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Referee report on the bachelor thesis of Mr. Dmitry Pavlovich Solovyov entitled "Crystal basis for spin chains in the AdS/CFT correspondence".

The bachelor thesis of Dmitry Solovyov is devoted to developing some representation theory tools for performing analytic computations in gauge and string theories related through the so-called AdS/CFT correspondence. Since its discovery in 1996, the AdS/CFT correspondence continues to play a prominent role in modern theoretical physics as its offers, in particular, an unprecedented possibility to access a strong coupling regime of gauge theories by using the corresponding string models. A remarkable feature discovered in recent years is that some of the gauge and string theories involved in this correspondence belong to a class of integrable dynamical systems. For these theories, this allows one to search for their exact solution by using ideas and methods developed in the theory of integrable systems (quantum groups, finite-gap integration, Bethe Ansatz, etc.). Many important results have been obtained world-wide in this way, and many of them rely on essential use of representation theory. This explains the relevance and promptness of the topic undertaken by Mr. Solovyov for his bachelor research.

The present thesis develops algorithmic tools and their implementation through computer simulations for tensor product decomposition of irreducible representations of Lie groups. In the context of the AdS/CFT correspondence this is needed for the following reasons. First of all, on the gauge theory side, the main theory under study is the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. This is a superconformal field theory in four dimensions. Its dilatation operator is known to admit an interpretation as the Hamiltonian of a certain spin chain. In the one-loop approximation this spin chain is nothing else but a certain generalisation of the famous Heisenberg $\mathrm{XXX}_{\frac{1}{2}}$ spin chain. For instance, restricting to a purely bosonic sector, this generalisation is an integrable SO(6) spin chain, the latter admits an integrable SU(3) subsector – the case relevant for the present thesis. To find the energy spectrum of this spin chain and degeneracies of states, one needs to know the decomposition of the multiple tensor product of irreducible representations of SO(6) (correspondingly SU(3)) into irreducible components. A very similar problem arises on the string side – this time for the (light-cone) Hamiltonian of closed IIB superstrings propagating in the Anti-de Sitter background.

Thus, the problem solved by Mr. Solovyov consists in finding the spectrum of a highly multiple tensor product of fundamental irreps of SU(3). On the string side, high multiplicity of the tensor product corresponds to considering *fastly* rotating strings, which is needed to be able to compare the string energy spectrum with that of the *one-loop* dilatation operator of the gauge theory. An effective strategy for such a tensor product decomposition is to use the techniques based on the crystal basis representation known from the theory of quantum groups. An implementation of an explicit algorithm for doing this is still rather tricky and I was impressed how Dmitry managed this task. The final outcome is the MATLAB program by Dmitry which, in addition to analytic results, produces beautiful pictures of irreducible crystal graphs.

Coming to other merits of the thesis, it contains an introduction in the theory of Lie algebras and quantum groups, necessary to appreciate an idea of the tensor product decomposition based on use of the crystal representation. Further, to motivate his research, Mr. Solovyov gives a fairly extended introduction into the AdS/CFT correspondence and its integrability aspects (those which actually allow us to make quantitative statements about the validity of the correspondence). Finally, a separate discussion is devoted to integrable spin chains and their solution via the algebraic Bethe Ansatz. The thesis is well structured and easy to read.

Concluding my assessment, this is a well-motivated thesis with an interesting result. In my opinion, it meets all the necessary requirements for bachelor theses. Therefore, I suggest to grade the present thesis as **excellent**. If you have any further questions, please do not hesitate to contact me.

Prof. Gleb Arutyunov