every moment only one task can be performed. When some task is completed, the next task can be any of the tasks that are immediate successors in the digraph or one among those of which the performance is independent of the task and does not block the performance of any immediate successor of that task.

In this paper we abide by the latter interpretation of a directed link. The main advantage of this approach is to introduce a single-valued solution for the class of TU games with limited cooperation represented by directed communication graphs which is component efficient. To define such a solution, we first introduce for any directed graph the set of so-called covering trees. The root of a covering tree of a digraph is one of the undominated players (nodes) of the digraph. The root has one of the undominated players in every component of the remaining players as immediate successors in the tree. On its turn each of these latter players has as immediate successors one of undominated players in each subcomponent of the remaining players in the component the player belongs to, and so on. Since every digraph on a finite set has at least one undominated node, the collection of covering trees defined in this way is nonempty. In every covering tree of a digraph, the dominance relation between players in the graph is preserved. The average covering tree value of a TU game with digraph communication structure is the average of marginal contribution vectors that correspond to all covering trees of the underlying digraph. We also give a convexity-type condition under which the solution is an element of the core and therefore cannot be blocked by any subset of connected players.

The Shapley Value for TU Games with Oriented Graph Cooperation Structures

Anna Khmelnitskaya[1], **Ozer Selcuk**[2] **and Dolf Talman**[3]

[1]*Saint-Petersburg State University, Russia*
a.khmelnitskaya@math.utwente.nl

[2,3]*Tilburg University, Netherlands*
[2]*ozeeer@gmail.com*
[3]*talman@uvt.nl*

Keywords: *Cooperative TU game, Oriented Digraph Cooperation Structure, the Shapley Value*

A situation in which a finite set of agents can obtain certain payoffs through cooperation can be described as a cooperative game with transferable utility or simply a TU game. In classical cooperative game theory it is assumed that any coalition of players may form. However, in many practical situations the collection of coalitions that can be formed is restricted by some social, economical, hierarchical, communication, or technical structure. The study of TU games with limited cooperation introduced by means of undirected communication graphs, further called graph games, was initiated by Myerson (1977). The best know solution concepts for TU games and graph games respectively are the Shapley value (Shapley, 1953) and Myerson value (Myerson, 1977), which in turn is the Shapley value for the Myerson restricted game. Behind the Shapley value stays the following probabilistic distribution mechanism: the players enter a room one by one in arbitrary order given by some permutation of players and each player as a payoff gets his expected marginal contribution to all his predecessors who entered the room before him, while all permutations are equally probable.

In the paper we consider a class of oriented graph games for which an oriented graph, that is a directed graph without directed cycles of length 2, prescribes subordination of players. It is assumed that for oriented graph games the cooperation among players is not restricted, i.e., all coalitions are feasible, but the subordination structure puts limitation on the coalition formation when the players want to form bigger coalitions, in particular, the grand coalition – each player may join only a coalition not containing his superiors. A typical example of such a situation is given by two firms, a market leading big firm and a small firm in some industry producing a homogenous good. The problem is how to distribute the monopoly profit after merging. The alternative for merging is the Stackelberg duopoly game where the big firm is the leader and receives by not cooperating the Stackelberg leader profit and the other firm receives the Stackelberg follower profit. This situation can be modeled as an oriented graph game where the big firm dominates the small firm. On the class of oriented graph games we introduce a new solution concept, the so-called Shapley value for oriented graph games. Similar to the Shapley value for TU games we define the Shapley value for oriented graph games assigning to each player as a payoff the average of the player's marginal contributions to coalitions of his predecessors, but now with respect not to all but to all consistent with the given oriented graph permutations of the player set. A permutation is consistent with an oriented graph if it does not violate the subordination of players prescribed by the graph.

We show that the Shapley value for oriented graph games satisfies efficiency, linearity, restricted null player property, restricted equal treatment property and is independent of inessential links. We also introduce a convexity type condition that guarantees the Shapley value for an oriented graph game to belong to the core of the given TU game. This convexity type condition is weaker than convexity that for TU games guarantees stability of the Shapley value.

Two-Factor Model of Monopolistic Competition and Market Integration

Sergey Kichko[1], **Sergey Kokovin**[2] **and Evgeny Zhelobodko**[3]

[1]*NRU HSE, Russia*
[2] *NSU, Russia*
[3]*Sobolev Institute of mathematics, Russia*

Keywords: *Monopolistic Competition, Market Integration, Variable Elasticity of Substitution*

We develop a two-factor, one-sector monopolistic competition model involving a variable elasticity of substitution. The goal is to show how market integration affects firm pricing policies, factor prices, and individuals' welfare. We study it with comparative statics of equilibria w. r. t. the country factor endowments that imitates market integration (shift from autarky to free trade).

The questions of gains for labor and capital from market integration or trade liberalization are traditional in international trade and agglomeration theories. Do workers or/and capital owners benefit in both countries from lowering the trade barriers, or these gains are subject to certain properties of demands, costs and market structure? What are the mechanisms of these gains/losses, what happens with number of firms, outputs, prices, and interest rates?

In our paper we develop a variable elasticity of substitution monopolistic competition model with only one sector and two production factors - labor and capital. Though having in mind trade and agglomeration topics, at this stage we develop a model imitating complete market integration. It means studying a closed economy and behavior of its equilibrium variables in response to increase in the market size (number of workers or capital owners), interpreted as integration of two countries, identical in preferences and technology. Besides, our comparative statics may mean the cross-countries comparisons.

There are two types of individuals, workers and capital-owners that share identical preferences and own one unit of labor and capital correspondingly. Producers bear a fixed cost in capital and a variable cost measured in labor. The utility function is additive, and taken from a wide class of functions that included HARA and CES functions

Our results include price changes. Increasing in labor endowment yields increasing both product and capital prices. On the other hand, when capital endowment increases product and capital prices are decreasing functions. In case increasing labor endowment impact on individuals' welfare is clear. Welfare of each worker decreases and welfare of each capitalist increases. In opposite, under increasing capital supply workers' welfare increases but welfare of capital owners behaves ambiguously. Not clear behavior of capital owner's utility could be explained as follows. From the one hand, increasing in capital endowment decreases capital price through more tough competition on capital market. From another hand, this increasing in capital endowment increases number of avaliable varieties at the market and it increases capitalist's welfare. The additional result is comparative statics of individual consumption of both agent types. Under increasing labor endowment capitalist individual consumption increases and workers individual consumption decreases. When capital endowment increases capitalist individual consumption decreases but behavior of workers individual consumption is ambiguous.

The next steps of research are:

- Study three agent types model adding middle class. Each middle class agent owns one unit of labor and $\alpha < 1$ units of capital (less than each capitalist ownes). Comparative statics w.r.t. number of middle class agents and share of capital α could show significancy level of middle class on equilibrium variables.

- Consider a trade model and study general trade equilibrium with Heckscher-Ohlin approach of differences in factor endowments between countries that involving in trade.

References

[1] Behrens, K. and Y. Murata, 2007, General equilibrium models of monopolistic competition: a new approach, Journal of Economic Theory 136(1): 776-87

[2] Brakman, S. and B.J. Heijdra, 2004, The Monopolistic Competition Revolution in Retrospect. Cambridge: Cambridge University Press

[3] Dixit, A.K. and J.E. Stiglitz, 1977, Monopolistic competition and optimum product diversity, American Economic Review 67: 297-308

[4] Krugman, P., 1979, Increasing returns, monopolistic competition, and international trade, Journal of international economics 9, 151-75.

[5] Ottaviano G.I.P., Tabuchi T., Thisse J.-F., 2002, Agglomeration and trade revisited. International Economic Review, 43: 409-36.

[6] Baldwin, R., Forslid R., Martin Ph., Ottaviano G.I.P., and Robert-Nicoud F., 2003, Economic Geography and Public Policy. Princeton, NJ: Princeton University Press.
[7] Martin P., Rogers C.A. Industrial location and public infrastructure, Journal of International Economics, 39, 1995.
[8] Helpman E., Krugman P. R. Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition, and the International Economy, MIT Press, 1987
[9] Bernard A. B., Jensen J.B., Redding S., Schott P.K. Firms in International Trade, CEP Discussion Paper No 795, 2007

Informed Middleman and Asymmetric Information

Irina Kirysheva

EUI, Italy
irina.kirysheva@eui.eu

Keywords: *Middleman, Asymmetric Information, Bilateral Deviations*

In this paper I look at the game of two-sided asymmetric information where agents make a collaboration decision not knowing types of each other. Intermediary has full knowledge about the types of agents and can make a decision that brings information to some types. However, once he puts information on the table agents are not committed to pay him, what undermines his incentive to participate in the first place. I find that intermediary still is welfare-improving and restores efficiency. He either brings information to the most vulnerable type or to nobody.

The situation is drastically different when I look at two informed intermediaries that compete in prices. In this case there is no equilibrium in pure strategies. Nonexistence in its sense is similar to that of screening models, though standard ways to deal with it (e.g. reactive equilibrium concept) do not work here. Once the competition between intermediaries sufficiently increases equilibrium happens to exist again. In this equilibrium there is partial specialisation between intermediaries - every pair of intermediaries set different wage and concentrate on some particular match. My model gives interesting insights into the market of talent agents. The concept of PBE stable to bilateral deviations is developed and applied in the paper.

Generalized Model of "Vanishing Cities"

Sergey Kokovin[1,2,3], Maxim Gorynov[4] and Evgeny Zhelobodko[1,2,3]

[1]*Sobolev Institute of Mathematics SB RAS, Novosibirsk, Russia*
[2]*NRU Novosibirsk State University, Novosibirsk, Russia*
[3]*NRU Higher School of Economics, St. Petersburg, Russia*
4CERGE University, Prague

Keywords: *Urban Economics, City System, General Preferences, Agglomeration Forces*

Motivation, literature and our goal. The *patterns of urbanization* in economic history and in contemporary economy allow for various explanations (see Handbook of Regional and Urban Economics, 2004). In particular, Anas (2004) developed a monopolistic competition model with CES utility describing such evolution. There the socially optimal distribution of population among cities of any closed country (or the globe) followed a dispersion patterns: each city size always decreases under growing population, because the number of cities grow disproportionally fast. Similar pattern arise when the trade costs decrease (globalization). The reason is that the only agglomeration force here is the economies of scale inherited in monopolistic competition, whereas related marginal payoff to a big city *declines* with the population size or globalization. By contrast, the dispersion force in the model – large commuting cost as a price of big city – remains unaffected by population of the globe or international trade. However, the model prediction – only vanishing cities – clearly contradicts the facts of historical urbanization. Anas used this contradiction in his reasoning about proper urban theory as an impossibility theorem: *urbanization cannot be explained by firm-level economies of scale only*, instead city-level economies should serve as the main explanation: externalities, public goods, etc.

Our hypothesis was that this methodological conclusion was incorrect, just vanishing cities followed from too restrictive simplifying assumptions of the Anas's model; notably, constant elasticity of substitution (CES) and very specific space representation: all the cities trade with each other through the "center", i.e., common hub. This suspicion motivated us to generalize the model in these two and other directions. Our hypothesis turned out wrong, we justified Anas's idea in broadersetting and we looked for new effects.

The methodological background for such generalization is the recent method of studying general monopolistic competition with variable elasticity of substitution: Zhelobodko et al. (2012), who extend the monopolistic competition model from Dixit and Stiglitz J. (1977), Krugman (1979). Expanding the new method from a closed economy – to the system of cities, we obtained several results.

International trade model with n cities.

Assume, population *P* of the world (identical consumers) is divided equally among n cities, so that $N=P/n$ is the city size. Whenever each (*i*-th) city produces l varieties ($k \le l$) of the differentiated good, a consumption of this variety by a consumer located in city j is denoted as x_{ikj}, and the price is p_{ikj}. Each citizen's utility maximization in this city i takes the form (each integral denotes summation across cities/varieties, to make n,l continuous variables):

$$\left\{ \int_0^n \int_0^l u\left(x_{ikj}\right) dkdj \quad \to \max_{x \ge 0} \; : \; \int_0^n \int_0^l p_{ikj} x_{ikj} dkdj \le I \right. ,$$

Here I is a disposable income that remains from the endowment of 1 unit of labor – after subtracting the commuting cost $C(N)$, that increases with the city size. By symmetry of cities, indices i,j can be dropped, but the varieties are divided into Home (domestic) consumption x^h and Foreign (imported) consumption x^f, the main variables of trade equilibrium.

Each of (identical) producers in each city bears marginal costs c, fixed cost f, and solves w.r.t. (x^h,h^f) the profit maximization problem

$$\pi = N(p\left(x^h\right) - c)x^h + (n-1)N(p\left(x^f\right) - tc)x^f - f$$

Here t is the (iceberg) trade cost coefficient and $\boldsymbol{p}$ denotes the inverse demand function derived from the consumer's optimization. Under given P,n, a trade equilibrium is a bundle $\left(x^H, x^F, l\right)$ satisfying all FOC, SOC, budgets, free entry condition $\pi=0$ and labor balances. (Second-best) social optimum is such number of cities n that maximizes each consumer's welfare under given world population P and respective trade equilibrium. This model differs from Anas's one by general utility u(x) instead of $u=x^r$ and general increasing commuting cost $C(N)$ instead of a square root.

Main Results. (i) We find in our generalized model the same Anas's effect of vanishing cities: when there is more than 1 city, their optimal size *decreases* w.r.t. world population P and/or decreasing trade cost t, tending to the "minimal admissible city size". This fact confirms, that agglomeration models generally need some city-level economies of scale, this need being independent of the functional forms of utilities and commuting costs.

(ii) We expanded the same conclusion to a version of the model where the cities trade with each other not through the common hub but in natural circular space.

(iii) By simulations with a broad class of utilities, we expanded the same conclusion to a version of the model where the cities sell their manufacturing goods not only to each other, but also to some immobile agricultural population, evenly distributed on the circular world.

(iv) The by-product of this study is the hypothesis (and a sketch of the proof) that the system of cities governed by their selfish "city developers" and migration – arrives at the same (second-best) social optimum as the world governed by a social planner, even under very general specifications of utilities.

We gratefully acknowledge the financial support from the Russian government grant No 11.G34.31.0059, Russian Foundation for Basic Research grant No 12-06-00174-a.

References

[1] Anas A. Vanishing cities: what does the new economic geography imply about the efficiency of urbanization? // Journal of Economic Geography 4, 2004, pp 181-199

[2] Dixit A., Stiglitz J. (1977) "Monopolistic Competition and Optimum Product Diversity", American Economic Review, V. 67, N. 3, p. 297-308.

[3] Handbook of Regional and Urban Economics, Volume 4 (2004) (edt. J.V.Henderson and J.-F.Thisse), Elsevier.

[4] Krugman P.R. (1979) "Increasing returns, monopolistic competition, and international trade", Journal of International Economics, V.9, p. 469-479.

[5] Zhelobodko E., Kokovin S., Parenti M., Thisse J.-F. (2012) "Monopolistic Competition: Beyond the Constant Elasticity of Substitution", Econometrica, Vol. 80, No. 6. – P. 2765–2784.

The Relationship Between Discrete and Continuous Equilibria in Bargaining Model

Aleksei Kondratev

Institute of Applied Mathematical Research
Karelian Research Center of RAS
185910, Russia, Petrozavodsk, Pushkinskaya str., 11
http://mathem.krc.karelia.ru
kondratev@krc.karelia.ru

Keywords: *Bargaining Model, Seller-Buyer Interactions, Differential Equations for Equilibrium*

We consider a game-theoretic bargaining model with incomplete information related with deals between buyers and sellers. A player (buyer or seller) has a private information about his reserved price. Reserved prices are random variables with known probability distributions. Each player declares a price which depends on his reserved price. If the bid price is above the ask price, the good is sold for the average of two prices. Otherwise, there is no deal. Two types of Bayes equilibrium are derived. One of them is a discrete form, another one is a solution of a system of differential equations.

When concluding deals on the market auctions are held. The participants are the sellers and buyers. There are different mechanisms of such auctions [1-6]. We research bargaining model, suggested by Chatterjee K. and Samuelson W. [1] and developed by Myerson R. [2,3].

This model is a game with incomplete information. Each player (buyer or seller) has private information about his reserved price $b \in [0,1]$ and $s \in [0,1]$ accordingly. Reserved prices have distribution functions $F(s)$, $G(b)$, and density functions $f(s)$, $g(b)$. Players declare prices $S(s)$ and $B(b)$. If $S \le B$ then the good is sold for the average price $(B+S)/2$.

In [6] we derive differential equations for Bayes equilibrium with continuous strategies

$$U'(t) = \frac{1-G(U(t))}{2(t-V(t))g(U(t))}, \tag{1}$$

$$V'(t) = \frac{F(V(t))}{2(U(t)-t)f(V(t))}, \qquad (2)$$

where $V = S^{-1}$, $U = B^{-1}$ are reversed functions.

Now we find finite difference schemes for (1)-(2), giving equations for Bayes equilibrium with piecewise constant strategies.

The work is supported by Russian Fund for Basic Research (project 13-01-91158-ГФЕН_а) and the Division of Mathematical Sciences of RAS (the program "Algebraic and combinatorial methods of mathematical cybernetics and new information system").

References

[1] Chatterjee K., Samuelson W. Bargaining under incomplete information // Operations Research. 1983. Vol. 31. N 5. P. 835--851.

[2] Myerson R., Satterthwait M.A. Efficient mechanisms for Bilateral Trading // Journal of Economic Theory. 1983. Vol. 29. P. 265–281.

[3] Myerson R., Two-Person Bargaining Problems with Incomplete Information // Econometrica. 1984. Vol. 52. P. 461–487.

[4] Klemperer P. The Economic Theory of Auction, Northampton, MA: Edward Elgar Publishing, Inc., 2000.

[5] Mazalov V. V., Tokareva J. S. Equilibrium in bargaining model with non-uniform distribution for reservation prices // Game theory and applications, 2011. Vol. 3. Issue 2. P. 37-49.

[6] Mazalov V. V., Kondratyev A. Y. Bargaining model with incomplete information // Vestnik St. Petersburg University. Ser. 10. 2012. Issue 1. P. 33–40.

Application of Stochastic Cooperative Games in the Analysis of the Interaction of Economic Agents

Pavel Konyukhovskiy[1] **and Alexandra Malova**[2]

[1,2]Saint-Petersburg State University
Russia
[1]aura2002@yandex.ru
[2]alex.malova@yandex.ru

Keywords: *Stochastic cooperative game, Stochastic imputation, Stochastic alpha-core*

Current economic conditions are characterized by quantitative and qualitative changes in the processes of interaction and cooperation of economic entities. This conclusion applies equally to both large corporations and relatively small companies. Problems and difficulties that are produced by the global trend of business climate deterioration, inevitably generate incentives for companies to seek for new forms of cooperation. Constructive analysis of these new forms suggests, itself, the development of economic and mathematical methods of the Cooperative Game Theory. It provides one with an adequate description cooperative interaction in the present world circumstances. Admittedly, a serious obstacle to the practical implementation of cooperative game-based models is the assumption of the characteristic function being a set of determined values/

This problem can be solved by switching from deterministic to stochastic cooperative games. Possible approaches to this class of games are considered in [1,2]. By the stochastic cooperative game we mean a pair of sets $\Gamma = (I, \tilde{v})$, where

- $I = \{1..m\}$ — a number of participants (players);
- $\tilde{v}(S)$ — random variables with known densities $p_{\tilde{v}(s)}(x)$, which are interpreted as the income (utility payments) players obtain participating the corresponding coalitions $S \subset I$.

The imputation in a stochastic cooperative game to a certain level of probability α can be defined as a vector $\mathbf{x}(\alpha) \in R^n$, satisfying the conditions:

(a) $$(\forall i \in I)\ \ \mathbf{P}\{x_i(\alpha) \geq \tilde{v}(i)\} \geq \alpha \tag{1}$$

– the stochastic analogue of individual rationality in the "classical" cooperative game,

(b) $$\mathbf{P}\{\sum_{i=1}^{m} x_i(\alpha) \leq \tilde{v}(I)\} \geq \alpha \tag{2}$$

– the stochastic analog of group rationality.

In essence, in the condition (1) i-th component of vector $\mathbf{x}(\alpha)$ compared with α-quantile $F_{\tilde{v}(i)}(x)$ — distribution function of the random variable $\tilde{v}(i)$. If we put $v_\alpha(i) = F^{-1}_{\tilde{v}(i)}(\alpha)$, then conditions (1)–(2) will take the form

(a) $$(\forall i \in I)\ x_i(\alpha) \geq v_\alpha(i)\,, \tag{3}$$

(b) $$\sum_{i=1}^{m} x_i(\alpha) \leq v_\alpha(I)\,. \tag{4}$$

In terms of modern Risk Management $v_\alpha(i)$ is VaR (value at risk) – the value of the characteristic function for i-th player. Thus, the proposed approach to the definition of concepts imputations offers great opportunities for meaningful interpretation of the subsequent results of studies the properties this class of games and concepts to determine their decisions.

Developing the proposed approach, we can introduce the concept of a stochastic analogy α–core (core in the game $(I,\tilde{v})$ to the level of probability α), defining it as a set of imputations (i.e. vectors $x(\alpha) \in R^m$, that satisfied conditions (3)–(4)), for which performed

$$(\forall S \subset I,\, S \neq \not\subset,\, S \neq I)\ \ x(S) \geq v_\alpha(S) \tag{5}$$

or

$$C_\alpha(\tilde{v}) = \{\mathbf{x} \in R^m \mid \ \ \forall S \subset I,\, S \neq \not\subset,\, S \neq I: \ \ \begin{array}{l} x(S) \geq v_\alpha(S); \\ x(I) \leq v_\alpha(I)\}. \end{array} \tag{6}$$

For the particular case of stochastic cooperative games, in which we assume that $\tilde{v}(S)$ are random normally distributed variables

$$\tilde{v}(S) \in N(v(S), \sigma_S^2)\,, \tag{7}$$

with $\alpha > 0.5$ we have

$$v_\alpha(S) > v(S)\,.$$

Furthermore, by comparing the conditions to be met by the division of belonging core of nonstochastic game with characteristic function $v(S)$, with condition (5), defining them as belonging to α –core of stochastic game, we come to conclusion that

$$x(S,\alpha) > x(S),$$

where $x(S,\alpha) = \sum_{i \in S} x_i(\alpha)$— a sum, distributed among members of the coalition S with imputation $\mathbf{x}(\alpha)$. Thus, with a probability level $\alpha > 0.5$

$$C_\alpha(\tilde{v}) \subset C(v) \qquad (8)$$

or, in other words, α –core encapsulated in the core. Aspects that influence the degree of this restriction are the standard deviation σ_s. Roughly speaking, the grate the value of σ_s, is (i.e. risk characteristics of deviation of realization random values from their expectations) the smaller the α –core becomes.

We are of the opinion that, it is quiet promising field of research to identify the scale which alterations in α affect the volume α –core. In essence, we have an additional tool to analyze the impact of risk factors on the «size of the area», in which there are opportunities to achieve the objective uncontested agreements between potential participants of economic alliances with a given level of probability.

This information is particularly significant and essential for situations in which the results, which collaborating participants obtain, depend on many imponderables.

References

[1] Konyukhovskiy P.V., Solovyeva M.Y. (2010) Stochastic cooperative games as an instrument for modeling of relations of public private partnership.//The Fourth International Conference Game Theory and Management. GTM2010. 28–30 June, 2010. St.Petersburg. P.99–101.

[2] Konyukhovskiy P.V. (2012) The application of stochastic cooperative games in studies of regularities in the realization of large-scale investment projects. // Contributions to Game Theory and Management. GTM2011. St. Petersburg. Graduate School of Management St. Petersburg University. 2012. P. 137–146.

Experiment with Efficient Groves-Ledyard Mechanism for Resource Allocation

Galina Boldyreva[1], Nikolay Korgin[2] and Vsevolod Korepanov[3]

[1,2]*Moscow Institute of Physics and Technology (State University)*
[1,2,3]*V.A.Trapeznikov Institute of Control Sciences of Russian Academy of Sciences*

Keywords: *Resource Allocation, Effective Groves-Ledyard Mechanism, Experiment*

The research is supported by the grant 12-07-31244 of Russian Foundation for Basic Research.

We consider the problem of resource allocation – a limited amount $R \in \mathbb{R}^1_+$ of an infinitely divisible good should be allotted among finite number of agents from the society $N = \{1,...,n\}$. Each agent $i \in N$ has an utility function $u_i(\bullet): \mathbb{R}^1_+ \to \mathbb{R}$:

1. concave and C^2;
2. it is not efficient to allocate all the good to one agent - that is $\forall \{i, j\} \in N^2$

$$\frac{\partial u_i}{\partial x_i}(0) \geq \frac{\partial u_j}{\partial x_j}(R);$$

3. there is exist possibility to transfer utility among agents.

Principal tries to maximize the total utility of all agents (effective resource allotment):

$$\sum_{i \in N} u_i(x_i) \xrightarrow[x \in A]{} \max,$$

but he doesn't know the agent's utility function.

To solve this problem it is proposed mechanism $\rho = (S = \times_{i \in N} S_i, (x,t))$ which is an extension of Groves-Ledyard mechanism (see [1, 2]):

$S_i = \{s_i \in \mathbb{R}^n : \sum_{j \in N} s_{ji} \leq R\}$ – a space of agent's actions,

$$x_i(s) = \frac{1}{n}\sum_{j=1}^{n} s_{ij},$$

$t_i(s) = p_i(s) - \frac{\alpha}{n}\sum_{j=1}^{n} p_j(s)$ – a balanced transfers, where $p_i(s) = \beta \sum_{j=1}^{n} (x_j(s) - s_{ji})^2$.

It was previously shown that the mechanism ρ:

- yields efficient allotment as the only Nash equilibrium [3] in game $\Gamma(\rho) = < N, S, f_{x,t} >$, where $f_{x,t} = \{ f_1, ..., f_n \}$ – profile of preferences of agents determined by their utility profile $u \in U$ and procedure $x(\bullet)$: $f_i(s) = u_i(x_i(s)) - t_i(s)$, $i \in N$;
- Nash equilibrium is reachable in Iterative bargaining process if agents behaviour is Cournot dynamics and holds some inequality on β [3].

We conducted a series experiment sessions for that mechanism with students, using the experimental software Ztree [4].

The games results showed that students' behavior is often different from the Cournot dynamics and game's outcome was effective only in some cases – primarily in cooperation of all students. Among models of behavior were cooperative, constant, Cournot dynamics and some others.

In this paper we present a models of students' behavior observed and properties of the mechanism when this behavior occurs.

References

[1] Groves, T., Ledyard, J. O., Optimal allocation of public goods: A solution to the 'freerider' problem. Econometrica – 1977 – 45 – pp. 783–809.

[2] Arifovic, J., Ledyard, J. O., A behavioral model for mechanism design: Individual evolutionary learning // Journal of Economic Behavior and Organization - 2011 - 78 - pp. 375–395.

[3] Korgin N.A., Efficient mechanism for resource allocation with quadratic payments and its realization via an iterative bargaining process. IFAC MIM '2013 proceedings, forthcoming.

[4] Fischbacher, U., 2007. z-Tree - Zurich Toolbox for Ready-made Economic Experiments. Experimental Economics 10(2), 171-78

Time Consistency of Strategic Alliances: Case Study Approach

Anastasia Koroleva

Graduate School of Management
Russia
anakorol@gmail.com

Keywords: *Strategic alliance, Time Consistency, Case Study*

In recent years the research in the field of strategic alliances has been focused on the problem of assessment of alliance performance. Different indicators of the performance are considered: profitability, alliance duration, alliance termination, achievement of strategic goals and also alliance stability. However, there is no common definition of alliance stability or instability in the scientific community. In the paper a game theory concept of time consistency (dynamic stability) is proposed to be implemented to real world strategic alliances.

In contrast to real life situations, in a direct theoretic problem of time consistency the dynamic cooperative game is supposed to be given. We consider a reverse problem: how to make conclusions about strategic alliance time consistency on the basis of their performance observations along cooperative trajectory. In the paper three cases of strategic alliances are considered: Renault-Nissan, Sony-Ericsson Joint Venture and DaimlerChrysler Company.

Design and Implementation of Optimal Natural Resource Management Systems

Serkan Kucuksenel

METU
Turkey
kuserkan@metu.edu.tr

Keywords: *Optimal Management Systems, Natural Resources, Mechanism Design*

In European fisheries, Maximum Sustainable Yield (MSY) has not been achieved for all economically valuable fish stocks. According to Facts and Figures on the Common Fisheries Policy (2012), only 11 fish stocks in the Atlantic shoreline and 21 fish stocks in the Mediterranean are fished at MSY. Most of the other fish stocks are outside the safe biological limits and overfished. This implies that some of the current fish stocks are not harvested at MSY levels and hence one of the most important environmental objectives of the Common Fisheries Policy (CFP), sustainable fish stock levels, has not been achieved. In the current reform proposals, one of the main topics is related to the achievement of MSY for all fish stocks. According to the CFP reform, all fish stocks should be harvested at MSY levels after 2015. The basic idea behind the reform proposal is that fishing is also an economic activity and hence maximum benefit for all agents in the fishing industry should be achieved given the sustainability requirement of fish stocks.

How can we achieve the MSY for all fish stocks? Firstly, we know that precise data about the structure of a given fish population is required to manage the fish stock at MSY. Then, we show that a well-designed rights-based management system is needed to align the interests of all agents in the fishing industry to implement the MSY. Given that MSY is calculated for a given fish stock, we present a rights-based management mechanism implementing the MSY levels in a simple age structured fish population model with three age classes, and without loss of generality with two fishing agents with different fishing technologies. The agents have different fishing technologies, which affect the by-catch ratio. In other words, the agents target the oldest age class but they also catch young immature fish due to imperfect fishing selectivity. We show that not only biological limitations due to structure of the fish population but also composition of fisheries or different catch technologies should be taken into consideration in determination of maximum catch limits (or property rights). We also show that initial allocation of quotas does matter to achieve the MSY and sustainable fisheries.

Journals in Game Theory

INTERNATIONAL JOURNAL OF GAME THEORY

Editor
Shmuel Zamir

SPRINGER

Switching Lines for Optimal Feedback Control in Game of Two Pursuers against One Evader*

Sergey Kumkov and Valerii Patsko[1]

Institute of Mathematics and Mechanics, Ural Branch,
Russian Academy of Sciences, Ekaterinburg, Russia,
Ural Federal University, Ekaterinburg, Russia,
[1]*patsko@imm.uran.ru*

In this paper, the authors continue the study of a model zero-sum differential game [1,2,3], in which three objects P_1, P_2, and E move in a straight line. The dynamics of the pursuers P_i, $i = 1,\ 2$, and the evader E are described by the relations

$$\begin{aligned} &\ddot{z}_{P_i} = a_{P_i}, && \ddot{z}_E = a_E, \\ &\dot{a}_{P_i} = (u_i - a_{P_i}) / l_{P_i}, && \dot{a}_E = (v - a_E) / l_E, \\ &a_{P_i}(t_0) = 0, |u_i| \le \mu_i, && a_E(t_0) = 0, |v| \le \nu. \end{aligned} \tag{1}$$

Here, z_{P_i} and z_E are the coordinates of geometric positions of the objects; a_{P_i} and a_{P_2} are their accelerations; the controls u_i and v have bounded absolute values; μ_i, ν, l_{P_i}, and l_E are parameters of the problem.

Let us fix two instants T_1 and T_2. At the instant T_i, we compute the miss of the pursuer P_i with respect to the evader E:

$$r_{P_i,E}(T_i) = |z_E(T_i) - z_{P_i}(T_i)|, \quad i = 1, 2. \tag{2}$$

Suppose that the pursuers act together. Therefore, we can join them into one player, which governs the vector control $u = (u_1, u_2)^{\mathrm{T}}$. Let us call it as the first player and let the evader E be the second player. The resultant miss is computed as

$$\phi = \min\{r_{P_1,E}(T_1), r_{P_2,E}(T_2)\}. \tag{3}$$

* This work was supported by Program of Presidium RAS \Dynamic Systems and Control Theory" under financial support of UrB RAS (project No.12-II-1-1002) and also by the Russian Foundation for Basic Research under grant No.12-01-00537.

At any current instant, both players know exactly phase coordinates z_{P_1}, $\dot{z}_{P_1}$, a_{P_1}, z_{P_2}, $\dot{z}_{P_2}$, a_{P_2}, z_E, $\dot{z}_E$, a_E. The first player generating his feedback control minimizes the miss ϕ, the second player maximizes it.

This work deals with constructing optimal strategy of the first player, which is composed of the pursuers, for the following cases of the game: 1) each pursuer has dynamic capabilities exceeding the ones of the evader; 2) each pursuer is ``weaker'' than the evader; 3) one of the pursuers is stronger and another one is weaker than the evader. In all mentioned cases (they are strictly defined by values of the parameters μ_i, ν, l_{P_i}, l_E), the optimal feedback control of the first player can be generated on the basis of switching lines depending on time and located in the two-dimensional state space of one-dimensional coordinates of the forecasted final misses (zero-effort miss coordinates).

Statements are given that characterize stability of the suggested control method with respect to small inaccuracies of numeric constructions of the switching lines and informational errors of measurements of the state position during generation of the control by the first player.

From the point of view of general theory of differential games [4], the obtained results show how in the framework of specific problems, optimal feedback control can be built on the basis of switching lines. These results can be useful during generating and analyzing non-orthodox guidance laws for the interception of maneuvering anti-surface missiles [5]. Also, they can widen modern area of studies of group pursuit games (multi-agent systems) [6,7,8,9].

References

[1] Le Ménec S. Linear differential game with two pursuers and one evader. In: Annals of the International Society of Dynamic Games, Vol. 11: Advances in Dynamic Games. Theory, Applications, and Numerical Methods for Differential and Stochastic Games. M. Breton, K. Szajowski (Eds.). Boston: Birkhauser, 2011. pp. 209--226.

[2] Ganebny S.A., Kumkov S.S., Le Ménec S., and Patsko V.S. Model Problem in a Line with Two Pursuers and One Eveder. In: Dynamic Games and Applications, No.2, 2012, pp. 228--257.

[3] Ganebny S.A., Kumkov S.S., Le Menec S., and Patsko V.S. Model Differential Game with Two Pursuers and One Evader. In: Contributions to game theory and management. Vol.V. Collected papers presented on the Fifth International Conference ``Game Theory and Management'', Leon A. Petrosyan, Nikolay A. Zenkevich (Eds), Graduate School of Management SPbU, Saint-Petersburg, 2012, pp. 83--96.

[4] Krasovskii N.N., Subbotin A.I. Game-Theoretical Control Problems, Springer-Verlag, New York, 1988.

[5] Shinar J., Shima T. Non-orthodox guidance law development approach for the interception of maneuvering anti-surface missiles. Journal of Guidance, Control, and Dynamics, Vol. 25, No 4, 2002, pp. 658--666.
[6] Petrosjan L.A. Differential Games of Pursuit. Leningrad University, Leningrad, 1977. (in Russian)
[7] Grigorenko N.L. The problem of pursuit by several objects. In: Differential Games – Developments in Modelling and Computation (Espoo, 1990), Lecture Notes in Control and Inform. Sci., Vol. 156, Springer, Berlin, 1991, pp. 71--80.
[8] Chikrii A.A. Conflict-Controlled Processes. Mathematics and its Applications, Vol. 405, Kluwer Academic Publishers Group, Dordrecht, 1997.
[9] Blagodatskih A.I., Petrov N.N. Conflict Interaction Controlled Objects Groups, Udmurt State University, Izhevsk, Russia, 2009. (in Russian)

An Axiomatization of the Nucleolus of Assignment Games

Francesc Llerena[1], **Marina Nunez**[2] **and Carles Rafels**[3]

[1]*University Rovira i Virgili, Spain*
francesc.llerena@urv.cat

[2,3]*University of Barcelona, Spain*
[2]*mnunez@ub.edu*
[3]*crafels@ub.edu*

Keywords: *Assignment Games, Core, Nucleolus*

The assignment game is a coalitional game that represents a two-sided market situation. In this market there exists a finite set of sellers, each one with an indivisible object on sell, and a finite set of buyers willing to buy at most one object each. Each agent has a reservation value that is what he or she obtains if not matched with an agent on the opposite side. Every buyer-seller pair (i,j) is attached to a real number aij that represents the value that this pair can attain if matched together. From these valuations, we obtain the assignment matrix A. The worth of each coalition is the total profit that can be obtained by optimally matching buyers and sellers in thecoalition. When reservation values are null and the assignment matrix is non-negative, our game is the one introduced by Shapley and Shubik (1972). Coalitional game theory analyzes how the agents can share the profit of an optimal pairing, taking into account the worth of all possible coalitions. The most studied solution concept in this model has been the core, the set of efficient allocations that are coalitionally rational. Shapley and Shubik prove that the core of the assignment game is non-empty and it can be described just in terms of the assignment matrix, with no need of the associated characteristic function. Other solutions have been considered for the assignment game: Thompson's fair division point (1981), the kernel or symmetrically pairwise bargained allocations (Rochford, 1984), the nucleolus (Solymosi and Raghavan, 1994), the Shapley value (Hoffmann and Sudhölter, 2007) and the von Neumann-Morgenstern stable sets (Nunez and Rafels, 2009). However, as far as we know, axiomatic characterizations of solutions in this framework have been focused on the core. Axiomatizations of the core of assignment games are due to Sasaki (1995) and Toda (2003 and 2005).

On the general class of coalitional games, the prenucleolus (that for the assignment game coincides with the nucleolus) has been axiomatized by Sobolev (1975) by means of covariance, anonimity and the reduced game property of Davis and Maschler (1965). Potters (1991) also characterizes the nucleolus on the class of balanced games by means of the above reduced game property. However, both aforementioned sets of axioms do not characterize the nucleolus on the class of assignment games since the Davis and Maschler reduced game of an assignment game needs not remain inside this class. Moreover, it seems desirable an axiomatization of the nucleolus of the assignment game in terms of axioms that are not stated by means of the characteristic function but by means of the data of the assignment market.

In the present paper, on the domain of assignment games, the nucleolus is uniquely determined by only two axioms: derived consistency and complaint monotonicity on sectors' size. Derived consistency is based on the derived game introduced by Owen (1992). Roughly speaking, complaint monotonicity on sectors' size only requires that at each solution outcome, the most dissatisfied agent on the short side of the market is at most as well off as the most dissatisfied agent on the large side of the market, where we interpret the dissatisfaction of an agent with a given outcome as the difference between his reservation value and the amount that this outcome allocates to him.

As a by-product of the axiomatization of the nucleolus, we obtain a geometric characterization of the nucleolus. Maschler et al. (1979) provide a geometrical characterization for the intersection of the kernel and the core of a coalitional game, showing that those allocations that lie in both sets are always the midpoint of certain bargaining range between each pair of players. In the case of the assignment game, this means that the kernel can be determined as those core allocations where the maximum amount that can be transferred, without getting outside the core, from one agent to his/her optimally matched partner equals the maximum amount that he/she can receive from this partner, also remaining inside the core (Rochford, 1984; Driessen, 1999). We now state that the nucleolus of the assignment game can be characterized by requiring this bisection property be satisfied not only for optimally matched pairs but also for optimally matched coalitions.

Determining the Optimal Strategies for Stochastic Positional Games

Dmitrii Lozovanu[1] **and Stefan Pickl**[2]

[1]*Institute of Mathematics and Computer Science, Academy of Sciences of Moldova, Academy str., 5, Chisinau, MD-2028, Moldova*
lozovanu@math.md
[2]*Institute for Theoretical Computer Science, Mathematics and Operations Research, Universitat der Bundeswehr, Miinchen 85577 Neubiberg-Munchen, Germany*
stefan.pickl@unibw.de

Keywords: *Stochastic Positional Game, Saddle Point Conditions, Computational Network*

In this contribution we formulate and study a class of stochastic positional games using the game-theoretical approach to finite state space Markov decision processes with average and discounted costs' optimization criteria. We consider Markov decision processes that may be controlled by several players. The set of states of the system in such processes is divided into several disjoint subsets which we regard as the position sets of the corresponding players. Each player has to determine which action should be taken in each state of his position set in order to minimize his own average cost per transition or discounted expected total cost. The cost of system's transition from one state to another in a Markov process is given for each player separately. In addition the set of actions, the transition probability functions and the starting state are known. We assume that players use stationary strategies, i.e. each player in an arbitrary position uses the same action for an arbitrary discrete moment of time. In the considered stochastic positional games we are seeking for a Nash equilibrium. Our main results are concerned with the existence of Nash equilibria in the considered games and an elaboration of algorithms for determining the optimal stationary strategies of the players. Additionally we consider the antagonistic stochastic positional games and formulate conditions for determining the saddle points in such games

We formulate he stochastic positional game with average payoff function using the framework of a Markov decision process (X, A, p, c) with a finite set of states X a finite set of actions, a transition probability function $p: X \times X \times A \to [0,1]$ that satisfies the condition $\sum_{y\in X} p^{a}_{x,y} = 1, \forall x \in X, \forall a \in A$ and a transition cost function $c^i : X \times X \to R$ (see [1]). For the non-cooperative game model with m players m transition cost functions $c^i : X \times X \to R, i = \overline{1,m}$, where $c^i_{x,y}$ expresses the cost of system's transition from the state $x \in X$ to the state $y \in Y$ for player i are given.

In addition we assume that the set of states X is divided into m subsets $X = X_1 \cup X_2 \cup \ldots \cup X_m \ (X_i \cup X_j) = \varnothing, \forall i \neq j)$, where X_i represents the position set of player i. We consider the stationary game model. We define the stationary strategies of players we define as m maps: $s^i : x \to a \in A^i(x)$ for $x \in X_i, i = \overline{1,n}$. If the players fix their stationary strategies $s^1, s^2, \ldots, s^m$ then we obtain a situation $s = (s^1, s^2, \ldots, s^m)$. This situation induces a simple Markov process determined by the probability distributions $p^{s^i(x)}_{x,y}$ in the states $x \in X_i, i = \overline{i,m}$. If we denote by $P^s = (p^s_{x,y})$ the matrix of probability transitions of this Markov process and take into account the transition cost matrix $C^i = (c^i_{x,y})$ for each player then for a given starting state x_0 we can determine the average cost per transition $M^i_{x_0}(s^1, s^2, \ldots, s^m)$ for $i = \overline{1,m}$. So, if on the set of situations $S = S_1 \times S_2 \times \ldots \times S_m$ we define the payoff functions $F^i_{x_0}(s^1, s^2, \ldots, s^m) = M^i_{x_0}(s^1, s^2, \ldots, s^m), i = \overline{1,m}$ then we obtain a discrete game in normal form. For this game we have proven the following results [2,3]:

- If an arbitrary situation $s \in S$ induces a probability matrix P^s that corresponds to a Markov uni-chain then for the stochastic positional game with an average payoff function there exists a Nash equilibrium;

- For an arbitrary antagonistic stochastic positional game with an average payoff function there exists the saddle point.

The stochastic positional game with discounted payoff functions can be defined in a similar way as the previous game. If for an arbitrary situation $s = (s^1, s^2, \ldots, s^m) \in S$ we consider the Markov process induced by the probability matrix $P^s = (p^s_{x,y})$ then for a given starting state x_0 and given discount factor $\lambda \ (0 < \lambda < 1)$ we can determine the expected total discounted cost $\sigma^i_{x_0}(s^1, s^2, \ldots, s^m)$ with respect to each player i. In such a way we can define the functions $\overline{F}_{x_0}^{i}(s^1, s^2, \ldots, s^m) = \sigma_{x_0}^{i}(s^1, s^2, \ldots, s^m)$ for $i = \overline{1, m}$. These function determine a discrete game in normal form. For this game we have proven the following result [2,3]: For an arbitrary stochastic positional game with discounted payoff function there exists Nash equilibrium. For the considered stochastic positional games elaborated algorithms for determining the optimal strategies of the players have been elaborated. Additionally the computational complexity aspects of the problem of determining the optimal strategies in stochastic antagonistic positional games have been studied [4].

References

[1] Puterman, M., Markov Decision Processes: Stochastic Dynamic Programming. John Wiley, New Jersey (2005).

[2] Lozovanu, D.: The game-theoretical approach to Markov decision problems and determining Nash equilibria for stochastic positional games. Int. J. Mathematical Modelling and Numerical Optimization, 2 (2), 162-164 (2011)

[3] Lozovanu, D., Pickl, S., Kropat, E.: Markov decision processes and determining Nash equilibria for stochastic positional games, Proceedings of the 18th World Cogress IFAC-2011, 320-327, (2011).

[4] Lozovanu, D., Pickl, S.: Discrete control and algorithms for solving antagonistic dynamic games on networks. Optimization, 58(6), 665-683, (2009).

Shapley Value for a Game with a Partially Ordered Set of Players

Michael Lutsenko[1] **and Natalija Shadrintseva**[2]

[1,2]Saint-Petersburg State Transport University
Russia
[1] ml4116@mail.ru

Keywords: *Cooperative Game, Shapley Value, Testing*

One of the problems of cooperative games is to find formal decisions acceptable to all game participants. Among the most popular solutions is the Shapley value, which determines the weight of each player in the game [5]. In this paper, we have found the Shapley value for the game with a partially ordered set of players and have shown how the methods of cooperative game theory can be applied for assigning weights in testing.

Testing as an instrument of measuring the knowledge level is used in teaching successfully for a long time. During the last years the interest to this problem has increased significantly, this interest being caused by the formalization of the educational process and the development of computer-based control of knowledge. The emergence of mathematical models of Rush, Birnbaum and others have substantially developed the theory and has made possible to use advanced mathematical models such as the item response theory [1], statistical games [3].

One of the problems of the item response theory is the determination of the item weights in test. It is important if we want to compare the students who had different tests.

A cooperative game is designed to assign weights to the test items. Value of the characteristic function on any subset of the test items is a time that is required for a student to find answers for items of this subset. In case of tree structure of the test a Shapley value and a Banzhaf power index are found by authors in [4]. The constructed Shapley and Banzhaf values are similar to the values proposed by Littlechild S.C. and Owen G. A. [2] for “Airport game”.

Let a set $A = \{\alpha_1,\ \alpha_2,...,\alpha_n\}$ be a set of parts of studying course and a set of test items. We define two functions: $S(\alpha)$, a_α. A set $S(\alpha)$ is a set of parts that precedes straightly the part α. A number a_α is the time needed to learn the part α if the student has learnt the test items from the set $S(\alpha)$.

Definition. If there are a sequence of test items (course parts) $\alpha = \gamma_1, \gamma_2, ..., \gamma_k = \beta$ for a test (course) $A = \{\alpha_1,\ \alpha_2,...,\alpha_n\}$ such that $\alpha \in S(\gamma_2),\ \gamma_2 \in S(\gamma_3),...,\gamma_{k-1} \in S(\beta)$ then we write $\alpha < \beta$ and say that the item α precedes the item β.

We denote by $V(\alpha) = \{\beta \in A \mid \beta < \alpha\}$ a set of test items (course parts) preceding the item α and by $\bar{V}(\alpha) = \{\beta \in A \mid \alpha < \beta\}$ a set of test items (course parts) following after the item α. For a set of item K we denote by $V(K) = \bigcup_{\alpha \in K} V(\alpha) = \{\beta \in A \mid \exists_{\alpha \in K}(\ \beta < \alpha)\}$ the set of test items preceded to items from the set K.

For a studying course A with two functions $S(\alpha)$, a_α we define a cooperative game $\Gamma = \langle A, v\rangle$ with characteristic function given by

$$v(K) = \sum_{\alpha \in K \cup V(K)} a_\alpha \qquad (1)$$

Therefore the value $v(K)$ is a time needed for learning course parts from the set K.

Theorem. If in a cooperative game $\Gamma = \langle A, v\rangle$ the characteristic function has the form given by eq.(1) and the number $\bar{m}_\alpha = \left|\bar{V}(\alpha)\right|$ is the power of set $\bar{V}(\alpha)$ then components of Shapley value can be found by the formulae

$$w_\alpha = \sum_{\beta \in \{\alpha\} \cup V(\alpha)} \frac{a_\beta}{\bar{m}_\beta + 1} \qquad (2)$$

The Shapley value has the following probabilistic interpretation. The component w_α of Shapley value (weight of test item α) is the mathematical expectation of time needed to a student for learning part α if all programs of the course studied have equal probability. A program of the study is a permutation of test parts $\alpha_1,\ \alpha_2,...,\alpha_n$.

Example. The test A consists of 6 items. Its structure is defined by a graph (see Fig.). The numbers $a_i \geq 0$, $i = \overline{1,6}$ determine the time that is necessary for studying the part i under the condition that all the previous parts of course have been studied. So the time needed for studying all the parts of course is $v(A) = a_1 + a_2 + \cdots + a_6$.

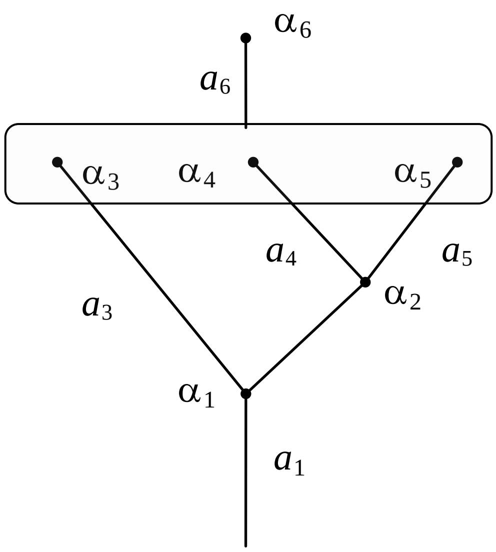


Fig.

According to the graph it is easy to construct function $S(\alpha)$. It is given by the following formulae: $S(\alpha_1) = \varnothing$, $S(\alpha_2) = \{\alpha_1\}$, $S(\alpha_3) = \{\alpha_1\}$, $S(\alpha_4) = \{\alpha_2\}$, $S(\alpha_5) = \{\alpha_2\}$, $S(\alpha_6) = \{\alpha_3, \alpha_4, \alpha_5\}$. For function $S(\alpha)$ on the set of the course we defend a ratio of the following items. It is not reflexive. Characteristic function $v(K)$ is constructed by (1) for all 64 subsets of set A.

The components w_α of the Shapley value can be found by formulae (2) and numbers

$\bar{m}_6 = 0, \bar{m}_5 = \bar{m}_4 = \bar{m}_3 = 1, \bar{m}_2 = 3$. They are given by

$w_1 = a_1/6$; $w_2 = a_1/6 + a_2/4$; $w_3 = a_1/6 + a_3/2$; $w_4 = a_1/6 + a_2/4 + a_4/2$;

$w_5 = a_1/6 + a_2/5 + a_5/2$; $w_6 = a_1/6 + a_2/4 + a_3/2 + a_4/2 + a_5/2 + a_6$.

References

[1] Handbook of Modern Item Response Theory. Edition Win. J. van der Linden, R.K. Hambleton, 1997, Springer – Verlag, New York, p.510.

[2] Littlechild S.C., Owen G. A Simple Expression for the Shapley Value in a Special Case// Management Sci., 20, p.370-372.

[3] Lutsenko M. M., Shadrintseva N. V. Reliability of test score, Proceedings of Petersburg transport university, ISSN 1815-588X, 2(31), 2012 cc. 178-186

[4] Lutsenko M. M., Shadrintseva N. V. The Shapley and Banzaf values for different courses of study, the Sixth International conference Game Theory and Management, Abstracts, St. Petersburg, 2012, p. 165-169..
[5] Weber R. Probabilistic Values for Games // The Shapley Value (Essays in Honor of Lloyd S Shapley/ A.E.Roth (ed.). 101-119, Cambridge Univ. Press. Cambridge, UK

ВЕСТНИК САНКТ–ПЕТЕРБУРГСКОГО УНИВЕРСИТЕТА

Серия 8
МЕНЕДЖМЕНТ

Ответственный редактор серии:
Ю.Е. Благов

Издательство Высшей школы менеджмента Санкт-Петербургского университета

Game-theoretic Solutions for Bargaining Problem

Andrey Lyapunov

St.Petersburg Institute for Economics and Mathematics
Russian Academy of Science,St.Petersburg, Russia
anlyapunov@yandex.ru

Keywords: *Bargaining Problem, Shapley Value, Fixed Point*

The surveys of the bargaining problems are contained in [1-2].

Let $X \subset \mathbb{R}_+^n$ be the set such that it is convex, bounded, closed and there is $x \in X$, $x > 0$. Moreover, X is comprehensive: if $x \in X$ and $0 \le y \le x$, then $y \in X$. (The point status quo is 0). The Pareto boundary of the set X will be denoted by $\mathcal{E}(X)$. in Let $\varphi : \mathbb{R}_+^n \to \mathbb{R}_+$ be the calibre function of the set X:

$$\varphi(x) = \inf\{\alpha \ge 0 \mid x \in \alpha X\}.$$

Let be $I = \{1,\dots,n\}$. For $x \in \mathbb{R}^n$, $S \subset I$ denote by x^S the vector with components

$$x_i^S = \begin{cases} x_i, & \text{if } i \in S \\ 0, & \text{if } i \notin S \end{cases}.$$

Let be $x \in \mathcal{E}(X)$. Define the cooperative game:

$$v(x,S) = \varphi(x^S).$$

Properties of the function $\boldsymbol{v}$.

1. $v(x,I) = 1$ for $x \in \mathcal{E}(X)$.
2. $v(x, S_1 \cup S_2) \le v(x,S_1) + v(x,S_2)$.
3. If $S_1 \subset S_2$ then $v(x,S_1) \le v(x,S_2)$.
4. For $\alpha \ge 0$ $v(\alpha x, S) = \alpha v(x,S)$.

Denote by $W(x)$ the Shapley value for game $v(x,\cdot)$.

Theorem. *On $\mathcal{E}(X)$ exists a fixed point x^* for map W: $x^* = W(x^*)$.*

Definitioin. *This fixed point is called the Shapley value of the bargaining problem.*

References

[1] Thomson W. Cooperative models of bargaining. Handbook of game theory with economic applications, v.II, eds. J.Aumann and S.Hart,1994,pp.1238 – 1284.

[2] Pechersky S.L., Yanovskaya E.B. Cooperative games: solutions and axioms. St.Petersburg, 2004, 459 p.(In Russian)

ВЕСТНИК САНКТ–ПЕТЕРБУРГСКОГО УНИВЕРСИТЕТА

Серия 10
ПРИКЛАДНАЯ МАТЕМАТИКА, ИНФОРМАТИКА, ПРОЦЕССЫ УПРАВЛЕНИЯ

Издательство Санкт-Петербургского университета

Auction with Downstream Market Interactions

Amarjyoti Mahanta

Centre for Economic Studies and Planning,
School of Social Sciences Jawaharlal Nehru University New Delhi, India
amarjyotimahanta@gmail.com

Keywords: *Auction, Oligopoly, Licensing*

We consider a two stage game in which in stage one two firms with private information regarding their cost of production participate in an auction to get the license to enter the other firm's market and in stage two there is Cournot competition between the winning firm and the losing firm. In such setting the post auction policy of information revelation regarding the bids of the firms have impact in the Cournot competition between the firms. We consider four types of information revelation scheme: i) Post auction no information regarding the bids of the firms is revealed and only the winning firm is privately asked to pay the bid. ii) All the bids are publicly announced. So all the firms participating in the auction come to know about the bids of the other firms. iii) Only the bid of the winning firm is publicly announced. iv) Only the bid of the losing firm is publicly announced. In scheme i), the post auction Cournot competition is still incomplete information game in first price auction and all pay auction. In second price auction, the post auction Cournot competition turns out to be asymmetric information game. In scheme ii), the post auction Cournot competition is complete information game in all these three auction formats. In scheme iii) and iv), the post auction Cournot competition is asymmetric information game in first price and all-pay auction. The post auction Cournot competition in second price auction is complete information games in scheme iii) and asymmetric information games in scheme iv). The pay-off of firms are different in these schemes of information revelation. Thus, the expected pay-off in stage one will be different in these four schemes of information revelation. Therefore, the bids will depend on the schemes of information revelation followed.

We look for symmetric pure strategy Bayesian Nash equilibrium in first price, second price and all-pay auctions in each of these four schemes. Our analysis assumes the types of the firms is denoted by their cost function (constant marginal cost)and uniformly distributed. We get that the expected revenue is higher when only the bid of the losing firm is revealed than when no information is revealed. The symmetric pure strategy Bayesian Nash equilibrium does not exist in scheme ii) (when all information is revealed) and scheme ii) (only bid of the winning firm is revealed). It may be considered as an answer to the question why information regarding amount of payment made while lobbying is not revealed.

МАТЕМАТИЧЕСКАЯ ТЕОРИЯ ИГР И ЕЕ ПРИЛОЖЕНИЯ

Выходит ежеквартально с января 2009 года

The Fight Against Cartels: a Transatlantic Perspective

Emilie Dargaud[1], Andrea Mantovaniy[2] and Carlo Reggianiz[3]

[1]*University of Lyon, F-69007, Lyon, France; CNRS, GATE Lyon St Etienne, 93, Chemin des Mouilles, F-69130, Ecully, France*
dargaud@gate.cnrs.fr

[2]*Department of Economics, University of Bologna, Strada Maggiore 45, 40125 Bologna, Italy*
a.mantovani@unibo.it

[3]*School of Social Sciences - Economics, University of Manchester, Manchester M13 9PL, United Kingdom*
carlo.reggiani@manchester.ac.uk

Keywords: *Cartels, Discount Factor, Repeated Games*

The fight against cartels is a priority for antitrust authorities on both sides of the Atlantic. The instruments used in the fight, however, are different. The target of the European Commission is merely the firm and its profits, while in the US managers are punished more directly and individual sanctions, including imprisonment, can be imposed. The aim of this paper is to highlight the effectiveness of different types of punishment in deterring cartels. To this goal we consider different types of fines leived on managerial firms: profit-based fines as opposed to delegation-based fines (hitting the managers more directly). The results suggest that the more suited instrument is related to the power delegated to managers: if delegation to managers is high, then a fine hitting them more directly is more effective in deterring cartel formation. If the fines have to be revenue equivalent, a non-distorsive profit based fine is more suited to deter cartels.

Introduction

The past decade has witnessed an increase in price-fixing, market-sharing and bid-rigging cartel behaviour across the two sides of the Atlantic. As a consequence, the fight against cartels evolved from transnational to `transatlantic', requiring unprecedented efforts and collaboration between the U.S. and the E.U. The respective legal and enforcement regimes are still quite different: in U.S. cartels are commonly treated with criminal sanctions whereas the principal sanction available to the Commission is the imposition of fines. Notwithstanding the different approaches, the fight against cartels has been recently very successful on both sides of the Atlantic. This success can be partly attributed to the "leniency revolution". Leniency policies provide discounted fines or even positive transfers (for only few systems) to companies willing to cooperate with the enforcing authorities.

Our paper, however, is not on leniency. Instead we aim to shed some light on the effectiveness of cartel deterrence in presence of managerial firms. The assumption of managerial firms reflects the observation that this is the nature of the majority of firms that raise the attention of antitrust authorities. Recent cases in the EU, like the TV and computer monitor tubes and in the US, as the LCD producers price fixing case clearly witness this. Besides reflecting different approaches to cartel punishment, these cases highlight how the corporate structure of the vast majority of the firms involved features managers who have been delegated decisions on behalf of the company by its owners or shareholders.

In this context, we aim to provide a comparison of the effectiveness of the different types of punishments for cartel deterrence. In our approach, an antitrust authority can deter the cartel adopting two types of fines. The first type hits the profits of the firm and it is the typical instrument considered in the literature (Motta and Polo, 2003). The second type of fine is instead focusing on the manager. The split between ownership and control can lead to non-profit-maximizing behaviour if the incentives of managers are not aligned with those of owners. This happens as managers may be interested in maximizing their own utility and do not necessarily share the profit maximizing goals of owners. A fine on the manager's contract and its delegated powers constitutes an hurdle to the goal of the manager of maximising the firm's size. This stylised modelling device does not fully capture the idea of criminal punishment through, for example, imprisonment; however, it constitutes a first attempt to evaluate a type of punishment targeted mainly at managers.

Model, Analysis, and Welfare Implications

We consider a duopoly in which two managerial firms 1 and 2 compete a la Cournot in the final product market. Managers aim to maximize a combination of firms' profits and sales and this is captured by the following objective function (Vickers, 1985; Lambertini and Trombetta, 2002):

$$M_i = \Pi_i + \theta_i q_i \quad i = 1, 2$$

where θ_i is the relevance of sales and it is a measure of delegation. For simplicity, we assume that the inverse demand is linear ($p = a - Q$) and the marginal cost is constant and equal to c for both firms.

Managers 1 and 2 can decide to reach a collusive agreement or to deviate (playing non-cooperatively). Collusion is sustained through a grim trigger strategy: if one firm deviates from the collusive agreement**,** it is punished by reverting to Cournot competition forever. Let $\alpha \in (0,1)$ be the common discount factor of the managers.

The policies adopted by the antitrust authority are intended to deter collusion**.** To this goal we suppose that the authority can use two types of fines. The first type hits directly the profits of the firm; the second type hits the contract delegating powers to the managers.

Starting from the profit-based fine, the two managers can reach collusive agreement which can be detected by the antitrust authority with the probability ρ. A successful prosecution of the cartel and the global amount of fine is denoted by F implying that each firm pays $F/2$. Given the probability of being caught ρ, we assume that the expected fine is not prohibitively high to make firms profits negative. Managers can sustain collusive agreement over time if and only if their discount factor is sufficiently high and satisfies the following condition:

$$\alpha \geq \alpha_{PB} = \frac{9}{17}\left(1 + \frac{32\rho F}{(A+\theta)^2}\right).$$

If managers are delegated decisions on behalf of the firm, the antitrust authority may punish them more directly. One way to model this scenario is to assume that the colluding managers' delegation is given by $\theta - \rho f$, where ρf is the expected delegation-based fine. Managers have an incentive to form a cartel if and only if:

$$\alpha \geq \alpha_{DB} = \frac{9\left[(A+\theta)^2 + 22\rho f(A+\theta) - 7(\rho f)^2\right]}{(A+\theta) + 3\rho f\left[17(A+\theta) + 3\rho f\right]}$$

The two types of fines can be compared and conditions can be found out for which one is more effective than the other in deterring collusion. In particular, we find that, the higher is the power that owners delegate to the managers, the more effective is a punishment that hits the managers more directly. More precisely, if owners retain the most of the decisional power, a profit based lump-sum fine is more likely to be more effective instrument in deterring cartels.

Take into account now revenue equivalence and suppose that the antitrust authority has limited resources only generated by the fines imposed to cartel members. In this context**,** the antitrust authority may be particularly interested finding the most effective instrument, given a value of the expected revenue it aims to achieve.

Proposition 2 *Under revenue equivalence, an antitrust authority can deter collusion between managerial firms more effectively using a delegation-based fine (more directly hitting managers) rather than a profit based lump-sum fine (more directly hitting shareholders).*

The previous proposition suggests that the delegation based fine is distortive but is more effective in deterring collusion than the lump-sum profit based fine. There is an intuitive explanation for this result: unlike the lump sum fine, the delegation based fine affects also the deviation profits; in particular, the best response output of the deviating firm is increased by the fact that the rival firms expects to be fined.

Finally, computing total welfare and assuming that no restriction on the two instruments is imposed, the results are similar in spirit to the analysis of deterrence in the previous section: the relative size determines which instrument leads to a higher expected welfare. More interesting is to perform the full comparison of the welfare effects under revenue equivalence, *i.e.* imposing that the revenue of the antitrust authority is identical in the two scenarios. The following result can then be stated:

Proposition 3*Under revenue equivalence, provided that fines are not prohibitive, a delegation based fine is welfare enhancing if and only if* $\rho f > \tilde{t}(\theta)$ *with* $\tilde{t}(\theta)$ *increasing in* θ*. Conversely, if* $0 < \rho f < \hat{t}(\theta) < \tilde{t}(\theta)$ *with* $\hat{t}(\theta)$ *increasing in* θ*, a delegation based fine is not only decreasing welfare compared to a profit based fine but also compared to the status quo of no intervention.*

Brief discussion

This paper addresses the effectiveness of different types of instruments in deterring cartels. In particular, we focus on fines levied on profits and fines hitting the delegation powers given to managers; with the latter, we aim to capture an instrument that hits more directly managers. As the majority of firms involved in cartel cases are managerial, we analyse these two types of antitrust instruments in the context of the delegation model of Vickers (1986). Our results indicate that the effectiveness of each type of instrument is closely related to the amount of powers delegated to the managers: intuitively, a delegation based fine is more likely to effectively deter collusion the more power owners have delegated to managers. This type of fine is the most effective if the antitrust authority aims at revenue equivalence: the distortionary effect of a delegation based fine reduces further the ability of managers to collude compared to a profit based fine. The distortion, however, has negative effects for consumers if managers are patient enough and firms still collude despite the antitrust efforts. In that case, we identify an important trade-off that has been largely overlooked in the existing literature between effective ex-ante deterrence and consumers' surplus and welfare if collusion still takes place.

References

[1] Aubert, C., Rey, P. and Kovacic, W.E. (2006). The Impact of leniency and whistleblowing programs on cartels. International Journal of Industrial Organization 24, 1264-1266.

[2] Lambertini, L. and Trombetta, M. (2002). Delegation and firms' ability to collude. Journal of Economic Behavior and Organization 47, 359-373

[3] Motta, M. and Polo, M. (2003). Leniency programs and cartel prosecution. International Journal of Industrial Organization 21, 347-379.

[4] Vickers, J. (1985). Delegation and the theory of the firm. The Economic Journal 95, 138-147.

Cooperative Assignment Games with the Inverse Monge Property

Javier Martinez-De-Albeniz[1] and Carles Rafels[2]

[1,2]*Universitat de Barcelona*
Spain
[1]*javier.martinezdealbeniz@ub.edu*
[2]*crafels@ub.edu*

Keywords: *Assignment Game, Core, Monge Matrix, Buyers-Optimal Core Allocation, Sellers-Optimal Core Allocation*

The Monge property on a matrix was named this way by Hoffman (1963) honoring the works of the 18th-century French mathematician G. Monge, who used the property in a context of a soil-transport problem. This property has been applied in many different areas such as operations research, coding theory, computational geometry, greedy algorithms, computational biology, statistics or economics. We refer to the surveys in Burkard (2007) or Burkard et al. (1996) for references, specific applications or properties. Our work addresses the following question posed in page 151 in Burkard et al. (1996): "Are there other fields where Monge matrices play a role?"

We try to contribute to this general question with a new work, mixing Monge matrices and assignment problems. Our main interest is to describe the core and two specific core allocations, the sectors-optimal allocations, in an easy and practical way, when dealing with Monge assignment cooperative games. Roughly speaking, our results agree with the "flavor" that adding the Monge conditions to the assignment matrix simplifies the analysis of the aforementioned solutions. The optimal (linear sum) assignment problem is that of finding an optimal matching, given a matrix that collects the potential profit of each pair of agents. Some examples are the placement of workers to jobs, of students to colleges, of physicians to hospitals or the pairing of men and women in marriage. Once an optimal matching has been found, one question arises: how to share the output among the partners. This question was first considered in Shapley and Shubik (1972).

They associate each assignment problem with a cooperative game, or game in coalitional form. In the assignment, each coalition of agents must consider the maximum profit they could attain by themselves as the worth of this coalition. The most relevant solution concept in cooperative games is the core. The core of a game consists of those allocations of the optimal profit (the worth of the grand coalition) such that no subcoalition can improve upon. Thus, we agree to share the profit of cooperation by means of a core allocation, no coalition has incentives to depart from the grand coalition and act on its own.

Shapley and Shubik prove that the core of the assignment game is a nonempty polyhedral convex set and it coincides with the set of solutions of the dual linear program related to the linear sum optimal assignment problem.

In this paper we study the assignment games, called Monge assignment games, where the matrix satisfies what is called the Monge property. Roughly speaking, the (inverse) Monge property is described by the fact that each 2×2 submatrix has an optimal matching in the main diagonal. This property can also be identified as the supermodularity of the matrix, interpreted as a function on the product of the set of indices with the usual order.

Monge matrices have been used in Operations Research, in different fields such as combinatorial optimization (see Burkard et al., 1996 or Burkard, 2007), coalitional game theory (see Okamoto, 2004), algorithmic issues (see Bein et al., 2005), or statistics (see Hou and Prékopa, 2007).

For square Monge assignment games, the central tridiagonal band of the matrix, that is the main diagonal, the upper diagonal and the lower diagonal, is sufficient to determine the core. As a result, and differently to the general case, not all inequalities are necessary to describe the core explicitly, and in this case the buyer-seller exact representative of the matrix (Núñez and Rafels, 2002b) can be computed by a closed formula. Two important points of the core, the buyers-optimal and the sellers-optimal core allocations, are computed with the aid of the previous representation.

A Dynamic Game of International Emissions under Uncertainty and Learning

Nahid Masoudi

HEC Montreal, Canada
nahid.masoudi@hec.ca

Keywords: *N-player Dynamic Game, International Emissions, Uncertainty, Bayesian Learning*

Abstract We introduce learning in an N-player dynamic game of international pollution, while planers are facing with ecological uncertainty. We analytically find and compare the feedback non-cooperative strategies of players under three different behavioral and information assumptions: Bayesian learner, adaptive learner and informed player. Besides we find out that while uncertainty due to anticipation of learning ends up to a decrease in total emissions, depending on functional form of distributions and beliefs itself, the effect of structural uncertainty could be either an increase, decrease, or even no change in the emissions of individual players and also total emissions. While the first observation is totally in line with the available literature the latter results are rather new and more controversial. Moreover, we find out that a more optimistic player may either emit less or more than a less optimistic one, depending on how beliefs affect their expectations about the unknown variable. Then we show that if one learner player feels more risk while others beliefs do not change, this player will decrease his emission and others react to this decrease by increasing their emission but they are never strong enough to overcome the direct effect.

Contests with Identity-Dependent Externalities

Alexander Matros

University of South Carolina
United States
alexander.matros@gmail.com

Keywords: *Contest, Externality, All-Pay Auction*

Traditionally, the focus of contest theory has been on models that view contests as isolated events. See, for example, Tullock (1980), Nitzan (1994), Congleton, Hillman, and Konrad (2008), Konrad (2009). However, most often contests are parts of larger environment. In many such economic environments, contestants care about not only winning the contest, but also the prize allocation in case of losing. This situation is known in the literature as identity-dependent externalities. There are many examples of contests with identity-dependent externalities in sports, rent seeking, lobbying etc. See examples in Funk (1996), Jehiel, Moldovanu, and Stacchetti (1996), Das Varma (2002), Klose and Kovenock (2011).

In this paper we analyze contests with identity-dependent externalities. We suggest a new approach how to analyze contests. Our approach is based on a particular set of determinants. First, we calculate each determinant in this set. Then, we show how to find a full-participation equilibrium based on the calculated determinants.

We demonstrate that our set of determinants can establish both the existence and uniqueness of the full-participation equilibrium. In order to guarantee a unique full-participation equilibrium, all determinants in our set of determinants have to have the same sign. This simple requirement and a standard assumption that all players prefer winning to losing are sufficient.

Our new approach is very general. We demonstrate how to find equilibria in the classic symmetric and asymmetric contests without externalities based on our set of determinants.

Finally, we analyze a particular n=3 player case. It turns out that in this case our approach helps to find a unique equilibrium which might or might not be the full-participation equilibrium.

Development on a Network with Positive Externalities: a Game with Transferable Values of Agents

Vladimir Matveenko

National Research University Higher School of Economics
Russia
vmatveenko@hse.ru

Keywords: *Transferable Values, Game, Oriented Graph, Dynamic Programming, Idempotent Operation, Structure of Optimal Paths, Dynamics of Agglomerations*

Matveenko [2] studied dynamics of a system of n agents with mutual positive externalities. Each agent i in period t is described by a positive number x_i^t referred as a *value*. Development of the i-th agent is being described as changes in her value. Discrete time periods $t = 0,1,...$ are considered and it is supposed that the value of the agent in the following period, $t+1$, depends on her own value and the values of some other agents in the previous period, t. The development of the system is constrained both by potential possibilities of the agents to grow, $x_i^{t+1} \le a_{ii} x_i^t$, and by the limitations of sizes of externalities created by other agents, $x_i^{t+1} \le a_{ij} x_j^{t+1}$, $i \ne j$. Here $a_{ij} = +\infty$ if agent j creates no externality used by agent i or if the externality created by j never becomes binding for i. A natural assumption of efficiency results in the following equations characterizing the equilibrium path given initial values $x_1^0,...,x_n^0$:

$$x_i^{t+1} = \min_{j=1,...,n} a_{ij} x_j^t, \ t = 0,1,...; i = 1,...,n,$$

which can be written as

$$x^{t+1} = A \otimes x^t, t = 0,1,..., \tag{1}$$

where x^t, x^{t+1} are the vectors of values, A is the matrix of coefficients, and $A \otimes x^t$ is a multiplication by use of operations of an idempotent semi-ring, $\otimes = \cdot, \oplus = \min$ (see the pioneering research by Vorobiev [5], and see e.g. [1] for recent references on the tropical (idempotent) mathematics). It is important to notice that an externality can be nonbinding until an agent i is "small", but when the agent develops the latter can face the limitation of insufficient development of another agent or a group of agents. The behavior of the system (1) is fully defined by properties of the matrix A. In the two-agent case, for example, the long-run growth factor is equal to $\alpha = \min\{a_{11}, a_{22}, \sqrt{a_{12}a_{21}}\}$ but the system can demonstrate different patterns, such as convergence to an eigenvector with stable growth factor, stable decline, convergence to a limit cycle, etc. An important result is that a small change in an element of the matrix A can lead to a radical change of a pattern of behavior of the system (so called butterfly effect or catastrophic bifurcation) [3]. Such system can be applied, e.g., to modeling agglomeration externalities [4].

In the present work we consider a *transferable value* version of the model formulated as the following game. Each player *i*, who possesses a value x_i^t chooses transfers y_{ij}^t of nonnegative parts of her value to other players, where $0 \le T_i^t = \sum_{j \in N, j \ne i} y_{ij}^t \le x_i^t$. The dynamics now is described by the system $x^{t+1} = A \otimes \overline{x}^t, t = 0,1,...,$ where $\overline{x}^t$ is the vector of after transfer values, $\overline{x}_i^t = x_i^t - T_i^t + \sum_{j \in N, j \ne i} y_{ji}^t, i = 1,...,n$. Each player i is interested in a short-run (stepwise) maximization of her growth factor, x_i^{t+1} / x_i^t, i.e. in a stepwise maximization of the value x_i^{t+1}, or in a long-run maximization of x_i^τ / x_i^0 where τ is a terminal period of time. We find conditions under which the short-run goals are consistent with the long-run goals. The game with the short-run maximization is studied by use of response curves. For the three-players game, it is shown that, under some conditions, the short-run Nash equilibrium is achieved also in a process of local adjustment of the transfers. In the two-players game, the cases of $DetA < 0$ and $DetA > 0$ are principally different, and under $DetA > 0$, which means small externalities, an inconsistency between the short-run and the long-run goals takes place; in this case a cooperative solution can be used.

References

[1] Litvinov G.L., Maslov V.P., Kushner A.G. and Sergeev S.N., eds. (2012) Tropical and Idempotent Mathematics. Moscow: French-Russian Laboratory "J.-V. Poncelet". Contemporary Mathematics, v. 495. American Mathematical Society.

[2] Matveenko V. (1995) Development with positive externalities: The case of the Russian economy. Journal of Policy Modeling 17(3), 207-221.

[3] Matveenko V. (2009) Optimal paths in oriented graphs and eigenvectors in max-$\otimes$ systems. Discrete Mathematics and Applications 19(4), 389-409.

Equilibrium in Cloud Computing Market

Vladimir Mazalov[1] and Audrey Lukyanenko[2]

[1]*Institute of Applied Mathematical Research Karelian Research Center of Russian Academy of Sciences*
vmazalov@krc.karelia.ru

[2]*University Aalto Helsinki, Finland,*
laser.ru@gmail.com

Hybrid cloud is a promising architecture to the modern market oriented cloud ecosystems. In this context public cloud with unlimited capacity provides solution to handle unexpected message peaks of variable Internet traffic and private cloud capacity for typical load.

We consider a non-cooperative game $\Gamma = < N, \{P_i\}_{i=1}^n, u_i(p) >$ with player set $N = \{1, 2, \ldots, n\}$, strategy sets $P_1 \times P_2 \times \ldots \times P_n$ and utility functions $u_i(p), i = 1, \ldots, n$

Here the players are the Private Clouds (PrC). The resources of the clouds are $r_i, i = 1, \ldots, n$. Each PrC chooses a price p_i for the resource. There is a demand R in the market. The customers observing the profile of the prices $p = (p_1, \ldots, p_n)$ are distributed among the PrC in logistic manner.

So, the portion

$$\gamma_i(p, r) = \frac{\exp\{-\alpha_i p_i + \beta_i r_i\}}{\sum_{k=1}^n \exp\{-\alpha_k p_k + \beta_k r_k\}}$$

of the demand is attractive to the i -th PrC, $i = 1, \ldots, n$. Introduce the costs c_i for i -PrC to serve the customers, $i = 1, \ldots, n$. Thus the net revenue of the PrC_i depending of the demand R is equal to

$$R\gamma_i(p, r)(p_i - c_i).$$

If the demand R becomes larger than the summarized resources of the PrC $r_1 + \ldots + r_n$ then the PrC-s have apply to Public Clouds (PuC) and buy additional resources in PuC. The price for the resource of the PuC is described by some function

$p = F(L)$ in dependence of the current load L of the PuC. $F(L)$ is non-decreasing continuous function.

We suppose that the demand is random variable distributed with CDF $G(R)$, so it is possible that the demand will be sufficient large. Suppose also that there are m PuCs $PuC_1,\ldots,PuC_m$ which have some initial loads $L_1,\ldots,L_m$. Without loss of generality suppose that $L_1 \leq \ldots \leq L_m \leq L_{m+1} = \infty$. So, the PrC-s apply at the beginning to the first cheapest PuC and after the load of PuC_1 reaches the value L_2 then they apply to both PuC_1, PuC_2 etc.

The equilibrium in this clod computing game is derived.

The research was supported by Russian Fund for Basic Research (project 13-01-91158) and Division of Mathematical Sciences of Russian Academy of Sciences (program Algebraic and combinatorial methods of mathematical cybernetics and new information systems).

Equilibria in two-sided Mate Choice Problem with Age Preferences

Vladimir Mazalov[1], Anna Ivashko[2] and Elena Konovalchikova[3]

[1,2]Institute of Applied Mathematical Research
Karelian Research Center of RAS
Russia, Petrozavodsk
[3]Transbaikal State University
Russia, Chita
[1]vmazalov@krc.karelia.ru
[2]aivashko@krc.karelia.ru
[3]konovalchikova_en@mail.ru

Keywords: *Mutual Mate Choice, Equilibrium, Threshold Strategy*

In the paper the two-sided mate choice model of Alpern, Katrantzi and Ramsey (2010) [3] is considered. The problem is following. The individuals from two groups (males and females) want to form a long-term relationship with a member of the other group, i.e. to form a couple. Each group has steady state distribution for the age of individuals. In the model males and females have lifetime m and n respectively. The total number of unmated males is greater than the total number of unmated females and $m \geq n$. The discrete time game is considered. In the game unmated individuals from different groups randomly meet each other in each period. If they accept each other, they form a couple and leave the game, otherwise they go into the next period unmated and older. It is assumed that individuals of both sexes enter the game at age 1 and stay until they are mated or males (females) pass the age $m\ (n)$. The aim of each player is to form a couple with individual of minimum age. Other two-sided mate choice model was considered in the papers [1, 2, 4-7]. Alpern, Katrantzi and Ramsey (2010) derive properties of equilibrium threshold strategies and analyse the model for small m and n. We derive analytically the equilibrium threshold strategies and investigate players' payoffs for the cases $n = 2,3$ and large m.

The work is supported by Russian Fund for Basic Research (project 13-01-91158-ГФЕН_a) and the Division of Mathematical Sciences of RAS (the program

"Algebraic and combinatorial methods of mathematical cybernetics and new information system").

References

[1] Alpern S., Reyniers D.J. Strategic mating with homotypic preferences. Journal of Theoretical Biology. 1999. N 198, 71--88.

[2] Alpern S., Reyniers D. Strategic mating with common preferences. Journal of Theoretical Biology, 2005, 237, 337--354.

[3] Alpern S., Katrantzi I., Ramsey D. Strategic mating with age dependent preferences. The London School of Economics and Political Science. 2010.

[4] Gale D., Shapley L.S. College Admissions and the Stability of Marriage. The American Mathematical Monthly. 1962. Vol. 69. N. 1, 9--15.

[5] Kalick S.M., Hamilton T.E. The mathing hypothesis reexamined. J. Personality Soc. Psychol. 1986 N 51, 673--682.

[6] Mazalov V., Falko A. Nash equilibrium in two-sided mate choice problem. International Game Theory Review. Vol. 10, N 4. 2008, 421--435.

[7] Roth A. Sotomayor M Two-sided matching: A study in game-theoretic modeling and analysis. Cambridge University Press. 1992.

Application of the Myerson Value for the Analysis of the Academic Sites

Vladimir Mazalov[1] **and Lyudmila Trukhina**[2]

[1]Institute of Applied Mathematical Research, Karelian Research Center
vmazalov@krc.karelia.ru
[2]Baikal State University of Economics and Low, Russia
lit-79@mail.ru

Keywords: *Partial Cooperation, Myerson Value, Communication Graph*

There are many real situations where there is a restriction in the coalition formation. Here we consider a TU-game with restricted cooperation which is presented by an undirected communication graph first developed by Myerson. The network relations are formally represented by graphs whose nodes are identified with the players and whose arcs capture the pairwise relations. The relation can be interpreted as an information transfer or resource distribution or transport connection. The nodes can be individuals or organizations or countries or web-pages.

It brings a communication (networking) game determined by a triple which consists on finite set of players, characteristic function and a graph of relations between players. A useful solution of the communication game is the Myerson value.

But computing the Myerson value is not easy problem. We propose a sufficient simple procedure to calculate the Myerson value for TU-game with special characteristic function.

Consider a game where the graph g is a tree which consists on n nodes and characteristic function is determined by the scheme proposed by Jackson: every direct connection gives to coalition S the impact r, where $0 \le r \le 1$ Moreover, players obtain an impact from non-direct connections. Each path of length 2 gives to coalition S the impact r^2 a path of length 3 gives to coalition the impact r^3 etc. So, for any coalition S we obtain

$$v(S) = a_1 r + a_2 r^2 + \ldots + a_k r^k + \ldots + a_L r^L = \sum_{k=1}^{L} a_k r^k ,$$

where L is a maximal distance between two nodes in the coalition; a_k s the number of paths of length k in this coalition.

$$v(i) = 0, \quad \forall i \in N.$$

We propose here the procedure of allocation the general gain $v(N)$ o each player $i \in N$ Stage 1. Two direct connected players obtain r. Individually, they don't receive nothing. So, each of them hopes to receive at least $r/2$ If player i has some direct connections then she receives the value $r/2$ imes the number of paths of length 1 which contain the node i.

Stage 2. Three connected players obtain r^2 so each of them must receive $r^2/3$ Arguing the same way we obtain the allocation rule of the following form:

$$Y_i\left(v, g\right) = \frac{A_1^i}{2} r + \frac{A_2^i}{3} r^2 + \ldots + \frac{A_L^i}{L+1} r^L = \sum_{k=1}^{L} \frac{A_k^i}{k+1} r^k,$$

where A_k^i s the number of all paths of length k which contain the node i.

We have proved that the determined allocation rule is coincides with the Myerson value.

For large n it is difficult to apply the formula (2) to compute the Myerson value. We propose to simplify the computing using the generating function approach.

Consider the tree $g_p = (N, E)$ ith the root in the node p. Introduce the generating function

$$\varphi_p(x) = \sum_{k=1}^{L} \alpha_k^p x^k$$

here α_k^p s the number of paths which consist on k nodes (length $k-1$and contain the node p.

To find this value we use modified algorithm proposed by Jamison for computing the generating function for the number of sub-trees of a tree g which contain k nodes of the tree g.

References

[1] Banzhaf, J.F. (1965). Weighted voting doesn’t work: mathematical analysis. Rutgers Law Review, 19, 317-343.

[2] Brams, S.F., Affuso, P.J. (1976). Power and Size: A new paradox. Theory and Decisions, 7, 29-56.

[3] Fernandez, J.R., Algaba, E., Bilbao, J.M., Jimenez, A., Jimenez, N., Lopez, J.J. (2002). Generation functions for computing the Myerson Value. Annals of operation research, 109, 143-158.
[4] Jackson, M.O. and Wolinsky, J. (1996). A strategic model of social and economic networks, Journal of Economi$n$-person games. Annals of Mathematical studies, 28, 307-317.
[9] Shapley, L.S., Shubik, M. (1954). A method for evaluating the distribution of power in a committee system. American Political Science Review, 48, 787-792.

On a Discrete Arbiration Procedure with Quadratic Payoff Function

Alexander Mentcher

Zabaikalsky State University
Alexandro-Zavodskaya str. 30, Chita, 672039, Russia
mentcherae@zabspu.ru

Keywords: *Arbitraton Procedure, Equilibrium, Mixed Strategies*

We consider a non-cooperative two players game, in wich we use an arbitration scheme with quadratic payoff function and noninform distribution of probabilities. The equilibrium in this game in mixed strategies is found.

We consider a non-cooperative zero-sume game in wich two players L и M (The Labour and the Manager) have a dispute on an improvement in the wage rate. The player L makes an offer x, and the player M --- an offer y, x and y. are arbitrary real numbers. If $x \le y$,there is no conflict, and the players agree on a payoff equal to $\frac{x+y}{2}$. If, otherewise, $x > y$, the players call in the arbitraror A. . Denote the arbitration decision by z. In the papers [1,2] we use for the settlement between players the final offer arbitration procedure. The payoff in this game has a form: $H(x,y) = EH_z(x,y)$. , where

$$H_z(x,y) = \begin{cases} \dfrac{x+y}{2}, & if \quad x \le y \\ x, & if \quad x > y, \ |x-z| < |y-z| \\ y, & if \quad x > y, \ |x-z| > |y-z| \\ z, & if \quad x > y, \ |x-z| = |y-z| \end{cases}. \tag{1}$$

The equilibrium in the game in mixed strategies was found.

Now we use an arbitration scheme wich quadratic payoff function. Namely, let $x \in [0;+\infty)$, $y \in (-\infty;0]$ and the arbitrator chooses the number 0 with the probability p

and the numbers -1 and 1 with equal probabilities $\frac{1-p}{2}$, where $0 < p < 1$. The payoff in this game has a form: $H(x,y) = EH_z(x,y)$, where

$$H_z(x,y) = \begin{cases} x^2, & if \quad |x-z| < |y-z| \\ -y^2, & if \quad |x-z| > |y-z| \\ z, & if \quad |x-z| = |y-z| \end{cases} \tag{2}$$

Denote by $f(x)$. and $g(y)$ the mixed strategies of the players L and M, respectively. We have,

$$f(x) \geq 0, \quad \int_0^{+\infty} f(x)dx = 1; \qquad g(y) \geq 0, \quad \int_{-\infty}^{0} g(y)dx = 1;$$

By symmetry it follows that the value of the game is equal to zero, and the optimal strategies must be symmetric in respect to y-axis i.e. $g(y) = f(-y)$. Hence, it suffices to construct the optimal strategy only for one player, for example L.

Theorem. If $p \in [\frac{1}{3};1)$, then for the player L the strategy

$$f(x) = \begin{cases} 0, & if \quad 0 \leq x < c, \\ \frac{1+p}{2p} \cdot \frac{c}{x^2}, & if \quad c < x < c+2 \\ 0, & if \quad c+2 < x < +\infty \end{cases} \tag{3}$$

where $c = \frac{1-p}{p}$, is optimal.

The research was supported by Ministry of Education and Science of the Russian Federation (project 8.3641.2011).

References

[1] Mazalov V.V., Mentcher A.E., Tokareva J.S. On a discrete arbitration procedure in three points // Game Theory and Applications. 2005. Vol. 11. p. 87--91.

[2] Mentcher A.E. On a discrete arbitration procedure wigh noninform distribution// Matematical Game Theory and Applications. 2009, Vol. 4. T.1. p-78-91 (in Russian)

Risk Aversion and Price Dynamics on the Stockmarket

Bernard De Meyer

PSE-Universite Paris 1

In [De Meyer (2010)], the market was modeled as a repeated exchange game between the informed sector and the remaining part of the market. In this setting, the informed agent is using his information strategically and this implies a very particular class of dynamics for the price process: The main result in that paper claims that, independently of the trading mechanism used to model the exchanges, the price process will be a so called Continuous Martingale of Maximal Variation (CMMV). This class of dynamics contains as a particular case the classical log normal dynamics of Black and Scholes. In that paper, the uninformed sector was modeled as a single risk neutral agent. In the present paper, we introduce risk aversion for the uninformed agent and consider one speci_c exchange mechanism. Prices at equilibrium are not martingales any more, but we prove that under a \martingale equivalent measure" depending on the utility function of the uninformed agent, the price process is a CMMV.

A Linear Programming Algorithm for an Undiscounted One Player Control Semi-Markov Game in the Unichain Case

Prasenjit Mondal and Sagnik Sinha

Mathematics Department, Jadavpur University Kolkata-700032, India

Keywords: *Semi-Markov Games, Limiting Average Payoffs, one Player Control Semi-Markov Games, Finite Step LP Algorithms for Semi-Markov Games*

Stochastic (Markov) games were introduced by Shapley (1953, Ref.4), whose original formulation has since been expanded by others to include a broader range of problems in game theory. Gillette (1957, Ref.2) introduced a limiting average payoff for non-terminating stochastic games with finite state and action spaces. In the theory of stochastic (Markov) games, Mertens and Neyman (1981, Ref.3) proved the existence of a value and near (epsilon) optimal strategies of the players in the limiting average payoff case. Stern (1975, Ref.5), in his Ph.D. thesis first studied one player control stochastic games and proved the existence of a value in a semi-Markov (stationary) strategies for limiting average payoffs. Parthasarathy and Raghavan (1981, Ref.8) proved the existence of value and stationary strategies for one player control stochastic games under both discounted and limiting average payoff criteria. They also proved the ordered field property for one player control stochastic games. Vrieze (1983, Ref.6) proposed a finite algorithm for one player control finite undiscounted stochastic games by solving one linear programme only.

Zero-sum two-person semi-Markov games were studied by Lal and Sinha (1992, Ref.1) which is a generalisation of stochastic (Markov) games. Under some ergodicity conditions, they proved the existence of value and optimal stationary strategies of the players for limiting average payoff semi-Markov games.

This paper considers two person zero-sum finite semi-Markov games under limiting average payoffs where the transition probabilities and the transition time distributions do not depend on the actions of a fixed player at all states. Though one player control undiscounted stochastic games have value and optimal stationary strategies for the players in the same Archimedean ordered field (Ref.8), it is not true for one player control undiscounted semi-Markov games in a general multichain structure (see Ref.7). So, we consider one player control undiscounted unichain semi-Markov games. Under unichain assumption, we prove the existence of value and stationary optimal strategies for the players by solving just one LP and its dual. The ordered field property follows from this LP algorithm.

References

[1] K. Lal and S. Sinha (1992): Zero-sum two-person semi-Markov games, J. Appl. Prob., 29, 56-72.

[2] D. Gillette (1957): Stochastic Games with Zero Stop Probabilities, Contributions to the Theory of Games, Vol.III, Princeton University Press, Princeton, pp. 179-187.

[3] J. F. Mertens and A. Neyman. (1981): S'tochastic Games. Internat. J. Game Theory, 10, 53-66.

[4] L. Shapley (1953): Stochastic Games. Proc. Natl. Acad. Sci. U.S.A. 39, 1095-1100.

[5] M. A. Stern (1975): On Stochastic Games with Limiting Average Payoff. University of Illinois at Chicago Circle, Chicago, Illinois, PhD Thesis.

[6] O. J. Vrieze (1983): Stochastic Games with Finite State and Action Spaces, Ph.D. Dissertation. Mathematical Centre, Amsterdam.

[7] P. Mondal and S. Sinha (2012): On Average Reward Semi-Markov Decision Processes: An Example, Forthcoming. to be appear in IGTR.

[8] T. Parthasarathy and T.E.S. Raghavan (1981): An Orderfield Property for Stochastic Games when One Player Controls Transition Probabilities, JOTA, Vol.33, No.3

Ordered Field Property in Subclasses of Finite Discounted AR-AT Semi-Markov Games

Prasenjit Mondal[1], Sagnik Sinha[2], Samir Kumar Neogy[3] and Arup Kumar Das[4]

[1,2]*Mathematics Department, Jadavpur University, Kolkata-700032, India*
[3]*Indian Statistical Institute, Delhi Centre, New Delhi-110016, India*
[4]*Indian Statistical Institute, Kolkata Centre, Kolkata-700108, India*
[1]*prasenjit1044@yahoo.com*

Keywords: *Semi-Markov Games, Additive Semi-Markov Games, Pure Stationary Optimal Strategies, Ordered Field Property, Applications of Subclasses of Additive Games, Finite Step Algorithms*

We study the ordered field property in the subclasses of two person finite AR-AT semi- Markov stochastic games with discounted payoffs. Though zero-sum discounted AR-AT stochastic games satisfy the ordered field property (Raghavan, Tijs and Vrieze [1985]), it is not true for AR-AT semi-Markov games, in general. Stochastic (Markov) games were introduced by Shapley [1953]. Semi-Markov games are generalizations of stochastic (Markov) games with random transition times which were studied by Lal and Sinha [1992]. They established the existence of a value and stationary optimal strategies of the two players in the discounted case.

In this paper, we restrict ourselves to a particular class of semi-Markov games which was

studied by Raghavan, Tijs and Vrieze [1985], in the Markov case and modify it to obtain some new subclasses which has the ordered field property. We have found two new subclasses namely AR-AT-AITT and AR-AIT-ATT of AR-AT semi-Markov games that satisfy the ordered field property. These two subclasses of semi-Markov games have application in our real life as we have shown by two examples in this paper.

References

[1] Filar, J. A. and Vrieze, K. (1997): Competitive Markov Decision Processes. Springer, New York.

[2] Kaplansky, I. (1945): A contribution to von Neumanns theory of games. Ann. of Math. (2), 46, 474-479.

[3] Lal, A. K. and Sinha, S. (1992): Zero-sum two-person semi-Markov games. J. Appl. Prob., 29, 56-72.
[4] Mondal, P and Sinha, S. (2013): Ordered Field Property in a Subclass of Finite SeR- SIT Semi-Markov Games. Accepted for publication in IGTR.
[5] Raghavan, T. E. S., Tijs, S. H., Vrieze, O. J. (1985): On stochastic games with a,dditive reward and tmnsition structure. J. Optim. Theory Appl. 47, 375-392
[6] Shapley, L. (1953): Stochastic Games. Proc. Natl. Acad. Sci. U.S.A. 39, 1095-1100.

On Uniqueness of Coalitional Equilibria for Cournot-like Games

Michael Finus[1], Pierre von Mouche[2] and Bianca Rundshagen[3]

[1]*University of Bath, Department of Economics, Bath BA2 7AY, United Kingdom.*
[2] *Wageningen Universiteit, Postbus 6700 EW Wageningen, The Netherlands.*
[3]*FernUniversitat in Hagen, Volkswirtschaftslehre, Profilstrasse 8, 58084 Hagen, Germany.*

Keywords: *Cournot Oligopoly, Public Good, Game in Strategic Form, Nash Equilibrium, Coalitional Equilibrium, (Semi-)Uniqueness.*

Introduction

The analysis of coalition formation - in particular in the context of externalities - has become an important topic in economics (see for instance [1] for an extensive overview). Examples not only include firms that coordinate their output or prices in oligopolistic markets (cartels), jointly invest in research assets (R&D-agreements) or completely merge (joint ventures), but also countries that coordinate their tariffs (trade agreements and customs unions) or their environmental policy (international environmental agreements). This article contributes to the so-called 'new approach' of coalition formation. This approach consists in modeling coalition formation as a two-stage game; the goal is to determine equilibrium coalition structures. Roughly speaking, in the first stage, each player chooses a membership action which leads to a coalition structure. In the second stage each coalition in this coalition structure chooses its 'physical' actions in a base game.*

*In this footnote we give a short description of this approach (see e.g. [3] for more). Suppose $N = \{1, \ldots, n\}$ is a set of players. A coalition structure is a sequence $\mathcal{C} = (\mathcal{C}_1, \ldots, \mathcal{C}_m)$ consisting of disjoint non-empty subsets of N whose union is N. Let C be the set of possible coalition structures. If Γ is a game in strategic form and $\mathcal{C} \in C$, then $\Gamma(\mathcal{C})$ denotes the game in strategic form with as players, also referred to as 'meta-players', the coalitions in $\mathcal{C}$, with for each meta-player as strategy set the Cartesian product of the strategy sets in Γ of the members of the meta-player, and with the payoff function of a meta-player the aggregate payoff function of its members. The Nash equilibria of $\Gamma(\mathcal{C})$ are called *coalitional equilibria*. Having said this, we now can formulate the two-stage game. Formally, the fundamental objects are: (1) a set of players N. (2) A game in strategic form Γ with these players, called the *base game*. (3) Sets M_i $(i \in N)$ and, with $\mathbf{M} := M_1 \times \cdots \times M_n$, a so-called *membership rule* $R : \mathbf{M} \to C$ (and may be also (4) a sharing rule). It is assumed that for each $\mathcal{C} \in C$ the game $\Gamma(\mathcal{C})$ has a unique coalitional

For the new approach it is important to have results that guarantee uniqueness of coali- tional equilibria. Conditions should be such that they can be easily checked for the base games that appeared so far in these models, like Cournot and public good games. As far as we know, general uniqueness results for coalitional equilibria of such games are not present in the literature. There it is just assumed that one deals with a situation where coalitional equilibria are unique or that one deals with a simple concrete example where uniqueness explicitly can be shown. The problem is not the existence but the semi-uniqueness, i.e. that there exist at most one equilibrium. Developing an abstract general uniqueness result is the main objective of this article.

Uniqueness comes down to existence and semi-uniqueness. For games in strategic form, continuous payoff functions together with quasi-concavity of conditional payoff functions implies (by virtue of the Nikaido-Isoda theorem or one of its slight extensions) existence of a Nash equilibrium under an appropriate compactness property. So for such games existence is not a real issue, but semi-uniqueness is. Semi-uniqueness for coalitional equilibria even is a larger issue as for handling such equilibria one has to leave the comfortable usual setting of one dimensional strategy sets. Indeed (as we have seen footnote 1) a coalition is formally treated as meta-player whose strategy set is the Cartesian product of the strategy sets of the players in this coalition.

In order to obtain our semi-uniqueness result for coalitional equilibria we develop a semiuniqueness result for Nash equilibria of games in strategic form with higher dimensional strategy sets. This result can be considered as a generalisation of a result in [2] to higher dimensions. It can handle various aggregative base-games with one-dimensional strategy sets. We identify a class of such games; this class contains various Cournot and public good games.

---

equilibrium. Let $V_i(\mathcal{C})$ be the individual payoff of player $i \in N$ in this equilibrium; this defines $V := (V_1, \ldots, V_n) : C \to \mathbb{R}^n$, called *valuation*. (A weaker appropriate assumption would be: each game $\Gamma(\mathcal{C})$ has at least one coalitional equilibrium and for each player it holds that he has the same payoff in each of these equilibria.) Thus the valuation V assigns to each coalition structure $\mathcal{C}$ for each player i an individual payoff $V_i(\mathcal{C})$. The game in strategic form G is defined as follows: its player set is N, the strategy set for player i is M_i and his payoff function is ... Finally, the Nash equilibria of G induce via the membership rule R the equilibrium coalition structures.

References

[1] F. Bloch. Non-cooperative models of coalition formation in games with spillovers. In C. Carraro, editor, Endogenous Formation of Economic Coalitions, chapter 2, pages 35-79. Edward Elgar, Cheltenham, 2003.

[2] L. C. Corchon. Theories of Imperfectly Competitive Markets, volume 442 of Lecture Notes in Economics and Mathematical Systems. Springer-Verlag, Berlin, second edition, 2001.

[3] M. Finus and B. Rundshagen. Membership rules and stability of coalition structures in positive externality games. Social Choice and Welfare, 32:389-406, 2009

How to arrange a Singles' Party: Coalition Formation in Matching Game

Joseph Mullat

Freelance researcher
Denmark
jm@mail-telia.dk

Keywords: *Stability Conditions, Game Theory, Coalition Formation*

Imagine a situation whereby, in order to find a suitable partner, five single women and five single men are going to participate in Singles Party. To cover arrangements expenses, refreshments, beverages, orchestra and rewards, the entrance fee is 50€ [*]. Thus, the cashier will be at the disposal of an amount of 500 €. Just before the party ended all have been kindly asked to disclose their priorities: women asked to disclose priorities about men, and men, visa versa, about woman. Those who agreed have been promised to collect a Box of Delights[†]; otherwise, the Box is not available. Players who hide their priorities are blind by default and cannot participate in the game. We also suppose that there was a plenty of time during the party making these priorities (preferences) clear to everyone. Below we continue to those who will participate in the game, in addition to delights, setting up the rules of payoffs in the form of mismatch-compensations and rewards. We call them all as participants. In particular, when all participants have gone on dating, the game ends, and, therefore, no reasonable justification exists for the payment of rewards and compensations.

We use index i for a woman notation, and an index j for a man. Let all the guests participate in the game; all five women, $\{1,...,i,...5\}$, and all five men, $\{1,..., j,...5\}$. Let during the evening the mutual priorities (preferences) have been

[*] Note that red colour points at negative number.
[†] In case the Box is undesirable it will be possible to get 10€ in return.

disclosed. Each woman i, $i=\overline{1,5}$, disclosed her priorities w_i towards men, and each man j, $j=\overline{1,5}$, towards women as m_j. Setting priorities straight, the priorities may be arranged into two 5×5 tables: $W=w_{i,j}$ and $M=m_{i,j}$: the rows in table W are some horizontal permutations w_i of numbers $\langle 1,2,3,4,5\rangle$; vertical permutations m_j, stand in columns of the table M. At the end of the evening, priorities w_i, m_j in writing, were handed over to the organizers. Below, in Table-1, priorities w_{ij} (numbers $\langle \overline{1,5}=1,2,3,4,5\rangle$) might repeat themselves in columns of table W. Repetitions may also happen in rows of the table M, i.e., a woman at identical priority level may be preferred by more than one man, and a man by many women. As proposed by Võhandu, * mutual regrets $r_{i,j}=w_{i,j}+m_{i,j}$ occupy the cells in table R.

$W=$

	M1	M2	M3	M4	M5
W1	1	5	3	2	4
W2	5	4	1	2	3
W3	3	5	4	2	1
W4	2	5	3	1	4
W5	4	3	1	2	5

Women Priorities

$+M=$

	M1	M2	M3	M4	M5
W1	3	4	2	1	2
W2	1	3	4	2	4
W3	5	2	3	4	3
W4	4	5	1	3	1
W5	2	1	5	5	5

Men Priorities

$=R=$

$=$

	M1	M2	M3	M4	M5
W1	4	9	5	3	6
W2	6	7	5	4	7
W3	8	7	7	6	4
W4	6	10	4	4	5
W5	6	4	6	7	10

Mutual Regrets

Table-1 1 1

Suppose that some fortunate participants, those with low level of mutual regrets, went to dating, and will receive in advance a reword – a prepaid ticket to happening, restaurant, concert, etc.

---

* Võhandu, L.V. (2010). Kõrgkooli vastuvõttu korraldamine stabiilse abielu mudeli rakendusena, Õpetajate Leht, reede, veebruar, nr.7/7.1, in Estonian.

Others, those unlucky ones, i.e., with higher levels of mutual regrets, may claim compensations because they were left empty-handed – only unsuitable partners remained. Suitable participants at the low levels of mutual regrets were already occupied.

What happens in case no one founds a partner? Let, for a moment, the mismatch compensation will be set up individually, and equals, $c_{i,j} = \frac{1}{2} r_{i,j} \cdot 10$ €. In fact, due to profit motive, it will be unwise to compensate participants in proportion to mutual regrets $r_{i,j}$, because of misrepresentation, cheating, imposture, and the like. Doing so, for example, couple $(5,5)$ may raise their compensation up to 50€! Therefore, arrangers of the party are ready to follow a principal of maximum upon minimum compensation.

In Table-1, the lowest mutual regret among all participants is $r_{1,4} = 3$. All not yet occupied men i and women j will receive, following our principal, the same compensation $\frac{1}{2} r_{1,4} \cdot 10 = 15$ €. It is also more profitable from the cashier point of view. Thus, the balance of payoffs for all participants, inclusive 10€ cost of delights, will be negative: 50€ + 15€ + 10€ = 25€, e.g., -50€ as the entrance fee, 15€ received as mismatch compensation, and 10€ as the cost of delights.

What happens in case the couple $(1,4)$ decides to date? Assume that, he/she, both of them,will receive a reward for a date. Let, for simplicity the reward equals to or is higher of a doubled grade of mismatch compensation. The entire table R should be dynamically reorganized to reflect the fact that $(1,4)$ are occupied, and the rest, i.e., the women $\{2,3,4,5\}$ and men $\{1,2,3,5\}$, no longer can count on $(1,4)$ as potential partners. Therefore, priorities will fall; the scale $\langle 1,2,3,4,5 \rangle$ packs together. We say the scale will dynamically shrink to $\langle 1,2,3,4 \rangle$. Table-1 transforms, into:

$$W = \begin{array}{c|ccccc} & M1 & M2 & M3 & M4 & M5 \\ W1 & & & & & \\ W2 & 4 & 3 & 1 & & 2 \\ W3 & 2 & 4 & 3 & & 1 \\ W4 & 1 & 4 & 2 & & 3 \\ W5 & 3 & 2 & 1 & & 4 \end{array} + M = \begin{array}{c|ccccc} & M1 & M2 & M3 & M4 & M5 \\ W1 & & & & & \\ W2 & 1 & 3 & 3 & & 3 \\ W3 & 4 & 2 & 2 & & 2 \\ W4 & 3 & 4 & 1 & & 1 \\ W5 & 2 & 1 & 4 & & 4 \end{array} = R =$$

Women Priorities　　　　Men Priorities

=

	M1	M2	M3	M4	M5
W1					
W2	5	6	4		5
W3	6	6	5		3
W4	4	8	3		4
W5	5	3	5		8

Mutual Regrets

Table-2 1 1

The mismatch compensation did not change and equals to $c_{3,5} = 15\,€$, while couple $(1,4)$ potential balance $-50\,€ + 10\,€ + 2 \cdot 15\,€ = 10€$ of payment improves. N.B., W1 and M4 each receive the reward to date of $30\,€$ in accord with the rule that the reward is always higher or equals to the doubled mismatch compensation! For the rest, the balance remains negative (in deficit) and equals 15€. The balance of payoffs improves as well in case the couple $(3,5)$ decides to date.

The party is over. Decisions have been made about who will enter the dating and who is not, and passed in writing to the organizers. What would be in accord with the rules and regulations of the game the best collective decision of participants based on the maximization principal of the minimum compensation? Who is then going to enter the coalition of dating participants?

Evolution of Agents Behavior in the Labor Market

Nikolai Balashov[1] and Maria Nastych[2]

[1,2]SPbSU
Russia
[2]nastych@bk.ru
[1]balashov.nd@gmail.com

Keywords: *Labor Market, Evolutionary Game Theory, Personnel Training*

One of the leading resources of Russian economic internal growth in modern conditions is the labor force. Currently the focus is on the problem of the economic growth in Russia associated with the energy dependency, whereas the problems connected with the dependence of the Russian economic system on the cost and quality of the labor force, unfairly in the shadows. The study of the reasons for the current state of the labor market is definitely important applied problems as well. However, the problem of explaining patterns of the labor market evolution is more significant on the theoretical level in the long term.

Urgent problem of the Russian economy is the lack of qualified personnel and imbalance of labor supply and labor demand relative to qualifications. Thus, there is excess of labor demand over its supply for the market of rare and needed professions, which causes the price of labor assignment by the candidates themselves. For the low-skilled labor market it can be observed the reverse situation. An enormous low-skilled labor supply leads to the situation where employers dictate most of the conditions.

Every person faces a problem of choosing a right path for his future career. In most of the cases decisions about profession and the quality of education are made based on the imperfect information and incorrect definition of objectives. Employees make decisions based not on the market indicators of demand and wages, but on the basis of subjective factors. In other words, candidates take into account not quantitative but qualitative indicators.

On the other hand, employers have a dilemma either to hire insufficient qualified personnel in a particular field and train until he or she reaches required level of qualification, or seek an opportunity to hire skilled personnel. In this case, firms should be guided by the motives for the optimization of personnel costs. Therefore, the objective of employers is the choice of their behavior, based on current market conditions and adequate quantitative assessment of possible alternatives.

Described problems reveal the need to study the behavior of agents in the labor market in order to find an equilibrium strategy in the long term. It is worth noting that the labor market agents also play one of the major roles in this area of Russian economy. Their recruiting strategy based on the appropriate quantitative indicators through their access to information could be an integral part of the process of equilibrium achieving.

The labor market can be divided into three sectors according to the level of personnel qualification: low-skilled, skilled and highly skilled. Each of these sectors has different sets of rules and conditions. Selection of an optimal strategy for responding to the behavior of other similar participants can facilitate a potential conflict of interests for both employers and candidates. Conflict character of participants' behavior stipulates the use of mathematical tools of game theory to the labor market study.

Each candidate has an alternative of either accept a job offer, which meets his current qualifications, or look for a job which will offer him or her an opportunity of getting trained by qualified personnel and in the long run give possibility of a career growth. Therefore, employees have two appropriate strategies: to hire already qualified personnel or to spend money on training non-qualified personnel. Since availability of training candidates can improve their skills and employers can improve the skills of their workers. Training and development can be considered within the evolutionary game theory. A propensity for trainings will be one of the factors which determines an evolutionarily stable strategy (ESS) in the game theory model for both employees and employers.

We introduce further notations for possible employees' strategies according to distinguished sectors: L,S and H, and notations for the average wage for each of the selected sectors: w^l, w^s, w^h for the low-skilled, skilled and highly skilled workforce, respectively. In terms of labor demand the set of possible strategies include two alternatives: Q for the strategy to hire already qualified personnel and E for the alternate behavior. Assume the cost of meeting the skilled and highly skilled level to be equal to c^s and c^h, respectively, for both employees and employers.

Let us consider firstly low-skilled labor market, where is no need to train employees. In non-crisis economic environment, individuals with higher qualifications will not compete for employment in this market. Then the payoff matrix in terms of employees could be described as shown in the Table 1. ESS of this game is obviously the situation (L,L). Possible to assume that in crisis economic environment all of the payoffs of such evolution game can be equal to $(w^l;w^l)$. A similar situation is in the game in terms of employers, where there is no need of any qualification level and, therefore, all of the payoffs is equal to $(w^l;w^l)$.

Table 1. The payoff matrix in terms of employees

	L	S	H
L	$(w^l;w^l)$	$(w^l;0)$	$(w^l;0)$
S	$(0;w^l)$	$(0;0)$	$(0;0)$
H	$(0;w^l)$	$(0;0)$	$(0;0)$

For modeling the games of the skilled and highly skilled labor markets it is necessary to introduce a parameter of propensity to education for workers θ^w and parameter of propensity to training for employers θ^e. It is shown in the article that there is exists ESS's dependence on these propensity parameters. Other result of such evolutionary games is the necessity for player to be guided by the unavailable information about the incentives of the opposite type players. The solution to this lack of information is the involvement of recruitment agencies that possess such data.

For practical application of this model, it is necessary to consider certain occupations for each of the sectors. Derived evolutionarily stable strategies determine the equilibrium behavior of the labor market with certain allowable intervals for the model parameters.

As a further development, we propose to introduce into the model additional parameters that affect the behavior of economic agents in the labor market, such as work experience. There are some occupations, in which more relevant is to get practical experience rather than to receive theorize education, and vice versa.

An Axiomatization of the Proportional Prenucleolus

Natalia Naumova

St. Petersburg State University
Russia
nataliai.naumova@mail.ru

Keywords: *Cooperative Games, Proportional Nucleolus, Prenucleolus, Consistency*

We consider the class of positive TU games

$$G^+ = \{(N,v) : |N| < \infty, v(S) > 0 \quad for \quad \emptyset \neq S \subset N\}.$$

For $(N,v) \in G^+$, $z \in R_{++}^{|N|}$, let the collection of coalitions $\{S : S \subset N, S \neq N, \emptyset\}$ be enumerated such that $z(S_i)/v(S_i) \leq z(S_{i+1})/v(S_{i+1})$. Denote

$$\theta((N,v),z) = \{z(S_i)/v(S_i)\}_{i=1}^{2^{|N|}-2}.$$

Then $y \in R_{++}^{|N|}$ with $y(N) = v(N)$ belongs to the proportional prenucleolus of (N,v) iff

$$\theta((N,v),y) \geq_{lex} \theta((N,v),z) \quad forall \quad z \in R_{++}^{|N|} \quad with \quad z(N) = v(N).$$

For each $(N,v) \in G^+$, the proportional prenucleolus of (N,v) is a singleton.

The following axiomatization of the proportional prenucleolus is a modification of Sobolev's axiomatization of the prenucleolus (see [2] or [1]).

Let a value f be defined on G^+. Consider the following axioms.

Efficiency. $f(N,v) \in R_{++}^{|N|}$ and $\sum_{i \in N} f_i(N,v) = v(N)$.

Proportionality. For any games $(N,v),(N,w) \in G^+$, any $x, y \in R_{++}^{|N|}$,

$$\frac{x(S)}{v(S)} = \frac{y(S)}{w(S)} \quad forall \quad S \subset N, S \neq \emptyset$$

implies

$$x = f(N,v) \quad iff \quad y = f(N,w).$$

Anonymity. Let for games (N,v) and (N',w) there exists a bijection

$\pi : N \to N'$ such that $v(S) = w(\pi S)$ for all $S \subset N$. Then $f_i(N,v) = f_{\pi i}(N',w)$.

Proportional DM consistency. Let $x = f(N,v)$, then for each $S \subset N$, $x_S = f(S, v^{x,S})$, where

$$v^{x,S}(Q) = \begin{cases} v(N) - x(N \setminus S) & for \quad Q = S, \\ \max\limits_{T \subset N \setminus S} \dfrac{v(Q \cup T)x(Q)}{x(Q \cup T)} & for \quad Q \subset S, Q \neq S. \end{cases}$$

Theorem 1. *The proportional prenucleolus is a unique value defined on* G^{+} *that satisfies efficiency, proportionality, anonymity, and proportional DM consistency properties.*

The proportionality property means that the value f depends only on the values of proportional excesses. It was used by Yanovskaya [3] for axiomatization of some proportional solutions instead of covariance property. The proportional DM consistency is a modification of Davis-Maschler consistency.

References

[1] Peleg,B. and Sudholter,P. (2003, 2007), Introduction to the theory of cooperative games. Springer-Verlag, Berlin Heidelberg New York.Chapter 6.

[2] Sobolev, A.I.(1975), The characterization of optimality principles in cooperative games by functional equations. Mathematical Methods in the Social Sciences, ed. by N.N.Vorobiev,Vol.6, 95-151, Vilnius,Academy of Sciences of the Lithuanian SSR, in Russian.

[3] Yanovskaya, E. (2002), Consistency for proportional solutions. International Game Theory Review, Volume 4, N3, 343-356.

A Distribution-free Newsvendor Problem with Nonlinear Holding Cost

Brojeswar Pal[1], Shib Sankar Sana[2] and Kripasindhu Chaudhuri[3]

[1,3]Department of Mathematics, Jadavpur University,
Kolkata - 700032, India.
[2]Department of Mathematics, Bhangar Mahavidyalaya, University of Calcutta
, Bhangar, Kolkata
743502, 24PGS (South), India
[1]brojo_math@yahoo.co.in
[2]shib_sankar@yahoo.com,
[3]chaudhuriks@gmail.com

Keywords: *Newsvendor Model, Customer Balking, Distribution Free, Partial Backlogging*

In this paper, we analyze the single period newsvendor model to determine the optimal order quantity where customer's balking occurs. This scenario occurs when the customers are opposed to buy a product for various reasons, like decreasing quality of product, product is not as good as fresh when it reaches under a threshold level, etc. The model is investigated by assuming that the holding cost function depends on order quantity and the inventory level at which customer balking occurs, depends on holding cost. We consider the holding cost which depends on lot-size (Q) (in such a special way that firstly holding cost increases with the higher inventory lot and at an inventory level, holding cost is maximum, then holding cost decreases with the inventory lot and lastly holding cost reach a saturated level). The holding cost per unit product is

$$h(Q) = h + \frac{Qe^{\frac{-Q}{\alpha}}}{\sqrt{2\alpha pi}}$$

Using the following Property 2.1, we have some characteristic of the above holding cost function as the holding cost function is increasing when $Q \le \alpha$ a and it is maximum at the point $Q = \alpha$ and the function is decreasing when $Q \ge \alpha$. Generally speaking, the constant part (h) is fixed for set up of the storage facility and the variable part varies with the lot size Q in practice. In our model, the inventory level $(K(Q)$ where balking starts depends on holding cost such as

$$K(Q) = K + \frac{\beta}{1 + h(Q)}$$

From the following two figures: holding cost per unit per unit time versus ordering size and the inventory level at which customer balking occurs versus holding cost per unit per unit time, taking the parameter values $\alpha = 400, h = 2, K = 100, \beta = 200,$ we may have a rough idea about the characteristics of holding cost per unit per unit time and the inventory level at which customer balking occurs in our model.

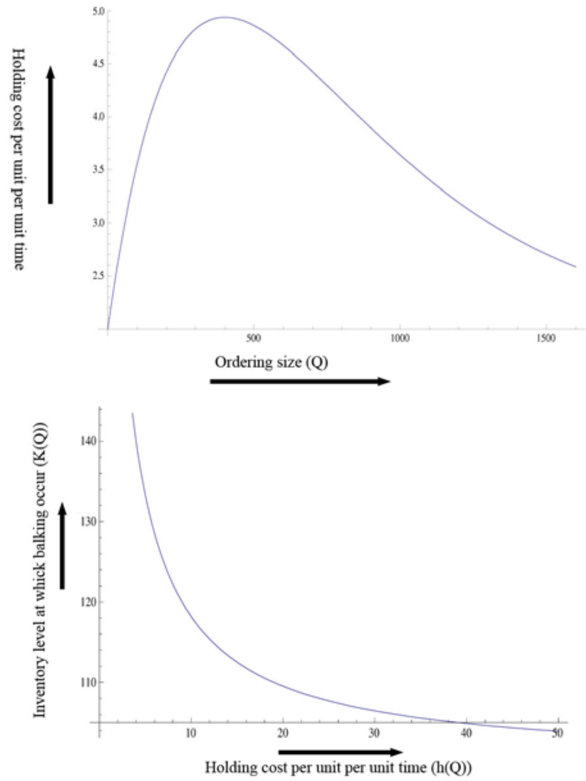


Fig. 2: Inventory level at which balking occur versus Holding Cost per unit per unit time

The model allows partial backlogging and permits part of the backlogged shortages to turn into lost sales. We develop the model without taking any specific distributional form of demand, only assuming the mean and the variance of the distribution of demand. Finally, we illustrate the model by numerical examples and compare our distribution-free model with the specific distributional form of demand.

Cooperative Interval Games: Forest Situations with Interval Data

Osman Palanci[1], Zeynep Alparslan Gok[2] and Gerhard Wilhelm Weber[3]

[1,2]Suleyman Demirel University, Faculty of Arts and Sciences,
Department of Mathematics, 32 260 Isparta, Turkey
[1]osmanpalanci@sdu.edu.tr
[2]zeynepalparslan@yahoo.com
[3]Institute of Applied Mathematics, Middle East Technical University, 06531,
Ankara, Turkey
gweber@metu.edu.tr

Keywords: *Cooperative Game Theory, Interval Uncertainty, Forest Situations*

In a classical mountain situation which is studied in [10] a group of people whose houses lie on mountains surrounding a valley or a part of a coast are considered. They want to be connected to a drainage system, where they have to empty their sewage. It is obvious that the sewage has to be purified before introduction into the environment. So, the sewage has to be collected downhill in a water purifier in the valley or along the coast. Consequently, each player wants to connect his house with a drain pipe to the water purifier.

The problem is the higher costs of direct connection to water purifier and pumping water from the houses at lower heights to the houses at upper heights. Further, being connected to the houses at the same height may be dangerous.

A mountain situation as described above leads to a connection problem of a directed graph without cycles and with some other properties [6]. A connection situation takes place in the presence of a group of agents, each of which needs to be connected to a source. If links are costly, then agents will evaluate the opportunity of cooperating in order to reduce costs. The cost allocation problem arising from a connection situation was introduced by [5] and has been studied with the aid of cooperative game theory [4] since the paper of [3]. We note that cost allocation problems may arise on many different physical networks such as telephone lines, highways, electric power systems, computer chips, water delivery systems, railways, etc. On the other hand, many authors have shown that to retrieve information needed to assess the exact cost of all the links of a real network is a very hard task [7, 8]. So, we argue that it is more realistic to imagine connection situations where the costs of links are identifiable at a level of uncertainty, i.e. only the range of the costs is known, and no probability information on the realization of costs is given. Such connection situations with uncertain costs may be represented using graphs where the costs associated to the edges are intervals of real numbers. In the sequel cost allocation problems arising from connection situations, where edge costs are closed intervals of real numbers are studied in [9].

Recently, [11] model mountain situations by introducing multiple sources called a forest situation. If we consider interval mountain situations with more purifiers, then we get special minimum interval cost connecting interval forest problems, where all houses are connected with at least one purifier.

In this study, we extend the obtained results in [11] to forest situations with interval data. We use the notion of cooperative interval games [1, 2] to tackle the cost sharing problem to a forest situation. We deal with cost monotonic allocation rules for forest situations. Further, we study optimal connection problems and related interval cost sharing problems of forest situations with interval data.

References

[1] Alparslan Gok S.Z., (2009). Cooperative interval games, PhD Dissertation Thesis, Institute of Applied Mathematics, Middle East Technical University.

[2] Alparslan Gok S.Z., (2010). Cooperative Interval Games: Theory and Applications, LAP-Lambert Academic Publishing house, Germany, (PROJECT-ID:5124, ISBN-NR: 978-3-8383-3430-1).

[3] Bird C.G., (1976). On cost allocation for a spanning tree: a game theoretic approach, Networks 6, 335-350.

[4] Branzei R., Dimitrov D., and Tijs S., (2005). Models in Cooperative Game Theory: Crisp, Fuzzy and Multi-Choice Games, Lecture Notes in Economics and Mathematical Systems, Springer-Verlag Berlin, Vol. 556.

[5] Claus A., Kleitman D.J., (1973). Cost allocation for a spanning tree, Networks (3):289-304.

[6] Diestel R., (2000). Graph Theory, Springer-Verlag.

[7] Janiak A., Kasperski A., (2008). The minimum spanning tree problem with fuzzy costs. Fuzzy Optimization and Decision Making, 7:105-18.

[8] Montemanni R., (2006). A Benders decomposition approach for the robust spanning tree problem with interval data, European Journal of Operational Research ,174:1479-90.

[9] Moretti S., Alparslan Gok S.Z., Branzei R., Tijs S., (2011). Connection Situations under Uncertainty and Cost Monotonic Solutions, Computers and Operations Research, Vol.38, issue 11,1638-1645.

[10] Moretti S., Norde H., Do K.H.P., and Tijs S., (2002). Connection problems in mountains and monotonic allocation schemes, Sociedad de Es- tadistica e Investigation Operativa, TOP, Vol. 10, No. 1, pp. 83-99.

[11] Palanci O., Alparslan-Gok S.Z., Weber G.-W., (2013) Forest situations and cost monotonic solutions, Institute of Applied Mathematics, Preprint number 2013-05.

Note on Disagreement Point Axioms and the Status Quo-proportional Bargaining Solution

Sergei Pechersky

St.Petersburg Institute for Economics and Mathematics RAS
European University at St.Petersburg;
1 Tchaikowski str., St.Petersburg, Russia 191187
Tel/fax: +7-812-2737953;
specher@eu.spb.ru

Keywords: *Bargaining, Egalitarian Solution, Status Quo-Proportional Solution, Disagreement Point Monotonicity*

This paper considers the disagreement point axioms to characterize the status quo-proportional solution and is inspired by paper [2], where these axioms are used to provide new characterizations of the egalitarian bargaining solution. Though the egalitarian and status quo-proportional solutions [1] are closely related (under logarithmic transformation of utilities the proportional excess transforms into the egalitarian excess due to Kalai) the problems appear because of absence of convexity requirement for definition of the status quo-proportional solution.

Disagreement point monotonicity is an axiom which requires a player's payoff to strictly increase in his disagreement payoff (status quo payoff).

We transfer some results by Rachmilevitch to our case, and provide another characterizations of the status quo-proportional solution.

References

[1] S.Pechersky (2007) Proportionality in NTU games: excesses and solutions. Game Theory and Management. Collected abstracts of papers presented on the International Conference Game Theory and Management. SPb.: Graduate School of Management. 2007. p. 105-108.

[2] S.Rachmilevitch (2011) Disagreement point axioms and the egalitarian bargaining solution. International Journal of Game Theory, v. 40, 1, 63 – 86.

Optimal Entering of Newcomer in the Voting Game

Ovanes Petrosian

Saint-Petersburg University
198504 Saint-Pe tersburg, Universitetsky Prospekt, 35
petrosian.ovanes@yandex.ru

Keywords: *Voting Game, Shapley-Shubic Power Index, Investment, Perspective Coalitions, Veto-Player, Monte-Karlo Method*

The new class of voting games, in which the number of players and their power indexes are changing coherently, is considered. As a power index Shapley–Shubik value is taken.

The new entering player is interested in receiving a given value of his component of Shapley–Shubik power index by investing $\alpha = (\alpha_1, \ldots, \alpha_n)$ in the game. We consider the problem, how to find a minimal investment, which guaranties the given value of Shapley–Shubik power index for the newcomer. The mathematical statement of the problem is given, some properties of the minimal investment are considered and Monte–Karlo method for the calculation of minimal investment is proposed.

The results are illustrated by numerical example.

References

[1] Petrosian O.L. Elements of dynamics in voting games // to appear in Mathematical Game Theory and Applications (in Russian). 2013.
[2] Shapley L.S., Shubik M. A Method for Evaluating the Distribution of Power in a Committee System // American Political Science Review. 1954. V. 48. N 3. P. 787–792.
[3] Hu X. An asymmetric Shapley-Shubik power index // International Journal of Game Theory. 2006. V. 34. N 1. P. 229–240.

On Axiomatizations of the Shapley Value for Assignment Games

Rene van den Brink[1] and Miklos Pinter[2]

[1]*University, Department of Econometrics and Tinbergen Institute, De Boelelaan 1105, 1081 HV Amsterdam,The Netherlands*
jrbrink@feweb.vu.nl

[2]*Corvinus University of Budapest, Department of Mathematics and MTA-BCE "Lendiilet" Strategic Interactions Research Group, 1093 Hungary, Budapest, Fovam ter 13-15.,*
miklos.pinter@uni-corvinus.hu

Keywords: *Game Theory, Assignment Game, Shapley Value, Communication Graph Game, Submarket Efficiency, Valuation Fairness*

The history of assignment games goes back to the XIX. century to Bohm-Bawerk's [1] horse market model. Later Shapley and Shubik [20] introduced the formal, modern concept of assignment games.

One of the most popular solution concepts for TU-games is the Shapley value (Shapley [18]). Numerous axiomatizations of the Shapley value are known in the literature. In this paper we focus on the following ones: (1) Shapley's original axiomatization [18] by efficiency, the null player property (originally stated together as the carrier axiom), symmetry and additivity (also discussed by Dubey [5] and Peleg and Sudholter [14]), (2) Young's [21] axiomatization replacing additivity and the null player property by strong monotonicity (also discussed by Moulin [11] and Pinter [15]), (3) Chun's [4] replacing strong monotonicity by coalitional strategic equivalence, (4) van den Brink's [2] replacing (in Shapley's original axiomatization) additivity and symmetry by fairness, and (5) Hart and Mas-Colell's [9] approaches using the potential function and a related reduced game consistency.

First, we examine these characterizations of the Shapley value on the class of assignment games, and conclude that none of these characterizations is valid on this class in the sense that they do not characterize a unique solution.

After these negative results, we show that when considering an assignment game as a communication graph game where the game is simply the assignment game and the graph is a corresponding bipartite graph where buyers are connected with sellers only, Myerson [12]'s component efficiency and fairness axioms do characterize the Shapley value on the class of assignment games. Moreover, the axioms have a natural interpretation for these games.

An assignment game is fully described by the *assignment situation* being a set of buyers, a set of sellers, and for every buyer a valuation of the good offered by each seller. Instead of defining an assignment game as a communication graph game, we will directly work on the class of these assignment situations. For such assignment situations, component efficiency of a graph game solution boils down to *submarket efficiency* stating that the sum of the payoffs of all players in a submarket equals the worth of that submarket, where a *submarket* in an assignment situation is a set of buyers and sellers such that all buyers in this set have zero valuation for the goods offered by the sellers outside the set, and all buyers outside the set have zero valuations for the goods offered by sellers inside the set.

Fairness of the graph game solution boils down to valuation fairness stating that only changing the valuation of one particular buyer for the good offered by a particular seller changes the payoffs of this buyer and seller by the same amount. We show that these two axioms do characterize the Shap- ley solution for assignment situations being the solution that is obtained by applying the Shapley value to the corresponding assignment game. We will refer to the Shapley solution for assignment games simply as their Shapley value. So, we obtain a positive result by viewing an assignment game as a communication graph game.

Besides introducing and axiomatizing his solution, Myerson [12] also shows that it is *stable* for superadditive graph games in the sense that two players never get worse off when building a link between them. The Shapley value for assignment situations is *valuation monotonic* in the sense that the payoffs of a buyer i and a seller j do not decrease if only the valuation of buyer i for the good offered by seller j increases.

In this paper first we apply the axiomatizations of the Shapley value for TU-games mentioned above to the class of assignment games, and show that they do not give uniqueness on this class. Then we consider assignment games as communication graph games and characterize the Shapley value for assignment situations by submarket efficiency and valuation fairness.

References

[1] Bohm-Bawerk, E. von: Positive theory of capital (translated by W. Smart), G.E. Steckert, New York, (1923, original publication 1891)

[2] van den Brink, R.: An axiomatization of the Shapley value using a fairness property, International Journal of Game Theory 30, 309-319. (2001)

[3] Chun, Y.: A New Axiomatization of the Shapley Value, Games and Economic Behavior 1, 119-130. (1989)

[4] Chun, Y.: On the Symmetric and Weighted Shapley Values, International Journal of Game Theory 20, 183-190. (1991)

[5] Dubey, P.: On the uniqueness of the Shapley value, International Journal of Game Theory 4, 131-139. (1975)

[6] Einy, E.: The Shapley value on some lattices of monotonic games, Mathematical Social Sciences 15, 1-10. (1988)

[7] Fernandez, S.M.: Contributions to Operations Research Games Ph.D. Thesis University of Barcelona (2005)

[8] Hart, S., A. Mas-Colell: The Potential of the Shapley Value, The Shapley Value, Alvin E. Roth (editor), Cambridge University Press (1988), 127137.

[9] Hart, S., A. Mas-Colell: Potential, value, and consistency, Econometrica 57, 589—614. (1989)

[10] Monderer, D., D. Samet: Variations on the Shapley Value in R. Au- mann, S. Hart, ed., Handbook of Game Theory with Economic Applications Volume 3, North Holland, 2055-2076. (1988)

[11] Moulin, H.: Axioms of cooperative decision making, Cambridge University Press (1988)

[12] Myerson, R.B.: Graphs and cooperation in games, Mathematics of Operations Research 2, 225—229. (1977)

[13] Neyman, A.: Uniqueness of the Shapley value, Games and Economic Behavior 1, 116-118. (1989)

[14] Peleg, B., P. Sudholter: Introduction to the Theory of Cooperative Games, Kluwer Academic Publishers, Boston / Dordrecht / London (2003)

[15] Pinter, M.: Young's axiomatization of the Shapley value - a new proof, working paper (2012)

[16] Roth, A.E.: The Shapley Value as a von Neumann-Morgenstern Utility, Econometrica 45, 657-664. (1977)

[17] Roth, A.E.: The expected value of playing a game in A.E. Roth, ed., The Shapley Value, Cambridge University Press, 51-70. (1988)

[18] Shapley, L.S., A Value for n-Person Games, Contributions to the Theory of Games Volume II (Annals of Mathematical Studies 28, editors: Kuhn, H.W.-Tucker, A.W.) 307-317. (1953)

[19] Shapley, L.S.: A Comparison of Power Indices and a Nonsymmetric Generalization, P-5872, The Rand Corporation, Santa Monica, CA. (1977)

[20] Shapley, L.S., M. Shubik: The assignment game I: the core, International Journal of Game Theory 1, 111—130. (1972)

[21] Young, H. P.: Monotonic Solutions of Cooperative Games, International Journal of Game Theory 14, 65-72. (1985)

[22] Young, H. P.: Individual contribution and just compensation in A.E. Roth, ed., The Shapley Value, Cambridge University Press, 267-278. (1988)

[23]Weber, R.J.: Probabilistic values for games in A.E. Roth, ed., The Shapley Value, Cambridge University Press, 101-119. (1988)

Kalai-Smorodinsky-Nash Robustness

Shiran Rachmilevitch

Department of Economics, University of Haifa,
Mount Carmel, Haifa, 31905, Israel.
shiranrach@econ.haifa.ac.il

Keywords: *Bargaining, Kalai-Smorodinsky Solution, Nash Solution*

Nash's (1950) bargaining problem is a fundamental problem in economics. It consists of two components: a feasible set of utility allocations, each of which can be achieved via cooperation, and one special utility allocation – the disagreement point – that prevails if the players do not cooperate. A solution is a function that picks a feasible utility allocation for every problem. The axiomatic approach to bargaining narrows down the set of "acceptable" solutions by imposing meaningful and desirable restrictions (axioms), to which the solution is required to adhere.

Weak and common restrictions are the following: weak Pareto optimality – the selected agreement should not be strictly dominated by another feasible agreement; symmetry – if the problem is symmetric with respect to the $45°$-line then the players should enjoy identical payoffs; independence of equivalent utility representations – the selected agreement should be invariant under positive affine transformations of the problem. I will call a solution that satisfies these three restrictions standard.

Two other principles that take center stage in this paper are independence and monotonicity. Informally, the former says that if some options are deleted from a given problem but the chosen agreement of this problem remains feasible (it is not deleted) then this agreement should also be chosen in the problem which corresponds to the post-deletion situation; the latter says that if a problem "expands" in such a way that the set of feasible utilities for player i remains the same, but given every utility-payoff in that set the maximum that player $j \neq i$ can now achieve is greater, then player j should not get hurt from this expansion. Within the class of standard solutions, these principles are incompatible: Nash (1950) showed that there is a unique standard independent solution, while Kalai and Smorodinsky (1975) showed that there exists a different solution, which is the unique standard and monotonic one. In light of these facts, the question I address is how can we reconcile independence and monotonicity in bargaining?

One way is to give up the restriction to standard solutions. Indeed, doing so resolves the issue: there are solutions that are both independent and monotonic, but are not standard. For example, the egalitarian solution (Kalai (1977)) satisfies all the aforementioned requirements except independence of equivalent utility representations[*] and the dictatorial solution satisfies all of them except symmetry[†] Neither one of these solutions offers a satisfactory resolution to our problem, as independent utility scales are necessary if there is no way to compare utilities interpersonally, and violations of symmetry are obviously not appealing. The remaining alternative, therefore, is to compromise on efficiency. This, in principle, seems reasonable, as there are many situations in economics in which some desired goal has a "price," which is expressed in terms of efficiency loss; for example, maximizing revenue in an auction environment with independent and private values can only be achieved by an inefficient allocation rule. Unfortunately however, in the bargaining model the willingness to compromise on efficiency does not take us very far: in Rachmilevitch (2013) I showed that the only solution that satisfies all the aforementioned properties except for efficiency is the disagreement solution – the solution that picks the disagreement point in every problem.

Therefore, in this paper I take a different approach, keeping the restriction to standard solutions in place. First, I introduce a ranking over solutions – the "more independent than" relation – and demand that the solution be at least as independent as the Kalai-Smorodinsky solution. I call this property minimal independence. Next, I introduce a weak version of monotonicity, which I call minimal monotonicity. It turns out that any standard solution that satisfies minimal independence and minimal monotonicity guarantees that each player receives at least the minimum of the payoffs that he would have received under the Nash and Kalai-Smorodinsky solutions. I call this latter property Kalai-Smorodinsky-Nash robustness, or KSNR for short.

KSNR captures much of the essence of the Nash and Kalai-Smorodinsky solutions: the former is the only independent solution that satisfies KSNR and the latter is the only monotonic solution that satisfies KSNR (i.e, "KSNR" can replace "standard" in the theorems of Nash and Kalai-Smorodinsky). There are many well-behaved standard

---

[1]is the unique continuous such solution.

[†]It is the unique continuous such solution.

solutions that satisfy KSNR. This is the case, for example, with a certain family of solutions that is due to Sobel (2001). However, I am not aware of a well-behaved interesting solution, other than the Nash or Kalai-Smorodinsky solution, that satisfies minimal independence and minimal monotonicity (artificial such solutions do exist). The message of the paper, therefore, is that KSNR offers a compromise between independence and monotonicity. It is also, I believe, of interest in its own right.

Fish Wars and Nash Bargaining Solution

Anna Rettieva

Institute of Applied Mathematical Research
Karelian Research Center of RAS
annaret@krc.karelia.ru

Keywords: *Bioresource Management Problem, Fish Wars, Nash Bargaining Solution*

Model with two players

Let two players (countries or fishing firms) exploit the fish stock during finite time horizon. The dynamics of the fishery is described by the equation

$$x_{t+1} = (\varepsilon x_t - u_{1t} - u_{2t})^{\alpha}, \quad x_0 = x,$$

where $x_t \geq 0$ – the size of population at a time t, $\varepsilon \in (0,1)$ – natural death rate, $\alpha \in (0,1)$ -- natural birth rate, $u_{it} \geq 0$ – the catch of player i, $i = 1,2$.

We suppose that the utility function of country i is logarithmic and players differ in their discount factors. The players' net revenues are

$$J_i = \sum_{t=0}^{n} \delta_i^t \ln(\mathrm{u}_{it}),$$

where $0 < \delta_i < 1$ – the discount factor for country i, $i = 1,2$.

The main question arising here is how to construct the value function for cooperative solution in the case when players have different discount factors. In the previous work [5] the authors showed how to determine the joint discount factor for the case when cooperative gain is distributed proportionally or in a some portion using Nash bargaining scheme. Here we will obtain the cooperative strategies without determining the joint discount factor using recursive Nash bargaining procedure.

The cooperative behavior

We will present two different approaches of bargaining procedure.

In the first one the cooperative strategies are determined as the Nash bargaining solution for the hole planning horizon. So we need to solve the next problem:

$$(V_1^n(x,\delta_1) - V_1(x,\delta_1))(V_2^n(x,\delta_2) - V_2(x,\delta_2)) =$$

$$= (\sum_{t=0}^{n} \delta_1^t \ln(u_{1t}^c) - V_1(x,\delta_1))(\sum_{t=0}^{n} \delta_2^t \ln(u_{2t}^c) - V_2(x,\delta_2)) \to \max \tag{1}$$

where $V_i(x,\delta_i)$ are the non-cooperative gains.

In the second approach, we construct the cooperative solution as a recursive bargaining procedure. On each time moment the cooperative strategies are determined as the Nash bargaining solution taking the non-cooperative profits as a status-quo point.

The results of numerical modelling and comparison of the schems are presented.

Also we use the presented approaches for the game where the players have different planning time horizons.

Acknowledgement

The research was supported by the Russian Fund for Basic Research, project 13-01-91158-ГФЕН_a.

References

[1] Breton M., Keoula M.Y. A great fish war model with asymmetric players // Cahiers du GERAD G-2010-73, December 2010.
[2] Levhari D., Mirman L.J. The great fish war: an example using a dynamic Cournot-Nash solution // The Bell Journal of Economic. 1980. V. 11. N 1. P. 322--334.
[3] Mazalov V.V., Rettieva A.N. Fish wars and cooperation maintenance // Ecological Modelling. 2010. V. 221. P. 1545--1553.
[4] Mazalov V.V., Rettieva A.N. Incentive conditions for rational behavior in discrete-time bioresource management problem // Doklady Mathematics. 2010. V. 81. N 3. P. 399--402.
[5] Rettieva A.N. Stable coalition structure in bioresource management problem // Ecological Modelling V. 235-236. 2012. P. 102-118.
[6] Rettieva A.N. Bioresource management problem with asymmetric players // Matematicheskaya Teoriya Igr i Prilozheniya. 2012. V. 4. N. 4. P. 63--72 (in Russian).

Balancedness Condition for Semi-symmetric Cooperative TU Games

Alexandra Zinchenko[1] **and Vasiliy Rodochenko**[2]

Southern Federal University
Russia
[1]*zinch46@gmail.com*
[2]*Rodochenko@yandex.ru*

Keywords: *Cooperative TU Game, Core, Balancedness, Symmetric Core*

Semi-symmetric named TU game having at least two symmetric players. We say that a game is s-symmetric if it is symmetric w.r.t. coalition S with cardinality $s \in \{2,...,n-1\}$. A necessary and sufficient condition for nonemptiness of the core of $(n-1)$-symmetric game are presented. This condition is simple and provides a natural extension of one for symmetric games. Three classes of such $(n-1)$-symmetric games that the core is a singleton and corresponding core-allocations are described. Keywords: cooperative TU game, core, balancedness, symmetric core.

Some of participants in a social, political or economic situation may have the identical power (strength). In associated cooperative game they are symmetric. Moreover, different agents in the underlying problem may become symmetric players in corresponding game. We focus on the study of n-person TU games with $(n-1)$ symmetric players. A few of their applications: patent licensing game with the firms each producing an identical commodity and a licensor of a patented technology; landlord game; weighted majority game with one large party and $n-1$ equal sized smaller parties; market with one seller and symmetric buyers; subclass of games related information collecting situations under uncertainty where an action taker can obtain more information from other agents; big boss games with symmetric powerless players.

Known that the core of a game (N,v), where $N=\{1,...,n\}$ and $v:2^N \to \mathbf{R}$, $v(\varnothing)=0$, is nonempty iff a game is balanced. The balancedness condition for n-person game is determined by a system of linear inequalities which coefficients correspond to the extreme points of special polytope $M^n = \{\lambda \in R_+^{2^n-2} : \sum_{K\in\Omega,\, i\in K}\lambda_K = 1, i \in N\}$, where

$\Omega = 2^N \setminus \{N, \varnothing\}$. The number $m_n = |ext(M^n)|$ of extreme points and their explicit representation known only for small n (m_3 =5, m_4=41, m_5=1291, m_6=200213). The symmetry of all players makes a game especially easy to handle. The criterion for existence of its core contains $n-1$ inequalities only. But as far as we know there is no a similar result for a game symmetric w.r.t. coalition with $(n-1)$ players. Using the concept of symmetric core we provide the simple condition for nonemptiness of the core $C(v)$ of such game.The notion of symmetric core $C_{sym}(v)$ was introduced in ([1])

$$C_{sym}(v) = \{x \in C(v) : x_i = x_j \text{ for symmetric players } i, j \in N\}.$$

A game is s-symmetric if it is symmetric w.r.t. coalition S, $|S| = s \in \{2,...,n-1\}$.

Denote the class of s-symmetric n-person TU games by G_s^n and without loss of generalities assume that $S = \{h, h+1..., n\}$, $h = n - s + 1$. The s-core of $v \in G_s^n$ is $C_s(v) = \{x \in C(v) : x_i = x_j; h \le i, j \le n\}$. A game $v \in G_s^n$ is s-balanced if $C_s(v) \ne \varnothing$.

The following lemma states that, in fact, the classes of balanced s-symmetric games and s-balanced games coincide. Let $v \in G_s^n$. Then $C(v) \ne \varnothing$ iff $C_s(v) \ne \varnothing$.

Since the core is relatively invariant w.r.t. strategic equivalence it is sufficient to study a game $v \in G_s^n$ in 0-form. Moreover, as a result of zero-normalization, the number of symmetric player can increase. Denote the set of $(n-1)$-symmetric, zero-normalized, non-negative games by $(G_{n-1}^n)_+^0$. A game $v \in (G_{n-1}^n)_+^0$ is determined by $2(n-2)$ numbers $v(K)$, $K \in \Omega_1 \cup \Omega_2$, where $\Omega_1 = \{\{2,3\},\{2,3,4\},...,\{2,...,n\}\}$, $\Omega_2 = \{\{1,2\},\{1,2,3\},...,\{1,...,n-1\}\}$. The next theorem characterizes $(n-1)$-symmetric games with a nonempty core. Let $n \ge 3$, $H \in \Omega_1$, $T \in \Omega_2$, $|H| = h$, $|T| = t$. The core of a game $v \in (G_{n-1}^n)_+^0$ is nonempty iff the system

$$v(T) + \frac{n-t}{h} v(H) \le v(N), \quad \frac{n-1}{h} v(H) \le v(N) \tag{1}$$

is consistent

Notice that (1) consists of $(n-1)(n-2)$ inequalities.

The core existence not always solves the problem of selecting a game outcome, because the core may be a relatively large set and even coincides with imputation set. Ideal is considered the case when the core contains one imputation only. But several examples have been built for which the unique core element provides a counter-intuitive payoff ([2]). The last theorem gives the sufficient conditions under which the core of $v \in (G^n_{n-1})^0_+$ is a singleton. Let $n \geq 4$; $v^1, v^2 \in (G^n_{n-1})^0_+$; $C(v^1) \neq \varnothing$, $C(v^2) \neq \varnothing$. Let also v^1 satisfies the equality

$$\frac{n-1}{n-2} v(N \setminus \{1,n\}) = v(N)$$

and v^2 satisfies at least one of two equalities

$$\frac{n-2}{n-1} v(N \setminus 1) + v(1,2) = v(N), \quad \frac{v(N \setminus 1)}{n-1} + v(N \setminus n) = v(N).$$

Then $C(v^1) = \{x^1\}$, $C(v^2) = \{x^2\}$, where

$$x^1 = (0, \frac{v^1(N)}{n-1}), \ldots, \frac{v^1(N)}{n-1}),$$

$$x^2 = (v^2(N) - v^2(N \setminus 1), \frac{v^2(N \setminus 1))}{n-1}), \ldots, \frac{v^2(N \setminus 1)}{n-1}).$$

References

[1] Norde H., V. Fragnelli, I. Garcia-Jurado, F. Patrone and S. Tijs (2002). Balancedness of infrastructure cost games. European Journal of Operational Research, 136, 635--654.

[2] Zinchenko A.B. (2012). On polytope of (0-1)-normal big boss games: redundancy and extreme points. Contributions to game theory and management. Collected papers of the Fifth International Conference "Game Theory and Management". 5, 386--397.

Consistency of the Shapley NTU Value

Miguel Ángel Hinojosa[1], **Eulalia Romero-Palacios**[2] **and Jose Manuel Zarzuelo**[3]

[1,2]*Dpt. of Economics, Quantitative Methods and Economic History.*
Pablo de Olavide University;
Ctra. de Utrera Km. 1; 41013 Sevilla, Spain.
[3]*Dpt. of Applied Economics IV; Fac. of Economics and Business Administration;*
The University of the Basque Country;
Av. Lehendakari Aguirre, 83; 48015 Bilbao, Spain.

Keywords: *Consistency, NTU Games, Shapley NTU Value, Shapley Value*

The Shapley NTU solution is shown to be consistent according to a generalization of the reduced game proposed by Hart and Mas-Colell on the subclass of TU games, by considering payoff configurations as solution outcomes. Moreover, the Shapley NTU solution is characterized on a wide class of NTU games by means of this consistency property plus certain plausible axioms, namely: maximality, covariance, symmetry, a null-player axiom, and an additional axiom requiring certain coherence in the payoffs of the intermediate coalitions.

Completions for Space of Preferences

Victor Rozen

Saratov State University,
Russia
Rozenvv@info.sgu.ru

Keywords: A *Space of Preferences, Quality Criteria, Completion for Space of Preferences*

We study some kinds of spaces of preferences which can be used in models of decision making with quality criteria. Formally, a space of preferences in the form of triple objects $< A, \alpha, \beta >$ is given, where A is an arbitrary set (a set of alternatives), α and β are binary relations on A satisfying the conditions: $\alpha \cap \alpha^{-1} = \varnothing$ (asymmetry), $\beta^{-1} = \beta$ (symmetry) and $\alpha \cap \beta = \varnothing$ (disjointness). The relation $\rho = \alpha \cup \beta$ is called a preference relation. Most important classes of spaces of preferences are the following:

1. The class of linear preferences;
2. The class of transitive preferences;
3. The class of acyclic preferences;
4. The class of order preferences.

Let $P =< A, \alpha, \beta >$ and $P_1 =< A, \alpha_1, \beta_1 >$ be two spaces of preferences. A space P_1 is said to be a completions for space P, if inclusions $\alpha \subseteq \alpha_1, \beta \subseteq \beta_1$ hold; a completion is a proper if at least one of these inclusions is strict.

We find conditions under which a given space of preferences has a completion belonging to indicated above classes of preferences and its combinations also. An algorithm for constructions of all completions of order preferences up to linear order preferences is given.

Unravelling Conditions for Successful Change Management

Michel Rudnianski[1] **and Cerasela Tanacescu**[2]

[1]*CNAM*
France
michel.rudnianski@cnam.fr

[2]*ESSEC*
France
b00207018@essec.edu

Keywords: *Change, Deterrence, Evolution, Incentives, Playability, Replicator Dynamics, Stability*

The ever increasing pace of ICT development and globalization generates a dramatic shortening of the products' life cycle, which in turn decreases the possibility of sustainable competitive advantages for the firms. Richard D'Aveni [1] has analyzed this phenomenon distinguishing different arenas of hyper-competition. One of the major elements highlighted is the core capacity of managing breakthroughs, in particular through the mastering of timing and knowhow associated with products and services. In this respect, there is no doubt that change management is a core competency for a firm which aim is a sustainable development. Now implementation of change management may present a variety of difficulties, among which the following ones, on which the present paper will focus: reluctance to share information and to change [4,6} . The works developed in Experimental Psychology, and especially Kahneman and Tversky's Prospect Theory [5} have highlighted decision biases like anchoring, procrastination, sensitivity to loss, stubbornness, mirroring, or status quo which all lead the individual under consideration to take inappropriate decisions, which most of the time have as hidden objective to comfort his/ her position and hence not accept to change. Whence the necessity for the management of the firm to proceed to an accurate cost-benefit analysis of change versus status quo for each decision maker. As a result of this analysis the management of then firm may decide to allocate incentives to the personnel concerned.

Considering the firm as structured in departments which have a relative autonomy in terms of information sharing, and change adoption, a previous paper has built a game theoretic model of the issues at stake [7}. This model has considered several hypotheses about the consequences for a department to receive information which is relevant from the global perspective of the company. The starting point was to consider that a department i can decide to send or not to send to a neighboring department j an information relevant to a possible change in the conduct of affairs, while department j if receiving such information from department i, may decide to act accordingly and especially to implement change in consistence with the information received. At the most elementary level, the problematic can be analyzed through a series of standard 2x2 games in which the players strategic sets could refer either to information sending or to change adoption. Various cases have been considered depending on the respective values for each player of costs, benefits, and incentives received from the company's general management. At a second level, the model has used the Replicator's Dynamics to define conditions under which cooperation between connected departments can prevail. At a third level, the initial model was extended to matrix games in which each party should simultaneously consider whether to send information or not and whether to adopt changes possibly stemming from the information received or not. An evolutionary analysis of these 2x4 games was not performed due to the possible difficulties to find an analytical solution of the dynamic system.

Now this obstacle can be removed, thanks to the results recently found by Ellison & Rudnianski about the existence of equivalence relations between standard quantitative games and a particular type of qualitative games called games of deterrence [2,3]. More precisely these equivalence relations enable to translate standard evolutionary games into evolutionary games of deterrence which display identical asymptotic properties. In turn, it has been shown [3] that the asymptotic properties of these evolutionary games of deterrence can be derived from the playability properties of the players strategies in the static game of deterrence. There is then no need to solve the corresponding dynamic system.

On these bases, the present paper will in a first part recall the results available in the analysis of conditions required for successful change management through the standard game theoretic approach. In a second part after having recalled the core properties of matrix games of deterrence, the paper will develop the equivalences between evolutionary standard matrix games and evolutionary matrix games of deterrence. A third part will then use these equivalences to analyze the conditions of success in non-elementary issues of change pervasion. In particular success of change pervasion will be associated with the playability properties of the games of deterrence under consideration.

References

[1] D'Aveni R. "Hypercompetition", Simon & Schuster, New York, 1994

[2] Ellison D., Rudnianski M. : « Is Deterrence Evolutionarily Stable », Annals of the International Society of Dynamic games, vol 10, Bernhard P., Gaitsgory V., Pourtaillier O. Eds, pp 357 – 375, Birkhauser, Berlin 2009

[3] Ellison D., Rudnianski M. : « Playability Properties in Games of Deterrence and Evolution in the Replicator's Dynamics", paper presented at GTM 2012, St Pestersburg, under current publication

[4] Fichman RG., Kemener CF., "The Assimilation of Software Process Innovations: An Organizational Learning Perspective", Management Science, 1345 – 1363, 1997

[5] Kahneman D. , Tversky A.;, « Prospect Theory: An Analysis of Decision under Risk », Econometrica vol. 47, no 2, March 1979, p. 263-291

[6] Rogers E. "Diffusion of Innovations". Free Press, New York, 1983

[7] Rudnianski M., Tanacescu C. : «Knowledge Management and Change Pervasion : a Game Theoretic Approach », paper presented at Cross-Border Innovation and Entrepreneurship Symposium, 3-5 December 2012 Singapore, under current publication

Pursuit-evasion Differential Games with Many Players and Integral Constraints

Mehdi Salimi[1] **and Gafurjan Ibragimov**[2]

[1]*Department of Mathematics, Toyserkan Branch,*
Islamic Azad University, Toyserkan, Iran
m.salimi@tuyiau.ac.ir

[2]*Department of Mathematics and Institute for Mathematical Research,*
Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, Malaysia
gafur@science.upm.edu.my

Keywords: *Differential Games, Pursuit-evasion Problems, Integral Constraints*

Introduction

What is a game? Mathematically speaking, a game has players, strategies and scores to explain why the players win or lose. Games which are modelled with differential equations are called differential games. Common examples are pursuit-evasion games which are abstract models for pursuers who try to catch evaders who are running away. E.g.,

Lady in the lake. A man is moving around a circle, and a woman is moving from the center of the circle heading to its boundary. The man is trying to have minimum distance to the woman, while the woman is maximizing the distance by moving accordingly.

Lion and man game. The goal is to find a strategy for the pursuer (lion) to capture the evader (man) in a given environment. Capturing means that the man and the lion are at the same position after a finite time. Besicovitch was the first who showed that with a certain strategy evasion from the lion is possible. The man wins the game if he can avoid capture indefinitely. Both the lion and the man have identical motion capabilities [2].

The pursuit-evasion differential games including several players with geometric or integral constraints were extensively studied. The development of the field could be traced through its history by consulting the classical researches on differential games, such as works by R. Isaacs and L.A. Petrosyan [3,4].

Main Result

Suppose a pursuit-evasion differential game of countably many players in Hilbert space with integral constraints. Motions of the players are described by the ordinary differential equations of second order. Indeed, in the space L_2 consisting of elements $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_k, \ldots)$, with $\sum_{k=1}^{\infty} \alpha_k^2 < \infty$, and inner product $(\alpha, \beta) = \sum_{k=1}^{\infty} \alpha_k \beta_k$, the motions of the countably many pursuers P_i and the evader E are defined by the equations

$$P_i : \ddot{x}_i(t) = u_i(t), \quad x_i(0) = x_i^0, \quad \dot{x}_i(0) = x_i^1,$$

$$E : \ddot{y}(t) = v(t), \quad y(0) = y^0, \quad \dot{y}(0) = y^1,$$

where $x_i, x_i^0, x_i^1, u_i, y, y^0, y^1, v \in L_2$, u_i is the control parameter of the pursuer P_i, and v is that of the evader E. The control functions of players are subject to integral constraint. The duration of the game is fixed. The payoff functional is the greatest lower bound of the distances between the pursuers and the evader when the game is terminated. The pursuers try to minimize the payoff functional, and the evader tries to maximize it. We fix the index i and study an auxiliary differential game of two players. We construct a suitable strategy for the pursuer and then we show that if the energy of the pursuer is more than that of the evader, then the pursuer catches the evader by using constructed strategies. Finally we define an amount as the value of the game and prove the accuracy of our guess.

In addition, we consider a non-inertial evasion differential game of several pursuers and one evader with simple motions and integral constraints on the control functions of the players. We find an adequate condition for the evader to escape from all pursuers by presenting explicit strategy for the evader. We show that the proposed escape is possible, no matter what control is adapted by the pursuers. The problem studied in this research is new. In the case of geometric constraints, a similar problem was solved by F.L. Chernous'ko [1], and later it was extended by V.L. Zak to many different differential games with geometric constraints [5]. However, in the case of integral constraints the problem was open.

References

[1] Chernous'ko, F.L. (1976). A problem of evasion from many pursuers. Prikladnaya Matematika I Mekhanika 40 (1): 14-24.

[2] Croft, H.T. (1964). Lion and Man: A Postscript. Journal London Mathematical Society 39 (1): 385-390.

[3] Isaacs, R. (1965). Differential Games. New York: John Wiley & Sons.

[4] Petrosyan, L.A. (1977). Differentsial'nye igry presledovaniya (Differential Pursuit Games). Leningrad: Leningr. Gos. Univ.

[5] Zak, V.L. (1978). On a problem of evading many pursuers, PMM 43 (3): 456-465.

On the Bidding with Asymmetric Information and the Finite Number of Repetition

Marina Sandomirskaia

St.Petersburg Institute for Economics and Mathematics, RAS
Russia
sandomirskaya_ms@mail.ru

Keywords: *Insider Trading, Incomplete Information, Repeated Games, the Simple Random Walk*

The first game-theoretical stock trading model with asymmetric information has been introduced in the paper of B. De Meyer and H. Moussa Saley [1]. The discrete modification of the repeated insider trading model with beforehand unlimited number of steps has been investigated in the paper of V. Domansky [2]. The choice of the infinite number of steps is caused by enormous combinatorial difficulties appropriated to the analysis of bidding with finite number of repetitions. Thus evaluation of the optimal parameters, such as the game value, the optimal strategies, for the games with finite number of steps seems to be impossible. However, in this paper we obtain the asymptotic formula for the error term of the insider's profit arising if the private information unexpectedly becomes common knowledge at step N (this automatically leads to the stopping of bidding).

The paper considers the n-stage bidding model with two risk neutral agents for the risky asset (the one type shares), solved in [2] for the case of $n = \infty$. The liquidation share price is determined by a chance move at the zero step for the whole bidding period and takes on two different integer values: either 0 with probability $1-p$ or m with probability p. Player 1 is informed about the chance choice, while Player 2 is not. Both players know p. Player 2 knows that Player 1 is the insider. Before bidding begins each player has sufficient amount of money and shares.

At each step of bidding both players simultaneously propose integer prices for one share. The player proposing the higher price buys one share from his opponent at this price. If proposed prices are equal then no transaction occurs. After each step prices proposed at this step are publicly announced. Each player aims to maximize his final portfolio (money plus money equivalent of acquired shares). The number of bidding stages is unlimited ($n = \infty$).

The model is reduced to the repeated zero-sum game with incomplete information on the side of Player 2. The game with infinite number of steps has been solved, namely the game value and the optimal strategies of both players have been found for any prior probability p. The insider's optimal strategy generates the simple random walk on lattice of posterior probabilities of high share price with absorption at extreme points. At each step until the absorption the insider's strategy guaranties him the benefit equal to 1/2. For the limited time the revealing of the insider's information occurs with probability 1.

This paper considers the situation of unexpected stopping of bidding at stage N. We obtain the formula for the profit of Player 1 in the N-stage game if he applies the strategy which is the optimal one for the game of unlimited duration.

Let the initial probability of the high share price be $p = k/m$. Let $\beta_N^m(k)$ denote the average number of steps of the simple random walk with absorption at 0 or 1 starting at the point k/m till a future moment N. Then the expected insider's profit $L_N^m(k/m) = \inf_\tau W_N^m(\sigma^m, \tau \mid k/m)$ is given by

$$L_N^m(k) = \frac{1}{2}\beta_N(k).$$

Here

$$\beta_N(k) = \mathbf{E}_k \Theta_k 1 1_{\{\Theta_k \le N\}} + N\mathbf{P}_k(\{\Theta_k \ge N\}),$$

where $\mathbf{E}_k, \mathbf{P}_k$ are the expectation and the probability, respectively, for the random walk starting at the point k/m, Θ_k is the random time of absorption.

The recursive equation on $\beta_N^m(k)$ holds

$$\beta_{N+1}^m(k) = \frac{1}{2}\beta_N^m(k+1) + \frac{1}{2}\beta_N^m(k-1) + 1$$

with the boundary conditions $\beta_N^m(0) \equiv 0, \quad \beta_N^m(m) \equiv 0.$

It is well known that

$$\beta_\infty(k) = (m-k)k.$$

The error term of the insider's profit arising in the case of revealing the private information at stage N is given by

$$\varepsilon_N^m(k) = \frac{1}{2}\left(\beta_\infty^m(k) - \beta_N^m(k)\right).$$

Applying the methods of solving partial differential equations we obtain the complete asymptotic expansion of $\varepsilon_N^m(k)$.

Thus we obtain the following theorem.

Theorem. If Player 1 exploits the strategy σ^m in the game $G_N^m(k/m)$, then his guaranteed gain $L_N^m(k/m)$ is given with the formula

$$L_N^m(k/m) = \frac{(m-k)k}{2} - \varepsilon_N^m(k),$$

where

$$\varepsilon_N^m(k) = \frac{1}{2m}\sum_{l=1}^{[m/2]} \cos^N \frac{\pi(2l-1)}{m} \sin\frac{\pi k(2l-1)}{m} \operatorname{ctg}\frac{\pi(2l-1)}{2m}\left(1+\operatorname{ctg}^2\frac{\pi(2l-1)}{2m}\right),$$

with $[\alpha]$ being the integer part of α.

Corollary. The strategy σ^m is a ε_N^m-optimal strategy of Player 1 for the finitely repeated game $G_N^m(p)$ of length N, where $\varepsilon_N^m = O(\cos^N \pi/m)$, i.e. the “error term” $\varepsilon_N^m(k)$ decreases exponentially.

It is to be noted that the value ε_N^m is to be regarded as the price of sudden revealing of insider information on stock market. If it is the case then ε_N^m means the insider's loss comparatively his expected profit in bidding of unlimited duration.

For $m=3$ the solution of the game $G_N^3(p)$ is found in terms of recursive sequences (see paper of V. Kreps [3]). We make a comparison of solution of this game (insider's profit in the case of his optimal behaviour) with his profit $L_N^3(k/3)$, $k=1,2$, obtained above. It is shown that for sufficiently large n the insider's optimal strategy in the bidding game of infinite duration proves to be a rather good approximation of his optimal strategy for the n-stage game.

The study was partially supported by the grant 13-01-00462-a of Russian Foundation for Basic Research.

References

[1] De Meyer B., Moussa Saley H. (2002) On the Strategic Origin of Brownian Motion in Finance. Int. J. Game Theory. Vol. 31, 285-319.

[2] Domansky V. (2007) Repeated games with asymmetric information and random price fluctuations at finance markets. Int. J. Game Theory, Vol. 36(2), 241-257.

[3] Kreps V. (2009) Repeated Games Simulating Exchange Auctions and Return Sequences. Journal of Computer and Systems Sciences International, Vol.48(4), 604-615.

Repeated Games with Incomplete Information and Slowly Growing Value

Fedor Sandomirskii

Chebyshev Laboratory SPbSU,
CEMI RAS
Russia
sandomirski@yandex.ru

Keywords: *Repeated Games, Incomplete Information, Error Term Behavior, Asymptotics of the Value, Maximal Variation, Measurevalued Martingales, Bayesian Learning*

Repeated zero-sum games with incomplete information (RGII) were introduced by R. Aumann and M. Maschler in the sixties (see [1]). Their aim was to develop a theory of multi-stage two-player interactions with participants having different amount of information about the interaction. Multi-stage structure allows players to obtain additional information about the interaction observing their opponents actions at previous stages.

Let us briefly describe such a game $G_N(p)$. Before the game starts a random state k is chosen from the set of states K according to a prior distribution p. Player 1 is informed of k and Player 2 is not. Then at each stage $n = 1,..N$ Player 1 and Player 2 simultaneously select their actions (i_n, j_n) from the sets of actions I and J, respectively, using the information they have at this stage. Before the next stage the selected actions are publicly announced. One-stage payoff (i.e., the contribution of this stage to the total gain of Player 1) is given by $A^k_{i_n, j_n}$ where A^k is one-stage payoff matrix at a state k. After the last stage Player 2 pays $\sum_{n=1}^{N} A^k_{i_n, j_n}$ to Player 1.

Of course, players are allowed to randomize their actions using behavioral strategies. Hence the value is defined as usual, i.e., $V[G_N(p)]$ is the expected total gain of Player 1 when both players play optimally.

The main feature of RGII is that the impact of information asymmetry on the game value represents the price of information in long interactions. So it is natural to consider only the games where the private information is the only strategic advantage of Player 1. In other words, we will assume that the non-revealing game, i.e., matrix game with the matrix $A(p) = \int_K dp(k) A^k$ (this game arise if Player 1 forgets his private information) has zero value for any $p \in \Delta(K)$. For such games $V[G_N(p)]$ is the price of information by itself. One of the main questions of the theory is about asymptotic behavior of $V[G_N(p)]$ as $N \to \infty$.

It is well-known (see [1] and [5]) that for some games with finite K, I and J the value can grow like $C\sqrt{N}$ and can not grow faster (this property is usually called ``$\sqrt{N}$-law"). It was recently shown in [6, 7] that for infinite $K$ ``$\sqrt{N}$-law" can be violated and examples of games with the value behaving like $CN^{\frac{1}{2}+\alpha}$ for any $\alpha \in (0,1)$ were constructed. On the other hand, it is interesting to describe classes of RGII with slower value growth. For the discrete models of finance market with asymmetric information it was recently observed (see [2] and [4]) that the value remains bounded as $N \to \infty$. Games considered in these papers have the following property: the optimal strategy of Player 2 in the non-revealing game is piecewise constant as a function of p. Our aim is to describe how this property affects the asymptotic behavior in general case. The main result is the following theorem.

Theorem: Assume that a RGII $G_N(p)$ with finite K, I and J has the following properties:

• the value of the non-revealing game $A(p) = \sum_{k \in K} p_k A^k$ is zero for any $p \in \Delta(K)$;

• there exists a piecewise constant function $y : \Delta(K) \to \Delta(J)$ with finite number of different values and such that the vector $y(p)$ is an optimal strategy of Player 2 in $A(p)$ for any p.

Then the value $V[G_N(p)]$ is uniformly bounded as a function of N and p.

It is important to note that V. Domansky and V. Kreps [3] showed that the conditions of the theorem are necessary and sufficient in the case of two-element I, J and K.

The study is supported by the grants 13-01-00462 and and 13-01-00784 of Russian Foundation for Basic Research and by the Chebyshev Laboratory (Faculty of

Mathematics and Mechanics, St.-Petersburg State University) under the grant 11.G34.31.0026 of the Government of the Russian Federation.

References:

[1] R. Aumann, M. Maschler (1995) Repeated Games with incomplete information. MIT, Cambridge, London.

[2] B. De Meyer, A. Marino (2005) Continuous versus discrete market game. Cowles Foundation discussion paper 1535.

[3] V. Domansky, V. Kreps (1994) ``Eventually revealing'' repeated games with incomplete information. Int. Journal of Game Theory 23:89-99.

[4] V. Domansky (2007) Repeated games with asymmetric information and random price fluctuations at finance markets. Int. Journal of Game Theory 36(i.2):241-257.

[5] J.-F. Mertens, S. Zamir (1976) The Normal Distribution and Repeated Games Int. Journal of Game Theory 4(i.4):187-197.

[6] F. Sandomirskii (2012) Variation of martingales taking their values in probability measures and repeated games with incomplete information. Doklady of Russian Ac. of Sci. V. 447. N3. (in Russian).

[7] F. Sandomirskii (2012) Variation of measure-valued martingales and repeated games with incomplete information. arXiv preprint: 1205.6791.

The Strategies that Dominate any Evolutionary Opponent in Infinitely Repeated Games

Tatiana Savina

National research university Higher School of Economics
Russia
suri-cat@yandex.ru

Keywords: *Repeated Game, Evolutionary Opponent, Dominating Strategy*

We consider infinitely repeated game of special type, that is 2 person game. The players can have short or long memory. In paper "Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent" by William H. Press and Freeman J. Dyson it has been shown that the player with long memory does not have any advantages and there exists a strategy that dominate any evolutionary opponent. The paper is dedicated to the expanding approach to other classes of infinitely repeated games "2 by 2".

Applying Game Theory in Procurement. An Approach for Coping with Dynamic Conditions in Supply Chains

Günther Schuh[1] **and Simone Runge**[2]

[1,2]*Institute for Industrial Management at RWTH Aachen University*
Aachen, Germany

Keywords: *Apply Game Theory in Procurement, Supply Chain Management, Adaptive Systems*

Today, companies face increasing dynamic conditions and volatile markets. Varying demands, shorter product lifecycle times and increasing diversity of products as well as unsteady economic situations force enterprises to react more flexible compared to some years ago (cf. Daxböck et al. 2011, p. 7; Schuh et al. 2012, p. 3 ; Stich et al. 2012, p. 123).

Central planning approaches regarding production- and logistics processes are matched to a particular moment of the enterprises conditions. These planning-oriented systems are not able to react spontaneously on changing boundary conditions (cf. Schmitt et al. 2011, p. 748). But exactly this ability becomes more and more a key factor of success for enterprises. Other approaches reduce central planning in favor of decentralized activities, which provide better possibilities to flexibly react according to the current situation (cf. Schmitt et al. 2011, p. 749). Thus integration into the value-added process is given in decentralized approaches. Nevertheless decentralized approaches have a big disadvantage as well. Central planning approaches often result in best solutions for the considered scope, a so called global optimum. Decentralized activities are performed with a smaller perspective as the decision-maker takes into consideration for example only the processes he is responsible for. Hence in decentral approaches a global optimum will often not be achieved.

Therefore for successful management of production- and logistics processes it is necessary to find an optimal way between detailed central planning and spontaneous decentral reactions to occurred changes. As a suitable approach for solving this problem, self-optimizing mechanisms could be integrated into the production planning and control as well as in supply chain processes (cf. Behnen et al. 2011, p. 103). Self-optimizing systems are “[…] systems that are able to effect independent (“endogenous”) changes of their inner states or structure based on varying input conditions or interferences. In production processes, according target values can be e.g. capacities, number of pieces, quality, costs or processing times” (Wagels; Schmitt 2012, p. 162). Self-optimizing systems have the capability to react autonomous and flexible to changing boundary conditions. They continuously carry out three activities (cf. Brecher et al. 2011, p. 13):

1. Analysis of the current situation,
2. Determination of objectives and
3. Adjustment of the behavior of the system to reach the defined objectives.

The project “Cognition-enhanced, Self-Optimizing Production Networks” which is part of the Aachen Cluster of Excellence (CoE) founded by the German Research Foundation (DFG, Deutsche Forschungsgemeinschaft) focusses on self-optimizing production planning and control from the level of machine control up to the level of supply chain management. In this regard both, human decision making as well as integrating the perspective of production and quality management, will be considered. Test-beds for experimental research in a real production environment to validate and enhance the outcome will be build. The objective is to develop prototypes of cybernetic solution components based on self-optimizing feedback loops.

When performing the research activities, in particular focusing on the supply chain level, parallels to **applying game theoretic models** became obvious. The parallels regarding the described need for a decentral approach and adequate consideration of dynamic boundary conditions will be described in more detail in the following:

- First of all supply chains which are composed of legally independent enterprises are decentral “by nature” as each partner of the supply chain pursues the goal of continuously optimizing with regard to his own objectives (cf. Hennet; Arda 2008, p. 399). Nevertheless everyone has to take into consideration decisions made by decision-makers of the other enterprises in the supply chain (cf. Reyes 2005, p. 1421). So each partner of the supply chain has to make the best decision for himself taking into account the other participants, as this is the case in game theoretic problems.
- In addition when taking into consideration dynamic boundary conditions as described before decisions should be made with regard to observed events beforehand (cf. Reyes 2005, p. 1430). As in game theoretic decision models

decisions often are predicated on an analysis of previous situations (cf. Hennig 2001, p. 6), another analogy to game theory is obvious.

Furthermore there are parallels between the above described activities of self-optimizing systems and dynamic games. In dynamic games

- an analysis of the current situation could be carried out (first attribute of self-optimizing Systems) and
- the decision-making could be adjusted to reach predefined objectives (cf. Holler; Illing 2006, p. 13) (third attribute of self-optimizing Systems).

When only these two aspects of self-optimization are covered a system is called an **adaptive system** (cf. Schmitt et al. 2011, p. 750). For a self-optimizing system only the determination of objectives is missing (second activity of self-optimizing systems). That means, the objectives are given in advance.

Hence inspired by the research activities in the project "Cognition-enhanced, Self-Optimizing Production Networks" and focusing on the application area of procurement it will now be examined how solution concepts of game theory can help to develop an adaptive method that can support planners in their decision-making better than conventional methods. With the described increasing dynamic conditions and volatile markets in mind, applying game theory in this application area seems to be a promising approach to reach better results as this is the case with conventional methods which are harmonized to a particular moment of the enterprise.

As a first step, the state of the art in this field was analyzed. In the context of these research activities, the focus will be on the process-related aspects of game theory and players, who are in successive value-added steps of a supply chain. First results of the survey of the state of the art in comparison to the planned research will be shown in the conference and the conference paper respectively. The results point out a deficit in the planned research area.

Further research activities will be carried out in three steps which will be explained very briefly in the following:

- In the first step the question of which game-theoretic solution concepts can be applied in the application area of procurement will be answered. For this purpose a list of existing generic game-theoretic concepts will be provided. Subsequently the solution concepts will be opposed to the application area by the use of a morphology. In this way it is ensured that exactly the solution concepts which are relevant in the application area of procurement are considered in the following research steps.

- Based on this analysis the aim of the second step will be to determine relevant logistics parameters and to analyze the correlation between them. This step gives a well-grounded basis for the derivation of instructions for setting up the decision model in the third step.
- The third and final step includes the development of a decision model based on the prior findings and therefore entails the setup of the adaptive method. In this regard it is planned to implement the method algorithmically and to investigate the advantage of the developed method based on game theoretic concepts by using data from a real production environment.

A more detailed description of the research activities as well as initial results will be presented in the conference and in the conference paper respectively.

References

[1] Behnen et al. 2011 Behnen, D.; Brecher, C.; Hammann, G.; Herfs, W.; Koeppe, R.; Pauly, D.; Possel-Dölken, F.; Verl, A.: Potenziale kognitiver Systeme in der Produktionstechnik. In: Wettbewerbsfaktor Produktionstechnik: Aachener Perspektiven. Aachener Werkzeugmaschinenkolloquium 2011. Edit.: C. Brecher; F. Klocke; R. Schmitt.; G. Schuh, G. Shaker, Aachen 2011, p. 101–123.

[2] Brecher et al. 2011 Brecher, C.; Kozielski, S.; Karmann, O.: Integrative Produktionstechnik für Hochlohnländer. In: Wettbewerbsfaktor Produktionstechnik: Aachener Perspektiven. Aachener Werkzeugmaschinenkolloquium 2011. Edit.: C. Brecher; F. Klocke; R. Schmitt, G. Schuh. Shaker, Aachen, p. 1–16.

[3] Daxböck et al. 2011 Daxböck, C.; Zinn, U.; Hunstock, T.: Supply Chain Performance Management. Über die intelligente Verknüpfung von Strategien, Prozessen und Controlling zur Steuerung der Wertschöpfungskette. Edit.: Horváth & Partner, Stuttgart 2011.

[4] Hennet; Arda 2008 Hennet, J.-C.; Arda, Y.: Supply chain coordination: A game-theory approach. In: Engineering Applications of Artificial Intelligence 21 (2008) 3, p. 399–405.

[5] Hennig 2001 Hennig, M.: Spieltheorie. http://www.marinahennig.de/PDF-Dateien/Spieltheorie.pdf. Stand: 28.06.2012.

[6] Holler, Illing 2006 Holler, M. J.; Illing, G.: Einführung in die Spieltheorie. Springer, Berlin [et al.] 2006.

[7] Reyes 2005 Reyes, P. M.: Logistics networks: A game theory application for solving the transshipment problem. In: Applied Mathematics and Computation (2005) 168, p. 1419–1431.

[8] Schmitt et al. 2011 Schmitt, R.; Brecher, C.; Corves, B.; Gries, T.; Jeschke, S.; Klocke, F.; Loosen, P.; Michaeli, W.; Müller, R.; Poprawe, R.; Reisgen, U.; Schlick, C., M; Schuh, G.; Auerbach, T.; Bauhoff, F.; Beckers, M.; Behnen, D.; Brosze, T.; Buchholz, G.; Büscher, C.; Eppelt, U.; Esser, M.; Ewert, D.; Fayzullin, K.; Freudenberg, R.; Fritz, P.; Fuchs, S.; Gloy, Y.-S.; Haag, S.; Hauck, E.; Herfs, W.; Hering, N.; Hüsing, M.; Isermann, M.; Janssen, M.; Kausch, B.; Kämpf, T.; Kratz, S.; Kuz, S.; Laass, M.; Lose, J.; Malik, A.; Mayer, M.; Molitor, T.; Müller, S.; Odenthal, B.; Pavim, A.; Petring, D.;

Potente, T,; Pyschny, N.; Reßmann, A.; Riedel, M.; Runge, S.; Schenuit, H.; Schilberg, D.; Schulz, W.; Schürmeyer, M.; Schüttler, J.; Thombansen, U.; Veselovac, D.; Vette, M.; Wagels, C.; Willms, K.: Selbstoptimierende Produktionssysteme. In: Integrative Produktionstechnik für Hochlohnländer. Edit.: C. Brecher. Springer, Heidelberg [et al.] 2011, p. 747–1057.

[9] Schuh et al. 2012 Schuh, G.; Stich, V.; Runge, S.: Einführung. In: Produktionsplanung und -steuerung 1. Grundlagen der PPS. Edit.: G. Schuh; V. Stich. Springer, Berlin [et al.], p. 3–7.

[10] Stich et al. 2011: Stich, V.; Brosze, T.; Bauhoff, F.; Gläsner, F.; Runge, S.; Groten, M.: High Resolution Supply Chain Management – A Structural Model for Optimized Planning Processes Based on Real-Time Data. In: Advances in Production Management Systems. Value Networks: Innovation, Technologies, and Management, IFIP WG 5.7 International Conference, APMS 2011, Stavanger, Norway, September 26-28, 2011, Revised selected papers. Edit.: J. Frick; B. T. Laugen. Springer, Berlin [et al.], p. 123–131.

[11] Wagels; Schmitt 2012 Wagels, C.; Schmitt, R.: Benchmarking of Methods and Instruments for Self-Optimization in Future Production Systems. In: Procedia CIRP (2012) 3, p. 161–166.

Compensation Plans, Management Turnover and Efficiency

Lubov Schukina

EDC
Russia
SchukinaLG@bke.ru

Keywords: *Agency Problem, Remuneration Plans, Management Incentives, Crisis Period, Compensation Strategy*

Double agency problem is not very much studied by economists. Just some it's aspects are reflected in literature, f.e.: results of incentive contracts usage, middle management remuneration problems, corporate conflicts influence on the management incentives.

Suggested game-theoretical model gives an opportunity to analyze the choice of efforts' level by employees and management turnover in case of business reorganization.

Shareholder is indifferent choosing the compensation scheme for his employees, but as for top-management, this choice is significant. If the company's profit during the game is constant, shareholder would be interested to be dishonest and not to pay bonuses to his employees and managers. But if the company's profit increases shareholder has no more incentives to be dishonest. Management change threat may be the reason for the worse compensation plan choice. This becomes a reason for the decrease of the employees efforts level.

The main tendencies of Russian and foreign oil companies remuneration during the crisis period of 2008-2010 were analyzed to illustrate some results of the model.

Trying to decrease their own risks top managers of the companies reduce the ratio of variable parts of their remuneration, which depends on their performance. Annual bonuses are paid for production, ecology and safety goals. LR plans are oriented mostly for financial parameters. That's why the shareholders' requirement to limit the option payments or to reinvest this remuneration part in company's shares seems to be significant. As for the model demonstrated it is important to note that all the players are oriented for the financial results of their work. It is significant for both: LR and SR periods.

Crisis period became the start point for the remuneration schemes change in a whole world . And this change gave different results for different groups of employees and managers (depended on the level of their motivation). Unfortunately shareholders could not react immediately and that became the reason for some inertia in changing compensation plans. This results seems to be the same as Casamatta and Gumboldt has got. They also mentioned the fact that shareholders change remuneration plans with some time lag.

ВЕСТНИК САНКТ–ПЕТЕРБУРГСКОГО УНИВЕРСИТЕТА

Серия 8
МЕНЕДЖМЕНТ

Ответственный редактор серии:
Ю.Е. Благов

Издательство Высшей школы менеджмента Санкт-Петербургского университета

Stable Cooperation with Communication Structure

Artem Sedakov[1] **and Elena Parilina**[2]

[1,2]*St Petersburg State University*
Faculty of Applied Mathematics and Control Processes, Russia
[1]*a.sedakov@yahoo.com*
[2]*elena.parilina@gmail.com*

Keywords: *Allocation, Characteristic Function, Communication Structure, Coalition Structure*

In the paper we study games with coalition structures. In general setting, it is supposed that players in one coalition can communicate with each other. However, if we assume that some of interactions between players in the coalition are not possible, we face the game with coalition structure but with restricted cooperation. In our setting restricted cooperation is associated with a network generating the structure of players communication. Therefore, having both network and coalition structure fixed, a payoff distribution can be proposed using auxiliary calculated worth of each coalition of players. Unfortunately, not always all coalition structures for a fixed network are appropriate for all players involved in the conflict. That is why it is reasonable to find ``proper'' or ``stable'' in some sense coalition structures and extract them from the bunch of all possible coalition structures and given network. In the paper we introduce the definition of stability of the coalition structure with respect to a cooperative allocation (e. g. the Shapley value, ES-value) for a given network and coalition structure of players. The results are based on theoretical propositions and illustrated with examples.

References

[1] Dutta B., van den Nouweland A., Tijs S. Link formation in cooperative situations. International Journal of Game Theory, 1998, 27:245--256.

[2] Jackson M. O. Allocation Rules for Network Games // Games and Economic Behavior, 2005, 51 (1), pp. 128--154.

[3] Myerson R. Graphs and cooperation in games. Mathematics of Operations Research, 1977, 2:225-229.

[4] Naumova, N. Generalized proportional solutions to games with restricted cooperation. In: Petrosyan, L. A., Zenkevich, N. (Eds.), Contributions to Game Theory and Management, 2012, vol. V. Graguate School of Management SPbU, pp. 230--242.

[5] Petrosyan L., Sedakov A. On A Multistage Link Formation Game // Contributions to Game Theory and Management. Collected papers. St Petersburg, 2011, pp. 367-377.

Completely Mixed Equilibrium in the Logistics Market with Asymmetrical Clients*

Vladimir Bure and Anna Sergeeva

St.Petersburg State University,
Faculty of Applied Mathematics and Control Processes
Russia

Keywords: *Nash Equilibrium, Logistics Market, Completely Mixed Strategies*

In this work the logistics market with the three carriers is considered. This market consists of a chain of several units of production and supply. In this article we will consider the group of the clients in need of services for the transportation of the goods and the supplier of this service. Customers who come to the market are asymmetrical because they have different characteristics. The paper presents the equilibrium in asymmetrical customers game in the market that contains three firms. Each firm is a service provider that fulfills for the client order and sets its own scheme of customer payment.

Let us introduce the parameters that describe each client. As each client is a company or a firm that is involved in a continuous cycle of production and delivery. Client has a set of obligations that must be done before other links in the chain. Denote by $r_i, i = 1,\dots,n$ specific losses of customer order fulfillment. These losses arise due to missed opportunities that could be implemented at the time while the client is waiting for the order. Let $R_i, i = 1,\dots,n$ be the penalty that may be incurred by the client if the total time of the order exceeds a certain expected limit T. This penalty is associated with the obligations of the customer to the another firms, clients, creditors, etc.

Let us introduce the parameters that describe each firm. We propose that all firms have a different customer order fulfillment scheme and also they have different price policies. The first firm serves customers in the order of queue and charges a fixed cost c_1 for the order fulfillment. The second firm serves all customers together. This firm charges a cost c_{22} for the unit time of service and also firm charges the fixed cost

* This paper was supported by grant RFBR, project number 12-01-00747a.

c_{21}. The third firm serves customers in the order of queue as a first firm and charges only a cost for the unit service time c_{32}.

So we can define the average losses of customer for the each firm respectively:

$$Q_{1i} = r_i(E\tau_i^{(11)} + E\tau_i^{(12)}) + c_1 + R_i EI\{\tau_i^{(1)}, T\},$$

$$Q_{2i} = (r_i + c_{22})E\tau_i^{(22)} + c_{21} + R_i EI\{\tau_i^{(2)}, T\},$$

$$Q_{3i} = r_i E\tau_i^{(31)} + (r_i + c_{32})E\tau_i^{(32)} + R_i EI\{\tau_i^{(3)}, T\},$$

where $\tau_1 = \tau_{11} + \tau_{12}$, $\tau_2 = \tau_{22}$, $\tau_3 = \tau_{31} + \tau_{32}$, τ_{11}, τ_{31} - are waiting service times for the first and third firms and $\tau_{12}, \tau_{22}, \tau_{32}$ – are maintenance times for each firm. τ_1, τ_2, τ_3 are random variables exponential distributed with parameters μ_1, μ_2, μ_3 respectively.

$$I\{t, T\} = \begin{cases} 1, & if \quad t \geq T, \\ 0, & if \quad t < T, \end{cases}$$

To solve the problem of finding equilibrium in the market that contains three logistics firms with asymmetrical clients we use game-theoretical approach.

Now we define the non-antagonistic game in normal form:

$\Gamma = < N, \{p_i^{(j)}\}_{i \in N}, \{H_i\}_{i \in N} >$, where

$N = \{1, \ldots, n\}$ - set of players,

$\{p_i^{(j)}\}_{i \in N}$ – set of strategies, $p_i = (p_i^{(1)}, p_i^{(2)}, p_i^{(3)})$, $p_i^{(j)} \in [0,1]$ – the probability of choice firm j by client i, $j = 1,2,3$

$\{H_i\}_{i \in N}$ – set of payoff functions.

$$H_i = -(p_i^{(1)} Q_{1i} + (1 - p_i^{(1)} - p_i^{(3)}) Q_{2i} + p_i^{(3)} Q_{3i}) =$$

$$= -(p_i^{(1)}(Q_{1i} - Q_{2i}) + p_i^{(3)}(Q_{3i} - Q_{2i}) + Q_{2i}).$$

In the work the Nash equilibria for this market are found. And uniqueness of these equilibria are proved. In some cases uniqueness are proved for completely mixed strategies.

Time-consistency Problem in Transportation Games

Ilya Seryakov

Saint-Petersburg State University
Russia
ilya.seryakov@gmail.com

***Keywords**: Network, Trancportation Games, Network Flows, Time-Consistency Problem*

The m-person transportation game over the network $G = (X, N)$ is considered. The players $i \in \{1, \ldots, m\} = M$ have to reach the target vertexes $z_i \in X$, from given initial vertexes $y_i \in X$, with minimal costs. The corresponding cooperative TU game is investigated, as solution the Shaply Value is choosen. Suppose $x^* = \left(y = x_1^*, \ldots, z = x_l^*\right)$ is cooperative trajectory minimizing total costs. Compution of Shaply Value for subgames along the cooperative trajectory x^* shows time-inconsistency of Shapley Value.

The regularisation procedure leading to the time-consistent Shaply Value is proposed.

Airline Networks under Price Competition

Anna Shchiptsova

Institute of Applied Mathematical Research of the Karelian Research Centre of the Russian Academy of Sciences
Russia
anna.shchiptsova1@gmail.com

Keywords: *Price Competition, Nash Equilibrium, Cooperative Game Theory, Airline Networks*

We consider a game-theoretic model of competition in airline market. Each airline $k \in K$ holds its route network and route assigned aircrafts. Particular route is characterized by origin-destination city pair, subset of legs in airline network and route price. Route price is taken as a sum of prices on each leg.

A multinomial logit model is used to describe passenger choice of the particular route between origin-destination city pair. Let's assume, that passenger preference depends on price and number of route legs

$$M_j = \frac{e^{-\alpha p_j - \beta n_j}}{e^{-\rho} + \sum_{j' \in R(v_O, v_D)} e^{-\alpha p_{j'} - \beta n_{j'}}}, \; j \in R(v_O, v_D)$$

We study Nash equilibrium in airline price game with payoff given as

$$\pi_k(p_k) = \sum_{e \in E^k} p_e^k \min\left\{ \sum_{j \in R(e)} D_j M_j, C_e^k \right\} - \sum_{e \in E^k} cost\left(v_O(e), v_D(e), C_e^k\right), k \in K$$

Airlines can cooperate and form alliances to merge their resources. Airline alliance is represented as a single player in airline price game. Following approach in [2], we study alliance revenue management problem as a two stage game with passenger demand described by multinomial logit model. In the first stage, airlines agree on the fixed proration rates. In the second stage, each airline tries to maximize its profit in (1) independently by allocating seats to each route in operating aircrafts

$$\pi_k(x, \beta) = \sum_{j \in L_k} p_j \min\left\{ D_j M_j, x_{kj} \right\} +$$

$$+ \sum_{j \in I_k} p_j \min\left\{ D_j M_j, \min_{k' \in K_j} x_{k'j} \right\} - \sum_{e \in E^S} cost\left(v_O(e), v_D(e), C_e^S\right), k \in K \qquad (1)$$

where L_k is a set of routes, which are operated only by k-th airline and I_k is a set of interline routes ($|K_j| \geq 2$).

The characteristic function is given by

$$v(s) = \max_{x^S}\left[\sum_{j\in R^S} p_j^* \min\left\{D_j M_j, x_j^S\right\} - \sum_{e\in E^S} cost\left(v_O(e), v_D(e), C_e^S\right)\right]$$

We present computational results in proposed model of price competition using route network in Russian airline market.

References

[1] Adler N., Smilowitz K. Hub-and-spoke network alliances and mergers: Price location competition in the airline industry // Transportation Research Part B. 2007. 41(4). P. 394-409.

[2] Hu X., Caldentey R., Vulcano G. Revenue Sharing in Airline Alliances. Forthcoming in Management Science.

[3] McFadden D. Conditional logit analysis of qualitative choice behavior // Ed. P. Zarembka. Frontiers in econometrics. 1973. New York: Academic Press. P. 105-142.

[4] Lederer P. J., Nambimadom R. S. Airline network design // Operations Research. 1998. Vol. 46. N. 6. P. 785-804.

ВЕСТНИК САНКТ–ПЕТЕРБУРГСКОГО УНИВЕРСИТЕТА

Серия 10
ПРИКЛАДНАЯ МАТЕМАТИКА, ИНФОРМАТИКА, ПРОЦЕССЫ УПРАВЛЕНИЯ

Ответственный редактор серии:
Л.А. Петросян
Зам. ответственного редактора:
Д.А. Овсянников
С.В. Чистяков

Издательство Санкт-Петербургского университета

Passing Between two Pursuers

Igor Shevchenko

Pacific Fisheries Research Center
4 Shevchenko Alley, Vladivostok, 690950, Russia
igor@tinro.ru
Far East Federal University
8 Sukhanova Street, Vladivostok, 690950, Russia
shevch@dvgu.ru

Keywords: *Breakout of Encirclement, Optimal Routing in Threaten Environment, Strategy with Memory*

Each of several evaders surrounded by a group of pursuers in the plane may safely break out of the encirclement only by passing between two particular pursuers. We discuss a basic game of a faster evader E and two slower pursuers P_1 a-nd P_2. E seeks to cross the intercept P_1P_2 once, to leave the pursuers far behind and, in addition, to maximize the closest distance to either of them during the period of manoeuvring. This game may be also considered as a generalization of the problem of optimal routing of an aircraft in a threaten environment with constraints on the path length for two ``moving" radars [1]. In [2], the game of E and $P = \{P_1, P_2\}$ has been studied for simple motions of all players. For a given safe distance $l > 0$, the optimal trajectories are constructed with use of the main equation and corresponding characteristics for the states on a barrier that bounds the region of escape [3]. Additionally it was assumed that all players travel on straight lines as long as $|EP_1|$ and $|EP_2|$ greater than l. After approaching one of the pursuers, e.g., P_1 to l, E maintains the constraint $|EP_1| = l$. In the process, E and P_1 move along curved paths and P_2 runs along the same straight line until the game ends at a state with $|EP_1| = |EP_2| = l$. The control variables may be derived from the path equations as the time derivatives. Assumed that solving the game for each $l > 0$, one fills out the playing space with the optimal trajectories and constructs an optimal feedback strategy. In our paper, we examine several evasion strategies that based on evaluations of the guaranteed results on straight line motions.

Let $z_E(t) \in \mathbb{R}^2$ and $z_{P_i}(t) \in \mathbb{R}^2$ obey the equations

$$\begin{aligned} \dot z_E(t) &= e(\psi(t)), \quad z_E(0)=z_E^0, \\ \dot z_{P_i}(t) &= \omega_i e(\varphi_i(t)), \quad z_{P_i}(0)=z_{P_i}^0, \end{aligned} \tag{1}$$

where $t \ge 0$, $e(\alpha)=(\cos(\alpha),\sin(\alpha))$, $\psi(t),\varphi_i(t)\in\mathbb{R}$, $0<\omega_i<1$, z_E^0 and $z_{P_i}^0$ are initial positions, $i=1,2$. Let $z^0=(z_E^0,z_{P_1}^0,z_{P_2}^0)\in Z=\{(z_E,z_{P_1},z_{P_2}): z_E\ne z_{P_1}, z_E\ne z_{P_2} z_{P_1}\ne z_{P_2}\}$, be the initial state and $\rho_i(t,\psi,\varphi_i,z^0)$ be the distance to P_i as a function of time when the players run in the directions corresponding to the constant angles ψ,φ_1 and φ_2.

First, we look at the one-on-one game where E maximizes the minimal distance to P_i with respect to ψ. Let $r_i^0=\| z_{P_i}^0 - z_E^0\|$,

$$\begin{aligned} &\tau_i(\psi,\varphi_i,z^0)\in \mathrm{Arg}\ \min_t \rho_i(t,\psi,\varphi_i,z^0), \\ &\varphi_i^0(\psi,z^0)\in \mathrm{Arg}\ \min_{\varphi_i} \rho_i(\tau_i(\psi,\varphi_i,z^0),\psi,\varphi_i,z^0), \\ &\psi_i^0(z^0)\in \mathrm{Arg}\ \max_{\psi} \rho_i(\tau_i(\psi,\varphi_i^0(\psi,z^0),z^0),\psi,\varphi_i^0(\psi,z^0),z^0), \\ &\tau_i^0(z^0)=\tau_i(\psi_i^0(z^0),\varphi_i^0(\psi_i^0(z^0),z^0),z^0), \\ &\rho_i^0(z^0)=\rho_i(\tau_i^0(z^0),\psi_i^0(z^0),\varphi_i^0(\psi_{\{1,2\}}^*,z^0),z^0), \quad z^0\in Z,\ i=1,2. \end{aligned} \tag{2}$$

Lemma 4 *When starting at some initial state* $z^0\in Z$, E *moves straight at the angle* $\psi_i^0(z^0)$, P_i *can't approach him closer than* $\rho_i^0(z^0)$ *if* $\tau_i^0(z^0)\ge 0$, *and closer than* r_i^0 *otherwise.*

Then, we define analogous concepts for the case when E maximizes the minimal distance to either of P_1 and P_2. Let $\Psi_{\{1,2\}}(z^0)$ be the set of all ψ such that E crosses $P_1^0P_2^0$ when moves in the direction determined by ψ,

$$\begin{aligned} &\psi_{\{1,2\}}^*(z^0)\in \mathrm{Arg}\ \max_{\psi\in\Psi_{\{1,2\}}(z^0)} \min_{i=1,2} \rho_i(\tau_i(\psi,\varphi_i^0(\psi,z^0),z^0),\psi,\varphi_i^0(\psi,z^0),z^0), \\ &\varphi_i^*(z^0)=\varphi_i^0(\psi_{\{1,2\}}^*(z^0),z^0), \\ &\rho_{\{1,2\}}^*(z^0)=\min_{t\ge 0}\min_{i=1,2}\rho_i(t,\psi_{\{1,2\}}^*(z^0),\varphi_i^*(z^0),z^0), \\ &\tau_i^*(z^0)=\tau_i(\psi_{\{1,2\}}^*(z^0),\varphi_i^*(z^0),z^0), \quad z^0\in Z,\ i=1,2. \end{aligned} \tag{3}$$

Lemma 5 *If* $\tau_1^*(z^0)\ge 0$ *and* $\tau_2^*(z^0)\ge 0$ *then*

$$\begin{aligned} \rho_{\{1,2\}}^*(z^0) &= \rho_1(\tau_1^*(z^0),\psi_{\{1,2\}}^*(z^0),\varphi_1^*(z^0),z^0) \\ &= \rho_2(\tau_2^*(z^0),\psi_{\{1,2\}}^*(z^0),\varphi_2^*(z^0),z^0). \end{aligned} \tag{4}$$

Theorem 6 *When starting at some initial state* $z^0\in Z$, E *moves straight at the angle* $\psi_{\{1,2\}}^*(z^0)$, *neither* P_1 *nor* P_2 *can approach him closer than* $\rho_{\{1,2\}}^*(z^0)$.

Actually, E may try to improve the described guaranteed result by updating the angle ψ as the game advances. In this case, he must use strategies with memory since the attained minimal distance to the pursuers as well as the constraints on ψ that ensure *exactly one* crossing of the intercept P_1P_2 depend on the game history.

References

[1] Zabarankin M., Uryasev S., Pardalos P. Optimal Risk Path Algorithms. Cooperative Control and Optimization. Eds. R. Murphey, P. Pardalos. Kluwer, P. 271–-303, 2002.
[2] Hagedorn P., Breakwell J.V. A Differential game with Two Pursuers and One Evader. JOTA, V. 18, No.1, Pp. 15--29, 1976.
[3] Isaacs R. Differential Games. Dover, 1999.

МАТЕМАТИЧЕСКАЯ ТЕОРИЯ ИГР И ЕЕ ПРИЛОЖЕНИЯ

Ответственный редактор:
Л.А. Петросян
Зам. ответственного редактора:
В.В. Мазалов
Ответственный секретарь:
Н.А. Зенкевич

Выходит ежеквартально с января 2009 года

Mechanisms of Endogenous Allocation of Firms and Workers in Urban Area: from Monocentric to Polycentric City

Alexander Sidorov

Sobolev Institute of Mathematics
Russian Federation
alex.v.sidorov@gmail.com

Keywords: *Secondary Business Center, Commuting, Trade, Communication Cost*

Spatial economics has acquired new life since publication of Krugman's pioneering papers. Combined increasing returns, imperfect competition, commodity trade and the mobility of production factors Krugman has formed his now famous “core-periphery” model. Such a combination has contradicted to the mainstream paradigm of constant returns and perfect competition, which has dominated in economic theory for a long time. Furthermore, to the trade-off between increasing returns and transport costs, Krugman has added a third factor – the size of spatially separated markets. The main achievement of New Economic Geography (NEG) was to show how market size interacts with scale economies internal to firms and transport costs to shape the space-economy.□□In NEG, the market outcome arises from the interplay between a dispersion force and an agglomeration force operating within a general equilibrium model. Due to Krugman, the dispersion force ensures from the spatial immobility of farmers. As for the agglomeration force, he noticed that circular causation takes place because the following two effects reinforce each other.□□In this framework, however, the internal structure of regions was not accounted for.

In the present paper we consider NEG models, which allows for the internal structure of urban agglomerations through the introduction of a land market. To be precise, we start by focusing on the causes and consequences of the internal structure of cities, because the way they are organized has a major impact of the well-being of people. In particular, housing and commuting costs, which we call urban costs, account for a large share of consumers' expenditures. At this point we are agree with Helpman, for whom urban costs are the main dispersion force at work in modern urbanized economies. In our setting, an agglomeration is structured as a monocentric city in which firms gather in a central business district. Competition for land among consumers gives rise to land rent and commuting costs that both increase with population size. In other words, our approach endows regions with an urban structure which is absent in standard NEG models.□□As a result, the space-economy is the outcome of the interaction between two types of mobility costs: the transport costs of commodities and the commuting costs borne by workers. Evolution of commuting costs within cities, instead of transport costs between cities, becomes the key-factor explaining how the space-economy is organized. Moreover, despite the many advantages provided by the inner city through an easy access to highly specialized services, the significant fall in communication costs has led firms or developers to form enterprise zones or edge cities. We then go one step further by allowing firms to form secondary business centers. This analysis shows how polycentricity alleviates the urban of urban costs, which allows a big city to retain its dominant position by accommodating a large share of activities.□□Creation of subcenters within a city, i.e. the formation of a polycentric city, appears to be a natural way to alleviate the burden of urban costs. Thus, the escalation of urban costs in large cities seems to prompt a redeployment of activities in a polycentric pattern, while smaller cities retain their monocentric shape. However, for this to happen, firms set up in the secondary centers must maintain a very good access to the main urban center, which requires low communication costs.□□

Trying to explain the emergence of cities with various sizes, our framework, allows cities to be polycentric. Moreover, in our treatment, there are no pre-specified locations or numbers of subcenters, and our model is a fully closed general equilibrium spatial economy. As mentioned above, emergence of additional job centers is based on the urge towards decreasing of urban costs. The feature of the presented paper is that we drop very convenient (yet non-realistic) assumption on “long narrow city.” Our analysis is extended to the two-dimension because the geographical space in the real world is better approximated by a two-dimensional space.

The model provides two types of agents: firms and workers. The firms produce horizontally differentiated good under monopolistic competition with increasing return to scale, choosing the location of production (in the Center or on the periphery) and maximizing net profit. However, placing the company on the periphery brings an additional communication cost. In turn, employees choose the place of job and residence, trying to reduce housing and commuting costs, or rather to maximize a difference between the wage and the urban costs. Moreover, workers are also consumers, demanding for goods of three types: land for housing, homogenous good produced outside the city (traditionally interpreted as agricultural good) and horizontally differentiated good. The general equilibrium in this model implies a lack of incentives for firms and workers to change position and clearing in all markets, including the housing and labor ones.

The following results are obtained:

1. Polycentric city structure may exist only if population of city exceeds the certain threshold, i.e., too small city cannot bear the burden of polycentricity. Moreover, increasing in number of Secondary Business Districts (SBD) implies that per capita urban costs strictly decrease. It results in increasing (ceteris paribus) of disposable income and indirect utility of the city residents, therefore, developing of the inner city structure may be an important policy instrument. □□

2. Disposable income is positive if and only if city population is not less than strictly positive lower threshold a do not exceeds the finite upper bound. It means that the effective production (with increasing return to scale) cannot be developed on the base of too small settlement, and, vice versa, very large city cannot survive because of too heavy burden of urban costs. Increasing in SBD number shifts up the upper threshold (i.e., increases city capacity), therefore, extensive development of the city structure can be an effective policy instrument for sufficiently large. It cannot help, however, small cities to survive as industrial settlements.

3. Sufficiently high level of trade openness (i.e., sufficiently small trade costs) shifts down to zero the lower threshold of city population. It means that under condition of almost free trade, small cities could survive as satellites of large ones. Another benefit of sufficiently free trade is that real wage (indirect utility) increases for residents in all cities, not depending on their sizes, although this effect is more significant for small cities. It increases the relative attractiveness for the labor inflow.

4. This inflow may result in overpopulation of city with given number of SBDs. To avoid this overpopulation, City Developer may increase the current SBD number, which increases city capacity. Mechanism of determining of endogenous minimum SBD number was suggested, which is consistent with empirical evidences.

This research was supported by the RFBR grant 12-06-00174-a and EERC grant 11-511.

The Set of α-prenucleoli in a 3-person Cooperative TU-Game

Nadezhda Smirnova[1], Svetlana Tarashnina[2] and Denis Kuzyutin[3]

[1,3]Russian Federation International Banking Institute
[2,3]St.Petersburg State University
Russia
[1]nadezhda.v.smirnova@gmail.com
[2]tarashnina@gmail.com
[3]kuzyutin@ibi.metrocom.ru

Keywords: *Cooperative Games, the α-Prenucleolus, the Prenucleolus, the SM-Nucleolus, the Shapley Value*

At first we produce the basic notations concerning to an α-prenucleolus for an arbitrary n-person cooperative TU-game. Consider a cooperative TU-game (N,v), where $N=\{1,2,\dots,n\}$ is the finite set of players and $v:2^N \mapsto R^1$ is a real valued function on the family of all subsets of N with $v(\varnothing)=0$. The dual game (N,v^*) of (N,v) is defined by

$$v^*(S)=v(N)-v(N\setminus S)$$

for all coalitions $S\subseteq N$.

Denote by $X^0(N,v)$ the set of preimputations of a game (N,v), i.e. $X^0(N,v)=\{x\in R^n : x(N)=v(N)\}$. Let x be a preimputation in a game (N,v).

The excess $e(x,v,S)$ of a coalition S at x is $e(x,v,S)=v(S)-x(S)$. So, the dual excess $e(x,v^*,S)$ of a coalition $S\subseteq N$ at x is $e(x,v^*,S)=v^*(S)-x(S)$.

For any fixed $\alpha\in R^1$ we consider the α-excess $e^\alpha(x,v,S)$ of coalition $S\subseteq N$ for each $x\in X^0(N,v)$ as follows

$$e^\alpha(x,v,S)=\alpha e(x,v,S)+(1-\alpha)e(x,v^*,S).$$

For any fixed $\alpha\in R^1$ the α-prenucleolus of a game (N,v) is defined as

$$\mathbf{N}^\alpha(N,v)=\{x\in X^0(N,v):\theta(e^\alpha(x,v,S))\succ_{lex}\theta(e^\alpha(y,v,S))\ for\ all\ y\in X^0(N,v)\},$$

where $\theta(e^\alpha(x,v,S)_{S\subset N})$ is a vector of α-excesses which components are arranged in nonincreasing order.

Denote by $\overline{\mathbf{N}}(N,v)$ a set of all α-prenucleoli of a game (N,v), then

$$\overline{\mathbf{N}}(N,v) = \bigcup_{\alpha \in R^1} \mathbf{N}^{\alpha}(N,v).$$

It was proved before that the α-prenucleolus of an arbitrary TU-game (N,v) coincides with the prenucleolus of a certain game (N,v^{α}), which is constructed as the weighted sum of the game and its dual:

$$v^{\alpha}(S) = \alpha v(S) + (1-\alpha)v^{*}(S),\ S \subseteq N.$$

Using that result, consider the set of α-prenucleoli in 3-person cooperative TU-games. Without loss of generality, we put the restriction on v:

$$\begin{cases} v(\{i\}) = 0,\ i = \overline{1,3}, \\ v(\{1,2\}) < v(\{1,3\}) < v(\{2,3\}). \end{cases}$$

Further we replace $v(\{i,j\})$ with note $v(ij)$. Denote the difference between $v(N)$ and $v(12)+v(13)+v(23)$ as $\Delta v(N)$, i.e. $\Delta v(N) = v(N) - v(12) - v(13) - v(23).$ Then the set of all α-prenucleoli of the game (N,v) is defined by formula $\overline{\mathbf{N}}(N,v) = \mathbf{N}_1^{\mathbf{a}} \cup \mathbf{N}_2^{\mathbf{a}} \cup \mathbf{N}_3^{\mathbf{a}} \cup \mathbf{N}_4^{\mathbf{a}} \cup \mathbf{N}_5^{\mathbf{a}}$, where $\mathbf{N}_{\mathbf{j}}^{\mathbf{a}},\ \mathbf{j} = \overline{1,5},$ is described by the following cases:

- for $\alpha \le \dfrac{1}{2} - \dfrac{|\Delta v(N)|}{2(v(23)-v(13))}$

$$\mathbf{N}_1^{\mathbf{a}} = \left\{ \left(\frac{v(12)+v(13)}{2}, \frac{v(23)+v(12)}{2} + \frac{\Delta v(N)}{2}, \frac{v(23)+v(13)}{2} + \frac{\Delta v(N)}{2} \right) \right\};$$

- for $\dfrac{1}{2} - \dfrac{|\Delta v(N)|}{2(v(23)-v(13))} \le \alpha \le \dfrac{1}{2} - \dfrac{|\Delta v(N)|}{2(v(23)+v(13)-2v(12))}$

$$\mathbf{N}_2^{\mathbf{a}} = \{(v_1^{\alpha}(N,v), v_2^{\alpha}(N,v), v_3^{\alpha}(N,v))\},\text{ where}$$

$$v_1^{\alpha}(N,v) = \frac{v(13)+v(12)}{2} + \frac{\Delta v(N)}{4} + sgn(\Delta v(N)) \frac{(2\alpha-1)(v(23)-v(13))}{4},$$

$$v_2^{\alpha}(N,v) = \frac{v(23)+v(12)}{2} + \frac{\Delta v(N)}{4} + sgn(\Delta v(N)) \frac{(1-2\alpha)(v(23)-v(13))}{4},$$

$$v_3^{\alpha}(N,v) = \frac{v(13)+v(23)}{2} + \frac{\Delta v(N)}{2};$$

- for $\dfrac{1}{2} - \dfrac{|\Delta v(N)|}{2(v(23)+v(13)-2v(12))} \le \alpha \le \dfrac{1}{2} + \dfrac{|\Delta v(N)|}{2(2v(23)-v(13)-v(12))}$

$$\mathbf{N}_3^{\mathbf{a}} = \{(v_1^{\alpha}(N,v), v_2^{\alpha}(N,v), v_3^{\alpha}(N,v))\},\text{ where}$$

$$v_1^{\alpha}(N,v) = \frac{v(13)+v(12)}{2} + \frac{\Delta v(N)}{3} + sgn(\Delta v(N)) \frac{(2\alpha-1)(2v(23)-v(13)-v(12))}{6},$$

$$\nu_2^\alpha(N,v) = \frac{v(23)+v(12)}{2} + \frac{\Delta v(N)}{3} + sgn(\Delta v(N))\frac{(2\alpha-1)(2v(13)-v(23)-v(12))}{6},$$

$$\nu_3^\alpha(N,v) = \frac{v(23)+v(13)}{2} + \frac{\Delta v(N)}{3} + sgn(\Delta v(N))\frac{(2\alpha-1)(2v(12)-v(23)-v(13))}{6};$$

• for $\frac{1}{2}+\frac{|\Delta v(N)|}{2(2v(23)-v(13)-v(12))} \le \alpha \le \frac{1}{2}+\frac{|\Delta v(N)|}{2(v(13)-v(12))}$

$$\mathbf{N_4^a} = \{(\nu_1^\alpha(N,v), \nu_2^\alpha(N,v), \nu_3^\alpha(N,v))\},\text{ where}$$

$$\nu_1^\alpha(N,v) = \frac{v(13)+v(12)}{2} + \frac{\Delta v(N)}{2},$$

$$\nu_2^\alpha(N,v) = \frac{v(23)+v(12)}{2} + \frac{\Delta v(N)}{4} + sgn(\Delta v(N))\frac{(2\alpha-1)(v(13)-v(12))}{4},$$

$$\nu_3^\alpha(N,v) = \frac{v(23)+v(13)}{2} + \frac{\Delta v(N)}{4} + sgn(\Delta v(N))\frac{(1-2\alpha)(v(13)-v(12))}{4};$$

• for $\alpha \ge \frac{1}{2}+\frac{|\Delta v(N)|}{2(v(13)-v(12))}$

$$\mathbf{N_5^a} = \left\{\left(\frac{v(13)+v(12)}{2}+\frac{\Delta v(N)}{2}, \frac{v(23)+v(12)}{2}+\frac{\Delta v(N)}{2}, \frac{v(13)+v(23)}{2}\right)\right\}.$$

These formulas help us to analyze what the effect the parameter α has on the players' payoffs. And which α is more profitable for each player.

Note that we have also obtained the formulas for the prenucleolus (in case of $\alpha=1$); for the Shapley value and SM -nucleolus (in case of $\alpha=\frac{1}{2}$); for the modiclus in the class of balanced games (in case of $\alpha=0$).

From Agency to Stewardship Theory: on the Role of Power and Satisfaction in Organizational Architectures

Frank Steffen

Professorship of International Governance, University of Bayreuth, Universitätsstraß e 30, D-95447 Bayreuth, Germany
Department of Economics, Finance and Accounting, The University of Liverpool Management School (ULMS), Chatham Street, Liverpool L69 7ZH, U.K.,
frank.steffen@uni-bayreuth.de

Keywords: *Agency Theory, Hierarchy, Organizational Architecture, Power, Satisfaction, Stewardship Theory*

In this paper we illustrate how measures of (positional) power and satisfaction - both belonging to the theory of voting power - can be used as instruments in order to support the implementation of a certain mode of individual behavior in organizations. We address the question of how organizational architectures, in terms of their resulting allocation of power and satisfaction, can contribute to an organizational environment which supports actors to behave according to the `model of man' underlying stewardship theory, rather then providing them incentives to follow the `model of man' as presupposed by agency theory.

Agency theory is the predominant theory of individual behavior is organizational economics. The `model of man' underlying this theory assumes that actors are `rational' and `self-interested': it is assumed that actors have preferences over a set of outcomes and will choose that (course of) action which allows them to reach their individually most preferred outcome. If there exists a contractual relationship between two actors such that one actor (the `agent') has formally agreed to always choose that action which will be in the best interest of another actor (the `principal) this will only work if the interest of both actors are either aligned or the principal fully monitors the behavior of its agent. It is assumed that the former is (usually) not the case as actors are considered to have an inherent dislike to perform tasks in the interest of someone else. The view regarding the latter is that this is either in contradiction to the idea of delegation underlying many contractual relationships between principals and agents and/or would be infeasible or too costly. As a result agency theory is mainly concerned with the design of mechanisms, usually some kind of payment scheme, in order to align the interest of the agent with that of its principal. Apart from this result a similar view can be found in organizational psychology where the above set of assumptions is known under `Theory X' (McGregor 1960). In this context, it is also argued that actors prefer to obtain clear and strict orders from their principals rather then being in an autonomous position which makes them responsible for their own choice of action.

About two decades ago Donaldson (1990a, 1990b) suggested an alternative theory of individual behavior in organizations which he called stewardship theory and has found its way in the standard textbooks on governance. This theory is based on an alternative 'model of man' originating from organizational sociology and psychology. Under stewardship theory the agent having signed an agreement with its principal has the incentive to fulfill its role as specified in the contract. Following McClelland (1961) and Herzberg et al. (1959) it is assumed that an agent gains intrinsic satisfaction from successfully performing inherently demanding work and from exercising responsibility and authority. Moreover, it is said that by gaining recognition from peers and superiors for such a behavior agents also gain extrinsic satisfaction. In contrast to agency theory no pecuniary component is considered here. Thus, even if from an individualistic point of view an action would be unattractive, i.e., an alternative action would result in an higher level of individual satisfaction, an agent will nevertheless carry out that the former action if its role would require this (Etzioni 1975). Again, a corresponding set of assumptions can be found in organizational psychology which is know under `Theory Y' (McGregor 1960). In the context of this theory it is also noted that the observed behavior of agents in line with `Theory X' does not imply that they dislike the performance of tasks in the interest of someone else as specified by their contracts. Rather the observed behavior is their reaction to their experience of being an agent in world which applies the agency theory paradigm. Thus, given the assumed absence of an inner motivational problem of agents, the question arises how those can be motivated to behave according to stewardship theory.

While it is well known from that `all people are different' and, hence, will not respond in the same way to the same stimuli (Davis 1981), we can still ask the question under what conditions it can be expected to be more likely that agents behave more in line with the stewardship theory. To put it in the terminology of Argyris and Schön (1992) our aim is to aline the `theory-in-use', i.e., the theory actors actually apply, and the `espoused theory', i.e., the words they use to convey what they think or do. While the former can be directly observed, the latter has to be identified by asking an actor how it would behave under certain circumstances. In this paper our conjecture is as follows: a necessary condition for the congruence of an agents theory-in-use and espoused theory is that an agent (within an organization) is able to met the tasks it is expected to perform on a sustainable basis; otherwise, i.e., if a task demands too much or if a task demands too little, an agent will show the patterns of behavior as assumed under agency theory.

Thus, in terms of organizational design, the task to be performed is to identify an organizational architecture, which produces the `right' balance of power and satisfaction among the actors within an organization. Making use of stylized examples of hierarchical organizational architectures and applying an `action-based approach' to the measurement of power and satisfaction in organizations (see van den Brink and Steffen 2008, 2012a, 2012b), we illustrate how institutional reforms may produce results, which instead of the `role-concurring' behavior are rather likely to induce 'self-interested' behavior, and vice versa.

References

[1] Argyris C and Schön DS (1992), Theory in Practice: Increasing Professional Effectiveness. San Francisco, Jossey-Bass Publishers.

[2] Brink R van den and Steffen F (2008), Positional Power in Hierarchies. In: Braham M, Steffen F (eds) Power, Freedom, and Voting, Berlin, Heidelberg, Springer, 57-81.

[3] Brink R van den and Steffen F (2012a), Axiomatizations of a Positional Power Score and Measure for Hierarchies. Public Choice 151, 757-787.

[4] Brink R van den and Steffen F (2012b), On the Measurement of Success and Satisfaction. In: Fara R et al. (eds), Voting Power and Procedures: Essays in Honor of Dan Felsenthal and Moshé Machover, Springer, (forthcoming).

[5] Davis K (1981), Human Behavior at Work: Organizational Behavior. New York, McGraw-Hill.

[6] Donaldson L (1990a), The Ethereal Hand: Organizational Economics and Management Theory. Academy of Management Review 15, 369-381.

[7] Donaldson L (1990b), A Rational Basis for Criticisms of Organizational Economics: A Reply to Barney. Academy of Management Review 15, 394-401.

[8] Etzioni A (1975), Comparative Analysis of Complex Organizations, revised and enlarged edn. New York, Free Press.

[9] Herzberg F, Mausner B and Snyderman B (1959) The Motivation to Work. New York, John Wiley.

[10] McClellend DC (1961), The Achieving Society. Princeton, Van Nostrand.

[11] McGregor D (1960), The Human Side of Enterprise. New York, Harper and Row.

Strong Stability in Networks and Matching Markets with Contracts

Alexander Teytelboym

Department of Economics, University of Oxford
alexander.teytelboym@economics.ox.ac.uk

Keywords: *Networks, Multilateral Matching, Stability, Complementarities, Spillovers, Contracts, Collective Bargaining, Unions*

We present a unified model of contractual networks with divisible surplus and matching markets with finite contracts. In both the network and the matching markets models, we depart from the usual restriction of bilateral contracts and allow for multilateral contracts [4]. We focus on strongly stable contract allocations. A contract allocation is strongly stable if it is robust to deviations by any group of agents (allowing them to keep any of their previous contracts while forming new ones only among themselves). We offer a necessary and sufficient condition for the existence of strongly stable contractual networks and contract allocations in multilateral matching markets in the presence of a sufficiently expressive contractual language.

In the network model, we drop the anonymity assumption on the allocation rule and generalise the results of Jackson and van den Nouweland [5] and Dutta and Mutuswami [1], which state that only egalitarian division of surplus on connected network components ensures strong stability. The key condition in this paper, which we call strong pairwise alignment, is an extension of the pairwise alignment condition introduced by Pycia [7]. Strong pairwise alignment states that for any two sets of contracts, any agents who are party to every contract in both sets, must have the same preferences over the two sets. This condition allows for the presence of technological spillovers and complementarities. The paper challenges recent results on the necessity of substitutability and coarseness of contractual language for the existence of strongly stable contract allocations in matching markets [3].

A common criticism of refinements of (pairwise) stable outcomes is that group deviations are hard to achieve since they require a lot of coordination on behalf of the deviating agents. However, a natural setting where group deviations may occur is in a bargaining process between firms and unionised workers (or workers, who are covered by a collective bargaining agreement and are not members of a union). The union acts as a credible coordinator of arbitrary deviations by workers: for example, it can ballot workers to go on strike. In this context, the set of pairwise stable outcomes may be an imprecise predictor of market outcomes and strongly stable outcomes are more accurate. The model presented in this paper allows firms to hire both unionised and non-unionised workers and allows workers to take on jobs outside the remit of their union. The results suggest strongly stable firm-union outcomes in thin labour markets are characterized by a generalized Nash bargaining solution. This provides the first micro-founded justification for the use of Nash bargaining solution in the analysis of firm-union bargaining [2, 6].

References

[1] Dutta, B. and S. Mutuswami (1997). Stable networks. Journal of Economic Theory 76(2), 322–344.

[2] Grout, P. (1984). Investment and wages in the absence of binding contracts: A nash bargaining approach. Econometrica 52(2), 449–460.

[3] Hatfield, J. W. and S. D. Kominers (2011a). Contract design and stability in matching markets. Working paper, Harvard Business School.

[4] Hatfield, J. W. and S. D. Kominers (2011b). Multilateral matching. Mimeo, Stanford GSB.

[5] Jackson, M. O. and A. van den Nouweland (2005). Strongly stable networks. Games and Economic Behavior 51(2), 420–444.

[6] Oswald, A. (1982). The microeconomic theory of the trade union. The Economic Journal 92(367), 576–595.

Cake Division Model with Non-symmetric Parameters

Julia Tokareva[1] **and Vladimir Mazalov**[2]

[1]*Transbaikal State University*
jtokareva2@mail.ru

[2]*Institute of Applied Mathematical Research, Karelian Research Center*
vmazalov@krc.karelia.ru

Keywords: *Cake Division Model, Consensus, Discounting, Dirichlet Distribution, Multistage Procedure, Random Offers, Threshold Strategy*

Multistep procedure of division of cake of the single size for three persons is considered. Participants of negotiations - players of I, II and III - on each step receive from the arbitrator of the offer (x_1^k, x_2^k, x_3^k), respectively. These are the random variables distributed on the law to Dirikhlet

$$f(x_1, x_2, x_3) = \frac{\Gamma(k_1 + k_2 + k_3)}{\Gamma(k_1)\Gamma(k_2)\Gamma(k_3)} x_1^{k_1-1} x_2^{k_2-1} x_3^{k_3-1},$$

where $x_1 + x_2 + x_3 = 1$.

Lets $k_1 = m$, $k_2 = k_3 = 1$. Then

$$f(x_1, x_2) = m(m+1)x_1^{m-1}, \quad x_1 + x_2 \le 1, x, y \ge 0.$$

Everyone players decide to accept the offer or to reject it. The final decision is made by consensus. If all players accept their offers then division (x, y, z). Otherwise, the proposal is ignored and the players come in to the next step $k-1$ hoping for the best offer in the future. Thus there is a discounting and on the following step players divide size cake $\delta < 1$. Bargaining continues until some player accepts the offer or the time of bargaining ends.

Let's enter into strategy consideration. Let $\mu_1(x_1)$ ($\mu_2(x_2), \mu_3(x_3) = \mu_3(1 - x_1 - x_2)$) is a probability of that the player of $I(II, III)$ will accept the proposal of the arbitrator $x_1(x_2, x_3)$. Owing to symmetry of game for the second and third player we believe $\mu_2(x_2) = \mu_3(1 - x_1 - x_2)$

Theorem. *The optimal strategies of players in* k *-th step are*

$$\mu_1(x_1) = I_{\{x_1 \ge \delta H^{(1)}_{k-1}\}}, \quad \mu_i(x_i) = I_{\{x_i \ge \delta H^{(i)}_{k-1}\}} \mathrm{mod} 0.5 cmi = 2,3 .$$

The value of the game satisfies to recurrence relation

$$H_k^{(1)} =$$

$$= \frac{1}{m+2} \cdot \left\{ (1 - 2\delta H_{k-1}^{(2)})^{m+1} \left[(1 - 2\delta H_{k-1}^{(2)})m - \delta H_{k-1}^{(1)}(m+2) \right] + (\delta H_{k-1}^{(1)})^{m+1} \left[(m+2)(1 - 2\delta H_{k-1}^{(2)}) - bm \right] \right\} + \delta H_{k-1}^{(1)}$$

$$H_k^{(2)} =$$

$$= \frac{1}{m+2} \{ (1 - 2\delta H_{k-1}^{(2)})^{m+2} - (\delta H_{k-1}^{(1)})^{m+1} (1 - 2\delta H_{k-1}^{(2)} - \delta H_{k-1}^{(1)})(m+2) +$$

$$+ (\delta H_{k-1}^{(1)})^m (\delta H_{k-1}^{(2)} - (\delta H_{k-1}^{(2)})^2 - 2\delta^2 H_{k-1}^{(2)} H_{k-1}^{(1)} - \frac{1}{2} + \delta H_{k-1}^{(1)} - \frac{1}{2} (\delta H_{k-1}^{(1)})^2)(m+2)(m+1) \} + \delta H_{k-1}^{(2)}.$$

If $\delta = 1$ and $k \to \infty$ when

$$H_k^{(1)} = \frac{1}{2} \left(-1 + 2H_k^{(2)} - H_k^{(1)} \right) \left(-1 + 2H_k^{(2)} + H_k^{(1)} \right)^3 + H_k^{(1)};$$

$$H_k^{(2)} = \frac{1}{4} \left(-1 + 2H_k^{(2)} + 3H_k^{(1)} \right) \left(-1 + 2H_k^{(2)} + H_k^{(1)} \right)^3 + H_k^{(2)}$$

and we have

$$H_k^{(2)} = \frac{1}{2} \left(1 - H_k^{(1)} \right).$$

The research was supported by Ministry of Education and Science of the Russian Federation (project 8.3641.2011)

References

[1] Farber H. An analysis of final-offer arbitration // Journal of conflict resolution. 1980. v. 24. P. 683 – 705.

[2] Mazalov V., Sakaguchi M. Location game on the plane // International game theory review. 2003. v. 5. P. 13–25.

[3] Mazalov V., Tokareva J. Bargaining model on the plane// Algorithmic and computational theory in algebra and languages. 2008. P. 42–49.

[4] Mazalov V., Tokareva J. Game-theoretic models of tenderвЂ™s design// The mathematical theory of games and its applications. 2010. P. 66–78.

On Solution of the Dynamical Reconstruction Problem for a Macroeconomic Model

Nina Subbotina[1], **Evgeniy Krupennikov**[2] **and Timofey Tokmantsev**[3]

[1,2,3]*Institute of Mathematics and Mechanics, UrB of RAS, Ekaterinburg*
[1]*subb@uran.ru*
[2]*mechokrupa@gmail.com*
[3]*tokmantsev@imm.uran.ru*

Keywords: *Macroeconomic Model, Statistic Data, Optimal Control Theory, Numerical Reconstructions*

A macroeconomic model due to E.G. Al'brekht [1] is under consideration. Symbol p denotes the gross product, q denotes the production costs, h denotes the profit. The function $G(p,q) = h$ is called the macroeconomic potential of the system. We assume that the function $G(p,q)$ has the form

$$G(p,q) = pq(a_0 + a_1 p + a_2 q),$$

where coefficients a_0, a_1, have to be defined.

We know the statistic data $(p^*(t_i), q^*(t_i), h^*(t_i)),\quad i = 0,\dots,N,$ with admissible error ξ.:

$$|p^*(t_i) - p(t_i)| \le \xi, \qquad |q^*(t_i) - q(t_i)| \le \xi, \qquad |h^*(t_i) - h(t_i)| \le \xi.$$

The dynamics of the macroeconomic system are described by the following differential equations

$$\frac{dp}{dt} = u_1(t)\frac{\partial G(p,q)}{\partial p}, \qquad \frac{dq}{dt} = u_2(t)\frac{\partial G(p,q)}{\partial q}, \quad t \in [0,T] = [t_0, t_N], \tag{1}$$

where controls $u_1(t)$, $u_2(t)$ have the following constraints:

$$|u_1| < U_1, |u_2| < U_2, \quad U_1 > 0, U_2 > 0. \tag{2}$$

We assume that the admissible controls $u_1(t)$, $u_2(t)$ are elements of the set

$$\mathbf{U_T} = \{\forall u(\cdot) = (u_1(\cdot), u_2(\cdot)) : [0,T] \mapsto [-U_1, U_1] \times [-U_2, U_2] \text{ are measurable}\}.$$

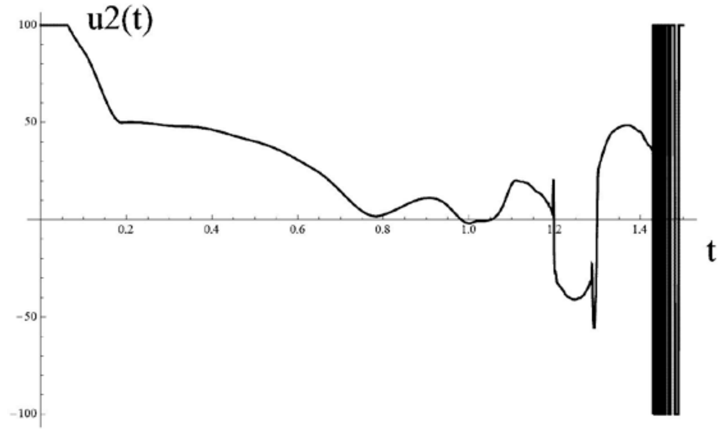
We define coefficients a_0, a_1, a_2 in $G(p,q)$ via applying the least-squares method to the relations

$$h^*(t_i) = G(p^*(t_i), q^*(t_i)), \quad i = 0,1,\ldots,N.$$

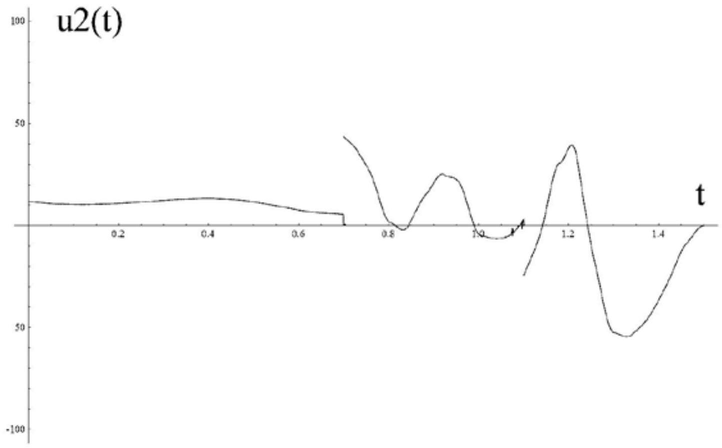
The problem of reconstruction of controls $u_1(t), u_2(t)$ generating motions $p(t), q(t)$ close to the statistic data is considered as the optimal control problem relative to the pay-off functional

$$I(p(\cdot), q(\cdot), u_1(\cdot), u_2(\cdot)) = \\ = \int_0^T \left[(p^*(t) - p(t))^2 + (q^*(t) - q(t))^2 + \varepsilon \frac{(u_1(t))^2 + (u_2(t))^2}{2} \right] dt. \tag{3}$$

Here $p^*(t)$, $q^*(t)$ are linear interpolations of the statistic data, ε is a small parameter. Solutions of the dynamical reconstruction problem are controls that are minimizing the pay-off on the set $\mathbf{U_T}$. Common sense recommends us to take into consideration only realistically reconstructed controls which have no sliding-mode intervals. The controls have to generate dynamics deviating from statistics not far from ξ. The results of numerical solution are exposed below for the case of stationary model (coefficients a_0, a_1, a_2 are constant) and for the nonstationary model (coefficients a_0, a_1, a_2 are peacewise constant).



A nonrealistic reconstructed control $u_2(t)$ for the stationary model



A realistic reconstructed control $u_2(t)$ for the nonstationary model

Relations of the obtained results with other approaches to solutions of ill-posed problems are discussed.

The work was supported by the Russian Foundation for Basic Researches (projects No. 11-01-00214, 12-01-31394) and by the Program of Presidium RAS for Basic Researches on Mathematical Control Theory

References

[1] Al'brekht E.G. ``Methods of construction and identification of mathematical models for macroeconomic processes'', electronic journal ``Investigated in Russia'' (http:://zhurnal.ape.relarn.ru/articles/2002/005.pdf) (in Russian).

[2] Pontryagin L.S., Boltyanskii V.G., Gamkrelidze R.V., Mishchenko E.F. Mathematical Theory of Optimal Processes. 1962. New York: Interscience.

[3] Subbotina N.N., Tokmantsev T.B. The Method of Characteristics in Macroeconomic Modeling. Contributions to Game Theory and Management, St. Petersburg: Graduated School of Management, vol.3, pp.399-408, 2009.

Most Dominant Imputations

Gabriel Turbay

Sociedad Colombiana de Economistas
Colombia
gt.gabrielturbay@gmail.com

Keywords: *Preference Measure, Most Dominant Imputations, Nash Bargaining Solution, one Point Solution Concept, n-Person Cooperative Game*

The Lebesgue measure of sets consisting of imputations that dominate or are dominated in zero-sum n-person cooperative games or that are preferred by or to a given imputation in pure-bargaining games, serves (1) to develop ordered relations and utility indicators that allow to narrow down the indeterminacy of vN-M stable sets; (2) to obtain most dominant and most preferent imputations for the above games ; (3) to obtain the Nash-bargaining product solution for 2-person pure-bargaining cooperative games as a particular case; and (4) to develop a one- point solution concept or value for general non-zero-sum n-person game. The solution concept is obtained optimizing a general welfare indicator measure function and can be expressed in terms of a basic strategic-equilibriums and the expected coalitional value to each player previous to considering the formation of the grand coalition plus a proportional part of the player?s added value to the grand coalition in case it forms. To measure preference or dominance in imputation simplexes becomes a relatively simple task when the measures of preference regions are made with the help of a self-similar geometry developed and introduce here for such task.

On Equilibrium Based Coalition Formation Likelihood

Gabriel Turbay[1] **and Guillermo Owen**[2]

[1]*Sociedad Colombiana de Economistas*
Colombia
gt.gabrielturbay@gmail.com

[2]*Naval Postgraduate School*
United States
gowen@nps.edu

Keywords: *Coalition Formationlikelihood, Extended-Imputation, Strategic-Equilibrium, Balanced Collections, Strategic-Core, Nash-Equilibrium, Harsanyi's P-Matrix, Game Value, Utility Functions*

Based on the concept of balanced collections of Bondareva and Shapley, the so defined fundamental strategic equilibrium that every n-person cooperative game possesses is shown to be a generalization von Neumann and Morgenstern “objective” equilibrium for the zero-sum 3-person cooperative game and it is used as an example to address the problem of measuring the likelihood of coalition formation. In particular, we depart from the defined fundamental equilibrium and its linear programming representation to determine the coalitions most likely to form. By assuming that player's coalition preferences may be described by player's choice probabilities, we assume the existence of players types in the form of Harsanyi's probability P-matrices to compute the probability of coalition formation and the player?s expected value for the game previous to the formation of the grand coalition. Based on these expected values for the players we look for Nash like P-matrix equilibrium in the sense that unilateral deviations do not improve the deviant player's expectation. Based on the described fundamental equilibrium and on the players expected value for the game, stronger and weaker core solution concepts are introduced.

Pareto-Nash-Stackelberg Linear Discrete-time Control Problem and Principles for its Solving

Valeriu Ungureanu

State University of Moldova,60, A. Mateevici str.,
Chi¸sin˘au, MD−2009, Moldova.
V.Ungureanu@ymail.com

Keywords: *Linear Discrete-Time Control Problem, Non-Cooperative Game, Multi-Criteria Strategic Game, Pareto-Nash-Stackelberg Control*

A direct-straightforward method for solving linear discrete-time optimal control problem is applied to solve control problem of a linear discrete-time system as a mixture of multi-criteria Stackelberg and Nash games. For simplicity, the exposure starts with the simplest case of linear discrete-time optimal control problem and, by sequential considering of more general cases, investigation finalizes with the highlighted Pareto-Nash-Stackelberg and set valued control problems. Different principles of solving are compared and their equivalence is proved.

Introduction

Optimal control theory which appeared due to Lev Pontryagin [2] and Richard Bellman [3], as natural extension of calculus of variations, often doesn't satisfy all requirements and needs for modelling and solving problems of real dynamic systems and processes. A situation of this type occurs for problem of linear discrete-time system control by a decision process that evolves as Pareto-Nash-Stackelberg game with constraints -- a mixture of hierarchical and simultaneous games For such system, the notion of optimal control evolves naturally to the notion of Pareto-Nash-Stackelberg type control and to the natural principle for solving the highlighted problem by applying a concept of Pareto-Nash-Stackelberg equilibrium [9] with a direct-straightforward principle for solving.

The direct method and principle for solving linear discrete-time optimal control problem is extended to control problem of a linear system in discrete time as a mixture of multi-criteria Stackelberg and Nash games [9] . The exposure starts with the simplest case of linear discrete-time optimal control problem [1] and, by sequential considering of more general cases, finalizes with the Pareto-Nash-Stackelberg and set valued control problems. The maximum principle of Pontryagin is formulated and proved for all the considered problems. Its equivalence with the direct-straightforward principle for solving is established.

Linear discrete-time Pareto-Nash-Stackelberg control problem

The conference presentation begins with the problem of optimal control and their optimal principles, and by considering different types of control and principles finalizes with the control of Pareto-Nash-Stackelberg type with T stages and $\nu_1 + \ldots + \nu_T$ players, where $\nu_1, \ldots, \nu_T$ are the correspondent numbers of players on stages $1, \ldots, T$. Every player is identified by two numbers: τ -- stage on which player selects his strategy and π -- player number at stage $.\tau$.. In such game, at each stage τ the players $1, 2, \ldots, \nu_\tau$ play a Pareto-Nash game by selecting simultaneously their strategies according to their criteria ($k_{\tau 1}, k_{\tau 2}, \ldots, k_{\tau \nu_\tau}$ are the numbers of criteria of respective players) and by communicating his and all precedent selected strategies to the following $\tau + 1$ stage players. After all stage strategy selections, all the players compute their gains on the resulting profile. Such type of control is named Pareto-Nash-Stackelberg control, and the corresponding problem -- linear discrete-time Pareto-Nash-Stackelberg control problem.

The decision control process may be modelled as:

$$
\begin{aligned}
& f_{\tau\pi}(x, u^{\tau\pi} \| u^{-\tau\pi}) = \sum_{t=1}^{T} \left(c^{\tau\pi t} x^t + \sum_{\mu=1}^{\nu_t} b^{\tau\pi t\mu} u^{t\mu} \right) \underset{u^{\tau\pi}}{\rightarrow} \mathrm{ef}\ \max, \\
& \tau = 1, \ldots, T, \pi = 1, \ldots, \nu_\tau, \\
& x^t = A^{t-1} x^{t-1} + \sum_{\pi=1}^{\nu_t} B^{t\pi} u^{t\pi}, t = 1, \ldots, T, \\
& D^{t\pi} u^{t\pi} \le d^{t\pi}, t = 1, \ldots, T, \pi = 1, \ldots, \nu_t,
\end{aligned}
\tag{1}
$$

where $x^0, x^t \in R^n$, $c^{\tau\pi t\mu} \in R^{k_{tp} \times n}$, $u^{\tau\pi} \in R^m$, $b^{\tau\pi t\mu} \in R^{k_{tp} \times n}$, $A^{t-1} \in R^{n \times n}$, $B^{\tau\pi} \in R^{n \times m}$, $d^{\tau\pi} \in R^k$, $D^{\tau\pi} \in R^{k \times n}$, $t, \tau = 1, \ldots, T, \pi = 1, \ldots, \nu_\tau, \mu = 1, \ldots, \nu_t$.

By performing direct transformation, (1) is reduced to a sequence of multi-criteria linear programming problems

$$
\begin{aligned}
& f(u^{\tau\pi} \| u^{-\tau\pi}) = \\
& = \left(c^{\tau\pi\tau} B^{\tau\pi} + c^{\tau\pi\tau+1} A^{\tau} B^{\tau\pi} + c^{\tau\pi\tau+2} A^{\tau+1} A^{\tau} B^{\tau\pi} + \ldots + \right. \\
& \left. + c^{\tau\pi T} A^{T-1} A^{T-2} \ldots A^{\tau} B^{\tau\pi} + b^{\tau\pi\tau\pi} \right) u^{\tau\pi} \underset{u^{\tau\pi}}{\rightarrow} \mathrm{e}f \max, \\
& D^{\tau\pi} u^{\tau\pi} \le d^{\tau\pi},
\end{aligned} \tag{2}
$$

$\tau = 1, \ldots, T, \pi = 1, \ldots, \nu_{\tau}$.

Equivalence of (1) and (2) proves the following Theorem 1.

Theorem 17 Let (1) be solvable. The sequence $\bar{u}^{11}, \bar{u}^{12}, \ldots, \bar{u}^{T\nu_T}$ forms a Pareto-Nash-Stackelberg equilibrium control in (1) if and only if $\bar{u}^{\tau\pi}$ is an efficient solution of multi-criteria linear programming problem (2), for $\tau = 1, \ldots, T, \pi = 1, \ldots, \nu_{\tau}$.

Pontryagin maximum principle may be generalized for (1). By considering recurrent relations

$$
p^{\tau\pi T} = c^{\tau\pi T}, \quad p^{\tau\pi t} = p^{\tau\pi t+1} A^{t} + c^{\tau\pi t}, \quad t = T-1, \ldots, 1, \tag{3}
$$

where $\tau = 1, \ldots, T, \pi = 1, \ldots, \nu_{\tau}$, Hamiltonian vector-functions are defined as

$$
H_{\tau\pi t}\left(u^{\tau\pi}\right) = \left\langle p^{\tau\pi t} B^{\tau\pi} + b^{\tau\pi\tau\pi}, u^{\tau\pi} \right\rangle, t = T, \ldots, 1,
$$

where $\tau = 1, \ldots, T, \pi = 1, \ldots, \nu_{\tau}$ and $p^{\tau\pi t}, t = T, \ldots, 1, \tau = 1, \ldots, T, \pi = 1, \ldots, \nu_{\tau}$.

Theorem 2 8 Let (1) be solvable. The sequence $\bar{u}^{11}, \bar{u}^{12}, \ldots, \bar{u}^{T\nu_T}$ forms a Pareto-Nash-Stackelberg equilibrium control if and only if

$$
\bar{u}^{\tau\pi} \in \underset{u^{\tau\pi}: D^{\tau\pi} u^{\tau\pi} \le d^{\tau\pi}}{\mathrm{A}rg \quad ef \max} \quad H_{\tau\pi t}\left(u^{\tau\pi}\right),
$$

for $t = T, \ldots, 1, \tau = 1, \ldots, T, \pi = 1, \ldots, \nu_{\tau}$.

Theorems are equivalent.

Concluding remarks

There are different types of processes control: optimal control, Stackelberg control, Pareto-Stackelberg control, Nash-Stackelberg control, Pareto-Nash-Stackelberg control, etc.

The direct-straightforward, dual and classical principles (Pontryagin and Bellman) may be applied for determining the desired control of dynamic processes. These principles are the bases for pseudo-polynomial methods, which are exposed as a consequence of theorems for linear discrete-time Pareto-Nash-Stackelberg control problems.

The direct-straightforward principle is applied for solving the problem of determining the optimal control of set-valued linear discrete-time processes. Pseudo-polynomial method of solving is constructed.

The results obtained for different types of set-valued control will be exposed in a future paper.

References

[1] S.A. Ashmanov, A.V. Timohov. The optimization theory in problems and exercises, Moscow, Nauka, 1991, pp. 142--143.

[2] L.S. Pontryagin, V.G. Boltyanskii, R.V. Gamkrelidze, E.F. Mishchenko. Mathematical theory of optimal processes, Moscow, Nauka, 1961 (in Russian).

[3] R. Bellman. Dynamic Programming, Princeton, New Jersey, Princeton University Press, 1957.

[4] T. Rockafellar. Convex Analysis, Princeton University Press, 1970.

[5] J. von Neuman, O. Morgenstern. Theory of Games and Economic Behavior, Annals Princeton University Press, Princeton, NJ, 1944, 2nd ed. 1947.

[6] J.F. Nash. Noncooperative game, Annals of Mathematics, Vol. 54, 1951, pp. 280--295.

[7] H. Von Stackelberg. Marktform und Gleichgewicht (Market Structure and Equilibrium), Springer Verlag, Vienna, 1934.

[8] G. Leitmann. On Generalized Stackelberg Strategies, Journal of Optimization Theory and Applications, Vol. 26, 1978, pp.637--648.

[9] V. Ungureanu. Solution principles for simultaneous and sequential games mixture, ROMAI Journal, Vol. 4, No.1, 2008, pp. 225--242.

NM-modified Generalized Raiffa Solution and its Application

Radim Valencik[1], Ondrej Cernik[2] and Petr Wawrosz[3]

[1,2,3]University of Finance and Administration, Czech Republic
[1]valencik@seznam.cz
[2]ondrej.cernik@mt-tech.cz
[3]petr.wawrosz@centrum.cz

Keywords: *Cooperative Games, Raiffa's Single Point Solution, N-M Modified Generalized Raiffa Solution, Tragedy of Commons*

The paper shows one of the possible modifications of Raiffa's single point solution (Mihola, Valencik, Vlach 2012), which has applications in the analysis of social networks associated with investing in social position and creating the structures based on mutual covering of violations of the generally accepted principles. These structures are formed on the basi of games of Tragedy of commons type when one player detects breaking the rules by another player. Hence the first player begins bribing the other player and simultaneously covering his back, one player is prejudiced in favour of another player. This gives a rise to social networks that significantly affect the formation of coalitions in various areas of the social system, including institutions whose mission is to protect society against violations of the generally accepted principles. Models follow the N-M modified generalized Raiffa solution to analyze the causes of redistence of corruption and with it the related social phenomena subsequently utilized to formulate measures for the restriction of corruption.

Raiffa's single point solution was proposed in the early 1950's. Raiffa (1953) suggested dynamic procedures for the cooperative bargaining in which the set S of possible alternatives is kept unchanged while the disagreement point gradually changes. Recently Diskin et al. (2011) have provided an axiomatization of a family of generalized Raiffa's discrete solutions. To make easier the presentation of modifications of Raiffa's discrete solution that are based on a different game theoretic model of systems, we first describe some details of solution concepts proposed in Diskin et al.

Diskin et al. deal with the bargaining problem B which is formed from instances (S,d) where each S is a nonempty, closed, convex, comprehensive, and positively bounded subset of R_n whose boundary points are Pareto optimal. They propose a solution concept which is composed of two solution functions. One solution function specifies an interim agreement and the other specifies the terminal agreement. Such a step-by-step solution concept can formally be defined as follows.

A pair (f,g) of functions from B into R_n is called the stepwise solution if both $f(S,d)$ and $g(S,d)$ belong to S for each instance (S,d) of B. Here, the first component specifies the interim agreement and the second component specifies the terminal agreement. The set of generalized Raiffa solutions is a family of stepwise bargaining solutions, where $f(S,d)$ is derived from $U(S,d)$. $U(S,d)$) is so called ideal or utopia point (Kalai, Smorodinsky 1975). Diskin et al. prove that a stepwise bargaining solution (f,g) satisfies sedmi Axioms and only if it is a generalized Raiffa solution.

Now we move on to the proposed generalized Raiffa solution for $n=3$ S is specified with the function $S(x,y,z)=0$ and contains all points on the surface and in the space below this surface defined by the function. We assume that this set is also a nonempty, closed, convex, comprehensive, and positively bounded subset of R_n whose boundary points are Pareto optimal. The disagreement point is set also $d_0(d_{01},d_{02},d_{03})$. Let S be defined as function $S(x,y,z)=0$ containing all points specified by the function on the surface and below the surface. We assume the set is also a nonempty, closed, convex, comprehensive, and positively bounded subset of Rn whose boundary points are Pareto optimal. The disagreement point $d_0(d_{01},d_{02},d_{03})$ is also given.

Let the solution of the system of equations

$$S(x,y,d_{01})=0$$

$$S(x,d_{02},z)=0$$

$$S(d_{03},y,z)=0$$

be $x=d_{11}, y=d_{12}, z=d_{13}$, which corresponds to point d_1.

Points with coordinates

$$(d_{11},d_{12},0)$$

$$(d_{11},0,d_{13})$$

$$(0,d_{12},d_{13})$$

(unlike in the original generalized Raiffa solution) $f(S,d)=d+2/3(NM(S,d)-d)$. If we keep in force any other definition of generalized Raiffa solution, we call this solution NM – modified generalized Raiffa solution for $n=3$. The solution in some way connects two cases:

1. In the first case the players (each of them) decide to create only a two-person fully discriminated coalition, i.e. two players who form a coalition, can give to the third player the smallest possible payoff. In our case this payoff equals d_{01}, d_{02}, d_{03}

2. In the second case the players form a grand coalition i.e. three-player coalition.

The connection between both the cases can be interpreted as follows: Payoffs of each player in the formation of fully discriminated coalitions can be seen from his perspective as an opportunity cost in relation to the possibility of creating a grand coalition. If players create a grand coalition, for obvious reasons they will require payoff higher or at least equal to the one they would have required in a two-person coalition. The problem is how to evaluate player's payoffs for the creation of fully discriminated two-person coalitions. Here we use (introduced by us) the term average expected payoff, which is a multiple of its payoff in a situation where the player is the member of the winning coalition, and the probability of this coalition i.e. $\frac{2}{3}d_{11}, \frac{2}{3}d_{12}, \frac{2}{3}d_{13}$.

"Bridge" by which we're connecting both the cases (formation of two-player coalitions and the grand coalitions above), ie, application of the principle of opportunity cost and the introduction of the concept of expected average payoff, implicitly contains input "step-by-step" process, which results in a single point solution in the case of grand coalition.

The following applies:

1. In case of two-person game the generalized Raiffa solution at $p=1/2$ coincides with the analogous NM-modified generalized Raiffa solution

2. If the sum of the payments of players is constant and we do not consider the difference between the payoff and the utility from the payoff, introduced NM-modified generalized Raiffa solution makes no sense because no Pareto improvement over the formation of a fully discriminated two-person coalitions exists.

One of the applications of our approach is the expression of external factors associated with the affinity between the players. By affinity of one player to another, we mean the utility which one player gains from formation of coalition with the second player, and this utility could be expressed in values in which we express payoffs in the original game. If there is utility for both players, we can talk about mutual affinity, but that may vary in size for each of the players. Positive affinity can also be called sympathy of one player to another, negative affinity is like antipathy of one player to another. The total payoff x_{ij}^* of a player in a coalition with a player with whom he has affinity is then equal to his payoff in the base game plus his payoff which corresponds with the utility of formation of the coalition s_{ij}:

$$x_{ij}^* = x_i + s_{ij}$$

The sign may be positive (positive affinity, i.e. sympathy) or negative (negative affinity, thus antipathy).

It would seem that if the difference between the two players is one-sided or there is mutual affinity and if between any of the players and the third player no affinity exists, then it is determined that there will be a coalition between the two players. However, that need not be if the third player lowers his payment and so compensates the positive affinity. Let's assume that all players are fully informed about the affinities among the players. Then the original system of equations is modified to the following system of eqations:

$$S(x_{12}^*, x_{21}^*, 0) = s_{12} + s_{21}$$

$$S(x_{13}^*, 0, x_{31}^*) = s_{13} + s_{31}$$

$$S(0, x_{23}^*, x_{32}^*) = s_{23} + s_{32}$$

The right side of the equation is interpreted in such a way that the game incurs additional payoffs based on the respective affinities. For payoffs in the original game, we get:

$$x_1 = \frac{1}{2}(x_{12}^* + s_{12} + x_{13}^* + s_{13}), etc.$$

From this result arise the following conclusions:

1. If one of the players (e.g. the first) has a positive affinity for the other player (such as the second player), then in the original game:

- Payoff of the first player is reduced in proportion to the size of this affinity.

- Payoff of the second player will increase in proportion to the size of this affinity.

- Payoff of the third player is reduced in proportion to the size of this affinity.

(The most, i.e. in the original game, gets the player who is likable for the other player, without feeling sympathy for that player or other player.)

2. If a player is informed about a positive affinity between the other two players, he may decrease the demand for his payoff in the original game, this affinity compensates and restores the situation in which discrete NM stable set in the extended game exists, which we considered in the interpretation of the formation of each of the two-person coalitions with equall probability. If the player is not aware of this affinity, a coalition, which he is not a member of is created.

References

[1] Diskin, A., Koppel, M., Samet, D.: Generalized Rai_a solutions, Games and Economic Behavior (2011), doi: 10.1016/j.geb.2011.04.002 Available online 14 April 2011, ISSN 0899-8256, DOI: 10.1016/j.geb.2011.04.002

[2] Kalai, E., Smorodinsky, M.: Other Solutions to Nash's Bargaining Problem, Econometrica, 43 (1975): 513-518

[3] Mihola, J., Valencik, R., Vlach, M. 2012: Game Theoretic Models of Distribution Systems

[4] Raiffa, H.: Arbitration Schemes for Generalized Two Person Games, Contributions to the Theory of Games II, H.W. Kuhn and A.W. Tucker eds., Princeton University Press, Princeton, New Jersey, 1953

A fuzzy-core Extension of Scarf Theorem

Valery Vasil'ev

Sobolev Institute of Mathematics,
Novosibirsk, Russia
vasilev@math.nsc.ru

Keywords: *Fuzzy Coalition, F-Balanced Collection, F-Balanced Game, Core*

The goal of the paper is to present a generalization of the famous Scarf theorem on the core of cooperative game [3] to the case of fuzzy domination. The approach proposed is heavily relies on the concept of balanced collection of fuzzy coalitions, introduced by the author [5]. This generalization of the well-known notion of balanced collection of standard coalitions makes It possible to introduce a natural analog of balancedness for so-called generalized cooperative games (fuzzy games without side payments, in terms of [1]). It turned out that similar to the standard games situation the new balancedness-like condition plays a crucial role in providing the core of the generalized game to be nonempty. Moreover, one of the main results of the paper demonstrates that the generalized balancedness assumption is a necessary and sufficient condition for the nonemptiness of the core of fuzzy game with side payments (an extension of the classic Bondareva-Shapley theorem on the core of a standard TU cooperative game [2,4]).

Some applications of the results obtained to the fuzzy-core allocation existence problem in the framework of general equilibrium theory are also considered.

1. Notations and definitions

Let $N = \{1,\dots,n\}$ be a set of players. Put $I^N = \{\tau \in \mathbf{R}^N \mid 0 \le \tau_i \le 1, i \in N\}$, and denote by σ_F the set $I^N \setminus \{0\}$ of *fuzzy coalitions* over N [1]. As usual, each component τ_i of $\tau = (\tau_1,\dots,\tau_n) \in \sigma_F$ is treated as the level of participation of player i in the fuzzy coalition τ. Remind [1], that any standard coalition $S \subseteq N$ is identified with the corresponding indicator function $e_S \in \mathbf{R}^N$, defined by the formula $(e_S)_i = 1$ if $i \in S$, and $(e_S)_i = 0$ if $i \notin S$. Further, for each $\tau = (\tau_1,\dots,\tau_n) \in \sigma_F$ denote by $N(\tau)$ the support of fuzzy coalition $\tau : N(\tau) = \{i \in N \mid \tau_i > 0\}$. Finally, in the notations given above, the definition of the games under consideration looks as follows: *a generalized cooperative game* (fuzzy game without side payments, according to [1]) is a set-valued map $\tau \mapsto G(\tau)$ that associates any fuzzy coalition $\tau \in \sigma_F$ with a subset $G(\tau)$ belonging to $\mathbf{R}^{N(\tau)}$. Each vector $(x_i)_{i \in N(\tau)} \in G(\tau)$ is called *an imputation of coalition* τ. Remind [5], that G is said to be *a regular game*, if the sets $G(e_{\{i\}})$ of imputations of singleton coalitions $e_{\{i\}}, i \in N,$ and the set of imputations $G(e_N)$ of "grand coalition" e_N are nonempty and closed. Following standard game-theoretic terminology, we say that generalized game G is *comprehensive from below*, if for any $\tau \in \sigma_F$ it holds: $x \in G(\tau)$ and $y \le x$ implies $y \in G(\tau)$.

To complete the section, we introduce one more characteristic of the generalized game, which makes it possible to propose the proper analog of standard balanced game. To this end we extend first the notion of balanced family to the case of fuzzy coalitions: a finite subset $\{\tau_k\}_{k \in K} \subseteq \sigma_F$ is an *F-balanced collection*, if $\sum_{k \in K} \lambda_k \tau_k = e_N$ for some balancing weights $\lambda_k \ge 0, k \in K$. A generalized cooperative game G is said to be *F-balanced game*, if any vector $x = (x_1, \ldots, x_n)$ belongs to $G(e_N)$ whenever its restrictions $x_{N(\tau_k)}$ belong to the corresponding sets $G(\tau_k), k \in K,$ for some F-balanced collection $\{\tau_k\}_{k \in K}$.

2. Main result

We say that $\tau \in \sigma_F$ can improve upon an imputation $x = (x_1, \ldots, x_n) \in G(e_N)$, if there exists an imputation $y \in G(\tau)$ such that $y_i > x_i$ for any $i \in N(\tau)$. One of the fundamental concepts of the paper is the following definition of the *core $C(G)$ of generalized game G*: the core of G is the set of imputations $x \in G(e_N)$ that no coalition $\tau \in \sigma_F$ can improve upon.

For any game G put $x^G = (x_1^G, \ldots, x_n^G)$ with $x_i^G = \sup\{x_i \in \mathbf{R} \mid x_i \in G(e_{\{i\}})\}, i \in N,$ and denote by $\widehat{G}(e_N)$ the set of individually rational imputations of "grand coalition" of the game $G : \widehat{G}(e_N) := \{x \in G(e_N) \mid x \ge x^G\}$.

The main result of the paper is the following extension of the famous Scarf theorem on the core [3] to the case of generalized games.

Theorem 9 *For any regular, comprehensive from below, and F-balanced generalized cooperative game G with $\widehat{G}(e_N)$ to be bounded from above, the core $C(G)$ is nonempty.*

3. A fuzzy-core extension of Bondareva-Shapley theorem

As a corollary of theorem 1 we propose an extension of the well-known Bondareva-Shapley criterion for nonemptiness of the core [2,4] to the case of generalized cooperative games with side payments (theorem 2, below). By our definition (cf. [1]), a generalized game G is a game with side payments, if there exists an extended characteristic function $v:\sigma_F \to \mathbf{R}$ such that $G = G_v$ with $G_v(\tau) = \{x \in \mathbf{R}^{N(\tau)} \mid \tau_{N(\tau)} \cdot x \le v(\tau)\}$ for any $\tau \in \sigma_F$ ($x \cdot y$ means inner product of x and y). We say that an extended characteristic function v is V*-balanced*, if for each F-balanced collection T, and each system $\{\lambda_\tau\}_T$ of balancing weights, it holds: $\sum_{\tau \in T} \lambda_\tau v(\tau) \le v(e_N)$. Note, that for generalized TU games the following analog of classic original holds: generalized cooperative game G_v is F-balanced iff v is V-balanced. By applying this analog and theorem 1 we get the following criterion.

Theorem 10 *The core $C(G_v)$ is nonempty iff v is V-balanced function.*

To conclude, let us mention a simple and useful extension of the classic description of the core of TU game, which ensures a direct proof of theorem 2.

Proposition 11 *The core $C(G_v)$ of generalized cooperative game with side payments, generated by characteristic function $v:\sigma_F \to \mathbf{R}$, has the following representation*

$$C(G_v) = \{x \in \mathbf{R}^N \mid x \cdot e_N = v(e_N), x \cdot \tau \ge v(\tau), \quad \tau \in \sigma_F\}.$$

References

[1] J.-P. Aubin, Optima and Equilibria, Springer-Verlag: Berlin-Heidelber g (1993).

[2] O.N. Bondareva, "Core theory for n-person game", Vestnik Leningrad. Univer., ser. matem., 17, 141–142 (1962).

[3] H.E. Scarf, "The core of an n-person game", Econometrica, 35, 50–69 (1967).

[4] L.S. Shapley, "On balanced sets and cores", Naval Research Logistics Quarterly, 14, 453–460 (1967).

[5] V.A.Vasil'ev, "On an extension of Scarf theorem on the nonemptiness of the core", Discussion Paper, Sobolev Institute of Mathematics, Siberian Branch of Russian Academy of Sciences, Novosibirsk, 283, 41 p. (2012)

Adaptive Dynamics in the Supply Function Auction for Oligopoly with Fixed Marginal Cost and Capacity Constraint

Alexander Vasin[1] **and Marina Dolmatova**[2]

[1,2]*Lomonosov Moscow State University, Moscow, Russia*

Keywords: *Auctions, Supply Function Equilibrium, Best Response Dynamics, Game Theory*

The basic mechanisms of electricity markets - day-ahead and real-time markets - perform as uniform price auctions where a producer's bid is a monotonous function that determines the supplied quantity depending on the price. One popular concept for modeling the uniform price auction in electricity markets is the supply function equilibrium (SFE) introduced by Klemperer and Meyer, 1989, and developed in many papers (see Anderson and Philpott, 2002, Baldick et al., 2000, Green and Newbery, 1992, Holmberg et al., 2008, Newbery, 2008). The original model permits a bid of a producer to be a monotone smooth function of the price and demand function to depend on an uncertain parameter besides the price. For each parameter value, the market clearing price is determined from the balance of the aggregate supply function and a current demand function. A bid profile is called a supply function equilibrium (SFE) if, for any demand realization, the bid of each firm maximizes its profit under fixed bids of other producers. The solution requires rather full information on the demand function and cost functions of all competitors. We study whether the actual behavior at the auction corresponds to this concept in the framework of adaptive and learning mechanisms' investigation.

We study existence and dynamics of *strong best responses*: a bid of a producer is a strong best response (SBR) to a bid profile of other participants if it maximizes his profit under any value of the uncertain parameter. We determine the set of SFE depending on the maximum demand value and study the SBRD for the symmetric duopoly with constrained capacity and generalize the results for the symmetric oligopoly.

We consider the symmetric duopoly with the cost function $C(q) = cq,\ c > 0$, constrained capacity $q \le Q$, and demand function $D(p,t) = \overline{D}(t) - dp$, where $d > 0$ and $\overline{D}(t)$ is a maximal demand value depending on random parameter t. SFE bid for this case is a continuous monotone function that meets equation : $S'(p) = \frac{S(p)}{(p-c)} - d$ until $S(p)$ equals Q for some p or reaches its maximum in p, then staying constant. General solution of the equation $S(p,A) = (p-c)(A - d\ln(p-c))$ depends on the integration constant A (see Newbery (2008)). This function reaches maximum value $q(A)$ under $p = p(A) \overset{def}{=} c + e^{\frac{A}{d}-1}$ at the point of intersection of its graph and the Cournot supply schedule $d(p-c)$. The inverse function is $A(q) = d(ln(q) - ln(d) + 1)$. Denote $D^* = \sup_t \overline{D}(t) - dc$.

Theorem 1. *If $D^* \ge 3Q$ then there exists a unique SFE in the model. The equilibrium bid is* $S^*(p) = \begin{cases} S(p, A(Q)) \ \textit{for}\ c \le p \le p(A) \\ Q,\ \textit{for}\ p \ge p(A) \end{cases}$

If $Q < D^* < 3Q$, then for any $A \in (A(D^*/3), A(Q))$, bid $\overline{S}(p,A) = \begin{cases} S(p,A), p \le p(A) \\ S(p(A),A), p \ge p(A) \end{cases}$ *determines SFE, and for any $A \in (A(Q), \overline{A}(D^*))$, where $\overline{A}(D^*)$ meets the equation $S(\tilde{p}(A,D^*))(\tilde{p}(A,D^*)) = \frac{(D^*-Q)^2}{4d}$, bid $\overline{\overline{S}}(p,A) = \min\{\overline{S}(p,A), Q)$ also determines SFE.*

If $D^* < Q$, then the capacity constraint is not binding for feasible bids. In this case $\forall A > A(D^*/3)$ the bid $S(p,A)$ determines SFE. For $A \to \infty$ SFE tends to Walrasian equilibrium.

For linear demand $D(p) = max\{0, \overline{D} - dp\}$ under fixed $\overline{D}$ and $c = 0$, we study the best response dynamics depending on the ratio between $\overline{D}$ and Q. At every stage $\tau = 1, 2, \ldots$, we search for the BR in the set of the supply functions $S(p,\tau) = min(k(\tau)p, Q)$. So the problem is to find the optimal slope $k(\tau)$.

Proposition 1: *The best response dynamics depends on the parameter values as follows:*

For $\overline{D} \geq 3Q$, $S(p,\tau) = min\{Q, d\tau p\}, \forall \tau$. *For any* τ *the SFE corresponds to the Cournot equilibrium. (Since the demand is high enough, the outcome coincides with the Walrasian equilibrium that is equal to the Cournot equilibrium in this case).*

For $Q < \overline{D} < 3Q$, $S(p,\tau) = min\{Q, d\tau p\}$ *at step* $\tau = 1,\ldots,T(\overline{D},Q)$, *then the best response functions repeat in cycle. The length* $T(\overline{D},Q)$ *of the cycle is the minimum integer number satisfying inequality:* $T > \left(\frac{\overline{D}}{\overline{D}-Q}\right)^2$, *for* $Q < \overline{D} \leq 2Q$, *and* $T > \frac{4Q}{\overline{D}-Q}$, *for* $2Q \leq \overline{D} < 3Q$.

For $\overline{D} \leq Q$, $S(p,\tau) = min\{Q, d\tau p\}, \forall \tau$. *For* $\tau \to \infty$ *the best response dynamics converges to the SFE with the outcome corresponding to the Walrasian supply function.*

The generalization of the results for the symmetric oligopoly with capapcity constraint.

Proposition 2: *The BRD for the model of n-firm oligopoly with constant fixed cost and capacity constraint depends on the ratio between parameters* $\overline{D}$ *and* Q *as follows:*

For $\overline{D} \leq (n-1)Q$, $S(p,\tau) = min\{Q, k(\tau)p\}, \forall \tau \geq 1$, *where* $k(\tau) = d\sum_{s=0}^{\tau-1}(n-1)^s$. *For* $\tau \to \infty$ *the BRD converges to the Walrasian supply function;*

For $(n-1)Q < \overline{D} < (n+1)Q$, *at the step* $\tau = 1,\ldots,T(\overline{D},Q)$ *the BR is* $S(p,\tau) = min\{Q, k(\tau)p\}$, *then the BR functions repeat in cycle. The length* $T(\overline{D},Q)$ *of the cycle is the minimum integer number satisfying inequality* $\sum_{s=0}^{T}(n-1)^s > \frac{4Q(\overline{D}-Q)}{(\overline{D}-(n-1)Q)^2}$;

For $(n+1)Q \leq \overline{D}$, $S(p,\tau) = min\{Q, kdp\}, \forall k \in [1,\infty) \forall \tau \geq 1$. *The BR is rather uncertain, but the outcome always corresponds to the Cournot equilibrium coinciding with the Walrasian equilibrium in this case.*

Our study together with the previous researches shows ambiguous results on the justification for the SFE concept. For the symmetric oligopoly with limited total capacity nQ and maximum demand $\overline{D}$, the SBRD exists and converges to the Walrasian equilibrium bid if $\overline{D} \leq (n-1)Q$ or $\overline{D} \geq (n+1)Q$. However, for each specified interval in the intermediate area $(n-1)Q < D < (n+1)Q$, the SBRD turns out to be cyclic with its special cycle. Under general assumptions on the demand shock distribution, the SBRD does not exist and a simple best response dynamics does not converge. Even for duopoly we do not observe the convergence to the unique SFE specified in Theorem 1 for $D^* > 3Q$. Our study shows that there is no ground to expect such kind of behavior in general: for every particular market structure, convergence of adaptive dynamics requires a special investigation.

Axiomatization of Two Dual Values with Associated Consistency

Wenna Wang[1], Genjiu Xu[2] and Hao Sun[3]

[1,2,3]Department of Applied Mathematics, Northwestern Polytechnical University Xi'an, Shaanxi 710072, P.R. China
[1]wenna_wang@yahoo.cn
[2]xugenjiu@nwpu.edu.cn
[3]hsun@nwpu.edu.cn

Keywords: *Associated Consistency, Axiomatization, the Equal Allocation of Nonseparable Cost Value, the Center of Gravity of Imputation Set Value*

This paper aims to introduce a new model of associated games, and then to characterize EANS value, as well as CIS value in the framework of Hamiache (2001), by means of associated consistency, continuity and the inessential game property.

The Bounded Core for Games with Restricted Cooperation

Elena Yanovskaya

St.Petersburg Institute for Economics and Mathematics
Russian Academy of Sciences
Russia
eyanov@emi.nw.ru

Keywords: *Restricted Cooperation, Feasible Coalition, Bounded Core*

The class of TU cooperative games G^r with with arbitrary collections of feasible coalitions is considered. The non-emptiness of the core for every game from this class depends both on the characteristic function and the collection of feasible coalitions. If the core is not empty, then it may be unbounded. Following Grabish and Sudholter (2012) the bounded core for the class G^r if defined as the union of all bounded faces of the core.

Two axiomatizations of the bounded core for this class of games are given: the first for the whole class $G\}^r$ (without non-emptiness axiom), and the second for the subclass $G\}^r$ of games with non-empty bounded cores (with non-emptiness axiom).

A Cooperative Stochastic Dynamic Game of Public Goods Provision

David Yeung[1] **and Leon Petrosyan**[2]

[1]*Center of Game Theory, Saint Petersburg State University; and SRS Consortium for Advanced Study in Dynamic Games, Hong Kong Shue Yan University*
dwkyeung@hksyu.edu

[2]*Faculty of Applied Mathematics-Control Processes, Saint Petersburg State University*
spbuoasis7@peterlink.ru

Keywords: *Public Goods, Stochastic Dynamic Games, Dynamic Cooperation, Subgame, Consistency*

The provision of public goods constitutes a classic case of market failure which calls for cooperative optimization. However, cooperation cannot be sustainable unless there is guarantee that the agreed-upon optimality principle can be maintained throughout the planning duration. This paper derives subgame consistent cooperative solutions for public goods provision by asymmetric agents with transferable payoffs in a stochastic dynamic game framework. This is the first time that discrete-time dynamic cooperative game in public goods provision is analyzed.

1 Introduction

Public goods, which are non-rival and non-excludable in consumption, are not uncommon in today’s economy. Examples of public goods include clean environment, national security, scientific knowledge, accessible public capital, technical know-how and public information. The non-exclusiveness and positive externalities of public goods constitutes major factors for market failure in their provision. In many contexts, the provision and use of public goods are carried out in an intertemporal framework. Cooperation suggests the possibility of socially optimal solutions in public goods provision problem. A discrete-time game framework is developed for theoretical analysis and practical applications.

2 Analytical Framework

Consider the case of the provision of a public good in which a group of n agents carry out a project by making continuous contributions of some inputs or investments to build up a productive stock of a public good. The game horizon consists of T stages. We use K_t denote the level of the productive stock and I_t^i denote the contribution to the public capital or investment by agent i at stage $t \in \{1, 2, ..., T\}$. The stock accumulation dynamics is then

$$K_{t+1} = \sum_{j=1}^{n} I_t^j - \delta K_t + \upsilon_t,\ K_1 = K^0 \text{, for } t \in \{1, 2, \ldots, T\} \tag{1}$$

where υ_t is a sequence of statistically independent random variables.

The payoff of player i at stage t is

$$R^i(K_t) - C^i(I_t^i),\ \text{i} \in \{1, 2, \ldots, n\} = N \tag{2}$$

where $R^i(K_t)$ the revenue/payoff to agent i, $C^i(I_t^i)$ is the cost of investing $I_t^i \in X^i$.

The objective of agent $i \in N$ is to maximize its expected net revenue over the planning horizon, that is

$$E_{\upsilon_1, \upsilon_2, \ldots, \upsilon_T} \left\{ \sum_{s=1}^{T} \left[R^i(K_s) - C^i(I_s^i) \right] (1+r)^{-(s-1)} + q^i(K_{T+1})(1+r)^{-T} \right\} \tag{3}$$

subject to the stock accumulation dynamics (1).

Non-cooperative solution to game (1)-(3) is characterized to reflect the market outcome.

3 Subgame Consistent Cooperative Scheme

It is well-known problem that noncooperative provision of goods with externalities, in general, would lead to dynamic inefficiency. Cooperative games suggest the possibility of socially optimal and group efficient solutions to decision problems involving strategic action. Now consider the case when the agents agree to cooperate and extract gains from cooperation. In particular, they act cooperatively and agree to distribute the joint payoff among themselves according to an optimality principle. If any agent disagrees and deviates from the cooperation scheme, all agents will revert to the noncooperative framework to counteract the free-rider problem in public goods provision. In particular, free-riding would lead to a lower future payoff due to the loss of cooperative gains. Thus a credible threat is in place.

To fulfil group optimality the agents would seek to maximize their expected joint payoff. Let $\psi_s^*(K) = \{\psi_s^{1*}(K), \psi_s^{2*}(K), \ldots, \psi_s^{n*}(K)\}$, for $s \in \{1, 2, \ldots, T\}$, denote a set of strategies that brings about an optimal cooperative solution. The optimal cooperative path can be derived as:

$$K_{t+1} = \left[\sum_{j=1}^{n} \psi_t^{j*}(K) - \delta K_t \right] + \upsilon_t, \ K_1 = K^0, \text{ for } t \in \{1, 2, \ldots, T\}. \tag{4}$$

We use X_s^* to denote the set of realizable values of K_s generated by (4) at stage s and use $K_s^* \in X_s^*$ to denote an element in the optimal set.

To enable a cooperation scheme to be sustainable throughout the agreement period, a stringent condition is needed – that of *subgame consistency*. This condition requires that the optimality principle agreed upon at the outset must remain effective in any subgame starting at a later starting time with a realizable state brought about by prior optimal behaviour. Hence the players do not possess incentives to deviate from the cooperative scheme throughout the cooperative duration.

Let $\xi(\cdot,\cdot)$ denote the agreed-upon imputation vector guiding the distribution of the total cooperative payoff under the agreed-upon optimality principle along the cooperative trajectory $\{K_s^*\}_{s=1}^{T+1}$. At stage s and if the productive stock is K_s^*, the imputation vector according to

$$\xi(s, K_s^*) = \left[\xi^1(s, K_s^*), \xi^2(s, K_s^*), \ldots, \xi^n(s, K_s^*) \right], \text{ for } s \in [0, T] \tag{5}$$

Subgame consistency requires the derivation a "payment-distribution-procedure" $B^i(s, K_s^*)$ for $i \in N$ and $s \in \{1, 2, \ldots, T\}$, which leads to the realization of imputation (5) and hence a subgame consistent cooperative solution can be obtained.

The paper would also consider the infinite horizon situation and provide applications with explicit function forms.

Navigation Strategies in Transportation Network

Victor Zakharov[1] and Alexander Krylatov[2]

[1,2]Saint-Petersburg State University
Russia
[1]mcvictor@mail.ru
[2]partizan-sasha@yandex.ru

Keywords: *Traffic Flow Allocation, Routing, Transport Management, Nash-Equilibrium*

The study of navigator's strategic behavior in traffic flow assignment is important in context of traffic jams. In a megalopolis, many drivers can use routs provided by different navigation systems (navigators) to reach their destinations with minimum driving time. Thereby, navigators are interested to provide their customers with higher level of service compare to other competitors.

In the present work the game of two navigation providers is considered. Each provider tries to assign the traffic flow of its customers from origin to destination through n paths so as to maximize the appropriate pay-offs. The strategy of Navigator j is $f^j = (f_1^j, ..., f_n^j)$ with $f_i^j \geq 0$

$$\sum_{i=1}^{n} f_i^j = F^j \tag{1}$$

where $F^j > 0$ for $j \in \{1,2\}$. For estimation of the pay-offs of Navigator j, the following utility function is considered that quantifies a trade-off between throughput and delay

$$v_j(f^1, f^2) = \frac{\left(\sum_{i=1}^{n} f_i^j\right)^{\beta}}{d_j(f^1, f^2)} \tag{2}$$

where $d_j(f^1, f^2)$ is the function that characterizes the average delay experienced by the Navigator j and β is a trade-off parameter. Such a utility function is commonly used in the literature in applications that are sensitive to throughput as well as delay. It consists of the ratio between the expected throughput and the expected delay. Thus it captures preferences towards higher throughputs and penalizes large delays.

The strategies (f^{1*}, f^{2*}) constitute a Nash equilibrium, if for any strategies (f^1, f^2) the following inequalities hold:

$$v_1(f^1, f^{2*}) \le v_1(f^{1*}, f^{2*}) \tag{3}$$

$$v_2(f^{1*}, f^2) \le v_2(f^{1*}, f^{2*}) \tag{4}$$

It is expected that delay function is not unique for all n paths of network but different for each of them. That means existence of vector $d^j = \left(d_1^j, ..., d_n^j\right)$ $d^j = \left(d_1^j, \ldots, d_n^j\right)$ where d_i^j characterize delay on i -th path for Navigator j. Another assumption is about the explicit form of delay function

$$d_i^j = \frac{f_i^m + h_i}{g_i}, \forall i = \overline{1,n} \tag{5}$$

where $j,m \in \{1,2\}$ when $j \neq m$ $j \neq m$., $h_i \ge 0$ characterizes the flow of drivers who are not customers of either Navigator 1 or Navigator 2 and $g_i > 0$ characterizes capacity of i -th path (larger g_i corresponds to larger capacity of i -th path).

Moreover, the following assumption about linear nature of pay-off holds

$$v_1(f^1, f^2) = \sum_{i=1}^{n} \frac{1}{d_i^1} f_i^1 \tag{6}$$

$$v_2(f^1, f^2) = \sum_{i=1}^{n} \frac{1}{d_i^2} f_i^2 \tag{7}$$

In case (6) and (7) are concave in $f^1$and f^2 respectively, it will be possible to exploit Kuhn – Tucker conditions for seeking equilibrium strategies (f^{1*}, f^{2*}). So there appeared intermediate result that state

(f^{1*}, f^{2*}) is a Nash equilibrium if and only if there are non-negative ω^1 and ω^2 (Lagrange multipliers) such that

$$\frac{g_i}{f_i^m + h_i} \begin{cases} = \omega^j \text{ for } f_i^{j*} > 0 \\ \le \omega^j \text{ for } f_i^{j*} = 0 \end{cases} \tag{8}$$

where $i = \overline{1,n}$, $j,m \in \{1,2\}$ and $j \neq m$.

Based on such an expression of equilibrium strategies the main result of this work (an explicit form of equilibrium strategies of navigation providers) can be formulated in the following form:

Nash equilibrium in a game of two Navigators with pay-offs (6) and (7) is achieved by the following strategies using

$$f_i^{1*} = \begin{cases} \dfrac{g_i F^1 + g_i \sum_{r=1}^{k_1} h_r}{\sum_{r=1}^{k_1} g_r} - h_i, \text{ if } i \le k_1 \\ 0\ , \quad if\ i > k_1 \end{cases} \tag{9}$$

$$f_i^{2*} = \begin{cases} \dfrac{g_i F^2 + g_i \sum_{r=1}^{k_2} h_r}{\sum_{r=1}^{k_2} g_r} - h_i, \text{ if } i \le k_2 \\ 0\ , \quad if\ i > k_2 \end{cases} \tag{10}$$

where k_1 and k_2 can be found from the following conditions:

$$\phi_{k_j} < F^j \le \phi_{k_j+1} \tag{11}$$

where $j \in \{1,2\}$ and

$$\phi_t = \sum_{i=1}^{t} g_i \left(\frac{h_t}{g_t} - \frac{h_i}{g_i} \right) \quad \text{for} \quad t \in \{1, n\} \tag{12}$$

and $k_j = n$ in case $F^j > \phi_n$.

It should be noted that this problem can be considered either in an optimization scenario. Moreover, as many navigation providers will understand behavior of other players, they can increase the level of service quality. We expect that the present approach will be able to generally to the case of more than two users. However, this is a topic for future research.

Motivating Informed Decisions

Andres Zambrano

Universidad de los Andes
azambrano@ucla.edu

Keywords: *Information Acquisition, Private Information, Contract, CEO Compensation*

This paper studies a principal-agent model where a risk-neutral principal delegates to a risk-neutral agent the decision of whether to pursue a risky project or a safe one. The return from the risky project is unknown and the agent can acquire costly unobservable information about it before taking the decision. The problem has features of moral hazard and hidden information since the acquisition of information and its content are unobservable to the principal. The optimal contract suggests that the principal should only reward the agent for outcomes that are significantly better than the safe return. It is also optimal to distort the project choice in favor of the risky one as a mechanism to induce the direct revelation of the uncertain state. In a managerial context, the findings explain why options and profit sharing compensation induce better decision making from CEOs, as well as why excessive risk taking might be optimal.

Strong Equilibria in the Vehicle Routing Game

Andrey Zyatchin

GSOM SPbSU
Russia
zyatchin@gsom.pu.ru

Keywords: *Strong Equilibrium, Cooperative Solution, Transportation Game*

This paper introduces a variant of the vehicle routing problem with several distributors under competition. Each customer is characterized by demand and wholesale price. Under this scenario a solution may contain unserved customers and elementary routes with no customer visits. The problem is described as a vehicle routing game (VRG) with coordinated strategies.

We provide a computable procedure to calculate strong equilibrium in the VRG, which is stable against deviation of any coalition. Following the procedure we solve iteratively optimization subproblems for a single distributor, reducing the set of unserved customers on each iteration.

We proof, that strong equilibrium of two types exists in a VRG and provide conditions for the third one to exist. We also introduce a semi-cooperative strong equilibrium which helps to reduce a set of strong equilibria in the vehicle routing game.

Our methodology is suited for parallel computing, and could be efficiently applied in routing of vehicles with a few compartments. It is calculated a numerical example for three person VRG with six cars and twelve customers.

Index

Game Theory and Management

Editors: Leon A. Petrosyan, Nikolay A. Zenkevich

Abstracts

Authorized: 18.06.2013. Author’s sheets: 13
150 copies
Publishing House of the Graduate School of Management, SPbU
Volkhovsky per. 3, St. Petersburg, 199004, Russia
tel. +7 (812) 323 84 60
www.gsom.spbu.ru