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Evaluation of Real Options Portfolio for Investment Projects

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ЗАЯВЛЕНИЕ О САМОСТОЯТЕЛЬНОМ ХАРАКТЕРЕ ВЫПОЛНЕНИЯ ВЫПУСКНОЙ КВАЛИФИКАЦИОННОЙ РАБОТЫ

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АННОТАЦИЯ

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Название магистерской диссертации	Оценка портфеля реальных опционов для инвестиционных проектов
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Описание цели, зада и основных результатов	Целью данной работы является предложение рекомендаций для оценки портфеля реальных опционов в инвестиционных проектах. Задачами исследования являются: Обзор метода реальных опционов; Анализ моделей оценки реальных опционов; Формулирование методологии для создания оценки портфеля реальных опционов; Применение указанной методики на симулированном проекте с использованием вводных данных, близких к практическим кейсам. Результатами работы является четко сформулированный подход к оценке портфеля реальных опционов в инвестиционных проектах, который протестирован на примере компании, занимающейся добычей сланцевой нефти. В итоге были выработаны рекомендации менеджерам, которые занимаются оценкой инвестиционных проектов, по механизму оценки портфеля реальных опционов и их прикладному применению.
Ключевые слова	Реальные опционы, инвестиционная оценка, портфель реальных опционов, сланцевая добыча нефти.

ABSTRACT

Master Student's name	Dzheikhun Nurullaev
Master thesis title	Evaluation of Real Options Portfolio for Investment Projects
Faculty	Graduate School of Management
Main field of study	Corporate Finance
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Academic Advisor's Name	Associate Professor, Anna E. Loukianova
Description of goal, tasks and results	The goal of this paper is to offer some recommendations for evaluating the portfolio of real options in investment projects. The objectives of the study are: Review of real options approach and methodology; Analysis of real options valuation models; Formulate methodology for creating and valuation real options portfolio; Application proposed methodology on simulated case study. The result of the paper is a clearly formulated approach to the evaluation of the real options portfolio in investment projects, which was tested using the example of a company that produces shale oil. As a result, the mechanism for assessing the portfolio of real options and their application were offered to managers who are engaged in the evaluation of investment projects.
Keywords	Real Options, valuation, risk-management, investments, real options portfolio, shale oil production.

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INTRODUCTION

Nowadays, there are many approaches of investments valuation. Traditional ones were used for long of time, but they have several drawbacks that do not allow managers to consider all possible risks within an investment and manage them. To response for needs of management about making investment projects valuation more flexible, new real options theory was created as an extension to financial options theory.

Since appearance first researches about real options theory many scientists and practitioners were encouraged with new approach. The reason for that is many advantages of real options that make decision-making process more flexible and allows to change some aspects of an investment project according to changing environment. Many new papers were written about this topic and many practical cases were considered by many researches.

At the same time, it was realized that financial options theory with minor changes to real investments cannot be applied in practical cases. According to financial options theory, investor asses an option with one underlying assets. The world of real investments is different, and some projects can include several real options simultaneously, another problem is that in the portfolio of investments the correlation of underlying assets is also possible that means that the value cannot be simple sum of options. That brought new problem to real options theory, the approach with one underlying asset and an option on it makes the theory practically inapplicable. Managers in practice could not use this idea to implement for decision-making process about investments and strategies.

The route of problem is that if options and assets were not correlated it would be simple just to add up their values to asses a project. In real life almost, all underlying assets and options are correlated, and additivity method is not applicable. Interdependencies of options and assets could not be ignored, because it influences significantly to a value of projects. As a response to this problem new articles appeared trying to explain the problem and propose a solution to the problem of options interrelation. One of the pioneers in this area was Trigeorgis with his articles devoted to the problem of real options portfolio within the investment projects. Also, other articles appeared trying to make real options theory more applicable to practical cases. At these articles considered some simulated projects that are very close to real life projects and value of portfolio of options that correlate within the project was assessed. They offered some solutions and methodology how to deal with the problem of interdependencies. Moreover, the scientific work of R. R. Brosch (2008) made a lot of contribution to the problem of portfolio of real options valuation. He accumulated main previous researches about problem of real options portfolio also created some methodology for portfolio valuation, and provided some numerical analysis applying proposed methodology. All of these points lead to the conclusion that valuation of real options portfolio is very important topic in real options theory due to practical applicability for real business cases.

In this paper, the real options analysis (ROA) is considered from portfolio point of view. The paper considers all relative articles about this approach and offer some methodology based on previous researches with some practical application based on simulated or real-life cases. The idea of the paper is to give some recommendation to managers that are responsible for investment decisions some tool that would help them to make valuation process more effective and clear. The *goal* of this paper is to propose recommendations of valuation real options portfolio for investment projects. By achieving this goal, we answer the main *research question* of the paper: *What is the value of the portfolio of options if there is interaction between them?* To reach this goal the following *objectives* should be accomplished:

- Review of real options approach and methodology;
- Analysis of real options valuation models;
- Formulate methodology for creating and valuation real options portfolio;
- Application proposed methodology on simulated case study.

The *hypothesis* of the paper is following. If the options interact in the project, the value of a portfolio cannot be the sum of individual values of option, but some synergy effect is obtained, positive or negative.

The Research is organized as follows. The first chapter of the paper is devoted to literature analysis of the topic. There is a review of real options theory, considered differences between traditional approaches and real options method and given main definitions and ideas about real options process. Then portfolio aspect of real options considered. Main articles and scientific works are analyzed about portfolio of real options and their aspects. Moreover, the attention was paid to the problem of interdependencies between options and underlying assets, and approach to the management of real options portfolio was under analysis.

The second chapter is devoted to valuation model of real options portfolio. In the first step real options valuation models were considered, like main approaches and methodology. The next step is extension of simple real options valuation methods to the portfolio of real options that have correlations within one project. And the third step differences between portfolio approach and simple ROA analyzed and emphasized main advantages of portfolio approach.

The last chapter considers several cases in contest of real options portfolio. In each case the real options portfolio creation is explained. Then the project is valuated using the methodology from the second chapter and at the final stage the value using portfolio approach is compared to the value of simple additivity method. In the result the formulated approach to portfolio of real options creating and valuation is proposed.

Finally, main conclusions are made about the nature of options interaction and their influence on the value of portfolio. Synergy effects and their type are stated.

CHAPTER 1. LITERATURE REVIEW

1.1 Real options approach

1.1.1 The concept of real options

The main issue for strategic management is about which choices and directions take to bring the company to a success (Rumelt, Schendel, & Teece, 1994). Companies involved in making strategic decisions concerning about internal investments in technological projects or some corporate activities, but main problem is providing enough resources for future investments and other activities under condition of uncertainty. So, for firms the problem of risk of making decision with uncertain future is the biggest deal of strategic management. There are many researches about making decisions under uncertainty and real option approach provides new way of treatment for such situations.

Traditional views claim that managerial decisions are limited in the face of uncertainty and organizational inertia dominates, real options theory insists that companies can deal with unpredictable future and benefit by applying option way thinking and managing investments under changing conditions (Kogut, 1991; Dixit & Pindyck, 1994).

Recent advances in strategy and finance have suggested that real options theory potentially offers a powerful valuation tool as well as a systematic strategy framework to evaluate and structure resource investments under uncertainty, and that successful use of real options can lead to the benefits of downside risk reduction and upside potential enhancement (Tony W. Tong & Jeffrey J. Reuer, 2015)

Now, it makes sense to give formal definition of real options and clarify the differences between real and financial options.

There are several definitions of real options, but simply defined real options theory is systematic approach that uses different sciences and analysis like economic, management, statistic, decision-making, and econometrics, to apply options theory in valuing real assets, as opposite to financial assets, in uncertain business reality, where the flexibility of decisions are assumed in the context of strategic capital investment decision-making, valuing investment opportunities and project capital expenditures (J. Mun, 2014).

So, as it was mentioned in definition, the real options valuation models are different from financial options' ones. And differences can be summarized in the following table.

Table1. Financial and Real options differences.

Parameters	Financial Options	Real options
Object	Securities	Real assets or project
Subject	Current price of the security	PV of expected cash flows
The premium for option	Exercise price	Investment costs
Time horizon	Time to expiry	Life of project opportunity
The reason of uncertainty	Stock price volatility	Uncertainty of the project

Source: Hull, 2014.

There is some motivation why managers and scientists started to seek for new tool of investments valuation different from DCF and other traditional approaches. The reason is that DCF approach has many limitations that cannot reflect some important realities and this approach ignores opportunity to be flexible in making investment decisions.

Table 2. DCF method limitations.

DCF assumptions	Realities
Decision for whole project are made now, cash flows are fixed for the entire life of the project	Cash flows are uncertain and can vary a lot from stated ones. Not all decisions are made today, managers can defer some decisions for future when some uncertainties will disappear
All projects are passively managed after launching	Projects are actually actively managed during the whole lifetime
Future free cash flows are predictable and deterministic	Future cash flows are volatile and risky and difficult to predict
Project discount rate is used as the opportunity cost of capital, which is proportional to idiosyncratic (non-diversifiable) risk	There are several sources for project risks with different features and some of them can be diversified applying flexible decision-making strategy
All risks are accounted in discount rate	During the life of the project the risks can change significantly
All factors that could affect the outcome of the project are reflected in the DCF model through the NPV and IRR	Project are complex and include some factors that cannot be quantified and so reflected in DCF models, but these factors can be significant and strategically important (e. g. strategic vision and entrepreneurial activity)
Unknown, intangible, or immeasurable factors are valued at zero	Many of the important benefits are intangible assets or qualitative strategic positions.

Source: J. Mun, 2014

The number of disadvantages of DCF methods brought to the investment valuation approach real option theory. This idea brings more flexibility do decision making process and deals with many uncertainties that managers meet during taking a decision about investment expenditures. As a result, many projects that under DCF model were unprofitable become good investment opportunity.

Real options approach is crucial in following situations:

- Under highly uncertain and dynamic business conditions, looking for and identifying corporate investments opportunities, pathways or projects;
- Valuing each of the strategic decisions pathways and what it represents in terms of financial viability and feasibility;
- Giving priorities to identified opportunities based on not only on quantitative, but also on qualitative metrics;
- Optimizing the value of investment decisions by evaluating different paths under certain conditions or determining how using a different sequence of paths can lead to optimal successful strategy
- Timing the effective execution of investments and finding the optimal trigger values and cost or revenue drivers;
- Managing existing or developing new optionalities and strategic decision paths for future opportunities (J. Mun, 2014).

Concluding the concept of real options, we can say that it is the approach that uses benefits from traditional approaches like DCF, but deals with limitations of them adding flexibility and allowing to consider non-quantitative information during strategic investment-decision making process.

1.1.2 Types of Real Options

To clarify the meaning of options and how do they work in real life we need some examples of main types of real options. Actually, types of real options are not restricted by the list provided in this paragraph, but it defines the most popular of them. In real life finding of real option depends only on creativity of manager who is seeking for flexibility opportunities. Real options' types described here will show how the idea if theory works in practice.

So, main examples of options embedded in projects are:

- *Abandonment option*. The option allows to investor in any time liquidate (sell, close) a project. The opportunity to do it at any time makes the changes it to American put option (right to use option up to expiration date). If in the contract the selling date is fixed, so option becomes European (right to use option only at expiration date). The strike price of the option is resale or liquidating value minus closing-down costs. The Abandonment option deal with projects that have high probability of poor outcomes in the future. So, applying this option increases initial value if the project.

- *Expansion option*. This option provides manager opportunity to increase future investments in the project if business conditions are favorable. It is an American call option that allows managers invest (buy) in future more capacity for expanding. The strike price of the call option is the cost of creating this additional capacity discounted to the time of option exercise. The strike price often depends on the initial investments. If manager initially decide to build small capacity and then increase it in future in better business conditions, the strike price can be relatively high.
- *Contraction option*. This option gives the managers opportunity to reduce the scale of a project's operation. This is an American put option, because allows to decrease (sell) some amount of production capacity. The strike price is present value of costs that would occur in future without applying this options in the time of exercising the option.
- *Option to defer*. This option allows managers to defer the start of the project to time when business conditions are more favorable. This is an American call option to start (buy) project later. The idea is very close to financial American call option. The strike price is the value of running the project in future.
- *Option to extend*. This option makes possible to extend the life of investment project if it is still profitable by paying some fixed amount. This is a European call options that gives the manager in a particular time of planned project's expiration date make additional investment and extend the life of the asset. (Hull, 2014)

Important things to be fixed in the option, when you decide to include it in your project, are:

- Form of option: Call or Put
- Sort of option: American or European
- Expiration date
- Strike price

Only options in which all characteristics are defined can be included in project valuation.

These are the main most used in business real options. As it was mentioned the list is open and only creativity of managers can add more value to the project by applying the theory.

1.1.3 Real Options Process

The process of implementation of real options is not easy and should analyzed several times. According to research paper of Bowman and Moskowitz (2001), there are several main steps in applying real options theory in strategic decision-making process.

In the first step, the managers must decide to buy or not the real option. The investments make sense only if the cost of real option is lower than expected value of an option according to real options valuation models.

Then, of managers decided to purchase an option about time of exercising an option if they exercise it at all. At this stage evaluation is based on the difference between the then-prevailing value of the project and the exercise price of an option. At this point option analysis is not included excepting multi-stage option investments (Edward h. Bowman & Gary t. Moskowitz, 2001).

This approach is rather the simples view on real options process. Going deeper to the real options process we can find more detailed approaches. M. Amram and N. Kulatilaka (1999) offer four step solution process for real options application.

Step 1: Frame Application. Real options are not clearly specified in contracts, but can be identified by analysis and judgment. So, a good application frame is the basic and the most important stage in the real options practice. It includes following elements:

- **The Decision**. Create a word picture: What are the possible decisions, when might they be made, and who is making them?
- **The Uncertainty.** Identify the form of evolution for each source of uncertainty and lay out any cash flows and/or convenience yields.
- **The Decision Rule.** Create a simple mathematical expression.
- **Look to the Financial Markets.** Which sources of uncertainty are private and which are market-priced? Is there an alternate application frame that better uses financial market information?
- **Review for Transparency and Simplicity.** Who would understand this application frame? Managers who make these kinds of decisions right now? An industry analyst?

Step 2: Implement the Option Valuation Model. Having all main features of options in a project, the next stage is implementing of best option valuation model for a project. It includes calculating current value of a project, volatility for each source of uncertainty and obtain other necessary data. Then using all available information calculate option value using predetermined model and obtain numerical results.

Step 3: Review the Results. Once calculating model was applied, different types of results might prove useful. It is important to remember that not only money value for option matters. Managers should consider also qualitative opportunities realizing after application of the approach, which also increases a value of a project.

Step 4: Redesign. After reviewing obtained results, managers should ask themselves are there any expand set of investment alternatives. So, if there is any opportunity to reconfigure or redesigning the investment strategy by adding new options to increase a value of project, managers must think about other options that can be added by staging or modularity. (M. Amram & N. Kulatilaka, 1999)

It is clear that real options application is complicated and multistage process that include some strategic view and numerical analysis. In every step of the process the company must consider all possible outcomes and use all available information to make the process of real options maximally effective.

1.2 Portfolio Approach to Real Options

1.2.1 Definition of Real Option Portfolio

To introduce to the portfolio approach of real options theory, it is necessary to define the term of portfolio. Simply defined portfolio is “a group of investments” (Farlex Financial Dictionary. 2012) or “a particular combination of assets in question” (Neftci 2015). So, portfolio is the object that consists of several elements that create a portfolio. In term of real options theory, these assets are real assets and real options written on these assets.

In real options models L. Trigeorgis showed that real options on the same underlying assets interact, requiring a simultaneous valuation of all options written on the same underlying asset. (R. Brosch, 2008). For example, there are two financial European call options on two different stocks with no correlation. The combined value of these options is a sum of options on each of these stocks. And opposite example to show what means interaction of options. There is a plant on which an investor has to options: defer further investments in the project up to time t and European option to abandon the project in time t+n. In terms od ROA these options correspond to each other because the decision can be made subsequently and the value of option to abandon increases because there is opportunity to defer and see what will go on in business environment.

So, we can see obvious interaction between these two options, because the combined value of options will be higher, because in isolation only defer does not give opportunity to abandon if business conditions are bad, and combining deferring and abandon options increase time horizon for put European option (abandon). Only in combination we get synergy effect from these two options.

Since both effects occur simultaneously, it is not possible to value defer and abandon in isolation. The decision about exercising the first option needs to explicitly take into account the existence of the subsequent option. This relationship is structurally akin to the valuation of compound options (cf. Geske 1979) Hence, the arising effect can conveniently be labelled as compoundness. Specifically, Trigeorgis (1993a) defines interactions between real options written on the same underlying asset as “intra–project compoundness”. Following the same logic, an analogous effect is identified for several, interdependent underlying assets which he denotes as “inter–project compoundness” (Trigeorgis 2012, pp. 132 f.). Both inter–project and intra–project compoundness must be considered in the context of portfolios of real options. (R. Brosch, 2008).

Real options as well as assets can be the objects of constraints. Real assets can have financial or operating or other types of interactions. As a result, some assets can be mutually exclusive or complementary ones. This affects the possibility of joint exercise of bundles of real options on different underlying assets. Likewise, the existence of constrained resources, e.g., funds available, influences the feasibility of joint option exercise. (R. Brosch, 2008).

Therefore, portfolios of real options here are defined as combinations of multiple risky assets and multiple real options written on these assets subject to constraints. Cases with only one underlying asset, or one real option, are special portfolio cases that reduce the scope of portfolio analysis dramatically. In order to seize all possible portfolio effects, it is important to analyze multiple underlying assets with multiple real options simultaneously. The usual “laboratory” setting for real options analysis with one underlying asset and one real option does not provide a structure capable of handling realistic decision problems. It is thus prone to ignore key portfolio effects with possibly substantial impact on (optimal) option exercise (R. Brosch, 2008).

Summarizing all main points, we can conclude that portfolio of real options is complicated phenomena in the theory of real options that includes not only assessing interacting real options within one option, but also simultaneous valuation number of underlying assets with several real options in each asset.

1.2.2 Portfolio approach

Now, we defined what real options mean portfolio. It is not only combination of several correlating real options on one underlying assets, but also several underlying assets that have relations with other assets and real options. This complicated structure makes valuation of real options multistep problem that must consider all dynamic interaction of all elements of a portfolio. Only complex analysis of portfolio that handles with all interactions between decisions over time provides fundamental way for effective decision-making process over investments in the company.

A suitable approach for portfolio analysis must include all main aspects that can influence the value of a project. From the definition of portfolios of real options, it follows directly that portfolio aspects can be attributed to the real assets involved, the real options involved, or constraints. Budget constraints are of special importance because they can have a considerable limiting impact and require a detailed modeling of the investment dynamics. Moreover, the ensuing budget levels over time are state– and path–dependent (R. Brosch, 2008). Based on these considerations, R. Bosch (2001) categorizes portfolio aspects as follows:

- *Interactions on real options level.* On this level, he defines intra-project compoundness and inter-project compoundness. The first one is about correlations and interdependencies of many real options written on one underlying assets, the last one differs from the first by taking into account correlations and interdependencies of several real options and several real assets simultaneously. For inter–project compoundness, the correlation between the underlying assets has to be modeled explicitly
- *Interactions on the real asset level.* On the assets level direct and indirect qualitative interactions are distinguished. Direct interactions are those which are inseparably connected to the underlying real assets, such as physical properties or operating synergies. Indirect interactions have their origin outside the strict asset level and are due to constraints, most prominently budget constraints. Both are qualitative in that they are not merely stochastic in nature, but result from the properties of projects or the specific background of the company (that would be different for another company).

As it was mentioned, all interactions take place in one moment of time and as result the separation of value impact of one single aspect is not possible to assess isolate. All effects must be valued only jointly in order to include synergy effect of combination of real assets and real options within the project. It is the reason why simultaneous modeling approach is required, such as compound option pricing one. Also, to handle all interactions, it is important to include all available constraints, such as budget or time, to the simultaneous modeling approach.

This complex approach allows formulate the model as one stochastic optimization problem subject to constraints. Interactions are captured through the interplay of constraints as well as state– and path–dependency of investment decisions and cash flows (R. Brosch, 2008).

1.2.3 Interdependencies of options and assets

Peirson and Bird (1981) prove that interdependence of assets in the portfolio cannot be assessed as two independent ones. They claim that for better quality of analysis deeper research is required. Betge (1995) offer some classifications why some assets can be interdependent. He offers following classification:

- *Direct qualitative interactions.* The situation when the source of interaction is not stochastic relationships, but originated from nature of investment plan. It means that physical properties of investments cause qualitative interaction between two investment assets. For example, they can be mutually exclusive (e.g. build office or residence in a land), complementary (e.g. build a plant and canalization system) or synergies (build a residence and school).
- *Indirect qualitative interactions.* This results from constraints included in investment plan. The relationship is indirect because they are not inseparably connected to the investment opportunities considered; qualitative because they do not result from stochastic relationships and cannot be avoided by diversification. These constraints are connected to the firm making investment decisions. Most importantly, indirect qualitative interactions stem from binding capital constraints, i.e., capital rationing.

Meier et al. in the paper (2011) discuss a problem of real options portfolio as subject to investments and offers two models for real options portfolios. The first model is a standard maximizing value problem. They calculate the value of portfolio as the sum of the values of options in the portfolio. Authors only replace NPV of the projects by options values and does not offer something new that could be different from basic traditional approach. In the model there is no any interdependence between real options and model is stated as static. In the second programming model, they use dynamic formulation of a problem. At this model investment decisions take place over time in binominal framework. All projects are related to the same underlying stochastic variable but are independent from one another. Further, only call options are considered which can be postponed indefinitely. Once an option is exercised, there is no further option available.

Therefore, the options are stand-alone options that do not interact. So, due to budget constraints are static and no interdependence between options, their paper does not explain main point about nature of the value of the portfolio of interacting options that are more realistic than independent ones.

Actually, the direct qualitative interaction between assets makes incorrect to calculate the value of portfolio only by additive method. The effect of synergy that comes from underlying mean–variance portfolio concept based on systematic risk, makes a lot of contribution to the real value of a portfolio that different from additive one. NPVs add up if (and only if) the only portfolio aspect is diversification and markets are perfect and complete. Other interdependencies, e.g. due to budget constraints, can cause deviations from additivity (R. Brosch, 2008).

Summarizing, we can conclude that the source and type of interaction matters not only for underlying assets, but also for options. Simple additive method of NPVs violates the real value of a portfolio due to ignoring mathematically significant mean-variance approach in valuing portfolios. The interaction models are actively used in financial options theory and it must be translated to the real options approach.

1.2.4 Real Options Analysis in a Portfolio Context

Many authors during long time state that portfolio of real options with interacting ones must be modeled explicitly. Following this argument several experts in real options theory made deeper research to the nature of real options. Thus, Trigeorgis (2012) defined two types of options interactions. The first one when several options are interacting based on the same underlying asset, the author named the phenomena "intra-project" interaction. The second way is interaction of several underlying assets, he named this "inter-project" interaction.

Also, R&D investments are studied by many authors paid a lot of attention to options interaction. Due to the exploratory nature of R&D projects which are typically multi–stage, early projects can generate insights about future projects. Moreover, in an R&D pipeline many projects are typically undertaken but only few make it to become a marketable product, such that the bundle of projects must be assessed from a portfolio point of view (R. Brosch, 2008). So inter-project interaction aspects become relevant to this kind of problems. Analyzing the problem, Childs et al. (1998) considered two mutually exclusive projects, where both of them run parallel but in final only one could be implemented. Their main conclusion was the idea that in highly correlated short-term projects with low volatility and requiring large capital expenses, sequential development is desirable. Similarly, Childs and Triantis (1999) as well as Lint and Pennings (2002) also analyzed the parallel development of two R&D projects that are mutually exclusive. There are several researches about similar topics. For example, Denardo et al. (2004) found out and proposed a stochastic search algorithm for R&D projects with a sequential compound decision process. Also, Gustafsson and Salo (2005) focus on project selection subject to budget constraints, which is achieved by explicitly modeling the decision maker's terminal utility (R. Brosch, 2008).

There is some motivation why, so many articles are about R&D projects. These types of projects are most risky. Outcomes are almost unpredictable at all. So, managers need some hedging flexible strategies to avoid so huge uncertainty. That explains also a number of papers devoted to R&D process. Huchzermeier and Loch (2001) in their research modeled a compound R&D decision problem where there is the option for management to take corrective action as a means of managing the risk involved in R&D projects which may stem from different sources (e.g., market payoffs and project schedules. Going deeper to this topic, Vassolo et al. (2004) revealed that in the biotechnology sector mutually competitive,correlated projects tend to be sub–additive, but that the sharing of resources among firms may create positive spill–overs resulting in super–additivity. Chien (2002) and Kavadias and Loch (2004) also analyzed some specific R&D investments from the point of inter-project correlation. Later, Smith and Thompson (2005) showed that for a risk–averse investor investing into highly correlated projects can be more desirable, because learning between correlated oil exploration activities may invalidate the conventional wisdom of diversification. For stronger argument to this position, Dias (2006) confirmed that positive correlations cause learning and synergy effects for company, as opposed to financial options where investors seek for diversification and avoid high correlated securities (R. Brosch, 2008).

There are a number of other models that analyze different project's interactions. Thus, Brown and Davis created an investment model in which a first can be followed up by one of two mutually exclusive projects. Following the framework designed by Trigeorgis (1993a) Rose (1998) and Bowe and Lee (2004) analyzed and made recommendations for intra–project interactions of options on infrastructure projects. Other two researches Triantis and Hodder (1990) in their paper considered very popular flexible production system option for switching the production mix among two products over time. Kogut and Kulatilaka (1994) and Huchzermeier and Cohen (1996) made big contribution to real options interaction theory by analyzing a global manufacturing network under exchange rate risk. The idea was that there is a switching options between different manufacturing strategies contingent on exchange rate realizations that decreased the level of risk for companies from the manufacturing industries dramatically. Wang and de Neufville (2004) value options inherent in the design of large physical systems, such as hydropower stations, modeled as path–dependent options. Approach offered by the authors has contribution because they created a model and valued option by stochastic mixed–integer program that can be implemented on commercially available optimization platforms. It is a great development in application theoretical basics of real options interactions to practical cases. Martzoukos et al. (2003) model a path–dependent investment problems that is essentially a switching model subject to stochastic switching costs. The idea of a research was proposing the algorithm that could keep track of all possible paths and possible early exercise on each path. They succeeded in their model and made it applicable and also named this as “exhaustive search”. Kamrad and Siddique (2004) considered the interactions problem from supply chain management point. They included in a model analysis of the interactions between a producer’s and a supplier’s investment decisions. For example, the research included such important options as order–quantity flexibility, supplier–switching, profit sharing, and supplier reaction options. The source of uncertainty was discretized exchange rate processes. They offered solution to the problem using numerical approach through a backward recursion in dynamic programming (R. Brosch, 2008).

All mentioned papers analyzed many complex practical and theoretical problems in the aspect of real options interactions, either intra–project or inter–project. The drawback of considered articles is that they did not take into account an explicit portfolio perspective on real options. Only Luehrman (1998) considered explicitly a portfolio perspective on real options, but only from qualitative pint of view. He chooses a gardening metaphor where the firm is behaving like a gardener who only picks tomatoes (= exercise options) which are “ripe and perfect” (= at the optimal time). He defined food and bad gardeners like active and passive ones. Active gardener is informed and know which tomatoes to pick, which ones to leave yet a little while, and which ones to pick even if not yet fully ripe, in order to prevent the squirrels (=the competitors) from stealing them. The bad gardener acts in different way and as a result suffer a lot of losses. This metaphoric presentation explicitly discovers the nature of real options portfolio, but do not consider any quantitative framework that could value the options interactions within the project or investment strategy (R. Brosch, 2008).

In the earlier paper of Trigeorgis (1988) and Trigeorgis and Kasanen (1991) portfolio perspective was considered more explicitly. They analyzed compound synergistic effects parallel projects. Also, growth options in inter-projects relations were considered by Kester (1984, 1993). Meanwhile, while Kasanen and Trigeorgis (1993) and Kasanen (1993) put more emphasis on modeling synergies. Mauer and Ott (1995) analyze replacement decisions as follow–up projects (R. Brosch, 2008).

After considering many related papers to real options interaction, we can conclude that the interaction problem occurs a lot in practical cases. It explains the reason why there are so many papers analyzing different investment cases related to real life. That support the argument that simple real options approach without portfolio view on projects makes the theory weak from practical point of view. And applying portfolio analysis the researches try to make real options theory applicable and more flexible, trying to offer some qualitative and quantitative framework.

1.2.5 Management of Portfolios of Real Options

Real options theory states that optimal management decisions must be taken simultaneously, considering all relevant portfolio aspects, to maximize a project's value. R. Brosch (2008) distinguishes two dimensions of managing portfolios of real options:

- *Portfolio design.* Optimal future exercising is supposed, so managers create the maximum value portfolio with this assumption
- *Portfolio execution.* Real options must be exercised equally in order to obtain full value of options

Moreover, portfolio design analyzes which assets and which options to include in the portfolio of real options. Decision based on the analysis of elements of portfolio and assessing the total value of different portfolio. The portfolio with a maximum yield should be included to a project. So, the basic rule for obtaining the best portfolio design is described in the figure 1. Choosing the optimal portfolio design, manager defines the future strategy of a company that maximizes the value of an entity. So, the approach helps to make decision based on quantitative analysis rather than subjective view of managers, which in options terms corresponds to assessing new real options and their interactions with the portfolio (R. Brosch, 2008).

Damisch (2002) also supposes that optimizing the value of a portfolio can be obtained not only by including new underlying assets or options, but also by changing assumptions about available underlying assets and embedded options, for example modifying volatility included or time of exploration.

So, just summarizing, the main idea is that optimal portfolio management is more about execution of existing design of a portfolio of real options. Assumptions about portfolio theory are based on the idea that options will be exercised in optimal point of time. And we can conclude that main challenge for management is to define this optional time. The information about this point of time should be included to the value of the portfolio in a valuation process, otherwise the real value of portfolio can be violated. Thus, the proposed model gives clear management recommendations about which options should be exercised, and when, suggesting to exercise real options as in the optimal policy (R. Brosch, 2008).

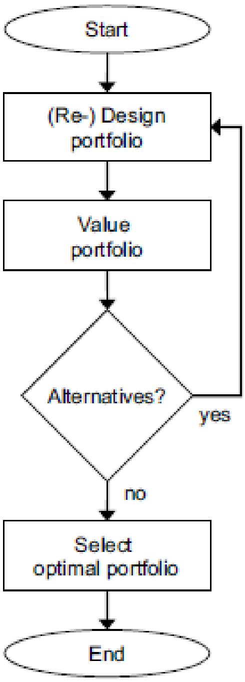

Figure 1. Portfolio design process (R. Brosch, 2008)

Generally, all main aspects of future portfolio execution are already included in the portfolio design because managers are supposed to use optimally created structure in future. On the other hand, during portfolio execution new unpredicted new information can be find out that opens new alternatives for the portfolio design. So, this proves again that manager should not be strictly stick to one chosen strategy. The mind of executor should be always flexible and open to accept new information and include it to strategies in order to maximize the value of a company.

Chapter Summary

The concept of real options is the theory that adds more flexibility do management decision when they consider investments in project and try to assess their value. These opportunities allow do deal with drawbacks of traditional approaches, like NPV and decrease risks for projects during realization.

At the same time the idea of real options also has some disadvantages in the case of interrelations of underlying assets (inter-relation) for projects or real options (intra-relation) within projects. The idea is that if there is some correlations of real options or assets additivity approach cannot be applied, because managers deal with a problem of portfolio of real options.

There are several very important papers that consider this problem and offer some reasonable solutions to this problem. The most famous ones are articles of Trigeorgis that analyzes insights of relations of projects. He, first, introduced classification of real options interactions and defined inter- and intra-relation types of interdependencies. More researches also contributed to the real options portfolio theory. They analyzed many practical cases where projects include several real options and there is some degree of interaction between them. Authors analyzed these cases and tried to assess the portfolio of options using non-additivity methods. One of the basic scientific work, that was used in this thesis was the Portfolios of Real options study written by Rainer Brosch. At his book he summarizes all main papers that are connected to real options portfolio theory and also provides valuation model for portfolio of real options that is used in the next chapters. Concluding all main points, we can say that valuation of real options portfolio is complicated process that need a lot of qualitative and quantitative analysis. The practical applicability of portfolio approach makes the theory more relevant for real life cases.

CHAPTER 2. METHODOLOGY

The chapter describes the valuation methodology portfolios of real options. In chapter there were introduced two types of interactions: inter-project (correlation of several underlying assets) and intra-project (correlation of options on the same underlying asset). For the analytical purpose of the research in this chapter we analyze only intra-project relations. The reason is that in the third chapter where the case is analyzed there is one main underlying asset. Moreover, in practice many projects are based on one underlying asset as the main source of cash inflows.

The chapter organized the following way. First, we introduce features, general assumptions and specifications for the model, that are necessary for calculations and applying formulas. Then binominal Option pricing model for one underlying assets is introduced as a starting point for general valuation model. And at final part the general valuation model for one underlying asset is presented.

2.1 General assumptions and features of the model

There are many questions about practical applicability of real options theory. Th e idea works well for financial assets but has some difficulties with real assets. The idea is that assumptions should agree on for financial assets and financial options. The assumptions for real options portfolio are the same as for single real options and presented here. They are standard for real options theory (Trigeorgis 2012; Copeland and Antikarov 2001). The assumptions are:

Perfectness and completeness of capital markets. Therefore, a spanning portfolio is available for each investment opportunity considered.

The risk–free interest rate is known for all market, constant, and does not depend on maturity.

- The values of the underlying assets follow discrete binomial processes.
- There are no any dividends on underlying assets.
- Investments are at least in part irreversible, unless corresponding real options exist.
- The decision maker has discretionary decision rights that can be interpreted as real options.
- The universe of all available options can be specified exhaustively.
- Options are proprietary options. There are no agency conflicts.
- All input parameters necessary for the purpose of option valuation are unambiguous and known. (R. Bosch, 2008)

The next step in specification of the model is defining the objective function. The main goal of decision maker is maximizing stockholders value, and hence, market value of equity of a firm (cf. Copeland et al. 2015, Part I). Translating this idea to the model, the objective function is maximizing portfolio pf real options on the project with one underlying asset and a several real options.

The portfolio value is calculated as the expected value of the optimal exercise policy of the portfolio of real options, dynamically over all managerial actions permitted and stochastic realizations across time. This optimized strategy of exercises corresponds to the investment program in terms of traditional capital budgeting theory (R. Bosch, 2008).

Also, important part for the model assumptions is risk preferences of investors. According to the financial options theory the expected value of an option is calculated as risk-neutral value (Hull, 2014). It means that the value of the portfolio does not depends on risk preferences of investors, because it does not need to be modeled explicitly.

The main part to be explained is actions of decision maker determining the utilization of the underlying assets in question, by investing and disinvesting. For any investment project managers can choose the most appropriate mode for specific situation that maximizes project value. In the theory the most representative modes for any model are:

- Mode 1. Money for project are not invested
- Mode 2. Money invested in relatively low capacity to maximum possible one
- Mode 3. Money invested in relatively high capacity to maximum possible one

So, we have several possible switches between modes:

- 1→1: Investor does not invest to project and stay with zero capacity
- 1→2: Invest to low capacity
- 2→1: Invest to high capacity
- 2→3: Switches from low capacity to high capacity
- 3→2: Decreases production capacity
- 2→1: Closes down the production from low capacity
- 3→1: Closes down the production from high capacity

So, also the model assumes that the process can be restated. For example, after closing down a project, the money can be reinvested to rerun the project with high or low capacity. This approach adds more flexibility to managers and reflects real life business cases when companies.

It is important t mention that switching cost can include additional switching costs that can be positive or negative (R. Bosch, 2008).

Stochastic process of underlying asset matters. In proposed model an underlying asset follows stochastic diffusion processes. The idea is that at each period the price, θ, for underlying asset moves up or down with probability *p* and *1-p* respectively.

For valuation purposes we use log-transformed version of binomial numerical analysis described in Trigeorgis (1991). Following standard practice in the real options literature, the gross project value (Vt) is assumed to follow a standard diffusion Wiener process given by

$$\frac{dV}{V} = (\alpha - \delta)dt + \sigma dz \quad (1)$$

Where,

α is actual expected return from a project

δ is return rate shortfall between the equilibrium return of a similar-risk traded financial asset. According some literature δ may also capture any proportional cash-flow payout on the operating project

σ is standard deviation of project value

dz is standard Wiener process. (Trigeorgis, 1993a).

The valuation process looks very similar to the theory of valuation of financial options, especially when we try to estimate a price for European call. At the same time, valuation of multiple real options on one underlying asset has some specifics that come from the nature of real options. Trigeorgis (1993a) and Kulatilaka (1995) showed that real options written on the same underlying asset actually interact and simple additivity does not hold anymore. So, valuation of these projects with multiple interacting options must be conducted as whole process of valuing the bundle of real options and underlying asset, which is similar in structure to valuing a compound option.

The intuition behind this idea is follows:

1. Real options written on the same underlying asset are connected through this asset.

2. If any option is exercised, the underlying asset as well as other options affected

3. The simplest example of option to abandon proves that. If investor abandons the project, all subsequent options are foregone.

There are more examples, when exercising real options affect the value of the whole project. For example, put exercise decision has to take the existence of subsequent options into account because these have a strictly positive value (Merton 1973b) which is forfeited (R. Bosch, 2008). At the same time the value of the value of subsequent options is affected by the possibility that the put may be exercised. This complicated nature of interactions demands new approach, different from traditional ones, like plain–vanilla financial option pricing (Trigeorgis 1991, 1993a; Brosch 2001). Trigeorgis (1993a) uses the model of numerical analysis of options interactions. Through explicit examples he proves that the value of interacting options is not additive, and interactions typically have negative sign. Also, he explains that interactions nature depends on several factors, like type, separation, degree of being in the money, and order of real options. This analysis finally shows that some usual options properties are preserved, e.g., sensitivity to time to maturity or volatility.

In the further paragraph there will be showed intra-project model of valuation for interacting real options. The model is based on previous researches of Trigeorgis (1993a) and R. Bosh (2008). They made great contribution to the theory of real options portfolio and their models are consistent and applicable.

2.2 Binominal Option pricing for one underlying asset

2.2.1 Real options valuation model

Let's start with general model of real options valuation, created in 80-th and developed later in 90-th. Trigeorgis (1993a) summarizes the main conceptions and theories about real options valuation.

The value of option is determined by be determined by discounting certainty-equivalent or risk-neutral expectations of future payoff at a given risk-free interest rate, r. Generally speaking, the price for any asset can be found by replacing the actual growth rate, α, with certainty-equivalent rate in the formula (2).

$$\hat{\alpha} = \alpha - RP \quad (2)$$

Where RP is an appropriate risk premium, and then behaving as if the world were risk neutral. (Hull 2014, Ch 7). According to the theory RP = Sσ, where S = (α-r)/σ is the asset's market price of risk or reward-to-variability ratio. ratio.

Given that α=α*- δ, then α-RP= (α*- RP)-δ=r-δ. This is equivalent to a risk-neutral valuation, where the actual drift (α) would be replaced by the risk-neutral equivalent drift, $\hat{\alpha}$=r-δ. For traded assets (in equilibrium) or for those real assets with no systematic risk (e.g., R&D projects, oil exploration, etc), α = r or δ = 0.

2.2.2 Stochastic Processes

In the model described in the paper the stochastic process of the underlying assets is assumed as discrete binominal process. The model specification based on main assumptions of financial options theory. First, the distribution of the stock price converges to the log–normal distribution in the limit, when $\Delta t \to 0$ (Kwok 2014, pp. 199 ff.). Also, geometric Brownian motions are widely used (Duffie 2001, p. 88). On the other hand, for model specifications, it is not necessary to converge to a log -normal distribution of the underlying assets (R Bosch, 2008).

The simplest and most clear way to present binominal model is standard version of binomial lattice trees. Horizontal dimension reflects time, vertical one represents up-down movements of an underlying asset. For the purposes of the model, new conventions are introduced. All variables are extended first by time t $(t=1, 2, \ldots, T)$, then by movement scenario s $(s_t=1, 2, \ldots, S(t))$, and then by actions (if necessary). So, the combination of time moment, scenario, and action (if included) are defined as "system state space". (R. Bosch, 2008). For example, the value of option in time 3, for scenario up→ up is V(3,1). More examples are in the table below.

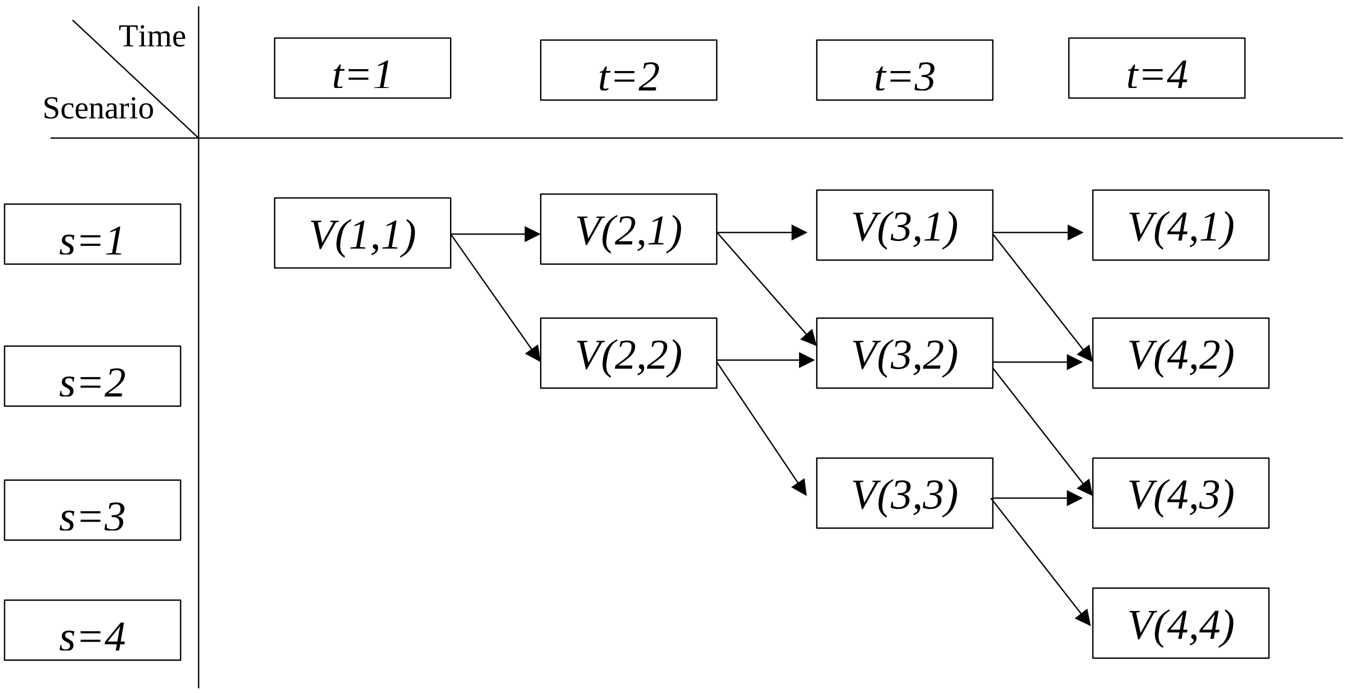

Figure 2. Binominal lattice tree example. (R. Bosch, 2008)

2.2.3 Binominal Option Pricing Model

The Cox-Ross-Rubinstein (CRR) and R. Bosch models are used to present the basic binominal option pricing model for the thesis. The notation come s from CRR derivation model, and model specifications and determinations come from R. Bosch (2008) Portfolios of real options analysis. Also, the model developed by R. Bosch is referred to CRR derivations in order to introduce the notation while the content of the CRR model is unchanged.

The binominal model based on several basic assumptions and notations:

- θ is value if underlying asset;
- The value θ follows a stationary time-discrete multiplicative binominal process;
- There are two factor of underlying asset movement: u (up) and d (down);
- Factors Up and Down are interrelated such that $u \times d = 1$;
- σ is the volatility of the underlying asset specified by Up and Down factors;
- In the limit the process converges to a log–normal distribution of the return of the underlying asset (Formula 3);

$$u = e^{\sigma\sqrt{\Delta t}}, and\ d = \frac{1}{u} \quad (3)$$

So, determined Up and Down factors are reasons for the stochastic process of the value of underlying asset, θ. So, based on the table example from previous paragraph, the system state space grows *in* t *(t=1, 2, …, T))* and s *(*s_t *=1, 2, …, S(t))* as follows in more general term:

$$\theta(t,s) = \theta(1,1) \times u^{t-s} \times d^{s-1} \quad (4)$$

The important part for the model is determining risk-neutral probabilities *p* and *1-p.* According to ROA theory, the Up scenario appears with probability *p* and the Down scenario with probability *1-p.* The risk-free rate is calculated as continuously compounded risk-free rate of return.

$$p = \frac{e^{r_f \Delta t} - d}{u - d} \quad (5)$$

Now, the basis for valuing of option with one underlying asset exemplified for European option can be obtained as follows. First, the boundary condition on option exercise at t = T is imposed. For example, let’s assume European Call with exercise price X, the terminal value condition will be defined as follows:

$$V(T,s) = [\theta(T,s) - X, 0], \qquad where\ s = 1,2,\dots,T \quad (6)$$

Finally, the model for binominal option pricing proceeds in backward recursion, discounting probability weighted expected values (Formula 7) to the first period when V (1,1).

$$V(t-1,s)=\frac{V(t,s)\times p+V(t,s+1)\times(1-p)}{e^{r_f\Delta t}},\quad (7)$$

$$where\ t=2,\dots,T;s=1,\dots,t$$

Now, we have the final version for option pricing with all specifications and model to expand these model for valuing the portfolio of real options written on one underlying asset.

2.3 Real options portfolio valuation model.

The algorithm for valuation of portfolio is based on general framework developed by Kulatilaka (1988) and Kulatilaka and Trigeorgis (1994). At any moment, the project can either continue operating in the current mode for one more period, receiving a short–term payoff (i.e., current cash flow) plus the long–term option value from optimal future switching, or switch immediately to a new mode by incurring specified switching costs (R. Bosch, 2008).

For each combination of time and state (t, s), and an underlying asset i, there is an entering mode $a_i(t,s)=1,2,\dots,A_i(t,s)\ \forall\ (t,s)$ and a leaving mode $a'_i(t,s)=1,2,\dots,A'_i(t,s)\ \forall\ (t,s)$. So, in any moment of time, a manager can take decision about switching from mode $a_i(t,s)$ to $a'_i(t,s)$, or stay the same mode so that $a_i(t,s)=a'_i(t,s)$

So, we have considered before three modes, and let's introduce them as mode specifications for the model for one underlying asset.

- $a'=1$: $waiting\ to\ invest\ (do\ not\ invest\ in\ moment\ of\ time)$
- $a'=2$: $invest\ (or\ continue\ )\ to\ operate\ in\ low\ capacity$
- $a'=3$: $invest\ (or\ continue) to\ operate\ in\ high\ capacity$

Associated with each switch is a cash flow realization $c(\theta(t,s),a')$. So, in each period realized cash flow from a project depends on the leaving operating mode a'. The amount of cash flows can be any function value of underlying asset $\theta(t,s)$. Without loss of generality, the cash flow function can be represented by a positive scalar, drawing on an analogy to levels of capacity:

- $c(\theta(t,s),a')=0$: $no\ investments$

- $0 < c(\theta(t,s),a') < 1{:}\, invested\ in\ low - capacity$
- $c(\theta(t,s),a') = 1{:}\, continue\ with\ low\ capacity$
- $c(\theta(t,s),a') > 1{:}\, invested\ in\ high\ capacity$

To make the model more realistic, the switching costs also must be considered as $I(a,a')\forall a,a'$. These costs represent additional investments company must incur to change the mode, or in terms of Real Options Analysis it is Exercise price. So, to be consistent with previous assumptions, we assume that not changing the mode does not cause any costs (costless). All other possible variations of investments for switching are presented below:

- $I(a,a') = \infty$: switching is not possible (applicable for European options, when switching is available only for one determined point of time)
- $I(a,a') = 0$: The mode is not changed, or the switching is costless
- $I(a,a') > 0$: Investments occur (analogous of call option)
- $I(a,a') < 0$:Disinvestments occur (selling the resources) is analogous of put option. (R. Bosch, 2008)

For the model we assume flexibility for switching algorithms, due to more applicability of a such approach. It can handle any structure of time–dependent symmetric or asymmetric switching costs, any cash flow payoff function and any options combinations in terms of type, sequence and option maturity. The switching algorithm is designed to model a bundle of real options that can be interpreted as a joint complex, compound option (Trigeorgis 2012, pp. 185 ff.). So, we come up with the model when the valuation cannot be achieved by valuing each option separately and then adding up the values. The main reason for that is Since each switching possibility represents a possible real option exercise and the switching costs matrix introduces an asymmetry. So, deviations from value additivity make the valuation and interpretation of incremental option contributions challenging (Trigeorgis 1993a).

And continuing with switching logic of the model, the valuation algorithm can be formalized as a backward recursion in a stochastic dynamic programming fashion. So it means that the algorithm consists double iteration:

1) It moves backward in time, applying the Bellman principle of optimality

2) At each point in time, it iterates over all entering modes, each time choosing the optimal leaving mode.

Speaking about the value of underlying asset, we have determined it as binominal process with probability of U movement *p*, and *(1-p)* for Down. Every decision at any point of time depends on two main factors:

- First, current cash flow per period which is realized after the switch, as a function of $\theta(t,s)$ and a'
- Second, expected value of future net cash flows, starting from the derived leaving mode.

All these assumptions and algorithms are translated into stochastic dynamic program based on backward recursion, which is derived similar in Trigeorgis (2012, p. 185) and R. Bosch (2008):

$$V(T,s,a) = \max_{a'}[c(\theta(T,s),a') - I(T,a,a')] \quad (8)$$

$$s = 1,2,\dots,T; a = 1,2,\dots,A$$

$$V(t-1,s,a) = $$

$$= \max_{a'}\left[c(\theta(T,s),a') - I(T,a,a') + \frac{V(t,s,a') \times p + V(t,s+1,a') \times (1-p)}{e^{r_f \Delta t}}\right] \quad (9)$$

$$t = 2,3,\dots,T; s = 1,2,\dots,t; a = 1,2,\dots,A$$

Where t Time indicator, $t = 1,2,\dots,T$,

s Scenario indicator, $s = 1,2,\dots,t$,

a Entering operating mode, $a = 1,2,\dots,A$,

a' Leaving operating mode, $a' = 1,2,\dots,A'$,

V Value of portfolio,

$c(\cdot, a')$ Cash flow function for operation in mode a', $a' = 1,2,\dots,A'$,

θ Stochastic value of underlying asset,

I Investment or switching costs,

p Risk–neutral probability for up–jump,

r_f Risk–free rate of return.

At every step of the system, for each entering mode, the stochastic programming mode determines the best leaving (or operating) mode, that maximizes the value of a project. Decision based on several inputs:

- Switching costs
- Expected future value given leaving mode.

The expected future value also includes all possible future switching decisions with the leaving mode the new starting point. Summarizing the model, we can determine logic sequence is as follows:

1) Observe the value of the underlying stochastic asset (variable);

2) Choose optimal switching policy;

3) Realize the cash flow in this period for the leaving mode. (R. Bosch, 2008)

The chosen leaving mode then will be entering mode for the next period. This stochastic dynamic programming formulation considers any possible structure of switching costs, including the non–symmetric case. So, in some steps (nodes), staying in the same mode can be optimal decision, because it can prevent a future costly switching back to the earlier mode. This approach represents hysteresis effect: even though immediate switching may appear profitable from a short–term perspective, for dynamic long–term considerations it can be optimal to remain in the original mode (Dixit and Pindyck 1994; Trigeorgis 2012).

Chapter Summary

The model for valuing multiple options on one underlying asset was introduced. The theoretical background of the model is based on researches of competent scientists in the field of real options analysis, such as Kulatilaka, Trigeorgis, and Bosch. The model is presented as stochastic dynamic programming approach that maximizes the value of a project by choosing the best leaving mode for every step. The advantage of the model is that it seeks not only short-term profitability, but also considers long–term perspectives, that make the model strategically optimizing. Also, the model is built on assumptions that can be applicable for real cases.

As well, as managers are seeking only practically applicable models, the idea is to prove that model can work on the cases that are very close to real life projects. So, in the third chapter we test the model on several modeled projects with assumptions that are similar with inputs for practical cases.

CHAPTER 3. CASE STUDY

3.1 Description of the project

3.1.1 Project background

The case study considered in the paper is devoted to shale oil extraction projects. There are several reasons, why this type of investment project is considered in the thesis. First, the shale miming nowadays one of the riskiest projects due to relatively high extraction costs and very volatile oil prices, the underlying asset is perfectly fits to the model assumptions, and it represents the bunch of projects where ROA is applicable and the models can be tested as the prove for general validity if the model.

The principle of shale oil extraction is different from traditional ones. The oil is extracted not from oil lakes, but from shale rock and other low permeability rock formations. It was made possible as technologies improved and the development of horizontal drilling techniques and hydraulic fracturing (“fracking”). Shale extraction has grown rapidly on the middle of XX century.

Despite being unconventional oil resources, shale oil and gas formations can be found around the world. In 2013, the US Energy Information Administration estimated that about 11% of total crude oil, or approximately 345 billion barrels are of shale oil from these formations. The countries with largest amount of technically recoverable shale oil resources include Russia, the United States, China, Argentina, and Libya. That means, that shale oil extraction projects can be realized in all continents, and it becomes more attractive as traditional crude oil resources are decreasing. (Shale Oil, Investopedia)

At the same time, shale mining is not widespread around the world. As it was mentioned before, the main reason for that is high production costs for this type of extraction. There is a trend for decreasing costs for shale extraction, but it never can be as cheap as traditional methods. So, as a result most companies prioritize conventional methods in the extraction, because they are less vulnerable to the oil price fluctuations. And oil prices instability during last 4-5 years made shale mining very risky.

In the following graphs, it is clear how oil prices change and how the cost per barrel also changes and it proves why assumptions about high risk of shale mining is true. Following the oil prices trend, the number of projects in shale extraction is also constantly changes, especially in the U.S, in the country where these unconventional methods are most developed.

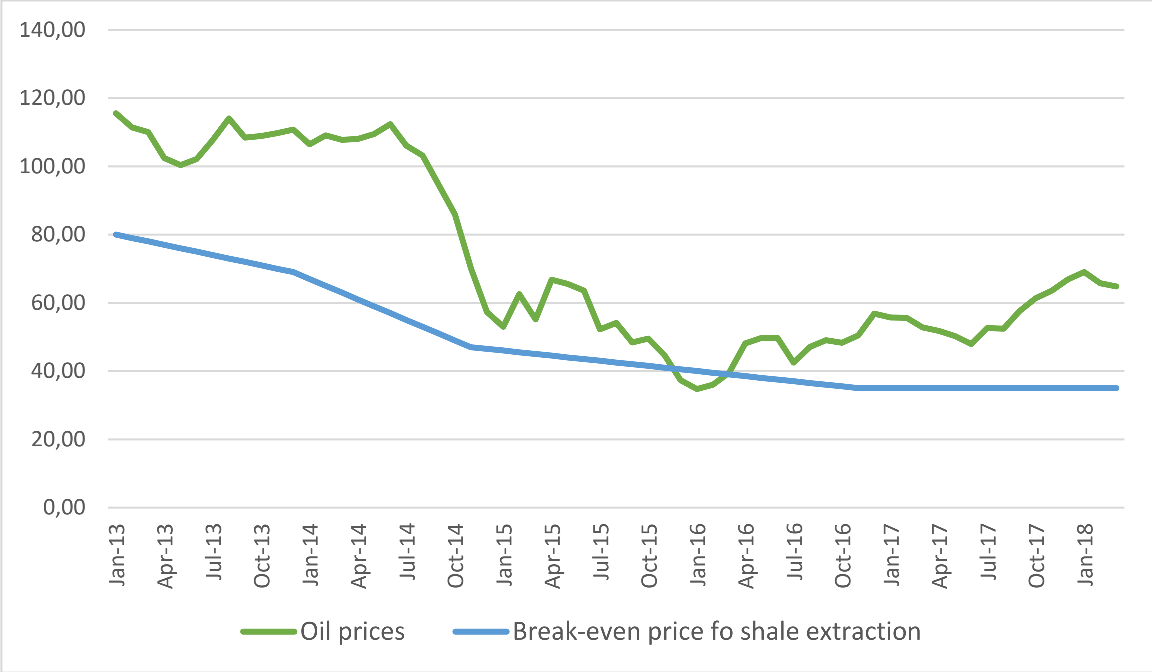

Figure 3. Oil prices and break-even price for extraction. (World Oil, 2018)

Since 2013, the average wellhead break-even price (BEP) for key shale plays has decreased from $80/bbl to $35/bbl. This represents a decrease of over 55%, on average. The wellhead BEP decreased across all key shale plays.

There are several reasons behind the observed drop in BEP. A part of it is attributable to the structural changes, such as improved well performance (which can be measured by improvements in the EUR); and the improved efficiency gains (which can be measured by the effect of lower drilling and completion cost, a result of more effective operations). Another set of drivers behind the falling BEP can be referred to as cyclical changes, which are driven by the industry cycle into which the oil industry entered in 2014, with a plummeting oil price.

We can see, that difference between oil prices and BEP is relatively small that and even in some points the BEP was lower than oil prices. If the trend will continue to more decreasing of oil prices, the shale projects can be even more risky, despite decreasing BEP. So, all these factors ask more flexibility for shale extraction companies in terms of production capacities. And real option approach that considers opportunity to be flexible in the unstable underlying assets case can maximize the value if such projects. If we apply standard NPV calculations, most of shale projects seems unprofitable, but they still operate, that proves another approach, like ROA is necessary for these types of projects. So, in the paragraphs it is clearly proved why ROA, especially portfolio approach for real options is best way to assess these investment projects.

3.1.2 Project inputs for analysis

First, main characteristics of the project will be implemented. The project is medium capacity shale oil extraction project in the North America. This region is selected as base case, because now most of the companies operating on the shale are placed in the US. On the other hand, it does not mean that these projects cannot be implemented in other countries, because the cost of investments and operational costs do not change significantly over the countries due to the same technologies and suppliers.

The shale oil resources are restricted in the capacity. As a rule, typical shale oil field can be classified as small sized according to international classification. A small sized crude oil field contains from 1 to 5 million ton of crude oil per field. Converting it to barrels of oil (crude oil prices are defined in U.S. dollars per barrel), in average as 7.5 barrels per one ton of crude oil (average rate for U. S WTI brand crude oil), we obtain from 7.5 million to 37.5 million of barrel. For our project we will take the average amount of 22.5 million barrels of crude oil. And, we consider a small field close to our that the project can expand in better conditions. The amount of crude oil at this field is about 3.5 million of barrels. (Society of Petroleum Engineers, 2016)

Considering, that in average the lifetime for small sized crude oil field is 15 years, it means that in average it is possible to produce 1.5 million of barrel per year, or 4000 barrels per day. For these purposes the company needs at least 40 oil drilling and extracting machines with average capacity of 100 barrels per day per machine (for contemporary oil wells, the capacity ranges from 90 to 110, so we can take also an average amount (AOGHS.org, 2010).

To sum up all necessary preliminary information about the project, look at the table below.

Table 3. Summary of project input.

Region	North America (U. S)
Crude oil brand	Brent
Shale field classification	Small seized
The amount of oil in the field	22,5 million of barrels
The field lifetime	15 years
Average amount of barrels per day	4000 barrels
Number of extracting machines	40

Source: author's calculations

Several main input factors are obtained. Now, it is necessary to value the amount of investments in U.S. dollars to the project. Construction of oil extraction plant is a multistep project. Though, the investment amount analysis goes by step-by step investments and analysis presented below.

Step 1. The first step is buying the field and all necessary licenses for the extraction. For our analysis we assume that the company already possesses the field and the main problem to make decision whether it is profitable to make investments or not. Also, important to mention, that all preliminary expenses about exploring the amount of oil in the field are not included in the investment analysis because it is common practice for oil companies. More complicated task is to value the amount of money company pays for licensing. The rules for obtaining licenses to explore on state mineral rights vary from state to state. Federal onshore exploration licenses are obtained through oral competitive bidding. So, it means that we cannot put this investment here, but after conducting all investment analysis, because we the company's maximum bid price cannot be higher than the value of the project.

Step 2. Buying all necessary machines and capacities for drilling. The least-expensive rigs are those classified as U.S. small footprint land rigs. U.S. shale-ready rigs tend to cost about $3 million to $5 million more than small footprint rigs. These small rigs are suitable for the project purposes due to small oil capacity in the field. Considering 40 drilling machines for the project we obtain about $120 million for main machines capacities. According to the statistics all surrounding expenses connected to services and other related expenses to run the production takes up to $130 million for small capacity onshore extraction. It means that total amount of investments to the project approximately is $250 million and can take up to 2 years for finishing the construction.

Step 3. Valuing operational costs for the plant. All operation costs are divided to fixed and variable costs. According to the industry specifics the fixed costs are about 15% of calculated average monthly revenue for the project. It means for our project we can assume that monthly fixed costs will be about $1.1 million per month. (Assuming $60 per barrel). Variable costs can be also defined as break-even price. It already demonstrated that the break-even prices for shale onshore extraction is approximately $35 for 2018.

Step 4. Risk-free rate. We need risk free rate (not WACC) for the project valuation according to the real options valuation model in binominal options pricing. The project take place in North America and the idea is to make the case analysis worldwide applicable, so we can use as risk-free rate international ones. In common world-wide practice as a risk free-rate we can take 1-year T-bills rate of U.S. They vary over time, but most companies take the rate as 1%. For the purposes of the hypothetical project this rate is valid as best fir and avoid 0% risk-free rate.

Now, after introducing all necessary for the proper investment analysis information, we can start to apply all the methodic and find maximum value of the project.

3.2 Investment analysis of the project

3.2.1 Binominal numerical analysis of the project: No options

First, to make binominal numerical analysis of the project we need to make some historical analysis of the underlying asset. The lifetime of the project is 15 years from start monthly, so to make the calculations close to real figures, we take 15 years retrospective analysis of Brent oil prices. This data is necessary for finding basic factors, like Up and Down factors that need standard deviation of underlying asset. Just to give overall view of oil prices movements, look at the graph below. (Chapter 2, Methodology)

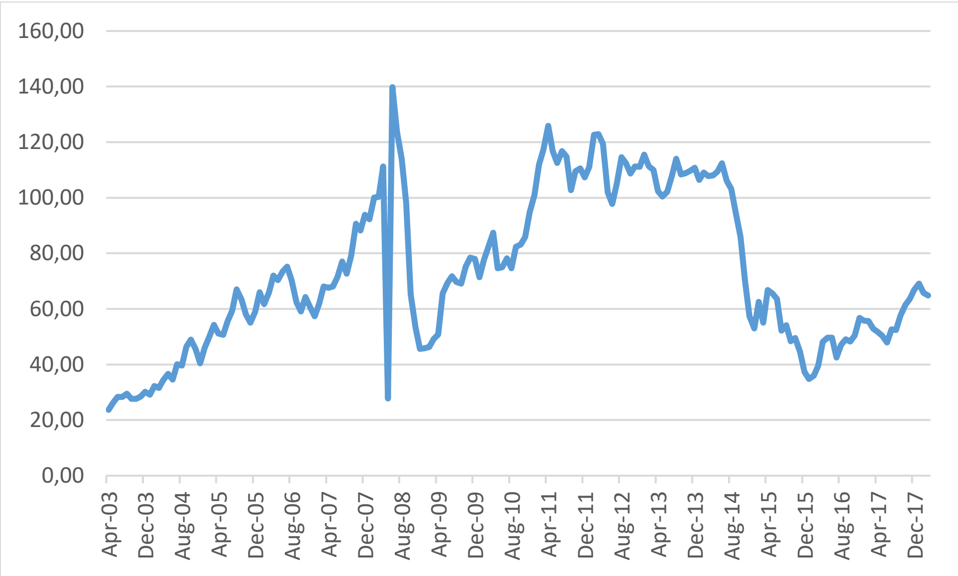

Figure 4. Oil prices dynamics (Investing.com, 2018)

We can see that in some periods we observe high deviations of monthly prices due to different shocks in row oil market. So, these outliers should be removed from the model otherwise we get some extremal (impossible) pricess in our binominal price tree due to high standard deviation.

After excluding outliers from the analysis, we obtain some basic factors, like Up and Down, and risk neutral probabilities necessary for the analysis of the project. So, the first step in valuing the project value without any options is to build binominal prices forecast for future 180 months (15 years). To be realistic, we should put some restrictions on maximum and minimum oil prices. In previous 15 years, the maximum oil rice was $139 per barrel and minimum $23.68 per barrel. Nobody knows the future, so to capture 99.99% of possible oil prices values, we put minimum price restriction of $10 per barrel and maximum $150 if ever prices in our model goes to those levels. Applying Up and Down factors and putting restrictions we obtain binominal prices model for underlying asset. (Appendix 1 and Application in Excel file).

The next step is based on forecasted oil prices, to forecast cash flows for every period for each possible state of oil prices. The cash flows are calculated in simplified way, like revenues minus expenses. All revenues come from selling crude oil in the amount of 4000 barrels per day or 120 000 barrels per month times forecasted monthly average oil price. Expenses are variable costs per barrel plus fixed costs. Variable costs are number of barrels sold times cost per barrel. Summarizing, the cash flows formula is presented below. Results of calculations are in Excel file application and Appendix 2.

$$CF_{it} = Numer\ of\ Barrels \times (Price\ per\ barrel_{it} - Variable\ cost\ per\ barrel) - Fixed\ costs \quad (10)$$

Obtained cash flows for each period and each forecasted oil prices are used further to value the project by discounting cumulated expected cash flows by backward induction to period 0. The method of backward induction and finding the value of the project is described in formal way in the Chapter 2. Methodology of this master thesis. Here, simplified and more understandable way is described using graphical presentation of the valuation model with numerical example.

Risk-free rate = 1% .

Risk neutral probabilities: P = 0.55 (Go Up) 1-P = 0.45 (Go Down)

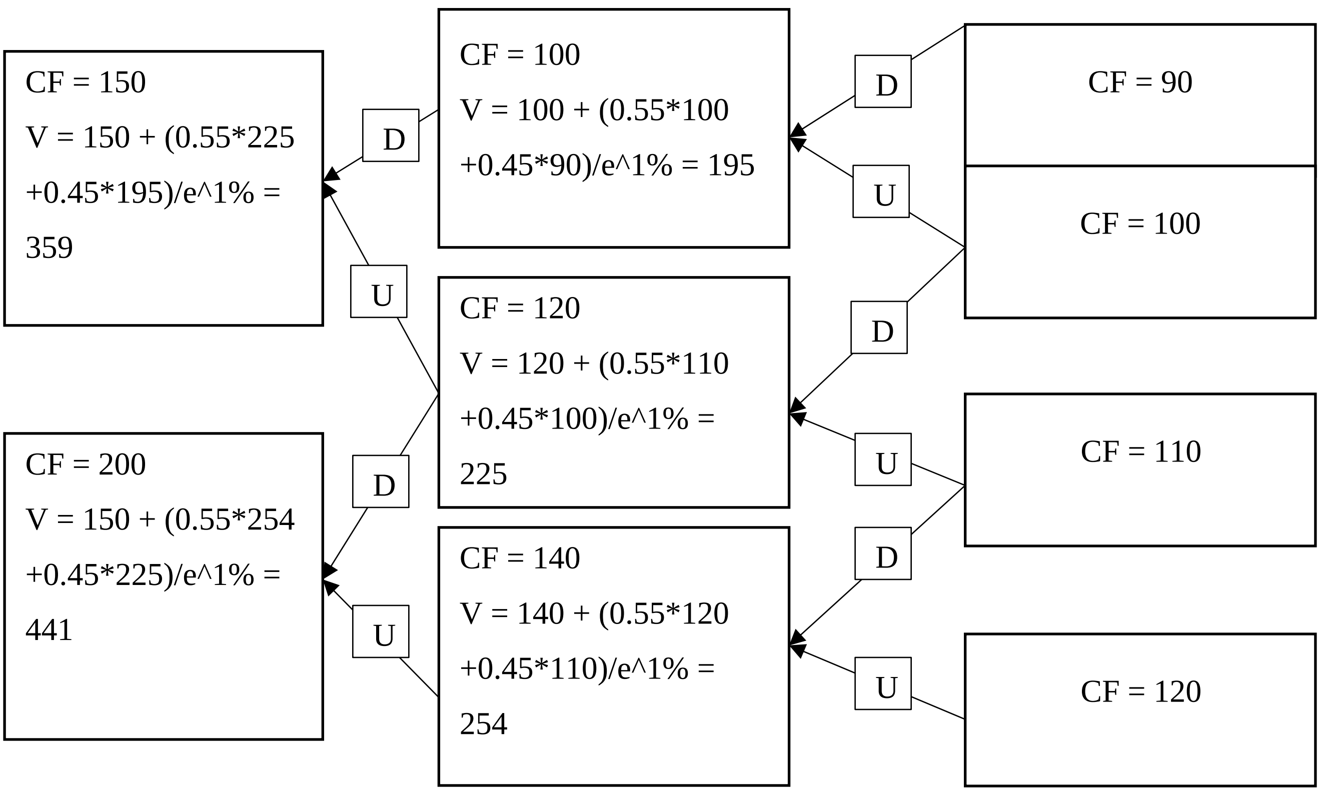

Figure 5. Binominal lattice tree in examples. (Author's calculations)

Discounting by the same logic over all 180 periods we end up with the value of the project in period 0. In the analysis we obtained the value of the project without any options of $49.249638 million. (Details in Appendix 3 and Excel file application). So, it means that this is the maximum amount the company can afford to bid for the license for oil drilling in a planned shale. What if other companies offer more? So, here we need more flexibility to maximize the value of the project and overbid competitors and buying license for profitable project. Here we have particular managerial application for the project. Now, let's move further and try to add maximum value to the project by creating the portfolio of options for the project.

3.2.1 Analysis of the project with options

In this paragraph we are going to consider how the value of the project changes with the implementation of different options separately. That will make a base for comparison of the sum options with the value of options and help make conclusions how the value of portfolio is different from the sum and which approach is better.

For crude oil extraction projects, it is possible to implement 4 basic options types:

- Option to defer the project. That means if circumstances on oil market are not appropriate, the project can be deferred to better conditions in future.
- Option to abandon the project. That means that if that crude oil prices in markets such low that make the project unprofitable, the company can close or sell pot the project.
- Option to expand the production capacities. That means that if oil prices are high enough company can consider the expansion of production to near small seized field.
- Option to contract the production capacities. That means that if conditions in crude oil markets are bad bur till make sense to produce with lower capacities, the company can reduce production capacities to decrease fixed costs of the project.

Option to defer. Option to defer the project can be considered if and only if the oil prices are low and cannot allow the producers start a profitable business. In the present conditions oil prices are high enough and even have a trend to increase in future, that means that option to defer the project will not be considered for the valuation of the project. It is better to start still prices are high enough than wait and end up without any projects. So, only remaining three options will be evaluated.

Option to abandon. The option to abandon is an analogue of European put option. If the company anodons the project, it sells out all the assets by terminal value for the moment of selling period. Moreover, company can choose the best period to exercise the option. The terminal value for the end of period 180 is 0, the assets loses the value linearly with lifetime of the project. That means that $250 million loos approximately $1.39 million dollars per month. To calculate the terminal value of the project for time *t* and oil prices in state *i*, look at the formula below.

$$TV_{ti} = 250 - \frac{250}{180} * t \quad (11)$$

So, it means that the option will be exercised only if cumulative cash flows for time *t* are lower that the terminal value of the project in time *t* and oil prices in state *i*. The idea is formally stated in the formula below.

$$V_{ti} = \max(Cumulative\ CF_{it}; TV) \quad (12)$$

It means that we have the maximization task the aim of which is to find the best exercise time for option to abandon that maximizes the value of the project in time $t = 0$. The graphical results of solving the problem are presented in Appendix 4. Solving the problem, we obtained the optimal exercise time for European put (abandon the project). In time t = 38 the value of the project in time 0 is maximum.

Summarizing, we can state that after 38 months after starting the project, the option must be exercised if Cumulative cash flows are less that TV of assets that equals to $197.6 million + project value of $35 million is $232,6 million. Simply speaking, if oil prices are lower than $55 per barrel of crude oil, it is better to sell out the business and maximize the value of the project. The value of project will increase to $49.4737million, and the value of option is $0.2280 million.

Option to contract. The option to contract is an analogue of European put option. The idea of the option is in opportunity to contract the production capacities of the field in the case of negative cash flows for the period, that will allow to decrease fixed costs twice, but the production capacities will decrease only by 1/4 by capacity per drilling machine from 70% to 90%. At the same time all free capacities can be sold out for its Terminal value. The tricky thing here is that, terminal value + contracted capacity can be even higher not only of cash flows become negative, but also in the case when it is higher than cumulative cash flows in particular period and oil prices period.

After making a number of calculation, we obtained several important moments. Contracting production in average decreases negative cash flow per period by 3/4, in case of positive cash flow, in average they decrease by 1/4.

This problem needs optimization task of finding the best expiration period for the option, that maximizes the project value. Applying a number of programmers, the best expiration period was find. It is equal to 78. Terminal Value of the assets to be sold are equal to $70 million. The option must be exercised if the oil prices in period 78 are less or equal to $57.8 per barrel.

Summarizing the option analysis results, we can conclude that European put option with exercise price of $70 million, and expiration time of 78 month after running the project, has the value of $3.2011 million and increases the value of the project to $52.4467 million. The option must be exercised if crude oil prices are less or equal to $57.8 per barrel. (Excel app)

Option to expand. The option to expand is an analogue of European call option. It allows the company to expand the project to the near field with additional 3.5 million of barrels. It can be possible of and only of oil prices are high enough to cover investments and make the project more profitable.

The expansion is possible not earlier than 5 years after running the project. This is common practice for new fields that are developed. In 5 years the project proves its validity, and additional expansion can be considered.

The expansion assumes of extraction all 3.5 million barrels of oil up to the end of the project. So, it means 0.028 million of barrels per month for 120 months. So, if the option is exercised, the extraction capacities will increase approximately to 0.15 million barrels per month. The expansion asks more investments. For simplicity. We can assume that increase in capacity for 15% per month ask the same rate of initial investment plus additional project fees (approximately 40%) for expansion, this is also common practice for industry. So, total investments are about 37.5 (share of initial investments) + 17.5 (fees) = $54.5 million of investments for expansion in total.

Expansion of the project in average increases the cash flow for the project to 27% per moths. So, it means that expansion to the project should at least as profitable as not expanding. Formal description is presented below in formula.

$$(13)\ Cunulative\ CF\ before\ expansion_{it} \leq Cumulative\ CF\ after\ expansion_{it} - Inv$$

Summarizing, we have European call, with expiration time equals to 60, that allows by investing additional $54.5 million earn additional 27% CF per month. So, the value of option equals to $10.4861 million, and increases the value of project to $59.7318 million. It is profitable to exercise the option if oil prices are more than $76 per barrel (Excel app).

All the summary about options values are presented in the table below.

Table 4. Summary of options analysis.

Option	Type	Exercise price	Expiration time	Value, million
Abandon	European put	$232.6 million	38	$0.228
Contract	European put	$70 million	78	$3.201
Expand	European call	$55 million	60	$10.486

Source: author's calculations.

We can conclude, that all options add some value to the portfolio. It means that all of them should be included to the portfolio to maximize the value of project. The most influence has the option to expand, the least one, option to abandon. Anyway, all of them add significant value and cannot be ignored.

3.2.2. Real options portfolio analysis

The basic part of the analysis is conducted. Now, it is a time to value what is the value of the portfolio of options. In the project we have 3 existing possible options with known characteristics: type, expiration type, amount of investments. The aim of creating the portfolio of options is to maximize the value of the project. So, the model based on the assumption that combining the options in a portfolio gives the value that is not simple sum their individual values. The main purpose here is to allocate the options on the time line so, that they will create positive synergy and maximize the value of options.

Moreover, different combinations of portfolio will be tested, because there is a possibility that portfolio of 2 options will add more synergy than group of 3. Having 3 options, following possible combinations are possible:

- Abandon & Contract
- Abandon & Expand
- Expand & Contract
- Abandon & Contract & Expand.

First step, is analyze their interactions. Look at the Appendix 6 to know whether these options interact or not. If they do not, there is no reason even to make further analysis, we just can sum up their values. Otherwise, we need to apply portfolio approach. Second step, all the options will be place on the time line according to their expiration times for individual valuations. Then, expiration times will be changed (considering all the possible restrictions), in order to find other possible value maximizing combination, if it does exist. So, then we can conclude whether chosen expiration times for individual value maximizing also best choice for portfolio or not.

Let's consider the first portfolio: Abandon & Contract. We can see clear interaction of the options. That means that we apply portfolio approach. (Appendix 6). Placing them with the same expiration rimes gives us the negative synergy of $0.1804 million. And, actually it is expectable. First, we consider abandoning the project, and only after that, contracting. At least, it looks illogical, because any manager first would consider contracting the expansion capacities, and then, if situation even worse, to close. So, logically, the abandon option should be exercised after contracting. We already know, that maximum value we obtain, if and only if we exercise the option do contract in the period 78. So, option to abandon should be considered after this time. The logic stands true, if the option to abandon exercised next period after the option to contract, the value of portfolio is maximum. The synergy still negative, but less than previous one and equals to $0.1654 million. We already have first evidence, that the value of portfolio is not equal to the sum of individuals values, that proves significant interaction between options. (Excel file)

The next portfolio to be considered is Abandon & Expand. They also interact. It means we cannot sum up their individual values. (Appendix 6) After conducting a number of analysis to find out the best placement of expiration period on project lifetime, it was clarified that original expiration periods give the maximum value for a portfolio. The value of the project reaches to $59.9068 million and the value of the portfolio is $10.6612million which is $0.0530 million less than the sum of these options. We can see significant increase in the value of the project, approximately by 25%, which can be crucial in bidding process. (Excel file)

The final two option portfolio is Expand & Contract option. The same logic applied here, and the same results about expiration time of options are obtained as in the previous portfolio. (Appendix 6). The value of the project reaches to $64.2827 million and the value of the portfolio is $15.0371 million which is $1.3498 million more than the sum of these options. And here we get positive synergy from combining two options. Total value of the project from these actions increased by 28.4%. So, we already have three proves that the value of portfolio cannot be considered as the sum of the values individually, and their combination can provide either positive or negative synergy, and this has to be considered to reflect actual value of the portfolio. (excel file)

As we have a task to maximize the value of the project, also combination of all three options in a portfolio must be considered. It is expectable, that if options interact 2 by2, so the portfolio of all 3 options also well interact. (Appendix 6) After applying the maximization techniques, we obtained that optimal expiration period for combination should be the following:

- Abandon and Contract must be placed as they were in Abandon & Contract portfolio
- Expand expiration time is equal to its original one.

So, the following results are obtained. The value of the project reaches $64.3575 million, and the value of portfolio is 15.1187, that $1.1966 million more than the simple sum of their values. Here we get the maximum value of the project which 28.8% higher than its original value. So, it means that the hypothesis is held, and the value of portfolio is not a sum of individual values, but their interaction influences the value of portfolio significantly (Excel file).

Chapter summary

Analyzing the results of the analysis presented in the table below, we can make several very important conclusions about the value of real options portfolio.

Table 5. Options portfolio analysis summary.

Portfolio	Value of Portfolio, million	Sum of options values, million	Difference, million
Contract & Abandon	$3.264	$3.429	$-0.1654
Abandon & Expand	$10.661	$10.714	$-0.053
Expand & Contract	$15.037	$13.687	$1.350
Expand & Abandon & Contract	$15.112	$13.915	$1.197

Source: author's calculations.

First, the difference between the sum of the options values and the portfolio values prove that interaction (overlapping) of the options influences significantly to the value of the portfolio. So, the hypothesis telling that the value of interacting options is not the simple sum of them holds.

Second, based on the analysis of previous paragraph, the value of the portfolio also can be violated by changing the expiration time of options. It explains that the nature if interaction also matters.

Third, we actually obtained better value for our project that allows to the company increase the bid price. At the same time, in both cases of negative or positive synergy, the value of portfolio must be considered not just as a sum. If we have negative synergy, it means we can overvalue the project by simply summing up the value and bid wrong price. If we have positive synergy, we can undervalue the project and bid lower price then we can afford and probably lose the auction for the field.

CONCLUSION

The goal of given research was to propose recommendations of valuation real options portfolio for investment projects. The idea of the goal is that, real options analysis not always is based on the one option, but also on many options, and these options can interact. The interaction can influence the value of the project, so the idea is to recommend some tools that allow managers make analysis of real options portfolio valid and effective considering all possible interactions. In order to achieve this goal, the number of objectives were accomplished. First, all the overview of real options approach and methodologies were provided. Then, the real options valuation methods were revised and presented in the paper. That means that the research step-by-step dived in the topic, trying to make the paper understandable and consistent. After this step, the main methodology for creating and valuation of real options portfolio was provided. It is the main model for the whole research and basic idea of it. And finally, the methodology was tested in the hypothetical case study. It is important to mention, that the case study is not just numbers from nowhere. The idea was to implement the technique to the simulated case but with the inputs very close to real life.

According to the first chapter, it was determined that real options analysis is complicated process and many approaches can be implemented. Analyzing the real option portfolio even is more complicated. The reviewed researches proved that existing interdependencies of options in the portfolio influence the value of the project. Moreover, the portfolio creation model is the optimization task the idea of which to maximize the value of the project. So, if any analyst can see any interdependence of options in the portfolio, he should understand that summing up their individual values is not applicable anymore.

As it was mentioned, there problem of portfolio is not new and already was considered by other scientists. That means some methodologies already exist. The idea here was to combine the best practices of them and offer for a reader the understandable methodology for the portfolio valuation. The methodology is introduced as a step-by-step guide. First, the general assumptions and the features of the model are formulated. Then, Binominal Option pricing model is presented. The binominal approach is the basic for the analysis in the given research. And finally, the model was expanded to the valuation of the portfolio of options. The model is presented as stochastic dynamic programming approach that maximizes the value of a project by choosing the best leaving mode for every step. The advantage of the model is that it seeks not only short-term profitability, but also considers long–term perspectives, that make the model strategically optimizing.

Finally, the methodology was applied to the simulated case study. At the same time, to make the case very close to real life, the prototype of real project was implemented. The prototype is the shale oil extraction project. All the inputs for the project are very close to real numbers but averaged due to limited access to real project information. Moreover, the crude oil as underlying asset was implemented to the model on the basis of real historic data. Based on conducted analysis following results for real options portfolio analysis are obtained.

1. The interaction of options influences the value of project significantly. The analysis results extensively showed if there is some interaction of real options in the project, the value of the portfolio never can be the sum of this options. The value of portfolio can be either positive or negative, depending on the type of interaction.
2. Allocation of real options on the lifetime of the project matters. When we place options together in one project as a portfolio, the value of the project changes. Moreover, the expiration time maximizing individual values of an option does not mean than the same expiration period will maximize the value of portfolio of options.
3. Ignoring the interaction of options can lead to mis value of the project. That means that if the portfolio approach is ignored, the value of project can be overvalued or undervalued, as a result can lead to either accepting unprofitable project or rejecting high potential project.
4. The maximization problem to be solved by optimal combination of real options in a portfolio and efficient allocation of them in time-line of the project. Managers should think flexible in terms of real options approach. They should not stick to a given inputs for options but try to maximize the value changing inputs that are changeable.

The main contribution of the paper is its managerial applicability of complicated theoretical model to practical cases. First of all, the methodology for valuing the portfolio of options was developed in the simplified form that makes the model understandable for managers without strong mathematical background. If any manager is considering the multi option project with interacting options in it he/she should apply the methodology, because only it considers all necessary aspects of options interaction. The advantage of this model is that anybody without strong mathematical background just read through the guide and apply it for any project that are fit for general assumptions for real options analysis. Secondly, the investment analysis was implemented for shale oil extraction project. These types of projects are widespread over the world due to shale mining boom in the world. At the same time, due to high cost of production and volatility of crude oil prices in world markets, shale mining is still high risky project that needs more flexibility to maximize the project value. This value can be used by managers to bid in an auction for the license for oil extraction in a particular field. Applying the portfolio analysis that reflects the real value of project, flexible managers can obtain the realistic value of the project that will allow them to bid optimal price than other managers. It means that they will bid only right price and will not overvalue or undervalue the project.

Possible limitations of the model are that it is not universal for all multi options project. If some of general assumptions of the model are not met, that means that the binominal valuation model is not applicable. In most cases this is biggest challenge for ROA. There are many businesses where underlying asset prices are regulated, or they do not follow stochastic process. Another limitation can be complexity of model for too many periods. Too volatile underlying asset prices can lead to extreme values of forecasted prices and make the model inapplicable. To overcome this limitation, it is necessary apply other models that are exist in the literature.

Overall, the paper makes contribution both for science of real options and managers. For the real options analysis it was an attempt to apply all the theoretical models to the case very close to practical conditions. From managerial side, the paper simplified complicated model and made is accessible for most of managers who are interested in applying real options analysis.

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APPENDICES

Appendix 1. The binominal prices model for underlying asset.

Example.

Metrics	Input		Calculated Factor	Result
Number of periods	180		Up	1,01
Avg Monthly St Dev (excl outliers)	0,9%		Down	0,99

Prices												
0	1	2	3	4	5	6	7	8	9	10	11	12
60	59,5	59,0	58,5	58,0	57,4	57,0	56,5	56,0	55,5	55,0	54,5	54,1
	60,5	60,0	59,5	59,0	58,5	58,0	57,4	57,0	56,5	56,0	55,5	55,0
		61,1	60,5	60,0	59,5	59,0	58,5	58,0	57,4	57,0	56,5	56,0
			61,6	61,1	60,5	60,0	59,5	59,0	58,5	58,0	57,4	57,0
				62,1	61,6	61,1	60,5	60,0	59,5	59,0	58,5	58,0
					62,7	62,1	61,6	61,1	60,5	60,0	59,5	59,0
						63,2	62,7	62,1	61,6	61,1	60,5	60,0
							63,8	63,2	62,7	62,1	61,6	61,1
								64,3	63,8	63,2	62,7	62,1
									64,9	64,3	63,8	63,2
										65,4	64,9	64,3
											66,0	65,4
												66,6

Source: author's calculations in Excel application of the paper.

Appendix 2. Cash flows for each period and each possible value of oil prices.

Example. (More details in Excel file application)

Forecasted CF for period, mln $												
0	1	2	3	4	5	6	7	8	9	10	11	12
-250,0	3,0	3,0	2,9	2,9	2,8	2,7	2,7	2,6	2,6	2,5	2,4	2,4
	3,2	3,1	3,0	3,0	2,9	2,9	2,8	2,7	2,7	2,6	2,6	2,5
		3,2	3,2	3,1	3,0	3,0	2,9	2,9	2,8	2,7	2,7	2,6
			3,3	3,2	3,2	3,1	3,0	3,0	2,9	2,9	2,8	2,7
				3,4	3,3	3,2	3,2	3,1	3,0	3,0	2,9	2,9
					3,4	3,4	3,3	3,2	3,2	3,1	3,0	3,0
						3,5	3,4	3,4	3,3	3,2	3,2	3,1
							3,6	3,5	3,4	3,4	3,3	3,2
								3,6	3,6	3,5	3,4	3,4
									3,7	3,6	3,6	3,5
										3,8	3,7	3,6
											3,8	3,8
												3,9

Source: author's calculations in Excel application of the paper.

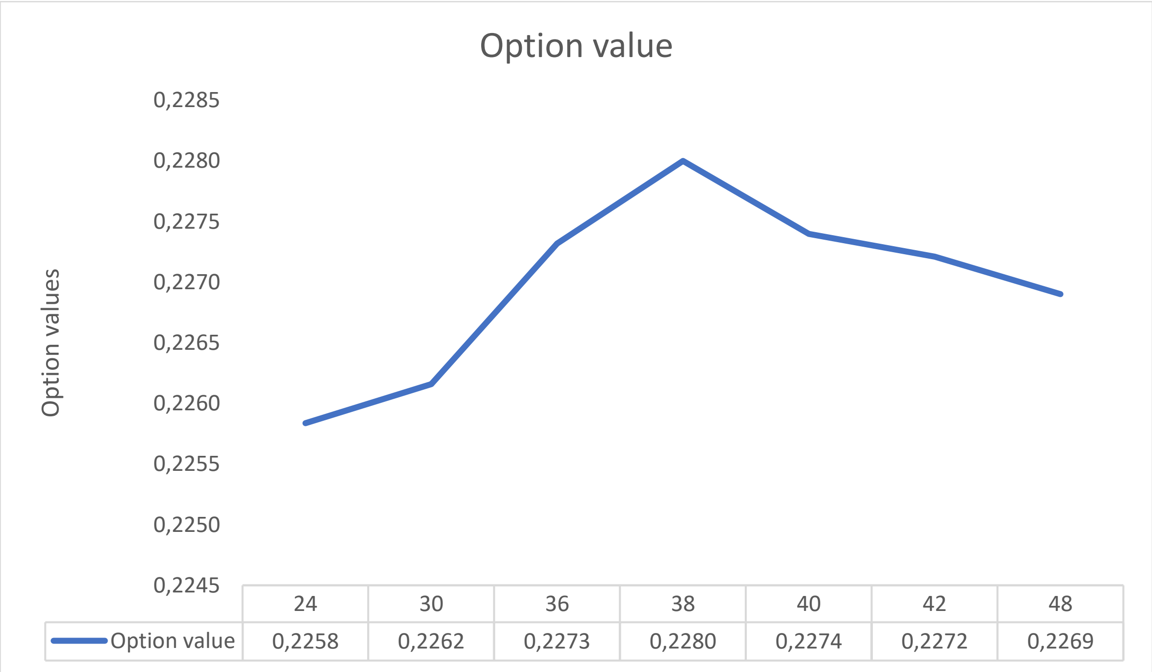
Appendix 3. The value of the project.

Example.

Calculated Factor	Result
Up	1,01
Down	0,99
P	0,56
1-P	0,44
Analysis input	**Amount**
Investments, $ mln	250
Monthly fixed cost, $	1,1
Variable costs per barrel, $	25
Barrels per month (avrg cap), mln	0,12
Monthly risk-free rate	1%

Source: Author's calculations in Excel application of the paper.

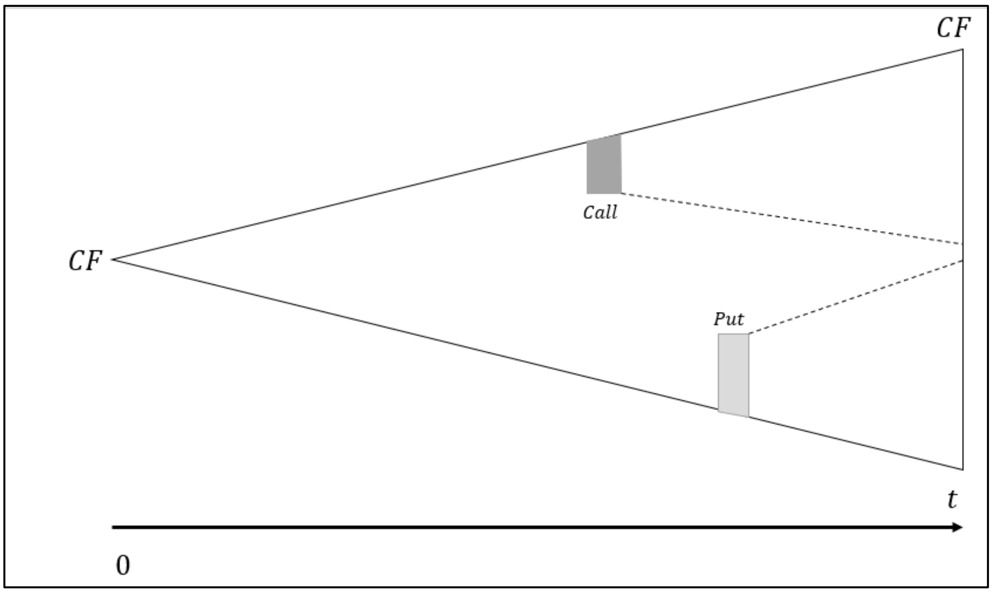
Appendix 4. Finding optimal exercise time for option to Abandon.



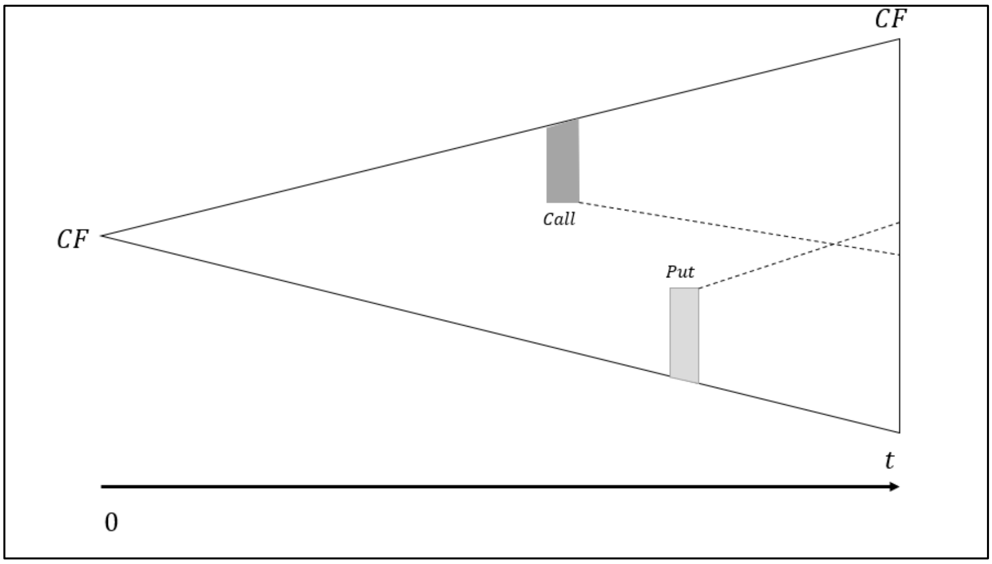
Source: author's calculations in Excel application of the paper.

Appendix 5. Example of not interacting and interacting options

1) Not interacting options



2) Interacting options.

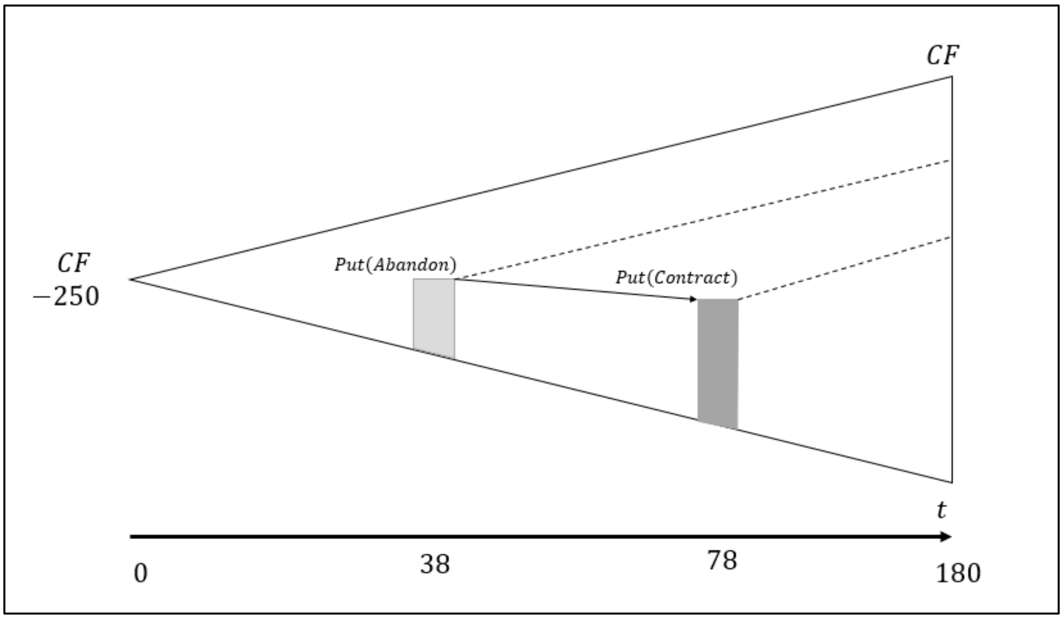


Source: author's analysis.

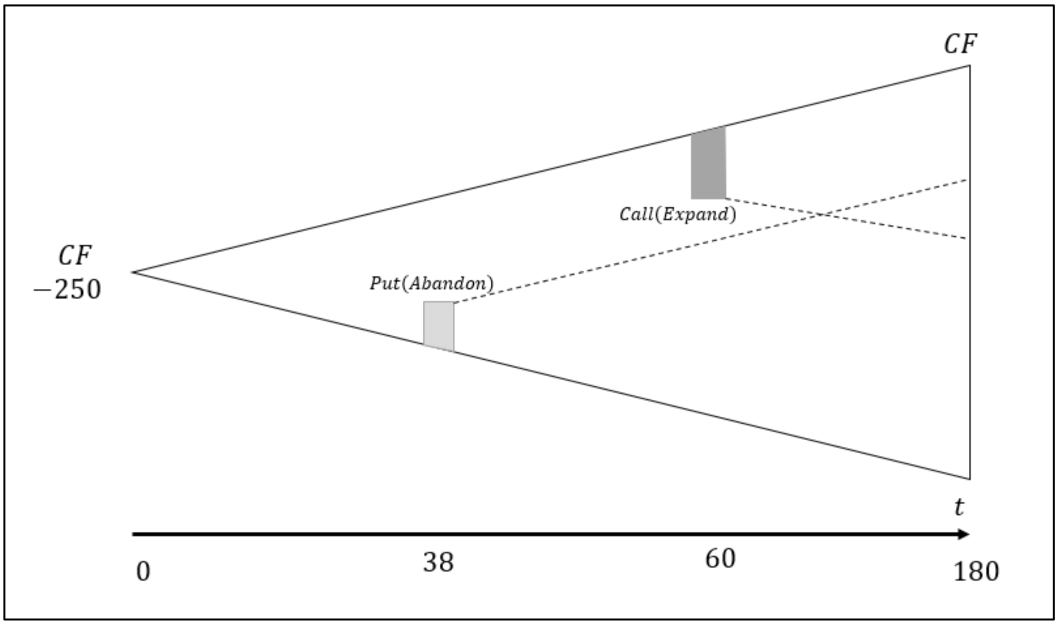
Appendix 6. Simplified presentation of options interaction analysis for the project.

(More details in Excel file)

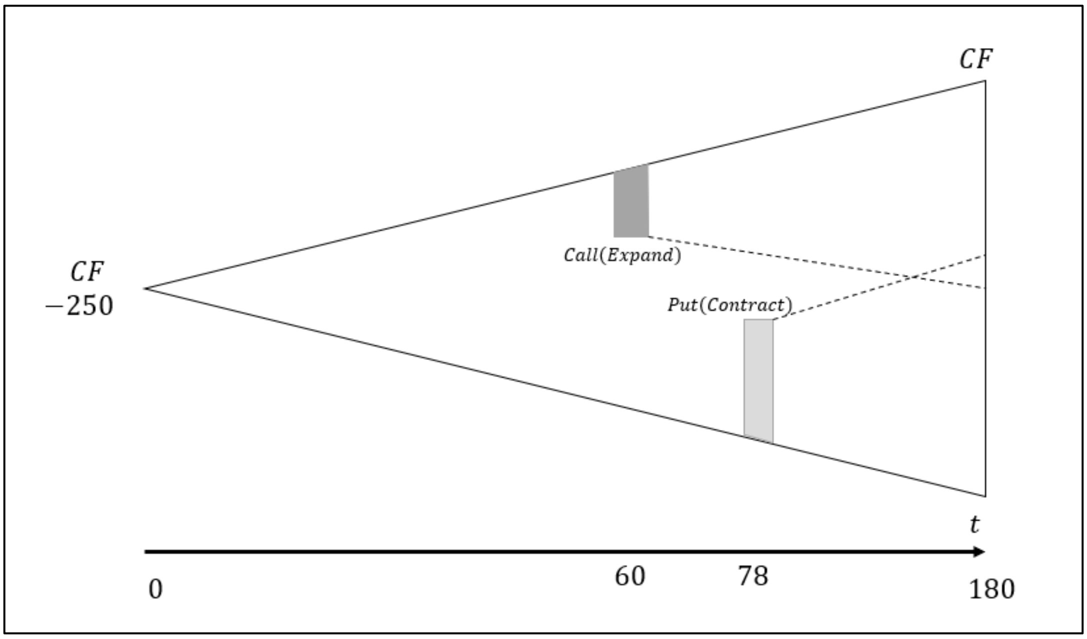
1) Abandon & Contract



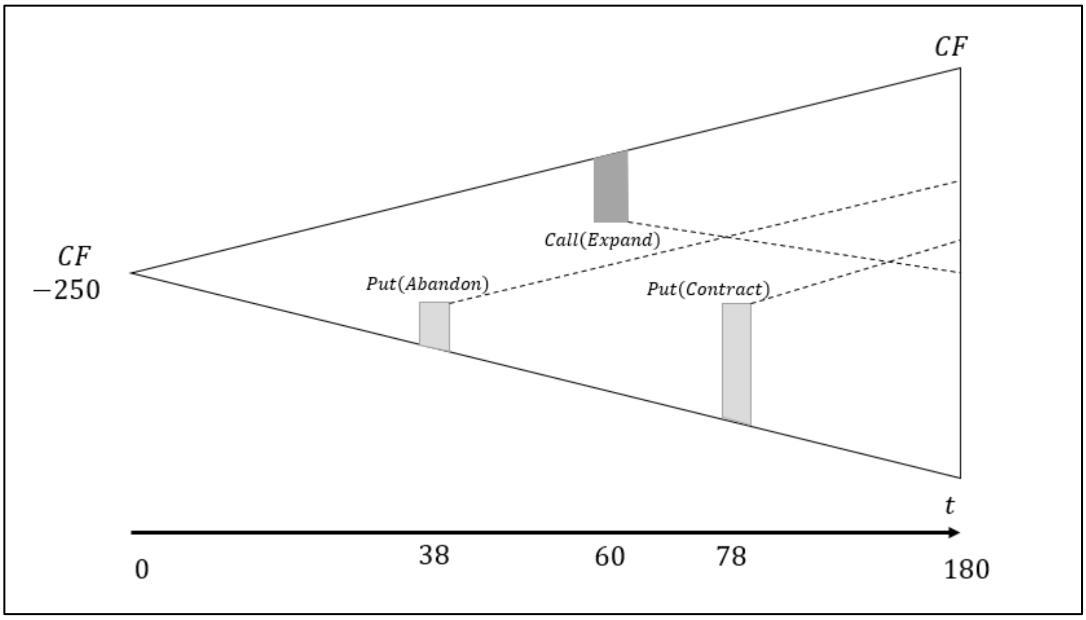
2) Abandon & Expand



3) Expand & Contract



4) Abandon & Expand & Contract



Source: author's analysis.